Computer algebra independent integration tests

`1_Algebraic_functions/1.2_Trinomial_products/1.2.2Quartic/1.2.2.4(fx)^m(d+ex^2)^q(a-`

Nasser M. Abbasi

December 15, 2018 Compiled on December 15, 2018 at 2:41am

Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

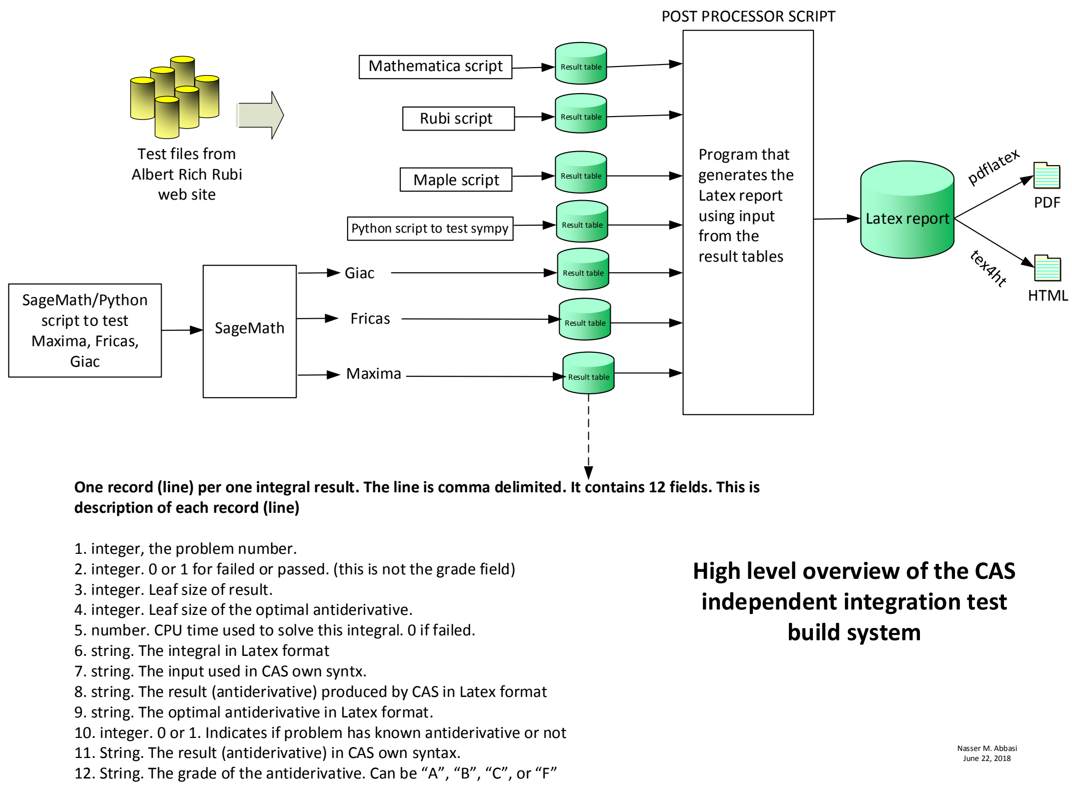
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (405)	% 0. (0)
Rubi in Sympy	% 75.8 (307)	% 24.2 (98)
Mathematica	% 90.62 (367)	% 9.38 (38)
Maple	% 90.86 (368)	% 9.14 (37)
Maxima	% 27.16 (110)	% 72.84 (295)
Fricas	% 66.17 (268)	% 33.83 (137)
Sympy	% 33.33 (135)	% 66.67 (270)
Giac	% 46.91 (190)	% 53.09 (215)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

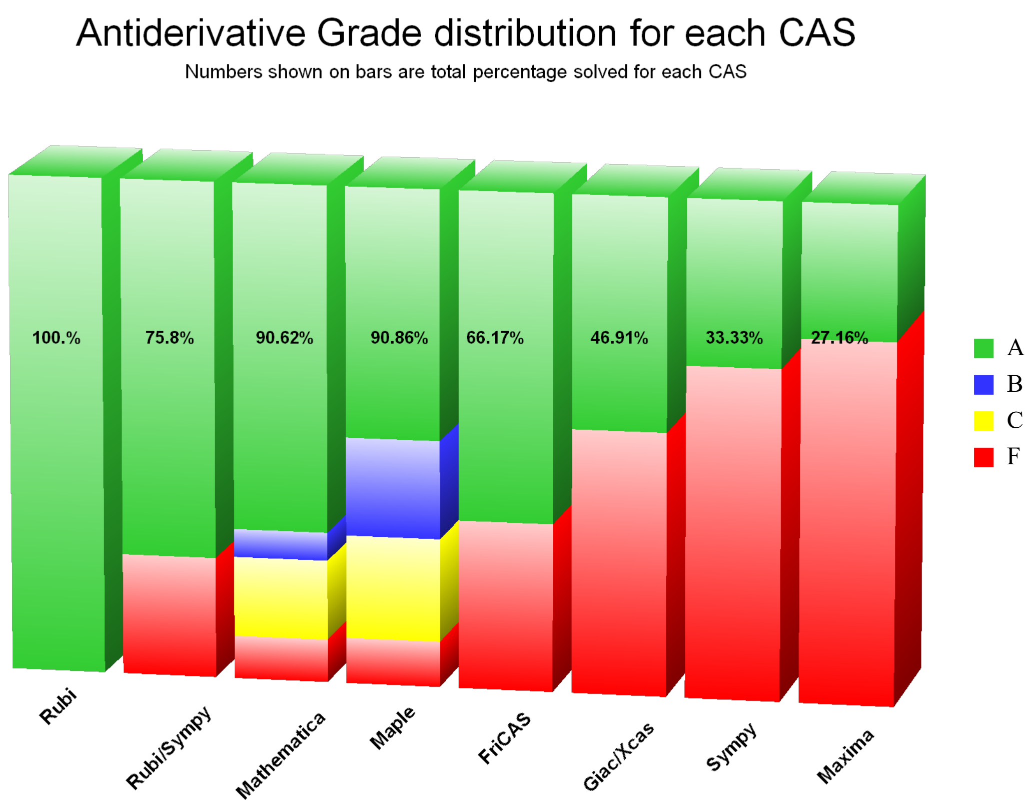
Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented.

For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

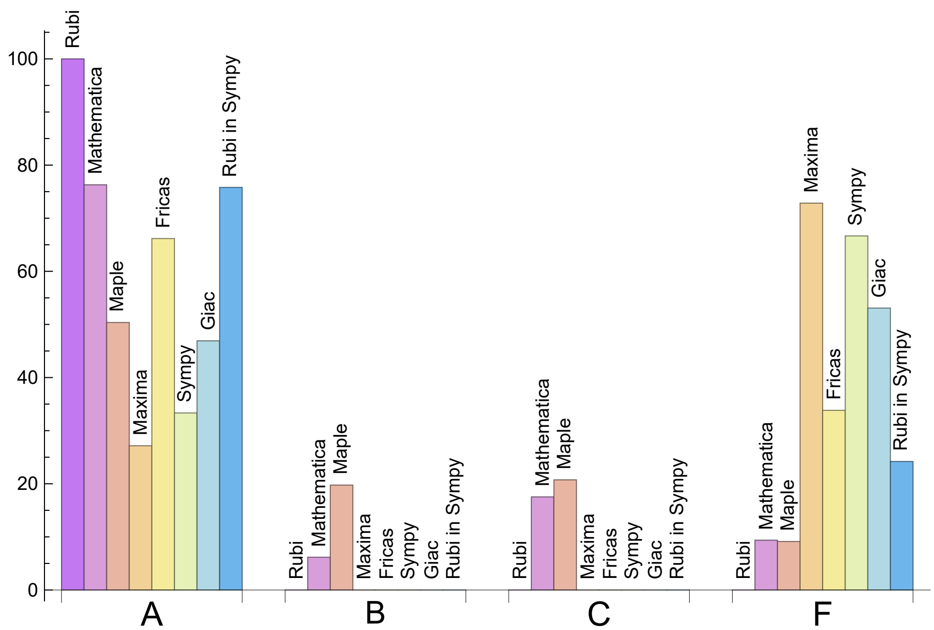
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Rubi in Sympy	75.8	0.	0.	24.2
Mathematica	76.3	6.17	17.53	9.38
Maple	50.37	19.75	20.74	9.14
Maxima	27.16	0.	0.	72.84
Fricas	66.17	0.	0.	33.83
Sympy	33.33	0.	0.	66.67
Giac	46.91	0.	0.	53.09

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	1.2	235.61	1.02	192.	1.
Rubi in Sympy	54.87	199.73	0.94	175.	0.91
Mathematica	0.65	276.47	1.23	151.	0.92
Maple	0.03	518.58	1.91	207.	1.05
Maxima	0.74	131.16	1.65	119.5	1.37
Fricas	7.24	1150.98	4.14	40.	0.56
Sympy	16.1	316.97	2.27	131.	1.22
Giac	1.61	194.73	1.37	131.5	1.22

1.8 list of integrals that has no closed form antiderivative

{}

1.9 list of integrals not solved by each system

Not solved by Rubi {}

Not solved by Rubi in Sympy {1, 3, 4, 5, 6, 7, 61, 62, 63, 64, 72, 74, 75, 77, 78, 79, 80, 81, 83, 85, 95, 97, 98, 99, 100, 101, 102, 103, 112, 118, 121, 122, 123, 124, 131, 132, 133, 135, 136, 224, 229, 230, 237, 238, 252, 257, 258, 261, 265, 267, 268, 269, 270, 271, 286, 287, 288, 294, 295, 296, 300, 301, 302, 318, 346, 347, 348, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 378, 384, 385, 386, 387, 388, 389, 390, 391, 404}

Not solved by Mathematica {353, 354, 355, 356, 357, 358, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 397, 399, 400, 401, 402, 403, 404}

Not solved by Maple {90, 91, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404}

Not solved by Maxima {15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 55, 65, 75, 77, 78, 79, 80, 81, 83, 84, 85, 86, 90, 91, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404}

Not solved by Fricas {15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 90, 91, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 176, 177, 178, 179, 180, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 250, 251, 287, 288, 292, 293, 294, 295, 296, 299, 300, 301, 302, 303, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 329, 330, 331, 332, 333, 340, 341, 342, 343, 344, 345, 346, 347, 351, 352, 359, 361, 362, 363, 378, 386, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404}

Not solved by Sympy {55, 65, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 106, 116, 117, 118, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405}

Not solved by Giac {11, 12, 13, 15, 16, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 39, 40, 41, 42, 43, 49, 50, 51, 52, 53, 54, 90, 91, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 133, 135, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 172, 173, 174, 175, 176, 177, 178, 179, 180, 185, 186, 187, 188, 189, 190, 191, 192, 193, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 327, 328, 329, 330, 331, 332, 333, 334, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {176, 177, 308, 309, 310, 311, 313, 319, 320, 321, 322, 323, 329, 332, 333, 340, 341, 342, 343, 344, 345}

Mathematica {151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 393}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	149	126	169	1	151	177	0
normalized size	1	1.	1.	0.85	1.13	0.01	1.01	1.19	0.
time (sec)	N/A	0.462	0.008	0.004	0.702	0.229	0.162	0.26	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	149	126	169	1	155	177	155
normalized size	1	1.	1.	0.85	1.13	0.01	1.04	1.19	1.04
time (sec)	N/A	0.237	0.006	0.002	0.706	0.24	0.162	0.26	31.156

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	146	125	167	1	150	176	0
normalized size	1	1.	1.64	1.4	1.88	0.01	1.69	1.98	0.
time (sec)	N/A	0.191	0.006	0.002	0.695	0.231	0.158	0.261	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	141	122	163	1	148	171	0
normalized size	1	1.	1.	0.87	1.16	0.01	1.05	1.21	0.
time (sec)	N/A	0.169	0.006	0.002	0.702	0.259	0.159	0.263	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	142	123	169	165	150	177	0
normalized size	1	1.	1.	0.87	1.19	1.16	1.06	1.25	0.
time (sec)	N/A	0.217	0.014	0.005	0.71	0.27	1.514	0.264	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	139	122	163	171	143	171	0
normalized size	1	1.	1.	0.88	1.17	1.23	1.03	1.23	0.
time (sec)	N/A	0.182	0.013	0.006	0.705	0.25	1.514	0.26	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	142	123	169	174	150	192	0
normalized size	1	1.	1.	0.87	1.19	1.23	1.06	1.35	0.
time (sec)	N/A	0.28	0.013	0.009	0.707	0.25	1.648	0.264	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	53	138	271	97	70	61
normalized size	1	1.	0.81	0.79	2.06	4.04	1.45	1.04	0.91
time (sec)	N/A	0.145	0.035	0.028	0.78	0.262	13.433	0.265	11.376

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	44	46	126	231	70	62	46
normalized size	1	1.	0.86	0.9	2.47	4.53	1.37	1.22	0.9
time (sec)	N/A	0.093	0.034	0.009	0.784	0.271	10.598	0.263	8.58

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	36	34	90	190	53	50	37
normalized size	1	1.	0.82	0.77	2.05	4.32	1.2	1.14	0.84
time (sec)	N/A	0.059	0.023	0.013	0.786	0.278	6.571	0.265	7.072

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	49	134	247	83	0	53
normalized size	1	1.	0.98	0.84	2.31	4.26	1.43	0.	0.91
time (sec)	N/A	0.144	0.094	0.017	0.783	0.272	12.298	0.	13.827

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	61	119	250	83	0	56
normalized size	1	1.	0.93	1.03	2.02	4.24	1.41	0.	0.95
time (sec)	N/A	0.148	0.091	0.019	0.782	0.303	11.939	0.	14.02

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	75	123	279	76	0	58
normalized size	1	1.	0.97	1.19	1.95	4.43	1.21	0.	0.92
time (sec)	N/A	0.149	0.103	0.018	0.784	0.304	12.495	0.	13.405

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	54	52	80	263	63	84	54
normalized size	1	1.	0.93	0.9	1.38	4.53	1.09	1.45	0.93
time (sec)	N/A	0.121	0.055	0.018	0.776	0.266	11.988	0.27	11.474

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	105	192	0	0	78	0	206
normalized size	1	1.	0.5	0.92	0.	0.	0.38	0.	0.99
time (sec)	N/A	0.29	0.212	0.072	0.	0.	4.553	0.	25.671

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	101	180	0	0	78	0	190
normalized size	1	1.	0.53	0.94	0.	0.	0.41	0.	0.99
time (sec)	N/A	0.225	0.19	0.017	0.	0.	3.673	0.	19.934

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	96	168	0	0	76	0	175
normalized size	1	1.	0.55	0.95	0.	0.	0.43	0.	0.99
time (sec)	N/A	0.134	0.199	0.015	0.	0.	3.215	0.	12.532

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	108	167	0	0	78	0	175
normalized size	1	1.	0.63	0.98	0.	0.	0.46	0.	1.02
time (sec)	N/A	0.155	0.162	0.022	0.	0.	3.846	0.	14.477

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	98	170	0	0	83	0	189
normalized size	1	1.	0.51	0.89	0.	0.	0.43	0.	0.98
time (sec)	N/A	0.203	0.224	0.023	0.	0.	4.209	0.	19.38

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	59	73	171	352	131	88	76
normalized size	1	1.	0.71	0.88	2.06	4.24	1.58	1.06	0.92
time (sec)	N/A	0.158	0.055	0.032	0.777	0.265	34.457	0.264	11.704

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	58	159	312	124	80	61
normalized size	1	1.	0.81	0.87	2.37	4.66	1.85	1.19	0.91
time (sec)	N/A	0.105	0.048	0.011	0.781	0.267	27.135	0.266	9.052

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	56	46	128	271	109	70	54
normalized size	1	1.	0.93	0.77	2.13	4.52	1.82	1.17	0.9
time (sec)	N/A	0.072	0.031	0.019	0.777	0.286	17.191	0.267	7.308

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	68	75	186	354	114	0	71
normalized size	1	1.	0.87	0.96	2.38	4.54	1.46	0.	0.91
time (sec)	N/A	0.197	0.1	0.021	0.785	0.289	22.755	0.	16.106

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	66	75	165	370	114	0	76
normalized size	1	1.	0.81	0.93	2.04	4.57	1.41	0.	0.94
time (sec)	N/A	0.19	0.123	0.023	0.789	0.299	18.937	0.	16.75

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	70	73	166	370	133	0	82
normalized size	1	1.	0.81	0.85	1.93	4.3	1.55	0.	0.95
time (sec)	N/A	0.198	0.112	0.023	0.786	0.299	23.105	0.	16.68

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	69	73	151	356	148	0	76
normalized size	1	1.	0.84	0.89	1.84	4.34	1.8	0.	0.93
time (sec)	N/A	0.196	0.118	0.024	0.78	0.294	22.712	0.	16.759

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	115	216	0	0	160	0	235
normalized size	1	1.	0.49	0.92	0.	0.	0.68	0.	1.
time (sec)	N/A	0.338	0.23	0.025	0.	0.	10.959	0.	31.03

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	110	204	0	0	160	0	219
normalized size	1	1.	0.5	0.93	0.	0.	0.73	0.	1.
time (sec)	N/A	0.297	0.231	0.017	0.	0.	8.016	0.	25.866

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	106	192	0	0	158	0	197
normalized size	1	1.	0.54	0.97	0.	0.	0.8	0.	1.
time (sec)	N/A	0.168	0.222	0.015	0.	0.	5.963	0.	16.012

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	125	192	0	0	160	0	197
normalized size	1	1.	0.63	0.96	0.	0.	0.8	0.	0.99
time (sec)	N/A	0.188	0.173	0.022	0.	0.	6.98	0.	17.597

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	124	192	0	0	163	0	199
normalized size	1	1.	0.62	0.96	0.	0.	0.81	0.	0.99
time (sec)	N/A	0.208	0.17	0.025	0.	0.	7.085	0.	18.466

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	44	51	140	224	85	62	60
normalized size	1	1.	0.66	0.76	2.09	3.34	1.27	0.93	0.9
time (sec)	N/A	0.185	0.042	0.022	0.781	0.261	15.607	0.262	13.113

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	36	39	103	190	66	50	44
normalized size	1	1.	0.71	0.76	2.02	3.73	1.29	0.98	0.86
time (sec)	N/A	0.134	0.029	0.016	0.778	0.296	10.518	0.264	10.665

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	32	88	142	53	45	31
normalized size	1	1.	0.97	0.91	2.51	4.06	1.51	1.29	0.89
time (sec)	N/A	0.087	0.029	0.011	0.782	0.267	8.272	0.265	7.997

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	20	57	88	22	35	22
normalized size	1	1.	1.	0.83	2.38	3.67	0.92	1.46	0.92
time (sec)	N/A	0.053	0.015	0.014	0.78	0.28	2.082	0.265	6.58

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	30	90	108	31	0	37
normalized size	1	1.	1.	0.79	2.37	2.84	0.82	0.	0.97
time (sec)	N/A	0.106	0.079	0.013	0.782	0.271	6.223	0.	10.276

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	31	63	140	31	65	39
normalized size	1	1.	1.	0.74	1.5	3.33	0.74	1.55	0.93
time (sec)	N/A	0.107	0.045	0.017	0.784	0.259	6.668	0.275	10.009

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	51	43	80	216	88	72	51
normalized size	1	1.	0.88	0.74	1.38	3.72	1.52	1.24	0.88
time (sec)	N/A	0.148	0.05	0.018	0.782	0.314	7.181	0.277	12.361

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	96	168	0	0	75	0	185
normalized size	1	1.	0.52	0.91	0.	0.	0.41	0.	1.
time (sec)	N/A	0.214	0.254	0.025	0.	0.	4.379	0.	18.908

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	71	155	0	0	75	0	167
normalized size	1	1.	0.43	0.93	0.	0.	0.45	0.	1.01
time (sec)	N/A	0.16	0.164	0.018	0.	0.	3.903	0.	13.606

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	62	146	0	0	73	0	158
normalized size	1	1.	0.4	0.94	0.	0.	0.47	0.	1.02
time (sec)	N/A	0.102	0.085	0.014	0.	0.	3.038	0.	8.428

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	81	158	0	0	75	0	175
normalized size	1	1.	0.47	0.91	0.	0.	0.43	0.	1.01
time (sec)	N/A	0.152	0.248	0.021	0.	0.	3.444	0.	13.26

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	97	170	0	0	80	0	190
normalized size	1	1.	0.51	0.9	0.	0.	0.42	0.	1.01
time (sec)	N/A	0.215	0.205	0.024	0.	0.	4.177	0.	17.989

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	44	50	120	205	66	61	51
normalized size	1	1.	0.76	0.86	2.07	3.53	1.14	1.05	0.88
time (sec)	N/A	0.146	0.059	0.026	0.774	0.297	30.356	0.269	11.231

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	36	37	85	151	48	53	41
normalized size	1	1.	0.8	0.82	1.89	3.36	1.07	1.18	0.91
time (sec)	N/A	0.12	0.048	0.018	0.783	0.327	20.569	0.27	10.518

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	34	73	95	39	45	31
normalized size	1	1.	1.	0.97	2.09	2.71	1.11	1.29	0.89
time (sec)	N/A	0.083	0.029	0.01	0.788	0.306	16.066	0.275	8.205

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	30	50	24	22	17
normalized size	1	1.	1.	0.85	1.5	2.5	1.2	1.1	0.85
time (sec)	N/A	0.051	0.016	0.006	0.777	0.308	13.152	0.271	6.244

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	40	76	170	212	72	39
normalized size	1	1.	1.	0.87	1.65	3.7	4.61	1.57	0.85
time (sec)	N/A	0.115	0.054	0.023	0.787	0.336	18.742	0.274	10.592

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	47	92	216	228	0	60
normalized size	1	1.	0.82	0.72	1.42	3.32	3.51	0.	0.92
time (sec)	N/A	0.157	0.092	0.019	0.78	0.299	27.01	0.	12.817

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	85	168	0	0	75	0	194
normalized size	1	1.	0.43	0.86	0.	0.	0.38	0.	0.99
time (sec)	N/A	0.229	0.198	0.028	0.	0.	13.094	0.	19.169

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	85	168	0	0	75	0	177
normalized size	1	1.	0.48	0.95	0.	0.	0.42	0.	1.
time (sec)	N/A	0.166	0.183	0.019	0.	0.	11.176	0.	14.171

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	86	168	0	0	73	0	182
normalized size	1	1.	0.48	0.93	0.	0.	0.41	0.	1.01
time (sec)	N/A	0.134	0.173	0.018	0.	0.	10.609	0.	11.72

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	108	180	0	0	75	0	194
normalized size	1	1.	0.55	0.92	0.	0.	0.38	0.	0.99
time (sec)	N/A	0.212	0.204	0.027	0.	0.	17.792	0.	18.692

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	119	192	0	0	80	0	209
normalized size	1	1.	0.56	0.9	0.	0.	0.37	0.	0.98
time (sec)	N/A	0.27	0.236	0.028	0.	0.	26.539	0.	23.426

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	189	2295	0	2121	0	1	241
normalized size	1	1.	0.7	8.53	0.	7.88	0.	0.	0.9
time (sec)	N/A	0.368	0.191	0.018	0.	0.309	0.	0.314	48.725

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	153	130	174	1	134	193	51
normalized size	1	1.	2.43	2.06	2.76	0.02	2.13	3.06	0.81
time (sec)	N/A	0.422	0.042	0.001	0.695	0.244	0.179	0.261	22.377

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	153	130	174	1	141	194	141
normalized size	1	1.	1.	0.85	1.14	0.01	0.92	1.27	0.92
time (sec)	N/A	0.277	0.04	0.002	0.699	0.23	0.178	0.262	25.836

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	151	130	174	1	136	194	36
normalized size	1	1.	3.36	2.89	3.87	0.02	3.02	4.31	0.8
time (sec)	N/A	0.334	0.038	0.001	0.702	0.239	0.174	0.263	19.205

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	153	130	174	1	139	194	139
normalized size	1	1.	1.	0.85	1.14	0.01	0.91	1.27	0.91
time (sec)	N/A	0.274	0.039	0.002	0.699	0.251	0.174	0.261	27.911

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	149	130	174	1	133	194	22
normalized size	1	1.	5.14	4.48	6.	0.03	4.59	6.69	0.76
time (sec)	N/A	0.13	0.026	0.002	0.694	0.239	0.176	0.261	16.177

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	143	127	169	1	134	190	0
normalized size	1	1.	1.	0.89	1.18	0.01	0.94	1.33	0.
time (sec)	N/A	0.205	0.038	0.001	0.704	0.235	0.183	0.272	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	149	132	176	171	131	196	0
normalized size	1	1.	1.6	1.42	1.89	1.84	1.41	2.11	0.
time (sec)	N/A	0.109	0.051	0.005	0.688	0.302	0.927	0.272	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	141	129	169	177	124	188	0
normalized size	1	1.	1.	0.91	1.2	1.26	0.88	1.33	0.
time (sec)	N/A	0.225	0.057	0.007	0.688	0.25	1.688	0.269	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	147	131	176	180	131	211	0
normalized size	1	1.	1.	0.89	1.2	1.22	0.89	1.44	0.
time (sec)	N/A	0.346	0.069	0.01	0.709	0.254	1.902	0.274	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	123	1121	0	1025	0	1	177
normalized size	1	1.	0.61	5.52	0.	5.05	0.	0.	0.87
time (sec)	N/A	0.17	0.057	0.015	0.	0.293	0.	0.296	35.597

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	85	62	82	1	76	82	24
normalized size	1	1.	2.5	1.82	2.41	0.03	2.24	2.41	0.71
time (sec)	N/A	0.094	0.004	0.002	0.699	0.258	0.108	0.269	10.466

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	62	82	1	75	82	75
normalized size	1	1.	1.	0.75	0.99	0.01	0.9	0.99	0.9
time (sec)	N/A	0.065	0.003	0.002	0.702	0.25	0.1	0.272	11.044

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	83	62	82	1	75	82	15
normalized size	1	1.	3.61	2.7	3.57	0.04	3.26	3.57	0.65
time (sec)	N/A	0.053	0.003	0.002	0.701	0.227	0.103	0.268	9.584

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	62	82	1	75	82	75
normalized size	1	1.	1.	0.75	0.99	0.01	0.9	0.99	0.9
time (sec)	N/A	0.059	0.002	0.002	0.696	0.226	0.101	0.268	11.216

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	62	82	1	71	82	7
normalized size	1	1.	1.	5.64	7.45	0.09	6.45	7.45	0.64
time (sec)	N/A	0.009	0.002	0.002	0.699	0.229	0.107	0.261	3.545

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	58	77	1	68	77	68
normalized size	1	1.	1.	0.79	1.05	0.01	0.93	1.05	0.93
time (sec)	N/A	0.045	0.001	0.002	0.699	0.256	0.11	0.261	9.602

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	59	84	78	75	84	0
normalized size	1	1.	1.	0.74	1.05	0.98	0.94	1.05	0.
time (sec)	N/A	0.061	0.005	0.004	0.699	0.25	0.205	0.262	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	60	80	84	66	80	66
normalized size	1	1.	1.	0.82	1.1	1.15	0.9	1.1	0.9
time (sec)	N/A	0.052	0.005	0.005	0.697	0.242	0.189	0.264	11.209

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	61	84	86	75	93	0
normalized size	1	1.	1.	0.76	1.05	1.08	0.94	1.16	0.
time (sec)	N/A	0.08	0.004	0.008	0.697	0.25	0.228	0.263	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	80	90	0	1	90	136	0
normalized size	1	1.	0.55	0.62	0.	0.01	0.62	0.94	0.
time (sec)	N/A	0.266	0.086	0.013	0.	0.26	1.76	0.265	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	51	55	92	39	27	57	76
normalized size	1	1.	0.61	0.66	1.11	0.47	0.33	0.69	0.92
time (sec)	N/A	0.192	0.039	0.01	0.711	0.29	1.515	0.264	24.596

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	69	62	0	1	82	80	0
normalized size	1	1.	0.71	0.64	0.	0.01	0.85	0.82	0.
time (sec)	N/A	0.122	0.051	0.009	0.	0.27	1.63	0.263	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	54	57	0	45	26	82	0
normalized size	1	1.	0.59	0.62	0.	0.49	0.28	0.89	0.
time (sec)	N/A	0.211	0.037	0.014	0.	0.256	2.191	0.264	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	72	67	0	1	82	84	0
normalized size	1	1.	0.71	0.66	0.	0.01	0.81	0.83	0.
time (sec)	N/A	0.194	0.058	0.013	0.	0.285	1.811	0.266	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	70	78	0	65	41	177	0
normalized size	1	1.	0.51	0.57	0.	0.47	0.3	1.29	0.
time (sec)	N/A	0.263	0.056	0.017	0.	0.267	2.969	0.265	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	108	188	0	1	0	4	0
normalized size	1	1.	0.71	1.23	0.	0.01	0.	0.03	0.
time (sec)	N/A	0.326	0.109	0.024	0.	0.267	0.	0.617	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	45	38	96	57	0	4	48
normalized size	1	1.	0.58	0.49	1.25	0.74	0.	0.05	0.62
time (sec)	N/A	0.18	0.042	0.008	0.709	0.259	0.	0.63	18.014

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	108	186	0	1	0	4	0
normalized size	1	1.	0.69	1.19	0.	0.01	0.	0.03	0.
time (sec)	N/A	0.202	0.097	0.02	0.	0.273	0.	0.628	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	92	132	0	161	0	4	163
normalized size	1	1.	0.57	0.82	0.	1.	0.	0.02	1.01
time (sec)	N/A	0.313	0.076	0.029	0.	0.266	0.	0.626	43.484

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	124	206	0	1	0	4	0
normalized size	1	1.	0.65	1.08	0.	0.01	0.	0.02	0.
time (sec)	N/A	0.472	0.139	0.027	0.	0.282	0.	0.637	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	130	249	0	277	0	4	223
normalized size	1	1.	0.58	1.12	0.	1.24	0.	0.02	1.
time (sec)	N/A	0.429	0.13	0.033	0.	0.274	0.	0.604	59.012

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	160	1099	663	1152	0	1	374
normalized size	1	1.	0.4	2.75	1.66	2.88	0.	0.	0.94
time (sec)	N/A	0.587	0.303	0.014	0.736	0.287	0.	0.308	85.596

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	112	495	328	514	0	1	255
normalized size	1	1.	0.41	1.79	1.19	1.86	0.	0.	0.92
time (sec)	N/A	0.399	0.18	0.012	0.713	0.284	0.	0.285	61.77

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	65	131	101	127	0	396	138
normalized size	1	1.	0.42	0.86	0.66	0.83	0.	2.59	0.9
time (sec)	N/A	0.221	0.103	0.007	0.719	0.286	0.	0.27	34.64

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	78	0	0	0	0	0	114
normalized size	1	1.	0.58	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.247	0.103	0.038	0.	0.	0.	0.	34.601

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	101	0	0	0	0	0	131
normalized size	1	1.	0.66	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.313	0.128	0.04	0.	0.	0.	0.	36.584

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	25	40	116	63	165	127	27
normalized size	1	1.	0.74	1.18	3.41	1.85	4.85	3.74	0.79
time (sec)	N/A	0.022	0.015	0.003	0.718	0.286	37.524	0.271	7.221

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	45	62	182	124	0	281	70
normalized size	1	1.	0.52	0.72	2.12	1.44	0.	3.27	0.81
time (sec)	N/A	0.211	0.035	0.007	0.721	0.284	0.	0.271	27.941

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	68	99	265	189	0	474	114
normalized size	1	1.	0.53	0.77	2.07	1.48	0.	3.7	0.89
time (sec)	N/A	0.288	0.052	0.009	0.718	0.278	0.	0.27	43.169

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	166	226	224	1	202	261	0
normalized size	1	1.	1.	1.36	1.35	0.01	1.22	1.57	0.
time (sec)	N/A	0.775	0.084	0.001	0.703	0.245	0.194	0.263	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	166	226	224	1	204	261	178
normalized size	1	1.	1.	1.36	1.35	0.01	1.23	1.57	1.07
time (sec)	N/A	0.367	0.094	0.002	0.696	0.233	0.201	0.263	51.437

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	154	226	224	1	199	261	0
normalized size	1	1.	0.93	1.36	1.35	0.01	1.2	1.57	0.
time (sec)	N/A	0.65	0.136	0.001	0.696	0.228	0.187	0.263	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	161	223	220	1	199	255	0
normalized size	1	1.	1.	1.39	1.37	0.01	1.24	1.58	0.
time (sec)	N/A	0.303	0.09	0.001	0.713	0.24	0.187	0.262	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	162	191	225	221	199	261	0
normalized size	1	1.	1.	1.18	1.39	1.36	1.23	1.61	0.
time (sec)	N/A	0.44	0.123	0.004	0.703	0.255	1.732	0.265	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	156	186	219	227	185	250	0
normalized size	1	1.	1.	1.19	1.4	1.46	1.19	1.6	0.
time (sec)	N/A	0.278	0.161	0.006	0.7	0.241	1.738	0.262	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	162	190	225	230	197	286	0
normalized size	1	1.	1.	1.17	1.39	1.42	1.22	1.77	0.
time (sec)	N/A	0.508	0.178	0.01	0.705	0.246	2.099	0.268	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	126	261	0	1	619	170	0
normalized size	1	1.	0.95	1.96	0.	0.01	4.65	1.28	0.
time (sec)	N/A	0.432	0.102	0.008	0.	0.28	24.859	0.295	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	93	175	0	1	434	123	0
normalized size	1	1.	0.96	1.8	0.	0.01	4.47	1.27	0.
time (sec)	N/A	0.259	0.123	0.005	0.	0.273	15.728	0.292	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	98	0	1	287	90	63
normalized size	1	1.	1.	1.38	0.	0.01	4.04	1.27	0.89
time (sec)	N/A	0.16	0.08	0.004	0.	0.272	7.682	0.292	23.239

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	128	105	0	1	330	105	73
normalized size	1	1.	1.64	1.35	0.	0.01	4.23	1.35	0.94
time (sec)	N/A	0.277	0.18	0.009	0.	0.318	133.358	0.289	35.578

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	186	191	0	1	0	167	105
normalized size	1	1.	1.66	1.71	0.	0.01	0.	1.49	0.94
time (sec)	N/A	0.457	0.424	0.013	0.	0.472	0.	0.29	51.711

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	327	825	0	6939	709	1	267
normalized size	1	1.	1.25	3.16	0.	26.59	2.72	0.	1.02
time (sec)	N/A	2.705	0.797	0.038	0.	1.37	81.888	1.302	119.699

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	251	560	0	3553	428	1	214
normalized size	1	1.	1.21	2.69	0.	17.08	2.06	0.	1.03
time (sec)	N/A	1.095	0.31	0.03	0.	0.454	32.171	1.21	67.239

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	173	328	0	2118	314	1	185
normalized size	1	1.	1.01	1.91	0.	12.31	1.83	0.01	1.08
time (sec)	N/A	0.416	0.168	0.024	0.	0.374	15.378	0.815	36.183

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	206	353	0	3934	490	1	194
normalized size	1	1.	1.09	1.87	0.	20.81	2.59	0.01	1.03
time (sec)	N/A	0.761	0.521	0.028	0.	0.507	36.276	0.884	64.017

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	267	611	0	7347	774	1	236
normalized size	1	1.	0.99	2.25	0.	27.11	2.86	0.	0.87
time (sec)	N/A	1.323	0.674	0.034	0.	1.404	90.055	1.259	111.36

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	208	1058	0	1	1266	0	0
normalized size	1	1.	0.98	4.99	0.	0.	5.97	0.	0.
time (sec)	N/A	0.762	0.526	0.026	0.	0.351	103.608	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	160	542	0	1	916	0	138
normalized size	1	1.	1.09	3.69	0.	0.01	6.23	0.	0.94
time (sec)	N/A	0.372	0.337	0.022	0.	0.284	54.662	0.	49.55

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	111	158	0	1	394	0	78
normalized size	1	1.	1.04	1.48	0.	0.01	3.68	0.	0.73
time (sec)	N/A	0.262	0.14	0.014	0.	0.286	20.585	0.	27.153

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	101	127	0	1	374	0	83
normalized size	1	1.	1.07	1.35	0.	0.01	3.98	0.	0.88
time (sec)	N/A	0.184	0.144	0.008	0.	0.291	12.211	0.	22.594

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	243	578	0	1	0	0	146
normalized size	1	1.	1.62	3.85	0.	0.01	0.	0.	0.97
time (sec)	N/A	0.649	0.624	0.024	0.	0.764	0.	0.	71.319

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	379	991	0	1	0	0	221
normalized size	1	1.	1.7	4.44	0.	0.	0.	0.	0.99
time (sec)	N/A	0.857	1.066	0.031	0.	1.665	0.	0.	169.834

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	455	4263	0	9790	0	0	0
normalized size	1	1.	1.07	10.03	0.	23.04	0.	0.	0.
time (sec)	N/A	7.939	2.379	0.087	0.	3.709	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	362	4009	0	6288	1129	0	330
normalized size	1	1.	1.08	11.93	0.	18.71	3.36	0.	0.98
time (sec)	N/A	3.343	1.688	0.082	0.	1.186	151.981	0.	127.181

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	298	2995	0	4680	923	0	264
normalized size	1	1.	1.08	10.85	0.	16.96	3.34	0.	0.96
time (sec)	N/A	1.065	1.202	0.059	0.	0.671	79.67	0.	172.384

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	304	1761	0	6595	1180	0	0
normalized size	1	1.	1.03	5.99	0.	22.43	4.01	0.	0.
time (sec)	N/A	1.729	1.472	0.095	0.	1.599	142.926	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	382	3752	0	10237	0	0	0
normalized size	1	1.	0.98	9.65	0.	26.32	0.	0.	0.
time (sec)	N/A	2.443	2.037	0.073	0.	4.258	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	487	4401	0	13757	0	0	0
normalized size	1	1.	0.93	8.43	0.	26.35	0.	0.	0.
time (sec)	N/A	2.927	2.403	0.083	0.	10.356	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	435	2916	0	1	0	807	0
normalized size	1	1.	1.19	7.99	0.	0.	0.	2.21	0.
time (sec)	N/A	2.714	1.384	0.034	0.	0.605	0.	15.976	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	354	1274	0	1	0	629	252
normalized size	1	1.	1.39	5.02	0.	0.	0.	2.48	0.99
time (sec)	N/A	0.802	0.866	0.034	0.	0.433	0.	15.75	94.604

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	261	398	0	1	0	429	138
normalized size	1	1.	1.79	2.73	0.	0.01	0.	2.94	0.95
time (sec)	N/A	0.309	0.475	0.022	0.	0.266	0.	15.977	38.493

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	233	411	0	1	833	362	153
normalized size	1	1.	1.26	2.22	0.	0.01	4.5	1.96	0.83
time (sec)	N/A	0.543	0.429	0.022	0.	0.306	172.415	15.973	49.873

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	172	379	0	1	789	308	160
normalized size	1	1.	1.01	2.23	0.	0.01	4.64	1.81	0.94
time (sec)	N/A	0.362	0.406	0.021	0.	0.298	85.159	15.728	46.902

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	142	262	0	1	661	281	131
normalized size	1	1.	1.02	1.88	0.	0.01	4.76	2.02	0.94
time (sec)	N/A	0.244	0.235	0.012	0.	0.268	50.162	15.953	30.05

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	396	1645	0	1	0	568	257
normalized size	1	1.	1.57	6.53	0.	0.	0.	2.25	1.02
time (sec)	N/A	1.061	1.161	0.033	0.	2.717	0.	15.965	137.877

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	642	2724	0	1	0	875	0
normalized size	1	1.	1.77	7.5	0.	0.	0.	2.41	0.
time (sec)	N/A	1.519	2.979	0.045	0.	6.151	0.	15.732	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	554	554	644	10352	0	13009	0	1	0
normalized size	1	1.	1.16	18.69	0.	23.48	0.	0.	0.
time (sec)	N/A	21.581	5.23	0.128	0.	9.199	0.	44.913	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	461	461	543	9168	0	9531	0	0	0
normalized size	1	1.	1.18	19.89	0.	20.67	0.	0.	0.
time (sec)	N/A	9.646	4.391	0.104	0.	3.004	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	447	7611	0	7628	0	1	408
normalized size	1	1.	1.18	20.03	0.	20.07	0.	0.	1.07
time (sec)	N/A	2.68	3.375	0.102	0.	2.092	0.	37.053	153.902

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	436	8433	0	9815	0	0	0
normalized size	1	1.	1.	19.25	0.	22.41	0.	0.	0.
time (sec)	N/A	2.127	2.893	0.202	0.	3.937	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	461	461	516	11936	0	13377	0	1	0
normalized size	1	1.	1.12	25.89	0.	29.02	0.	0.	0.
time (sec)	N/A	2.857	4.421	0.286	0.	10.218	0.	39.469	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	23	23	17	26	17
normalized size	1	1.	1.	0.72	0.92	0.92	0.68	1.04	0.68
time (sec)	N/A	0.051	0.01	0.009	0.702	0.253	0.232	0.29	11.855

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	34	23	17	26	17
normalized size	1	1.	1.	0.72	1.36	0.92	0.68	1.04	0.68
time (sec)	N/A	0.052	0.008	0.008	0.697	0.251	0.218	0.272	13.583

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	41	41	37	41	34
normalized size	1	1.	1.	0.84	1.11	1.11	1.	1.11	0.92
time (sec)	N/A	0.073	0.016	0.004	0.785	0.254	0.249	0.273	11.276

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	72	41	37	41	34
normalized size	1	1.	1.	0.84	1.95	1.11	1.	1.11	0.92
time (sec)	N/A	0.07	0.009	0.003	0.796	0.25	0.225	0.272	11.905

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	41	51	70	44	51	41
normalized size	1	1.	1.	0.91	1.13	1.56	0.98	1.13	0.91
time (sec)	N/A	0.091	0.047	0.011	0.825	0.281	0.377	0.271	13.923

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	69	91	140	360	0	90	95
normalized size	1	1.	0.68	0.89	1.37	3.53	0.	0.88	0.93
time (sec)	N/A	0.205	0.053	0.033	0.768	0.26	0.	0.28	19.887

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	64	74	117	306	0	81	73
normalized size	1	1.	0.79	0.91	1.44	3.78	0.	1.	0.9
time (sec)	N/A	0.141	0.038	0.021	0.734	0.276	0.	0.276	14.819

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	61	57	95	252	0	72	65
normalized size	1	1.	0.82	0.77	1.28	3.41	0.	0.97	0.88
time (sec)	N/A	0.102	0.038	0.016	0.731	0.262	0.	0.275	12.07

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	100	85	120	356	0	0	82
normalized size	1	1.	1.06	0.9	1.28	3.79	0.	0.	0.87
time (sec)	N/A	0.2	0.127	0.018	0.859	0.266	0.	0.	21.185

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	96	104	120	419	0	0	88
normalized size	1	1.	0.99	1.07	1.24	4.32	0.	0.	0.91
time (sec)	N/A	0.208	0.131	0.022	0.833	0.28	0.	0.	21.116

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	102	121	143	405	0	0	88
normalized size	1	1.	1.03	1.22	1.44	4.09	0.	0.	0.89
time (sec)	N/A	0.207	0.157	0.022	0.819	0.27	0.	0.	20.953

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	79	118	134	362	0	0	80
normalized size	1	1.	0.88	1.31	1.49	4.02	0.	0.	0.89
time (sec)	N/A	0.163	0.101	0.02	0.807	0.275	0.	0.	18.302

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	87	135	157	413	0	0	102
normalized size	1	1.	0.78	1.22	1.41	3.72	0.	0.	0.92
time (sec)	N/A	0.216	0.086	0.023	0.835	0.274	0.	0.	22.835

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	89	152	180	470	0	0	121
normalized size	1	1.	0.67	1.15	1.36	3.56	0.	0.	0.92
time (sec)	N/A	0.269	0.124	0.025	0.815	0.275	0.	0.	27.37

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	237	260	0	0	0	0	301
normalized size	1	1.	0.74	0.81	0.	0.	0.	0.	0.93
time (sec)	N/A	0.603	0.551	0.3	0.	0.	0.	0.	42.963

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	234	243	0	0	0	0	280
normalized size	1	1.	0.77	0.8	0.	0.	0.	0.	0.92
time (sec)	N/A	0.465	0.523	0.019	0.	0.	0.	0.	37.894

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	229	226	0	0	0	0	258
normalized size	1	1.	0.82	0.81	0.	0.	0.	0.	0.92
time (sec)	N/A	0.285	0.516	0.016	0.	0.	0.	0.	23.464

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	231	225	0	0	0	0	260
normalized size	1	1.	0.81	0.79	0.	0.	0.	0.	0.92
time (sec)	N/A	0.297	0.529	0.025	0.	0.	0.	0.	25.257

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	237	228	0	0	0	0	280
normalized size	1	1.	0.78	0.75	0.	0.	0.	0.	0.92
time (sec)	N/A	0.378	0.55	0.025	0.	0.	0.	0.	32.629

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	79	138	182	468	0	109	119
normalized size	1	1.	0.62	1.09	1.43	3.69	0.	0.86	0.94
time (sec)	N/A	0.23	0.067	0.047	0.74	0.262	0.	0.282	21.194

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	74	121	159	414	0	100	97
normalized size	1	1.	0.7	1.14	1.5	3.91	0.	0.94	0.92
time (sec)	N/A	0.167	0.062	0.023	0.743	0.254	0.	0.276	15.594

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	69	104	136	360	0	90	90
normalized size	1	1.	0.7	1.05	1.37	3.64	0.	0.91	0.91
time (sec)	N/A	0.127	0.046	0.019	0.723	0.283	0.	0.276	12.871

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	107	117	162	491	0	0	110
normalized size	1	1.	0.9	0.98	1.36	4.13	0.	0.	0.92
time (sec)	N/A	0.256	0.139	0.022	0.819	0.266	0.	0.	27.012

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	107	117	162	524	0	0	110
normalized size	1	1.	0.88	0.96	1.33	4.3	0.	0.	0.9
time (sec)	N/A	0.266	0.161	0.025	0.82	0.279	0.	0.	26.5

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	110	117	185	524	0	0	116
normalized size	1	1.	0.87	0.92	1.46	4.13	0.	0.	0.91
time (sec)	N/A	0.267	0.231	0.027	0.819	0.273	0.	0.	26.616

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	112	117	208	554	0	0	114
normalized size	1	1.	0.88	0.92	1.64	4.36	0.	0.	0.9
time (sec)	N/A	0.263	0.193	0.026	0.827	0.273	0.	0.	26.639

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	249	294	0	0	0	0	326
normalized size	1	1.	0.7	0.83	0.	0.	0.	0.	0.92
time (sec)	N/A	0.643	0.655	0.027	0.	0.	0.	0.	50.582

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	244	277	0	0	0	0	306
normalized size	1	1.	0.74	0.84	0.	0.	0.	0.	0.92
time (sec)	N/A	0.535	0.557	0.021	0.	0.	0.	0.	47.355

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	239	260	0	0	0	0	284
normalized size	1	1.	0.78	0.84	0.	0.	0.	0.	0.92
time (sec)	N/A	0.346	0.501	0.017	0.	0.	0.	0.	29.493

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	235	260	0	0	0	0	284
normalized size	1	1.	0.75	0.83	0.	0.	0.	0.	0.91
time (sec)	N/A	0.357	0.559	0.024	0.	0.	0.	0.	31.223

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	247	260	0	0	0	0	286
normalized size	1	1.	0.79	0.83	0.	0.	0.	0.	0.91
time (sec)	N/A	0.4	0.566	0.025	0.	0.	0.	0.	33.158

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	244	259	0	0	0	0	306
normalized size	1	1.	0.74	0.78	0.	0.	0.	0.	0.92
time (sec)	N/A	0.494	0.605	0.028	0.	0.	0.	0.	39.894

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	137	286	0	1	0	203	153
normalized size	1	1.	0.9	1.87	0.	0.01	0.	1.33	1.
time (sec)	N/A	0.507	0.163	0.015	0.	0.359	0.	0.306	35.478

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	99	176	0	1	0	132	97
normalized size	1	1.	0.99	1.76	0.	0.01	0.	1.32	0.97
time (sec)	N/A	0.253	0.106	0.012	0.	0.342	0.	0.296	21.57

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	75	93	0	1	0	93	66
normalized size	1	1.	0.99	1.22	0.	0.01	0.	1.22	0.87
time (sec)	N/A	0.161	0.064	0.009	0.	0.314	0.	0.296	17.407

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	96	76	0	1	0	0	80
normalized size	1	1.	1.07	0.84	0.	0.01	0.	0.	0.89
time (sec)	N/A	0.248	0.3	0.009	0.	0.371	0.	0.	23.153

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	87	104	0	1	0	0	70
normalized size	1	1.	1.09	1.3	0.	0.01	0.	0.	0.88
time (sec)	N/A	0.232	0.222	0.013	0.	0.322	0.	0.	21.861

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	112	194	0	1	0	0	114
normalized size	1	1.	0.9	1.56	0.	0.01	0.	0.	0.92
time (sec)	N/A	0.382	0.196	0.014	0.	0.347	0.	0.	34.623

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	146	311	0	1	0	0	172
normalized size	1	1.	0.82	1.76	0.	0.01	0.	0.	0.97
time (sec)	N/A	0.6	0.266	0.016	0.	0.358	0.	0.	54.457

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	532	815	0	0	0	0	381
normalized size	1	1.	1.32	2.02	0.	0.	0.	0.	0.95
time (sec)	N/A	0.671	3.772	0.012	0.	0.	0.	0.	78.283

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	479	607	0	0	0	0	308
normalized size	1	1.	1.43	1.81	0.	0.	0.	0.	0.92
time (sec)	N/A	0.393	2.405	0.01	0.	0.	0.	0.	51.819

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	302	362	0	0	0	0	257
normalized size	1	1.	1.07	1.28	0.	0.	0.	0.	0.91
time (sec)	N/A	0.225	0.395	0.007	0.	0.	0.	0.	33.415

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	448	386	0	0	0	0	279
normalized size	1	1.	1.44	1.24	0.	0.	0.	0.	0.89
time (sec)	N/A	0.344	1.835	0.012	0.	0.	0.	0.	50.283

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	373	656	0	0	0	0	345
normalized size	1	1.	0.99	1.74	0.	0.	0.	0.	0.92
time (sec)	N/A	0.564	1.164	0.013	0.	0.	0.	0.	74.28

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	64	87	122	306	0	81	92
normalized size	1	1.	0.65	0.89	1.24	3.12	0.	0.83	0.94
time (sec)	N/A	0.248	0.053	0.027	0.707	0.273	0.	0.282	26.061

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	61	70	99	252	0	72	70
normalized size	1	1.	0.79	0.91	1.29	3.27	0.	0.94	0.91
time (sec)	N/A	0.184	0.041	0.017	0.703	0.267	0.	0.283	19.491

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	53	76	198	0	62	49
normalized size	1	1.	0.96	0.95	1.36	3.54	0.	1.11	0.88
time (sec)	N/A	0.124	0.025	0.017	0.703	0.271	0.	0.277	14.491

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	47	36	53	139	0	53	42
normalized size	1	1.	0.96	0.73	1.08	2.84	0.	1.08	0.86
time (sec)	N/A	0.081	0.021	0.015	0.698	0.266	0.	0.277	12.229

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	78	52	78	151	0	0	61
normalized size	1	1.	1.13	0.75	1.13	2.19	0.	0.	0.88
time (sec)	N/A	0.146	0.112	0.016	0.78	0.281	0.	0.	16.805

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	67	49	69	239	0	0	56
normalized size	1	1.	1.08	0.79	1.11	3.85	0.	0.	0.9
time (sec)	N/A	0.135	0.067	0.021	0.771	0.268	0.	0.	16.268

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	66	92	290	0	0	71
normalized size	1	1.	0.87	0.8	1.11	3.49	0.	0.	0.86
time (sec)	N/A	0.187	0.09	0.019	0.78	0.268	0.	0.	20.341

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	83	83	115	359	0	0	94
normalized size	1	1.	0.8	0.8	1.11	3.45	0.	0.	0.9
time (sec)	N/A	0.238	0.094	0.019	0.783	0.264	0.	0.	24.956

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	229	226	0	0	0	0	275
normalized size	1	1.	0.77	0.76	0.	0.	0.	0.	0.92
time (sec)	N/A	0.484	0.528	0.026	0.	0.	0.	0.	34.415

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	222	208	0	0	0	0	248
normalized size	1	1.	0.82	0.77	0.	0.	0.	0.	0.92
time (sec)	N/A	0.333	0.494	0.019	0.	0.	0.	0.	26.412

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	159	194	0	0	0	0	233
normalized size	1	1.	0.62	0.75	0.	0.	0.	0.	0.91
time (sec)	N/A	0.211	0.181	0.016	0.	0.	0.	0.	16.691

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	224	211	0	0	0	0	250
normalized size	1	1.	0.81	0.76	0.	0.	0.	0.	0.9
time (sec)	N/A	0.302	0.535	0.024	0.	0.	0.	0.	24.146

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	237	228	0	0	0	0	274
normalized size	1	1.	0.78	0.75	0.	0.	0.	0.	0.91
time (sec)	N/A	0.404	0.549	0.027	0.	0.	0.	0.	32.129

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	59	91	99	225	0	70	68
normalized size	1	1.	0.77	1.18	1.29	2.92	0.	0.91	0.88
time (sec)	N/A	0.165	0.054	0.029	0.696	0.263	0.	0.288	18.58

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	95	76	155	0	62	48
normalized size	1	1.	0.96	1.7	1.36	2.77	0.	1.11	0.86
time (sec)	N/A	0.119	0.036	0.017	0.707	0.27	0.	0.283	14.386

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	43	77	0	28	20
normalized size	1	1.	1.	0.88	1.72	3.08	0.	1.12	0.8
time (sec)	N/A	0.064	0.022	0.007	0.706	0.282	0.	0.282	10.831

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	71	67	88	248	0	0	58
normalized size	1	1.	1.08	1.02	1.33	3.76	0.	0.	0.88
time (sec)	N/A	0.149	0.128	0.027	0.774	0.269	0.	0.	17.702

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	79	84	111	333	0	0	80
normalized size	1	1.	0.88	0.93	1.23	3.7	0.	0.	0.89
time (sec)	N/A	0.202	0.091	0.023	0.781	0.283	0.	0.	20.911

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	219	240	0	0	0	0	280
normalized size	1	1.	0.71	0.78	0.	0.	0.	0.	0.91
time (sec)	N/A	0.419	0.495	0.033	0.	0.	0.	0.	34.783

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	219	240	0	0	0	0	260
normalized size	1	1.	0.77	0.84	0.	0.	0.	0.	0.91
time (sec)	N/A	0.318	0.495	0.022	0.	0.	0.	0.	27.73

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	219	240	0	0	0	0	260
normalized size	1	1.	0.78	0.85	0.	0.	0.	0.	0.92
time (sec)	N/A	0.266	0.492	0.021	0.	0.	0.	0.	23.357

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	228	257	0	0	0	0	279
normalized size	1	1.	0.74	0.83	0.	0.	0.	0.	0.9
time (sec)	N/A	0.401	0.538	0.029	0.	0.	0.	0.	33.101

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	234	274	0	0	0	0	301
normalized size	1	1.	0.72	0.84	0.	0.	0.	0.	0.92
time (sec)	N/A	0.507	0.552	0.031	0.	0.	0.	0.	40.213

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	2835	0	0	0	0	0	269
normalized size	1	1.	9.55	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	1.123	7.466	0.073	0.	0.	0.	0.	80.841

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	1717	0	0	0	0	0	269
normalized size	1	1.	5.78	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.999	4.835	0.062	0.	0.	0.	0.	79.755

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	1717	0	0	0	0	0	267
normalized size	1	1.	5.82	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.995	4.939	0.059	0.	0.	0.	0.	79.883

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	1383	0	0	0	0	0	269
normalized size	1	1.	4.69	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.999	2.74	0.064	0.	0.	0.	0.	80.816

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	4499	0	0	0	0	1	272
normalized size	1	1.	15.05	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	1.005	6.168	0.058	0.	0.	0.	0.797	91.962

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	3656	0	0	0	0	1	272
normalized size	1	1.	12.23	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	1.009	6.151	0.056	0.	0.	0.	0.755	91.888

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	3656	0	0	0	0	1	270
normalized size	1	1.	12.31	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	1.014	6.143	0.059	0.	0.	0.	0.692	90.851

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	2839	0	0	0	0	0	272
normalized size	1	1.	9.56	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	1.028	6.144	0.067	0.	0.	0.	0.	91.834

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	1037	0	0	0	0	0	265
normalized size	1	1.	3.49	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	1.001	2.29	0.053	0.	0.	0.	0.	81.53

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	642	0	0	0	0	0	265
normalized size	1	1.	2.16	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	1.014	0.797	0.029	0.	0.	0.	0.	82.859

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	642	0	0	0	0	0	264
normalized size	1	1.	2.18	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	1.009	1.002	0.035	0.	0.	0.	0.	83.208

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	1049	0	0	0	0	0	265
normalized size	1	1.	3.56	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	1.017	1.653	0.062	0.	0.	0.	0.	82.709

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	1404	0	0	0	0	0	269
normalized size	1	1.	4.63	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	1.029	3.769	0.037	0.	0.	0.	0.	102.03

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	1740	0	0	0	0	0	269
normalized size	1	1.	5.74	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	1.004	4.525	0.032	0.	0.	0.	0.	101.061

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	1740	0	0	0	0	0	267
normalized size	1	1.	5.78	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	1.006	3.966	0.051	0.	0.	0.	0.	99.151

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	2959	0	0	0	0	0	269
normalized size	1	1.	9.83	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	1.019	6.188	0.082	0.	0.	0.	0.	99.432

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	191	1935	0	1832	11538	1	238
normalized size	1	1.	0.79	7.96	0.	7.54	47.48	0.	0.98
time (sec)	N/A	0.4	0.529	0.013	0.	0.315	39.984	0.293	72.137

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	117	783	0	774	4190	1	146
normalized size	1	1.	0.75	5.05	0.	4.99	27.03	0.01	0.94
time (sec)	N/A	0.237	0.244	0.011	0.	0.313	16.854	0.277	43.503

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	59	221	0	231	1056	537	71
normalized size	1	1.	0.71	2.66	0.	2.78	12.72	6.47	0.86
time (sec)	N/A	0.113	0.081	0.006	0.	0.268	5.424	0.276	21.12

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	316	0	0	0	0	0	185
normalized size	1	1.	1.63	0.	0.	0.	0.	0.	0.95
time (sec)	N/A	0.628	0.475	0.04	0.	0.	0.	0.	48.509

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	392	358	692	0	0	0	0	0	0
normalized size	1	0.91	1.77	0.	0.	0.	0.	0.	0.
time (sec)	N/A	5.094	1.939	0.037	0.	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	2559	0	0	0	0	0	286
normalized size	1	1.	8.02	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	1.106	5.373	0.015	0.	0.	0.	0.	94.899

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	755	0	0	0	0	0	282
normalized size	1	1.	2.38	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	1.064	0.563	0.015	0.	0.	0.	0.	82.534

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	728	0	0	0	0	0	279
normalized size	1	1.	2.3	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	1.061	2.67	0.019	0.	0.	0.	0.	86.83

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	728	0	0	0	0	0	282
normalized size	1	1.	2.25	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	1.085	3.028	0.016	0.	0.	0.	0.	111.897

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	134	122	0	1	0	163	0
normalized size	1	1.	1.	0.91	0.	0.01	0.	1.22	0.
time (sec)	N/A	0.322	0.107	0.016	0.	5.198	0.	0.276	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	99	108	0	1	0	142	0
normalized size	1	1.	0.84	0.92	0.	0.01	0.	1.2	0.
time (sec)	N/A	0.278	0.16	0.012	0.	1.836	0.	0.274	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	77	92	0	1	0	122	88
normalized size	1	1.	0.73	0.88	0.	0.01	0.	1.16	0.84
time (sec)	N/A	0.251	0.058	0.01	0.	1.082	0.	0.276	35.602

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	66	83	0	1	0	116	85
normalized size	1	1.	0.69	0.86	0.	0.01	0.	1.21	0.89
time (sec)	N/A	0.198	0.066	0.009	0.	0.444	0.	0.275	27.189

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	67	83	0	1	0	115	85
normalized size	1	1.	0.7	0.86	0.	0.01	0.	1.2	0.89
time (sec)	N/A	0.139	0.06	0.009	0.	0.555	0.	0.275	25.002

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	134	101	0	1	0	138	99
normalized size	1	1.	1.18	0.89	0.	0.01	0.	1.21	0.87
time (sec)	N/A	0.263	0.118	0.014	0.	7.646	0.	0.274	43.954

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	169	119	0	1	0	178	114
normalized size	1	1.	1.31	0.92	0.	0.01	0.	1.38	0.88
time (sec)	N/A	0.307	0.175	0.016	0.	39.863	0.	0.276	45.033

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	209	145	0	1	0	227	139
normalized size	1	1.	1.34	0.93	0.	0.01	0.	1.46	0.89
time (sec)	N/A	0.374	0.161	0.017	0.	87.552	0.	0.277	54.421

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	344	405	0	1	0	490	0
normalized size	1	1.	0.96	1.13	0.	0.	0.	1.36	0.
time (sec)	N/A	0.654	0.865	0.014	0.	8.756	0.	0.283	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	373	387	0	1	0	450	0
normalized size	1	1.	1.08	1.12	0.	0.	0.	1.3	0.
time (sec)	N/A	0.577	0.474	0.011	0.	1.458	0.	0.282	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	233	363	0	1	0	441	304
normalized size	1	1.	0.69	1.08	0.	0.	0.	1.31	0.9
time (sec)	N/A	0.535	0.364	0.01	0.	0.514	0.	0.281	100.285

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	232	351	0	1	0	454	304
normalized size	1	1.	0.69	1.04	0.	0.	0.	1.35	0.9
time (sec)	N/A	0.519	0.266	0.01	0.	0.323	0.	0.282	91.733

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	234	363	0	1	0	458	304
normalized size	1	1.	0.7	1.08	0.	0.	0.	1.36	0.9
time (sec)	N/A	0.495	0.316	0.003	0.	0.853	0.	0.282	95.347

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	389	390	0	1	0	470	311
normalized size	1	1.	1.12	1.12	0.	0.	0.	1.35	0.89
time (sec)	N/A	0.58	0.458	0.013	0.	2.953	0.	0.281	109.165

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	367	406	0	1	0	491	323
normalized size	1	1.	1.02	1.13	0.	0.	0.	1.36	0.9
time (sec)	N/A	0.602	0.997	0.017	0.	10.075	0.	0.282	119.6

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	135	309	0	1	0	339	192
normalized size	1	1.	0.8	1.83	0.	0.01	0.	2.01	1.14
time (sec)	N/A	0.631	0.366	0.033	0.	15.344	0.	0.277	67.401

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	142	262	0	1	0	301	170
normalized size	1	1.	0.95	1.75	0.	0.01	0.	2.01	1.13
time (sec)	N/A	0.485	0.191	0.023	0.	7.11	0.	0.274	70.393

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	120	252	0	1	0	297	165
normalized size	1	1.	0.78	1.65	0.	0.01	0.	1.94	1.08
time (sec)	N/A	0.483	0.274	0.023	0.	2.842	0.	0.276	55.587

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	114	247	0	1	0	254	129
normalized size	1	1.	0.77	1.67	0.	0.01	0.	1.72	0.87
time (sec)	N/A	0.388	0.259	0.021	0.	2.82	0.	0.276	67.431

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	117	257	0	1	0	269	133
normalized size	1	1.	0.77	1.7	0.	0.01	0.	1.78	0.88
time (sec)	N/A	0.363	0.249	0.023	0.	7.248	0.	0.276	58.776

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	241	309	0	1	0	377	189
normalized size	1	1.	1.15	1.48	0.	0.	0.	1.8	0.9
time (sec)	N/A	0.485	0.321	0.029	0.	130.493	0.	0.277	74.112

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	248	332	0	0	0	464	221
normalized size	1	1.	1.05	1.41	0.	0.	0.	1.97	0.94
time (sec)	N/A	0.534	1.305	0.032	0.	0.	0.	0.276	76.223

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	278	363	0	0	0	473	246
normalized size	1	1.	1.05	1.37	0.	0.	0.	1.78	0.93
time (sec)	N/A	0.664	0.803	0.035	0.	0.	0.	0.278	96.419

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	712	712	431	873	0	1	0	784	0
normalized size	1	1.	0.61	1.23	0.	0.	0.	1.1	0.
time (sec)	N/A	1.38	0.535	0.021	0.	15.962	0.	0.288	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	687	687	428	852	0	1	0	803	631
normalized size	1	1.	0.62	1.24	0.	0.	0.	1.17	0.92
time (sec)	N/A	1.196	0.629	0.02	0.	11.363	0.	0.287	161.664

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	685	685	423	848	0	1	0	791	632
normalized size	1	1.	0.62	1.24	0.	0.	0.	1.15	0.92
time (sec)	N/A	1.15	0.487	0.019	0.	9.199	0.	0.286	163.398

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	685	685	428	852	0	1	0	814	638
normalized size	1	1.	0.62	1.24	0.	0.	0.	1.19	0.93
time (sec)	N/A	1.096	0.494	0.02	0.	10.812	0.	0.287	160.217

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	689	689	429	873	0	1	0	814	636
normalized size	1	1.	0.62	1.27	0.	0.	0.	1.18	0.92
time (sec)	N/A	1.139	0.523	0.002	0.	21.088	0.	0.286	171.294

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	745	745	499	911	0	1	0	863	0
normalized size	1	1.	0.67	1.22	0.	0.	0.	1.16	0.
time (sec)	N/A	1.46	0.692	0.025	0.	46.78	0.	0.289	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	751	751	513	932	0	1	0	848	0
normalized size	1	1.	0.68	1.24	0.	0.	0.	1.13	0.
time (sec)	N/A	1.324	0.714	0.03	0.	106.225	0.	0.286	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	121	164	0	1	0	211	182
normalized size	1	1.	0.5	0.67	0.	0.	0.	0.87	0.75
time (sec)	N/A	0.398	0.138	0.016	0.	0.295	0.	0.277	27.609

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	56	51	68	68	0	85	75
normalized size	1	1.	0.52	0.47	0.63	0.63	0.	0.79	0.69
time (sec)	N/A	0.28	0.058	0.006	0.709	0.27	0.	0.272	18.133

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	98	122	0	1	0	147	0
normalized size	1	1.	0.55	0.69	0.	0.01	0.	0.83	0.
time (sec)	N/A	0.19	0.084	0.01	0.	0.286	0.	0.277	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	96	79	0	1	0	113	105
normalized size	1	1.	0.63	0.52	0.	0.01	0.	0.74	0.69
time (sec)	N/A	0.305	0.114	0.01	0.	0.279	0.	0.276	20.097

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	93	128	0	1	0	157	128
normalized size	1	1.	0.53	0.72	0.	0.01	0.	0.89	0.72
time (sec)	N/A	0.288	0.08	0.013	0.	0.289	0.	0.282	17.751

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	109	139	0	1	0	135	124
normalized size	1	1.	0.62	0.79	0.	0.01	0.	0.76	0.7
time (sec)	N/A	0.363	0.111	0.015	0.	0.287	0.	0.277	22.799

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	72	73	97	1	76	107	0
normalized size	1	1.	0.92	0.94	1.24	0.01	0.97	1.37	0.
time (sec)	N/A	0.323	0.04	0.002	0.702	0.233	0.128	0.269	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	73	97	1	82	107	75
normalized size	1	1.	1.	0.94	1.24	0.01	1.05	1.37	0.96
time (sec)	N/A	0.166	0.027	0.001	0.702	0.229	0.131	0.267	23.864

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	72	73	97	1	76	107	0
normalized size	1	1.	0.96	0.97	1.29	0.01	1.01	1.43	0.
time (sec)	N/A	0.287	0.04	0.001	0.694	0.23	0.131	0.268	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	93	1	78	103	0
normalized size	1	1.	1.	0.96	1.27	0.01	1.07	1.41	0.
time (sec)	N/A	0.116	0.028	0.	0.7	0.246	0.127	0.269	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	77	99	95	73	107	0
normalized size	1	1.	1.	1.04	1.34	1.28	0.99	1.45	0.
time (sec)	N/A	0.194	0.039	0.004	0.7	0.252	1.262	0.269	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	75	93	100	73	100	0
normalized size	1	1.	1.	1.06	1.31	1.41	1.03	1.41	0.
time (sec)	N/A	0.125	0.056	0.006	0.7	0.245	1.255	0.267	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	71	76	99	103	71	131	0
normalized size	1	1.	0.96	1.03	1.34	1.39	0.96	1.77	0.
time (sec)	N/A	0.23	0.083	0.011	0.691	0.259	1.593	0.271	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	165	214	0	1	313	216	158
normalized size	1	1.	0.91	1.18	0.	0.01	1.73	1.19	0.87
time (sec)	N/A	0.505	0.265	0.015	0.	0.266	5.518	0.27	86.314

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	133	176	0	1	184	169	131
normalized size	1	1.	0.88	1.17	0.	0.01	1.22	1.12	0.87
time (sec)	N/A	0.43	0.132	0.014	0.	0.277	5.121	0.27	64.152

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	102	141	0	1	160	123	95
normalized size	1	1.	0.84	1.16	0.	0.01	1.31	1.01	0.78
time (sec)	N/A	0.334	0.115	0.013	0.	0.271	4.512	0.27	51.79

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	88	118	0	1	153	101	78
normalized size	1	1.	1.06	1.42	0.	0.01	1.84	1.22	0.94
time (sec)	N/A	0.146	0.089	0.	0.	0.274	3.778	0.271	32.929

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	89	121	0	1	155	112	80
normalized size	1	1.	1.03	1.41	0.	0.01	1.8	1.3	0.93
time (sec)	N/A	0.203	0.109	0.016	0.	0.267	5.076	0.271	25.027

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	105	146	0	1	167	127	95
normalized size	1	1.	0.99	1.38	0.	0.01	1.58	1.2	0.9
time (sec)	N/A	0.294	0.103	0.018	0.	0.281	6.89	0.271	38.966

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	135	183	0	1	284	177	126
normalized size	1	1.	0.99	1.35	0.	0.01	2.09	1.3	0.93
time (sec)	N/A	0.423	0.154	0.021	0.	0.274	9.701	0.27	66.28

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	166	221	0	1	328	221	158
normalized size	1	1.	0.99	1.32	0.	0.01	1.96	1.32	0.95
time (sec)	N/A	0.585	0.166	0.023	0.	0.267	13.021	0.27	88.32

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	170	239	0	1	233	216	165
normalized size	1	1.	0.91	1.28	0.	0.01	1.25	1.16	0.88
time (sec)	N/A	0.6	0.19	0.018	0.	0.268	15.891	0.272	129.758

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	141	202	0	1	211	169	134
normalized size	1	1.	0.89	1.28	0.	0.01	1.34	1.07	0.85
time (sec)	N/A	0.477	0.162	0.016	0.	0.28	14.455	0.273	97.309

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	122	179	0	1	201	144	114
normalized size	1	1.	0.98	1.44	0.	0.01	1.62	1.16	0.92
time (sec)	N/A	0.367	0.185	0.015	0.	0.269	11.183	0.272	54.623

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	110	131	0	1	196	136	110
normalized size	1	1.	0.96	1.14	0.	0.01	1.7	1.18	0.96
time (sec)	N/A	0.206	0.168	0.	0.	0.278	6.638	0.272	34.075

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	124	182	0	1	202	149	116
normalized size	1	1.	1.	1.47	0.	0.01	1.63	1.2	0.94
time (sec)	N/A	0.35	0.256	0.018	0.	0.272	9.167	0.272	37.784

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	141	207	0	1	214	173	133
normalized size	1	1.	0.99	1.46	0.	0.01	1.51	1.22	0.94
time (sec)	N/A	0.424	0.145	0.022	0.	0.278	12.595	0.272	62.88

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	173	245	0	1	330	221	0
normalized size	1	1.	1.01	1.43	0.	0.01	1.93	1.29	0.
time (sec)	N/A	0.584	0.193	0.023	0.	0.273	18.27	0.272	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	228	538	0	0	0	319	0
normalized size	1	1.	0.99	2.34	0.	0.	0.	1.39	0.
time (sec)	N/A	0.926	0.41	0.019	0.	0.	0.	0.301	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	186	408	0	0	0	262	0
normalized size	1	1.	0.98	2.16	0.	0.	0.	1.39	0.
time (sec)	N/A	0.598	0.312	0.014	0.	0.	0.	0.302	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	139	289	0	1	0	212	136
normalized size	1	1.	0.88	1.83	0.	0.01	0.	1.34	0.86
time (sec)	N/A	0.468	0.179	0.012	0.	56.341	0.	0.302	66.387

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	114	176	0	1	0	180	124
normalized size	1	1.	0.86	1.33	0.	0.01	0.	1.36	0.94
time (sec)	N/A	0.331	0.122	0.012	0.	18.297	0.	0.301	47.958

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	112	176	0	1	0	181	124
normalized size	1	1.	0.84	1.32	0.	0.01	0.	1.36	0.93
time (sec)	N/A	0.251	0.115	0.011	0.	12.993	0.	0.302	47.282

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	242	298	0	0	0	232	148
normalized size	1	1.	1.45	1.78	0.	0.	0.	1.39	0.89
time (sec)	N/A	0.571	0.592	0.016	0.	0.	0.	0.302	100.157

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	331	430	0	0	0	320	194
normalized size	1	1.	1.61	2.1	0.	0.	0.	1.56	0.95
time (sec)	N/A	0.787	0.574	0.021	0.	0.	0.	0.303	157.915

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	426	584	0	0	0	448	0
normalized size	1	1.	1.59	2.18	0.	0.	0.	1.67	0.
time (sec)	N/A	1.069	0.795	0.028	0.	0.	0.	0.303	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	463	1449	0	0	0	0	0
normalized size	1	1.	1.2	3.74	0.	0.	0.	0.	0.
time (sec)	N/A	8.195	1.165	0.051	0.	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	385	1098	0	0	0	0	0
normalized size	1	1.	1.19	3.4	0.	0.	0.	0.	0.
time (sec)	N/A	3.296	0.947	0.042	0.	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	323	764	0	1	0	0	279
normalized size	1	1.	1.15	2.73	0.	0.	0.	0.	1.
time (sec)	N/A	1.878	0.579	0.034	0.	3.772	0.	0.	113.205

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	277	478	0	1	0	0	255
normalized size	1	1.	1.1	1.9	0.	0.	0.	0.	1.02
time (sec)	N/A	0.871	0.993	0.028	0.	1.453	0.	0.	105.017

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	274	480	0	0	0	0	255
normalized size	1	1.	1.08	1.89	0.	0.	0.	0.	1.
time (sec)	N/A	1.078	0.424	0.002	0.	0.	0.	0.	102.417

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	340	817	0	0	0	0	0
normalized size	1	1.	1.14	2.74	0.	0.	0.	0.	0.
time (sec)	N/A	1.801	0.774	0.039	0.	0.	0.	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	410	1160	0	0	0	0	0
normalized size	1	1.	1.18	3.33	0.	0.	0.	0.	0.
time (sec)	N/A	3.327	0.993	0.048	0.	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	866	866	267	336	0	0	0	0	0
normalized size	1	1.	0.31	0.39	0.	0.	0.	0.	0.
time (sec)	N/A	4.775	0.514	0.103	0.	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	301	1049	0	0	0	0	350
normalized size	1	1.	1.11	3.86	0.	0.	0.	0.	1.29
time (sec)	N/A	1.167	1.021	0.03	0.	0.	0.	0.	104.923

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	237	887	0	1	0	0	190
normalized size	1	1.	1.14	4.26	0.	0.	0.	0.	0.91
time (sec)	N/A	0.713	0.558	0.015	0.	18.512	0.	0.	68.107

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	199	757	0	1	0	0	150
normalized size	1	1.	1.18	4.51	0.	0.01	0.	0.	0.89
time (sec)	N/A	0.494	0.246	0.009	0.	1.262	0.	0.	61.506

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	214	851	0	1	0	0	162
normalized size	1	1.	1.15	4.58	0.	0.01	0.	0.	0.87
time (sec)	N/A	0.614	0.793	0.016	0.	73.241	0.	0.	72.819

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	244	1009	0	1	0	0	326
normalized size	1	1.	0.68	2.8	0.	0.	0.	0.	0.9
time (sec)	N/A	1.186	1.171	0.017	0.	0.478	0.	0.	125.688

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	632	209	528	0	0	0	0	571
normalized size	1	1.49	0.49	1.25	0.	0.	0.	0.	1.35
time (sec)	N/A	0.996	0.215	0.093	0.	0.	0.	0.	80.407

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	604	204	509	0	0	0	0	544
normalized size	1	1.45	0.49	1.22	0.	0.	0.	0.	1.3
time (sec)	N/A	0.811	0.2	0.012	0.	0.	0.	0.	74.91

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	483	127	341	0	0	0	0	432
normalized size	1	1.27	0.33	0.9	0.	0.	0.	0.	1.13
time (sec)	N/A	0.399	0.101	0.007	0.	0.	0.	0.	45.029

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	509	208	511	0	0	0	0	450
normalized size	1	1.24	0.5	1.24	0.	0.	0.	0.	1.09
time (sec)	N/A	0.491	0.19	0.021	0.	0.	0.	0.	53.195

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	154	448	0	0	0	0	330
normalized size	1	1.	0.41	1.2	0.	0.	0.	0.	0.88
time (sec)	N/A	0.392	0.165	0.024	0.	0.	0.	0.	37.133

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	647	647	224	549	0	0	0	0	581
normalized size	1	1.	0.35	0.85	0.	0.	0.	0.	0.9
time (sec)	N/A	1.039	0.236	0.028	0.	0.	0.	0.	95.453

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	482	492	2068	0	0	0	0	551
normalized size	1	1.	1.02	4.29	0.	0.	0.	0.	1.14
time (sec)	N/A	2.16	1.107	0.045	0.	0.	0.	0.	178.164

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	376	1696	0	0	0	0	388
normalized size	1	1.	1.04	4.71	0.	0.	0.	0.	1.08
time (sec)	N/A	1.493	0.474	0.017	0.	0.	0.	0.	127.517

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	276	1411	0	0	0	0	253
normalized size	1	1.	1.03	5.25	0.	0.	0.	0.	0.94
time (sec)	N/A	1.004	0.416	0.011	0.	0.	0.	0.	109.583

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	292	1270	0	0	0	0	332
normalized size	1	1.	0.83	3.63	0.	0.	0.	0.	0.95
time (sec)	N/A	1.27	2.257	0.017	0.	0.	0.	0.	130.837

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	562	562	284	1207	0	0	0	0	0
normalized size	1	1.	0.51	2.15	0.	0.	0.	0.	0.
time (sec)	N/A	2.192	1.766	0.018	0.	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	888	214	547	0	0	0	0	796
normalized size	1	1.92	0.46	1.18	0.	0.	0.	0.	1.72
time (sec)	N/A	1.192	0.213	0.032	0.	0.	0.	0.	134.069

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	428	615	209	377	0	0	0	0	663
normalized size	1	1.44	0.49	0.88	0.	0.	0.	0.	1.55
time (sec)	N/A	0.654	0.198	0.009	0.	0.	0.	0.	97.272

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	735	735	213	528	0	0	0	0	544
normalized size	1	1.	0.29	0.72	0.	0.	0.	0.	0.74
time (sec)	N/A	0.758	0.195	0.021	0.	0.	0.	0.	62.902

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	638	638	219	530	0	0	0	0	570
normalized size	1	1.	0.34	0.83	0.	0.	0.	0.	0.89
time (sec)	N/A	0.799	0.206	0.024	0.	0.	0.	0.	67.298

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	665	665	224	549	0	0	0	0	597
normalized size	1	1.	0.34	0.83	0.	0.	0.	0.	0.9
time (sec)	N/A	0.957	0.246	0.027	0.	0.	0.	0.	86.904

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	196	267	0	1	0	0	190
normalized size	1	1.	1.13	1.54	0.	0.01	0.	0.	1.1
time (sec)	N/A	0.757	0.759	0.026	0.	21.367	0.	0.	62.115

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	161	204	0	1	0	0	121
normalized size	1	1.	1.18	1.49	0.	0.01	0.	0.	0.88
time (sec)	N/A	0.445	0.284	0.014	0.	1.425	0.	0.	44.933

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	96	165	0	1	0	101	78
normalized size	1	1.	1.12	1.92	0.	0.01	0.	1.17	0.91
time (sec)	N/A	0.244	0.091	0.008	0.	0.36	0.	0.288	30.993

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	174	207	0	1	0	0	121
normalized size	1	1.	1.26	1.5	0.	0.01	0.	0.	0.88
time (sec)	N/A	0.52	0.942	0.017	0.	0.487	0.	0.	53.656

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	256	276	0	1	0	0	194
normalized size	1	1.	1.17	1.27	0.	0.	0.	0.	0.89
time (sec)	N/A	0.701	1.852	0.019	0.	0.951	0.	0.	64.385

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	488	127	222	0	0	0	0	507
normalized size	1	1.23	0.32	0.56	0.	0.	0.	0.	1.27
time (sec)	N/A	0.462	0.143	0.03	0.	0.	0.	0.	43.751

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	99	134	0	0	0	0	308
normalized size	1	1.	0.35	0.48	0.	0.	0.	0.	1.1
time (sec)	N/A	0.396	0.095	0.011	0.	0.	0.	0.	29.152

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	80	70	0	0	0	0	226
normalized size	1	1.	0.31	0.27	0.	0.	0.	0.	0.88
time (sec)	N/A	0.193	0.037	0.007	0.	0.	0.	0.	15.235

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	500	500	147	178	0	0	0	0	439
normalized size	1	1.	0.29	0.36	0.	0.	0.	0.	0.88
time (sec)	N/A	0.622	0.266	0.022	0.	0.	0.	0.	47.349

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	622	622	219	260	0	0	0	0	556
normalized size	1	1.	0.35	0.42	0.	0.	0.	0.	0.89
time (sec)	N/A	0.936	0.176	0.023	0.	0.	0.	0.	69.829

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	271	720	0	1	0	0	348
normalized size	1	1.	1.15	3.05	0.	0.	0.	0.	1.47
time (sec)	N/A	1.113	0.634	0.08	0.	69.96	0.	0.	134.65

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	204	613	0	1	0	536	235
normalized size	1	1.	1.22	3.67	0.	0.01	0.	3.21	1.41
time (sec)	N/A	0.634	0.736	0.019	0.	0.634	0.	0.352	105.888

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	225	506	0	1	0	595	143
normalized size	1	1.	1.42	3.18	0.	0.01	0.	3.74	0.9
time (sec)	N/A	0.521	0.511	0.016	0.	0.621	0.	0.319	64.413

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	201	454	0	1	0	613	150
normalized size	1	1.	1.21	2.73	0.	0.01	0.	3.69	0.9
time (sec)	N/A	0.437	0.618	0.011	0.	0.633	0.	0.32	61.421

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	378	612	0	1	0	0	238
normalized size	1	1.	1.42	2.3	0.	0.	0.	0.	0.89
time (sec)	N/A	0.923	3.097	0.018	0.	3.514	0.	0.	111.14

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	349	863	0	1	0	0	384
normalized size	1	1.	0.83	2.06	0.	0.	0.	0.	0.92
time (sec)	N/A	1.344	2.93	0.019	0.	8.485	0.	0.	148.256

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	636	199	603	0	0	0	0	643
normalized size	1	1.42	0.44	1.34	0.	0.	0.	0.	1.43
time (sec)	N/A	0.992	0.203	0.052	0.	0.	0.	0.	84.541

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	536	199	586	0	0	0	0	541
normalized size	1	1.27	0.47	1.39	0.	0.	0.	0.	1.28
time (sec)	N/A	0.825	0.183	0.015	0.	0.	0.	0.	65.212

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	514	199	561	0	0	0	0	459
normalized size	1	1.22	0.47	1.33	0.	0.	0.	0.	1.09
time (sec)	N/A	0.581	0.217	0.013	0.	0.	0.	0.	47.45

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	516	199	536	0	0	0	0	456
normalized size	1	1.22	0.47	1.27	0.	0.	0.	0.	1.08
time (sec)	N/A	0.564	0.182	0.013	0.	0.	0.	0.	50.597

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	514	199	366	0	0	0	0	530
normalized size	1	1.22	0.47	0.87	0.	0.	0.	0.	1.26
time (sec)	N/A	0.509	0.162	0.009	0.	0.	0.	0.	96.282

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	657	211	553	0	0	0	0	658
normalized size	1	1.4	0.45	1.18	0.	0.	0.	0.	1.41
time (sec)	N/A	1.079	0.237	0.024	0.	0.	0.	0.	133.09

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	476	496	0	0	0	0	0
normalized size	1	1.	1.17	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	17.863	1.52	0.084	0.	0.	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	383	332	0	0	0	0	0
normalized size	1	1.	1.18	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	6.786	1.511	0.038	0.	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	310	275	0	3287	0	0	0
normalized size	1	1.	1.06	0.94	0.	11.26	0.	0.	0.
time (sec)	N/A	8.745	0.537	0.031	0.	53.871	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	179	177	0	1465	0	0	192
normalized size	1	1.	0.89	0.88	0.	7.25	0.	0.	0.95
time (sec)	N/A	0.892	0.407	0.019	0.	11.617	0.	0.	75.734

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	269	294	0	1	0	0	0
normalized size	1	1.	0.96	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	2.877	1.195	0.034	0.	92.386	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	370	351	401	0	0	0	0	0
normalized size	1	0.97	0.92	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	8.186	1.704	0.038	0.	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	468	655	0	0	0	0	0
normalized size	1	1.	0.85	1.19	0.	0.	0.	0.	0.
time (sec)	N/A	9.29	2.958	0.047	0.	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	0	290	0	1	0	0	0
normalized size	1	1.	0.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	6.529	0.809	0.048	0.	25.828	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	0	224	0	1	0	0	0
normalized size	1	1.	0.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	3.462	0.587	0.033	0.	3.863	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	0	161	0	1330	0	0	219
normalized size	1	1.	0.	0.67	0.	5.54	0.	0.	0.91
time (sec)	N/A	0.926	0.126	0.02	0.	1.045	0.	0.	103.247

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	0	272	0	3243	0	0	0
normalized size	1	1.	0.	0.93	0.	11.14	0.	0.	0.
time (sec)	N/A	1.638	0.596	0.038	0.	2.273	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	0	322	0	5528	0	0	0
normalized size	1	1.	0.	0.86	0.	14.82	0.	0.	0.
time (sec)	N/A	5.635	0.782	0.042	0.	9.84	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	0	503	0	7794	0	0	0
normalized size	1	1.	0.	0.98	0.	15.22	0.	0.	0.
time (sec)	N/A	12.826	1.006	0.051	0.	12.149	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	448	490	0	0	0	0	0
normalized size	1	1.	0.97	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	12.065	1.706	0.041	0.	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	326	279	0	5999	0	0	0
normalized size	1	1.	1.	0.85	0.	18.35	0.	0.	0.
time (sec)	N/A	3.192	0.762	0.028	0.	129.552	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	333	388	0	0	0	0	0
normalized size	1	1.	0.96	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	3.908	1.842	0.039	0.	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	416	382	555	0	0	0	0	0
normalized size	1	1.	0.92	1.33	0.	0.	0.	0.	0.
time (sec)	N/A	7.542	2.017	0.046	0.	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	595	595	0	516	0	0	0	0	0
normalized size	1	1.	0.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	7.253	1.247	0.048	0.	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	491	491	0	382	0	1	0	0	0
normalized size	1	1.	0.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	3.996	0.954	0.042	0.	99.554	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	0	217	0	1	0	0	0
normalized size	1	1.	0.	0.45	0.	0.	0.	0.	0.
time (sec)	N/A	4.226	0.672	0.029	0.	27.132	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	260	432	0	360	0	5480	0	0	0
normalized size	1	1.66	0.	1.38	0.	21.08	0.	0.	0.
time (sec)	N/A	2.16	0.736	0.043	0.	12.386	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	523	523	0	511	0	10571	0	0	0
normalized size	1	1.	0.	0.98	0.	20.21	0.	0.	0.
time (sec)	N/A	5.569	0.912	0.047	0.	31.01	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	354	2134	0	5154	0	0	0
normalized size	1	1.	1.26	7.59	0.	18.34	0.	0.	0.
time (sec)	N/A	14.23	0.787	0.119	0.	5.482	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	279	1223	0	2858	0	0	243
normalized size	1	1.	1.22	5.34	0.	12.48	0.	0.	1.06
time (sec)	N/A	3.56	0.482	0.063	0.	2.027	0.	0.	152.828

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	169	1167	0	1176	0	0	168
normalized size	1	1.	0.93	6.41	0.	6.46	0.	0.	0.92
time (sec)	N/A	0.619	0.26	0.052	0.	0.937	0.	0.	57.688

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	234	2099	0	1663	0	0	218
normalized size	1	1.	0.97	8.71	0.	6.9	0.	0.	0.9
time (sec)	N/A	3.569	0.676	0.071	0.	3.948	0.	0.	143.468

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	298	2770	0	4031	0	0	0
normalized size	1	1.	1.03	9.55	0.	13.9	0.	0.	0.
time (sec)	N/A	5.237	1.008	0.087	0.	11.349	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	0	222	0	4086	0	0	0
normalized size	1	1.	0.	0.68	0.	12.57	0.	0.	0.
time (sec)	N/A	11.687	0.658	0.045	0.	1.778	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	0	175	0	1931	0	0	0
normalized size	1	1.	0.	0.67	0.	7.34	0.	0.	0.
time (sec)	N/A	4.313	0.486	0.03	0.	0.78	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	0	130	0	1025	0	0	196
normalized size	1	1.	0.	0.59	0.	4.66	0.	0.	0.89
time (sec)	N/A	0.726	0.1	0.016	0.	0.378	0.	0.	78.493

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	0	217	0	2804	0	0	248
normalized size	1	1.	0.	0.82	0.	10.58	0.	0.	0.94
time (sec)	N/A	1.848	0.493	0.035	0.	0.488	0.	0.	154.285

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	0	160	0	475	0	282	128
normalized size	1	1.	0.	1.67	0.	4.95	0.	2.94	1.33
time (sec)	N/A	0.42	0.264	0.101	0.	0.299	0.	0.342	59.137

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	0	377	0	0	0	0	0
normalized size	1	1.	0.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	3.981	0.897	0.04	0.	0.	0.	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	0	269	0	1	0	0	359
normalized size	1	1.	0.	0.73	0.	0.	0.	0.	0.98
time (sec)	N/A	2.599	0.699	0.035	0.	89.9	0.	0.	179.501

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	0	200	0	1	0	0	292
normalized size	1	1.	0.	0.67	0.	0.	0.	0.	0.98
time (sec)	N/A	1.758	0.517	0.028	0.	15.299	0.	0.	173.891

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	0	161	0	4583	0	0	219
normalized size	1	1.	0.	0.67	0.	19.1	0.	0.	0.91
time (sec)	N/A	0.908	0.184	0.024	0.	4.082	0.	0.	78.566

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	0	151	0	6152	0	0	226
normalized size	1	1.	0.	0.62	0.	25.32	0.	0.	0.93
time (sec)	N/A	0.46	0.1	0.021	0.	10.395	0.	0.	70.873

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	0	197	0	8682	0	0	269
normalized size	1	1.	0.	0.7	0.	31.01	0.	0.	0.96
time (sec)	N/A	1.362	0.607	0.031	0.	6.184	0.	0.	152.541

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	0	248	0	11052	0	0	0
normalized size	1	1.	0.	0.73	0.	32.41	0.	0.	0.
time (sec)	N/A	1.774	0.768	0.05	0.	58.107	0.	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	0	350	0	13497	0	0	0
normalized size	1	1.	0.	0.79	0.	30.47	0.	0.	0.
time (sec)	N/A	3.271	0.943	0.042	0.	94.507	0.	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	350	507	0	480	0	0	0	0	0
normalized size	1	1.45	0.	1.37	0.	0.	0.	0.	0.
time (sec)	N/A	10.553	1.085	0.051	0.	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	0	338	0	19524	0	0	0
normalized size	1	1.	0.	0.94	0.	54.23	0.	0.	0.
time (sec)	N/A	3.237	0.808	0.042	0.	69.281	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	0	252	0	19097	0	0	0
normalized size	1	1.	0.	0.76	0.	57.35	0.	0.	0.
time (sec)	N/A	1.704	0.687	0.037	0.	89.878	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	0	246	0	0	0	0	0
normalized size	1	1.	0.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	2.024	0.735	0.03	0.	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	339	462	0	387	0	0	0	0	0
normalized size	1	1.36	0.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	6.188	1.125	0.046	0.	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	419	647	0	541	0	0	0	0	0
normalized size	1	1.54	0.	1.29	0.	0.	0.	0.	0.
time (sec)	N/A	12.814	1.361	0.052	0.	0.	0.	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	0	0	0	0	0	0	202
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.83
time (sec)	N/A	1.489	0.106	0.092	0.	0.	0.	0.	114.954

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	367	0	0	0	0	0	289
normalized size	1	1.	1.18	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	1.801	1.351	0.096	0.	0.	0.	0.	96.978

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	362	0	0	0	0	0	241
normalized size	1	1.	1.42	0.	0.	0.	0.	0.	0.95
time (sec)	N/A	1.16	1.891	0.083	0.	0.	0.	0.	93.411

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	303	0	0	0	0	0	206
normalized size	1	1.	1.45	0.	0.	0.	0.	0.	0.99
time (sec)	N/A	0.721	0.715	0.086	0.	0.	0.	0.	66.313

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	261	0	0	0	0	0	175
normalized size	1	1.	1.32	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.82	0.485	0.068	0.	0.	0.	0.	71.855

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	0	0	0	0	0	0	243
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	1.216	0.11	0.051	0.	0.	0.	0.	105.931

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	420	0	0	0	0	0	303
normalized size	1	1.	1.31	0.	0.	0.	0.	0.	0.94
time (sec)	N/A	1.57	0.989	0.121	0.	0.	0.	0.	141.294

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	0	0	0	0	0	0	308
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	1.416	0.314	0.082	0.	0.	0.	0.	163.142

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	0	0	0	0	0	0	240
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	1.194	0.136	0.07	0.	0.	0.	0.	150.094

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	0	0	0	0	0	0	134
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.83
time (sec)	N/A	0.73	0.105	0.078	0.	0.	0.	0.	69.108

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	0	0	0	0	0	0	163
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.716	0.048	0.065	0.	0.	0.	0.	61.759

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	0	0	0	0	0	0	228
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	1.167	0.103	0.091	0.	0.	0.	0.	138.045

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.313	0.295	0.065	0.	0.	0.	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	79	101	42	162	0	78	56
normalized size	1	1.1	1.98	2.52	1.05	4.05	0.	1.95	1.4
time (sec)	N/A	0.275	0.077	0.083	0.763	0.292	0.	0.299	20.008

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [252] had the largest ratio of [0.5]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.	20	0.1
2	A	2	1	1.	20	0.05
3	A	4	3	1.	18	0.167
4	A	2	1	1.	17	0.059
5	A	3	2	1.	20	0.1
6	A	2	1	1.	20	0.05
7	A	3	2	1.	20	0.1
8	A	5	5	1.	20	0.25
9	A	4	4	1.	20	0.2
10	A	4	4	1.	18	0.222
11	A	7	7	1.	20	0.35
12	A	7	7	1.	20	0.35
13	A	7	7	1.	20	0.35
14	A	6	6	1.	20	0.3
15	A	6	5	1.	20	0.25
16	A	5	5	1.	20	0.25
17	A	4	4	1.	17	0.235
18	A	4	4	1.	20	0.2

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
19	A	5	5	1.	20	0.25
20	A	6	5	1.	20	0.25
21	A	5	4	1.	20	0.2
22	A	5	4	1.	18	0.222
23	A	8	7	1.	20	0.35
24	A	8	8	1.	20	0.4
25	A	8	7	1.	20	0.35
26	A	8	8	1.	20	0.4
27	A	7	5	1.	20	0.25
28	A	6	5	1.	20	0.25
29	A	5	4	1.	17	0.235
30	A	5	5	1.	20	0.25
31	A	5	4	1.	20	0.2
32	A	5	4	1.	20	0.2
33	A	4	4	1.	20	0.2
34	A	3	3	1.	20	0.15
35	A	3	3	1.	18	0.167
36	A	6	6	1.	20	0.3
37	A	5	5	1.	20	0.25
38	A	6	6	1.	20	0.3
39	A	5	4	1.	20	0.2
40	A	4	4	1.	20	0.2
41	A	3	3	1.	17	0.176
42	A	4	4	1.	20	0.2
43	A	5	4	1.	20	0.2
44	A	4	4	1.	20	0.2
45	A	4	4	1.	20	0.2
46	A	3	3	1.	20	0.15
47	A	2	2	1.	18	0.111
48	A	6	6	1.	20	0.3
49	A	6	6	1.	20	0.3
50	A	5	5	1.	20	0.25
51	A	4	4	1.	20	0.2
52	A	4	4	1.	17	0.235

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
53	A	5	5	1.	20	0.25
54	A	6	5	1.	20	0.25
55	A	3	2	1.	25	0.08
56	A	4	3	1.	23	0.13
57	A	3	2	1.	23	0.087
58	A	4	3	1.	23	0.13
59	A	3	2	1.	23	0.087
60	A	4	3	1.	21	0.143
61	A	3	2	1.	20	0.1
62	A	5	4	1.	23	0.174
63	A	3	2	1.	23	0.087
64	A	4	3	1.	23	0.13
65	A	3	2	1.	23	0.087
66	A	4	3	1.	21	0.143
67	A	3	2	1.	21	0.095
68	A	4	3	1.	21	0.143
69	A	3	2	1.	21	0.095
70	A	2	2	1.	19	0.105
71	A	3	2	1.	18	0.111
72	A	4	3	1.	21	0.143
73	A	3	2	1.	21	0.095
74	A	4	3	1.	21	0.143
75	A	4	4	1.	33	0.121
76	A	4	4	1.	31	0.129
77	A	3	3	1.	30	0.1
78	A	4	3	1.	33	0.091
79	A	3	3	1.	33	0.091
80	A	4	3	1.	33	0.091
81	A	4	4	1.	33	0.121
82	A	3	3	1.	31	0.097
83	A	4	4	1.	30	0.133
84	A	4	3	1.	33	0.091
85	A	5	4	1.	33	0.121
86	A	4	3	1.	33	0.091

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
87	A	3	2	1.	35	0.057
88	A	3	2	1.	35	0.057
89	A	3	2	1.	35	0.057
90	A	3	3	1.	35	0.086
91	A	3	3	1.	35	0.086
92	A	1	1	1.	29	0.034
93	A	5	4	1.	31	0.129
94	A	5	4	1.	31	0.129
95	A	3	2	1.	25	0.08
96	A	2	1	1.	25	0.04
97	A	3	2	1.	23	0.087
98	A	2	1	1.	22	0.045
99	A	3	2	1.	25	0.08
100	A	2	1	1.	25	0.04
101	A	3	2	1.	25	0.08
102	A	7	6	1.	25	0.24
103	A	6	6	1.	25	0.24
104	A	5	5	1.	23	0.217
105	A	7	6	1.	25	0.24
106	A	7	6	1.	25	0.24
107	A	5	3	1.	25	0.12
108	A	4	3	1.	25	0.12
109	A	3	2	1.	22	0.091
110	A	4	3	1.	25	0.12
111	A	5	3	1.	25	0.12
112	A	7	7	1.	25	0.28
113	A	6	6	1.	25	0.24
114	A	4	4	1.	25	0.16
115	A	4	4	1.	23	0.174
116	A	8	7	1.	25	0.28
117	A	8	7	1.	25	0.28
118	A	6	4	1.	25	0.16
119	A	5	4	1.	25	0.16
120	A	4	3	1.	25	0.12

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
121	A	4	3	1.	22	0.136
122	A	5	4	1.	25	0.16
123	A	6	4	1.	25	0.16
124	A	8	7	1.	25	0.28
125	A	7	6	1.	25	0.24
126	A	5	5	1.	25	0.2
127	A	5	5	1.	25	0.2
128	A	5	5	1.	25	0.2
129	A	5	5	1.	23	0.217
130	A	9	7	1.	25	0.28
131	A	9	7	1.	25	0.28
132	A	7	4	1.	25	0.16
133	A	6	4	1.	25	0.16
134	A	5	3	1.	25	0.12
135	A	5	4	1.	25	0.16
136	A	5	3	1.	22	0.136
137	A	4	3	1.	21	0.143
138	A	5	4	1.	22	0.182
139	A	5	5	1.	17	0.294
140	A	6	6	1.	18	0.333
141	A	5	5	1.	22	0.227
142	A	6	6	1.	25	0.24
143	A	5	5	1.	25	0.2
144	A	5	5	1.	23	0.217
145	A	7	6	1.	25	0.24
146	A	7	6	1.	25	0.24
147	A	7	6	1.	25	0.24
148	A	5	5	1.	25	0.2
149	A	6	6	1.	25	0.24
150	A	7	6	1.	25	0.24
151	A	6	5	1.	25	0.2
152	A	5	5	1.	25	0.2
153	A	4	4	1.	22	0.182
154	A	4	4	1.	25	0.16

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
155	A	5	5	1.	25	0.2
156	A	7	6	1.	25	0.24
157	A	6	5	1.	25	0.2
158	A	6	5	1.	23	0.217
159	A	8	6	1.	25	0.24
160	A	8	7	1.	25	0.28
161	A	8	6	1.	25	0.24
162	A	8	7	1.	25	0.28
163	A	7	5	1.	25	0.2
164	A	6	5	1.	25	0.2
165	A	5	4	1.	22	0.182
166	A	5	5	1.	25	0.2
167	A	5	4	1.	25	0.16
168	A	6	5	1.	25	0.2
169	A	5	5	1.	27	0.185
170	A	4	4	1.	27	0.148
171	A	4	4	1.	25	0.16
172	A	6	5	1.	27	0.185
173	A	4	4	1.	27	0.148
174	A	5	5	1.	27	0.185
175	A	6	5	1.	27	0.185
176	A	5	4	1.	27	0.148
177	A	4	4	1.	27	0.148
178	A	3	3	1.	24	0.125
179	A	4	4	1.	27	0.148
180	A	5	4	1.	27	0.148
181	A	6	5	1.	25	0.2
182	A	5	5	1.	25	0.2
183	A	4	4	1.	25	0.16
184	A	4	4	1.	23	0.174
185	A	6	5	1.	25	0.2
186	A	4	4	1.	25	0.16
187	A	5	5	1.	25	0.2
188	A	6	5	1.	25	0.2

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
189	A	5	4	1.	25	0.16
190	A	4	4	1.	25	0.16
191	A	3	3	1.	22	0.136
192	A	4	4	1.	25	0.16
193	A	5	4	1.	25	0.16
194	A	5	5	1.	25	0.2
195	A	4	4	1.	25	0.16
196	A	2	2	1.	23	0.087
197	A	5	5	1.	25	0.2
198	A	5	5	1.	25	0.2
199	A	5	5	1.	25	0.2
200	A	4	4	1.	25	0.16
201	A	4	4	1.	22	0.182
202	A	5	5	1.	25	0.2
203	A	6	5	1.	25	0.2
204	A	6	3	1.	31	0.097
205	A	6	3	1.	31	0.097
206	A	6	3	1.	31	0.097
207	A	6	3	1.	31	0.097
208	A	6	3	1.	31	0.097
209	A	6	3	1.	31	0.097
210	A	6	3	1.	31	0.097
211	A	6	3	1.	31	0.097
212	A	6	3	1.	31	0.097
213	A	6	3	1.	31	0.097
214	A	6	3	1.	31	0.097
215	A	6	3	1.	31	0.097
216	A	6	3	1.	31	0.097
217	A	6	3	1.	31	0.097
218	A	6	3	1.	31	0.097
219	A	6	3	1.	31	0.097
220	A	2	1	1.	27	0.037
221	A	2	1	1.	27	0.037
222	A	2	1	1.	25	0.04

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
223	A	3	2	1.	27	0.074
224	A	4	3	0.91	27	0.111
225	A	6	3	1.	29	0.103
226	A	6	3	1.	29	0.103
227	A	6	3	1.	29	0.103
228	A	6	3	1.	29	0.103
229	A	6	5	1.	22	0.227
230	A	6	5	1.	22	0.227
231	A	6	5	1.	22	0.227
232	A	6	5	1.	22	0.227
233	A	6	6	1.	20	0.3
234	A	6	5	1.	22	0.227
235	A	6	5	1.	22	0.227
236	A	6	5	1.	22	0.227
237	A	12	8	1.	22	0.364
238	A	12	8	1.	22	0.364
239	A	12	8	1.	22	0.364
240	A	12	8	1.	22	0.364
241	A	12	8	1.	19	0.421
242	A	12	8	1.	22	0.364
243	A	12	8	1.	22	0.364
244	A	7	6	1.	22	0.273
245	A	7	6	1.	22	0.273
246	A	7	6	1.	22	0.273
247	A	7	6	1.	22	0.273
248	A	7	6	1.	20	0.3
249	A	8	6	1.	22	0.273
250	A	8	6	1.	22	0.273
251	A	8	6	1.	22	0.273
252	A	24	11	1.	22	0.5
253	A	23	10	1.	22	0.454
254	A	23	10	1.	22	0.454
255	A	23	10	1.	22	0.454
256	A	22	9	1.	19	0.474

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
257	A	22	9	1.	22	0.409
258	A	22	9	1.	22	0.409
259	A	6	6	1.	37	0.162
260	A	4	3	1.	35	0.086
261	A	5	5	1.	34	0.147
262	A	6	6	1.	37	0.162
263	A	5	5	1.	37	0.135
264	A	6	6	1.	37	0.162
265	A	3	2	1.	25	0.08
266	A	2	1	1.	25	0.04
267	A	3	2	1.	23	0.087
268	A	2	1	1.	22	0.045
269	A	3	2	1.	25	0.08
270	A	2	1	1.	25	0.04
271	A	3	2	1.	25	0.08
272	A	6	5	1.	25	0.2
273	A	6	5	1.	25	0.2
274	A	5	5	1.	25	0.2
275	A	3	3	1.	22	0.136
276	A	3	3	1.	25	0.12
277	A	4	3	1.	25	0.12
278	A	4	3	1.	25	0.12
279	A	4	3	1.	25	0.12
280	A	6	5	1.	25	0.2
281	A	6	5	1.	25	0.2
282	A	5	5	1.	25	0.2
283	A	3	3	1.	22	0.136
284	A	4	4	1.	25	0.16
285	A	5	3	1.	25	0.12
286	A	5	3	1.	25	0.12
287	A	7	6	1.	27	0.222
288	A	7	6	1.	27	0.222
289	A	7	6	1.	27	0.222
290	A	7	6	1.	27	0.222

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
291	A	7	7	1.	25	0.28
292	A	7	6	1.	27	0.222
293	A	7	6	1.	27	0.222
294	A	7	6	1.	27	0.222
295	A	6	3	1.	27	0.111
296	A	6	3	1.	27	0.111
297	A	6	3	1.	27	0.111
298	A	6	3	1.	27	0.111
299	A	6	3	1.	24	0.125
300	A	6	3	1.	27	0.111
301	A	6	3	1.	27	0.111
302	A	19	12	1.	31	0.387
303	A	8	7	1.	29	0.241
304	A	7	6	1.	29	0.207
305	A	7	6	1.	27	0.222
306	A	9	6	1.	29	0.207
307	A	21	8	1.	29	0.276
308	A	16	8	1.49	29	0.276
309	A	12	7	1.45	29	0.241
310	A	6	5	1.27	26	0.192
311	A	7	6	1.24	29	0.207
312	A	6	5	1.	29	0.172
313	A	16	10	1.	29	0.345
314	A	9	7	1.	29	0.241
315	A	8	6	1.	29	0.207
316	A	8	7	1.	27	0.259
317	A	14	8	1.	29	0.276
318	A	24	9	1.	29	0.31
319	A	18	8	1.92	29	0.276
320	A	11	6	1.44	26	0.231
321	A	12	9	1.	29	0.31
322	A	12	8	1.	29	0.276
323	A	14	8	1.	29	0.276
324	A	7	6	1.	29	0.207

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
325	A	6	5	1.	29	0.172
326	A	3	3	1.	27	0.111
327	A	7	4	1.	29	0.138
328	A	10	5	1.	29	0.172
329	A	7	6	1.23	29	0.207
330	A	3	3	1.	29	0.103
331	A	2	2	1.	26	0.077
332	A	9	7	1.	29	0.241
333	A	14	9	1.	29	0.31
334	A	7	6	1.	29	0.207
335	A	5	5	1.	29	0.172
336	A	5	5	1.	29	0.172
337	A	5	5	1.	27	0.185
338	A	11	6	1.	29	0.207
339	A	15	7	1.	29	0.241
340	A	13	9	1.42	29	0.31
341	A	8	7	1.27	29	0.241
342	A	7	6	1.22	29	0.207
343	A	7	6	1.22	29	0.207
344	A	7	6	1.22	26	0.231
345	A	14	9	1.4	29	0.31
346	A	7	5	1.	29	0.172
347	A	7	5	1.	29	0.172
348	A	6	5	1.	29	0.172
349	A	5	4	1.	27	0.148
350	A	8	6	1.	29	0.207
351	A	10	7	0.97	29	0.241
352	A	13	7	1.	29	0.241
353	A	10	7	1.	29	0.241
354	A	9	6	1.	29	0.207
355	A	11	6	1.	26	0.231
356	A	8	5	1.	29	0.172
357	A	12	7	1.	29	0.241
358	A	15	7	1.	29	0.241

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
359	A	7	5	1.	29	0.172
360	A	6	5	1.	27	0.185
361	A	8	6	1.	29	0.207
362	A	10	7	1.	29	0.241
363	A	17	9	1.	29	0.31
364	A	16	8	1.	29	0.276
365	A	13	7	1.	26	0.269
366	A	16	8	1.66	29	0.276
367	A	19	10	1.	29	0.345
368	A	7	5	1.	29	0.172
369	A	6	5	1.	29	0.172
370	A	5	4	1.	27	0.148
371	A	8	6	1.	29	0.207
372	A	8	6	1.	29	0.207
373	A	9	6	1.	29	0.207
374	A	8	5	1.	29	0.172
375	A	9	5	1.	26	0.192
376	A	8	5	1.	29	0.172
377	A	8	6	1.	25	0.24
378	A	17	7	1.	29	0.241
379	A	13	7	1.	29	0.241
380	A	10	6	1.	29	0.207
381	A	6	3	1.	29	0.103
382	A	5	3	1.	26	0.115
383	A	9	5	1.	29	0.172
384	A	11	6	1.	29	0.207
385	A	14	6	1.	29	0.207
386	A	14	7	1.45	29	0.241
387	A	8	5	1.	29	0.172
388	A	8	5	1.	29	0.172
389	A	8	5	1.	26	0.192
390	A	12	8	1.36	29	0.276
391	A	15	8	1.54	29	0.276
392	A	6	3	1.	29	0.103

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
393	A	5	3	1.	27	0.111
394	A	5	3	1.	27	0.111
395	A	5	3	1.	27	0.111
396	A	5	3	1.	25	0.12
397	A	8	5	1.	27	0.185
398	A	9	6	1.	27	0.222
399	A	12	8	1.	27	0.296
400	A	10	6	1.	27	0.222
401	A	6	3	1.	27	0.111
402	A	5	3	1.	24	0.125
403	A	10	6	1.	27	0.222
404	A	12	6	1.	27	0.222
405	A	5	5	1.1	28	0.179

3 Listing of integrals

3.1 $\int x^3 (d + ex^2) (a + cx^4)^5 \, dx$

Optimal. Leaf size=149

$$\frac{1}{4}a^5dx^4 + \frac{1}{6}a^5ex^6 + \frac{5}{8}a^4cdx^8 + \frac{1}{2}a^4cex^{10} + \frac{5}{6}a^3c^2dx^{12} + \frac{5}{7}a^3c^2ex^{14} + \frac{5}{8}a^2c^3dx^{16} + \frac{5}{9}a^2c^3ex^{18} + \frac{1}{4}ac^4dx^{20} + \frac{5}{22}ac^4ex^{22} + \frac{1}{24}c^5dx^{24} + \frac{1}{26}c^5ex^{26}$$

```
[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (5*a^4*c*d*x^8)/8 + (a^4*c*e*x^10
)/2 + (5*a^3*c^2*d*x^12)/6 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*d*
x^16)/8 + (5*a^2*c^3*e*x^18)/9 + (a*c^4*d*x^20)/4 + (5*a*c^4*e*x^
22)/22 + (c^5*d*x^24)/24 + (c^5*e*x^26)/26
```

Rubi [A] time = 0.461615, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{1}{4}a^5dx^4 + \frac{1}{6}a^5ex^6 + \frac{5}{8}a^4cdx^8 + \frac{1}{2}a^4cex^{10} + \frac{5}{6}a^3c^2dx^{12} + \frac{5}{7}a^3c^2ex^{14} + \frac{5}{8}a^2c^3dx^{16} + \frac{5}{9}a^2c^3ex^{18} + \frac{1}{4}ac^4dx^{20} + \frac{5}{22}ac^4ex^{22} + \frac{1}{24}c^5dx^{24} + \frac{1}{26}c^5ex^{26}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (5*a^4*c*d*x^8)/8 + (a^4*c*e*x^10
)/2 + (5*a^3*c^2*d*x^12)/6 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*d*
x^16)/8 + (5*a^2*c^3*e*x^18)/9 + (a*c^4*d*x^20)/4 + (5*a*c^4*e*x^
22)/22 + (c^5*d*x^24)/24 + (c^5*e*x^26)/26
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^5d\int^{x^2} x\,dx}{2} + \frac{a^5ex^6}{6} + \frac{5a^4cdx^8}{8} + \frac{a^4cex^{10}}{2} + \frac{5a^3c^2dx^{12}}{6} + \frac{5a^3c^2ex^{14}}{7} + \frac{5a^2c^3dx^{16}}{8} + \frac{5a^2c^3ex^{18}}{9} + \frac{ac^4dx^{20}}{4} + \frac{5ac^4ex^{22}}{22} + \frac{c^5dx^{24}}{24} + \frac{c^5ex^{26}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*Integral(x, (x, x**2))/2 + a**5*e*x**6/6 + 5*a**4*c*d*x**8
/8 + a**4*c*e*x**10/2 + 5*a**3*c**2*d*x**12/6 + 5*a**3*c**2*e*x**
14/7 + 5*a**2*c**3*d*x**16/8 + 5*a**2*c**3*e*x**18/9 + a*c**4*d*x
**20/4 + 5*a*c**4*e*x**22/22 + c**5*d*x**24/24 + c**5*e*x**26/26
```

Mathematica [A] time = 0.00782934, size = 149, normalized size = 1.

$$\frac{1}{4}a^5dx^4+\frac{1}{6}a^5ex^6+\frac{5}{8}a^4cdx^8+\frac{1}{2}a^4cex^{10}+\frac{5}{6}a^3c^2dx^{12}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{8}a^2c^3dx^{16}+\frac{5}{9}a^2c^3ex^{18}+\frac{1}{4}ac^4dx^{20}+\frac{5}{22}ac^4ex^{22}+\frac{1}{24}c^5dx^{24}+\frac{1}{26}c^5ex^{26}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (5*a^4*c*d*x^8)/8 + (a^4*c*e*x^10
)/2 + (5*a^3*c^2*d*x^12)/6 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*d*
x^16)/8 + (5*a^2*c^3*e*x^18)/9 + (a*c^4*d*x^20)/4 + (5*a*c^4*e*x^
22)/22 + (c^5*d*x^24)/24 + (c^5*e*x^26)/26
```

Maple [A] time = 0.004, size = 126, normalized size = 0.9

$$\frac{a^5dx^4}{4}+\frac{a^5ex^6}{6}+\frac{5\,a^4cdx^8}{8}+\frac{a^4cex^{10}}{2}+\frac{5\,a^3c^2dx^{12}}{6}+\frac{5\,a^3c^2ex^{14}}{7}+\frac{5\,a^2c^3dx^{16}}{8}+\frac{5\,a^2c^3ex^{18}}{9}+\frac{ac^4dx^{20}}{4}+\frac{5\,ac^4ex^{22}}{22}+\frac{c^5dx^{24}}{24}+\frac{c^5ex^{26}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(e*x^2+d)*(c*x^4+a)^5,x)
```

```
[Out] 1/4*a^5*d*x^4+1/6*a^5*e*x^6+5/8*a^4*c*d*x^8+1/2*a^4*c*e*x^10+5/6*
a^3*c^2*d*x^12+5/7*a^3*c^2*e*x^14+5/8*a^2*c^3*d*x^16+5/9*a^2*c^3*
e*x^18+1/4*a*c^4*d*x^20+5/22*a*c^4*e*x^22+1/24*c^5*d*x^24+1/26*c^
5*e*x^26
```

Maxima [A] time = 0.702379, size = 169, normalized size = 1.13

$$\frac{1}{26}c^5ex^{26}+\frac{1}{24}c^5dx^{24}+\frac{5}{22}ac^4ex^{22}+\frac{1}{4}ac^4dx^{20}+\frac{5}{9}a^2c^3ex^{18}+\frac{5}{8}a^2c^3dx^{16}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{6}a^3c^2dx^{12}+\frac{1}{2}a^4cex^{10}+\frac{5}{8}a^4cdx^8+\frac{1}{6}a^5ex^6+\frac{1}{4}a^5dx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d)*x^3,x, algorithm="maxima")
```

```
[Out] 1/26*c^5*e*x^26 + 1/24*c^5*d*x^24 + 5/22*a*c^4*e*x^22 + 1/4*a*c^4
*d*x^20 + 5/9*a^2*c^3*e*x^18 + 5/8*a^2*c^3*d*x^16 + 5/7*a^3*c^2*e
*x^14 + 5/6*a^3*c^2*d*x^12 + 1/2*a^4*c*e*x^10 + 5/8*a^4*c*d*x^8 +
 1/6*a^5*e*x^6 + 1/4*a^5*d*x^4
```

Fricas [A] time = 0.229426, size = 1, normalized size = 0.01

$$\frac{1}{26}x^{26}ec^5+\frac{1}{24}x^{24}dc^5+\frac{5}{22}x^{22}ec^4a+\frac{1}{4}x^{20}dc^4a+\frac{5}{9}x^{18}ec^3a^2+\frac{5}{8}x^{16}dc^3a^2+\frac{5}{7}x^{14}ec^2a^3+\frac{5}{6}x^{12}dc^2a^3+\frac{1}{2}x^{10}eca^4+\frac{5}{8}x^8dca^4+\frac{1}{6}x^6ea^5+\frac{1}{4}x^4da^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d)*x^3,x, algorithm="fricas")
```

```
[Out] 1/26*x^26*e*c^5 + 1/24*x^24*d*c^5 + 5/22*x^22*e*c^4*a + 1/4*x^20*
d*c^4*a + 5/9*x^18*e*c^3*a^2 + 5/8*x^16*d*c^3*a^2 + 5/7*x^14*e*c^
2*a^3 + 5/6*x^12*d*c^2*a^3 + 1/2*x^10*e*c*a^4 + 5/8*x^8*d*c*a^4 +
 1/6*x^6*e*a^5 + 1/4*x^4*d*a^5
```

Sympy [A] time = 0.161928, size = 151, normalized size = 1.01

$$\frac{a^5dx^4}{4}+\frac{a^5ex^6}{6}+\frac{5a^4cdx^8}{8}+\frac{a^4cex^{10}}{2}+\frac{5a^3c^2dx^{12}}{6}+\frac{5a^3c^2ex^{14}}{7}+\frac{5a^2c^3dx^{16}}{8}+\frac{5a^2c^3ex^{18}}{9}+\frac{ac^4dx^{20}}{4}+\frac{5ac^4ex^{22}}{22}+\frac{c^5dx^{24}}{24}+\frac{c^5ex^{26}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x**4/4 + a**5*e*x**6/6 + 5*a**4*c*d*x**8/8 + a**4*c*e*x**1
0/2 + 5*a**3*c**2*d*x**12/6 + 5*a**3*c**2*e*x**14/7 + 5*a**2*c**3
*d*x**16/8 + 5*a**2*c**3*e*x**18/9 + a*c**4*d*x**20/4 + 5*a*c**4*
e*x**22/22 + c**5*d*x**24/24 + c**5*e*x**26/26
```

GIAC/XCAS [A] time = 0.260016, size = 177, normalized size = 1.19

$$\frac{1}{26}c^5x^{26}e + \frac{1}{24}c^5dx^{24} + \frac{5}{22}ac^4x^{22}e + \frac{1}{4}ac^4dx^{20} + \frac{5}{9}a^2c^3x^{18}e + \frac{5}{8}a^2c^3dx^{16} + \frac{5}{7}a^3c^2x^{14}e + \frac{5}{6}a^3c^2dx^{12} + \frac{1}{2}a^4cx^{10}e + \frac{5}{8}a^4cdx^8 + \frac{1}{6}a^5x^6e + \frac{1}{4}a^5dx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d)*x^3,x, algorithm="giac")
```

```
[Out] 1/26*c^5*x^26*e + 1/24*c^5*d*x^24 + 5/22*a*c^4*x^22*e + 1/4*a*c^4
*d*x^20 + 5/9*a^2*c^3*x^18*e + 5/8*a^2*c^3*d*x^16 + 5/7*a^3*c^2*x
^14*e + 5/6*a^3*c^2*d*x^12 + 1/2*a^4*c*x^10*e + 5/8*a^4*c*d*x^8 +
 1/6*a^5*x^6*e + 1/4*a^5*d*x^4
```

3.2 $\int x^2 \left(d+ex^2\right) \left(a+cx^4\right)^5 \, dx$

Optimal. Leaf size=149

$$\frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{13}a^3c^2ex^{13}+\frac{2}{3}a^2c^3dx^{15}+\frac{10}{17}a^2c^3ex^{17}+\frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4ex^{21}+\frac{1}{23}c^5dx^{23}+\frac{1}{25}c^5ex^{25}$$

```
[Out] (a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (5*a^4*c*d*x^7)/7 + (5*a^4*c*e*x^
9)/9 + (10*a^3*c^2*d*x^11)/11 + (10*a^3*c^2*e*x^13)/13 + (2*a^2*c
^3*d*x^15)/3 + (10*a^2*c^3*e*x^17)/17 + (5*a*c^4*d*x^19)/19 + (5*
a*c^4*e*x^21)/21 + (c^5*d*x^23)/23 + (c^5*e*x^25)/25
```

Rubi [A] time = 0.237103, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{13}a^3c^2ex^{13}+\frac{2}{3}a^2c^3dx^{15}+\frac{10}{17}a^2c^3ex^{17}+\frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4ex^{21}+\frac{1}{23}c^5dx^{23}+\frac{1}{25}c^5ex^{25}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (5*a^4*c*d*x^7)/7 + (5*a^4*c*e*x^
9)/9 + (10*a^3*c^2*d*x^11)/11 + (10*a^3*c^2*e*x^13)/13 + (2*a^2*c
^3*d*x^15)/3 + (10*a^2*c^3*e*x^17)/17 + (5*a*c^4*d*x^19)/19 + (5*
a*c^4*e*x^21)/21 + (c^5*d*x^23)/23 + (c^5*e*x^25)/25
```

Rubi in Sympy [A] time = 31.1564, size = 155, normalized size = 1.04

$$\frac{a^5dx^3}{3}+\frac{a^5ex^5}{5}+\frac{5a^4cdx^7}{7}+\frac{5a^4cex^9}{9}+\frac{10a^3c^2dx^{11}}{11}+\frac{10a^3c^2ex^{13}}{13}+\frac{2a^2c^3dx^{15}}{3}+\frac{10a^2c^3ex^{17}}{17}+\frac{5ac^4dx^{19}}{19}+\frac{5ac^4ex^{21}}{21}+\frac{c^5dx^{23}}{23}+\frac{c^5ex^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x**3/3 + a**5*e*x**5/5 + 5*a**4*c*d*x**7/7 + 5*a**4*c*e*x*
*9/9 + 10*a**3*c**2*d*x**11/11 + 10*a**3*c**2*e*x**13/13 + 2*a**2
*c**3*d*x**15/3 + 10*a**2*c**3*e*x**17/17 + 5*a*c**4*d*x**19/19 +
 5*a*c**4*e*x**21/21 + c**5*d*x**23/23 + c**5*e*x**25/25
```

Mathematica [A] time = 0.00615455, size = 149, normalized size = 1.

$$\frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{13}a^3c^2ex^{13}+\frac{2}{3}a^2c^3dx^{15}+\frac{10}{17}a^2c^3ex^{17}+\frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4ex^{21}+\frac{1}{23}c^5dx^{23}+\frac{1}{25}c^5ex^{25}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (5*a^4*c*d*x^7)/7 + (5*a^4*c*e*x^
9)/9 + (10*a^3*c^2*d*x^11)/11 + (10*a^3*c^2*e*x^13)/13 + (2*a^2*c
^3*d*x^15)/3 + (10*a^2*c^3*e*x^17)/17 + (5*a*c^4*d*x^19)/19 + (5*
a*c^4*e*x^21)/21 + (c^5*d*x^23)/23 + (c^5*e*x^25)/25
```

Maple [A] time = 0.002, size = 126, normalized size = 0.9

$$\frac{a^5dx^3}{3}+\frac{a^5ex^5}{5}+\frac{5\,a^4cdx^7}{7}+\frac{5\,a^4cex^9}{9}+\frac{10\,a^3c^2dx^{11}}{11}+\frac{10\,a^3c^2ex^{13}}{13}+\frac{2\,a^2c^3dx^{15}}{3}+\frac{10\,a^2c^3ex^{17}}{17}+\frac{5\,ac^4dx^{19}}{19}+\frac{5\,ac^4ex^{21}}{21}+\frac{c^5dx^{23}}{23}+\frac{c^5ex^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^2+d)*(c*x^4+a)^5,x)
```

```
[Out] 1/3*a^5*d*x^3+1/5*a^5*e*x^5+5/7*a^4*c*d*x^7+5/9*a^4*c*e*x^9+10/11
*a^3*c^2*d*x^11+10/13*a^3*c^2*e*x^13+2/3*a^2*c^3*d*x^15+10/17*a^2
*c^3*e*x^17+5/19*a*c^4*d*x^19+5/21*a*c^4*e*x^21+1/23*c^5*d*x^23+1
/25*c^5*e*x^25
```

Maxima [A] time = 0.70569, size = 169, normalized size = 1.13

$$\frac{1}{25}\,c^5ex^{25}+\frac{1}{23}\,c^5dx^{23}+\frac{5}{21}\,ac^4ex^{21}+\frac{5}{19}\,ac^4dx^{19}+\frac{10}{17}\,a^2c^3ex^{17}+\frac{2}{3}\,a^2c^3dx^{15}+\frac{10}{13}\,a^3c^2ex^{13}+\frac{10}{11}\,a^3c^2dx^{11}+\frac{5}{9}\,a^4cex^9+\frac{5}{7}\,a^4cdx^7+\frac{1}{5}\,a^5ex^5+\frac{1}{3}\,a^5dx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d)*x^2,x, algorithm="maxima")
```

```
[Out] 1/25*c^5*e*x^25 + 1/23*c^5*d*x^23 + 5/21*a*c^4*e*x^21 + 5/19*a*c^
4*d*x^19 + 10/17*a^2*c^3*e*x^17 + 2/3*a^2*c^3*d*x^15 + 10/13*a^3*
c^2*e*x^13 + 10/11*a^3*c^2*d*x^11 + 5/9*a^4*c*e*x^9 + 5/7*a^4*c*d
*x^7 + 1/5*a^5*e*x^5 + 1/3*a^5*d*x^3
```

Fricas [A] time = 0.239731, size = 1, normalized size = 0.01

$$\frac{1}{25}x^{25}ec^5 + \frac{1}{23}x^{23}dc^5 + \frac{5}{21}x^{21}ec^4a + \frac{5}{19}x^{19}dc^4a + \frac{10}{17}x^{17}ec^3a^2 + \frac{2}{3}x^{15}dc^3a^2 + \frac{10}{13}x^{13}ec^2a^3 + \frac{10}{11}x^{11}dc^2a^3 + \frac{5}{9}x^9eca^4 + \frac{5}{7}x^7dca^4 + \frac{1}{5}x^5ea^5 + \frac{1}{3}x^3da^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d)*x^2,x, algorithm="fricas")
```

```
[Out] 1/25*x^25*e*c^5 + 1/23*x^23*d*c^5 + 5/21*x^21*e*c^4*a + 5/19*x^19
*d*c^4*a + 10/17*x^17*e*c^3*a^2 + 2/3*x^15*d*c^3*a^2 + 10/13*x^13
*e*c^2*a^3 + 10/11*x^11*d*c^2*a^3 + 5/9*x^9*e*c*a^4 + 5/7*x^7*d*c
*a^4 + 1/5*x^5*e*a^5 + 1/3*x^3*d*a^5
```

Sympy [A] time = 0.162444, size = 155, normalized size = 1.04

$$\frac{a^5dx^3}{3} + \frac{a^5ex^5}{5} + \frac{5a^4cdx^7}{7} + \frac{5a^4cex^9}{9} + \frac{10a^3c^2dx^{11}}{11} + \frac{10a^3c^2ex^{13}}{13} + \frac{2a^2c^3dx^{15}}{3} + \frac{10a^2c^3ex^{17}}{17} + \frac{5ac^4dx^{19}}{19} + \frac{5ac^4ex^{21}}{21} + \frac{c^5dx^{23}}{23} + \frac{c^5ex^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x**3/3 + a**5*e*x**5/5 + 5*a**4*c*d*x**7/7 + 5*a**4*c*e*x*
*9/9 + 10*a**3*c**2*d*x**11/11 + 10*a**3*c**2*e*x**13/13 + 2*a**2
*c**3*d*x**15/3 + 10*a**2*c**3*e*x**17/17 + 5*a*c**4*d*x**19/19 +
 5*a*c**4*e*x**21/21 + c**5*d*x**23/23 + c**5*e*x**25/25
```

GIAC/XCAS [A] time = 0.260036, size = 177, normalized size = 1.19

$$\frac{1}{25}\,c^{5}x^{25}e + \frac{1}{23}\,c^{5}dx^{23} + \frac{5}{21}\,ac^{4}x^{21}e + \frac{5}{19}\,ac^{4}dx^{19} + \frac{10}{17}\,a^{2}c^{3}x^{17}e + \frac{2}{3}\,a^{2}c^{3}dx^{15} + \frac{10}{13}\,a^{3}c^{2}x^{13}e + \frac{10}{11}\,a^{3}c^{2}dx^{11} + \frac{5}{9}\,a^{4}cx^{9}e + \frac{5}{7}\,a^{4}cdx^{7} + \frac{1}{5}\,a^{5}x^{5}e + \frac{1}{3}\,a^{5}dx^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d)*x^2,x, algorithm="giac")
```

```
[Out] 1/25*c^5*x^25*e + 1/23*c^5*d*x^23 + 5/21*a*c^4*x^21*e + 5/19*a*c^
4*d*x^19 + 10/17*a^2*c^3*x^17*e + 2/3*a^2*c^3*d*x^15 + 10/13*a^3*
c^2*x^13*e + 10/11*a^3*c^2*d*x^11 + 5/9*a^4*c*x^9*e + 5/7*a^4*c*d
*x^7 + 1/5*a^5*x^5*e + 1/3*a^5*d*x^3
```

3.3 $\int x\left(d+ex^2\right)\left(a+cx^4\right)^5\,dx$

Optimal. Leaf size=89

$$\frac{1}{2}a^5dx^2+\frac{5}{6}a^4cdx^6+a^3c^2dx^{10}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{18}ac^4dx^{18}+\frac{e\left(a+cx^4\right)^6}{24c}+\frac{1}{22}c^5dx^{22}$$

```
[Out] (a^5*d*x^2)/2 + (5*a^4*c*d*x^6)/6 + a^3*c^2*d*x^10 + (5*a^2*c^3*d
*x^14)/7 + (5*a*c^4*d*x^18)/18 + (c^5*d*x^22)/22 + (e*(a + c*x^4)
^6)/(24*c)
```

Rubi [A] time = 0.190976, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{1}{2}a^5dx^2+\frac{5}{6}a^4cdx^6+a^3c^2dx^{10}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{18}ac^4dx^{18}+\frac{e\left(a+cx^4\right)^6}{24c}+\frac{1}{22}c^5dx^{22}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^2)/2 + (5*a^4*c*d*x^6)/6 + a^3*c^2*d*x^10 + (5*a^2*c^3*d
*x^14)/7 + (5*a*c^4*d*x^18)/18 + (c^5*d*x^22)/22 + (e*(a + c*x^4)
^6)/(24*c)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{5a^4cdx^6}{6}+a^3c^2dx^{10}+\frac{5a^2c^3dx^{14}}{7}+\frac{5ac^4dx^{18}}{18}+\frac{c^5dx^{22}}{22}+\frac{d\int^{x^2}a^5\,dx}{2}+\frac{e\left(a+cx^4\right)^6}{24c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] 5*a**4*c*d*x**6/6 + a**3*c**2*d*x**10 + 5*a**2*c**3*d*x**14/7 + 5
*a*c**4*d*x**18/18 + c**5*d*x**22/22 + d*Integral(a**5, (x, x**2)
)/2 + e*(a + c*x**4)**6/(24*c)
```

Mathematica [A] time = 0.00580161, size = 146, normalized size = 1.64

$$\frac{1}{2}a^5dx^2+\frac{1}{4}a^5ex^4+\frac{5}{6}a^4cdx^6+\frac{5}{8}a^4cex^8+a^3c^2dx^{10}+\frac{5}{6}a^3c^2ex^{12}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{8}a^2c^3ex^{16}+\frac{5}{18}ac^4dx^{18}+\frac{1}{4}ac^4ex^{20}+\frac{1}{22}c^5dx^{22}+\frac{1}{24}c^5ex^{24}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^2)/2 + (a^5*e*x^4)/4 + (5*a^4*c*d*x^6)/6 + (5*a^4*c*e*x^
8)/8 + a^3*c^2*d*x^10 + (5*a^3*c^2*e*x^12)/6 + (5*a^2*c^3*d*x^14)
/7 + (5*a^2*c^3*e*x^16)/8 + (5*a*c^4*d*x^18)/18 + (a*c^4*e*x^20)/
4 + (c^5*d*x^22)/22 + (c^5*e*x^24)/24
```

Maple [A] time = 0.002, size = 125, normalized size = 1.4

$$\frac{c^5ex^{24}}{24}+\frac{c^5dx^{22}}{22}+\frac{ac^4ex^{20}}{4}+\frac{5\,ac^4dx^{18}}{18}+\frac{5\,a^2c^3ex^{16}}{8}+\frac{5\,a^2c^3dx^{14}}{7}+\frac{5\,a^3c^2ex^{12}}{6}+a^3c^2dx^{10}+\frac{5\,a^4cex^8}{8}+\frac{5\,a^4cdx^6}{6}+\frac{a^5ex^4}{4}+\frac{a^5dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d)*(c*x^4+a)^5,x)
```

```
[Out] 1/24*c^5*e*x^24+1/22*c^5*d*x^22+1/4*a*c^4*e*x^20+5/18*a*c^4*d*x^1
8+5/8*a^2*c^3*e*x^16+5/7*a^2*c^3*d*x^14+5/6*a^3*c^2*e*x^12+a^3*c^
2*d*x^10+5/8*a^4*c*e*x^8+5/6*a^4*c*d*x^6+1/4*a^5*e*x^4+1/2*a^5*d*
x^2
```

Maxima [A] time = 0.695389, size = 167, normalized size = 1.88

$$\frac{1}{24}\,c^5ex^{24}+\frac{1}{22}\,c^5dx^{22}+\frac{1}{4}\,ac^4ex^{20}+\frac{5}{18}\,ac^4dx^{18}+\frac{5}{8}\,a^2c^3ex^{16}+\frac{5}{7}\,a^2c^3dx^{14}+\frac{5}{6}\,a^3c^2ex^{12}+a^3c^2dx^{10}+\frac{5}{8}\,a^4cex^8+\frac{5}{6}\,a^4cdx^6+\frac{1}{4}\,a^5ex^4+\frac{1}{2}\,a^5dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d)*x,x, algorithm="maxima")
```

```
[Out] 1/24*c^5*e*x^24 + 1/22*c^5*d*x^22 + 1/4*a*c^4*e*x^20 + 5/18*a*c^4
*d*x^18 + 5/8*a^2*c^3*e*x^16 + 5/7*a^2*c^3*d*x^14 + 5/6*a^3*c^2*e
*x^12 + a^3*c^2*d*x^10 + 5/8*a^4*c*e*x^8 + 5/6*a^4*c*d*x^6 + 1/4*
a^5*e*x^4 + 1/2*a^5*d*x^2
```

Fricas [A] time = 0.230683, size = 1, normalized size = 0.01

$$\frac{1}{24}x^{24}ec^5 + \frac{1}{22}x^{22}dc^5 + \frac{1}{4}x^{20}ec^4a + \frac{5}{18}x^{18}dc^4a + \frac{5}{8}x^{16}ec^3a^2 + \frac{5}{7}x^{14}dc^3a^2 + \frac{5}{6}x^{12}ec^2a^3 + x^{10}dc^2a^3 + \frac{5}{8}x^8eca^4 + \frac{5}{6}x^6dca^4 + \frac{1}{4}x^4ea^5 + \frac{1}{2}x^2da^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d)*x,x, algorithm="fricas")
```

```
[Out] 1/24*x^24*e*c^5 + 1/22*x^22*d*c^5 + 1/4*x^20*e*c^4*a + 5/18*x^18*
d*c^4*a + 5/8*x^16*e*c^3*a^2 + 5/7*x^14*d*c^3*a^2 + 5/6*x^12*e*c^
2*a^3 + x^10*d*c^2*a^3 + 5/8*x^8*e*c*a^4 + 5/6*x^6*d*c*a^4 + 1/4*
x^4*e*a^5 + 1/2*x^2*d*a^5
```

Sympy [A] time = 0.157902, size = 150, normalized size = 1.69

$$\frac{a^5dx^2}{2} + \frac{a^5ex^4}{4} + \frac{5a^4cdx^6}{6} + \frac{5a^4cex^8}{8} + a^3c^2dx^{10} + \frac{5a^3c^2ex^{12}}{6} + \frac{5a^2c^3dx^{14}}{7} + \frac{5a^2c^3ex^{16}}{8} + \frac{5ac^4dx^{18}}{18} + \frac{ac^4ex^{20}}{4} + \frac{c^5dx^{22}}{22} + \frac{c^5ex^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x**2/2 + a**5*e*x**4/4 + 5*a**4*c*d*x**6/6 + 5*a**4*c*e*x*
*8/8 + a**3*c**2*d*x**10 + 5*a**3*c**2*e*x**12/6 + 5*a**2*c**3*d*
x**14/7 + 5*a**2*c**3*e*x**16/8 + 5*a*c**4*d*x**18/18 + a*c**4*e*
x**20/4 + c**5*d*x**22/22 + c**5*e*x**24/24
```

GIAC/XCAS [A] time = 0.260696, size = 176, normalized size = 1.98

$$\frac{1}{24}c^5x^{24}e + \frac{1}{22}c^5dx^{22} + \frac{1}{4}ac^4x^{20}e + \frac{5}{18}ac^4dx^{18} + \frac{5}{8}a^2c^3x^{16}e + \frac{5}{7}a^2c^3dx^{14} + \frac{5}{6}a^3c^2x^{12}e + a^3c^2dx^{10} + \frac{5}{8}a^4cx^8e + \frac{5}{6}a^4cdx^6 + \frac{1}{4}a^5x^4e + \frac{1}{2}a^5dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d)*x,x, algorithm="giac")
```

```
[Out] 1/24*c^5*x^24*e + 1/22*c^5*d*x^22 + 1/4*a*c^4*x^20*e + 5/18*a*c^4
*d*x^18 + 5/8*a^2*c^3*x^16*e + 5/7*a^2*c^3*d*x^14 + 5/6*a^3*c^2*x
^12*e + a^3*c^2*d*x^10 + 5/8*a^4*c*x^8*e + 5/6*a^4*c*d*x^6 + 1/4*
a^5*x^4*e + 1/2*a^5*d*x^2
```

3.4 $\int (d+ex^2) (a+cx^4)^5 \, dx$

Optimal. Leaf size=141

$$a^5dx+\frac{1}{3}a^5ex^3+a^4cdx^5+\frac{5}{7}a^4cex^7+\frac{10}{9}a^3c^2dx^9+\frac{10}{11}a^3c^2ex^{11}+\frac{10}{13}a^2c^3dx^{13}+\frac{2}{3}a^2c^3ex^{15}+\frac{5}{17}ac^4dx^{17}+\frac{5}{19}ac^4ex^{19}+\frac{1}{21}c^5dx^{21}+\frac{1}{23}c^5ex^{23}$$

```
[Out] a^5*d*x + (a^5*e*x^3)/3 + a^4*c*d*x^5 + (5*a^4*c*e*x^7)/7 + (10*a
^3*c^2*d*x^9)/9 + (10*a^3*c^2*e*x^11)/11 + (10*a^2*c^3*d*x^13)/13
 + (2*a^2*c^3*e*x^15)/3 + (5*a*c^4*d*x^17)/17 + (5*a*c^4*e*x^19)/
19 + (c^5*d*x^21)/21 + (c^5*e*x^23)/23
```

Rubi [A] time = 0.168695, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$a^5dx+\frac{1}{3}a^5ex^3+a^4cdx^5+\frac{5}{7}a^4cex^7+\frac{10}{9}a^3c^2dx^9+\frac{10}{11}a^3c^2ex^{11}+\frac{10}{13}a^2c^3dx^{13}+\frac{2}{3}a^2c^3ex^{15}+\frac{5}{17}ac^4dx^{17}+\frac{5}{19}ac^4ex^{19}+\frac{1}{21}c^5dx^{21}+\frac{1}{23}c^5ex^{23}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] a^5*d*x + (a^5*e*x^3)/3 + a^4*c*d*x^5 + (5*a^4*c*e*x^7)/7 + (10*a
^3*c^2*d*x^9)/9 + (10*a^3*c^2*e*x^11)/11 + (10*a^2*c^3*d*x^13)/13
 + (2*a^2*c^3*e*x^15)/3 + (5*a*c^4*d*x^17)/17 + (5*a*c^4*e*x^19)/
19 + (c^5*d*x^21)/21 + (c^5*e*x^23)/23
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^5ex^3}{3}+a^5\int d\,dx+a^4cdx^5+\frac{5a^4cex^7}{7}+\frac{10a^3c^2dx^9}{9}+\frac{10a^3c^2ex^{11}}{11}+\frac{10a^2c^3dx^{13}}{13}+\frac{2a^2c^3ex^{15}}{3}+\frac{5ac^4dx^{17}}{17}+\frac{5ac^4ex^{19}}{19}+\frac{c^5dx^{21}}{21}+\frac{c^5ex^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*e*x**3/3 + a**5*Integral(d, x) + a**4*c*d*x**5 + 5*a**4*c*e*
x**7/7 + 10*a**3*c**2*d*x**9/9 + 10*a**3*c**2*e*x**11/11 + 10*a**
2*c**3*d*x**13/13 + 2*a**2*c**3*e*x**15/3 + 5*a*c**4*d*x**17/17 +
 5*a*c**4*e*x**19/19 + c**5*d*x**21/21 + c**5*e*x**23/23
```

Mathematica [A] time = 0.00553059, size = 141, normalized size = 1.

$$a^5dx + \frac{1}{3}a^5ex^3 + a^4cdx^5 + \frac{5}{7}a^4cex^7 + \frac{10}{9}a^3c^2dx^9 + \frac{10}{11}a^3c^2ex^{11} + \frac{10}{13}a^2c^3dx^{13} + \frac{2}{3}a^2c^3ex^{15} + \frac{5}{17}ac^4dx^{17} + \frac{5}{19}ac^4ex^{19} + \frac{1}{21}c^5dx^{21} + \frac{1}{23}c^5ex^{23}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] a^5*d*x + (a^5*e*x^3)/3 + a^4*c*d*x^5 + (5*a^4*c*e*x^7)/7 + (10*a
^3*c^2*d*x^9)/9 + (10*a^3*c^2*e*x^11)/11 + (10*a^2*c^3*d*x^13)/13
 + (2*a^2*c^3*e*x^15)/3 + (5*a*c^4*d*x^17)/17 + (5*a*c^4*e*x^19)/
19 + (c^5*d*x^21)/21 + (c^5*e*x^23)/23
```

Maple [A] time = 0.002, size = 122, normalized size = 0.9

$$a^5dx + \frac{a^5ex^3}{3} + a^4cdx^5 + \frac{5\,a^4cex^7}{7} + \frac{10\,a^3c^2dx^9}{9} + \frac{10\,a^3c^2ex^{11}}{11} + \frac{10\,a^2c^3dx^{13}}{13} + \frac{2\,a^2c^3ex^{15}}{3} + \frac{5\,ac^4dx^{17}}{17} + \frac{5\,ac^4ex^{19}}{19} + \frac{c^5dx^{21}}{21} + \frac{c^5ex^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)*(c*x^4+a)^5,x)
```

```
[Out] a^5*d*x+1/3*a^5*e*x^3+a^4*c*d*x^5+5/7*a^4*c*e*x^7+10/9*a^3*c^2*d*
x^9+10/11*a^3*c^2*e*x^11+10/13*a^2*c^3*d*x^13+2/3*a^2*c^3*e*x^15+
5/17*a*c^4*d*x^17+5/19*a*c^4*e*x^19+1/21*c^5*d*x^21+1/23*c^5*e*x^
23
```

Maxima [A] time = 0.702475, size = 163, normalized size = 1.16

$$\frac{1}{23}\,c^5ex^{23} + \frac{1}{21}\,c^5dx^{21} + \frac{5}{19}\,ac^4ex^{19} + \frac{5}{17}\,ac^4dx^{17} + \frac{2}{3}\,a^2c^3ex^{15} + \frac{10}{13}\,a^2c^3dx^{13} + \frac{10}{11}\,a^3c^2ex^{11} + \frac{10}{9}\,a^3c^2dx^9 + \frac{5}{7}\,a^4cex^7 + a^4cdx^5 + \frac{1}{3}\,a^5ex^3 + a^5dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d),x, algorithm="maxima")
```

```
[Out] 1/23*c^5*e*x^23 + 1/21*c^5*d*x^21 + 5/19*a*c^4*e*x^19 + 5/17*a*c^
4*d*x^17 + 2/3*a^2*c^3*e*x^15 + 10/13*a^2*c^3*d*x^13 + 10/11*a^3*
c^2*e*x^11 + 10/9*a^3*c^2*d*x^9 + 5/7*a^4*c*e*x^7 + a^4*c*d*x^5 +
 1/3*a^5*e*x^3 + a^5*d*x
```

Fricas [A] time = 0.258749, size = 1, normalized size = 0.01

$$\frac{1}{23}x^{23}ec^5 + \frac{1}{21}x^{21}dc^5 + \frac{5}{19}x^{19}ec^4a + \frac{5}{17}x^{17}dc^4a + \frac{2}{3}x^{15}ec^3a^2 + \frac{10}{13}x^{13}dc^3a^2 + \frac{10}{11}x^{11}ec^2a^3 + \frac{10}{9}x^9dc^2a^3 + \frac{5}{7}x^7eca^4 + x^5dca^4 + \frac{1}{3}x^3ea^5 + xda^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d),x, algorithm="fricas")
```

```
[Out] 1/23*x^23*e*c^5 + 1/21*x^21*d*c^5 + 5/19*x^19*e*c^4*a + 5/17*x^17
*d*c^4*a + 2/3*x^15*e*c^3*a^2 + 10/13*x^13*d*c^3*a^2 + 10/11*x^11
*e*c^2*a^3 + 10/9*x^9*d*c^2*a^3 + 5/7*x^7*e*c*a^4 + x^5*d*c*a^4 +
 1/3*x^3*e*a^5 + x*d*a^5
```

Sympy [A] time = 0.158536, size = 148, normalized size = 1.05

$$a^5dx + \frac{a^5ex^3}{3} + a^4cdx^5 + \frac{5a^4cex^7}{7} + \frac{10a^3c^2dx^9}{9} + \frac{10a^3c^2ex^{11}}{11} + \frac{10a^2c^3dx^{13}}{13} + \frac{2a^2c^3ex^{15}}{3} + \frac{5ac^4dx^{17}}{17} + \frac{5ac^4ex^{19}}{19} + \frac{c^5dx^{21}}{21} + \frac{c^5ex^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x + a**5*e*x**3/3 + a**4*c*d*x**5 + 5*a**4*c*e*x**7/7 + 10
*a**3*c**2*d*x**9/9 + 10*a**3*c**2*e*x**11/11 + 10*a**2*c**3*d*x*
*13/13 + 2*a**2*c**3*e*x**15/3 + 5*a*c**4*d*x**17/17 + 5*a*c**4*e
*x**19/19 + c**5*d*x**21/21 + c**5*e*x**23/23
```

GIAC/XCAS [A] time = 0.262686, size = 171, normalized size = 1.21

$$\frac{1}{23}\,c^{5}x^{23}e+\frac{1}{21}\,c^{5}dx^{21}+\frac{5}{19}\,ac^{4}x^{19}e+\frac{5}{17}\,ac^{4}dx^{17}+\frac{2}{3}\,a^{2}c^{3}x^{15}e+\frac{10}{13}\,a^{2}c^{3}dx^{13}+\frac{10}{11}\,a^{3}c^{2}x^{11}e+\frac{10}{9}\,a^{3}c^{2}dx^{9}+\frac{5}{7}\,a^{4}cx^{7}e+a^{4}cdx^{5}+\frac{1}{3}\,a^{5}x^{3}e+a^{5}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d),x, algorithm="giac")
```

```
[Out] 1/23*c^5*x^23*e + 1/21*c^5*d*x^21 + 5/19*a*c^4*x^19*e + 5/17*a*c^
4*d*x^17 + 2/3*a^2*c^3*x^15*e + 10/13*a^2*c^3*d*x^13 + 10/11*a^3*
c^2*x^11*e + 10/9*a^3*c^2*d*x^9 + 5/7*a^4*c*x^7*e + a^4*c*d*x^5 +
 1/3*a^5*x^3*e + a^5*d*x
```

3.5 $\int \frac{\left(d+ex^2\right)\left(a+cx^4\right)^5}{x}\,dx$

Optimal. Leaf size=142

$$a^5d\log(x)+\frac{1}{2}a^5ex^2+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+\frac{5}{4}a^3c^2dx^8+a^3c^2ex^{10}+\frac{5}{6}a^2c^3dx^{12}+\frac{5}{7}a^2c^3ex^{14}+\frac{5}{16}ac^4dx^{16}+\frac{5}{18}ac^4ex^{18}+\frac{1}{20}c^5dx^{20}+\frac{1}{22}c^5ex^{22}$$

```
[Out] (a^5*e*x^2)/2 + (5*a^4*c*d*x^4)/4 + (5*a^4*c*e*x^6)/6 + (5*a^3*c^
2*d*x^8)/4 + a^3*c^2*e*x^10 + (5*a^2*c^3*d*x^12)/6 + (5*a^2*c^3*e
*x^14)/7 + (5*a*c^4*d*x^16)/16 + (5*a*c^4*e*x^18)/18 + (c^5*d*x^2
0)/20 + (c^5*e*x^22)/22 + a^5*d*Log[x]
```

Rubi [A] time = 0.217483, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$a^5d\log(x)+\frac{1}{2}a^5ex^2+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+\frac{5}{4}a^3c^2dx^8+a^3c^2ex^{10}+\frac{5}{6}a^2c^3dx^{12}+\frac{5}{7}a^2c^3ex^{14}+\frac{5}{16}ac^4dx^{16}+\frac{5}{18}ac^4ex^{18}+\frac{1}{20}c^5dx^{20}+\frac{1}{22}c^5ex^{22}$$

Antiderivative was successfully verified.

```
[In]  Int[((d + e*x^2)*(a + c*x^4)^5)/x,x]
```

```
[Out] (a^5*e*x^2)/2 + (5*a^4*c*d*x^4)/4 + (5*a^4*c*e*x^6)/6 + (5*a^3*c^
2*d*x^8)/4 + a^3*c^2*e*x^10 + (5*a^2*c^3*d*x^12)/6 + (5*a^2*c^3*e
*x^14)/7 + (5*a*c^4*d*x^16)/16 + (5*a*c^4*e*x^18)/18 + (c^5*d*x^2
0)/20 + (c^5*e*x^22)/22 + a^5*d*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^5d\log\left(x^2\right)}{2}+\frac{a^5\int^{x^2}e\,dx}{2}+\frac{5a^4cd\int^{x^2}x\,dx}{2}+\frac{5a^4cex^6}{6}+\frac{5a^3c^2dx^8}{4}+a^3c^2ex^{10}+\frac{5a^2c^3dx^{12}}{6}+\frac{5a^2c^3ex^{14}}{7}+\frac{5ac^4dx^{16}}{16}+\frac{5ac^4ex^{18}}{18}+\frac{c^5dx^{20}}{20}+\frac{c^5ex^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)*(c*x**4+a)**5/x,x)
```

```
[Out] a**5*d*log(x**2)/2 + a**5*Integral(e, (x, x**2))/2 + 5*a**4*c*d*I
ntegral(x, (x, x**2))/2 + 5*a**4*c*e*x**6/6 + 5*a**3*c**2*d*x**8/
4 + a**3*c**2*e*x**10 + 5*a**2*c**3*d*x**12/6 + 5*a**2*c**3*e*x**
14/7 + 5*a*c**4*d*x**16/16 + 5*a*c**4*e*x**18/18 + c**5*d*x**20/2
0 + c**5*e*x**22/22
```

Mathematica [A] time = 0.0135551, size = 142, normalized size = 1.

$$a^5 d\log(x)+\frac{1}{2}a^5ex^2+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+\frac{5}{4}a^3c^2dx^8+a^3c^2ex^{10}+\frac{5}{6}a^2c^3dx^{12}+\frac{5}{7}a^2c^3ex^{14}+\frac{5}{16}ac^4dx^{16}+\frac{5}{18}ac^4ex^{18}+\frac{1}{20}c^5dx^{20}+\frac{1}{22}c^5ex^{22}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((d + e*x^2)*(a + c*x^4)^5)/x,x]
```

```
[Out] (a^5*e*x^2)/2 + (5*a^4*c*d*x^4)/4 + (5*a^4*c*e*x^6)/6 + (5*a^3*c^
2*d*x^8)/4 + a^3*c^2*e*x^10 + (5*a^2*c^3*d*x^12)/6 + (5*a^2*c^3*e
*x^14)/7 + (5*a*c^4*d*x^16)/16 + (5*a*c^4*e*x^18)/18 + (c^5*d*x^2
0)/20 + (c^5*e*x^22)/22 + a^5*d*Log[x]
```

Maple [A] time = 0.005, size = 123, normalized size = 0.9

$$\frac{a^5ex^2}{2}+\frac{5\,a^4cdx^4}{4}+\frac{5\,a^4cex^6}{6}+\frac{5\,a^3c^2dx^8}{4}+a^3c^2ex^{10}+\frac{5\,a^2c^3dx^{12}}{6}+\frac{5\,a^2c^3ex^{14}}{7}+\frac{5\,ac^4dx^{16}}{16}+\frac{5\,ac^4ex^{18}}{18}+\frac{c^5dx^{20}}{20}+\frac{c^5ex^{22}}{22}+a^5d\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)*(c*x^4+a)^5/x,x)
```

```
[Out] 1/2*a^5*e*x^2+5/4*a^4*c*d*x^4+5/6*a^4*c*e*x^6+5/4*a^3*c^2*d*x^8+a
^3*c^2*e*x^10+5/6*a^2*c^3*d*x^12+5/7*a^2*c^3*e*x^14+5/16*a*c^4*d*
x^16+5/18*a*c^4*e*x^18+1/20*c^5*d*x^20+1/22*c^5*e*x^22+a^5*d*ln(x
)
```

Maxima [A] time = 0.710135, size = 169, normalized size = 1.19

$$\frac{1}{22}\,c^5ex^{22}+\frac{1}{20}\,c^5dx^{20}+\frac{5}{18}\,ac^4ex^{18}+\frac{5}{16}\,ac^4dx^{16}+\frac{5}{7}\,a^2c^3ex^{14}+\frac{5}{6}\,a^2c^3dx^{12}+a^3c^2ex^{10}+\frac{5}{4}\,a^3c^2dx^8+\frac{5}{6}\,a^4cex^6+\frac{5}{4}\,a^4cdx^4+\frac{1}{2}\,a^5ex^2+\frac{1}{2}\,a^5d\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d)/x,x, algorithm="maxima")
```

```
[Out] 1/22*c^5*e*x^22 + 1/20*c^5*d*x^20 + 5/18*a*c^4*e*x^18 + 5/16*a*c^
4*d*x^16 + 5/7*a^2*c^3*e*x^14 + 5/6*a^2*c^3*d*x^12 + a^3*c^2*e*x^
10 + 5/4*a^3*c^2*d*x^8 + 5/6*a^4*c*e*x^6 + 5/4*a^4*c*d*x^4 + 1/2*
a^5*e*x^2 + 1/2*a^5*d*log(x^2)
```

Fricas [A] time = 0.269625, size = 165, normalized size = 1.16

$$\frac{1}{22}\,c^5ex^{22}+\frac{1}{20}\,c^5dx^{20}+\frac{5}{18}\,ac^4ex^{18}+\frac{5}{16}\,ac^4dx^{16}+\frac{5}{7}\,a^2c^3ex^{14}+\frac{5}{6}\,a^2c^3dx^{12}+a^3c^2ex^{10}+\frac{5}{4}\,a^3c^2dx^8+\frac{5}{6}\,a^4cex^6+\frac{5}{4}\,a^4cdx^4+\frac{1}{2}\,a^5ex^2+a^5d\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d)/x,x, algorithm="fricas")
```

```
[Out] 1/22*c^5*e*x^22 + 1/20*c^5*d*x^20 + 5/18*a*c^4*e*x^18 + 5/16*a*c^
4*d*x^16 + 5/7*a^2*c^3*e*x^14 + 5/6*a^2*c^3*d*x^12 + a^3*c^2*e*x^
10 + 5/4*a^3*c^2*d*x^8 + 5/6*a^4*c*e*x^6 + 5/4*a^4*c*d*x^4 + 1/2*
a^5*e*x^2 + a^5*d*log(x)
```

Sympy [A] time = 1.51428, size = 150, normalized size = 1.06

$$a^5d\log(x)+\frac{a^5ex^2}{2}+\frac{5a^4cdx^4}{4}+\frac{5a^4cex^6}{6}+\frac{5a^3c^2dx^8}{4}+a^3c^2ex^{10}+\frac{5a^2c^3dx^{12}}{6}+\frac{5a^2c^3ex^{14}}{7}+\frac{5ac^4dx^{16}}{16}+\frac{5ac^4ex^{18}}{18}+\frac{c^5dx^{20}}{20}+\frac{c^5ex^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)*(c*x**4+a)**5/x,x)
```

```
[Out] a**5*d*log(x) + a**5*e*x**2/2 + 5*a**4*c*d*x**4/4 + 5*a**4*c*e*x*
*6/6 + 5*a**3*c**2*d*x**8/4 + a**3*c**2*e*x**10 + 5*a**2*c**3*d*x
**12/6 + 5*a**2*c**3*e*x**14/7 + 5*a*c**4*d*x**16/16 + 5*a*c**4*e
*x**18/18 + c**5*d*x**20/20 + c**5*e*x**22/22
```

<u>GIAC/XCAS</u> [A] time = 0.264024, size = 177, normalized size = 1.25

$$\frac{1}{22}\,c^{5}x^{22}e+\frac{1}{20}\,c^{5}dx^{20}+\frac{5}{18}\,ac^{4}x^{18}e+\frac{5}{16}\,ac^{4}dx^{16}+\frac{5}{7}\,a^{2}c^{3}x^{14}e+\frac{5}{6}\,a^{2}c^{3}dx^{12}+a^{3}c^{2}x^{10}e+\frac{5}{4}\,a^{3}c^{2}dx^{8}+\frac{5}{6}\,a^{4}cx^{6}e+\frac{5}{4}\,a^{4}cdx^{4}+\frac{1}{2}\,a^{5}x^{2}e+\frac{1}{2}\,a^{5}d\ln\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d)/x,x, algorithm="giac")
```

```
[Out] 1/22*c^5*x^22*e + 1/20*c^5*d*x^20 + 5/18*a*c^4*x^18*e + 5/16*a*c^
4*d*x^16 + 5/7*a^2*c^3*x^14*e + 5/6*a^2*c^3*d*x^12 + a^3*c^2*x^10
*e + 5/4*a^3*c^2*d*x^8 + 5/6*a^4*c*x^6*e + 5/4*a^4*c*d*x^4 + 1/2*
a^5*x^2*e + 1/2*a^5*d*ln(x^2)
```

3.6 $\int \frac{(d+ex^2)(a+cx^4)^5}{x^2}\,dx$

Optimal. Leaf size=139

$$-\frac{a^5d}{x}+a^5ex+\frac{5}{3}a^4cdx^3+a^4cex^5+\frac{10}{7}a^3c^2dx^7+\frac{10}{9}a^3c^2ex^9+\frac{10}{11}a^2c^3dx^{11}+\frac{10}{13}a^2c^3ex^{13}+\frac{1}{3}ac^4dx^{15}+\frac{5}{17}ac^4ex^{17}+\frac{1}{19}c^5dx^{19}+\frac{1}{21}c^5ex^{21}$$

```
[Out] -((a^5*d)/x) + a^5*e*x + (5*a^4*c*d*x^3)/3 + a^4*c*e*x^5 + (10*a^
3*c^2*d*x^7)/7 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*d*x^11)/11 +
(10*a^2*c^3*e*x^13)/13 + (a*c^4*d*x^15)/3 + (5*a*c^4*e*x^17)/17 +
 (c^5*d*x^19)/19 + (c^5*e*x^21)/21
```

Rubi [A] time = 0.182175, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{a^5d}{x}+a^5ex+\frac{5}{3}a^4cdx^3+a^4cex^5+\frac{10}{7}a^3c^2dx^7+\frac{10}{9}a^3c^2ex^9+\frac{10}{11}a^2c^3dx^{11}+\frac{10}{13}a^2c^3ex^{13}+\frac{1}{3}ac^4dx^{15}+\frac{5}{17}ac^4ex^{17}+\frac{1}{19}c^5dx^{19}+\frac{1}{21}c^5ex^{21}$$

Antiderivative was successfully verified.

```
[In]  Int[((d + e*x^2)*(a + c*x^4)^5)/x^2,x]
```

```
[Out] -((a^5*d)/x) + a^5*e*x + (5*a^4*c*d*x^3)/3 + a^4*c*e*x^5 + (10*a^
3*c^2*d*x^7)/7 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*d*x^11)/11 +
(10*a^2*c^3*e*x^13)/13 + (a*c^4*d*x^15)/3 + (5*a*c^4*e*x^17)/17 +
 (c^5*d*x^19)/19 + (c^5*e*x^21)/21
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^5d}{x}+a^5\int e\,dx+\frac{5a^4cdx^3}{3}+a^4cex^5+\frac{10a^3c^2dx^7}{7}+\frac{10a^3c^2ex^9}{9}+\frac{10a^2c^3dx^{11}}{11}+\frac{10a^2c^3ex^{13}}{13}+\frac{ac^4dx^{15}}{3}+\frac{5ac^4ex^{17}}{17}+\frac{c^5dx^{19}}{19}+\frac{c^5ex^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)*(c*x**4+a)**5/x**2,x)
```

```
[Out] -a**5*d/x + a**5*Integral(e, x) + 5*a**4*c*d*x**3/3 + a**4*c*e*x*
*5 + 10*a**3*c**2*d*x**7/7 + 10*a**3*c**2*e*x**9/9 + 10*a**2*c**3
*d*x**11/11 + 10*a**2*c**3*e*x**13/13 + a*c**4*d*x**15/3 + 5*a*c*
*4*e*x**17/17 + c**5*d*x**19/19 + c**5*e*x**21/21
```

Mathematica [A] time = 0.0129878, size = 139, normalized size = 1.

$$-\frac{a^5 d}{x}+a^5 e x+\frac{5}{3}a^4 c d x^3+a^4 c e x^5+\frac{10}{7}a^3 c^2 d x^7+\frac{10}{9}a^3 c^2 e x^9+\frac{10}{11}a^2 c^3 d x^{11}+\frac{10}{13}a^2 c^3 e x^{13}+\frac{1}{3}a c^4 d x^{15}+\frac{5}{17}a c^4 e x^{17}+\frac{1}{19}c^5 d x^{19}+\frac{1}{21}c^5 e x^{21}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((d + e*x^2)*(a + c*x^4)^5)/x^2,x]
```

```
[Out] -((a^5*d)/x) + a^5*e*x + (5*a^4*c*d*x^3)/3 + a^4*c*e*x^5 + (10*a^
3*c^2*d*x^7)/7 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*d*x^11)/11 +
(10*a^2*c^3*e*x^13)/13 + (a*c^4*d*x^15)/3 + (5*a*c^4*e*x^17)/17 +
 (c^5*d*x^19)/19 + (c^5*e*x^21)/21
```

Maple [A] time = 0.006, size = 122, normalized size = 0.9

$$-\frac{a^5 d}{x}+a^5 e x+\frac{5\,a^4 c d x^3}{3}+a^4 c e x^5+\frac{10\,a^3 c^2 d x^7}{7}+\frac{10\,a^3 c^2 e x^9}{9}+\frac{10\,a^2 c^3 d x^{11}}{11}+\frac{10\,a^2 c^3 e x^{13}}{13}+\frac{a c^4 d x^{15}}{3}+\frac{5\,a c^4 e x^{17}}{17}+\frac{c^5 d x^{19}}{19}+\frac{c^5 e x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)*(c*x^4+a)^5/x^2,x)
```

```
[Out] -a^5*d/x+a^5*e*x+5/3*a^4*c*d*x^3+a^4*c*e*x^5+10/7*a^3*c^2*d*x^7+1
0/9*a^3*c^2*e*x^9+10/11*a^2*c^3*d*x^11+10/13*a^2*c^3*e*x^13+1/3*a
*c^4*d*x^15+5/17*a*c^4*e*x^17+1/19*c^5*d*x^19+1/21*c^5*e*x^21
```

Maxima [A] time = 0.705077, size = 163, normalized size = 1.17

$$\frac{1}{21}\,c^5 e x^{21}+\frac{1}{19}\,c^5 d x^{19}+\frac{5}{17}\,a c^4 e x^{17}+\frac{1}{3}\,a c^4 d x^{15}+\frac{10}{13}\,a^2 c^3 e x^{13}+\frac{10}{11}\,a^2 c^3 d x^{11}+\frac{10}{9}\,a^3 c^2 e x^9+\frac{10}{7}\,a^3 c^2 d x^7+a^4 c e x^5+\frac{5}{3}\,a^4 c d x^3+a^5 e x-\frac{a^5 d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d)/x^2,x, algorithm="maxima")
```

```
[Out] 1/21*c^5*e*x^21 + 1/19*c^5*d*x^19 + 5/17*a*c^4*e*x^17 + 1/3*a*c^4
*d*x^15 + 10/13*a^2*c^3*e*x^13 + 10/11*a^2*c^3*d*x^11 + 10/9*a^3*
c^2*e*x^9 + 10/7*a^3*c^2*d*x^7 + a^4*c*e*x^5 + 5/3*a^4*c*d*x^3 +
a^5*e*x - a^5*d/x
```

Fricas [A] time = 0.249982, size = 171, normalized size = 1.23

$$\frac{138567\,c^5ex^{22} + 153153\,c^5dx^{20} + 855855\,ac^4ex^{18} + 969969\,ac^4dx^{16} + 2238390\,a^2c^3ex^{14} + 2645370\,a^2c^3dx^{12} + 3233230\,a^3c^2e}{2909907\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2909907*(138567*c^5*e*x^22 + 153153*c^5*d*x^20 + 855855*a*c^4*e
*x^18 + 969969*a*c^4*d*x^16 + 2238390*a^2*c^3*e*x^14 + 2645370*a^
2*c^3*d*x^12 + 3233230*a^3*c^2*e*x^10 + 4157010*a^3*c^2*d*x^8 + 2
909907*a^4*c*e*x^6 + 4849845*a^4*c*d*x^4 + 2909907*a^5*e*x^2 - 29
09907*a^5*d)/x
```

Sympy [A] time = 1.51429, size = 143, normalized size = 1.03

$$-\frac{a^5d}{x} + a^5ex + \frac{5a^4cdx^3}{3} + a^4cex^5 + \frac{10a^3c^2dx^7}{7} + \frac{10a^3c^2ex^9}{9} + \frac{10a^2c^3dx^{11}}{11} + \frac{10a^2c^3ex^{13}}{13} + \frac{ac^4dx^{15}}{3} + \frac{5ac^4ex^{17}}{17} + \frac{c^5dx^{19}}{19} + \frac{c^5ex^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)*(c*x**4+a)**5/x**2,x)
```

```
[Out] -a**5*d/x + a**5*e*x + 5*a**4*c*d*x**3/3 + a**4*c*e*x**5 + 10*a**
3*c**2*d*x**7/7 + 10*a**3*c**2*e*x**9/9 + 10*a**2*c**3*d*x**11/11
 + 10*a**2*c**3*e*x**13/13 + a*c**4*d*x**15/3 + 5*a*c**4*e*x**17/
17 + c**5*d*x**19/19 + c**5*e*x**21/21
```

GIAC/XCAS [A] time = 0.260405, size = 171, normalized size = 1.23

$$\frac{1}{21}\,c^5x^{21}e+\frac{1}{19}\,c^5dx^{19}+\frac{5}{17}\,ac^4x^{17}e+\frac{1}{3}\,ac^4dx^{15}+\frac{10}{13}\,a^2c^3x^{13}e+\frac{10}{11}\,a^2c^3dx^{11}+\frac{10}{9}\,a^3c^2x^9e+\frac{10}{7}\,a^3c^2dx^7+a^4cx^5e+\frac{5}{3}\,a^4cdx^3+a^5xe-\frac{a^5d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d)/x^2,x, algorithm="giac")
```

```
[Out] 1/21*c^5*x^21*e + 1/19*c^5*d*x^19 + 5/17*a*c^4*x^17*e + 1/3*a*c^4
*d*x^15 + 10/13*a^2*c^3*x^13*e + 10/11*a^2*c^3*d*x^11 + 10/9*a^3*
c^2*x^9*e + 10/7*a^3*c^2*d*x^7 + a^4*c*x^5*e + 5/3*a^4*c*d*x^3 +
a^5*x*e - a^5*d/x
```

3.7 $\int \frac{(d+ex^2)(a+cx^4)^5}{x^3} \, dx$

Optimal. Leaf size=142

$$-\frac{a^5 d}{2x^2} + a^5 e \log(x) + \frac{5}{2} a^4 c d x^2 + \frac{5}{4} a^4 c e x^4 + \frac{5}{3} a^3 c^2 d x^6 + \frac{5}{4} a^3 c^2 e x^8 + a^2 c^3 d x^{10} + \frac{5}{6} a^2 c^3 e x^{12} + \frac{5}{14} a c^4 d x^{14} + \frac{5}{16} a c^4 e x^{16} + \frac{1}{18} c^5 d x^{18} + \frac{1}{20} c^5 e x^{20}$$

```
[Out] -(a^5*d)/(2*x^2) + (5*a^4*c*d*x^2)/2 + (5*a^4*c*e*x^4)/4 + (5*a^3
*c^2*d*x^6)/3 + (5*a^3*c^2*e*x^8)/4 + a^2*c^3*d*x^10 + (5*a^2*c^3
*e*x^12)/6 + (5*a*c^4*d*x^14)/14 + (5*a*c^4*e*x^16)/16 + (c^5*d*x
^18)/18 + (c^5*e*x^20)/20 + a^5*e*Log[x]
```

Rubi [A] time = 0.279526, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{a^5 d}{2x^2} + a^5 e \log(x) + \frac{5}{2} a^4 c d x^2 + \frac{5}{4} a^4 c e x^4 + \frac{5}{3} a^3 c^2 d x^6 + \frac{5}{4} a^3 c^2 e x^8 + a^2 c^3 d x^{10} + \frac{5}{6} a^2 c^3 e x^{12} + \frac{5}{14} a c^4 d x^{14} + \frac{5}{16} a c^4 e x^{16} + \frac{1}{18} c^5 d x^{18} + \frac{1}{20} c^5 e x^{20}$$

Antiderivative was successfully verified.

```
[In]  Int[((d + e*x^2)*(a + c*x^4)^5)/x^3,x]
```

```
[Out] -(a^5*d)/(2*x^2) + (5*a^4*c*d*x^2)/2 + (5*a^4*c*e*x^4)/4 + (5*a^3
*c^2*d*x^6)/3 + (5*a^3*c^2*e*x^8)/4 + a^2*c^3*d*x^10 + (5*a^2*c^3
*e*x^12)/6 + (5*a*c^4*d*x^14)/14 + (5*a*c^4*e*x^16)/16 + (c^5*d*x
^18)/18 + (c^5*e*x^20)/20 + a^5*e*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^5 d}{2x^2} + \frac{a^5 e \log(x^2)}{2} + \frac{5a^4 c d x^2}{2} + \frac{5a^4 c e \int^{x^2} x \, dx}{2} + \frac{5a^3 c^2 d x^6}{3} + \frac{5a^3 c^2 e x^8}{4} + a^2 c^3 d x^{10} + \frac{5a^2 c^3 e x^{12}}{6} + \frac{5a c^4 d x^{14}}{14} + \frac{5a c^4 e x^{16}}{16} + \frac{c^5 d x^{18}}{18} + \frac{c^5 e x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)*(c*x**4+a)**5/x**3,x)
```

```
[Out] -a**5*d/(2*x**2) + a**5*e*log(x**2)/2 + 5*a**4*c*d*x**2/2 + 5*a**
4*c*e*Integral(x, (x, x**2))/2 + 5*a**3*c**2*d*x**6/3 + 5*a**3*c*
*2*e*x**8/4 + a**2*c**3*d*x**10 + 5*a**2*c**3*e*x**12/6 + 5*a*c**
4*d*x**14/14 + 5*a*c**4*e*x**16/16 + c**5*d*x**18/18 + c**5*e*x**
20/20
```

Mathematica [A] time = 0.0131955, size = 142, normalized size = 1.

$$-\frac{a^5d}{2x^2}+a^5e\log(x)+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4+\frac{5}{3}a^3c^2dx^6+\frac{5}{4}a^3c^2ex^8+a^2c^3dx^{10}+\frac{5}{6}a^2c^3ex^{12}+\frac{5}{14}ac^4dx^{14}+\frac{5}{16}ac^4ex^{16}+\frac{1}{18}c^5dx^{18}+\frac{1}{20}c^5ex^{20}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((d + e*x^2)*(a + c*x^4)^5)/x^3,x]
```

```
[Out] -(a^5*d)/(2*x^2) + (5*a^4*c*d*x^2)/2 + (5*a^4*c*e*x^4)/4 + (5*a^3
*c^2*d*x^6)/3 + (5*a^3*c^2*e*x^8)/4 + a^2*c^3*d*x^10 + (5*a^2*c^3
*e*x^12)/6 + (5*a*c^4*d*x^14)/14 + (5*a*c^4*e*x^16)/16 + (c^5*d*x
^18)/18 + (c^5*e*x^20)/20 + a^5*e*Log[x]
```

Maple [A] time = 0.009, size = 123, normalized size = 0.9

$$-\frac{a^5d}{2\,x^2}+\frac{5\,a^4cdx^2}{2}+\frac{5\,a^4cex^4}{4}+\frac{5\,a^3c^2dx^6}{3}+\frac{5\,a^3c^2ex^8}{4}+a^2c^3dx^{10}+\frac{5\,a^2c^3ex^{12}}{6}+\frac{5\,ac^4dx^{14}}{14}+\frac{5\,ac^4ex^{16}}{16}+\frac{c^5dx^{18}}{18}+\frac{c^5ex^{20}}{20}+a^5e\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)*(c*x^4+a)^5/x^3,x)
```

```
[Out] -1/2*a^5*d/x^2+5/2*a^4*c*d*x^2+5/4*a^4*c*e*x^4+5/3*a^3*c^2*d*x^6+
5/4*a^3*c^2*e*x^8+a^2*c^3*d*x^10+5/6*a^2*c^3*e*x^12+5/14*a*c^4*d*
x^14+5/16*a*c^4*e*x^16+1/18*c^5*d*x^18+1/20*c^5*e*x^20+a^5*e*ln(x
)
```

Maxima [A] time = 0.707444, size = 169, normalized size = 1.19

$$\frac{1}{20}\,c^5ex^{20}+\frac{1}{18}\,c^5dx^{18}+\frac{5}{16}\,ac^4ex^{16}+\frac{5}{14}\,ac^4dx^{14}+\frac{5}{6}\,a^2c^3ex^{12}+a^2c^3dx^{10}+\frac{5}{4}\,a^3c^2ex^8+\frac{5}{3}\,a^3c^2dx^6+\frac{5}{4}\,a^4cex^4+\frac{5}{2}\,a^4cdx^2+\frac{1}{2}\,a^5e\log\left(x^2\right)-\frac{a^5d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d)/x^3,x, algorithm="maxima")
```

```
[Out] 1/20*c^5*e*x^20 + 1/18*c^5*d*x^18 + 5/16*a*c^4*e*x^16 + 5/14*a*c^
4*d*x^14 + 5/6*a^2*c^3*e*x^12 + a^2*c^3*d*x^10 + 5/4*a^3*c^2*e*x^
8 + 5/3*a^3*c^2*d*x^6 + 5/4*a^4*c*e*x^4 + 5/2*a^4*c*d*x^2 + 1/2*a
^5*e*log(x^2) - 1/2*a^5*d/x^2
```

Fricas [A] time = 0.250074, size = 174, normalized size = 1.23

$$\frac{252\,c^5ex^{22}+280\,c^5dx^{20}+1575\,ac^4ex^{18}+1800\,ac^4dx^{16}+4200\,a^2c^3ex^{14}+5040\,a^2c^3dx^{12}+6300\,a^3c^2ex^{10}+8400\,a^3c^2dx^8+6}{5040\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d)/x^3,x, algorithm="fricas")
```

```
[Out] 1/5040*(252*c^5*e*x^22 + 280*c^5*d*x^20 + 1575*a*c^4*e*x^18 + 180
0*a*c^4*d*x^16 + 4200*a^2*c^3*e*x^14 + 5040*a^2*c^3*d*x^12 + 6300
*a^3*c^2*e*x^10 + 8400*a^3*c^2*d*x^8 + 6300*a^4*c*e*x^6 + 12600*a
^4*c*d*x^4 + 5040*a^5*e*x^2*log(x) - 2520*a^5*d)/x^2
```

Sympy [A] time = 1.64821, size = 150, normalized size = 1.06

$$-\frac{a^5d}{2x^2}+a^5e\log\left(x\right)+\frac{5a^4cdx^2}{2}+\frac{5a^4cex^4}{4}+\frac{5a^3c^2dx^6}{3}+\frac{5a^3c^2ex^8}{4}+a^2c^3dx^{10}+\frac{5a^2c^3ex^{12}}{6}+\frac{5ac^4dx^{14}}{14}+\frac{5ac^4ex^{16}}{16}+\frac{c^5dx^{18}}{18}+\frac{c^5ex^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)*(c*x**4+a)**5/x**3,x)
```

```
[Out] -a**5*d/(2*x**2) + a**5*e*log(x) + 5*a**4*c*d*x**2/2 + 5*a**4*c*e
*x**4/4 + 5*a**3*c**2*d*x**6/3 + 5*a**3*c**2*e*x**8/4 + a**2*c**3
```

```
*d*x**10 + 5*a**2*c**3*e*x**12/6 + 5*a*c**4*d*x**14/14 + 5*a*c**4
*e*x**16/16 + c**5*d*x**18/18 + c**5*e*x**20/20
```

GIAC/XCAS [A] time = 0.263759, size = 192, normalized size = 1.35

$$\frac{1}{20}\,c^5x^{20}e+\frac{1}{18}\,c^5dx^{18}+\frac{5}{16}\,ac^4x^{16}e+\frac{5}{14}\,ac^4dx^{14}+\frac{5}{6}\,a^2c^3x^{12}e+a^2c^3dx^{10}+\frac{5}{4}\,a^3c^2x^8e+\frac{5}{3}\,a^3c^2dx^6+\frac{5}{4}\,a^4cx^4e+\frac{5}{2}\,a^4cdx^2+\frac{1}{2}\,a^5e\ln\left(x^2\right)-\frac{a^5x^2e+a^5d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + a)^5*(e*x^2 + d)/x^3,x, algorithm="giac")
```

```
[Out] 1/20*c^5*x^20*e + 1/18*c^5*d*x^18 + 5/16*a*c^4*x^16*e + 5/14*a*c^
4*d*x^14 + 5/6*a^2*c^3*x^12*e + a^2*c^3*d*x^10 + 5/4*a^3*c^2*x^8*
e + 5/3*a^3*c^2*d*x^6 + 5/4*a^4*c*x^4*e + 5/2*a^4*c*d*x^2 + 1/2*a
^5*e*ln(x^2) - 1/2*(a^5*x^2*e + a^5*d)/x^2
```

3.8 $\int x^5 \left(2+3x^2\right) \sqrt{5+x^4} \, dx$

Optimal. Leaf size=67

$$\frac{3}{10} \left(x^4+5\right)^{3/2} x^4-\frac{25}{8} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{5}{8} \sqrt{x^4+5} x^2-\frac{1}{4} \left(4-x^2\right) \left(x^4+5\right)^{3/2}$$

```
[Out] (-5*x^2*Sqrt[5 + x^4])/8 + (3*x^4*(5 + x^4)^(3/2))/10 - ((4 - x^2
)*(5 + x^4)^(3/2))/4 - (25*ArcSinh[x^2/Sqrt[5]])/8
```

Rubi [A] time = 0.144855, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{3}{10} \left(x^4+5\right)^{3/2} x^4-\frac{25}{8} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{5}{8} \sqrt{x^4+5} x^2-\frac{1}{4} \left(4-x^2\right) \left(x^4+5\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (-5*x^2*Sqrt[5 + x^4])/8 + (3*x^4*(5 + x^4)^(3/2))/10 - ((4 - x^2
)*(5 + x^4)^(3/2))/4 - (25*ArcSinh[x^2/Sqrt[5]])/8
```

Rubi in Sympy [A] time = 11.3758, size = 61, normalized size = 0.91

$$\frac{3x^4\left(x^4+5\right)^{\frac{3}{2}}}{10}-\frac{5x^2\sqrt{x^4+5}}{8}-\frac{\left(-30x^2+120\right)\left(x^4+5\right)^{\frac{3}{2}}}{120}-\frac{25\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] 3*x**4*(x**4 + 5)**(3/2)/10 - 5*x**2*sqrt(x**4 + 5)/8 - (-30*x**2
 + 120)*(x**4 + 5)**(3/2)/120 - 25*asinh(sqrt(5)*x**2/5)/8
```

Mathematica [A] time = 0.0345895, size = 54, normalized size = 0.81

$$\frac{1}{2}\sqrt{x^4+5}\left(\frac{3x^8}{5}+\frac{x^6}{2}+x^4+\frac{5x^2}{4}-10\right)-\frac{25}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (Sqrt[5 + x^4]*(-10 + (5*x^2)/4 + x^4 + x^6/2 + (3*x^8)/5))/2 - (
25*ArcSinh[x^2/Sqrt[5]])/8
```

Maple [A]   time = 0.028, size = 53, normalized size = 0.8

$$\frac{x^2}{4}\left(x^4+5\right)^{\frac{3}{2}}-\frac{5x^2}{8}\sqrt{x^4+5}-\frac{25}{8}Arcsinh\left(\frac{\sqrt{5}x^2}{5}\right)+\frac{3x^4-10}{10}\left(x^4+5\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(3*x^2+2)*(x^4+5)^(1/2),x)
```

```
[Out] 1/4*x^2*(x^4+5)^(3/2)-5/8*x^2*(x^4+5)^(1/2)-25/8*arcsinh(1/5*5^(1
/2)*x^2)+1/10*(x^4+5)^(3/2)*(3*x^4-10)
```

Maxima [A]   time = 0.779942, size = 138, normalized size = 2.06

$$\frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}}-\frac{5}{2}\left(x^4+5\right)^{\frac{3}{2}}-\frac{25\left(\frac{\sqrt{x^4+5}}{x^2}+\frac{\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{8\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)}-\frac{25}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{25}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^5,x, algorithm="maxima")
```

```
[Out] 3/10*(x^4 + 5)^(5/2) - 5/2*(x^4 + 5)^(3/2) - 25/8*(sqrt(x^4 + 5)/
x^2 + (x^4 + 5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1
) - 25/16*log(sqrt(x^4 + 5)/x^2 + 1) + 25/16*log(sqrt(x^4 + 5)/x^
2 - 1)
```

Fricas [A]   time = 0.262343, size = 271, normalized size = 4.04

$$-\frac{192x^{20}+160x^{18}+2000x^{16}+1800x^{14}+3500x^{12}+6750x^{10}-20000x^8+9375x^6-62500x^4+3125x^2-125\left(16x^{10}+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^5,x, algorithm="fricas")
```

```
[Out] -1/40*(192*x^20 + 160*x^18 + 2000*x^16 + 1800*x^14 + 3500*x^12 +
6750*x^10 - 20000*x^8 + 9375*x^6 - 62500*x^4 + 3125*x^2 - 125*(16
*x^10 + 100*x^6 + 125*x^2 - (16*x^8 + 60*x^4 + 25)*sqrt(x^4 + 5))
*log(-x^2 + sqrt(x^4 + 5)) - (192*x^18 + 160*x^16 + 1520*x^14 + 1
400*x^12 + 300*x^10 + 3750*x^8 - 17500*x^6 + 3125*x^4 - 25000*x^2
)*sqrt(x^4 + 5) - 25000)/(16*x^10 + 100*x^6 + 125*x^2 - (16*x^8 +
 60*x^4 + 25)*sqrt(x^4 + 5))
```

Sympy [A] time = 13.4333, size = 97, normalized size = 1.45

$$\frac{x^{10}}{4\sqrt{x^4+5}} + \frac{3x^8\sqrt{x^4+5}}{10} + \frac{15x^6}{8\sqrt{x^4+5}} + \frac{x^4\sqrt{x^4+5}}{2} + \frac{25x^2}{8\sqrt{x^4+5}} - 5\sqrt{x^4+5} - \frac{25\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] x**10/(4*sqrt(x**4 + 5)) + 3*x**8*sqrt(x**4 + 5)/10 + 15*x**6/(8*
sqrt(x**4 + 5)) + x**4*sqrt(x**4 + 5)/2 + 25*x**2/(8*sqrt(x**4 +
5)) - 5*sqrt(x**4 + 5) - 25*asinh(sqrt(5)*x**2/5)/8
```

GIAC/XCAS [A] time = 0.264544, size = 70, normalized size = 1.04

$$\frac{1}{40}\sqrt{x^4+5}\left(\left(2\left(\left(6x^2+5\right)x^2+10\right)x^2+25\right)x^2-200\right) + \frac{25}{8}\ln\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^5,x, algorithm="giac")
```

```
[Out] 1/40*sqrt(x^4 + 5)*((2*((6*x^2 + 5)*x^2 + 10)*x^2 + 25)*x^2 - 200
) + 25/8*ln(-x^2 + sqrt(x^4 + 5))
```

3.9 $\int x^3 \left(2+3x^2\right) \sqrt{5+x^4}\,dx$

Optimal. Leaf size=51

$$-\frac{75}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{15}{16}\sqrt{x^4+5}x^2+\frac{1}{24}\left(9x^2+8\right)\left(x^4+5\right)^{3/2}$$

```
[Out] (-15*x^2*Sqrt[5 + x^4])/16 + ((8 + 9*x^2)*(5 + x^4)^(3/2))/24 - (
75*ArcSinh[x^2/Sqrt[5]])/16
```

Rubi [A] time = 0.0932686, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{75}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{15}{16}\sqrt{x^4+5}x^2+\frac{1}{24}\left(9x^2+8\right)\left(x^4+5\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (-15*x^2*Sqrt[5 + x^4])/16 + ((8 + 9*x^2)*(5 + x^4)^(3/2))/24 - (
75*ArcSinh[x^2/Sqrt[5]])/16
```

Rubi in Sympy [A] time = 8.58032, size = 46, normalized size = 0.9

$$-\frac{15x^2\sqrt{x^4+5}}{16}+\frac{\left(9x^2+8\right)\left(x^4+5\right)^{\frac{3}{2}}}{24}-\frac{75\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] -15*x**2*sqrt(x**4 + 5)/16 + (9*x**2 + 8)*(x**4 + 5)**(3/2)/24 -
75*asinh(sqrt(5)*x**2/5)/16
```

Mathematica [A] time = 0.0343582, size = 44, normalized size = 0.86

$$\frac{1}{48}\left(\sqrt{x^4+5}\left(18x^6+16x^4+45x^2+80\right)-225\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (Sqrt[5 + x^4]*(80 + 45*x^2 + 16*x^4 + 18*x^6) - 225*ArcSinh[x^2/
Sqrt[5]])/48
```

Maple [A] time = 0.009, size = 46, normalized size = 0.9

$$\frac{1}{3}\left(x^{4}+5\right)^{\frac{3}{2}}+\frac{3\,x^{2}}{8}\left(x^{4}+5\right)^{\frac{3}{2}}-\frac{15\,x^{2}}{16}\sqrt{x^{4}+5}-\frac{75}{16}\mathit{Arcsinh}\left(\frac{\sqrt{5}x^{2}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(3*x^2+2)*(x^4+5)^(1/2),x)
```

```
[Out] 1/3*(x^4+5)^(3/2)+3/8*x^2*(x^4+5)^(3/2)-15/16*x^2*(x^4+5)^(1/2)-7
5/16*arcsinh(1/5*5^(1/2)*x^2)
```

Maxima [A] time = 0.784207, size = 126, normalized size = 2.47

$$\frac{1}{3}\left(x^{4}+5\right)^{\frac{3}{2}}-\frac{75\left(\frac{\sqrt{x^{4}+5}}{x^{2}}+\frac{\left(x^{4}+5\right)^{\frac{3}{2}}}{x^{6}}\right)}{16\left(\frac{2\left(x^{4}+5\right)}{x^{4}}-\frac{\left(x^{4}+5\right)^{2}}{x^{8}}-1\right)}-\frac{75}{32}\log\left(\frac{\sqrt{x^{4}+5}}{x^{2}}+1\right)+\frac{75}{32}\log\left(\frac{\sqrt{x^{4}+5}}{x^{2}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^3,x, algorithm="maxima")
```

```
[Out] 1/3*(x^4 + 5)^(3/2) - 75/16*(sqrt(x^4 + 5)/x^2 + (x^4 + 5)^(3/2)/
x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) - 75/32*log(sqrt(x^4
 + 5)/x^2 + 1) + 75/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 0.270577, size = 231, normalized size = 4.53

$$-\frac{144\,x^{16}+128\,x^{14}+1440\,x^{12}+1600\,x^{10}+4500\,x^{8}+6400\,x^{6}+4500\,x^{4}+8000\,x^{2}-225\left(8\,x^{8}+40\,x^{4}-4\left(2\,x^{6}+5\,x^{2}\right)\sqrt{x^{4}-}\right.}{48\left(8\,x^{8}+40\,x^{4}-4\left(2\,x^{6}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^3,x, algorithm="fricas")
```

```
[Out] -1/48*(144*x^16 + 128*x^14 + 1440*x^12 + 1600*x^10 + 4500*x^8 + 6
400*x^6 + 4500*x^4 + 8000*x^2 - 225*(8*x^8 + 40*x^4 - 4*(2*x^6 +
5*x^2)*sqrt(x^4 + 5) + 25)*log(-x^2 + sqrt(x^4 + 5)) - (144*x^14
+ 128*x^12 + 1080*x^10 + 1280*x^8 + 2250*x^6 + 3600*x^4 + 1125*x^
2 + 2000)*sqrt(x^4 + 5))/(8*x^8 + 40*x^4 - 4*(2*x^6 + 5*x^2)*sqrt
(x^4 + 5) + 25)
```

Sympy [A] time = 10.5978, size = 70, normalized size = 1.37

$$\frac{3x^{10}}{8\sqrt{x^4+5}} + \frac{45x^6}{16\sqrt{x^4+5}} + \frac{75x^2}{16\sqrt{x^4+5}} + \frac{\left(x^4+5\right)^{\frac{3}{2}}}{3} - \frac{75\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] 3*x**10/(8*sqrt(x**4 + 5)) + 45*x**6/(16*sqrt(x**4 + 5)) + 75*x**
2/(16*sqrt(x**4 + 5)) + (x**4 + 5)**(3/2)/3 - 75*asinh(sqrt(5)*x*
*2/5)/16
```

GIAC/XCAS [A] time = 0.263236, size = 62, normalized size = 1.22

$$\frac{1}{48}\sqrt{x^4+5}\left(\left(2\left(9x^2+8\right)x^2+45\right)x^2+80\right) + \frac{75}{16}\ln\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^3,x, algorithm="giac")
```

```
[Out] 1/48*sqrt(x^4 + 5)*((2*(9*x^2 + 8)*x^2 + 45)*x^2 + 80) + 75/16*ln
(-x^2 + sqrt(x^4 + 5))
```

3.10 $\int x \left(2+3x^2\right) \sqrt{5+x^4} \, dx$

Optimal. Leaf size=44

$$\frac{1}{2}\left(x^4+5\right)^{3/2}+\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^4+5}x^2$$

```
[Out] (x^2*Sqrt[5 + x^4])/2 + (5 + x^4)^(3/2)/2 + (5*ArcSinh[x^2/Sqrt[5
]])/2
```

Rubi [A] time = 0.0585607, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{1}{2}\left(x^4+5\right)^{3/2}+\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^4+5}x^2$$

Antiderivative was successfully verified.

```
[In]  Int[x*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (x^2*Sqrt[5 + x^4])/2 + (5 + x^4)^(3/2)/2 + (5*ArcSinh[x^2/Sqrt[5
]])/2
```

Rubi in Sympy [A] time = 7.07213, size = 37, normalized size = 0.84

$$\frac{x^2\sqrt{x^4+5}}{2}+\frac{\left(x^4+5\right)^{\frac{3}{2}}}{2}+\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] x**2*sqrt(x**4 + 5)/2 + (x**4 + 5)**(3/2)/2 + 5*asinh(sqrt(5)*x**
2/5)/2
```

Mathematica [A] time = 0.0233472, size = 36, normalized size = 0.82

$$\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^4+5}\left(x^4+x^2+5\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (Sqrt[5 + x^4]*(5 + x^2 + x^4))/2 + (5*ArcSinh[x^2/Sqrt[5]])/2
```

Maple [A] time = 0.013, size = 34, normalized size = 0.8

$$\frac{1}{2}\left(x^{4}+5\right)^{\frac{3}{2}}+\frac{5}{2}\mathit{Arcsinh}\left(\frac{\sqrt{5}x^{2}}{5}\right)+\frac{x^{2}}{2}\sqrt{x^{4}+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(3*x^2+2)*(x^4+5)^(1/2),x)
```

```
[Out] 1/2*(x^4+5)^(3/2)+5/2*arcsinh(1/5*5^(1/2)*x^2)+1/2*x^2*(x^4+5)^(1
/2)
```

Maxima [A] time = 0.785638, size = 90, normalized size = 2.05

$$\frac{1}{2}\left(x^{4}+5\right)^{\frac{3}{2}}+\frac{5\sqrt{x^{4}+5}}{2x^{2}\left(\frac{x^{4}+5}{x^{4}}-1\right)}+\frac{5}{4}\log\left(\frac{\sqrt{x^{4}+5}}{x^{2}}+1\right)-\frac{5}{4}\log\left(\frac{\sqrt{x^{4}+5}}{x^{2}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*(x^4 + 5)^(3/2) + 5/2*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1))
 + 5/4*log(sqrt(x^4 + 5)/x^2 + 1) - 5/4*log(sqrt(x^4 + 5)/x^2 - 1
)
```

Fricas [A] time = 0.277566, size = 190, normalized size = 4.32

$$-\frac{4x^{12}+4x^{10}+45x^{8}+25x^{6}+150x^{4}+25x^{2}+5\left(4x^{6}+15x^{2}-\left(4x^{4}+5\right)\sqrt{x^{4}+5}\right)\log\left(-x^{2}+\sqrt{x^{4}+5}\right)-\left(4x^{10}+4x^{8}+\right.}{2\left(4x^{6}+15x^{2}-(4x^{4}+5)\sqrt{x^{4}+5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x,x, algorithm="fricas")
```

```
[Out] -1/2*(4*x^12 + 4*x^10 + 45*x^8 + 25*x^6 + 150*x^4 + 25*x^2 + 5*(4
*x^6 + 15*x^2 - (4*x^4 + 5)*sqrt(x^4 + 5))*log(-x^2 + sqrt(x^4 +
5)) - (4*x^10 + 4*x^8 + 35*x^6 + 15*x^4 + 75*x^2)*sqrt(x^4 + 5) +
 125)/(4*x^6 + 15*x^2 - (4*x^4 + 5)*sqrt(x^4 + 5))
```

Sympy [A] time = 6.5713, size = 53, normalized size = 1.2

$$\frac{x^6}{2\sqrt{x^4+5}} + \frac{5x^2}{2\sqrt{x^4+5}} + \frac{\left(x^4+5\right)^{\frac{3}{2}}}{2} + \frac{5\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] x**6/(2*sqrt(x**4 + 5)) + 5*x**2/(2*sqrt(x**4 + 5)) + (x**4 + 5)*
*(3/2)/2 + 5*asinh(sqrt(5)*x**2/5)/2
```

GIAC/XCAS [A] time = 0.264549, size = 50, normalized size = 1.14

$$\frac{1}{2}\sqrt{x^4+5}\left(\left(x^2+1\right)x^2+5\right) - \frac{5}{2}\ln\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^4 + 5)*((x^2 + 1)*x^2 + 5) - 5/2*ln(-x^2 + sqrt(x^4 +
5))
```

3.11 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx$

Optimal. Leaf size=58

$$-\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)$$

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 + (15*ArcSinh[x^2/Sqrt[5]])/4 - Sqr
t[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]
```

Rubi [A] time = 0.143838, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$-\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x,x]
```

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 + (15*ArcSinh[x^2/Sqrt[5]])/4 - Sqr
t[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]
```

Rubi in Sympy [A] time = 13.8267, size = 53, normalized size = 0.91

$$\frac{\left(3x^2+4\right)\sqrt{x^4+5}}{4}+\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}-\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5)**(1/2)/x,x)
```

```
[Out] (3*x**2 + 4)*sqrt(x**4 + 5)/4 + 15*asinh(sqrt(5)*x**2/5)/4 - sqrt
(5)*atanh(sqrt(5)*sqrt(x**4 + 5)/5)
```

Mathematica [A] time = 0.0938827, size = 57, normalized size = 0.98

$$-\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\sqrt{x^4+5}\left(\frac{3x^2}{4}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x,x]
```

```
[Out] (1 + (3*x^2)/4)*Sqrt[5 + x^4] + (15*ArcSinh[x^2/Sqrt[5]])/4 - Sqr
t[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]
```

Maple [A] time = 0.017, size = 49, normalized size = 0.8

$$\sqrt{x^4+5}-\sqrt{5}\mathit{Artanh}\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)+\frac{3\,x^2}{4}\sqrt{x^4+5}+\frac{15}{4}\mathit{Arcsinh}\left(\frac{\sqrt{5}x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5)^(1/2)/x,x)
```

```
[Out] (x^4+5)^(1/2)-5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))+3/4*x^2*(x^4
+5)^(1/2)+15/4*arcsinh(1/5*5^(1/2)*x^2)
```

Maxima [A] time = 0.782895, size = 134, normalized size = 2.31

$$\frac{1}{2}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\sqrt{x^4+5}+\frac{15\sqrt{x^4+5}}{4\,x^2\left(\frac{x^4+5}{x^4}-1\right)}+\frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 +
5))) + sqrt(x^4 + 5) + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1
)) + 15/8*log(sqrt(x^4 + 5)/x^2 + 1) - 15/8*log(sqrt(x^4 + 5)/x^2
 - 1)
```

Fricas [A] time = 0.272107, size = 247, normalized size = 4.26

$$-\frac{6\,x^{8}+8\,x^{6}+30\,x^{4}+40\,x^{2}+15\left(2\,x^{4}-2\,\sqrt{x^{4}+5}x^{2}+5\right)\log\left(-x^{2}+\sqrt{x^{4}+5}\right)+4\left(2\,\sqrt{5}\sqrt{x^{4}+5}x^{2}-\sqrt{5}\left(2\,x^{4}+5\right)\right)\log\left(\frac{x}{-}\right.}{4\left(2\,x^{4}-2\,\sqrt{x^{4}+5}x^{2}+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x,x, algorithm="fricas")
```

```
[Out] -1/4*(6*x^8 + 8*x^6 + 30*x^4 + 40*x^2 + 15*(2*x^4 - 2*sqrt(x^4 +
5)*x^2 + 5)*log(-x^2 + sqrt(x^4 + 5)) + 4*(2*sqrt(5)*sqrt(x^4 + 5
)*x^2 - sqrt(5)*(2*x^4 + 5))*log((x^4 + sqrt(5)*x^2 - sqrt(x^4 +
5)*(x^2 + sqrt(5)) + 5)/(x^4 - sqrt(x^4 + 5)*x^2)) - (6*x^6 + 8*x
^4 + 15*x^2 + 20)*sqrt(x^4 + 5))/(2*x^4 - 2*sqrt(x^4 + 5)*x^2 + 5
)
```

Sympy [A] time = 12.2978, size = 83, normalized size = 1.43

$$\frac{3x^{6}}{4\sqrt{x^{4}+5}}+\frac{15x^{2}}{4\sqrt{x^{4}+5}}+\sqrt{x^{4}+5}+\frac{\sqrt{5}\log\left(x^{4}\right)}{2}-\sqrt{5}\log\left(\sqrt{\frac{x^{4}}{5}+1}+1\right)+\frac{15\operatorname{asinh}\left(\frac{\sqrt{5}x^{2}}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5)**(1/2)/x,x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) + 15*x**2/(4*sqrt(x**4 + 5)) + sqrt(x**
4 + 5) + sqrt(5)*log(x**4)/2 - sqrt(5)*log(sqrt(x**4/5 + 1) + 1)
+ 15*asinh(sqrt(5)*x**2/5)/4
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^{4}+5}\left(3\,x^{2}+2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x, x)
```

3.12 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx$

Optimal. Leaf size=59

$$-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}\,(2-3x^2)}{2x^2}$$

```
[Out] -((2 - 3*x^2)*Sqrt[5 + x^4])/(2*x^2) + ArcSinh[x^2/Sqrt[5]] - (3*
Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2
```

Rubi [A] time = 0.147822, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}\,(2-3x^2)}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^3,x]
```

```
[Out] -((2 - 3*x^2)*Sqrt[5 + x^4])/(2*x^2) + ArcSinh[x^2/Sqrt[5]] - (3*
Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2
```

Rubi in Sympy [A] time = 14.02, size = 56, normalized size = 0.95

$$\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)-\frac{3\sqrt{5}\operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)}{2}-\frac{(-3x^2+2)\sqrt{x^4+5}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5)**(1/2)/x**3,x)
```

```
[Out] asinh(sqrt(5)*x**2/5) - 3*sqrt(5)*atanh(sqrt(5)*sqrt(x**4 + 5)/5)
/2 - (-3*x**2 + 2)*sqrt(x**4 + 5)/(2*x**2)
```

Mathematica [A] time = 0.0914243, size = 55, normalized size = 0.93

$$-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\sqrt{x^4+5}\left(\frac{3}{2}-\frac{1}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^3,x]
```

```
[Out] (3/2 - x^(-2))*Sqrt[5 + x^4] + ArcSinh[x^2/Sqrt[5]] - (3*Sqrt[5]*
ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2
```

Maple [A] time = 0.019, size = 61, normalized size = 1.

$$-\frac{1}{5x^2}\left(x^4+5\right)^{\frac{3}{2}}+\frac{x^2}{5}\sqrt{x^4+5}+Arcsinh\left(\frac{\sqrt{5}x^2}{5}\right)+\frac{3}{2}\sqrt{x^4+5}-\frac{3\sqrt{5}}{2}Artanh\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5)^(1/2)/x^3,x)
```

```
[Out] -1/5/x^2*(x^4+5)^(3/2)+1/5*x^2*(x^4+5)^(1/2)+arcsinh(1/5*5^(1/2)*
x^2)+3/2*(x^4+5)^(1/2)-3/2*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))
```

Maxima [A] time = 0.78202, size = 119, normalized size = 2.02

$$\frac{3}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{3}{2}\sqrt{x^4+5}-\frac{\sqrt{x^4+5}}{x^2}+\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 +
5))) + 3/2*sqrt(x^4 + 5) - sqrt(x^4 + 5)/x^2 + 1/2*log(sqrt(x^4 +
 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 0.302511, size = 250, normalized size = 4.24

$$-\frac{6x^8+30x^4-10x^2+2\left(2x^6-2\sqrt{x^4+5}x^4+5x^2\right)\log\left(-x^2+\sqrt{x^4+5}\right)+3\left(2\sqrt{5}\sqrt{x^4+5}x^4-\sqrt{5}\left(2x^6+5x^2\right)\right)\log\left(\frac{x^4+\cdots}{\cdots}\right.}{2\left(2x^6-2\sqrt{x^4+5}x^4+5x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*x^8 + 30*x^4 - 10*x^2 + 2*(2*x^6 - 2*sqrt(x^4 + 5)*x^4 +
5*x^2)*log(-x^2 + sqrt(x^4 + 5)) + 3*(2*sqrt(5)*sqrt(x^4 + 5)*x^4
 - sqrt(5)*(2*x^6 + 5*x^2))*log((x^4 + sqrt(5)*x^2 - sqrt(x^4 + 5
)*(x^2 + sqrt(5)) + 5)/(x^4 - sqrt(x^4 + 5)*x^2)) - (6*x^6 + 15*x
^2 - 10)*sqrt(x^4 + 5))/(2*x^6 - 2*sqrt(x^4 + 5)*x^4 + 5*x^2)
```

Sympy [A] time = 11.9394, size = 83, normalized size = 1.41

$$-\frac{x^2}{\sqrt{x^4+5}}+\frac{3\sqrt{x^4+5}}{2}+\frac{3\sqrt{5}\log\left(x^4\right)}{4}-\frac{3\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2}+\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)-\frac{5}{x^2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5)**(1/2)/x**3,x)
```

```
[Out] -x**2/sqrt(x**4 + 5) + 3*sqrt(x**4 + 5)/2 + 3*sqrt(5)*log(x**4)/4
 - 3*sqrt(5)*log(sqrt(x**4/5 + 1) + 1)/2 + asinh(sqrt(5)*x**2/5)
- 5/(x**2*sqrt(x**4 + 5))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5}\left(3\,x^2+2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^3, x)
```

3.13 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5}\,dx$

Optimal. Leaf size=63

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}\,(3x^2+1)}{2x^4}$$

```
[Out] -((1 + 3*x^2)*Sqrt[5 + x^4])/(2*x^4) + (3*ArcSinh[x^2/Sqrt[5]])/2
 - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/(2*Sqrt[5])
```

Rubi [A] time = 0.148627, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}\,(3x^2+1)}{2x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^5,x]
```

```
[Out] -((1 + 3*x^2)*Sqrt[5 + x^4])/(2*x^4) + (3*ArcSinh[x^2/Sqrt[5]])/2
 - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/(2*Sqrt[5])
```

Rubi in Sympy [A] time = 13.4055, size = 58, normalized size = 0.92

$$\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}-\frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)}{10}-\frac{(30x^2+10)\sqrt{x^4+5}}{20x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5)**(1/2)/x**5,x)
```

```
[Out] 3*asinh(sqrt(5)*x**2/5)/2 - sqrt(5)*atanh(sqrt(5)*sqrt(x**4 + 5)/
5)/10 - (30*x**2 + 10)*sqrt(x**4 + 5)/(20*x**4)
```

Mathematica [A] time = 0.103358, size = 61, normalized size = 0.97

$$\frac{1}{10}\left(-\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+15\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{5\sqrt{x^4+5}\,(3x^2+1)}{x^4}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^5,x]
```

```
[Out] ((-5*(1 + 3*x^2)*Sqrt[5 + x^4])/x^4 + 15*ArcSinh[x^2/Sqrt[5]] - S
qrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/10
```

Maple [A] time = 0.018, size = 75, normalized size = 1.2

$$-\frac{1}{10\,x^4}\left(x^4+5\right)^{\frac{3}{2}}+\frac{1}{10}\sqrt{x^4+5}-\frac{\sqrt{5}}{10}Artanh\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)$$
$$-\frac{3}{10\,x^2}\left(x^4+5\right)^{\frac{3}{2}}+\frac{3\,x^2}{10}\sqrt{x^4+5}+\frac{3}{2}Arcsinh\left(\frac{\sqrt{5}x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5)^(1/2)/x^5,x)
```

```
[Out] -1/10/x^4*(x^4+5)^(3/2)+1/10*(x^4+5)^(1/2)-1/10*5^(1/2)*arctanh(5
^(1/2)/(x^4+5)^(1/2))-3/10/x^2*(x^4+5)^(3/2)+3/10*x^2*(x^4+5)^(1/
2)+3/2*arcsinh(1/5*5^(1/2)*x^2)
```

Maxima [A] time = 0.783888, size = 123, normalized size = 1.95

$$\frac{1}{20}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{3\sqrt{x^4+5}}{2\,x^2}-\frac{\sqrt{x^4+5}}{2\,x^4}+\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 +
 5))) - 3/2*sqrt(x^4 + 5)/x^2 - 1/2*sqrt(x^4 + 5)/x^4 + 3/4*log(s
qrt(x^4 + 5)/x^2 + 1) - 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 0.303752, size = 279, normalized size = 4.43

$$\frac{\sqrt{5}\left(2\,x^4+15\,x^2+5\right)\sqrt{x^4+5}-3\left(2\,\sqrt{5}\sqrt{x^4+5}x^6-\sqrt{5}\left(2\,x^8+5\,x^4\right)\right)\log\left(-x^2+\sqrt{x^4+5}\right)-\left(2\,x^8-2\,\sqrt{x^4+5}x^6+5\,x^4\right)\log}{2\left(2\,\sqrt{5}\sqrt{x^4+5}x^6-\sqrt{5}\left(2\,x^8+5\,x^4\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(5)*(2*x^4 + 15*x^2 + 5)*sqrt(x^4 + 5) - 3*(2*sqrt(5)*sq
rt(x^4 + 5)*x^6 - sqrt(5)*(2*x^8 + 5*x^4))*log(-x^2 + sqrt(x^4 +
5)) - (2*x^8 - 2*sqrt(x^4 + 5)*x^6 + 5*x^4)*log((5*x^2 + sqrt(5)*
(x^4 + 5) - sqrt(x^4 + 5)*(sqrt(5)*x^2 + 5))/(x^4 - sqrt(x^4 + 5)
*x^2)) - sqrt(5)*(2*x^6 + 15*x^4 + 10*x^2))/(2*sqrt(5)*sqrt(x^4 +
 5)*x^6 - sqrt(5)*(2*x^8 + 5*x^4))
```

Sympy [A] time = 12.4948, size = 76, normalized size = 1.21

$$-\frac{3x^2}{2\sqrt{x^4+5}}-\frac{\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{10}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}-\frac{\sqrt{1+\frac{5}{x^4}}}{2x^2}-\frac{15}{2x^2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5)**(1/2)/x**5,x)
```

```
[Out] -3*x**2/(2*sqrt(x**4 + 5)) - sqrt(5)*asinh(sqrt(5)/x**2)/10 + 3*a
sinh(sqrt(5)*x**2/5)/2 - sqrt(1 + 5/x**4)/(2*x**2) - 15/(2*x**2*s
qrt(x**4 + 5))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5}\left(3\,x^2+2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^5, x)
```

3.14 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sqrt{x^4+5}}{4x^4}-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{4\sqrt{5}}-\frac{\left(x^4+5\right)^{3/2}}{15x^6}$$

```
[Out] (-3*Sqrt[5 + x^4])/(4*x^4) - (5 + x^4)^(3/2)/(15*x^6) - (3*ArcTan
h[Sqrt[5 + x^4]/Sqrt[5]])/(4*Sqrt[5])
```

Rubi [A] time = 0.121051, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{3\sqrt{x^4+5}}{4x^4}-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{4\sqrt{5}}-\frac{\left(x^4+5\right)^{3/2}}{15x^6}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^7,x]
```

```
[Out] (-3*Sqrt[5 + x^4])/(4*x^4) - (5 + x^4)^(3/2)/(15*x^6) - (3*ArcTan
h[Sqrt[5 + x^4]/Sqrt[5]])/(4*Sqrt[5])
```

Rubi in Sympy [A] time = 11.4744, size = 54, normalized size = 0.93

$$-\frac{3\sqrt{5}\,\operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)}{20}-\frac{3\sqrt{x^4+5}}{4x^4}-\frac{\left(x^4+5\right)^{\frac{3}{2}}}{15x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5)**(1/2)/x**7,x)
```

```
[Out] -3*sqrt(5)*atanh(sqrt(5)*sqrt(x**4 + 5)/5)/20 - 3*sqrt(x**4 + 5)/
(4*x**4) - (x**4 + 5)**(3/2)/(15*x**6)
```

Mathematica [A] time = 0.0549388, size = 54, normalized size = 0.93

$$\frac{1}{60}\left(-9\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}\left(4x^4+45x^2+20\right)}{x^6}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^7,x]
```

```
[Out] (-((Sqrt[5 + x^4]*(20 + 45*x^2 + 4*x^4))/x^6) - 9*Sqrt[5]*ArcTanh
[Sqrt[5 + x^4]/Sqrt[5]])/60
```

Maple [A] time = 0.018, size = 52, normalized size = 0.9

$$-\frac{1}{15\,x^{6}}\left(x^{4}+5\right)^{\frac{3}{2}}-\frac{3}{20\,x^{4}}\left(x^{4}+5\right)^{\frac{3}{2}}+\frac{3}{20}\sqrt{x^{4}+5}-\frac{3\sqrt{5}}{20}\mathit{Artanh}\left(\sqrt{5}\frac{1}{\sqrt{x^{4}+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5)^(1/2)/x^7,x)
```

```
[Out] -1/15*(x^4+5)^(3/2)/x^6-3/20/x^4*(x^4+5)^(3/2)+3/20*(x^4+5)^(1/2)
-3/20*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))
```

Maxima [A] time = 0.776061, size = 80, normalized size = 1.38

$$\frac{3}{40}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^{4}+5}}{\sqrt{5}+\sqrt{x^{4}+5}}\right)-\frac{3\sqrt{x^{4}+5}}{4\,x^{4}}-\frac{\left(x^{4}+5\right)^{\frac{3}{2}}}{15\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^7,x, algorithm="maxima")
```

```
[Out] 3/40*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 +
 5))) - 3/4*sqrt(x^4 + 5)/x^4 - 1/15*(x^4 + 5)^(3/2)/x^6
```

Fricas [A] time = 0.266378, size = 263, normalized size = 4.53

$$\frac{3\sqrt{5}\left(12\,x^{8}+8\,x^{6}+45\,x^{4}+20\,x^{2}\right)\sqrt{x^{4}+5}-9\left(4\,x^{12}+15\,x^{8}-\left(4\,x^{10}+5\,x^{6}\right)\sqrt{x^{4}+5}\right)\log\left(\frac{5\,x^{2}+\sqrt{5}\left(x^{4}+5\right)-\sqrt{x^{4}+5}\left(\sqrt{5}x^{2}+5\right)}{x^{4}-\sqrt{x^{4}+5}x^{2}}\right)-\sqrt{}}{12\left(\sqrt{5}(4\,x^{10}+5\,x^{6})\sqrt{x^{4}+5}-\sqrt{5}(4\,x^{12}+15\,x^{8})\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(5)*(12*x^8 + 8*x^6 + 45*x^4 + 20*x^2)*sqrt(x^4 + 5)
- 9*(4*x^12 + 15*x^8 - (4*x^10 + 5*x^6)*sqrt(x^4 + 5))*log((5*x^2
 + sqrt(5)*(x^4 + 5) - sqrt(x^4 + 5)*(sqrt(5)*x^2 + 5))/(x^4 - sq
rt(x^4 + 5)*x^2)) - sqrt(5)*(36*x^10 + 24*x^8 + 225*x^6 + 120*x^4
 + 225*x^2 + 100))/(sqrt(5)*(4*x^10 + 5*x^6)*sqrt(x^4 + 5) - sqrt
(5)*(4*x^12 + 15*x^8))
```

Sympy [A] time = 11.988, size = 63, normalized size = 1.09

$$-\frac{\sqrt{1+\frac{5}{x^4}}}{15}-\frac{3\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{20}-\frac{3\sqrt{1+\frac{5}{x^4}}}{4x^2}-\frac{\sqrt{1+\frac{5}{x^4}}}{3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5)**(1/2)/x**7,x)
```

```
[Out] -sqrt(1 + 5/x**4)/15 - 3*sqrt(5)*asinh(sqrt(5)/x**2)/20 - 3*sqrt(
1 + 5/x**4)/(4*x**2) - sqrt(1 + 5/x**4)/(3*x**4)
```

GIAC/XCAS [A] time = 0.269881, size = 84, normalized size = 1.45

$$-\frac{1}{60}\left(\frac{5\left(\frac{4}{x^2}+9\right)}{x^2}+4\right)\sqrt{\frac{5}{x^4}+1}-\frac{3}{40}\sqrt{5}\ln\left(\sqrt{5}+\sqrt{x^4+5}\right)+\frac{3}{40}\sqrt{5}\ln\left(-\sqrt{5}+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^7,x, algorithm="giac")
```

```
[Out] -1/60*(5*(4/x^2 + 9)/x^2 + 4)*sqrt(5/x^4 + 1) - 3/40*sqrt(5)*ln(s
qrt(5) + sqrt(x^4 + 5)) + 3/40*sqrt(5)*ln(-sqrt(5) + sqrt(x^4 + 5
))
```

3.15 $\int x^4 \left(2+3x^2\right) \sqrt{5+x^4}\, dx$

Optimal. Leaf size=208

$$\frac{20}{21}\sqrt{x^4+5}x+\frac{2}{3}\sqrt{x^4+5}x^3-\frac{10\sqrt{x^4+5}x}{x^2+\sqrt{5}}-\frac{5\sqrt[4]{5}\left(21+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{21\sqrt{x^4+5}}+\frac{10\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}+\frac{1}{21}\left(7x^2+6\right)\sqrt{x^4+5}x^5$$

```
[Out] (20*x*Sqrt[5 + x^4])/21 + (2*x^3*Sqrt[5 + x^4])/3 - (10*x*Sqrt[5
+ x^4])/(Sqrt[5] + x^2) + (x^5*(6 + 7*x^2)*Sqrt[5 + x^4])/21 + (1
0*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Ellip
ticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] - (5*5^(1/4)*(21 +
2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Elli
pticF[2*ArcTan[x/5^(1/4)], 1/2])/(21*Sqrt[5 + x^4])
```

Rubi [A] time = 0.290105, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{20}{21}\sqrt{x^4+5}x+\frac{2}{3}\sqrt{x^4+5}x^3-\frac{10\sqrt{x^4+5}x}{x^2+\sqrt{5}}-\frac{5\sqrt[4]{5}\left(21+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{21\sqrt{x^4+5}}+\frac{10\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}+\frac{1}{21}\left(7x^2+6\right)\sqrt{x^4+5}x^5$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (20*x*Sqrt[5 + x^4])/21 + (2*x^3*Sqrt[5 + x^4])/3 - (10*x*Sqrt[5
+ x^4])/(Sqrt[5] + x^2) + (x^5*(6 + 7*x^2)*Sqrt[5 + x^4])/21 + (1
0*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Ellip
ticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] - (5*5^(1/4)*(21 +
2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Elli
```

```
pticF[2*ArcTan[x/5^(1/4)], 1/2])/(21*Sqrt[5 + x^4])
```

Rubi in Sympy [A] time = 25.6711, size = 206, normalized size = 0.99

$$\frac{x^5\left(21x^2+18\right)\sqrt{x^4+5}}{63}+\frac{2x^3\sqrt{x^4+5}}{3}+\frac{20x\sqrt{x^4+5}}{21}-\frac{10x\sqrt{x^4+5}}{x^2+\sqrt{5}}+\frac{10\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{\sqrt{x^4+5}}-\frac{\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(90\sqrt{5}+945\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{189\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] x**5*(21*x**2 + 18)*sqrt(x**4 + 5)/63 + 2*x**3*sqrt(x**4 + 5)/3 +
 20*x*sqrt(x**4 + 5)/21 - 10*x*sqrt(x**4 + 5)/(x**2 + sqrt(5)) +
10*5**(1/4)*sqrt((x**4 + 5)/(sqrt(5)*x**2/5 + 1)**2)*(sqrt(5)*x**
2/5 + 1)*elliptic_e(2*atan(5**(3/4)*x/5), 1/2)/sqrt(x**4 + 5) - 5
**(1/4)*sqrt((x**4 + 5)/(sqrt(5)*x**2/5 + 1)**2)*(90*sqrt(5) + 94
5)*(sqrt(5)*x**2/5 + 1)*elliptic_f(2*atan(5**(3/4)*x/5), 1/2)/(18
9*sqrt(x**4 + 5))
```

Mathematica [C] time = 0.211888, size = 105, normalized size = 0.5

$$\frac{1}{21}\left(\frac{x\left(7x^{10}+6x^8+49x^6+50x^4+70x^2+100\right)}{\sqrt{x^4+5}}+10\sqrt[4]{-5}\left(2\sqrt{5}-21i\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)+210(-1)^{3/4}\sqrt[4]{5}E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] ((x*(100 + 70*x^2 + 50*x^4 + 49*x^6 + 6*x^8 + 7*x^10))/Sqrt[5 + x
^4] + 210*(-1)^(3/4)*5^(1/4)*EllipticE[I*ArcSinh[(-1/5)^(1/4)*x],
 -1] + 10*(-5)^(1/4)*(-21*I + 2*Sqrt[5])*EllipticF[I*ArcSinh[(-1/
5)^(1/4)*x], -1])/21
```

Maple [C] time = 0.072, size = 192, normalized size = 0.9

$$\frac{2x^5}{7}\sqrt{x^4+5}+\frac{20x}{21}\sqrt{x^4+5}-\frac{4\sqrt{5}}{21\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\,EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}}+\frac{x^7}{3}\sqrt{x^4+5}+\frac{2x^3}{3}\sqrt{x^4+5}-\frac{2i}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-EllipticE\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(3*x^2+2)*(x^4+5)^(1/2),x)
```

```
[Out] 2/7*x^5*(x^4+5)^(1/2)+20/21*x*(x^4+5)^(1/2)-4/21*5^(1/2)/(I*5^(1/
2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(
x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)+1/3*x^7
*(x^4+5)^(1/2)+2/3*x^3*(x^4+5)^(1/2)-2*I/(I*5^(1/2))^(1/2)*(25-5*
I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(El
lipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2
)*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+5}\left(3x^2+2\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(3x^6+2x^4\right)\sqrt{x^4+5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^4,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4)*sqrt(x^4 + 5), x)
```

Sympy [A] time = 4.55302, size = 78, normalized size = 0.38

$$\frac{3\sqrt{5}x^7\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\,\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{5}x^5\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), x**4*exp_po
lar(I*pi)/5)/(4*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((-1/
2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(9/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+5}\left(3\,x^2+2\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^4, x)
```

3.16 $\int x^2 \left(2+3x^2\right) \sqrt{5+x^4}\,dx$

Optimal. Leaf size=192

$$\frac{10}{7}\sqrt{x^4+5}x+\frac{4\sqrt{x^4+5}x}{x^2+\sqrt{5}}+\frac{\sqrt[4]{5}\left(14-5\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{7\sqrt{x^4+5}}-\frac{4\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}+\frac{1}{35}\left(15x^2+14\right)\sqrt{x^4+5}x^3$$

```
[Out] (10*x*Sqrt[5 + x^4])/7 + (4*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (x
^3*(14 + 15*x^2)*Sqrt[5 + x^4])/35 - (4*5^(1/4)*(Sqrt[5] + x^2)*S
qrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1
/2])/Sqrt[5 + x^4] + (5^(1/4)*(14 - 5*Sqrt[5])*(Sqrt[5] + x^2)*Sq
rt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/
2])/(7*Sqrt[5 + x^4])
```

Rubi [A] time = 0.22468, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{10}{7}\sqrt{x^4+5}x+\frac{4\sqrt{x^4+5}x}{x^2+\sqrt{5}}+\frac{\sqrt[4]{5}\left(14-5\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{7\sqrt{x^4+5}}-\frac{4\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}+\frac{1}{35}\left(15x^2+14\right)\sqrt{x^4+5}x^3$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (10*x*Sqrt[5 + x^4])/7 + (4*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (x
^3*(14 + 15*x^2)*Sqrt[5 + x^4])/35 - (4*5^(1/4)*(Sqrt[5] + x^2)*S
qrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1
/2])/Sqrt[5 + x^4] + (5^(1/4)*(14 - 5*Sqrt[5])*(Sqrt[5] + x^2)*Sq
rt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/
2])/(7*Sqrt[5 + x^4])
```

Rubi in Sympy [A] time = 19.9341, size = 190, normalized size = 0.99

$$\frac{x^3\left(15x^2+14\right)\sqrt{x^4+5}}{35}+\frac{10x\sqrt{x^4+5}}{7}+\frac{4x\sqrt{x^4+5}}{x^2+\sqrt{5}}-\frac{4\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\,\mathrm{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{\sqrt{x^4+5}}+\frac{\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(-15\sqrt{5}+42\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\,\mathrm{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{21\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] x**3*(15*x**2 + 14)*sqrt(x**4 + 5)/35 + 10*x*sqrt(x**4 + 5)/7 + 4
*x*sqrt(x**4 + 5)/(x**2 + sqrt(5)) - 4*5**(1/4)*sqrt((x**4 + 5)/(
sqrt(5)*x**2/5 + 1)**2)*(sqrt(5)*x**2/5 + 1)*elliptic_e(2*atan(5*
*(3/4)*x/5), 1/2)/sqrt(x**4 + 5) + 5**(1/4)*sqrt((x**4 + 5)/(sqrt
(5)*x**2/5 + 1)**2)*(-15*sqrt(5) + 42)*(sqrt(5)*x**2/5 + 1)*ellip
tic_f(2*atan(5**(3/4)*x/5), 1/2)/(21*sqrt(x**4 + 5))
```

Mathematica [C] time = 0.189907, size = 101, normalized size = 0.53

$$\frac{x\left(15x^8+14x^6+125x^4+70x^2+250\right)}{35\sqrt{x^4+5}}+\frac{2}{7}\sqrt[4]{-5}\left(5\sqrt{5}+14i\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)-4(-1)^{3/4}\sqrt[4]{5}E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (x*(250 + 70*x^2 + 125*x^4 + 14*x^6 + 15*x^8))/(35*Sqrt[5 + x^4])
 - 4*(-1)^(3/4)*5^(1/4)*EllipticE[I*ArcSinh[(-1/5)^(1/4)*x], -1]
+ (2*(-5)^(1/4)*(14*I + 5*Sqrt[5])*EllipticF[I*ArcSinh[(-1/5)^(1/
4)*x], -1])/7
```

Maple [C] time = 0.017, size = 180, normalized size = 0.9

$$\frac{2x^3}{5}\sqrt{x^4+5} + \frac{\frac{4i}{5}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-EllipticE\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}} + \frac{3x^5}{7}\sqrt{x^4+5}+\frac{10x}{7}\sqrt{x^4+5}-\frac{2\sqrt{5}}{7\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(3*x^2+2)*(x^4+5)^(1/2),x)
```

```
[Out] 2/5*x^3*(x^4+5)^(1/2)+4/5*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2
)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x
*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))
^(1/2),I))+3/7*x^5*(x^4+5)^(1/2)+10/7*x*(x^4+5)^(1/2)-2/7*5^(1/2)
/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2
)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+5}\left(3x^2+2\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(3x^4+2x^2\right)\sqrt{x^4+5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^2,x, algorithm="fricas")
```

```
[Out] integral((3*x^4 + 2*x^2)*sqrt(x^4 + 5), x)
```

Sympy [A] time = 3.67311, size = 78, normalized size = 0.41

$$\frac{3\sqrt{5}x^5\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\,\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{5}x^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_pol
ar(I*pi)/5)/(4*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((-1/2,
 3/4), (7/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+5}\left(3\,x^2+2\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^2, x)
```

3.17 $\int \left(2+3x^2\right)\sqrt{5+x^4}\,dx$

Optimal. Leaf size=176

$$\frac{1}{15}\left(9x^2+10\right)\sqrt{x^4+5}x+\frac{6\sqrt{x^4+5}x}{x^2+\sqrt{5}}+\frac{\sqrt[4]{5}\left(9+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt{x^4+5}}-\frac{6\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}$$

```
[Out] (6*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (x*(10 + 9*x^2)*Sqrt[5 + x^
4])/15 - (6*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2
)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4
)*(9 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^
2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(3*Sqrt[5 + x^4])
```

Rubi [A] time = 0.134484, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{1}{15}\left(9x^2+10\right)\sqrt{x^4+5}x+\frac{6\sqrt{x^4+5}x}{x^2+\sqrt{5}}+\frac{\sqrt[4]{5}\left(9+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt{x^4+5}}-\frac{6\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)*Sqrt[5 + x^4],x]
```


```
[Out] (6*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (x*(10 + 9*x^2)*Sqrt[5 + x^
4])/15 - (6*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2
)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4
)*(9 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^
2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(3*Sqrt[5 + x^4])
```

Rubi in Sympy [A] time = 12.5323, size = 175, normalized size = 0.99

$$\frac{x\left(9x^2+10\right)\sqrt{x^4+5}}{15}+\frac{6x\sqrt{x^4+5}}{x^2+\sqrt{5}}-\frac{6\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{\sqrt{x^4+5}}$$
$$+\frac{\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(2\sqrt{5}+9\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{3\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] x*(9*x**2 + 10)*sqrt(x**4 + 5)/15 + 6*x*sqrt(x**4 + 5)/(x**2 + sq
rt(5)) - 6*5**(1/4)*sqrt((x**4 + 5)/(sqrt(5)*x**2/5 + 1)**2)*(sqr
t(5)*x**2/5 + 1)*elliptic_e(2*atan(5**(3/4)*x/5), 1/2)/sqrt(x**4
+ 5) + 5**(1/4)*sqrt((x**4 + 5)/(sqrt(5)*x**2/5 + 1)**2)*(2*sqrt(
5) + 9)*(sqrt(5)*x**2/5 + 1)*elliptic_f(2*atan(5**(3/4)*x/5), 1/2
)/(3*sqrt(x**4 + 5))
```

Mathematica [C] time = 0.199219, size = 96, normalized size = 0.55

$$\frac{x\left(9x^6+10x^4+45x^2+50\right)}{15\sqrt{x^4+5}}+\frac{2}{3}\sqrt[4]{-5}\left(-2\sqrt{5}+9i\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)$$
$$-6(-1)^{3/4}\sqrt[4]{5}E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (x*(50 + 45*x^2 + 10*x^4 + 9*x^6))/(15*Sqrt[5 + x^4]) - 6*(-1)^(3
/4)*5^(1/4)*EllipticE[I*ArcSinh[(-1/5)^(1/4)*x], -1] + (2*(-5)^(1
/4)*(9*I - 2*Sqrt[5])*EllipticF[I*ArcSinh[(-1/5)^(1/4)*x], -1])/3
```

Maple [C] time = 0.015, size = 168, normalized size = 1.

$$\frac{2x}{3}\sqrt{x^4+5}+\frac{4\sqrt{5}}{15\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\mathit{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}}+\frac{3x^3}{5}\sqrt{x^4+5}+\frac{\frac{6i}{5}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(\mathit{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-\mathit{EllipticE}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5)^(1/2),x)
```

```
[Out] 2/3*x*(x^4+5)^(1/2)+4/15*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2
)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1
/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)+3/5*x^3*(x^4+5)^(1/2)+6/5*I/(I*
5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1
/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-E
llipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+5}\left(3x^2+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{x^4+5}\left(3x^2+2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2), x)
```

Sympy [A] time = 3.21481, size = 76, normalized size = 0.43

$$\frac{3\sqrt{5}x^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\,\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_pol
ar(I*pi)/5)/(4*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/
4), (5/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+5}\left(3\,x^2+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2), x)
```

3.18 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx$

Optimal. Leaf size=171

$$\frac{4\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}-\frac{(2-x^2)\sqrt{x^4+5}}{x}+\frac{\sqrt[4]{5}\left(2+\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}-\frac{4\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}$$

```
[Out] -(((2 - x^2)*Sqrt[5 + x^4])/x) + (4*x*Sqrt[5 + x^4])/(Sqrt[5] + x
^2) - (4*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2
]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4)*(
2 + Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*El
lipticF[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]
```

Rubi [A] time = 0.154998, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{4\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}-\frac{(2-x^2)\sqrt{x^4+5}}{x}+\frac{\sqrt[4]{5}\left(2+\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}-\frac{4\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^2,x]
```


```
[Out] -(((2 - x^2)*Sqrt[5 + x^4])/x) + (4*x*Sqrt[5 + x^4])/(Sqrt[5] + x
^2) - (4*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2
]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4)*(
2 + Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*El
lipticF[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]
```

Rubi in Sympy [A] time = 14.4775, size = 175, normalized size = 1.02

$$\frac{4x\sqrt{x^4+5}}{x^2+\sqrt{5}} - \frac{4\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{\sqrt{x^4+5}} + \frac{\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(6+3\sqrt{5}\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{3\sqrt{x^4+5}} - \frac{\left(-3x^2+6\right)\sqrt{x^4+5}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5)**(1/2)/x**2,x)
```

```
[Out] 4*x*sqrt(x**4 + 5)/(x**2 + sqrt(5)) - 4*5**(1/4)*sqrt((x**4 + 5)/
(sqrt(5)*x**2/5 + 1)**2)*(sqrt(5)*x**2/5 + 1)*elliptic_e(2*atan(5
**(3/4)*x/5), 1/2)/sqrt(x**4 + 5) + 5**(1/4)*sqrt((x**4 + 5)/(sqr
t(5)*x**2/5 + 1)**2)*(6 + 3*sqrt(5))*(sqrt(5)*x**2/5 + 1)*ellipti
c_f(2*atan(5**(3/4)*x/5), 1/2)/(3*sqrt(x**4 + 5)) - (-3*x**2 + 6)
*sqrt(x**4 + 5)/(3*x)
```

Mathematica [C] time = 0.161813, size = 108, normalized size = 0.63

$$\frac{x^6 - 2x^4 - 2\sqrt[4]{-5}\left(\sqrt{5}-2i\right)\sqrt{x^4+5}\,xF\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right) - 4(-1)^{3/4}\sqrt[4]{5}\sqrt{x^4+5}\,xE\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right) + 5x^2 \cdots}{x\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^2,x]
```

```
[Out] (-10 + 5*x^2 - 2*x^4 + x^6 - 4*(-1)^(3/4)*5^(1/4)*x*Sqrt[5 + x^4]
*EllipticE[I*ArcSinh[(-1/5)^(1/4)*x], -1] - 2*(-5)^(1/4)*(-2*I +
Sqrt[5])*x*Sqrt[5 + x^4]*EllipticF[I*ArcSinh[(-1/5)^(1/4)*x], -1]
)/(x*Sqrt[5 + x^4])
```

Maple [C] time = 0.022, size = 167, normalized size = 1.

$$x\sqrt{x^4+5} + \frac{2\sqrt{5}}{5\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\,EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}} - 2\frac{\sqrt{x^4+5}}{x} + \frac{\frac{4i}{5}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right) - EllipticE\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5)^(1/2)/x^2,x)
```

```
[Out] x*(x^4+5)^(1/2)+2/5*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2
)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*
5^(1/2)*(I*5^(1/2))^(1/2),I)-2*(x^4+5)^(1/2)/x+4/5*I/(I*5^(1/2))^
(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+
5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(
1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5}\,(3\,x^2+2)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+5}\,(3\,x^2+2)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2, x)
```

Sympy [A] time = 3.84646, size = 78, normalized size = 0.46

$$\frac{3\sqrt{5}x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\,\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2x\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5)**(1/2)/x**2,x)
```

```
[Out] 3*sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp_polar(
I*pi)/5)/(4*gamma(5/4)) + sqrt(5)*gamma(-1/4)*hyper((-1/2, -1/4),
 (3/4,), x**4*exp_polar(I*pi)/5)/(2*x*gamma(3/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5}\left(3\,x^2+2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2, x)
```

3.19 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx$

Optimal. Leaf size=192

$$-\frac{6\sqrt{x^4+5}}{x}+\frac{6\sqrt{x^4+5}x}{x^2+\sqrt{5}}+\frac{\left(2+9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{5}\sqrt{x^4+5}}-\frac{6\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}-\frac{\left(2-9x^2\right)\sqrt{x^4+5}}{3x^3}$$

```
[Out] (-6*Sqrt[5 + x^4])/x - ((2 - 9*x^2)*Sqrt[5 + x^4])/(3*x^3) + (6*x
*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (6*5^(1/4)*(Sqrt[5] + x^2)*Sqrt
[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2]
)/Sqrt[5 + x^4] + ((2 + 9*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)
/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(3*5^(1/
4)*Sqrt[5 + x^4])
```

Rubi [A] time = 0.202889, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{6\sqrt{x^4+5}}{x}+\frac{6\sqrt{x^4+5}x}{x^2+\sqrt{5}}+\frac{\left(2+9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{5}\sqrt{x^4+5}}-\frac{6\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}-\frac{\left(2-9x^2\right)\sqrt{x^4+5}}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^4,x]
```


```
[Out] (-6*Sqrt[5 + x^4])/x - ((2 - 9*x^2)*Sqrt[5 + x^4])/(3*x^3) + (6*x
*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (6*5^(1/4)*(Sqrt[5] + x^2)*Sqrt
[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2]
)/Sqrt[5 + x^4] + ((2 + 9*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)
/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(3*5^(1/
4)*Sqrt[5 + x^4])
```

Rubi in Sympy [A] time = 19.3798, size = 189, normalized size = 0.98

$$\frac{6x\sqrt{x^4+5}}{x^2+\sqrt{5}} - \frac{6\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{\sqrt{x^4+5}} + \frac{\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(2\sqrt{5}+45\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{15\sqrt{x^4+5}} - \frac{6\sqrt{x^4+5}}{x} - \frac{\left(-9x^2+2\right)\sqrt{x^4+5}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5)**(1/2)/x**4,x)
```

```
[Out] 6*x*sqrt(x**4 + 5)/(x**2 + sqrt(5)) - 6*5**(1/4)*sqrt((x**4 + 5)/
(sqrt(5)*x**2/5 + 1)**2)*(sqrt(5)*x**2/5 + 1)*elliptic_e(2*atan(5
**(3/4)*x/5), 1/2)/sqrt(x**4 + 5) + 5**(1/4)*sqrt((x**4 + 5)/(sqr
t(5)*x**2/5 + 1)**2)*(2*sqrt(5) + 45)*(sqrt(5)*x**2/5 + 1)*ellipt
ic_f(2*atan(5**(3/4)*x/5), 1/2)/(15*sqrt(x**4 + 5)) - 6*sqrt(x**4
 + 5)/x - (-9*x**2 + 2)*sqrt(x**4 + 5)/(3*x**3)
```

Mathematica [C] time = 0.223878, size = 98, normalized size = 0.51

$$\frac{1}{15}\left(-\frac{5\left(9x^6+2x^4+45x^2+10\right)}{x^3\sqrt{x^4+5}} + 2\sqrt[4]{-5}\left(-2\sqrt{5}+45i\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right) - 90(-1)^{3/4}\sqrt[4]{5}E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^4,x]
```

```
[Out] ((-5*(10 + 45*x^2 + 2*x^4 + 9*x^6))/(x^3*Sqrt[5 + x^4]) - 90*(-1)
^(3/4)*5^(1/4)*EllipticE[I*ArcSinh[(-1/5)^(1/4)*x], -1] + 2*(-5)^
(1/4)*(45*I - 2*Sqrt[5])*EllipticF[I*ArcSinh[(-1/5)^(1/4)*x], -1]
)/15
```

Maple [C] time = 0.023, size = 170, normalized size = 0.9

$$-\frac{2}{3\,x^{3}}\sqrt{x^{4}+5}+\frac{4\sqrt{5}}{75\sqrt{i\sqrt{5}}}\sqrt{25-5\,i\sqrt{5}x^{2}}\sqrt{25+5\,i\sqrt{5}x^{2}}\mathit{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^{4}+5}}-3\,\frac{\sqrt{x^{4}+5}}{x}+\frac{\frac{6\,i}{5}}{\sqrt{i\sqrt{5}}}\sqrt{25-5\,i\sqrt{5}x^{2}}\sqrt{25+5\,i\sqrt{5}x^{2}}\left(\mathit{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-\mathit{EllipticE}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^{4}+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5)^(1/2)/x^4,x)
```

```
[Out] -2/3*(x^4+5)^(1/2)/x^3+4/75*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(
1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*Elliptic
F(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-3*(x^4+5)^(1/2)/x+6/5*I/(I*5
^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/
2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-El
lipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^{4}+5}\left(3\,x^{2}+2\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^{4}+5}\left(3\,x^{2}+2\right)}{x^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4, x)
```

Sympy [A] time = 4.2087, size = 83, normalized size = 0.43

$$\frac{3\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4x\,\Gamma\left(\frac{3}{4}\right)}+\frac{\sqrt{5}\,\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2x^3\,\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5)**(1/2)/x**4,x)
```

```
[Out] 3*sqrt(5)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), x**4*exp_polar(
I*pi)/5)/(4*x*gamma(3/4)) + sqrt(5)*gamma(-3/4)*hyper((-3/4, -1/2
), (1/4,), x**4*exp_polar(I*pi)/5)/(2*x**3*gamma(1/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5}\left(3\,x^2+2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4, x)
```

3.20 $\int x^5 \left(2+3x^2\right) \left(5+x^4\right)^{3/2} dx$

Optimal. Leaf size=83

$$\frac{3}{14}\left(x^4+5\right)^{5/2}x^4-\frac{125}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{5}{24}\left(x^4+5\right)^{3/2}x^2-\frac{25}{16}\sqrt{x^4+5}x^2-\frac{1}{42}\left(18-7x^2\right)\left(x^4+5\right)^{5/2}$$

```
[Out] (-25*x^2*Sqrt[5 + x^4])/16 - (5*x^2*(5 + x^4)^(3/2))/24 + (3*x^4*
(5 + x^4)^(5/2))/14 - ((18 - 7*x^2)*(5 + x^4)^(5/2))/42 - (125*Ar
cSinh[x^2/Sqrt[5]])/16
```

Rubi [A] time = 0.157682, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{3}{14}\left(x^4+5\right)^{5/2}x^4-\frac{125}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{5}{24}\left(x^4+5\right)^{3/2}x^2-\frac{25}{16}\sqrt{x^4+5}x^2-\frac{1}{42}\left(18-7x^2\right)\left(x^4+5\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (-25*x^2*Sqrt[5 + x^4])/16 - (5*x^2*(5 + x^4)^(3/2))/24 + (3*x^4*
(5 + x^4)^(5/2))/14 - ((18 - 7*x^2)*(5 + x^4)^(5/2))/42 - (125*Ar
cSinh[x^2/Sqrt[5]])/16
```

Rubi in Sympy [A] time = 11.7038, size = 76, normalized size = 0.92

$$\frac{3x^4\left(x^4+5\right)^{\frac{5}{2}}}{14}-\frac{5x^2\left(x^4+5\right)^{\frac{3}{2}}}{24}-\frac{25x^2\sqrt{x^4+5}}{16}-\frac{\left(-70x^2+180\right)\left(x^4+5\right)^{\frac{5}{2}}}{420}-\frac{125\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] 3*x**4*(x**4 + 5)**(5/2)/14 - 5*x**2*(x**4 + 5)**(3/2)/24 - 25*x*
*2*sqrt(x**4 + 5)/16 - (-70*x**2 + 180)*(x**4 + 5)**(5/2)/420 - 1
25*asinh(sqrt(5)*x**2/5)/16
```

Mathematica [A] time = 0.0546489, size = 59, normalized size = 0.71

$$\frac{1}{336}\left(\sqrt{x^4+5}\left(72x^{12}+56x^{10}+576x^8+490x^6+360x^4+525x^2-3600\right)-2625\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[5 + x^4]*(-3600 + 525*x^2 + 360*x^4 + 490*x^6 + 576*x^8 + 5
6*x^10 + 72*x^12) - 2625*ArcSinh[x^2/Sqrt[5]])/336
```

Maple [A] time = 0.032, size = 73, normalized size = 0.9

$$\frac{x^{10}}{6}\sqrt{x^4+5}+\frac{35\,x^6}{24}\sqrt{x^4+5}+\frac{25\,x^2}{16}\sqrt{x^4+5}-\frac{125}{16}Arcsinh\left(\frac{\sqrt{5}x^2}{5}\right)+\frac{\left(3\,x^4-6\right)\left(x^8+10\,x^4+25\right)}{14}\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 1/6*x^10*(x^4+5)^(1/2)+35/24*x^6*(x^4+5)^(1/2)+25/16*x^2*(x^4+5)^
(1/2)-125/16*arcsinh(1/5*5^(1/2)*x^2)+3/14*(x^4+5)^(1/2)*(x^4-2)*
(x^8+10*x^4+25)
```

Maxima [A] time = 0.776728, size = 171, normalized size = 2.06

$$\frac{3}{14}\left(x^4+5\right)^{\frac{7}{2}}-\frac{3}{2}\left(x^4+5\right)^{\frac{5}{2}}-\frac{125\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{8\left(x^4+5\right)^{\frac{3}{2}}}{x^6}-\frac{3\left(x^4+5\right)^{\frac{5}{2}}}{x^{10}}\right)}{48\left(\frac{3\left(x^4+5\right)}{x^4}-\frac{3\left(x^4+5\right)^2}{x^8}+\frac{\left(x^4+5\right)^3}{x^{12}}-1\right)}-\frac{125}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{125}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^5,x, algorithm="maxima")
```

```
[Out] 3/14*(x^4 + 5)^(7/2) - 3/2*(x^4 + 5)^(5/2) - 125/48*(3*sqrt(x^4 +
 5)/x^2 - 8*(x^4 + 5)^(3/2)/x^6 - 3*(x^4 + 5)^(5/2)/x^10)/(3*(x^4
 + 5)/x^4 - 3*(x^4 + 5)^2/x^8 + (x^4 + 5)^3/x^12 - 1) - 125/32*lo
```

```
g(sqrt(x^4 + 5)/x^2 + 1) + 125/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 0.264553, size = 352, normalized size = 4.24

$$-\frac{4608\,x^{28} + 3584\,x^{26} + 88704\,x^{24} + 71680\,x^{22} + 624960\,x^{20} + 532000\,x^{18} + 1751400\,x^{16} + 1827000\,x^{14} + 189000\,x^{12} + 29312}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^5,x, algorithm="fricas")
```

```
[Out] -1/336*(4608*x^28 + 3584*x^26 + 88704*x^24 + 71680*x^22 + 624960*
x^20 + 532000*x^18 + 1751400*x^16 + 1827000*x^14 + 189000*x^12 +
2931250*x^10 - 7875000*x^8 + 1946875*x^6 - 11025000*x^4 + 328125*
x^2 - 2625*(64*x^14 + 560*x^10 + 1400*x^6 + 875*x^2 - (64*x^12 +
400*x^8 + 600*x^4 + 125)*sqrt(x^4 + 5))*log(-x^2 + sqrt(x^4 + 5))
 - (4608*x^26 + 3584*x^24 + 77184*x^22 + 62720*x^20 + 446400*x^18
 + 386400*x^16 + 840600*x^14 + 1029000*x^12 - 1008000*x^10 + 1163
750*x^8 - 4725000*x^6 + 459375*x^4 - 3150000*x^2)*sqrt(x^4 + 5) -
 2250000)/(64*x^14 + 560*x^10 + 1400*x^6 + 875*x^2 - (64*x^12 + 4
00*x^8 + 600*x^4 + 125)*sqrt(x^4 + 5))
```

Sympy [A] time = 34.4574, size = 131, normalized size = 1.58

$$\frac{x^{14}}{6\sqrt{x^4+5}} + \frac{3x^{12}\sqrt{x^4+5}}{14} + \frac{55x^{10}}{24\sqrt{x^4+5}} + \frac{12x^8\sqrt{x^4+5}}{7} + \frac{425x^6}{48\sqrt{x^4+5}} + \frac{15x^4\sqrt{x^4+5}}{14} + \frac{125x^2}{16\sqrt{x^4+5}} - \frac{75\sqrt{x^4+5}}{7} - \frac{125\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] x**14/(6*sqrt(x**4 + 5)) + 3*x**12*sqrt(x**4 + 5)/14 + 55*x**10/(
24*sqrt(x**4 + 5)) + 12*x**8*sqrt(x**4 + 5)/7 + 425*x**6/(48*sqrt
(x**4 + 5)) + 15*x**4*sqrt(x**4 + 5)/14 + 125*x**2/(16*sqrt(x**4
+ 5)) - 75*sqrt(x**4 + 5)/7 - 125*asinh(sqrt(5)*x**2/5)/16
```

GIAC/XCAS [A] time = 0.263861, size = 88, normalized size = 1.06

$$\frac{1}{336}\sqrt{x^4+5}\left(\left(2\left(\left(4\left(\left(9x^2+7\right)x^2+72\right)x^2+245\right)x^2+180\right)x^2+525\right)x^2-3600\right)+\frac{125}{16}\ln\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^5,x, algorithm="giac")
```

```
[Out] 1/336*sqrt(x^4 + 5)*((2*((4*((9*x^2 + 7)*x^2 + 72)*x^2 + 245)*x^2
 + 180)*x^2 + 525)*x^2 - 3600) + 125/16*ln(-x^2 + sqrt(x^4 + 5))
```

3.21 $\int x^3 \left(2+3x^2\right) \left(5+x^4\right)^{3/2} dx$

Optimal. Leaf size=67

$$-\frac{375}{32}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{20}\left(5x^2+4\right)\left(x^4+5\right)^{5/2}-\frac{5}{16}x^2\left(x^4+5\right)^{3/2}-\frac{75}{32}x^2\sqrt{x^4+5}$$

```
[Out] (-75*x^2*Sqrt[5 + x^4])/32 - (5*x^2*(5 + x^4)^(3/2))/16 + ((4 + 5
*x^2)*(5 + x^4)^(5/2))/20 - (375*ArcSinh[x^2/Sqrt[5]])/32
```

Rubi [A] time = 0.10457, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{375}{32}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{20}\left(5x^2+4\right)\left(x^4+5\right)^{5/2}-\frac{5}{16}x^2\left(x^4+5\right)^{3/2}-\frac{75}{32}x^2\sqrt{x^4+5}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (-75*x^2*Sqrt[5 + x^4])/32 - (5*x^2*(5 + x^4)^(3/2))/16 + ((4 + 5
*x^2)*(5 + x^4)^(5/2))/20 - (375*ArcSinh[x^2/Sqrt[5]])/32
```

Rubi in Sympy [A] time = 9.05241, size = 61, normalized size = 0.91

$$-\frac{5x^2\left(x^4+5\right)^{\frac{3}{2}}}{16}-\frac{75x^2\sqrt{x^4+5}}{32}+\frac{\left(15x^2+12\right)\left(x^4+5\right)^{\frac{5}{2}}}{60}-\frac{375\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] -5*x**2*(x**4 + 5)**(3/2)/16 - 75*x**2*sqrt(x**4 + 5)/32 + (15*x*
*2 + 12)*(x**4 + 5)**(5/2)/60 - 375*asinh(sqrt(5)*x**2/5)/32
```

Mathematica [A] time = 0.0484758, size = 54, normalized size = 0.81

$$\frac{1}{160}\left(\sqrt{x^4+5}\left(40x^{10}+32x^8+350x^6+320x^4+375x^2+800\right)-1875\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[5 + x^4]*(800 + 375*x^2 + 320*x^4 + 350*x^6 + 32*x^8 + 40*x
^10) - 1875*ArcSinh[x^2/Sqrt[5]])/160
```

Maple [A] time = 0.011, size = 58, normalized size = 0.9

$$\frac{1}{5}\left(x^{4}+5\right)^{\frac{5}{2}}+\frac{x^{10}}{4}\sqrt{x^{4}+5}+\frac{35\,x^{6}}{16}\sqrt{x^{4}+5}+\frac{75\,x^{2}}{32}\sqrt{x^{4}+5}-\frac{375}{32}Arcsinh\left(\frac{\sqrt{5}x^{2}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 1/5*(x^4+5)^(5/2)+1/4*x^10*(x^4+5)^(1/2)+35/16*x^6*(x^4+5)^(1/2)+
75/32*x^2*(x^4+5)^(1/2)-375/32*arcsinh(1/5*5^(1/2)*x^2)
```

Maxima [A] time = 0.781443, size = 159, normalized size = 2.37

$$\frac{1}{5}\left(x^{4}+5\right)^{\frac{5}{2}}-\frac{125\left(\frac{3\sqrt{x^{4}+5}}{x^{2}}-\frac{8\left(x^{4}+5\right)^{\frac{3}{2}}}{x^{6}}-\frac{3\left(x^{4}+5\right)^{\frac{5}{2}}}{x^{10}}\right)}{32\left(\frac{3\left(x^{4}+5\right)}{x^{4}}-\frac{3\left(x^{4}+5\right)^{2}}{x^{8}}+\frac{\left(x^{4}+5\right)^{3}}{x^{12}}-1\right)}-\frac{375}{64}\log\left(\frac{\sqrt{x^{4}+5}}{x^{2}}+1\right)+\frac{375}{64}\log\left(\frac{\sqrt{x^{4}+5}}{x^{2}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^3,x, algorithm="maxima")
```

```
[Out] 1/5*(x^4 + 5)^(5/2) - 125/32*(3*sqrt(x^4 + 5)/x^2 - 8*(x^4 + 5)^(
3/2)/x^6 - 3*(x^4 + 5)^(5/2)/x^10)/(3*(x^4 + 5)/x^4 - 3*(x^4 + 5)
^2/x^8 + (x^4 + 5)^3/x^12 - 1) - 375/64*log(sqrt(x^4 + 5)/x^2 + 1
) + 375/64*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 0.267292, size = 312, normalized size = 4.66

$$-\frac{1280\,x^{24}+1024\,x^{22}+24000\,x^{20}+20480\,x^{18}+162000\,x^{16}+158400\,x^{14}+482500\,x^{12}+584000\,x^{10}+618750\,x^{8}+1000000\,x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^3,x, algorithm="fricas")
```

```
[Out] -1/160*(1280*x^24 + 1024*x^22 + 24000*x^20 + 20480*x^18 + 162000*
x^16 + 158400*x^14 + 482500*x^12 + 584000*x^10 + 618750*x^8 + 100
0000*x^6 + 281250*x^4 + 600000*x^2 - 1875*(32*x^12 + 240*x^8 + 45
0*x^4 - 2*(16*x^10 + 80*x^6 + 75*x^2)*sqrt(x^4 + 5) + 125)*log(-x
^2 + sqrt(x^4 + 5)) - (1280*x^22 + 1024*x^20 + 20800*x^18 + 17920
*x^16 + 114000*x^14 + 116800*x^12 + 252500*x^10 + 340000*x^8 + 21
2500*x^6 + 400000*x^4 + 46875*x^2 + 100000)*sqrt(x^4 + 5))/(32*x^
12 + 240*x^8 + 450*x^4 - 2*(16*x^10 + 80*x^6 + 75*x^2)*sqrt(x^4 +
 5) + 125)
```

Sympy [A] time = 27.1352, size = 124, normalized size = 1.85

$$\frac{x^{14}}{4\sqrt{x^4+5}}+\frac{55x^{10}}{16\sqrt{x^4+5}}+\frac{x^8\sqrt{x^4+5}}{5}+\frac{425x^6}{32\sqrt{x^4+5}}+\frac{x^4\sqrt{x^4+5}}{3}+\frac{375x^2}{32\sqrt{x^4+5}}+\frac{5\left(x^4+5\right)^{\frac{3}{2}}}{3}-\frac{10\sqrt{x^4+5}}{3}-\frac{375\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] x**14/(4*sqrt(x**4 + 5)) + 55*x**10/(16*sqrt(x**4 + 5)) + x**8*sq
rt(x**4 + 5)/5 + 425*x**6/(32*sqrt(x**4 + 5)) + x**4*sqrt(x**4 +
5)/3 + 375*x**2/(32*sqrt(x**4 + 5)) + 5*(x**4 + 5)**(3/2)/3 - 10*
sqrt(x**4 + 5)/3 - 375*asinh(sqrt(5)*x**2/5)/32
```

GIAC/XCAS [A] time = 0.265655, size = 80, normalized size = 1.19

$$\frac{1}{160}\sqrt{x^4+5}\left(\left(2\left(\left(4\left(5x^2+4\right)x^2+175\right)x^2+160\right)x^2+375\right)x^2+800\right)+\frac{375}{32}\ln\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^3,x, algorithm="giac")
```

```
[Out] 1/160*sqrt(x^4 + 5)*((2*((4*(5*x^2 + 4)*x^2 + 175)*x^2 + 160)*x^2
 + 375)*x^2 + 800) + 375/32*ln(-x^2 + sqrt(x^4 + 5))
```

3.22 $\int x\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=60

$$\frac{3}{10}\left(x^4+5\right)^{5/2}+\frac{75}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{4}x^2\left(x^4+5\right)^{3/2}+\frac{15}{8}x^2\sqrt{x^4+5}$$

```
[Out] (15*x^2*Sqrt[5 + x^4])/8 + (x^2*(5 + x^4)^(3/2))/4 + (3*(5 + x^4)
^(5/2))/10 + (75*ArcSinh[x^2/Sqrt[5]])/8
```

Rubi [A] time = 0.0720775, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{3}{10}\left(x^4+5\right)^{5/2}+\frac{75}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{4}x^2\left(x^4+5\right)^{3/2}+\frac{15}{8}x^2\sqrt{x^4+5}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (15*x^2*Sqrt[5 + x^4])/8 + (x^2*(5 + x^4)^(3/2))/4 + (3*(5 + x^4)
^(5/2))/10 + (75*ArcSinh[x^2/Sqrt[5]])/8
```

Rubi in Sympy [A] time = 7.308, size = 54, normalized size = 0.9

$$\frac{x^2\left(x^4+5\right)^{\frac{3}{2}}}{4}+\frac{15x^2\sqrt{x^4+5}}{8}+\frac{3\left(x^4+5\right)^{\frac{5}{2}}}{10}+\frac{75\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] x**2*(x**4 + 5)**(3/2)/4 + 15*x**2*sqrt(x**4 + 5)/8 + 3*(x**4 + 5
)**(5/2)/10 + 75*asinh(sqrt(5)*x**2/5)/8
```

Mathematica [A] time = 0.031343, size = 56, normalized size = 0.93

$$\frac{75}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^4+5}\left(\frac{3x^8}{5}+\frac{x^6}{2}+6x^4+\frac{25x^2}{4}+15\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[5 + x^4]*(15 + (25*x^2)/4 + 6*x^4 + x^6/2 + (3*x^8)/5))/2 +
 (75*ArcSinh[x^2/Sqrt[5]])/8
```

Maple [A] time = 0.019, size = 46, normalized size = 0.8

$$\frac{x^6}{4}\sqrt{x^4+5}+\frac{25\,x^2}{8}\sqrt{x^4+5}+\frac{75}{8}Arcsinh\left(\frac{\sqrt{5}x^2}{5}\right)+\frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 1/4*x^6*(x^4+5)^(1/2)+25/8*x^2*(x^4+5)^(1/2)+75/8*arcsinh(1/5*5^(
1/2)*x^2)+3/10*(x^4+5)^(5/2)
```

Maxima [A] time = 0.776508, size = 128, normalized size = 2.13

$$\frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}}+\frac{25\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{5\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{8\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)}+\frac{75}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{75}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x,x, algorithm="maxima")
```

```
[Out] 3/10*(x^4 + 5)^(5/2) + 25/8*(3*sqrt(x^4 + 5)/x^2 - 5*(x^4 + 5)^(3
/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 75/16*log(sqrt
(x^4 + 5)/x^2 + 1) - 75/16*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 0.28594, size = 271, normalized size = 4.52

$$-\frac{192\,x^{20}+160\,x^{18}+3600\,x^{16}+3400\,x^{14}+25500\,x^{12}+20750\,x^{10}+82500\,x^{8}+41875\,x^{6}+112500\,x^{4}+15625\,x^{2}+375\left(16\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x,x, algorithm="fricas")
```

```
[Out] -1/40*(192*x^20 + 160*x^18 + 3600*x^16 + 3400*x^14 + 25500*x^12 +
 20750*x^10 + 82500*x^8 + 41875*x^6 + 112500*x^4 + 15625*x^2 + 37
5*(16*x^10 + 100*x^6 + 125*x^2 - (16*x^8 + 60*x^4 + 25)*sqrt(x^4
+ 5))*log(-x^2 + sqrt(x^4 + 5)) - (192*x^18 + 160*x^16 + 3120*x^1
4 + 3000*x^12 + 18300*x^10 + 13750*x^8 + 45000*x^6 + 15625*x^4 +
37500*x^2)*sqrt(x^4 + 5) + 37500)/(16*x^10 + 100*x^6 + 125*x^2 -
(16*x^8 + 60*x^4 + 25)*sqrt(x^4 + 5))
```

Sympy [A] time = 17.1906, size = 109, normalized size = 1.82

$$\frac{x^{10}}{4\sqrt{x^4+5}}+\frac{3x^8\sqrt{x^4+5}}{10}+\frac{35x^6}{8\sqrt{x^4+5}}+\frac{x^4\sqrt{x^4+5}}{2}+\frac{125x^2}{8\sqrt{x^4+5}}+\frac{5\left(x^4+5\right)^{\frac{3}{2}}}{2}-5\sqrt{x^4+5}+\frac{75\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] x**10/(4*sqrt(x**4 + 5)) + 3*x**8*sqrt(x**4 + 5)/10 + 35*x**6/(8*
sqrt(x**4 + 5)) + x**4*sqrt(x**4 + 5)/2 + 125*x**2/(8*sqrt(x**4 +
 5)) + 5*(x**4 + 5)**(3/2)/2 - 5*sqrt(x**4 + 5) + 75*asinh(sqrt(5
)*x**2/5)/8
```

GIAC/XCAS [A] time = 0.267085, size = 70, normalized size = 1.17

$$\frac{1}{40}\sqrt{x^4+5}\left(\left(2\left(\left(6x^2+5\right)x^2+60\right)x^2+125\right)x^2+300\right)-\frac{75}{8}\ln\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x,x, algorithm="giac")
```

```
[Out] 1/40*sqrt(x^4 + 5)*((2*((6*x^2 + 5)*x^2 + 60)*x^2 + 125)*x^2 + 30
0) - 75/8*ln(-x^2 + sqrt(x^4 + 5))
```

3.23 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x}\,dx$

Optimal. Leaf size=78

$$-5\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{24}\left(9x^2+8\right)\left(x^4+5\right)^{3/2}+\frac{5}{16}\left(9x^2+16\right)\sqrt{x^4+5}$$

```
[Out] (5*(16 + 9*x^2)*Sqrt[5 + x^4])/16 + ((8 + 9*x^2)*(5 + x^4)^(3/2))
/24 + (225*ArcSinh[x^2/Sqrt[5]])/16 - 5*Sqrt[5]*ArcTanh[Sqrt[5 +
x^4]/Sqrt[5]]
```

Rubi [A] time = 0.197324, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$-5\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{24}\left(9x^2+8\right)\left(x^4+5\right)^{3/2}+\frac{5}{16}\left(9x^2+16\right)\sqrt{x^4+5}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x,x]
```

```
[Out] (5*(16 + 9*x^2)*Sqrt[5 + x^4])/16 + ((8 + 9*x^2)*(5 + x^4)^(3/2))
/24 + (225*ArcSinh[x^2/Sqrt[5]])/16 - 5*Sqrt[5]*ArcTanh[Sqrt[5 +
x^4]/Sqrt[5]]
```

Rubi in Sympy [A] time = 16.1055, size = 71, normalized size = 0.91

$$\frac{\left(9x^2+8\right)\left(x^4+5\right)^{\frac{3}{2}}}{24}+\frac{\left(45x^2+80\right)\sqrt{x^4+5}}{16}+\frac{225\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}-5\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5)**(3/2)/x,x)
```

```
[Out] (9*x**2 + 8)*(x**4 + 5)**(3/2)/24 + (45*x**2 + 80)*sqrt(x**4 + 5)
/16 + 225*asinh(sqrt(5)*x**2/5)/16 - 5*sqrt(5)*atanh(sqrt(5)*sqrt
(x**4 + 5)/5)
```

Mathematica [A] time = 0.100323, size = 68, normalized size = 0.87

$$-5\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{48}\sqrt{x^4+5}\left(18x^6+16x^4+225x^2+320\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x,x]
```

```
[Out] (Sqrt[5 + x^4]*(320 + 225*x^2 + 16*x^4 + 18*x^6))/48 + (225*ArcSi
nh[x^2/Sqrt[5]])/16 - 5*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]
```

Maple [A] time = 0.021, size = 75, normalized size = 1.

$$\frac{x^4}{3}\sqrt{x^4+5}+\frac{20}{3}\sqrt{x^4+5}-5\sqrt{5}Artanh\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)+\frac{3x^6}{8}\sqrt{x^4+5}+\frac{75x^2}{16}\sqrt{x^4+5}+\frac{225}{16}Arcsinh\left(\frac{\sqrt{5}x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5)^(3/2)/x,x)
```

```
[Out] 1/3*x^4*(x^4+5)^(1/2)+20/3*(x^4+5)^(1/2)-5*5^(1/2)*arctanh(5^(1/2
)/(x^4+5)^(1/2))+3/8*x^6*(x^4+5)^(1/2)+75/16*x^2*(x^4+5)^(1/2)+22
5/16*arcsinh(1/5*5^(1/2)*x^2)
```

Maxima [A] time = 0.785344, size = 186, normalized size = 2.38

$$\frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}}+\frac{5}{2}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+5\sqrt{x^4+5}+\frac{75\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{5\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{16\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)}$$
$$+\frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(x^4 + 5)^(3/2) + 5/2*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/
(sqrt(5) + sqrt(x^4 + 5))) + 5*sqrt(x^4 + 5) + 75/16*(3*sqrt(x^4
```

```
+ 5)/x^2 - 5*(x^4 + 5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/
x^8 - 1) + 225/32*log(sqrt(x^4 + 5)/x^2 + 1) - 225/32*log(sqrt(x^
4 + 5)/x^2 - 1)
```

Fricas [A] time = 0.289188, size = 354, normalized size = 4.54

$$-\frac{144\,x^{16} + 128\,x^{14} + 2880\,x^{12} + 3520\,x^{10} + 15300\,x^{8} + 20800\,x^{6} + 22500\,x^{4} + 32000\,x^{2} + 675\,\left(8\,x^{8} + 40\,x^{4} - 4\,\left(2\,x^{6} + 5\,x^{2}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x,x, algorithm="fricas")
```

```
[Out] -1/48*(144*x^16 + 128*x^14 + 2880*x^12 + 3520*x^10 + 15300*x^8 +
20800*x^6 + 22500*x^4 + 32000*x^2 + 675*(8*x^8 + 40*x^4 - 4*(2*x^
6 + 5*x^2)*sqrt(x^4 + 5) + 25)*log(-x^2 + sqrt(x^4 + 5)) + 240*(4
*sqrt(5)*(2*x^6 + 5*x^2)*sqrt(x^4 + 5) - sqrt(5)*(8*x^8 + 40*x^4
+ 25))*log((x^4 + sqrt(5)*x^2 - sqrt(x^4 + 5)*(x^2 + sqrt(5)) + 5
)/(x^4 - sqrt(x^4 + 5)*x^2)) - (144*x^14 + 128*x^12 + 2520*x^10 +
 3200*x^8 + 9450*x^6 + 13200*x^4 + 5625*x^2 + 8000)*sqrt(x^4 + 5)
)/(8*x^8 + 40*x^4 - 4*(2*x^6 + 5*x^2)*sqrt(x^4 + 5) + 25)
```

Sympy [A] time = 22.7554, size = 114, normalized size = 1.46

$$\frac{3x^{10}}{8\sqrt{x^4+5}} + \frac{105x^6}{16\sqrt{x^4+5}} + \frac{375x^2}{16\sqrt{x^4+5}} + \frac{\left(x^4+5\right)^{\frac{3}{2}}}{3} + 5\sqrt{x^4+5} + \frac{5\sqrt{5}\log\left(x^4\right)}{2} - 5\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right) + \frac{225\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5)**(3/2)/x,x)
```

```
[Out] 3*x**10/(8*sqrt(x**4 + 5)) + 105*x**6/(16*sqrt(x**4 + 5)) + 375*x
**2/(16*sqrt(x**4 + 5)) + (x**4 + 5)**(3/2)/3 + 5*sqrt(x**4 + 5)
+ 5*sqrt(5)*log(x**4)/2 - 5*sqrt(5)*log(sqrt(x**4/5 + 1) + 1) + 2
25*asinh(sqrt(5)*x**2/5)/16
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x, x)
```

3.24 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=81

$$-\frac{15}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(2-x^2)(x^4+5)^{3/2}}{2x^2}+\frac{3}{2}(x^2+5)\sqrt{x^4+5}$$

```
[Out] (3*(5 + x^2)*Sqrt[5 + x^4])/2 - ((2 - x^2)*(5 + x^4)^(3/2))/(2*x^
2) + (15*ArcSinh[x^2/Sqrt[5]])/2 - (15*Sqrt[5]*ArcTanh[Sqrt[5 + x
^4]/Sqrt[5]])/2
```

Rubi [A] time = 0.190416, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{15}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(2-x^2)(x^4+5)^{3/2}}{2x^2}+\frac{3}{2}(x^2+5)\sqrt{x^4+5}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^3,x]
```

```
[Out] (3*(5 + x^2)*Sqrt[5 + x^4])/2 - ((2 - x^2)*(5 + x^4)^(3/2))/(2*x^
2) + (15*ArcSinh[x^2/Sqrt[5]])/2 - (15*Sqrt[5]*ArcTanh[Sqrt[5 + x
^4]/Sqrt[5]])/2
```

Rubi in Sympy [A] time = 16.7501, size = 76, normalized size = 0.94

$$\frac{(12x^2+60)\sqrt{x^4+5}}{8}+\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}-\frac{15\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)}{2}-\frac{(-3x^2+6)(x^4+5)^{\frac{3}{2}}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5)**(3/2)/x**3,x)
```

```
[Out] (12*x**2 + 60)*sqrt(x**4 + 5)/8 + 15*asinh(sqrt(5)*x**2/5)/2 - 15
*sqrt(5)*atanh(sqrt(5)*sqrt(x**4 + 5)/5)/2 - (-3*x**2 + 6)*(x**4
+ 5)**(3/2)/(6*x**2)
```

Mathematica [A] time = 0.122688, size = 66, normalized size = 0.81

$$\frac{1}{2}\left(-15\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+15\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{\sqrt{x^4+5}\,(x^6+x^4+20x^2-10)}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^3,x]
```

```
[Out] ((Sqrt[5 + x^4]*(-10 + 20*x^2 + x^4 + x^6))/x^2 + 15*ArcSinh[x^2/
Sqrt[5]] - 15*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2
```

Maple [A] time = 0.023, size = 75, normalized size = 0.9

$$\frac{x^2}{2}\sqrt{x^4+5}+\frac{15}{2}Arcsinh\left(\frac{\sqrt{5}x^2}{5}\right)-5\frac{\sqrt{x^4+5}}{x^2}+\frac{x^4}{2}\sqrt{x^4+5}+10\sqrt{x^4+5}-\frac{15\sqrt{5}}{2}Artanh\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5)^(3/2)/x^3,x)
```

```
[Out] 1/2*x^2*(x^4+5)^(1/2)+15/2*arcsinh(1/5*5^(1/2)*x^2)-5*(x^4+5)^(1/
2)/x^2+1/2*x^4*(x^4+5)^(1/2)+10*(x^4+5)^(1/2)-15/2*5^(1/2)*arctan
h(5^(1/2)/(x^4+5)^(1/2))
```

Maxima [A] time = 0.7891, size = 165, normalized size = 2.04

$$\frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}}+\frac{15}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{15}{2}\sqrt{x^4+5}-\frac{5\sqrt{x^4+5}}{x^2}+\frac{5\sqrt{x^4+5}}{2x^2\left(\frac{x^4+5}{x^4}-1\right)}+\frac{15}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{15}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(x^4 + 5)^(3/2) + 15/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))
/(sqrt(5) + sqrt(x^4 + 5))) + 15/2*sqrt(x^4 + 5) - 5*sqrt(x^4 + 5
)/x^2 + 5/2*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 15/4*log(sq
```

```
rt(x^4 + 5)/x^2 + 1) - 15/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 0.299306, size = 370, normalized size = 4.57

$$-\frac{8\,x^{16} + 8\,x^{14} + 220\,x^{12} + 60\,x^{10} + 1300\,x^{8} - 100\,x^{6} + 2000\,x^{4} - 750\,x^{2} + 15\left(8\,x^{10} + 40\,x^{6} + 25\,x^{2} - 4\left(2\,x^{8} + 5\,x^{4}\right)\sqrt{x^{4} + 5}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(8*x^16 + 8*x^14 + 220*x^12 + 60*x^10 + 1300*x^8 - 100*x^6 +
 2000*x^4 - 750*x^2 + 15*(8*x^10 + 40*x^6 + 25*x^2 - 4*(2*x^8 + 5
*x^4)*sqrt(x^4 + 5))*log(-x^2 + sqrt(x^4 + 5)) + 15*(4*sqrt(5)*(2
*x^8 + 5*x^4)*sqrt(x^4 + 5) - sqrt(5)*(8*x^10 + 40*x^6 + 25*x^2))
*log((x^4 + sqrt(5)*x^2 - sqrt(x^4 + 5)*(x^2 + sqrt(5)) + 5)/(x^4
 - sqrt(x^4 + 5)*x^2)) - (8*x^14 + 8*x^12 + 200*x^10 + 40*x^8 + 8
25*x^6 - 175*x^4 + 500*x^2 - 250)*sqrt(x^4 + 5))/(8*x^10 + 40*x^6
 + 25*x^2 - 4*(2*x^8 + 5*x^4)*sqrt(x^4 + 5))
```

Sympy [A] time = 18.9371, size = 114, normalized size = 1.41

$$\frac{x^{6}}{2\sqrt{x^{4} + 5}} - \frac{5x^{2}}{2\sqrt{x^{4} + 5}} + \frac{\left(x^{4} + 5\right)^{\frac{3}{2}}}{2} + \frac{15\sqrt{x^{4} + 5}}{2} + \frac{15\sqrt{5}\log\left(x^{4}\right)}{4} - \frac{15\sqrt{5}\log\left(\sqrt{\frac{x^{4}}{5} + 1} + 1\right)}{2} + \frac{15\operatorname{asinh}\left(\frac{\sqrt{5}x^{2}}{5}\right)}{2} - \frac{25}{x^{2}\sqrt{x^{4} + 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5)**(3/2)/x**3,x)
```

```
[Out] x**6/(2*sqrt(x**4 + 5)) - 5*x**2/(2*sqrt(x**4 + 5)) + (x**4 + 5)*
*(3/2)/2 + 15*sqrt(x**4 + 5)/2 + 15*sqrt(5)*log(x**4)/4 - 15*sqrt
(5)*log(sqrt(x**4/5 + 1) + 1)/2 + 15*asinh(sqrt(5)*x**2/5)/2 - 25
/(x**2*sqrt(x**4 + 5))
```

<u>GIAC/XCAS</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+5\right)^{\frac{3}{2}} \left(3 x^2+2\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^3,x, algorithm="giac")

[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^3, x)
```

3.25 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=86

$$-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(2-3x^2)(x^4+5)^{3/2}}{4x^4}-\frac{3(15-2x^2)\sqrt{x^4+5}}{4x^2}$$

```
[Out] (-3*(15 - 2*x^2)*Sqrt[5 + x^4])/(4*x^2) - ((2 - 3*x^2)*(5 + x^4)^
(3/2))/(4*x^4) + (45*ArcSinh[x^2/Sqrt[5]])/4 - (3*Sqrt[5]*ArcTanh
[Sqrt[5 + x^4]/Sqrt[5]])/2
```

Rubi [A] time = 0.198322, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(2-3x^2)(x^4+5)^{3/2}}{4x^4}-\frac{3(15-2x^2)\sqrt{x^4+5}}{4x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^5,x]
```


```
[Out] (-3*(15 - 2*x^2)*Sqrt[5 + x^4])/(4*x^2) - ((2 - 3*x^2)*(5 + x^4)^
(3/2))/(4*x^4) + (45*ArcSinh[x^2/Sqrt[5]])/4 - (3*Sqrt[5]*ArcTanh
[Sqrt[5 + x^4]/Sqrt[5]])/2
```

Rubi in Sympy [A] time = 16.6795, size = 82, normalized size = 0.95

$$\frac{45\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}-\frac{3\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)}{2}-\frac{3(-8x^2+60)\sqrt{x^4+5}}{16x^2}-\frac{(-6x^2+4)(x^4+5)^{\frac{3}{2}}}{8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5)**(3/2)/x**5,x)
```

```
[Out] 45*asinh(sqrt(5)*x**2/5)/4 - 3*sqrt(5)*atanh(sqrt(5)*sqrt(x**4 +
5)/5)/2 - 3*(-8*x**2 + 60)*sqrt(x**4 + 5)/(16*x**2) - (-6*x**2 +
4)*(x**4 + 5)**(3/2)/(8*x**4)
```

Mathematica [A] time = 0.111896, size = 70, normalized size = 0.81

$$\frac{1}{4}\left(-6\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+45\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{\sqrt{x^4+5}\left(3x^6+4x^4-30x^2-10\right)}{x^4}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^5,x]
```

```
[Out] ((Sqrt[5 + x^4]*(-10 - 30*x^2 + 4*x^4 + 3*x^6))/x^4 + 45*ArcSinh[
x^2/Sqrt[5]] - 6*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/4
```

Maple [A] time = 0.023, size = 73, normalized size = 0.9

$$\sqrt{x^4+5}-\frac{5}{2\,x^4}\sqrt{x^4+5}-\frac{3\,\sqrt{5}}{2}Artanh\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)+\frac{3\,x^2}{4}\sqrt{x^4+5}+\frac{45}{4}Arcsinh\left(\frac{\sqrt{5}x^2}{5}\right)-\frac{15}{2\,x^2}\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5)^(3/2)/x^5,x)
```

```
[Out] (x^4+5)^(1/2)-5/2*(x^4+5)^(1/2)/x^4-3/2*5^(1/2)*arctanh(5^(1/2)/(
x^4+5)^(1/2))+3/4*x^2*(x^4+5)^(1/2)+45/4*arcsinh(1/5*5^(1/2)*x^2)
-15/2*(x^4+5)^(1/2)/x^2
```

Maxima [A] time = 0.785619, size = 166, normalized size = 1.93

$$\frac{3}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\sqrt{x^4+5}-\frac{15\sqrt{x^4+5}}{2\,x^2}+\frac{15\sqrt{x^4+5}}{4\,x^2\left(\frac{x^4+5}{x^4}-1\right)}-\frac{5\sqrt{x^4+5}}{2\,x^4}+\frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^5,x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 +
5))) + sqrt(x^4 + 5) - 15/2*sqrt(x^4 + 5)/x^2 + 15/4*sqrt(x^4 + 5
)/(x^2*((x^4 + 5)/x^4 - 1)) - 5/2*sqrt(x^4 + 5)/x^4 + 45/8*log(sq
```

```
rt(x^4 + 5)/x^2 + 1) - 45/8*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 0.298746, size = 370, normalized size = 4.3

$$-\frac{24\,x^{16} + 32\,x^{14} + 180\,x^{12} + 160\,x^{10} - 300\,x^{8} - 200\,x^{6} - 2250\,x^{4} - 1000\,x^{2} + 45\left(8\,x^{12} + 40\,x^{8} + 25\,x^{4} - 4\left(2\,x^{10} + 5\,x^{6}\right)\sqrt{x^{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(24*x^16 + 32*x^14 + 180*x^12 + 160*x^10 - 300*x^8 - 200*x^6
 - 2250*x^4 - 1000*x^2 + 45*(8*x^12 + 40*x^8 + 25*x^4 - 4*(2*x^10
 + 5*x^6)*sqrt(x^4 + 5))*log(-x^2 + sqrt(x^4 + 5)) + 6*(4*sqrt(5)
*(2*x^10 + 5*x^6)*sqrt(x^4 + 5) - sqrt(5)*(8*x^12 + 40*x^8 + 25*x
^4))*log((x^4 + sqrt(5)*x^2 - sqrt(x^4 + 5)*(x^2 + sqrt(5)) + 5)/
(x^4 - sqrt(x^4 + 5)*x^2)) - (24*x^14 + 32*x^12 + 120*x^10 + 80*x
^8 - 525*x^6 - 300*x^4 - 750*x^2 - 250)*sqrt(x^4 + 5))/(8*x^12 +
40*x^8 + 25*x^4 - 4*(2*x^10 + 5*x^6)*sqrt(x^4 + 5))
```

Sympy [A] time = 23.105, size = 133, normalized size = 1.55

$$\frac{3x^{6}}{4\sqrt{x^{4}+5}} - \frac{15x^{2}}{4\sqrt{x^{4}+5}} + \sqrt{x^{4}+5} + \frac{\sqrt{5}\log\left(x^{4}\right)}{2} - \sqrt{5}\log\left(\sqrt{\frac{x^{4}}{5}+1}+1\right) - \frac{\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^{2}}\right)}{2} + \frac{45\operatorname{asinh}\left(\frac{\sqrt{5}x^{2}}{5}\right)}{4} - \frac{5\sqrt{1+\frac{5}{x^{4}}}}{2x^{2}} - \frac{75}{2x^{2}\sqrt{x^{4}+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5)**(3/2)/x**5,x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) - 15*x**2/(4*sqrt(x**4 + 5)) + sqrt(x**
4 + 5) + sqrt(5)*log(x**4)/2 - sqrt(5)*log(sqrt(x**4/5 + 1) + 1)
- sqrt(5)*asinh(sqrt(5)/x**2)/2 + 45*asinh(sqrt(5)*x**2/5)/4 - 5*
sqrt(1 + 5/x**4)/(2*x**2) - 75/(2*x**2*sqrt(x**4 + 5))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^5,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^5, x)
```

3.26 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7} dx$

Optimal. Leaf size=82

$$-\frac{9}{4}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(4-9x^2)\sqrt{x^4+5}}{4x^2}-\frac{(9x^2+4)(x^4+5)^{3/2}}{12x^6}$$

```
[Out] -((4 - 9*x^2)*Sqrt[5 + x^4])/(4*x^2) - ((4 + 9*x^2)*(5 + x^4)^(3/
2))/(12*x^6) + ArcSinh[x^2/Sqrt[5]] - (9*Sqrt[5]*ArcTanh[Sqrt[5 +
 x^4]/Sqrt[5]])/4
```

Rubi [A] time = 0.195773, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{9}{4}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(4-9x^2)\sqrt{x^4+5}}{4x^2}-\frac{(9x^2+4)(x^4+5)^{3/2}}{12x^6}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^7,x]
```

```
[Out] -((4 - 9*x^2)*Sqrt[5 + x^4])/(4*x^2) - ((4 + 9*x^2)*(5 + x^4)^(3/
2))/(12*x^6) + ArcSinh[x^2/Sqrt[5]] - (9*Sqrt[5]*ArcTanh[Sqrt[5 +
 x^4]/Sqrt[5]])/4
```

Rubi in Sympy [A] time = 16.7594, size = 76, normalized size = 0.93

$$\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)-\frac{9\sqrt{5}\operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)}{4}-\frac{(-90x^2+40)\sqrt{x^4+5}}{40x^2}-\frac{(45x^2+20)(x^4+5)^{\frac{3}{2}}}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5)**(3/2)/x**7,x)
```

```
[Out] asinh(sqrt(5)*x**2/5) - 9*sqrt(5)*atanh(sqrt(5)*sqrt(x**4 + 5)/5)
/4 - (-90*x**2 + 40)*sqrt(x**4 + 5)/(40*x**2) - (45*x**2 + 20)*(x
**4 + 5)**(3/2)/(60*x**6)
```

Mathematica [A] time = 0.117886, size = 69, normalized size = 0.84

$$-\frac{9}{4}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{\sqrt{x^4+5}\left(18x^6-16x^4-45x^2-20\right)}{12x^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^7,x]
```

```
[Out] (Sqrt[5 + x^4]*(-20 - 45*x^2 - 16*x^4 + 18*x^6))/(12*x^6) + ArcSi
nh[x^2/Sqrt[5]] - (9*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/4
```

Maple [A] time = 0.024, size = 73, normalized size = 0.9

$$\mathit{Arcsinh}\left(\frac{\sqrt{5}x^2}{5}\right)-\frac{5}{3\,x^6}\sqrt{x^4+5}-\frac{4}{3\,x^2}\sqrt{x^4+5}+\frac{3}{2}\sqrt{x^4+5}-\frac{15}{4\,x^4}\sqrt{x^4+5}-\frac{9\,\sqrt{5}}{4}\mathit{Artanh}\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5)^(3/2)/x^7,x)
```

```
[Out] arcsinh(1/5*5^(1/2)*x^2)-5/3*(x^4+5)^(1/2)/x^6-4/3*(x^4+5)^(1/2)/
x^2+3/2*(x^4+5)^(1/2)-15/4*(x^4+5)^(1/2)/x^4-9/4*5^(1/2)*arctanh(
5^(1/2)/(x^4+5)^(1/2))
```

Maxima [A] time = 0.780308, size = 151, normalized size = 1.84

$$\frac{9}{8}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{3}{2}\sqrt{x^4+5}-\frac{\sqrt{x^4+5}}{x^2}-\frac{15\sqrt{x^4+5}}{4\,x^4}-\frac{\left(x^4+5\right)^{\frac{3}{2}}}{3\,x^6}+\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^7,x, algorithm="maxima")
```

```
[Out] 9/8*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 +
5))) + 3/2*sqrt(x^4 + 5) - sqrt(x^4 + 5)/x^2 - 15/4*sqrt(x^4 + 5)
/x^4 - 1/3*(x^4 + 5)^(3/2)/x^6 + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) -
```

```
1/2*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 0.29371, size = 356, normalized size = 4.34

$$-\frac{144\,x^{16} + 720\,x^{12} - 480\,x^{10} - 900\,x^{8} - 2400\,x^{6} - 4500\,x^{4} - 2000\,x^{2} + 12\left(8\,x^{14} + 40\,x^{10} + 25\,x^{6} - 4\left(2\,x^{12} + 5\,x^{8}\right)\sqrt{x^{4} + 5}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(144*x^16 + 720*x^12 - 480*x^10 - 900*x^8 - 2400*x^6 - 4500
*x^4 - 2000*x^2 + 12*(8*x^14 + 40*x^10 + 25*x^6 - 4*(2*x^12 + 5*x
^8)*sqrt(x^4 + 5))*log(-x^2 + sqrt(x^4 + 5)) + 27*(4*sqrt(5)*(2*x
^12 + 5*x^8)*sqrt(x^4 + 5) - sqrt(5)*(8*x^14 + 40*x^10 + 25*x^6))
*log((x^4 + sqrt(5)*x^2 - sqrt(x^4 + 5)*(x^2 + sqrt(5)) + 5)/(x^4
 - sqrt(x^4 + 5)*x^2)) - (144*x^14 + 360*x^10 - 480*x^8 - 1350*x^
6 - 1200*x^4 - 1125*x^2 - 500)*sqrt(x^4 + 5))/(8*x^14 + 40*x^10 +
 25*x^6 - 4*(2*x^12 + 5*x^8)*sqrt(x^4 + 5))
```

Sympy [A] time = 22.7124, size = 148, normalized size = 1.8

$$-\frac{x^{2}}{\sqrt{x^{4} + 5}} - \frac{\sqrt{1 + \frac{5}{x^{4}}}}{3} + \frac{3\sqrt{x^{4} + 5}}{2} + \frac{3\sqrt{5}\log\left(x^{4}\right)}{4} - \frac{3\sqrt{5}\log\left(\sqrt{\frac{x^{4}}{5} + 1} + 1\right)}{2} - \frac{3\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^{2}}\right)}{4} + \operatorname{asinh}\left(\frac{\sqrt{5}x^{2}}{5}\right) - \frac{15\sqrt{1 + \frac{5}{x^{4}}}}{4x^{2}} - \frac{5}{x^{2}\sqrt{x^{4} + 5}} - \frac{5\sqrt{1 + \frac{5}{x^{4}}}}{3x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5)**(3/2)/x**7,x)
```

```
[Out] -x**2/sqrt(x**4 + 5) - sqrt(1 + 5/x**4)/3 + 3*sqrt(x**4 + 5)/2 +
3*sqrt(5)*log(x**4)/4 - 3*sqrt(5)*log(sqrt(x**4/5 + 1) + 1)/2 - 3
*sqrt(5)*asinh(sqrt(5)/x**2)/4 + asinh(sqrt(5)*x**2/5) - 15*sqrt(
1 + 5/x**4)/(4*x**2) - 5/(x**2*sqrt(x**4 + 5)) - 5*sqrt(1 + 5/x**
4)/(3*x**4)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^7,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^7, x)
```

3.27 $\int x^4 \left(2+3 x^2\right) \left(5+x^4\right)^{3/2} \, dx$

Optimal. Leaf size=235

$$\frac{200}{77}\sqrt{x^4+5}x+\frac{20}{13}\sqrt{x^4+5}x^3-\frac{300\sqrt{x^4+5}x}{13\left(x^2+\sqrt{5}\right)} -\frac{50\sqrt[4]{5}\left(231+26\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{1001\sqrt{x^4+5}} +\frac{300\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{13\sqrt{x^4+5}} +\frac{1}{143}\left(33x^2+26\right)\left(x^4+5\right)^{3/2}x^5+\frac{10\left(77x^2+78\right)\sqrt{x^4+5}x^5}{1001}$$

```
[Out] (200*x*Sqrt[5 + x^4])/77 + (20*x^3*Sqrt[5 + x^4])/13 - (300*x*Sqrt[5 + x^4])/(13*(Sqrt[5] + x^2)) + (10*x^5*(78 + 77*x^2)*Sqrt[5 + x^4])/1001 + (x^5*(26 + 33*x^2)*(5 + x^4)^(3/2))/143 + (300*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(13*Sqrt[5 + x^4]) - (50*5^(1/4)*(231 + 26*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(1001*Sqrt[5 + x^4])
```

Rubi [A] time = 0.337565, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{200}{77}\sqrt{x^4+5}x+\frac{20}{13}\sqrt{x^4+5}x^3-\frac{300\sqrt{x^4+5}x}{13\left(x^2+\sqrt{5}\right)} -\frac{50\sqrt[4]{5}\left(231+26\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{1001\sqrt{x^4+5}} +\frac{300\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{13\sqrt{x^4+5}} +\frac{1}{143}\left(33x^2+26\right)\left(x^4+5\right)^{3/2}x^5+\frac{10\left(77x^2+78\right)\sqrt{x^4+5}x^5}{1001}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (200*x*Sqrt[5 + x^4])/77 + (20*x^3*Sqrt[5 + x^4])/13 - (300*x*Sqr
t[5 + x^4])/(13*(Sqrt[5] + x^2)) + (10*x^5*(78 + 77*x^2)*Sqrt[5 +
 x^4])/1001 + (x^5*(26 + 33*x^2)*(5 + x^4)^(3/2))/143 + (300*5^(1
/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2
*ArcTan[x/5^(1/4)], 1/2])/(13*Sqrt[5 + x^4]) - (50*5^(1/4)*(231 +
 26*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*El
lipticF[2*ArcTan[x/5^(1/4)], 1/2])/(1001*Sqrt[5 + x^4])
```

Rubi in Sympy [A] time = 31.0299, size = 235, normalized size = 1.

$$\frac{x^5\left(33x^2+26\right)\left(x^4+5\right)^{\frac{3}{2}}}{143}+\frac{10x^5\left(231x^2+234\right)\sqrt{x^4+5}}{3003}+\frac{20x^3\sqrt{x^4+5}}{13}+\frac{200x\sqrt{x^4+5}}{77}-\frac{300x\sqrt{x^4+5}}{13\left(x^2+\sqrt{5}\right)}+\frac{300\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{13\sqrt{x^4+5}}-\frac{10\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(1170\sqrt{5}+10395\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{9009\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] x**5*(33*x**2 + 26)*(x**4 + 5)**(3/2)/143 + 10*x**5*(231*x**2 + 2
34)*sqrt(x**4 + 5)/3003 + 20*x**3*sqrt(x**4 + 5)/13 + 200*x*sqrt(
x**4 + 5)/77 - 300*x*sqrt(x**4 + 5)/(13*(x**2 + sqrt(5))) + 300*5
**(1/4)*sqrt((x**4 + 5)/(sqrt(5)*x**2/5 + 1)**2)*(sqrt(5)*x**2/5
+ 1)*elliptic_e(2*atan(5**(3/4)*x/5), 1/2)/(13*sqrt(x**4 + 5)) -
10*5**(1/4)*sqrt((x**4 + 5)/(sqrt(5)*x**2/5 + 1)**2)*(1170*sqrt(5
) + 10395)*(sqrt(5)*x**2/5 + 1)*elliptic_f(2*atan(5**(3/4)*x/5),
1/2)/(9009*sqrt(x**4 + 5))
```

Mathematica [C] time = 0.229889, size = 115, normalized size = 0.49

$$\frac{\frac{x\left(231x^{14}+182x^{12}+3080x^{10}+2600x^8+11165x^6+11050x^4+7700x^2+13000\right)}{\sqrt{x^4+5}}+100\sqrt[4]{-5}\left(26\sqrt{5}-231i\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)+23100(-1)^{3}}{1001}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] ((x*(13000 + 7700*x^2 + 11050*x^4 + 11165*x^6 + 2600*x^8 + 3080*x
^10 + 182*x^12 + 231*x^14))/Sqrt[5 + x^4] + 23100*(-1)^(3/4)*5^(1
/4)*EllipticE[I*ArcSinh[(-1/5)^(1/4)*x], -1] + 100*(-5)^(1/4)*(-2
31*I + 26*Sqrt[5])*EllipticF[I*ArcSinh[(-1/5)^(1/4)*x], -1])/1001
```

Maple [C] time = 0.025, size = 216, normalized size = 0.9

$$\frac{2x^9}{11}\sqrt{x^4+5}+\frac{130x^5}{77}\sqrt{x^4+5}+\frac{200x}{77}\sqrt{x^4+5} - \frac{40\sqrt{5}}{77\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\mathit{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}} + \frac{3x^{11}}{13}\sqrt{x^4+5}+\frac{25x^7}{13}\sqrt{x^4+5}+\frac{20x^3}{13}\sqrt{x^4+5} - \frac{\frac{60i}{13}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(\mathit{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-\mathit{EllipticE}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 2/11*x^9*(x^4+5)^(1/2)+130/77*x^5*(x^4+5)^(1/2)+200/77*x*(x^4+5)^
(1/2)-40/77*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*
(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*
(I*5^(1/2))^(1/2),I)+3/13*x^11*(x^4+5)^(1/2)+25/13*x^7*(x^4+5)^(1
/2)+20/13*x^3*(x^4+5)^(1/2)-60/13*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(
1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(Ellipti
cF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*
5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+5\right)^{\frac{3}{2}}\left(3x^2+2\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^4,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(3\,x^{10}+2\,x^{8}+15\,x^{6}+10\,x^{4}\right)\sqrt{x^{4}+5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^4,x, algorithm="fricas")
```

```
[Out] integral((3*x^10 + 2*x^8 + 15*x^6 + 10*x^4)*sqrt(x^4 + 5), x)
```

Sympy [A] time = 10.9589, size = 160, normalized size = 0.68

$$\frac{3\sqrt{5}x^{11}\,\Gamma\left(\frac{11}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\,\Gamma\left(\frac{15}{4}\right)}+\frac{\sqrt{5}x^{9}\,\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\,\Gamma\left(\frac{13}{4}\right)}+\frac{15\sqrt{5}x^{7}\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\,\Gamma\left(\frac{11}{4}\right)}+\frac{5\sqrt{5}x^{5}\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), x**4*exp
_polar(I*pi)/5)/(4*gamma(15/4)) + sqrt(5)*x**9*gamma(9/4)*hyper((
-1/2, 9/4), (13/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(13/4)) + 15
*sqrt(5)*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), x**4*exp_pol
ar(I*pi)/5)/(4*gamma(11/4)) + 5*sqrt(5)*x**5*gamma(5/4)*hyper((-1
/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(9/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^{4}+5\right)^{\frac{3}{2}}\left(3\,x^{2}+2\right)x^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^4,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^4, x)
```

3.28 $\int x^2 \left(2+3x^2\right) \left(5+x^4\right)^{3/2} \, dx$

Optimal. Leaf size=219

$$\frac{300}{77}\sqrt{x^4+5}x+\frac{40\sqrt{x^4+5}x}{3\left(x^2+\sqrt{5}\right)}+\frac{10\sqrt[4]{5}\left(154-45\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{231\sqrt{x^4+5}}$$
$$-\frac{40\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt{x^4+5}}$$
$$+\frac{1}{99}\left(27x^2+22\right)\left(x^4+5\right)^{3/2}x^3+\frac{2}{231}\left(135x^2+154\right)\sqrt{x^4+5}x^3$$

```
[Out] (300*x*Sqrt[5 + x^4])/77 + (40*x*Sqrt[5 + x^4])/(3*(Sqrt[5] + x^2
)) + (2*x^3*(154 + 135*x^2)*Sqrt[5 + x^4])/231 + (x^3*(22 + 27*x^
2)*(5 + x^4)^(3/2))/99 - (40*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^
4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(3*Sqr
t[5 + x^4]) + (10*5^(1/4)*(154 - 45*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt
[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2]
)/(231*Sqrt[5 + x^4])
```

Rubi [A] time = 0.29666, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{300}{77}\sqrt{x^4+5}x+\frac{40\sqrt{x^4+5}x}{3\left(x^2+\sqrt{5}\right)}+\frac{10\sqrt[4]{5}\left(154-45\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{231\sqrt{x^4+5}}$$
$$-\frac{40\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt{x^4+5}}$$
$$+\frac{1}{99}\left(27x^2+22\right)\left(x^4+5\right)^{3/2}x^3+\frac{2}{231}\left(135x^2+154\right)\sqrt{x^4+5}x^3$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```


```
[Out] (300*x*Sqrt[5 + x^4])/77 + (40*x*Sqrt[5 + x^4])/(3*(Sqrt[5] + x^2
)) + (2*x^3*(154 + 135*x^2)*Sqrt[5 + x^4])/231 + (x^3*(22 + 27*x^
2)*(5 + x^4)^(3/2))/99 - (40*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^
4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(3*Sqr
t[5 + x^4]) + (10*5^(1/4)*(154 - 45*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt
```

```
[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2]
)/(231*Sqrt[5 + x^4])
```

Rubi in Sympy [A] time = 25.8658, size = 219, normalized size = 1.

$$\frac{x^3\left(27x^2+22\right)\left(x^4+5\right)^{\frac{3}{2}}}{99}+\frac{2x^3\left(135x^2+154\right)\sqrt{x^4+5}}{231}+\frac{300x\sqrt{x^4+5}}{77}+\frac{40x\sqrt{x^4+5}}{3\left(x^2+\sqrt{5}\right)}-\frac{40\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{3\sqrt{x^4+5}}+\frac{10\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(-135\sqrt{5}+462\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{693\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] x**3*(27*x**2 + 22)*(x**4 + 5)**(3/2)/99 + 2*x**3*(135*x**2 + 154
)*sqrt(x**4 + 5)/231 + 300*x*sqrt(x**4 + 5)/77 + 40*x*sqrt(x**4 +
 5)/(3*(x**2 + sqrt(5))) - 40*5**(1/4)*sqrt((x**4 + 5)/(sqrt(5)*x
**2/5 + 1)**2)*(sqrt(5)*x**2/5 + 1)*elliptic_e(2*atan(5**(3/4)*x/
5), 1/2)/(3*sqrt(x**4 + 5)) + 10*5**(1/4)*sqrt((x**4 + 5)/(sqrt(5
)*x**2/5 + 1)**2)*(-135*sqrt(5) + 462)*(sqrt(5)*x**2/5 + 1)*ellip
tic_f(2*atan(5**(3/4)*x/5), 1/2)/(693*sqrt(x**4 + 5))
```

Mathematica [C] time = 0.230519, size = 110, normalized size = 0.5

$$\frac{1}{693}\left(\frac{x\left(189x^{12}+154x^{10}+2700x^8+2464x^6+11475x^4+8470x^2+13500\right)}{\sqrt{x^4+5}}+60\sqrt[4]{-5}\left(45\sqrt{5}+154i\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)-9240(-1)^{3/4}\sqrt[4]{5}E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] ((x*(13500 + 8470*x^2 + 11475*x^4 + 2464*x^6 + 2700*x^8 + 154*x^1
0 + 189*x^12))/Sqrt[5 + x^4] - 9240*(-1)^(3/4)*5^(1/4)*EllipticE[
I*ArcSinh[(-1/5)^(1/4)*x], -1] + 60*(-5)^(1/4)*(154*I + 45*Sqrt[5
```

```
])*EllipticF[I*ArcSinh[(-1/5)^(1/4)*x], -1])/693
```

Maple [C] time = 0.017, size = 204, normalized size = 0.9

$$\frac{2x^7}{9}\sqrt{x^4+5}+\frac{22x^3}{9}\sqrt{x^4+5} + \frac{\frac{8i}{3}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-EllipticE\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}} + \frac{3x^9}{11}\sqrt{x^4+5}+\frac{195x^5}{77}\sqrt{x^4+5}+\frac{300x}{77}\sqrt{x^4+5} - \frac{60\sqrt{5}}{77\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 2/9*x^7*(x^4+5)^(1/2)+22/9*x^3*(x^4+5)^(1/2)+8/3*I/(I*5^(1/2))^(1
/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)
^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/
5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))+3/11*x^9*(x^4+5)^(1/2)+195/77*x
^5*(x^4+5)^(1/2)+300/77*x*(x^4+5)^(1/2)-60/77*5^(1/2)/(I*5^(1/2))
^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4
+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+5\right)^{\frac{3}{2}}\left(3x^2+2\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(3x^8+2x^6+15x^4+10x^2\right)\sqrt{x^4+5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^2,x, algorithm="fricas")
```

```
[Out] integral((3*x^8 + 2*x^6 + 15*x^4 + 10*x^2)*sqrt(x^4 + 5), x)
```

Sympy [A] time = 8.01579, size = 160, normalized size = 0.73

$$\frac{3\sqrt{5}x^9\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\left(\frac{13}{4}\right)}+\frac{\sqrt{5}x^7\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\left(\frac{11}{4}\right)}+\frac{15\sqrt{5}x^5\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\left(\frac{9}{4}\right)}+\frac{5\sqrt{5}x^3\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), x**4*exp_po
lar(I*pi)/5)/(4*gamma(13/4)) + sqrt(5)*x**7*gamma(7/4)*hyper((-1/
2, 7/4), (11/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(11/4)) + 15*sq
rt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_polar(I
*pi)/5)/(4*gamma(9/4)) + 5*sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3
/4), (7/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^2,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^2, x)
```

3.29 $\int \left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=197

$$\frac{1}{21}x\left(7x^2+6\right)\left(x^4+5\right)^{3/2}+\frac{2}{7}x\left(7x^2+10\right)\sqrt{x^4+5}+\frac{20x\sqrt{x^4+5}}{x^2+\sqrt{5}}+\frac{10\sqrt[4]{5}\left(7+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{7\sqrt{x^4+5}}-\frac{20\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}$$

```
[Out] (20*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (2*x*(10 + 7*x^2)*Sqrt[5 +
 x^4])/7 + (x*(6 + 7*x^2)*(5 + x^4)^(3/2))/21 - (20*5^(1/4)*(Sqrt
[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x
/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (10*5^(1/4)*(7 + 2*Sqrt[5])*(Sqr
t[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[
x/5^(1/4)], 1/2])/(7*Sqrt[5 + x^4])
```

Rubi [A] time = 0.167943, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{1}{21}x\left(7x^2+6\right)\left(x^4+5\right)^{3/2}+\frac{2}{7}x\left(7x^2+10\right)\sqrt{x^4+5}+\frac{20x\sqrt{x^4+5}}{x^2+\sqrt{5}}+\frac{10\sqrt[4]{5}\left(7+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{7\sqrt{x^4+5}}-\frac{20\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```


```
[Out] (20*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (2*x*(10 + 7*x^2)*Sqrt[5 +
 x^4])/7 + (x*(6 + 7*x^2)*(5 + x^4)^(3/2))/21 - (20*5^(1/4)*(Sqrt
[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x
/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (10*5^(1/4)*(7 + 2*Sqrt[5])*(Sqr
t[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[
x/5^(1/4)], 1/2])/(7*Sqrt[5 + x^4])
```

Rubi in Sympy [A] time = 16.0123, size = 197, normalized size = 1.

$$\frac{x\left(21x^2+18\right)\left(x^4+5\right)^{\frac{3}{2}}}{63}+\frac{2x\left(63x^2+90\right)\sqrt{x^4+5}}{63}+\frac{20x\sqrt{x^4+5}}{x^2+\sqrt{5}}-\frac{20\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\,\mathrm{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{\sqrt{x^4+5}}+\frac{10\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(18\sqrt{5}+63\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\,\mathrm{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{63\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] x*(21*x**2 + 18)*(x**4 + 5)**(3/2)/63 + 2*x*(63*x**2 + 90)*sqrt(x
**4 + 5)/63 + 20*x*sqrt(x**4 + 5)/(x**2 + sqrt(5)) - 20*5**(1/4)*
sqrt((x**4 + 5)/(sqrt(5)*x**2/5 + 1)**2)*(sqrt(5)*x**2/5 + 1)*ell
iptic_e(2*atan(5**(3/4)*x/5), 1/2)/sqrt(x**4 + 5) + 10*5**(1/4)*s
qrt((x**4 + 5)/(sqrt(5)*x**2/5 + 1)**2)*(18*sqrt(5) + 63)*(sqrt(5
)*x**2/5 + 1)*elliptic_f(2*atan(5**(3/4)*x/5), 1/2)/(63*sqrt(x**4
 + 5))
```

Mathematica [C] time = 0.221681, size = 106, normalized size = 0.54

$$\frac{x\left(7x^{10}+6x^8+112x^6+120x^4+385x^2+450\right)}{21\sqrt{x^4+5}}+\frac{20}{7}\sqrt[4]{-5}\left(-2\sqrt{5}+7i\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)-20(-1)^{3/4}\sqrt[4]{5}E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (x*(450 + 385*x^2 + 120*x^4 + 112*x^6 + 6*x^8 + 7*x^10))/(21*Sqrt
[5 + x^4]) - 20*(-1)^(3/4)*5^(1/4)*EllipticE[I*ArcSinh[(-1/5)^(1/
4)*x], -1] + (20*(-5)^(1/4)*(7*I - 2*Sqrt[5])*EllipticF[I*ArcSinh
[(-1/5)^(1/4)*x], -1])/7
```

Maple [C] time = 0.015, size = 192, normalized size = 1.

$$\frac{2x^5}{7}\sqrt{x^4+5}+\frac{30x}{7}\sqrt{x^4+5}+\frac{8\sqrt{5}}{7\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\,EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}}$$
$$+\frac{x^7}{3}\sqrt{x^4+5}+\frac{11x^3}{3}\sqrt{x^4+5}$$
$$+\frac{4i}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-EllipticE\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 2/7*x^5*(x^4+5)^(1/2)+30/7*x*(x^4+5)^(1/2)+8/7*5^(1/2)/(I*5^(1/2)
)^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^
4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)+1/3*x^7*(
x^4+5)^(1/2)+11/3*x^3*(x^4+5)^(1/2)+4*I/(I*5^(1/2))^(1/2)*(25-5*I
*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(Ell
ipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)
*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^4+5\right)^{\frac{3}{2}}\left(3x^2+2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(3x^6+2x^4+15x^2+10\right)\sqrt{x^4+5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4 + 15*x^2 + 10)*sqrt(x^4 + 5), x)
```

Sympy [A] time = 5.96342, size = 158, normalized size = 0.8

$$\frac{3\sqrt{5}x^{7}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{x^{4}e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{5}x^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{x^{4}e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{9}{4}\right)}+\frac{15\sqrt{5}x^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{x^{4}e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{5\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{x^{4}e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), x**4*exp_po
lar(I*pi)/5)/(4*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((-1/
2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(9/4)) + 15*sqrt
(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_polar(I*p
i)/5)/(4*gamma(7/4)) + 5*sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/4),
(5/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^{4}+5\right)^{\frac{3}{2}}\left(3\,x^{2}+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

3.30 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=199

$$-\frac{(14-3x^2)(x^4+5)^{3/2}}{7x}+\frac{6}{35}x(14x^2+25)\sqrt{x^4+5}+\frac{24x\sqrt{x^4+5}}{x^2+\sqrt{5}}+\frac{6\sqrt[4]{5}(14+5\sqrt{5})(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{7\sqrt{x^4+5}}-\frac{24\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}$$

```
[Out] (24*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (6*x*(25 + 14*x^2)*Sqrt[5
+ x^4])/35 - ((14 - 3*x^2)*(5 + x^4)^(3/2))/(7*x) - (24*5^(1/4)*(
Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcT
an[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (6*5^(1/4)*(14 + 5*Sqrt[5])*
(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*Arc
Tan[x/5^(1/4)], 1/2])/(7*Sqrt[5 + x^4])
```

Rubi [A] time = 0.188189, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{(14-3x^2)(x^4+5)^{3/2}}{7x}+\frac{6}{35}x(14x^2+25)\sqrt{x^4+5}+\frac{24x\sqrt{x^4+5}}{x^2+\sqrt{5}}+\frac{6\sqrt[4]{5}(14+5\sqrt{5})(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{7\sqrt{x^4+5}}-\frac{24\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^2,x]
```


```
[Out] (24*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (6*x*(25 + 14*x^2)*Sqrt[5
+ x^4])/35 - ((14 - 3*x^2)*(5 + x^4)^(3/2))/(7*x) - (24*5^(1/4)*(
Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcT
an[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (6*5^(1/4)*(14 + 5*Sqrt[5])*
(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*Arc
```

```
Tan[x/5^(1/4)], 1/2])/(7*Sqrt[5 + x^4])
```

Rubi in Sympy [A] time = 17.5969, size = 197, normalized size = 0.99

$$\frac{2x\left(42x^2+75\right)\sqrt{x^4+5}}{35}+\frac{24x\sqrt{x^4+5}}{x^2+\sqrt{5}}-\frac{24\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\,\mathrm{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{\sqrt{x^4+5}}+\frac{2\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(15\sqrt{5}+42\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\,\mathrm{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{7\sqrt{x^4+5}}-\frac{\left(-3x^2+14\right)\left(x^4+5\right)^{\frac{3}{2}}}{7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5)**(3/2)/x**2,x)
```

```
[Out] 2*x*(42*x**2 + 75)*sqrt(x**4 + 5)/35 + 24*x*sqrt(x**4 + 5)/(x**2
+ sqrt(5)) - 24*5**(1/4)*sqrt((x**4 + 5)/(sqrt(5)*x**2/5 + 1)**2)
*(sqrt(5)*x**2/5 + 1)*elliptic_e(2*atan(5**(3/4)*x/5), 1/2)/sqrt(
x**4 + 5) + 2*5**(1/4)*sqrt((x**4 + 5)/(sqrt(5)*x**2/5 + 1)**2)*(
15*sqrt(5) + 42)*(sqrt(5)*x**2/5 + 1)*elliptic_f(2*atan(5**(3/4)*
x/5), 1/2)/(7*sqrt(x**4 + 5)) - (-3*x**2 + 14)*(x**4 + 5)**(3/2)/
(7*x)
```

Mathematica [C] time = 0.172841, size = 125, normalized size = 0.63

$$\frac{15x^{10}+14x^8+300x^6-280x^4+60\sqrt[4]{-5}\left(-5\sqrt{5}+14i\right)\sqrt{x^4+5}xF\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)-840(-1)^{3/4}\sqrt[4]{5}\sqrt{x^4+5}xE\left(i\,\mathrm{si}\right.}{35x\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^2,x]
```

```
[Out] (-1750 + 1125*x^2 - 280*x^4 + 300*x^6 + 14*x^8 + 15*x^10 - 840*(-
1)^(3/4)*5^(1/4)*x*Sqrt[5 + x^4]*EllipticE[I*ArcSinh[(-1/5)^(1/4)
*x], -1] + 60*(-5)^(1/4)*(14*I - 5*Sqrt[5])*x*Sqrt[5 + x^4]*Ellip
ticF[I*ArcSinh[(-1/5)^(1/4)*x], -1])/(35*x*Sqrt[5 + x^4])
```

Maple [C] time = 0.022, size = 192, normalized size = 1.

$$\frac{3x^5}{7}\sqrt{x^4+5}+\frac{45x}{7}\sqrt{x^4+5}+\frac{12\sqrt{5}}{7\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\,EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}}$$
$$-10\frac{\sqrt{x^4+5}}{x}+\frac{2x^3}{5}\sqrt{x^4+5}$$
$$+\frac{\frac{24i}{5}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-EllipticE\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5)^(3/2)/x^2,x)
```

```
[Out] 3/7*x^5*(x^4+5)^(1/2)+45/7*x*(x^4+5)^(1/2)+12/7*5^(1/2)/(I*5^(1/2
))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x
^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-10*(x^4+
5)^(1/2)/x+2/5*x^3*(x^4+5)^(1/2)+24/5*I/(I*5^(1/2))^(1/2)*(25-5*I
*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(Ell
ipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)
*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(x^4+5)^{\frac{3}{2}}(3x^2+2)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(3x^6+2x^4+15x^2+10)\sqrt{x^4+5}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4 + 15*x^2 + 10)*sqrt(x^4 + 5)/x^2, x)
```

Sympy [A] time = 6.98045, size = 160, normalized size = 0.8

$$\frac{3\sqrt{5}x^{5}\,\Gamma\left(\frac{5}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^{4}e^{i\pi}}{5}\right)}{4\,\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{5}x^{3}\,\Gamma\left(\frac{3}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^{4}e^{i\pi}}{5}\right)}{2\,\Gamma\left(\frac{7}{4}\right)}+\frac{15\sqrt{5}x\,\Gamma\left(\frac{1}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^{4}e^{i\pi}}{5}\right)}{4\,\Gamma\left(\frac{5}{4}\right)}+\frac{5\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^{4}e^{i\pi}}{5}\right)}{2x\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5)**(3/2)/x**2,x)
```

```
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_pol
ar(I*pi)/5)/(4*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((-1/2,
 3/4), (7/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(7/4)) + 15*sqrt(5
)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp_polar(I*pi)/5)
/(4*gamma(5/4)) + 5*sqrt(5)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,
), x**4*exp_polar(I*pi)/5)/(2*x*gamma(3/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^{4}+5\right)^{\frac{3}{2}}\left(3\,x^{2}+2\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^2, x)
```

3.31 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=201

$$-\frac{2(27-2x^2)\sqrt{x^4+5}}{3x}+\frac{36x\sqrt{x^4+5}}{x^2+\sqrt{5}}+\frac{2\sqrt[4]{5}\left(27+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt{x^4+5}}$$
$$-\frac{36\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}-\frac{(10-9x^2)(x^4+5)^{3/2}}{15x^3}$$

```
[Out] (-2*(27 - 2*x^2)*Sqrt[5 + x^4])/(3*x) + (36*x*Sqrt[5 + x^4])/(Sqr
t[5] + x^2) - ((10 - 9*x^2)*(5 + x^4)^(3/2))/(15*x^3) - (36*5^(1/
4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*
ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (2*5^(1/4)*(27 + 2*Sqrt[
5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2
*ArcTan[x/5^(1/4)], 1/2])/(3*Sqrt[5 + x^4])
```

Rubi [A] time = 0.207704, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{2(27-2x^2)\sqrt{x^4+5}}{3x}+\frac{36x\sqrt{x^4+5}}{x^2+\sqrt{5}}+\frac{2\sqrt[4]{5}\left(27+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt{x^4+5}}$$
$$-\frac{36\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}-\frac{(10-9x^2)(x^4+5)^{3/2}}{15x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^4,x]
```


```
[Out] (-2*(27 - 2*x^2)*Sqrt[5 + x^4])/(3*x) + (36*x*Sqrt[5 + x^4])/(Sqr
t[5] + x^2) - ((10 - 9*x^2)*(5 + x^4)^(3/2))/(15*x^3) - (36*5^(1/
4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*
ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (2*5^(1/4)*(27 + 2*Sqrt[
5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2
*ArcTan[x/5^(1/4)], 1/2])/(3*Sqrt[5 + x^4])
```

Rubi in Sympy [A] time = 18.466, size = 199, normalized size = 0.99

$$\frac{36x\sqrt{x^4+5}}{x^2+\sqrt{5}} - \frac{36\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{\sqrt{x^4+5}} + \frac{2\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(10\sqrt{5}+135\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{15\sqrt{x^4+5}} - \frac{2\left(-10x^2+135\right)\sqrt{x^4+5}}{15x} - \frac{\left(-9x^2+10\right)\left(x^4+5\right)^{\frac{3}{2}}}{15x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5)**(3/2)/x**4,x)
```

```
[Out] 36*x*sqrt(x**4 + 5)/(x**2 + sqrt(5)) - 36*5**(1/4)*sqrt((x**4 + 5
)/(sqrt(5)*x**2/5 + 1)**2)*(sqrt(5)*x**2/5 + 1)*elliptic_e(2*atan
(5**(3/4)*x/5), 1/2)/sqrt(x**4 + 5) + 2*5**(1/4)*sqrt((x**4 + 5)/
(sqrt(5)*x**2/5 + 1)**2)*(10*sqrt(5) + 135)*(sqrt(5)*x**2/5 + 1)*
elliptic_f(2*atan(5**(3/4)*x/5), 1/2)/(15*sqrt(x**4 + 5)) - 2*(-1
0*x**2 + 135)*sqrt(x**4 + 5)/(15*x) - (-9*x**2 + 10)*(x**4 + 5)**
(3/2)/(15*x**3)
```

Mathematica [C] time = 0.169942, size = 124, normalized size = 0.62

$$\frac{9x^{10}+10x^8-180x^6-1125x^2+20\sqrt[4]{-5}\left(-2\sqrt{5}+27i\right)\sqrt{x^4+5}x^3F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)-540(-1)^{3/4}\sqrt[4]{5}\sqrt{x^4+5}x^3E\left(i\right.}{15x^3\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^4,x]
```

```
[Out] (-250 - 1125*x^2 - 180*x^6 + 10*x^8 + 9*x^10 - 540*(-1)^(3/4)*5^(
1/4)*x^3*Sqrt[5 + x^4]*EllipticE[I*ArcSinh[(-1/5)^(1/4)*x], -1] +
 20*(-5)^(1/4)*(27*I - 2*Sqrt[5])*x^3*Sqrt[5 + x^4]*EllipticF[I*A
rcSinh[(-1/5)^(1/4)*x], -1])/(15*x^3*Sqrt[5 + x^4])
```

Maple [C] time = 0.025, size = 192, normalized size = 1.

$$-\frac{10}{3x^{3}}\sqrt{x^{4}+5}+\frac{2x}{3}\sqrt{x^{4}+5}+\frac{8\sqrt{5}}{15\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^{2}}\sqrt{25+5i\sqrt{5}x^{2}}\mathit{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^{4}+5}}$$
$$-15\frac{\sqrt{x^{4}+5}}{x}+\frac{3x^{3}}{5}\sqrt{x^{4}+5}$$
$$+\frac{\frac{36i}{5}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^{2}}\sqrt{25+5i\sqrt{5}x^{2}}\left(\mathit{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-\mathit{EllipticE}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^{4}+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5)^(3/2)/x^4,x)
```

```
[Out] -10/3*(x^4+5)^(1/2)/x^3+2/3*x*(x^4+5)^(1/2)+8/15*5^(1/2)/(I*5^(1/
2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(
x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-15*(x^4
+5)^(1/2)/x+3/5*x^3*(x^4+5)^(1/2)+36/5*I/(I*5^(1/2))^(1/2)*(25-5*
I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(El
lipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2
)*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^{4}+5\right)^{\frac{3}{2}}\left(3x^{2}+2\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3x^{6}+2x^{4}+15x^{2}+10\right)\sqrt{x^{4}+5}}{x^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4 + 15*x^2 + 10)*sqrt(x^4 + 5)/x^4, x)
```

Sympy [A] time = 7.08515, size = 163, normalized size = 0.81

$$\frac{3\sqrt{5}x^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\,\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\,\Gamma\left(\frac{5}{4}\right)}+\frac{15\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4x\,\Gamma\left(\frac{3}{4}\right)}+\frac{5\sqrt{5}\,\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2x^3\,\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5)**(3/2)/x**4,x)
```

```
[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_pol
ar(I*pi)/5)/(4*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/
4), (5/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(5/4)) + 15*sqrt(5)*g
amma(-1/4)*hyper((-1/2, -1/4), (3/4,), x**4*exp_polar(I*pi)/5)/(4
*x*gamma(3/4)) + 5*sqrt(5)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,)
, x**4*exp_polar(I*pi)/5)/(2*x**3*gamma(1/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^4, x)
```

3.32 $\int \frac{x^7\left(2+3x^2\right)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=67

$$\frac{1}{3}\sqrt{x^4+5}x^4+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{3}{8}\sqrt{x^4+5}x^6-\frac{5}{48}\left(27x^2+32\right)\sqrt{x^4+5}$$

```
[Out] (x^4*Sqrt[5 + x^4])/3 + (3*x^6*Sqrt[5 + x^4])/8 - (5*(32 + 27*x^2
)*Sqrt[5 + x^4])/48 + (225*ArcSinh[x^2/Sqrt[5]])/16
```

Rubi [A] time = 0.185368, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{1}{3}\sqrt{x^4+5}x^4+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{3}{8}\sqrt{x^4+5}x^6-\frac{5}{48}\left(27x^2+32\right)\sqrt{x^4+5}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^7*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (x^4*Sqrt[5 + x^4])/3 + (3*x^6*Sqrt[5 + x^4])/8 - (5*(32 + 27*x^2
)*Sqrt[5 + x^4])/48 + (225*ArcSinh[x^2/Sqrt[5]])/16
```

Rubi in Sympy [A] time = 13.1133, size = 60, normalized size = 0.9

$$\frac{3x^6\sqrt{x^4+5}}{8}+\frac{x^4\sqrt{x^4+5}}{3}-\frac{\left(135x^2+160\right)\sqrt{x^4+5}}{48}+\frac{225\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*x**6*sqrt(x**4 + 5)/8 + x**4*sqrt(x**4 + 5)/3 - (135*x**2 + 160
)*sqrt(x**4 + 5)/48 + 225*asinh(sqrt(5)*x**2/5)/16
```

Mathematica [A] time = 0.0423699, size = 44, normalized size = 0.66

$$\frac{1}{48}\left(675\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\sqrt{x^4+5}\left(18x^6+16x^4-135x^2-160\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^7*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (Sqrt[5 + x^4]*(-160 - 135*x^2 + 16*x^4 + 18*x^6) + 675*ArcSinh[x
^2/Sqrt[5]])/48
```

Maple [A] time = 0.022, size = 51, normalized size = 0.8

$$\frac{x^4-10}{3}\sqrt{x^4+5}+\frac{3\,x^6}{8}\sqrt{x^4+5}-\frac{45\,x^2}{16}\sqrt{x^4+5}+\frac{225}{16}Arcsinh\left(\frac{\sqrt{5}x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(3*x^2+2)/(x^4+5)^(1/2),x)
```

```
[Out] 1/3*(x^4+5)^(1/2)*(x^4-10)+3/8*x^6*(x^4+5)^(1/2)-45/16*x^2*(x^4+5
)^(1/2)+225/16*arcsinh(1/5*5^(1/2)*x^2)
```

Maxima [A] time = 0.78087, size = 140, normalized size = 2.09

$$\frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}}-5\sqrt{x^4+5}-\frac{75\left(\frac{5\sqrt{x^4+5}}{x^2}-\frac{3\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{16\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)}+\frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^7/sqrt(x^4 + 5),x, algorithm="maxima")
```

```
[Out] 1/3*(x^4 + 5)^(3/2) - 5*sqrt(x^4 + 5) - 75/16*(5*sqrt(x^4 + 5)/x^
2 - 3*(x^4 + 5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1
) + 225/32*log(sqrt(x^4 + 5)/x^2 + 1) - 225/32*log(sqrt(x^4 + 5)/
x^2 - 1)
```

Fricas [A] time = 0.261317, size = 224, normalized size = 3.34

$$-\frac{144\,x^{16}+128\,x^{14}-320\,x^{10}-6300\,x^8-8000\,x^6-13500\,x^4-16000\,x^2+675\left(8\,x^8+40\,x^4-4\left(2\,x^6+5\,x^2\right)\sqrt{x^4+5}+25\right)}{48\left(8\,x^8+40\,x^4-4\left(2\,x^6+5\,x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^7/sqrt(x^4 + 5),x, algorithm="fricas")
```

```
[Out] -1/48*(144*x^16 + 128*x^14 - 320*x^10 - 6300*x^8 - 8000*x^6 - 135
00*x^4 - 16000*x^2 + 675*(8*x^8 + 40*x^4 - 4*(2*x^6 + 5*x^2)*sqrt
(x^4 + 5) + 25)*log(-x^2 + sqrt(x^4 + 5)) - (144*x^14 + 128*x^12
- 360*x^10 - 640*x^8 - 4950*x^6 - 6000*x^4 - 3375*x^2 - 4000)*sqr
t(x^4 + 5))/(8*x^8 + 40*x^4 - 4*(2*x^6 + 5*x^2)*sqrt(x^4 + 5) + 2
5)
```

Sympy [A] time = 15.6066, size = 85, normalized size = 1.27

$$\frac{3x^{10}}{8\sqrt{x^4+5}} - \frac{15x^6}{16\sqrt{x^4+5}} + \frac{x^4\sqrt{x^4+5}}{3} - \frac{225x^2}{16\sqrt{x^4+5}} - \frac{10\sqrt{x^4+5}}{3} + \frac{225\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*x**10/(8*sqrt(x**4 + 5)) - 15*x**6/(16*sqrt(x**4 + 5)) + x**4*s
qrt(x**4 + 5)/3 - 225*x**2/(16*sqrt(x**4 + 5)) - 10*sqrt(x**4 + 5
)/3 + 225*asinh(sqrt(5)*x**2/5)/16
```

GIAC/XCAS [A] time = 0.261964, size = 62, normalized size = 0.93

$$\frac{1}{48}\sqrt{x^4+5}\left(\left(2\left(9x^2+8\right)x^2-135\right)x^2-160\right) - \frac{225}{16}\ln\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^7/sqrt(x^4 + 5),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(x^4 + 5)*((2*(9*x^2 + 8)*x^2 - 135)*x^2 - 160) - 225/16
*ln(-x^2 + sqrt(x^4 + 5))
```

3.33 $\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=51

$$\frac{1}{2}\sqrt{x^4+5}x^4-\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{1}{2}\left(10-x^2\right)\sqrt{x^4+5}$$

```
[Out] (x^4*Sqrt[5 + x^4])/2 - ((10 - x^2)*Sqrt[5 + x^4])/2 - (5*ArcSinh
[x^2/Sqrt[5]])/2
```

Rubi [A] time = 0.134439, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{1}{2}\sqrt{x^4+5}x^4-\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{1}{2}\left(10-x^2\right)\sqrt{x^4+5}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (x^4*Sqrt[5 + x^4])/2 - ((10 - x^2)*Sqrt[5 + x^4])/2 - (5*ArcSinh
[x^2/Sqrt[5]])/2
```

Rubi in Sympy [A] time = 10.6654, size = 44, normalized size = 0.86

$$\frac{x^4\sqrt{x^4+5}}{2}-\frac{\left(-6x^2+60\right)\sqrt{x^4+5}}{12}-\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] x**4*sqrt(x**4 + 5)/2 - (-6*x**2 + 60)*sqrt(x**4 + 5)/12 - 5*asin
h(sqrt(5)*x**2/5)/2
```

Mathematica [A] time = 0.029074, size = 36, normalized size = 0.71

$$\frac{1}{2}\sqrt{x^4+5}\left(x^4+x^2-10\right)-\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (Sqrt[5 + x^4]*(-10 + x^2 + x^4))/2 - (5*ArcSinh[x^2/Sqrt[5]])/2
```

Maple [A] time = 0.016, size = 39, normalized size = 0.8

$$\frac{x^2}{2}\sqrt{x^4+5} - \frac{5}{2}Arcsinh\left(\frac{\sqrt{5}x^2}{5}\right) + \frac{x^4-10}{2}\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(3*x^2+2)/(x^4+5)^(1/2),x)
```

```
[Out] 1/2*x^2*(x^4+5)^(1/2)-5/2*arcsinh(1/5*5^(1/2)*x^2)+1/2*(x^4+5)^(1
/2)*(x^4-10)
```

Maxima [A] time = 0.778367, size = 103, normalized size = 2.02

$$\frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}} - \frac{15}{2}\sqrt{x^4+5} + \frac{5\sqrt{x^4+5}}{2x^2\left(\frac{x^4+5}{x^4}-1\right)} - \frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) + \frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^5/sqrt(x^4 + 5),x, algorithm="maxima")
```

```
[Out] 1/2*(x^4 + 5)^(3/2) - 15/2*sqrt(x^4 + 5) + 5/2*sqrt(x^4 + 5)/(x^2
*((x^4 + 5)/x^4 - 1)) - 5/4*log(sqrt(x^4 + 5)/x^2 + 1) + 5/4*log(
sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 0.296281, size = 190, normalized size = 3.73

$$-\frac{4x^{12} + 4x^{10} - 15x^8 + 25x^6 - 225x^4 + 25x^2 - 5\left(4x^6 + 15x^2 - \left(4x^4+5\right)\sqrt{x^4+5}\right)\log\left(-x^2+\sqrt{x^4+5}\right) - \left(4x^{10} + 4x^8 -\right.}{2\left(4x^6 + 15x^2 - (4x^4+5)\sqrt{x^4+5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^5/sqrt(x^4 + 5),x, algorithm="fricas")
```

```
[Out] -1/2*(4*x^12 + 4*x^10 - 15*x^8 + 25*x^6 - 225*x^4 + 25*x^2 - 5*(4
*x^6 + 15*x^2 - (4*x^4 + 5)*sqrt(x^4 + 5))*log(-x^2 + sqrt(x^4 +
5)) - (4*x^10 + 4*x^8 - 25*x^6 + 15*x^4 - 150*x^2)*sqrt(x^4 + 5)
- 250)/(4*x^6 + 15*x^2 - (4*x^4 + 5)*sqrt(x^4 + 5))
```

Sympy [A] time = 10.5183, size = 66, normalized size = 1.29

$$\frac{x^6}{2\sqrt{x^4+5}}+\frac{x^4\sqrt{x^4+5}}{2}+\frac{5x^2}{2\sqrt{x^4+5}}-5\sqrt{x^4+5}-\frac{5\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] x**6/(2*sqrt(x**4 + 5)) + x**4*sqrt(x**4 + 5)/2 + 5*x**2/(2*sqrt(
x**4 + 5)) - 5*sqrt(x**4 + 5) - 5*asinh(sqrt(5)*x**2/5)/2
```

GIAC/XCAS [A] time = 0.264367, size = 50, normalized size = 0.98

$$\frac{1}{2}\sqrt{x^4+5}\left(\left(x^2+1\right)x^2-10\right)+\frac{5}{2}\ln\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^5/sqrt(x^4 + 5),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^4 + 5)*((x^2 + 1)*x^2 - 10) + 5/2*ln(-x^2 + sqrt(x^4 +
 5))
```

3.34 $\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=35

$$\frac{1}{4}\left(3x^2+4\right)\sqrt{x^4+5}-\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 - (15*ArcSinh[x^2/Sqrt[5]])/4
```

Rubi [A] time = 0.0872677, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{1}{4}\left(3x^2+4\right)\sqrt{x^4+5}-\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(2 + 3*x^2))/Sqrt[5 + x^4],x]

[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 - (15*ArcSinh[x^2/Sqrt[5]])/4
```

Rubi in Sympy [A] time = 7.99697, size = 31, normalized size = 0.89

$$\frac{\left(3x^2+4\right)\sqrt{x^4+5}}{4}-\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(3*x**2+2)/(x**4+5)**(1/2),x)

[Out] (3*x**2 + 4)*sqrt(x**4 + 5)/4 - 15*asinh(sqrt(5)*x**2/5)/4
```

Mathematica [A] time = 0.0287086, size = 34, normalized size = 0.97

$$\frac{1}{4}\left(\left(3x^2+4\right)\sqrt{x^4+5}-15\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4] - 15*ArcSinh[x^2/Sqrt[5]])/4
```

Maple [A] time = 0.011, size = 32, normalized size = 0.9

$$\sqrt{x^4+5}+\frac{3\,x^2}{4}\sqrt{x^4+5}-\frac{15}{4}Arcsinh\left(\frac{\sqrt{5}x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(3*x^2+2)/(x^4+5)^(1/2),x)
```

```
[Out] (x^4+5)^(1/2)+3/4*x^2*(x^4+5)^(1/2)-15/4*arcsinh(1/5*5^(1/2)*x^2)
```

Maxima [A] time = 0.782286, size = 88, normalized size = 2.51

$$\sqrt{x^4+5}+\frac{15\,\sqrt{x^4+5}}{4\,x^2\left(\frac{x^4+5}{x^4}-1\right)}-\frac{15}{8}\,\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{15}{8}\,\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^3/sqrt(x^4 + 5),x, algorithm="maxima")
```

```
[Out] sqrt(x^4 + 5) + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) - 15
/8*log(sqrt(x^4 + 5)/x^2 + 1) + 15/8*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 0.266802, size = 142, normalized size = 4.06

$$-\frac{6\,x^8+8\,x^6+30\,x^4+40\,x^2-15\left(2\,x^4-2\,\sqrt{x^4+5}x^2+5\right)\log\left(-x^2+\sqrt{x^4+5}\right)-\left(6\,x^6+8\,x^4+15\,x^2+20\right)\sqrt{x^4+5}}{4\left(2\,x^4-2\,\sqrt{x^4+5}x^2+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^3/sqrt(x^4 + 5),x, algorithm="fricas")
```

```
[Out] -1/4*(6*x^8 + 8*x^6 + 30*x^4 + 40*x^2 - 15*(2*x^4 - 2*sqrt(x^4 +
5)*x^2 + 5)*log(-x^2 + sqrt(x^4 + 5)) - (6*x^6 + 8*x^4 + 15*x^2 +
 20)*sqrt(x^4 + 5))/(2*x^4 - 2*sqrt(x^4 + 5)*x^2 + 5)
```

Sympy [A] time = 8.27188, size = 53, normalized size = 1.51

$$\frac{3x^6}{4\sqrt{x^4+5}} + \frac{15x^2}{4\sqrt{x^4+5}} + \sqrt{x^4+5} - \frac{15\,\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) + 15*x**2/(4*sqrt(x**4 + 5)) + sqrt(x**
4 + 5) - 15*asinh(sqrt(5)*x**2/5)/4
```

GIAC/XCAS [A] time = 0.265266, size = 45, normalized size = 1.29

$$\frac{1}{4}\sqrt{x^4+5}\left(3\,x^2+4\right) + \frac{15}{4}\ln\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^3/sqrt(x^4 + 5),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 15/4*ln(-x^2 + sqrt(x^4 + 5))
```

3.35 $\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=24

$$\frac{3\sqrt{x^4+5}}{2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] (3*Sqrt[5 + x^4])/2 + ArcSinh[x^2/Sqrt[5]]
```

Rubi [A] time = 0.052905, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{3\sqrt{x^4+5}}{2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (3*Sqrt[5 + x^4])/2 + ArcSinh[x^2/Sqrt[5]]
```

Rubi in Sympy [A] time = 6.57961, size = 22, normalized size = 0.92

$$\frac{3\sqrt{x^4+5}}{2}+\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(x**4 + 5)/2 + asinh(sqrt(5)*x**2/5)
```

Mathematica [A] time = 0.0145477, size = 24, normalized size = 1.

$$\frac{3\sqrt{x^4+5}}{2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(2 + 3*x^2))/Sqrt[5 + x^4],x]

[Out] (3*Sqrt[5 + x^4])/2 + ArcSinh[x^2/Sqrt[5]]
```

Maple [A] time = 0.014, size = 20, normalized size = 0.8

$$Arcsinh\left(\frac{\sqrt{5}x^2}{5}\right) + \frac{3}{2}\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(3*x^2+2)/(x^4+5)^(1/2),x)

[Out] arcsinh(1/5*5^(1/2)*x^2)+3/2*(x^4+5)^(1/2)
```

Maxima [A] time = 0.779643, size = 57, normalized size = 2.38

$$\frac{3}{2}\sqrt{x^4+5} + \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x/sqrt(x^4 + 5),x, algorithm="maxima")

[Out] 3/2*sqrt(x^4 + 5) + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*log(sqrt
(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 0.280068, size = 88, normalized size = 3.67

$$-\frac{3\,x^4 - 3\,\sqrt{x^4+5}x^2 + 2\left(x^2-\sqrt{x^4+5}\right)\log\left(-x^2+\sqrt{x^4+5}\right) + 15}{2\left(x^2-\sqrt{x^4+5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x/sqrt(x^4 + 5),x, algorithm="fricas")
```

```
[Out] -1/2*(3*x^4 - 3*sqrt(x^4 + 5)*x^2 + 2*(x^2 - sqrt(x^4 + 5))*log(-
x^2 + sqrt(x^4 + 5)) + 15)/(x^2 - sqrt(x^4 + 5))
```

Sympy [A] time = 2.08159, size = 22, normalized size = 0.92

$$\frac{3\sqrt{x^4+5}}{2}+\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(x**4 + 5)/2 + asinh(sqrt(5)*x**2/5)
```

GIAC/XCAS [A] time = 0.265035, size = 35, normalized size = 1.46

$$\frac{3}{2}\sqrt{x^4+5}-\ln\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x/sqrt(x^4 + 5),x, algorithm="giac")
```

```
[Out] 3/2*sqrt(x^4 + 5) - ln(-x^2 + sqrt(x^4 + 5))
```

3.36 $\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=38

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/Sqrt[
5]
```

Rubi [A] time = 0.106172, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(x*Sqrt[5 + x^4]),x]
```

```
[Out] (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/Sqrt[
5]
```

Rubi in Sympy [A] time = 10.2756, size = 37, normalized size = 0.97

$$\frac{3\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}-\frac{\sqrt{5}\operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/x/(x**4+5)**(1/2),x)
```

```
[Out] 3*asinh(sqrt(5)*x**2/5)/2 - sqrt(5)*atanh(sqrt(5)*sqrt(x**4 + 5)/
5)/5
```

Mathematica [A] time = 0.0794236, size = 38, normalized size = 1.

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)/(x*Sqrt[5 + x^4]),x]
```

```
[Out] (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/Sqrt[
5]
```

Maple [A] time = 0.013, size = 30, normalized size = 0.8

$$-\frac{\sqrt{5}}{5}\mathit{Artanh}\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)+\frac{3}{2}\mathit{Arcsinh}\left(\frac{\sqrt{5}x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/x/(x^4+5)^(1/2),x)
```

```
[Out] -1/5*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))+3/2*arcsinh(1/5*5^(1/
2)*x^2)
```

Maxima [A] time = 0.782211, size = 90, normalized size = 2.37

$$\frac{1}{10}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 +
 5))) + 3/4*log(sqrt(x^4 + 5)/x^2 + 1) - 3/4*log(sqrt(x^4 + 5)/x^
2 - 1)
```

Fricas [A] time = 0.270603, size = 108, normalized size = 2.84

$$-\frac{1}{10}\sqrt{5}\left(3\sqrt{5}\log\left(-x^2+\sqrt{x^4+5}\right)-2\log\left(\frac{5x^2+\sqrt{5}\left(x^4+5\right)-\sqrt{x^4+5}\left(\sqrt{5}x^2+5\right)}{x^4-\sqrt{x^4+5}x^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(5)*(3*sqrt(5)*log(-x^2 + sqrt(x^4 + 5)) - 2*log((5*x^2
 + sqrt(5)*(x^4 + 5) - sqrt(x^4 + 5)*(sqrt(5)*x^2 + 5))/(x^4 - sq
rt(x^4 + 5)*x^2)))
```

Sympy [A] time = 6.22333, size = 31, normalized size = 0.82

$$-\frac{\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{5}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)/x/(x**4+5)**(1/2),x)
```

```
[Out] -sqrt(5)*asinh(sqrt(5)/x**2)/5 + 3*asinh(sqrt(5)*x**2/5)/2
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x), x)
```

3.37 $\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=42

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}-\frac{\sqrt{x^4+5}}{5x^2}$$

```
[Out] -Sqrt[5 + x^4]/(5*x^2) - (3*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/(2*Sq
rt[5])
```

Rubi [A] time = 0.107084, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}-\frac{\sqrt{x^4+5}}{5x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(x^3*Sqrt[5 + x^4]),x]
```

```
[Out] -Sqrt[5 + x^4]/(5*x^2) - (3*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/(2*Sq
rt[5])
```

Rubi in Sympy [A] time = 10.0094, size = 39, normalized size = 0.93

$$-\frac{3\sqrt{5}\,\operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)}{10}-\frac{\sqrt{x^4+5}}{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/x**3/(x**4+5)**(1/2),x)
```

```
[Out] -3*sqrt(5)*atanh(sqrt(5)*sqrt(x**4 + 5)/5)/10 - sqrt(x**4 + 5)/(5
*x**2)
```

Mathematica [A] time = 0.0453208, size = 42, normalized size = 1.

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}-\frac{\sqrt{x^4+5}}{5x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)/(x^3*Sqrt[5 + x^4]),x]
```

```
[Out] -Sqrt[5 + x^4]/(5*x^2) - (3*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/(2*Sq
rt[5])
```

Maple [A] time = 0.017, size = 31, normalized size = 0.7

$$-\frac{1}{5\,x^{2}}\sqrt{x^{4}+5}-\frac{3\,\sqrt{5}}{10}\mathit{Artanh}\left(\sqrt{5}\frac{1}{\sqrt{x^{4}+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/x^3/(x^4+5)^(1/2),x)
```

```
[Out] -1/5*(x^4+5)^(1/2)/x^2-3/10*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2)
)
```

Maxima [A] time = 0.784281, size = 63, normalized size = 1.5

$$\frac{3}{20}\,\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^{4}+5}}{\sqrt{5}+\sqrt{x^{4}+5}}\right)-\frac{\sqrt{x^{4}+5}}{5\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^3),x, algorithm="maxima")
```

```
[Out] 3/20*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 +
 5))) - 1/5*sqrt(x^4 + 5)/x^2
```

Fricas [A] time = 0.259365, size = 140, normalized size = 3.33

$$\frac{3\left(x^{4}-\sqrt{x^{4}+5}x^{2}\right)\log\left(\frac{5\,x^{2}+\sqrt{5}\left(x^{4}+5\right)-\sqrt{x^{4}+5}\left(\sqrt{5}x^{2}+5\right)}{x^{4}-\sqrt{x^{4}+5}x^{2}}\right)+2\,\sqrt{5}}{2\left(\sqrt{5}x^{4}-\sqrt{5}\sqrt{x^{4}+5}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^3),x, algorithm="fricas")
```

```
[Out] 1/2*(3*(x^4 - sqrt(x^4 + 5)*x^2)*log((5*x^2 + sqrt(5)*(x^4 + 5) -
 sqrt(x^4 + 5)*(sqrt(5)*x^2 + 5))/(x^4 - sqrt(x^4 + 5)*x^2)) + 2*
sqrt(5))/(sqrt(5)*x^4 - sqrt(5)*sqrt(x^4 + 5)*x^2)
```

Sympy [A] time = 6.66834, size = 31, normalized size = 0.74

$$-\frac{\sqrt{1+\frac{5}{x^4}}}{5}-\frac{3\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)/x**3/(x**4+5)**(1/2),x)
```

```
[Out] -sqrt(1 + 5/x**4)/5 - 3*sqrt(5)*asinh(sqrt(5)/x**2)/10
```

GIAC/XCAS [A] time = 0.275485, size = 65, normalized size = 1.55

$$-\frac{3}{20}\sqrt{5}\ln\left(\sqrt{5}+\sqrt{x^4+5}\right)+\frac{3}{20}\sqrt{5}\ln\left(-\sqrt{5}+\sqrt{x^4+5}\right)-\frac{1}{5}\sqrt{\frac{5}{x^4}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^3),x, algorithm="giac")
```

```
[Out] -3/20*sqrt(5)*ln(sqrt(5) + sqrt(x^4 + 5)) + 3/20*sqrt(5)*ln(-sqrt
(5) + sqrt(x^4 + 5)) - 1/5*sqrt(5/x^4 + 1)
```

3.38 $\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=58

$$-\frac{\sqrt{x^4+5}}{10x^4}+\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}-\frac{3\sqrt{x^4+5}}{10x^2}$$

```
[Out] -Sqrt[5 + x^4]/(10*x^4) - (3*Sqrt[5 + x^4])/(10*x^2) + ArcTanh[Sq
rt[5 + x^4]/Sqrt[5]]/(10*Sqrt[5])
```

Rubi [A] time = 0.14823, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{\sqrt{x^4+5}}{10x^4}+\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}-\frac{3\sqrt{x^4+5}}{10x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(x^5*Sqrt[5 + x^4]),x]
```

```
[Out] -Sqrt[5 + x^4]/(10*x^4) - (3*Sqrt[5 + x^4])/(10*x^2) + ArcTanh[Sq
rt[5 + x^4]/Sqrt[5]]/(10*Sqrt[5])
```

Rubi in Sympy [A] time = 12.361, size = 51, normalized size = 0.88

$$\frac{\sqrt{5}\,\operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)}{50}-\frac{3\sqrt{x^4+5}}{10x^2}-\frac{\sqrt{x^4+5}}{10x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/x**5/(x**4+5)**(1/2),x)
```

```
[Out] sqrt(5)*atanh(sqrt(5)*sqrt(x**4 + 5)/5)/50 - 3*sqrt(x**4 + 5)/(10
*x**2) - sqrt(x**4 + 5)/(10*x**4)
```

Mathematica [A] time = 0.0496393, size = 51, normalized size = 0.88

$$\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}+\sqrt{x^4+5}\left(-\frac{1}{10x^4}-\frac{3}{10x^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)/(x^5*Sqrt[5 + x^4]),x]
```

```
[Out] (-1/(10*x^4) - 3/(10*x^2))*Sqrt[5 + x^4] + ArcTanh[Sqrt[5 + x^4]/
Sqrt[5]]/(10*Sqrt[5])
```

Maple [A] time = 0.018, size = 43, normalized size = 0.7

$$-\frac{1}{10\,x^4}\sqrt{x^4+5}+\frac{\sqrt{5}}{50}\mathit{Artanh}\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)-\frac{3}{10\,x^2}\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/x^5/(x^4+5)^(1/2),x)
```

```
[Out] -1/10*(x^4+5)^(1/2)/x^4+1/50*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2
))-3/10*(x^4+5)^(1/2)/x^2
```

Maxima [A] time = 0.781845, size = 80, normalized size = 1.38

$$-\frac{1}{100}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{3\sqrt{x^4+5}}{10\,x^2}-\frac{\sqrt{x^4+5}}{10\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^5),x, algorithm="maxima")
```

```
[Out] -1/100*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4
 + 5))) - 3/10*sqrt(x^4 + 5)/x^2 - 1/10*sqrt(x^4 + 5)/x^4
```

Fricas [A] time = 0.314246, size = 216, normalized size = 3.72

$$\frac{\sqrt{5}\left(2\,x^4+15\,x^2+5\right)\sqrt{x^4+5}-\left(2\,x^8-2\,\sqrt{x^4+5}x^6+5\,x^4\right)\log\left(-\frac{5\,x^2-\sqrt{5}\left(x^4+5\right)+\sqrt{x^4+5}\left(\sqrt{5}x^2-5\right)}{x^4-\sqrt{x^4+5}x^2}\right)-\sqrt{5}\left(2\,x^6+15\,x^4+10\,x^2\right)}{10\left(2\,\sqrt{5}\sqrt{x^4+5}x^6-\sqrt{5}(2\,x^8+5\,x^4)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^5),x, algorithm="fricas")
```

```
[Out] 1/10*(sqrt(5)*(2*x^4 + 15*x^2 + 5)*sqrt(x^4 + 5) - (2*x^8 - 2*sqr
t(x^4 + 5)*x^6 + 5*x^4)*log(-(5*x^2 - sqrt(5)*(x^4 + 5) + sqrt(x^
4 + 5)*(sqrt(5)*x^2 - 5))/(x^4 - sqrt(x^4 + 5)*x^2)) - sqrt(5)*(2
*x^6 + 15*x^4 + 10*x^2))/(2*sqrt(5)*sqrt(x^4 + 5)*x^6 - sqrt(5)*(
2*x^8 + 5*x^4))
```

Sympy [A] time = 7.18127, size = 88, normalized size = 1.52

$$\frac{\sqrt{5}\left(-\frac{\log\left(\sqrt{\frac{x^4}{5}+1}-1\right)}{4}+\frac{\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{4}-\frac{1}{4\left(\sqrt{\frac{x^4}{5}+1}+1\right)}-\frac{1}{4\left(\sqrt{\frac{x^4}{5}+1}-1\right)}\right)}{25}-\frac{3\sqrt{5}\sqrt{5x^4+25}}{50x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)/x**5/(x**4+5)**(1/2),x)
```

```
[Out] sqrt(5)*(-log(sqrt(x**4/5 + 1) - 1)/4 + log(sqrt(x**4/5 + 1) + 1)
/4 - 1/(4*(sqrt(x**4/5 + 1) + 1)) - 1/(4*(sqrt(x**4/5 + 1) - 1)))
/25 - 3*sqrt(5)*sqrt(5*x**4 + 25)/(50*x**2)
```

GIAC/XCAS [A] time = 0.27716, size = 72, normalized size = 1.24

$$-\frac{1}{10}\left(\frac{1}{x^2}+3\right)\sqrt{\frac{5}{x^4}+1}+\frac{1}{100}\sqrt{5}\ln\left(\sqrt{5}+\sqrt{x^4+5}\right)-\frac{1}{100}\sqrt{5}\ln\left(-\sqrt{5}+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^5),x, algorithm="giac")
```

```
[Out] -1/10*(1/x^2 + 3)*sqrt(5/x^4 + 1) + 1/100*sqrt(5)*ln(sqrt(5) + sq
rt(x^4 + 5)) - 1/100*sqrt(5)*ln(-sqrt(5) + sqrt(x^4 + 5))
```

3.39 $\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=185

$$\frac{2}{3}\sqrt{x^4+5}x+\frac{3}{5}\sqrt{x^4+5}x^3-\frac{9\sqrt{x^4+5}x}{x^2+\sqrt{5}}-\frac{\sqrt[4]{5}\left(27+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{6\sqrt{x^4+5}}+\frac{9\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}$$

```
[Out] (2*x*Sqrt[5 + x^4])/3 + (3*x^3*Sqrt[5 + x^4])/5 - (9*x*Sqrt[5 + x
^4])/(Sqrt[5] + x^2) + (9*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/
(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 +
x^4] - (5^(1/4)*(27 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(
Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(6*Sqrt[5
+ x^4])
```

Rubi [A] time = 0.213611, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{2}{3}\sqrt{x^4+5}x+\frac{3}{5}\sqrt{x^4+5}x^3-\frac{9\sqrt{x^4+5}x}{x^2+\sqrt{5}}-\frac{\sqrt[4]{5}\left(27+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{6\sqrt{x^4+5}}+\frac{9\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (2*x*Sqrt[5 + x^4])/3 + (3*x^3*Sqrt[5 + x^4])/5 - (9*x*Sqrt[5 + x
^4])/(Sqrt[5] + x^2) + (9*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/
(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 +
x^4] - (5^(1/4)*(27 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(
Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(6*Sqrt[5
+ x^4])
```

Rubi in Sympy [A] time = 18.9082, size = 185, normalized size = 1.

$$\frac{3x^3\sqrt{x^4+5}}{5}+\frac{2x\sqrt{x^4+5}}{3}-\frac{9x\sqrt{x^4+5}}{x^2+\sqrt{5}}+\frac{9\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\,\mathrm{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{\sqrt{x^4+5}}$$
$$-\frac{\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(10\sqrt{5}+135\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\,\mathrm{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{30\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*x**3*sqrt(x**4 + 5)/5 + 2*x*sqrt(x**4 + 5)/3 - 9*x*sqrt(x**4 +
5)/(x**2 + sqrt(5)) + 9*5**(1/4)*sqrt((x**4 + 5)/(sqrt(5)*x**2/5
+ 1)**2)*(sqrt(5)*x**2/5 + 1)*elliptic_e(2*atan(5**(3/4)*x/5), 1/
2)/sqrt(x**4 + 5) - 5**(1/4)*sqrt((x**4 + 5)/(sqrt(5)*x**2/5 + 1)
**2)*(10*sqrt(5) + 135)*(sqrt(5)*x**2/5 + 1)*elliptic_f(2*atan(5*
*(3/4)*x/5), 1/2)/(30*sqrt(x**4 + 5))
```

Mathematica [C] time = 0.254498, size = 96, normalized size = 0.52

$$\frac{1}{15}\left(\frac{x\left(9x^6+10x^4+45x^2+50\right)}{\sqrt{x^4+5}}+5\sqrt[4]{-5}\left(2\sqrt{5}-27i\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)\right.$$
$$\left.+9(-1)^{3/4}\sqrt[4]{5}E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] 9*(-1)^(3/4)*5^(1/4)*EllipticE[I*ArcSinh[(-1/5)^(1/4)*x], -1] + (
(x*(50 + 45*x^2 + 10*x^4 + 9*x^6))/Sqrt[5 + x^4] + 5*(-5)^(1/4)*(
-27*I + 2*Sqrt[5])*EllipticF[I*ArcSinh[(-1/5)^(1/4)*x], -1])/15
```

Maple [C] time = 0.025, size = 168, normalized size = 0.9

$$\frac{2x}{3}\sqrt{x^4+5}-\frac{2\sqrt{5}}{15\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\,EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}}+\frac{3x^3}{5}\sqrt{x^4+5}-\frac{\frac{9i}{5}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-EllipticE\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(3*x^2+2)/(x^4+5)^(1/2),x)
```

```
[Out] 2/3*x*(x^4+5)^(1/2)-2/15*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2
)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1
/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)+3/5*x^3*(x^4+5)^(1/2)-9/5*I/(I*
5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1
/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-E
llipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)x^4}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{3x^6+2x^4}{\sqrt{x^4+5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5),x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4)/sqrt(x^4 + 5), x)
```

Sympy [A] time = 4.37928, size = 75, normalized size = 0.41

$$\frac{3\sqrt{5}x^{7}\,\Gamma\left(\frac{7}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^{4}e^{i\pi}}{5}\right)}{20\,\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{5}x^{5}\,\Gamma\left(\frac{5}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^{4}e^{i\pi}}{5}\right)}{10\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), x**4*exp_pol
ar(I*pi)/5)/(20*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((1/2
, 5/4), (9/4,), x**4*exp_polar(I*pi)/5)/(10*gamma(9/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3\,x^{2}+2\right)x^{4}}{\sqrt{x^{4}+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5), x)
```

3.40 $\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=166

$$\sqrt{x^4+5}x+\frac{2\sqrt{x^4+5}x}{x^2+\sqrt{5}}+\frac{\sqrt[4]{5}\left(2-\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt{x^4+5}}-\frac{2\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}$$

```
[Out] x*Sqrt[5 + x^4] + (2*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (2*5^(1/4
)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*A
rcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4)*(2 - Sqrt[5])*(S
qrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTa
n[x/5^(1/4)], 1/2])/(2*Sqrt[5 + x^4])
```

Rubi [A] time = 0.159616, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\sqrt{x^4+5}x+\frac{2\sqrt{x^4+5}x}{x^2+\sqrt{5}}+\frac{\sqrt[4]{5}\left(2-\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt{x^4+5}}-\frac{2\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] x*Sqrt[5 + x^4] + (2*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (2*5^(1/4
)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*A
rcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4)*(2 - Sqrt[5])*(S
qrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTa
n[x/5^(1/4)], 1/2])/(2*Sqrt[5 + x^4])
```

Rubi in Sympy [A] time = 13.6059, size = 167, normalized size = 1.01

$$x\sqrt{x^4+5}+\frac{2x\sqrt{x^4+5}}{x^2+\sqrt{5}}-\frac{2\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\,\mathrm{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{\sqrt{x^4+5}}+\frac{\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(-3\sqrt{5}+6\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\,\mathrm{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{6\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] x*sqrt(x**4 + 5) + 2*x*sqrt(x**4 + 5)/(x**2 + sqrt(5)) - 2*5**(1/
4)*sqrt((x**4 + 5)/(sqrt(5)*x**2/5 + 1)**2)*(sqrt(5)*x**2/5 + 1)*
elliptic_e(2*atan(5**(3/4)*x/5), 1/2)/sqrt(x**4 + 5) + 5**(1/4)*s
qrt((x**4 + 5)/(sqrt(5)*x**2/5 + 1)**2)*(-3*sqrt(5) + 6)*(sqrt(5)
*x**2/5 + 1)*elliptic_f(2*atan(5**(3/4)*x/5), 1/2)/(6*sqrt(x**4 +
 5))
```

Mathematica [C] time = 0.164497, size = 71, normalized size = 0.43

$$\sqrt{x^4+5}x+\sqrt[4]{-5}\left(\sqrt{5}+2i\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)-2(-1)^{3/4}\sqrt[4]{5}E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] x*Sqrt[5 + x^4] - 2*(-1)^(3/4)*5^(1/4)*EllipticE[I*ArcSinh[(-1/5)
^(1/4)*x], -1] + (-5)^(1/4)*(2*I + Sqrt[5])*EllipticF[I*ArcSinh[(
-1/5)^(1/4)*x], -1]
```

Maple [C] time = 0.018, size = 155, normalized size = 0.9

$$\frac{\frac{2i}{5}}{\sqrt{i\sqrt{5}}}\sqrt{25-5\,i\sqrt{5}x^2}\sqrt{25+5\,i\sqrt{5}x^2}\left(\mathit{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-\mathit{EllipticE}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}+x\sqrt{x^4+5}-\frac{\sqrt{5}}{5\sqrt{i\sqrt{5}}}\sqrt{25-5\,i\sqrt{5}x^2}\sqrt{25+5\,i\sqrt{5}x^2}\mathit{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(3*x^2+2)/(x^4+5)^(1/2),x)
```

```
[Out] 2/5*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2
)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(
1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))+x*(x^4+5)^(
1/2)-1/5*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25
+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*
5^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3\,x^2+2\right)x^2}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{3\,x^4+2\,x^2}{\sqrt{x^4+5}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5),x, algorithm="fricas")
```

```
[Out] integral((3*x^4 + 2*x^2)/sqrt(x^4 + 5), x)
```

Sympy [A] time = 3.90261, size = 75, normalized size = 0.45

$$\frac{3\sqrt{5}x^5\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{20\,\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{5}x^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{10\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**4*exp_pola
r(I*pi)/5)/(20*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((1/2,
3/4), (7/4,), x**4*exp_polar(I*pi)/5)/(10*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3\,x^2+2\right)x^2}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5), x)
```

3.41 $\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=155

$$\frac{3\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}+\frac{\left(2+3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-\frac{3\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}$$

```
[Out] (3*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (3*5^(1/4)*(Sqrt[5] + x^2)*
Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)],
1/2])/Sqrt[5 + x^4] + ((2 + 3*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 +
x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5
^(1/4)*Sqrt[5 + x^4])
```

Rubi [A] time = 0.102151, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{3\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}+\frac{\left(2+3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-\frac{3\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/Sqrt[5 + x^4],x]
```

```
[Out] (3*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (3*5^(1/4)*(Sqrt[5] + x^2)*
Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)],
1/2])/Sqrt[5 + x^4] + ((2 + 3*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 +
x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5
^(1/4)*Sqrt[5 + x^4])
```

Rubi in Sympy [A] time = 8.42789, size = 158, normalized size = 1.02

$$\frac{3x\sqrt{x^4+5}}{x^2+\sqrt{5}} - \frac{3\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\,\mathrm{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{\sqrt{x^4+5}} + \frac{\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{2\sqrt{5}}{5}+3\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\,\mathrm{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*x*sqrt(x**4 + 5)/(x**2 + sqrt(5)) - 3*5**(1/4)*sqrt((x**4 + 5)/
(sqrt(5)*x**2/5 + 1)**2)*(sqrt(5)*x**2/5 + 1)*elliptic_e(2*atan(5
**(3/4)*x/5), 1/2)/sqrt(x**4 + 5) + 5**(1/4)*sqrt((x**4 + 5)/(sqr
t(5)*x**2/5 + 1)**2)*(2*sqrt(5)/5 + 3)*(sqrt(5)*x**2/5 + 1)*ellip
tic_f(2*atan(5**(3/4)*x/5), 1/2)/(2*sqrt(x**4 + 5))
```

Mathematica [C] time = 0.0849446, size = 62, normalized size = 0.4

$$\sqrt[4]{-\frac{1}{5}}\left(\left(-2+3i\sqrt{5}\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)-3i\sqrt{5}E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)/Sqrt[5 + x^4],x]
```

```
[Out] (-1/5)^(1/4)*((-3*I)*Sqrt[5]*EllipticE[I*ArcSinh[(-1/5)^(1/4)*x],
 -1] + (-2 + (3*I)*Sqrt[5])*EllipticF[I*ArcSinh[(-1/5)^(1/4)*x],
-1])
```

Maple [C] time = 0.014, size = 146, normalized size = 0.9

$$\frac{2\sqrt{5}}{25\sqrt{i\sqrt{5}}}\sqrt{25-5\,i\sqrt{5}x^2}\sqrt{25+5\,i\sqrt{5}x^2}\,EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}} + \frac{\frac{3\,i}{5}}{\sqrt{i\sqrt{5}}}\sqrt{25-5\,i\sqrt{5}x^2}\sqrt{25+5\,i\sqrt{5}x^2}\left(EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-EllipticE\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/(x^4+5)^(1/2),x)
```

```
[Out] 2/25*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I
*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1
/2))^(1/2),I)+3/5*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*
(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)
*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/sqrt(x^4 + 5),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,x^2+2}{\sqrt{x^4+5}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/sqrt(x^4 + 5),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)/sqrt(x^4 + 5), x)
```

Sympy [A] time = 3.03769, size = 73, normalized size = 0.47

$$\frac{3\sqrt{5}x^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20\,\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_pola
r(I*pi)/5)/(20*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((1/4, 1/2
), (5/4,), x**4*exp_polar(I*pi)/5)/(10*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/sqrt(x^4 + 5),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5), x)
```

3.42 $\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=173

$$-\frac{2\sqrt{x^4+5}}{5x}+\frac{2\sqrt{x^4+5}x}{5\left(x^2+\sqrt{5}\right)}+\frac{\left(2+3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\,5^{3/4}\sqrt{x^4+5}}-\frac{2\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{x^4+5}}$$

```
[Out] (-2*Sqrt[5 + x^4])/(5*x) + (2*x*Sqrt[5 + x^4])/(5*(Sqrt[5] + x^2)
) - (2*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Elliptic
E[2*ArcTan[x/5^(1/4)], 1/2])/(5^(3/4)*Sqrt[5 + x^4]) + ((2 + 3*Sq
rt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Elliptic
F[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(3/4)*Sqrt[5 + x^4])
```

Rubi [A] time = 0.152372, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{2\sqrt{x^4+5}}{5x}+\frac{2\sqrt{x^4+5}x}{5\left(x^2+\sqrt{5}\right)}+\frac{\left(2+3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\,5^{3/4}\sqrt{x^4+5}}-\frac{2\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(x^2*Sqrt[5 + x^4]),x]
```

```
[Out] (-2*Sqrt[5 + x^4])/(5*x) + (2*x*Sqrt[5 + x^4])/(5*(Sqrt[5] + x^2)
) - (2*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Elliptic
E[2*ArcTan[x/5^(1/4)], 1/2])/(5^(3/4)*Sqrt[5 + x^4]) + ((2 + 3*Sq
rt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Elliptic
F[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(3/4)*Sqrt[5 + x^4])
```

Rubi in Sympy [A] time = 13.2602, size = 175, normalized size = 1.01

$$\frac{2x\sqrt{x^4+5}}{5\left(x^2+\sqrt{5}\right)} - \frac{2\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{5\sqrt{x^4+5}} + \frac{\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(2+3\sqrt{5}\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{10\sqrt{x^4+5}} - \frac{2\sqrt{x^4+5}}{5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/x**2/(x**4+5)**(1/2),x)
```

```
[Out] 2*x*sqrt(x**4 + 5)/(5*(x**2 + sqrt(5))) - 2*5**(1/4)*sqrt((x**4 +
 5)/(sqrt(5)*x**2/5 + 1)**2)*(sqrt(5)*x**2/5 + 1)*elliptic_e(2*at
an(5**(3/4)*x/5), 1/2)/(5*sqrt(x**4 + 5)) + 5**(1/4)*sqrt((x**4 +
 5)/(sqrt(5)*x**2/5 + 1)**2)*(2 + 3*sqrt(5))*(sqrt(5)*x**2/5 + 1)
*elliptic_f(2*atan(5**(3/4)*x/5), 1/2)/(10*sqrt(x**4 + 5)) - 2*sq
rt(x**4 + 5)/(5*x)
```

Mathematica [C] time = 0.24759, size = 81, normalized size = 0.47

$$\frac{1}{5}\left(-\frac{2\sqrt{x^4+5}}{x} - \sqrt[4]{-5}\left(3\sqrt{5}-2i\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right) - 2(-1)^{3/4}\sqrt[4]{5}E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)/(x^2*Sqrt[5 + x^4]),x]
```

```
[Out] ((-2*Sqrt[5 + x^4])/x - 2*(-1)^(3/4)*5^(1/4)*EllipticE[I*ArcSinh[
(-1/5)^(1/4)*x], -1] - (-5)^(1/4)*(-2*I + 3*Sqrt[5])*EllipticF[I*
ArcSinh[(-1/5)^(1/4)*x], -1])/5
```

Maple [C] time = 0.021, size = 158, normalized size = 0.9

$$\frac{3\sqrt{5}}{25\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}} - \frac{2}{5x}\sqrt{x^4+5} + \frac{\frac{2i}{25}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right) - EllipticE\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/x^2/(x^4+5)^(1/2),x)
```

```
[Out] 3/25*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I
*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1
/2))^(1/2),I)-2/5*(x^4+5)^(1/2)/x+2/25*I/(I*5^(1/2))^(1/2)*(25-5*
I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(El
lipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2
)*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,x^2+2}{\sqrt{x^4+5}x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^2),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)/(sqrt(x^4 + 5)*x^2), x)
```

Sympy [A] time = 3.44405, size = 75, normalized size = 0.43

$$\frac{3\sqrt{5}x\,\Gamma\left(\frac{1}{4}\right)\,{}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{20\,\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right)\,{}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{2}\\\frac{3}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{10x\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)/x**2/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(I
*pi)/5)/(20*gamma(5/4)) + sqrt(5)*gamma(-1/4)*hyper((-1/4, 1/2),
(3/4,), x**4*exp_polar(I*pi)/5)/(10*x*gamma(3/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^2), x)
```

3.43 $\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=189

$$-\frac{3\sqrt{x^4+5}}{5x}-\frac{2\sqrt{x^4+5}}{15x^3}+\frac{3\sqrt{x^4+5}\,x}{5\left(x^2+\sqrt{5}\right)}-\frac{\left(2-9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{30\sqrt[4]{5}\sqrt{x^4+5}}-\frac{3\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{x^4+5}}$$

```
[Out] (-2*Sqrt[5 + x^4])/(15*x^3) - (3*Sqrt[5 + x^4])/(5*x) + (3*x*Sqrt
[5 + x^4])/(5*(Sqrt[5] + x^2)) - (3*(Sqrt[5] + x^2)*Sqrt[(5 + x^4
)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(5^(3/4
)*Sqrt[5 + x^4]) - ((2 - 9*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4
)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(30*5^(
1/4)*Sqrt[5 + x^4])
```

Rubi [A] time = 0.214632, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{3\sqrt{x^4+5}}{5x}-\frac{2\sqrt{x^4+5}}{15x^3}+\frac{3\sqrt{x^4+5}\,x}{5\left(x^2+\sqrt{5}\right)}-\frac{\left(2-9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{30\sqrt[4]{5}\sqrt{x^4+5}}-\frac{3\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(x^4*Sqrt[5 + x^4]),x]
```


```
[Out] (-2*Sqrt[5 + x^4])/(15*x^3) - (3*Sqrt[5 + x^4])/(5*x) + (3*x*Sqrt
[5 + x^4])/(5*(Sqrt[5] + x^2)) - (3*(Sqrt[5] + x^2)*Sqrt[(5 + x^4
)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(5^(3/4
)*Sqrt[5 + x^4]) - ((2 - 9*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4
)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(30*5^(
1/4)*Sqrt[5 + x^4])
```

Rubi in Sympy [A] time = 17.9886, size = 190, normalized size = 1.01

$$\frac{3x\sqrt{x^4+5}}{5\left(x^2+\sqrt{5}\right)}-\frac{3\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{5\sqrt{x^4+5}}+\frac{\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(-2\sqrt{5}+45\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{150\sqrt{x^4+5}}-\frac{3\sqrt{x^4+5}}{5x}-\frac{2\sqrt{x^4+5}}{15x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/x**4/(x**4+5)**(1/2),x)
```

```
[Out] 3*x*sqrt(x**4 + 5)/(5*(x**2 + sqrt(5))) - 3*5**(1/4)*sqrt((x**4 +
 5)/(sqrt(5)*x**2/5 + 1)**2)*(sqrt(5)*x**2/5 + 1)*elliptic_e(2*at
an(5**(3/4)*x/5), 1/2)/(5*sqrt(x**4 + 5)) + 5**(1/4)*sqrt((x**4 +
 5)/(sqrt(5)*x**2/5 + 1)**2)*(-2*sqrt(5) + 45)*(sqrt(5)*x**2/5 +
1)*elliptic_f(2*atan(5**(3/4)*x/5), 1/2)/(150*sqrt(x**4 + 5)) - 3
*sqrt(x**4 + 5)/(5*x) - 2*sqrt(x**4 + 5)/(15*x**3)
```

Mathematica [C] time = 0.204651, size = 97, normalized size = 0.51

$$\frac{1}{75}\left(-\frac{5\left(9x^6+2x^4+45x^2+10\right)}{x^3\sqrt{x^4+5}}+\sqrt[4]{-5}\left(2\sqrt{5}+45i\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)-45(-1)^{3/4}\sqrt[4]{5}E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)/(x^4*Sqrt[5 + x^4]),x]
```

```
[Out] ((-5*(10 + 45*x^2 + 2*x^4 + 9*x^6))/(x^3*Sqrt[5 + x^4]) - 45*(-1)
^(3/4)*5^(1/4)*EllipticE[I*ArcSinh[(-1/5)^(1/4)*x], -1] + (-5)^(1
/4)*(45*I + 2*Sqrt[5])*EllipticF[I*ArcSinh[(-1/5)^(1/4)*x], -1])/
75
```

Maple [C] time = 0.024, size = 170, normalized size = 0.9

$$-\frac{2}{15 x^{3}}\sqrt{x^{4}+5}-\frac{2\sqrt{5}}{375\sqrt{i\sqrt{5}}}\sqrt{25-5 i\sqrt{5}x^{2}}\sqrt{25+5 i\sqrt{5}x^{2}}EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^{4}+5}}-\frac{3}{5 x}\sqrt{x^{4}+5}+\frac{\frac{3 i}{25}}{\sqrt{i\sqrt{5}}}\sqrt{25-5 i\sqrt{5}x^{2}}\sqrt{25+5 i\sqrt{5}x^{2}}\left(EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-EllipticE\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^{4}+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/x^4/(x^4+5)^(1/2),x)
```

```
[Out] -2/15*(x^4+5)^(1/2)/x^3-2/375*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5
^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*Ellipt
icF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-3/5*(x^4+5)^(1/2)/x+3/25*I
/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2
)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),
I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3 x^{2}+2}{\sqrt{x^{4}+5}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^4),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3 x^{2}+2}{\sqrt{x^{4}+5}x^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)/(sqrt(x^4 + 5)*x^4), x)
```

Sympy [A] time = 4.17723, size = 80, normalized size = 0.42

$$\frac{3\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20x\,\Gamma\left(\frac{3}{4}\right)}+\frac{\sqrt{5}\,\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10x^3\,\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)/x**4/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4*exp_polar(I
*pi)/5)/(20*x*gamma(3/4)) + sqrt(5)*gamma(-3/4)*hyper((-3/4, 1/2)
, (1/4,), x**4*exp_polar(I*pi)/5)/(10*x**3*gamma(1/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^4),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^4), x)
```

3.44 $\int \frac{x^7\left(2+3x^2\right)}{\left(5+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=58

$$-\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\left(3x^2+2\right)x^4}{2\sqrt{x^4+5}}+\frac{1}{4}\left(9x^2+8\right)\sqrt{x^4+5}$$

```
[Out] -(x^4*(2 + 3*x^2))/(2*Sqrt[5 + x^4]) + ((8 + 9*x^2)*Sqrt[5 + x^4]
)/4 - (45*ArcSinh[x^2/Sqrt[5]])/4
```

Rubi [A] time = 0.145792, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\left(3x^2+2\right)x^4}{2\sqrt{x^4+5}}+\frac{1}{4}\left(9x^2+8\right)\sqrt{x^4+5}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^7*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -(x^4*(2 + 3*x^2))/(2*Sqrt[5 + x^4]) + ((8 + 9*x^2)*Sqrt[5 + x^4]
)/4 - (45*ArcSinh[x^2/Sqrt[5]])/4
```

Rubi in Sympy [A] time = 11.2306, size = 51, normalized size = 0.88

$$-\frac{x^4\left(30x^2+20\right)}{20\sqrt{x^4+5}}+\frac{\left(90x^2+80\right)\sqrt{x^4+5}}{40}-\frac{45\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] -x**4*(30*x**2 + 20)/(20*sqrt(x**4 + 5)) + (90*x**2 + 80)*sqrt(x*
*4 + 5)/40 - 45*asinh(sqrt(5)*x**2/5)/4
```

Mathematica [A] time = 0.0589764, size = 44, normalized size = 0.76

$$\frac{1}{4}\left(\frac{3x^6+4x^4+45x^2+40}{\sqrt{x^4+5}}-45\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^7*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] ((40 + 45*x^2 + 4*x^4 + 3*x^6)/Sqrt[5 + x^4] - 45*ArcSinh[x^2/Sqr
t[5]])/4
```

Maple [A] time = 0.026, size = 50, normalized size = 0.9

$$(x^4+10)\frac{1}{\sqrt{x^4+5}}+\frac{3\,x^6}{4}\frac{1}{\sqrt{x^4+5}}+\frac{45\,x^2}{4}\frac{1}{\sqrt{x^4+5}}-\frac{45}{4}\mathit{Arcsinh}\left(\frac{\sqrt{5}x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] 1/(x^4+5)^(1/2)*(x^4+10)+3/4*x^6/(x^4+5)^(1/2)+45/4*x^2/(x^4+5)^(
1/2)-45/4*arcsinh(1/5*5^(1/2)*x^2)
```

Maxima [A] time = 0.774079, size = 120, normalized size = 2.07

$$\sqrt{x^4+5}-\frac{15\left(\frac{3\,(x^4+5)}{x^4}-2\right)}{4\left(\frac{\sqrt{x^4+5}}{x^2}-\frac{(x^4+5)^{\frac{3}{2}}}{x^6}\right)}+\frac{5}{\sqrt{x^4+5}}-\frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^7/(x^4 + 5)^(3/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^4 + 5) - 15/4*(3*(x^4 + 5)/x^4 - 2)/(sqrt(x^4 + 5)/x^2 - (
x^4 + 5)^(3/2)/x^6) + 5/sqrt(x^4 + 5) - 45/8*log(sqrt(x^4 + 5)/x^
2 + 1) + 45/8*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 0.297073, size = 205, normalized size = 3.53

$$-\frac{12\,x^{12}+16\,x^{10}+105\,x^8+220\,x^6-75\,x^4+600\,x^2-45\left(4\,x^8+25\,x^4-\left(4\,x^6+15\,x^2\right)\sqrt{x^4+5}+25\right)\log\left(-x^2+\sqrt{x^4+5}\right)-}{4\left(4\,x^8+25\,x^4-(4\,x^6+15\,x^2)\sqrt{x^4+5}+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^7/(x^4 + 5)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(12*x^12 + 16*x^10 + 105*x^8 + 220*x^6 - 75*x^4 + 600*x^2 -
45*(4*x^8 + 25*x^4 - (4*x^6 + 15*x^2)*sqrt(x^4 + 5) + 25)*log(-x^
2 + sqrt(x^4 + 5)) - (12*x^10 + 16*x^8 + 75*x^6 + 180*x^4 - 225*x
^2 + 200)*sqrt(x^4 + 5) - 750)/(4*x^8 + 25*x^4 - (4*x^6 + 15*x^2)
*sqrt(x^4 + 5) + 25)
```

Sympy [A] time = 30.3557, size = 66, normalized size = 1.14

$$\frac{3x^6}{4\sqrt{x^4+5}} + \frac{x^4}{\sqrt{x^4+5}} + \frac{45x^2}{4\sqrt{x^4+5}} - \frac{45\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4} + \frac{10}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) + x**4/sqrt(x**4 + 5) + 45*x**2/(4*sqrt
(x**4 + 5)) - 45*asinh(sqrt(5)*x**2/5)/4 + 10/sqrt(x**4 + 5)
```

GIAC/XCAS [A] time = 0.268689, size = 61, normalized size = 1.05

$$\frac{\left(\left(3\,x^2+4\right)x^2+45\right)x^2+40}{4\,\sqrt{x^4+5}} + \frac{45}{4}\ln\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^7/(x^4 + 5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(((3*x^2 + 4)*x^2 + 45)*x^2 + 40)/sqrt(x^4 + 5) + 45/4*ln(-x^
2 + sqrt(x^4 + 5))
```

3.45 $\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=45

$$3\sqrt{x^4+5}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(3x^2+2)\,x^2}{2\sqrt{x^4+5}}$$

```
[Out] -(x^2*(2 + 3*x^2))/(2*Sqrt[5 + x^4]) + 3*Sqrt[5 + x^4] + ArcSinh[
x^2/Sqrt[5]]
```

Rubi [A] time = 0.120434, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$3\sqrt{x^4+5}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(3x^2+2)\,x^2}{2\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -(x^2*(2 + 3*x^2))/(2*Sqrt[5 + x^4]) + 3*Sqrt[5 + x^4] + ArcSinh[
x^2/Sqrt[5]]
```

Rubi in Sympy [A] time = 10.5176, size = 41, normalized size = 0.91

$$-\frac{x^2\left(30x^2+20\right)}{20\sqrt{x^4+5}}+3\sqrt{x^4+5}+\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] -x**2*(30*x**2 + 20)/(20*sqrt(x**4 + 5)) + 3*sqrt(x**4 + 5) + asi
nh(sqrt(5)*x**2/5)
```

Mathematica [A] time = 0.0475588, size = 36, normalized size = 0.8

$$\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{3x^4-2x^2+30}{2\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (30 - 2*x^2 + 3*x^4)/(2*Sqrt[5 + x^4]) + ArcSinh[x^2/Sqrt[5]]
```

Maple [A] time = 0.018, size = 37, normalized size = 0.8

$$-x^2\frac{1}{\sqrt{x^4+5}}+Arcsinh\left(\frac{\sqrt{5}x^2}{5}\right)+\frac{3\,x^4+30}{2}\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] -x^2/(x^4+5)^(1/2)+arcsinh(1/5*5^(1/2)*x^2)+3/2/(x^4+5)^(1/2)*(x^
4+10)
```

Maxima [A] time = 0.783187, size = 85, normalized size = 1.89

$$-\frac{x^2}{\sqrt{x^4+5}}+\frac{3}{2}\sqrt{x^4+5}+\frac{15}{2\sqrt{x^4+5}}+\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^5/(x^4 + 5)^(3/2),x, algorithm="maxima")
```

```
[Out] -x^2/sqrt(x^4 + 5) + 3/2*sqrt(x^4 + 5) + 15/2/sqrt(x^4 + 5) + 1/2
*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 0.326741, size = 151, normalized size = 3.36

$$-\frac{6\,x^8+75\,x^4+10\,x^2+2\left(2\,x^6+10\,x^2-\left(2\,x^4+5\right)\sqrt{x^4+5}\right)\log\left(-x^2+\sqrt{x^4+5}\right)-2\left(3\,x^6+30\,x^2+5\right)\sqrt{x^4+5}+150}{2\left(2\,x^6+10\,x^2-(2\,x^4+5)\sqrt{x^4+5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^5/(x^4 + 5)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(6*x^8 + 75*x^4 + 10*x^2 + 2*(2*x^6 + 10*x^2 - (2*x^4 + 5)*s
qrt(x^4 + 5))*log(-x^2 + sqrt(x^4 + 5)) - 2*(3*x^6 + 30*x^2 + 5)*
sqrt(x^4 + 5) + 150)/(2*x^6 + 10*x^2 - (2*x^4 + 5)*sqrt(x^4 + 5))
```

Sympy [A] time = 20.5688, size = 48, normalized size = 1.07

$$\frac{3x^4}{2\sqrt{x^4+5}} - \frac{x^2}{\sqrt{x^4+5}} + \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right) + \frac{15}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*x**4/(2*sqrt(x**4 + 5)) - x**2/sqrt(x**4 + 5) + asinh(sqrt(5)*x
**2/5) + 15/sqrt(x**4 + 5)
```

GIAC/XCAS [A] time = 0.269716, size = 53, normalized size = 1.18

$$\frac{\left(3\,x^2-2\right)x^2+30}{2\,\sqrt{x^4+5}} - \ln\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^5/(x^4 + 5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((3*x^2 - 2)*x^2 + 30)/sqrt(x^4 + 5) - ln(-x^2 + sqrt(x^4 + 5
))
```

3.46 $\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{3x^2+2}{2\sqrt{x^4+5}}$$

```
[Out] -(2 + 3*x^2)/(2*Sqrt[5 + x^4]) + (3*ArcSinh[x^2/Sqrt[5]])/2
```

Rubi [A] time = 0.0831988, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{3x^2+2}{2\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(2 + 3*x^2))/(5 + x^4)^(3/2),x]

[Out] -(2 + 3*x^2)/(2*Sqrt[5 + x^4]) + (3*ArcSinh[x^2/Sqrt[5]])/2
```

Rubi in Sympy [A] time = 8.20522, size = 31, normalized size = 0.89

$$-\frac{15x^2+10}{10\sqrt{x^4+5}}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(3*x**2+2)/(x**4+5)**(3/2),x)

[Out] -(15*x**2 + 10)/(10*sqrt(x**4 + 5)) + 3*asinh(sqrt(5)*x**2/5)/2
```

Mathematica [A] time = 0.028633, size = 35, normalized size = 1.

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{-3x^2-2}{2\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (-2 - 3*x^2)/(2*Sqrt[5 + x^4]) + (3*ArcSinh[x^2/Sqrt[5]])/2
```

Maple [A] time = 0.01, size = 34, normalized size = 1.

$$-\frac{1}{\sqrt{x^4+5}}-\frac{3\,x^2}{2}\frac{1}{\sqrt{x^4+5}}+\frac{3}{2}\mathit{Arcsinh}\left(\frac{\sqrt{5}x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] -1/(x^4+5)^(1/2)-3/2*x^2/(x^4+5)^(1/2)+3/2*arcsinh(1/5*5^(1/2)*x^
2)
```

Maxima [A] time = 0.787725, size = 73, normalized size = 2.09

$$-\frac{3\,x^2}{2\,\sqrt{x^4+5}}-\frac{1}{\sqrt{x^4+5}}+\frac{3}{4}\,\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{3}{4}\,\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^3/(x^4 + 5)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/2*x^2/sqrt(x^4 + 5) - 1/sqrt(x^4 + 5) + 3/4*log(sqrt(x^4 + 5)/
x^2 + 1) - 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 0.306065, size = 95, normalized size = 2.71

$$\frac{2\,x^2-3\left(x^4-\sqrt{x^4+5}x^2+5\right)\log\left(-x^2+\sqrt{x^4+5}\right)-2\,\sqrt{x^4+5}-15}{2\left(x^4-\sqrt{x^4+5}x^2+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^3/(x^4 + 5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^2 - 3*(x^4 - sqrt(x^4 + 5)*x^2 + 5)*log(-x^2 + sqrt(x^4
+ 5)) - 2*sqrt(x^4 + 5) - 15)/(x^4 - sqrt(x^4 + 5)*x^2 + 5)
```

Sympy [A] time = 16.0662, size = 39, normalized size = 1.11

$$-\frac{3x^2}{2\sqrt{x^4+5}}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}-\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] -3*x**2/(2*sqrt(x**4 + 5)) + 3*asinh(sqrt(5)*x**2/5)/2 - 1/sqrt(x
**4 + 5)
```

GIAC/XCAS [A] time = 0.274721, size = 45, normalized size = 1.29

$$-\frac{3\,x^2+2}{2\,\sqrt{x^4+5}}-\frac{3}{2}\ln\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^3/(x^4 + 5)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*x^2 + 2)/sqrt(x^4 + 5) - 3/2*ln(-x^2 + sqrt(x^4 + 5))
```

3.47 $\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=20

$$-\frac{15-2x^2}{10\sqrt{x^4+5}}$$

```
[Out] -(15 - 2*x^2)/(10*Sqrt[5 + x^4])
```

Rubi [A] time = 0.0505839, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{15-2x^2}{10\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(2 + 3*x^2))/(5 + x^4)^(3/2),x]

[Out] -(15 - 2*x^2)/(10*Sqrt[5 + x^4])
```

Rubi in Sympy [A] time = 6.2442, size = 17, normalized size = 0.85

$$-\frac{-2x^2+15}{10\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(3*x**2+2)/(x**4+5)**(3/2),x)

[Out] -(-2*x**2 + 15)/(10*sqrt(x**4 + 5))
```

Mathematica [A] time = 0.0157912, size = 20, normalized size = 1.

$$\frac{2x^2-15}{10\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(2 + 3*x^2))/(5 + x^4)^(3/2),x]

[Out] (-15 + 2*x^2)/(10*Sqrt[5 + x^4])
```

Maple [A] time = 0.006, size = 17, normalized size = 0.9

$$\frac{2\,x^2-15}{10}\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(3*x^2+2)/(x^4+5)^(3/2),x)

[Out] 1/10*(2*x^2-15)/(x^4+5)^(1/2)
```

Maxima [A] time = 0.776759, size = 30, normalized size = 1.5

$$\frac{x^2}{5\,\sqrt{x^4+5}}-\frac{3}{2\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x/(x^4 + 5)^(3/2),x, algorithm="maxima")

[Out] 1/5*x^2/sqrt(x^4 + 5) - 3/2/sqrt(x^4 + 5)
```

Fricas [A] time = 0.308302, size = 50, normalized size = 2.5

$$\frac{3\,x^2-3\,\sqrt{x^4+5}+2}{2\left(x^4-\sqrt{x^4+5}x^2+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x/(x^4 + 5)^(3/2),x, algorithm="fricas")

[Out] 1/2*(3*x^2 - 3*sqrt(x^4 + 5) + 2)/(x^4 - sqrt(x^4 + 5)*x^2 + 5)
```

Sympy [A] time = 13.1521, size = 24, normalized size = 1.2

$$\frac{x^2}{5\sqrt{x^4+5}} - \frac{3}{2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(3*x**2+2)/(x**4+5)**(3/2),x)

[Out] x**2/(5*sqrt(x**4 + 5)) - 3/(2*sqrt(x**4 + 5))
```

GIAC/XCAS [A] time = 0.270664, size = 22, normalized size = 1.1

$$\frac{2\,x^2 - 15}{10\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x/(x^4 + 5)^(3/2),x, algorithm="giac")

[Out] 1/10*(2*x^2 - 15)/sqrt(x^4 + 5)
```

3.48 $\int \frac{2+3x^2}{x(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{3x^2+2}{10\sqrt{x^4+5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

```
[Out] (2 + 3*x^2)/(10*Sqrt[5 + x^4]) - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/(
5*Sqrt[5])
```

Rubi [A] time = 0.115096, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{3x^2+2}{10\sqrt{x^4+5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(x*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*Sqrt[5 + x^4]) - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/(
5*Sqrt[5])
```

Rubi in Sympy [A] time = 10.5916, size = 39, normalized size = 0.85

$$\frac{15x^2+10}{50\sqrt{x^4+5}}-\frac{\sqrt{5}\,\operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/x/(x**4+5)**(3/2),x)
```

```
[Out] (15*x**2 + 10)/(50*sqrt(x**4 + 5)) - sqrt(5)*atanh(sqrt(5)*sqrt(x
**4 + 5)/5)/25
```

Mathematica [A] time = 0.0540103, size = 46, normalized size = 1.

$$\frac{3x^2+2}{10\sqrt{x^4+5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)/(x*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*Sqrt[5 + x^4]) - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/(
5*Sqrt[5])
```

Maple [A] time = 0.023, size = 40, normalized size = 0.9

$$\frac{1}{5}\frac{1}{\sqrt{x^4+5}}-\frac{\sqrt{5}}{25}Artanh\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)+\frac{3\,x^2}{10}\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/x/(x^4+5)^(3/2),x)
```

```
[Out] 1/5/(x^4+5)^(1/2)-1/25*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))+3/1
0*x^2/(x^4+5)^(1/2)
```

Maxima [A] time = 0.787468, size = 76, normalized size = 1.65

$$\frac{3\,x^2}{10\,\sqrt{x^4+5}}+\frac{1}{50}\,\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{1}{5\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] 3/10*x^2/sqrt(x^4 + 5) + 1/50*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 +
5))/(sqrt(5) + sqrt(x^4 + 5))) + 1/5/sqrt(x^4 + 5)
```

Fricas [A] time = 0.335621, size = 170, normalized size = 3.7

$$-\frac{2\left(x^{4}-\sqrt{x^{4}+5}x^{2}+5\right)\log\left(\frac{5x^{2}+\sqrt{5}(x^{4}+5)-\sqrt{x^{4}+5}\left(\sqrt{5}x^{2}+5\right)}{x^{4}-\sqrt{x^{4}+5}x^{2}}\right)-\sqrt{5}\left(2x^{2}-15\right)+2\sqrt{5}\sqrt{x^{4}+5}}{10\left(\sqrt{5}\sqrt{x^{4}+5}x^{2}-\sqrt{5}(x^{4}+5)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] -1/10*(2*(x^4 - sqrt(x^4 + 5)*x^2 + 5)*log((5*x^2 + sqrt(5)*(x^4
+ 5) - sqrt(x^4 + 5)*(sqrt(5)*x^2 + 5))/(x^4 - sqrt(x^4 + 5)*x^2)
) - sqrt(5)*(2*x^2 - 15) + 2*sqrt(5)*sqrt(x^4 + 5))/(sqrt(5)*sqrt
(x^4 + 5)*x^2 - sqrt(5)*(x^4 + 5))
```

Sympy [A] time = 18.7425, size = 212, normalized size = 4.61

$$\frac{2x^{4}\log\left(x^{4}\right)}{20\sqrt{5}x^{4}+100\sqrt{5}}-\frac{4x^{4}\log\left(\sqrt{\frac{x^{4}}{5}+1}+1\right)}{20\sqrt{5}x^{4}+100\sqrt{5}}-\frac{2x^{4}\log\left(5\right)}{20\sqrt{5}x^{4}+100\sqrt{5}}+\frac{3x^{2}}{10\sqrt{x^{4}+5}}+\frac{4\sqrt{5}\sqrt{x^{4}+5}}{20\sqrt{5}x^{4}+100\sqrt{5}}+\frac{10\log\left(x^{4}\right)}{20\sqrt{5}x^{4}+100\sqrt{5}}-\frac{20\log\left(\sqrt{\frac{x^{4}}{5}+1}+1\right)}{20\sqrt{5}x^{4}+100\sqrt{5}}-\frac{10\log\left(5\right)}{20\sqrt{5}x^{4}+100\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)/x/(x**4+5)**(3/2),x)
```

```
[Out] 2*x**4*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 4*x**4*log(sqr
t(x**4/5 + 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 2*x**4*log(5
)/(20*sqrt(5)*x**4 + 100*sqrt(5)) + 3*x**2/(10*sqrt(x**4 + 5)) +
4*sqrt(5)*sqrt(x**4 + 5)/(20*sqrt(5)*x**4 + 100*sqrt(5)) + 10*log
(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 20*log(sqrt(x**4/5 + 1)
+ 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 10*log(5)/(20*sqrt(5)*x**4
 + 100*sqrt(5))
```

GIAC/XCAS [A] time = 0.274245, size = 72, normalized size = 1.57

$$-\frac{1}{50}\sqrt{5}\ln\left(\sqrt{5}+\sqrt{x^{4}+5}\right)+\frac{1}{50}\sqrt{5}\ln\left(-\sqrt{5}+\sqrt{x^{4}+5}\right)+\frac{3x^{2}+2}{10\sqrt{x^{4}+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x),x, algorithm="giac")
```

```
[Out] -1/50*sqrt(5)*ln(sqrt(5) + sqrt(x^4 + 5)) + 1/50*sqrt(5)*ln(-sqrt
(5) + sqrt(x^4 + 5)) + 1/10*(3*x^2 + 2)/sqrt(x^4 + 5)
```

3.49 $\int \frac{2+3x^2}{x^3(5+x^4)^{3/2}} \, dx$

Optimal. Leaf size=65

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}+\frac{3x^2+2}{10x^2\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{25x^2}$$

```
[Out] (2 + 3*x^2)/(10*x^2*Sqrt[5 + x^4]) - (2*Sqrt[5 + x^4])/(25*x^2) -
 (3*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/(10*Sqrt[5])
```

Rubi [A] time = 0.157011, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}+\frac{3x^2+2}{10x^2\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{25x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(x^3*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*x^2*Sqrt[5 + x^4]) - (2*Sqrt[5 + x^4])/(25*x^2) -
 (3*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/(10*Sqrt[5])
```

Rubi in Sympy [A] time = 12.8169, size = 60, normalized size = 0.92

$$-\frac{3\sqrt{5}\operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)}{50}+\frac{15x^2+10}{50x^2\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{25x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/x**3/(x**4+5)**(3/2),x)
```

```
[Out] -3*sqrt(5)*atanh(sqrt(5)*sqrt(x**4 + 5)/5)/50 + (15*x**2 + 10)/(5
0*x**2*sqrt(x**4 + 5)) - 2*sqrt(x**4 + 5)/(25*x**2)
```

Mathematica [A] time = 0.0921717, size = 53, normalized size = 0.82

$$\frac{1}{50}\left(\frac{-4x^4+15x^2-10}{x^2\sqrt{x^4+5}}-3\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)/(x^3*(5 + x^4)^(3/2)),x]
```

```
[Out] ((-10 + 15*x^2 - 4*x^4)/(x^2*Sqrt[5 + x^4]) - 3*Sqrt[5]*ArcTanh[S
qrt[5 + x^4]/Sqrt[5]])/50
```

Maple [A] time = 0.019, size = 47, normalized size = 0.7

$$-\frac{2x^4+5}{25x^2}\frac{1}{\sqrt{x^4+5}}+\frac{3}{10}\frac{1}{\sqrt{x^4+5}}-\frac{3\sqrt{5}}{50}Artanh\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/x^3/(x^4+5)^(3/2),x)
```

```
[Out] -1/25/x^2*(2*x^4+5)/(x^4+5)^(1/2)+3/10/(x^4+5)^(1/2)-3/50*5^(1/2)
*arctanh(5^(1/2)/(x^4+5)^(1/2))
```

Maxima [A] time = 0.779939, size = 92, normalized size = 1.42

$$-\frac{x^2}{25\sqrt{x^4+5}}+\frac{3}{100}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{3}{10\sqrt{x^4+5}}-\frac{\sqrt{x^4+5}}{25x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^3),x, algorithm="maxima")
```

```
[Out] -1/25*x^2/sqrt(x^4 + 5) + 3/100*sqrt(5)*log(-(sqrt(5) - sqrt(x^4
+ 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3/10/sqrt(x^4 + 5) - 1/25*sqrt
(x^4 + 5)/x^2
```

Fricas [A] time = 0.299276, size = 216, normalized size = 3.32

$$-\frac{6\sqrt{5}\sqrt{x^4+5}x^4+3\left(2x^8+10x^4-\left(2x^6+5x^2\right)\sqrt{x^4+5}\right)\log\left(\frac{5x^2+\sqrt{5}\left(x^4+5\right)-\sqrt{x^4+5}\left(\sqrt{5}x^2+5\right)}{x^4-\sqrt{x^4+5}x^2}\right)-\sqrt{5}\left(6x^6+15x^2-10\right)}{10\left(\sqrt{5}(2x^6+5x^2)\sqrt{x^4+5}-2\sqrt{5}(x^8+5x^4)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^3),x, algorithm="fricas")
```

```
[Out] -1/10*(6*sqrt(5)*sqrt(x^4 + 5)*x^4 + 3*(2*x^8 + 10*x^4 - (2*x^6 +
 5*x^2)*sqrt(x^4 + 5))*log((5*x^2 + sqrt(5)*(x^4 + 5) - sqrt(x^4
+ 5)*(sqrt(5)*x^2 + 5))/(x^4 - sqrt(x^4 + 5)*x^2)) - sqrt(5)*(6*x
^6 + 15*x^2 - 10))/(sqrt(5)*(2*x^6 + 5*x^2)*sqrt(x^4 + 5) - 2*sqr
t(5)*(x^8 + 5*x^4))
```

Sympy [A] time = 27.01, size = 228, normalized size = 3.51

$$\frac{3x^4\log\left(x^4\right)}{20\sqrt{5}x^4+100\sqrt{5}}-\frac{6x^4\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{20\sqrt{5}x^4+100\sqrt{5}}-\frac{3x^4\log\left(5\right)}{20\sqrt{5}x^4+100\sqrt{5}}+\frac{6\sqrt{5}\sqrt{x^4+5}}{20\sqrt{5}x^4+100\sqrt{5}}+\frac{15\log\left(x^4\right)}{20\sqrt{5}x^4+100\sqrt{5}}-\frac{30\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{20\sqrt{5}x^4+100\sqrt{5}}-\frac{15\log\left(5\right)}{20\sqrt{5}x^4+100\sqrt{5}}-\frac{2}{25\sqrt{1+\frac{5}{x^4}}}-\frac{1}{5x^4\sqrt{1+\frac{5}{x^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)/x**3/(x**4+5)**(3/2),x)
```

```
[Out] 3*x**4*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 6*x**4*log(sqr
t(x**4/5 + 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 3*x**4*log(5
)/(20*sqrt(5)*x**4 + 100*sqrt(5)) + 6*sqrt(5)*sqrt(x**4 + 5)/(20*
sqrt(5)*x**4 + 100*sqrt(5)) + 15*log(x**4)/(20*sqrt(5)*x**4 + 100
*sqrt(5)) - 30*log(sqrt(x**4/5 + 1) + 1)/(20*sqrt(5)*x**4 + 100*s
qrt(5)) - 15*log(5)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 2/(25*sqrt(
1 + 5/x**4)) - 1/(5*x**4*sqrt(1 + 5/x**4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{(x^4+5)^{\frac{3}{2}}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^3),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^3), x)
```

3.50 $\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=196

$$-\frac{1}{5}\sqrt{x^4+5}x+\frac{9\sqrt{x^4+5}x}{2\left(x^2+\sqrt{5}\right)}+\frac{\left(2+9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{5}\sqrt{x^4+5}}-\frac{9\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt{x^4+5}}-\frac{\left(15-2x^2\right)x^3}{10\sqrt{x^4+5}}$$

```
[Out] -(x^3*(15 - 2*x^2))/(10*Sqrt[5 + x^4]) - (x*Sqrt[5 + x^4])/5 + (9
*x*Sqrt[5 + x^4])/(2*(Sqrt[5] + x^2)) - (9*5^(1/4)*(Sqrt[5] + x^2
)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)]
, 1/2])/(2*Sqrt[5 + x^4]) + ((2 + 9*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt
[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2]
)/(4*5^(1/4)*Sqrt[5 + x^4])
```

Rubi [A] time = 0.228814, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{1}{5}\sqrt{x^4+5}x+\frac{9\sqrt{x^4+5}x}{2\left(x^2+\sqrt{5}\right)}+\frac{\left(2+9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{5}\sqrt{x^4+5}}-\frac{9\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt{x^4+5}}-\frac{\left(15-2x^2\right)x^3}{10\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```


```
[Out] -(x^3*(15 - 2*x^2))/(10*Sqrt[5 + x^4]) - (x*Sqrt[5 + x^4])/5 + (9
*x*Sqrt[5 + x^4])/(2*(Sqrt[5] + x^2)) - (9*5^(1/4)*(Sqrt[5] + x^2
)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)]
, 1/2])/(2*Sqrt[5 + x^4]) + ((2 + 9*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt
[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2]
)/(4*5^(1/4)*Sqrt[5 + x^4])
```

Rubi in Sympy [A] time = 19.1688, size = 194, normalized size = 0.99

$$-\frac{x^3\left(-2x^2+15\right)}{10\sqrt{x^4+5}}-\frac{x\sqrt{x^4+5}}{5}+\frac{9x\sqrt{x^4+5}}{2\left(x^2+\sqrt{5}\right)}-\frac{9\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{2\sqrt{x^4+5}}$$
$$+\frac{\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(6\sqrt{5}+135\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{60\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] -x**3*(-2*x**2 + 15)/(10*sqrt(x**4 + 5)) - x*sqrt(x**4 + 5)/5 + 9
*x*sqrt(x**4 + 5)/(2*(x**2 + sqrt(5))) - 9*5**(1/4)*sqrt((x**4 +
5)/(sqrt(5)*x**2/5 + 1)**2)*(sqrt(5)*x**2/5 + 1)*elliptic_e(2*ata
n(5**(3/4)*x/5), 1/2)/(2*sqrt(x**4 + 5)) + 5**(1/4)*sqrt((x**4 +
5)/(sqrt(5)*x**2/5 + 1)**2)*(6*sqrt(5) + 135)*(sqrt(5)*x**2/5 + 1
)*elliptic_f(2*atan(5**(3/4)*x/5), 1/2)/(60*sqrt(x**4 + 5))
```

Mathematica [C] time = 0.197549, size = 85, normalized size = 0.43

$$\frac{1}{10}\left(-\frac{5x\left(3x^2+2\right)}{\sqrt{x^4+5}}+\sqrt[4]{-5}\left(-2\sqrt{5}+45i\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)\right.$$
$$\left.-45(-1)^{3/4}\sqrt[4]{5}E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] ((-5*x*(2 + 3*x^2))/Sqrt[5 + x^4] - 45*(-1)^(3/4)*5^(1/4)*Ellipti
cE[I*ArcSinh[(-1/5)^(1/4)*x], -1] + (-5)^(1/4)*(45*I - 2*Sqrt[5])
*EllipticF[I*ArcSinh[(-1/5)^(1/4)*x], -1])/10
```

Maple [C] time = 0.028, size = 168, normalized size = 0.9

$$-x\frac{1}{\sqrt{x^4+5}}+\frac{\sqrt{5}}{25\sqrt{i\sqrt{5}}}\sqrt{25-5\,i\sqrt{5}x^2}\sqrt{25+5\,i\sqrt{5}x^2}\mathit{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}}-\frac{3\,x^3}{2}\frac{1}{\sqrt{x^4+5}}+\frac{\frac{9\,i}{10}}{\sqrt{i\sqrt{5}}}\sqrt{25-5\,i\sqrt{5}x^2}\sqrt{25+5\,i\sqrt{5}x^2}\left(\mathit{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-\mathit{EllipticE}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] -x/(x^4+5)^(1/2)+1/25*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x
^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*
x*5^(1/2)*(I*5^(1/2))^(1/2),I)-3/2*x^3/(x^4+5)^(1/2)+9/10*I/(I*5^
(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2
)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-Ell
ipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3\,x^2+2)\,x^4}{(x^4+5)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^4/(x^4 + 5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{3\,x^6+2\,x^4}{(x^4+5)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^4/(x^4 + 5)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4)/(x^4 + 5)^(3/2), x)
```

Sympy [A] time = 13.0936, size = 75, normalized size = 0.38

$$\frac{3\sqrt{5}x^{7}\,\Gamma\left(\frac{7}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{x^{4}e^{i\pi}}{5}\right)}{100\,\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{5}x^{5}\,\Gamma\left(\frac{5}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\middle|\frac{x^{4}e^{i\pi}}{5}\right)}{50\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), x**4*exp_pol
ar(I*pi)/5)/(100*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((5/
4, 3/2), (9/4,), x**4*exp_polar(I*pi)/5)/(50*gamma(9/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3\,x^{2}+2\right)x^{4}}{\left(x^{4}+5\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^4/(x^4 + 5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5)^(3/2), x)
```

3.51 $\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{\sqrt{x^4+5}x}{5\left(x^2+\sqrt{5}\right)}-\frac{\left(15-2x^2\right)x}{10\sqrt{x^4+5}}-\frac{\left(2-3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{4\,5^{3/4}\sqrt{x^4+5}}+\frac{\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{x^4+5}}$$

```
[Out] -(x*(15 - 2*x^2))/(10*Sqrt[5 + x^4]) - (x*Sqrt[5 + x^4])/(5*(Sqrt
[5] + x^2)) + ((Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*
EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(5^(3/4)*Sqrt[5 + x^4]) - ((
2 - 3*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*
EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(4*5^(3/4)*Sqrt[5 + x^4])
```

Rubi [A] time = 0.166431, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{\sqrt{x^4+5}x}{5\left(x^2+\sqrt{5}\right)}-\frac{\left(15-2x^2\right)x}{10\sqrt{x^4+5}}-\frac{\left(2-3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{4\,5^{3/4}\sqrt{x^4+5}}+\frac{\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -(x*(15 - 2*x^2))/(10*Sqrt[5 + x^4]) - (x*Sqrt[5 + x^4])/(5*(Sqrt
[5] + x^2)) + ((Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*
EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(5^(3/4)*Sqrt[5 + x^4]) - ((
2 - 3*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*
EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(4*5^(3/4)*Sqrt[5 + x^4])
```

Rubi in Sympy [A] time = 14.1709, size = 177, normalized size = 1.

$$-\frac{x\left(-2x^2+15\right)}{10\sqrt{x^4+5}}-\frac{x\sqrt{x^4+5}}{5\left(x^2+\sqrt{5}\right)}+\frac{\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{5\sqrt{x^4+5}}$$
$$-\frac{\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(-3\sqrt{5}+2\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{20\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] -x*(-2*x**2 + 15)/(10*sqrt(x**4 + 5)) - x*sqrt(x**4 + 5)/(5*(x**2
 + sqrt(5))) + 5**(1/4)*sqrt((x**4 + 5)/(sqrt(5)*x**2/5 + 1)**2)*
(sqrt(5)*x**2/5 + 1)*elliptic_e(2*atan(5**(3/4)*x/5), 1/2)/(5*sqr
t(x**4 + 5)) - 5**(1/4)*sqrt((x**4 + 5)/(sqrt(5)*x**2/5 + 1)**2)*
(-3*sqrt(5) + 2)*(sqrt(5)*x**2/5 + 1)*elliptic_f(2*atan(5**(3/4)*
x/5), 1/2)/(20*sqrt(x**4 + 5))
```

Mathematica [C] time = 0.18264, size = 85, normalized size = 0.48

$$\frac{1}{10}\left(\frac{x\left(2x^2-15\right)}{\sqrt{x^4+5}}-\sqrt[4]{-5}\left(3\sqrt{5}+2i\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)+2(-1)^{3/4}\sqrt[4]{5}E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] ((x*(-15 + 2*x^2))/Sqrt[5 + x^4] + 2*(-1)^(3/4)*5^(1/4)*EllipticE
[I*ArcSinh[(-1/5)^(1/4)*x], -1] - (-5)^(1/4)*(2*I + 3*Sqrt[5])*El
lipticF[I*ArcSinh[(-1/5)^(1/4)*x], -1])/10
```

Maple [C] time = 0.019, size = 168, normalized size = 1.

$$\frac{x^3}{5}\frac{1}{\sqrt{x^4+5}} - \frac{\frac{i}{25}}{\sqrt{i\sqrt{5}}}\sqrt{25-5\,i\sqrt{5}x^2}\sqrt{25+5\,i\sqrt{5}x^2}\left(EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-EllipticE\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}} - \frac{3\,x}{2}\frac{1}{\sqrt{x^4+5}} + \frac{3\sqrt{5}}{50\sqrt{i\sqrt{5}}}\sqrt{25-5\,i\sqrt{5}x^2}\sqrt{25+5\,i\sqrt{5}x^2}EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] 1/5*x^3/(x^4+5)^(1/2)-1/25*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^
2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*
x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2)
)^(1/2),I))-3/2*x/(x^4+5)^(1/2)+3/50*5^(1/2)/(I*5^(1/2))^(1/2)*(2
5-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)
*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3\,x^2+2)\,x^2}{(x^4+5)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^2/(x^4 + 5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,x^4+2\,x^2}{(x^4+5)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^2/(x^4 + 5)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^4 + 2*x^2)/(x^4 + 5)^(3/2), x)
```

Sympy [A] time = 11.1758, size = 75, normalized size = 0.42

$$\frac{3\sqrt{5}x^5\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\,\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{5}x^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**4*exp_pola
r(I*pi)/5)/(100*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((3/4,
 3/2), (7/4,), x**4*exp_polar(I*pi)/5)/(50*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3\,x^2+2\right)x^2}{\left(x^4+5\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^2/(x^4 + 5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5)^(3/2), x)
```

3.52 $\int \frac{2+3x^2}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=180

$$-\frac{3\sqrt{x^4+5}\,x}{10\left(x^2+\sqrt{5}\right)}+\frac{\left(3x^2+2\right)x}{10\sqrt{x^4+5}}+\frac{\left(2-3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{20\sqrt[4]{5}\sqrt{x^4+5}}+\frac{3\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\,5^{3/4}\sqrt{x^4+5}}$$

```
[Out] (x*(2 + 3*x^2))/(10*Sqrt[5 + x^4]) - (3*x*Sqrt[5 + x^4])/(10*(Sqr
t[5] + x^2)) + (3*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^
2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(3/4)*Sqrt[5 + x^4])
 + ((2 - 3*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2
)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(20*5^(1/4)*Sqrt[5 + x^
4])
```

Rubi [A] time = 0.13435, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$-\frac{3\sqrt{x^4+5}\,x}{10\left(x^2+\sqrt{5}\right)}+\frac{\left(3x^2+2\right)x}{10\sqrt{x^4+5}}+\frac{\left(2-3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{20\sqrt[4]{5}\sqrt{x^4+5}}+\frac{3\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\,5^{3/4}\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(5 + x^4)^(3/2),x]
```

```
[Out] (x*(2 + 3*x^2))/(10*Sqrt[5 + x^4]) - (3*x*Sqrt[5 + x^4])/(10*(Sqr
t[5] + x^2)) + (3*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^
2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(3/4)*Sqrt[5 + x^4])
 + ((2 - 3*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2
)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(20*5^(1/4)*Sqrt[5 + x^
4])
```

Rubi in Sympy [A] time = 11.7199, size = 182, normalized size = 1.01

$$\frac{x\left(3x^2+2\right)}{10\sqrt{x^4+5}} - \frac{3x\sqrt{x^4+5}}{10\left(x^2+\sqrt{5}\right)} + \frac{3\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{10\sqrt{x^4+5}} - \frac{\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(-\frac{2\sqrt{5}}{5}+3\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\operatorname{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{20\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] x*(3*x**2 + 2)/(10*sqrt(x**4 + 5)) - 3*x*sqrt(x**4 + 5)/(10*(x**2
 + sqrt(5))) + 3*5**(1/4)*sqrt((x**4 + 5)/(sqrt(5)*x**2/5 + 1)**2
)*(sqrt(5)*x**2/5 + 1)*elliptic_e(2*atan(5**(3/4)*x/5), 1/2)/(10*
sqrt(x**4 + 5)) - 5**(1/4)*sqrt((x**4 + 5)/(sqrt(5)*x**2/5 + 1)**
2)*(-2*sqrt(5)/5 + 3)*(sqrt(5)*x**2/5 + 1)*elliptic_f(2*atan(5**(
3/4)*x/5), 1/2)/(20*sqrt(x**4 + 5))
```

Mathematica [C] time = 0.172831, size = 86, normalized size = 0.48

$$\frac{1}{50}\left(\frac{5x\left(3x^2+2\right)}{\sqrt{x^4+5}} - \sqrt[4]{-5}\left(2\sqrt{5}+15i\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right) + 15(-1)^{3/4}\sqrt[4]{5}E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)/(5 + x^4)^(3/2),x]
```

```
[Out] ((5*x*(2 + 3*x^2))/Sqrt[5 + x^4] + 15*(-1)^(3/4)*5^(1/4)*Elliptic
E[I*ArcSinh[(-1/5)^(1/4)*x], -1] - (-5)^(1/4)*(15*I + 2*Sqrt[5])*
EllipticF[I*ArcSinh[(-1/5)^(1/4)*x], -1])/50
```

Maple [C] time = 0.018, size = 168, normalized size = 0.9

$$\frac{x}{5}\frac{1}{\sqrt{x^4+5}} + \frac{\sqrt{5}}{125\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}} + \frac{3x^3}{10}\frac{1}{\sqrt{x^4+5}} - \frac{\frac{3i}{50}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right) - EllipticE\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] 1/5*x/(x^4+5)^(1/2)+1/125*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/
2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(
1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)+3/10*x^3/(x^4+5)^(1/2)-3/50*I/
(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)
^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I
)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\left(x^4+5\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(x^4 + 5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(x^4 + 5)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,x^2+2}{\left(x^4+5\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(x^4 + 5)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)/(x^4 + 5)^(3/2), x)
```

Sympy [A] time = 10.6093, size = 73, normalized size = 0.41

$$\frac{3\sqrt{5}x^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\,\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), x**4*exp_pola
r(I*pi)/5)/(100*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((1/4, 3/
2), (5/4,), x**4*exp_polar(I*pi)/5)/(50*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{(x^4+5)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(x^4 + 5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(x^4 + 5)^(3/2), x)
```

3.53 $\int \frac{2+3x^2}{x^2(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=196

$$-\frac{3\sqrt{x^4+5}}{25x}+\frac{3\sqrt{x^4+5}\,x}{25\left(x^2+\sqrt{5}\right)}+\frac{3x^2+2}{10\sqrt{x^4+5}\,x}+\frac{3\left(2+\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{20\,5^{3/4}\sqrt{x^4+5}}-\frac{3\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5\,5^{3/4}\sqrt{x^4+5}}$$

```
[Out] (2 + 3*x^2)/(10*x*Sqrt[5 + x^4]) - (3*Sqrt[5 + x^4])/(25*x) + (3*
x*Sqrt[5 + x^4])/(25*(Sqrt[5] + x^2)) - (3*(Sqrt[5] + x^2)*Sqrt[(
5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/
(5*5^(3/4)*Sqrt[5 + x^4]) + (3*(2 + Sqrt[5])*(Sqrt[5] + x^2)*Sqrt
[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2]
)/(20*5^(3/4)*Sqrt[5 + x^4])
```

Rubi [A] time = 0.211718, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{3\sqrt{x^4+5}}{25x}+\frac{3\sqrt{x^4+5}\,x}{25\left(x^2+\sqrt{5}\right)}+\frac{3x^2+2}{10\sqrt{x^4+5}\,x}+\frac{3\left(2+\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{20\,5^{3/4}\sqrt{x^4+5}}-\frac{3\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5\,5^{3/4}\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(x^2*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*x*Sqrt[5 + x^4]) - (3*Sqrt[5 + x^4])/(25*x) + (3*
x*Sqrt[5 + x^4])/(25*(Sqrt[5] + x^2)) - (3*(Sqrt[5] + x^2)*Sqrt[(
5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/
(5*5^(3/4)*Sqrt[5 + x^4]) + (3*(2 + Sqrt[5])*(Sqrt[5] + x^2)*Sqrt
[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2]
)/(20*5^(3/4)*Sqrt[5 + x^4])
```

Rubi in Sympy [A] time = 18.6915, size = 194, normalized size = 0.99

$$\frac{3x\sqrt{x^4+5}}{25\left(x^2+\sqrt{5}\right)}-\frac{3\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\,\mathrm{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{25\sqrt{x^4+5}}+\frac{\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(6+3\sqrt{5}\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\,\mathrm{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{100\sqrt{x^4+5}}+\frac{3x^2+2}{10x\sqrt{x^4+5}}-\frac{3\sqrt{x^4+5}}{25x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/x**2/(x**4+5)**(3/2),x)
```

```
[Out] 3*x*sqrt(x**4 + 5)/(25*(x**2 + sqrt(5))) - 3*5**(1/4)*sqrt((x**4
+ 5)/(sqrt(5)*x**2/5 + 1)**2)*(sqrt(5)*x**2/5 + 1)*elliptic_e(2*a
tan(5**(3/4)*x/5), 1/2)/(25*sqrt(x**4 + 5)) + 5**(1/4)*sqrt((x**4
 + 5)/(sqrt(5)*x**2/5 + 1)**2)*(6 + 3*sqrt(5))*(sqrt(5)*x**2/5 +
1)*elliptic_f(2*atan(5**(3/4)*x/5), 1/2)/(100*sqrt(x**4 + 5)) + (
3*x**2 + 2)/(10*x*sqrt(x**4 + 5)) - 3*sqrt(x**4 + 5)/(25*x)
```

Mathematica [C] time = 0.204498, size = 108, normalized size = 0.55

$$-\frac{6x^4+3\sqrt[4]{-5}\left(\sqrt{5}-2i\right)\sqrt{x^4+5}\,xF\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)+6(-1)^{3/4}\sqrt[4]{5}\sqrt{x^4+5}\,xE\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)-15x^2+2}{50x\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)/(x^2*(5 + x^4)^(3/2)),x]
```

```
[Out] -(20 - 15*x^2 + 6*x^4 + 6*(-1)^(3/4)*5^(1/4)*x*Sqrt[5 + x^4]*Elli
pticE[I*ArcSinh[(-1/5)^(1/4)*x], -1] + 3*(-5)^(1/4)*(-2*I + Sqrt[
5])*x*Sqrt[5 + x^4]*EllipticF[I*ArcSinh[(-1/5)^(1/4)*x], -1])/(50
*x*Sqrt[5 + x^4])
```

Maple [C] time = 0.027, size = 180, normalized size = 0.9

$$\frac{3x}{10}\frac{1}{\sqrt{x^4+5}}+\frac{3\sqrt{5}}{250\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\,EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}}$$
$$-\frac{x^3}{25}\frac{1}{\sqrt{x^4+5}}-\frac{2}{25x}\sqrt{x^4+5}$$
$$+\frac{\frac{3i}{125}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(EllipticF\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-EllipticE\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/x^2/(x^4+5)^(3/2),x)
```

```
[Out] 3/10*x/(x^4+5)^(1/2)+3/250*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1
/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF
(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-1/25*x^3/(x^4+5)^(1/2)-2/25*(
x^4+5)^(1/2)/x+3/125*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/
2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1
/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2
),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{(x^4+5)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{3x^2+2}{(x^6+5x^2)\sqrt{x^4+5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^2),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)/((x^6 + 5*x^2)*sqrt(x^4 + 5)), x)
```

Sympy [A] time = 17.7922, size = 75, normalized size = 0.38

$$\frac{3\sqrt{5}x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\,\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\\frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50x\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)/x**2/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(I
*pi)/5)/(100*gamma(5/4)) + sqrt(5)*gamma(-1/4)*hyper((-1/4, 3/2),
 (3/4,), x**4*exp_polar(I*pi)/5)/(50*x*gamma(3/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{(x^4+5)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^2), x)
```

3.54 $\int \frac{2+3x^2}{x^4(5+x^4)^{3/2}}\, dx$

Optimal. Leaf size=214

$$-\frac{9\sqrt{x^4+5}}{50x}-\frac{\sqrt{x^4+5}}{15x^3}+\frac{9\sqrt{x^4+5}\,x}{50\left(x^2+\sqrt{5}\right)}+\frac{\left(27-2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{60\ 5^{3/4}\sqrt{x^4+5}}-\frac{9\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{10\ 5^{3/4}\sqrt{x^4+5}}+\frac{3x^2+2}{10\sqrt{x^4+5}\,x^3}$$

```
[Out] (2 + 3*x^2)/(10*x^3*Sqrt[5 + x^4]) - Sqrt[5 + x^4]/(15*x^3) - (9*
Sqrt[5 + x^4])/(50*x) + (9*x*Sqrt[5 + x^4])/(50*(Sqrt[5] + x^2))
- (9*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[
2*ArcTan[x/5^(1/4)], 1/2])/(10*5^(3/4)*Sqrt[5 + x^4]) + ((27 - 2*
Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Ellipt
icF[2*ArcTan[x/5^(1/4)], 1/2])/(60*5^(3/4)*Sqrt[5 + x^4])
```

Rubi [A] time = 0.270152, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{9\sqrt{x^4+5}}{50x}-\frac{\sqrt{x^4+5}}{15x^3}+\frac{9\sqrt{x^4+5}\,x}{50\left(x^2+\sqrt{5}\right)}+\frac{\left(27-2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{60\ 5^{3/4}\sqrt{x^4+5}}-\frac{9\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{10\ 5^{3/4}\sqrt{x^4+5}}+\frac{3x^2+2}{10\sqrt{x^4+5}\,x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(x^4*(5 + x^4)^(3/2)),x]
```


```
[Out] (2 + 3*x^2)/(10*x^3*Sqrt[5 + x^4]) - Sqrt[5 + x^4]/(15*x^3) - (9*
Sqrt[5 + x^4])/(50*x) + (9*x*Sqrt[5 + x^4])/(50*(Sqrt[5] + x^2))
- (9*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[
2*ArcTan[x/5^(1/4)], 1/2])/(10*5^(3/4)*Sqrt[5 + x^4]) + ((27 - 2*
Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Ellipt
icF[2*ArcTan[x/5^(1/4)], 1/2])/(60*5^(3/4)*Sqrt[5 + x^4])
```

Rubi in Sympy [A] time = 23.4258, size = 209, normalized size = 0.98

$$\frac{9x\sqrt{x^4+5}}{50\left(x^2+\sqrt{5}\right)} - \frac{9\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(\frac{\sqrt{5}x^2}{5}+1\right)E\left(2\,\mathrm{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{50\sqrt{x^4+5}} + \frac{\sqrt[4]{5}\sqrt{\frac{x^4+5}{\left(\frac{\sqrt{5}x^2}{5}+1\right)^2}}\left(-10\sqrt{5}+135\right)\left(\frac{\sqrt{5}x^2}{5}+1\right)F\left(2\,\mathrm{atan}\left(\frac{5^{\frac{3}{4}}x}{5}\right)\middle|\frac{1}{2}\right)}{1500\sqrt{x^4+5}} - \frac{9\sqrt{x^4+5}}{50x} + \frac{3x^2+2}{10x^3\sqrt{x^4+5}} - \frac{\sqrt{x^4+5}}{15x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/x**4/(x**4+5)**(3/2),x)
```

```
[Out] 9*x*sqrt(x**4 + 5)/(50*(x**2 + sqrt(5))) - 9*5**(1/4)*sqrt((x**4
+ 5)/(sqrt(5)*x**2/5 + 1)**2)*(sqrt(5)*x**2/5 + 1)*elliptic_e(2*a
tan(5**(3/4)*x/5), 1/2)/(50*sqrt(x**4 + 5)) + 5**(1/4)*sqrt((x**4
 + 5)/(sqrt(5)*x**2/5 + 1)**2)*(-10*sqrt(5) + 135)*(sqrt(5)*x**2/
5 + 1)*elliptic_f(2*atan(5**(3/4)*x/5), 1/2)/(1500*sqrt(x**4 + 5)
) - 9*sqrt(x**4 + 5)/(50*x) + (3*x**2 + 2)/(10*x**3*sqrt(x**4 + 5
)) - sqrt(x**4 + 5)/(15*x**3)
```

Mathematica [C] time = 0.236388, size = 119, normalized size = 0.56

$$-\frac{27x^6+10x^4+90x^2-\sqrt[4]{-5}\left(2\sqrt{5}+27i\right)\sqrt{x^4+5}x^3F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}x\right)\middle|-1\right)+27(-1)^{3/4}\sqrt[4]{5}\sqrt{x^4+5}x^3E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}\right.\right.}{150x^3\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)/(x^4*(5 + x^4)^(3/2)),x]
```

```
[Out] -(20 + 90*x^2 + 10*x^4 + 27*x^6 + 27*(-1)^(3/4)*5^(1/4)*x^3*Sqrt[
5 + x^4]*EllipticE[I*ArcSinh[(-1/5)^(1/4)*x], -1] - (-5)^(1/4)*(2
7*I + 2*Sqrt[5])*x^3*Sqrt[5 + x^4]*EllipticF[I*ArcSinh[(-1/5)^(1/
4)*x], -1])/(150*x^3*Sqrt[5 + x^4])
```

Maple [C] time = 0.028, size = 192, normalized size = 0.9

$$-\frac{2}{75\,x^3}\sqrt{x^4+5}-\frac{x}{25}\frac{1}{\sqrt{x^4+5}}-\frac{\sqrt{5}}{375\,\sqrt{i\sqrt{5}}}\sqrt{25-5\,i\sqrt{5}x^2}\sqrt{25+5\,i\sqrt{5}x^2}\mathit{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}}$$
$$-\frac{3\,x^3}{50}\frac{1}{\sqrt{x^4+5}}-\frac{3}{25\,x}\sqrt{x^4+5}$$
$$+\frac{\frac{9\,i}{250}}{\sqrt{i\sqrt{5}}}\sqrt{25-5\,i\sqrt{5}x^2}\sqrt{25+5\,i\sqrt{5}x^2}\left(\mathit{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-\mathit{EllipticE}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/x^4/(x^4+5)^(3/2),x)
```

```
[Out] -2/75*(x^4+5)^(1/2)/x^3-1/25*x/(x^4+5)^(1/2)-1/375*5^(1/2)/(I*5^(
1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)
/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-3/50*
x^3/(x^4+5)^(1/2)-3/25*(x^4+5)^(1/2)/x+9/250*I/(I*5^(1/2))^(1/2)*
(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/
2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*
5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{3\,x^2+2}{(x^4+5)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^4),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{3\,x^2+2}{(x^8+5\,x^4)\sqrt{x^4+5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)/((x^8 + 5*x^4)*sqrt(x^4 + 5)), x)
```

Sympy [A] time = 26.5395, size = 80, normalized size = 0.37

$$\frac{3\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100x\,\Gamma\left(\frac{3}{4}\right)}+\frac{\sqrt{5}\,\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50x^3\,\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)/x**4/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), x**4*exp_polar(I
*pi)/5)/(100*x*gamma(3/4)) + sqrt(5)*gamma(-3/4)*hyper((-3/4, 3/2
), (1/4,), x**4*exp_polar(I*pi)/5)/(50*x**3*gamma(1/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{(x^4+5)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^4),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^4), x)
```

3.55 $\int (fx)^m \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=269

$$\frac{(d+10e)(fx)^{m+21}}{f^{21}(m+21)}+\frac{5(2d+9e)(fx)^{m+19}}{f^{19}(m+19)}+\frac{15(3d+8e)(fx)^{m+17}}{f^{17}(m+17)}+\frac{30(4d+7e)(fx)^{m+15}}{f^{15}(m+15)}+\frac{42(5d+6e)(fx)^{m+13}}{f^{13}(m+13)}+\frac{42(6d+5e)(fx)^{m+11}}{f^{11}(m+11)}+\frac{30(7d+4e)(fx)^{m+9}}{f^9(m+9)}+\frac{15(8d+3e)(fx)^{m+7}}{f^7(m+7)}+\frac{5(9d+2e)(fx)^{m+5}}{f^5(m+5)}+\frac{(10d+e)(fx)^{m+3}}{f^3(m+3)}+\frac{d(fx)^{m+1}}{f(m+1)}+\frac{e(fx)^{m+23}}{f^{23}(m+23)}$$

```
[Out] (d*(f*x)^(1 + m))/(f*(1 + m)) + ((10*d + e)*(f*x)^(3 + m))/(f^3*(
3 + m)) + (5*(9*d + 2*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (15*(8*d
+ 3*e)*(f*x)^(7 + m))/(f^7*(7 + m)) + (30*(7*d + 4*e)*(f*x)^(9 +
m))/(f^9*(9 + m)) + (42*(6*d + 5*e)*(f*x)^(11 + m))/(f^11*(11 + m
)) + (42*(5*d + 6*e)*(f*x)^(13 + m))/(f^13*(13 + m)) + (30*(4*d +
 7*e)*(f*x)^(15 + m))/(f^15*(15 + m)) + (15*(3*d + 8*e)*(f*x)^(17
 + m))/(f^17*(17 + m)) + (5*(2*d + 9*e)*(f*x)^(19 + m))/(f^19*(19
 + m)) + ((d + 10*e)*(f*x)^(21 + m))/(f^21*(21 + m)) + (e*(f*x)^(
23 + m))/(f^23*(23 + m))
```

Rubi [A] time = 0.367987, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{(d+10e)(fx)^{m+21}}{f^{21}(m+21)}+\frac{5(2d+9e)(fx)^{m+19}}{f^{19}(m+19)}+\frac{15(3d+8e)(fx)^{m+17}}{f^{17}(m+17)}+\frac{30(4d+7e)(fx)^{m+15}}{f^{15}(m+15)}+\frac{42(5d+6e)(fx)^{m+13}}{f^{13}(m+13)}+\frac{42(6d+5e)(fx)^{m+11}}{f^{11}(m+11)}+\frac{30(7d+4e)(fx)^{m+9}}{f^9(m+9)}+\frac{15(8d+3e)(fx)^{m+7}}{f^7(m+7)}+\frac{5(9d+2e)(fx)^{m+5}}{f^5(m+5)}+\frac{(10d+e)(fx)^{m+3}}{f^3(m+3)}+\frac{d(fx)^{m+1}}{f(m+1)}+\frac{e(fx)^{m+23}}{f^{23}(m+23)}$$

Antiderivative was successfully verified.

```
[In]  Int[(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```


```
[Out] (d*(f*x)^(1 + m))/(f*(1 + m)) + ((10*d + e)*(f*x)^(3 + m))/(f^3*(
3 + m)) + (5*(9*d + 2*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (15*(8*d
+ 3*e)*(f*x)^(7 + m))/(f^7*(7 + m)) + (30*(7*d + 4*e)*(f*x)^(9 +
m))/(f^9*(9 + m)) + (42*(6*d + 5*e)*(f*x)^(11 + m))/(f^11*(11 + m
)) + (42*(5*d + 6*e)*(f*x)^(13 + m))/(f^13*(13 + m)) + (30*(4*d +
 7*e)*(f*x)^(15 + m))/(f^15*(15 + m)) + (15*(3*d + 8*e)*(f*x)^(17
 + m))/(f^17*(17 + m)) + (5*(2*d + 9*e)*(f*x)^(19 + m))/(f^19*(19
 + m)) + ((d + 10*e)*(f*x)^(21 + m))/(f^21*(21 + m)) + (e*(f*x)^(
23 + m))/(f^23*(23 + m))
```

Rubi in Sympy [A] time = 48.7252, size = 241, normalized size = 0.9

$$\frac{d(fx)^{m+1}}{f(m+1)}+\frac{e(fx)^{m+23}}{f^{23}(m+23)}+\frac{(fx)^{m+3}(10d+e)}{f^3(m+3)}+\frac{5(fx)^{m+5}(9d+2e)}{f^5(m+5)}+\frac{15(fx)^{m+7}(8d+3e)}{f^7(m+7)}+\frac{30(fx)^{m+9}(7d+4e)}{f^9(m+9)}+\frac{42(fx)^{m+11}(6d+5e)}{f^{11}(m+11)}+\frac{42(fx)^{m+13}(5d+6e)}{f^{13}(m+13)}+\frac{30(fx)^{m+15}(4d+7e)}{f^{15}(m+15)}+\frac{15(fx)^{m+17}(3d+8e)}{f^{17}(m+17)}+\frac{5(fx)^{m+19}(2d+9e)}{f^{19}(m+19)}+\frac{(fx)^{m+21}(d+10e)}{f^{21}(m+21)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*(f*x)**(m + 1)/(f*(m + 1)) + e*(f*x)**(m + 23)/(f**23*(m + 23))
 + (f*x)**(m + 3)*(10*d + e)/(f**3*(m + 3)) + 5*(f*x)**(m + 5)*(9
*d + 2*e)/(f**5*(m + 5)) + 15*(f*x)**(m + 7)*(8*d + 3*e)/(f**7*(m
 + 7)) + 30*(f*x)**(m + 9)*(7*d + 4*e)/(f**9*(m + 9)) + 42*(f*x)*
*(m + 11)*(6*d + 5*e)/(f**11*(m + 11)) + 42*(f*x)**(m + 13)*(5*d
+ 6*e)/(f**13*(m + 13)) + 30*(f*x)**(m + 15)*(4*d + 7*e)/(f**15*(
m + 15)) + 15*(f*x)**(m + 17)*(3*d + 8*e)/(f**17*(m + 17)) + 5*(f
*x)**(m + 19)*(2*d + 9*e)/(f**19*(m + 19)) + (f*x)**(m + 21)*(d +
 10*e)/(f**21*(m + 21))
```

Mathematica [A] time = 0.190964, size = 189, normalized size = 0.7

$$(fx)^m\left(\frac{x^{21}(d+10e)}{m+21}+\frac{5x^{19}(2d+9e)}{m+19}+\frac{15x^{17}(3d+8e)}{m+17}+\frac{30x^{15}(4d+7e)}{m+15}+\frac{42x^{13}(5d+6e)}{m+13}+\frac{42x^{11}(6d+5e)}{m+11}+\frac{30x^9(7d+4e)}{m+9}+\frac{15x^7(8d+3e)}{m+7}+\frac{5x^5(9d+2e)}{m+5}+\frac{x^3(10d+e)}{m+3}+\frac{dx}{m+1}+\frac{ex^{23}}{m+23}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (f*x)^m*((d*x)/(1 + m) + ((10*d + e)*x^3)/(3 + m) + (5*(9*d + 2*e
)*x^5)/(5 + m) + (15*(8*d + 3*e)*x^7)/(7 + m) + (30*(7*d + 4*e)*x
^9)/(9 + m) + (42*(6*d + 5*e)*x^11)/(11 + m) + (42*(5*d + 6*e)*x^
13)/(13 + m) + (30*(4*d + 7*e)*x^15)/(15 + m) + (15*(3*d + 8*e)*x
^17)/(17 + m) + (5*(2*d + 9*e)*x^19)/(19 + m) + ((d + 10*e)*x^21)
/(21 + m) + (e*x^23)/(23 + m))
```

Maple [B] time = 0.018, size = 2295, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] (f*x)^m*(e*m^11*x^22+121*e*m^10*x^22+d*m^11*x^20+10*e*m^11*x^20+6
435*e*m^9*x^22+123*d*m^10*x^20+1230*e*m^10*x^20+197835*e*m^8*x^22
+10*d*m^11*x^18+6635*d*m^9*x^20+45*e*m^11*x^18+66350*e*m^9*x^20+3
889578*e*m^7*x^22+1250*d*m^10*x^18+206505*d*m^8*x^20+5625*e*m^10*
x^18+2065050*e*m^8*x^20+51069018*e*m^6*x^22+45*d*m^11*x^16+68430*
d*m^9*x^18+4103178*d*m^7*x^20+120*e*m^11*x^16+307935*e*m^9*x^18+4
1031780*e*m^7*x^20+453714470*e*m^5*x^22+5715*d*m^10*x^16+2158230*
d*m^8*x^18+54362574*d*m^6*x^20+15240*e*m^10*x^16+9712035*e*m^8*x^
18+543625740*e*m^6*x^20+2702025590*e*m^4*x^22+120*d*m^11*x^14+317
655*d*m^9*x^16+43391460*d*m^7*x^18+486687830*d*m^5*x^20+210*e*m^1
1*x^14+847080*e*m^9*x^16+195261570*e*m^7*x^18+4866878300*e*m^5*x^
20+10431670821*e*m^3*x^22+15480*d*m^10*x^14+10162665*d*m^8*x^16+5
80855380*d*m^6*x^18+2917013970*d*m^4*x^20+27090*e*m^10*x^14+27100
440*e*m^8*x^16+2613849210*e*m^6*x^18+29170139700*e*m^4*x^20+24372
200061*e*m^2*x^22+210*d*m^11*x^12+873960*d*m^9*x^14+207024930*d*m
^7*x^16+5246766620*d*m^5*x^18+11320966021*d*m^3*x^20+252*e*m^11*x
^12+1529430*e*m^9*x^14+552066480*e*m^7*x^16+23610449790*e*m^5*x^1
8+113209660210*e*m^3*x^20+29985521895*e*m*x^22+27510*d*m^10*x^12+
28391400*d*m^8*x^14+2804395230*d*m^6*x^16+31686018220*d*m^4*x^18+
26560342503*d*m^2*x^20+33012*e*m^10*x^12+49684950*e*m^8*x^14+7478
387280*e*m^6*x^16+142587081990*e*m^4*x^18+265603425030*e*m^2*x^20
+13749310575*e*x^22+252*d*m^11*x^10+1578150*d*m^9*x^12+586902960*
d*m^7*x^14+25598865870*d*m^5*x^16+123748247730*d*m^3*x^18+3277893
0735*d*m*x^20+210*e*m^11*x^10+1893780*e*m^9*x^12+1027080180*e*m^7
*x^14+68263642320*e*m^5*x^16+556867114785*e*m^3*x^18+327789307350
*e*m*x^20+33516*d*m^10*x^10+52110450*d*m^8*x^12+8059973040*d*m^6*
x^14+156004908210*d*m^4*x^16+291789582570*d*m^2*x^18+15058768725*
d*x^20+27930*e*m^10*x^10+62532540*e*m^8*x^12+14104952820*e*m^6*x^
14+416013088560*e*m^4*x^16+1313053121565*e*m^2*x^18+150587687250*
e*x^20+210*d*m^11*x^8+1954260*d*m^9*x^10+1094918580*d*m^7*x^12+74
496630480*d*m^5*x^14+613938233025*d*m^3*x^16+361459164150*d*m*x^1
8+120*e*m^11*x^8+1628550*e*m^9*x^10+1313902296*e*m^7*x^12+1303691
03340*e*m^5*x^14+1637168621400*e*m^3*x^16+1626566238675*e*m*x^18+
28350*d*m^10*x^8+65654820*d*m^8*x^10+15277213980*d*m^6*x^12+45904
5550800*d*m^4*x^14+1456578341055*d*m^2*x^16+166439022750*d*x^18+1
6200*e*m^10*x^8+54712350*e*m^8*x^10+18332656776*e*m^6*x^12+803329
713900*e*m^4*x^14+3884208909480*e*m^2*x^16+748975602375*e*x^18+12
0*d*m^11*x^6+1680630*d*m^9*x^8+1404622296*d*m^7*x^10+143339613900
*d*m^5*x^12+1823707864920*d*m^3*x^14+1812743750475*d*m*x^16+45*e*
m^11*x^6+960360*e*m^9*x^8+1170518580*e*m^7*x^10+172007536680*e*m^
5*x^12+3191488763610*e*m^3*x^14+4833983334600*e*m*x^16+16440*d*m^
10*x^6+57500730*d*m^8*x^8+19962541368*d*m^6*x^10+895451283300*d*m
^4*x^12+4360457499480*d*m^2*x^14+837090379125*d*x^16+6165*e*m^10*
x^6+32857560*e*m^8*x^8+16635451140*e*m^6*x^10+1074541539960*e*m^4
```

```
*x^12+7630800624090*e*m^2*x^14+2232241011000*e*x^16+45*d*m^11*x^4
+991080*d*m^9*x^6+1254847860*d*m^7*x^8+190744119720*d*m^5*x^10+36
00567789210*d*m^3*x^12+5458672303560*d*m*x^14+10*e*m^11*x^4+37165
5*e*m^9*x^6+717055920*e*m^7*x^8+158953433100*e*m^5*x^10+432068134
7052*e*m^3*x^12+9552676531230*e*m*x^14+6255*d*m^10*x^4+34563240*d
*m^8*x^6+18217524780*d*m^6*x^8+1212454199880*d*m^4*x^10+869575081
8510*d*m^2*x^12+2529873145800*d*x^14+1390*e*m^10*x^4+12961215*e*m
^8*x^6+10410014160*e*m^6*x^8+1010378499900*e*m^4*x^10+10434900982
212*e*m^2*x^12+4427278005150*e*x^14+10*d*m^11*x^2+383535*d*m^9*x^
4+770831280*d*m^7*x^6+177985672620*d*m^5*x^8+4952725167852*d*m^3*
x^10+10969925251950*d*m*x^12+e*m^11*x^2+85230*e*m^9*x^4+289061730
*e*m^7*x^6+101706098640*e*m^5*x^8+4127270973210*e*m^3*x^10+131639
10302340*e*m*x^12+1410*d*m^10*x^2+13645125*d*m^8*x^4+11467698480*
d*m^6*x^6+1156995210420*d*m^4*x^8+12123781647516*d*m^2*x^10+51083
97698250*d*x^12+141*e*m^10*x^2+3032250*e*m^8*x^4+4300386930*e*m^6
*x^6+661140120240*e*m^4*x^8+10103151372930*e*m^2*x^10+61300772379
00*e*x^12+d*m^11+87950*d*m^9*x^2+311564610*d*m^7*x^4+115122336720
*d*m^5*x^6+4828477578330*d*m^3*x^8+15456024948420*d*m*x^10+8795*e
*m^9*x^2+69236580*e*m^7*x^4+43170876270*e*m^5*x^6+2759130044760*e
*m^3*x^8+12880020790350*e*m*x^10+143*d*m^10+3194550*d*m^8*x^2+476
5995990*d*m^6*x^4+770638650960*d*m^4*x^6+12046833873270*d*m^2*x^8
+7244636735700*d*x^10+319455*e*m^8*x^2+1059110220*e*m^6*x^4+28898
9494110*e*m^4*x^6+6883905070440*e*m^2*x^8+6037197279750*e*x^10+90
75*d*m^9+74814180*d*m^7*x^2+49443604830*d*m^5*x^4+3314920570200*d
*m^3*x^6+15593181033150*d*m*x^8+7481418*e*m^7*x^2+10987467740*e*m
^5*x^4+1243095213825*e*m^3*x^6+8910389161800*e*m*x^8+336765*d*m^8
+1180850580*d*m^6*x^2+343967603850*d*m^4*x^4+8511631481880*d*m^2*
x^6+7378796675250*d*x^8+118085058*e*m^6*x^2+76437245300*e*m^4*x^4
+3191861805705*e*m^2*x^6+4216455243000*e*x^8+8103018*d*m^7+127404
67100*d*m^5*x^2+1546183653345*d*m^3*x^4+11284114422600*d*m*x^6+12
74046710*e*m^5*x^2+343596367410*e*m^3*x^4+4231542908475*e*m*x^6+1
32426294*d*m^6+93153182700*d*m^4*x^2+4162610035755*d*m^2*x^4+5421
156741000*d*x^6+9315318270*e*m^4*x^2+925024452390*e*m^2*x^4+20329
33777875*e*x^6+1495875590*d*m^5+446323045810*d*m^3*x^2+5761525369
635*d*m*x^4+44632304581*e*m^3*x^2+1280338971030*e*m*x^4+116415828
10*d*m^4+1304037152010*d*m^2*x^2+2846107289025*d*x^4+130403715201
*e*m^2*x^2+632468286450*e*x^4+60936676581*d*m^3+1993349776950*d*m
*x^2+199334977695*e*m*x^2+203363952363*d*m^2+1054113810750*d*x^2+
105411381075*e*x^2+387182170935*d*m+316234143225*d)*x/(1+m)/(3+m)
/(5+m)/(7+m)/(9+m)/(11+m)/(13+m)/(15+m)/(17+m)/(19+m)/(21+m)/(23+
m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)*(f*x)^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.308509, size = 2121, normalized size = 7.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)*(f*x)^m,x, algorithm="fricas")
```

```
[Out] ((e*m^11 + 121*e*m^10 + 6435*e*m^9 + 197835*e*m^8 + 3889578*e*m^7
 + 51069018*e*m^6 + 453714470*e*m^5 + 2702025590*e*m^4 + 10431670
821*e*m^3 + 24372200061*e*m^2 + 29985521895*e*m + 13749310575*e)*
x^23 + ((d + 10*e)*m^11 + 123*(d + 10*e)*m^10 + 6635*(d + 10*e)*m
^9 + 206505*(d + 10*e)*m^8 + 4103178*(d + 10*e)*m^7 + 54362574*(d
 + 10*e)*m^6 + 486687830*(d + 10*e)*m^5 + 2917013970*(d + 10*e)*m
^4 + 11320966021*(d + 10*e)*m^3 + 26560342503*(d + 10*e)*m^2 + 32
778930735*(d + 10*e)*m + 15058768725*d + 150587687250*e)*x^21 + 5
*((2*d + 9*e)*m^11 + 125*(2*d + 9*e)*m^10 + 6843*(2*d + 9*e)*m^9
+ 215823*(2*d + 9*e)*m^8 + 4339146*(2*d + 9*e)*m^7 + 58085538*(2*
d + 9*e)*m^6 + 524676662*(2*d + 9*e)*m^5 + 3168601822*(2*d + 9*e)
*m^4 + 12374824773*(2*d + 9*e)*m^3 + 29178958257*(2*d + 9*e)*m^2
+ 36145916415*(2*d + 9*e)*m + 33287804550*d + 149795120475*e)*x^1
9 + 15*((3*d + 8*e)*m^11 + 127*(3*d + 8*e)*m^10 + 7059*(3*d + 8*e
)*m^9 + 225837*(3*d + 8*e)*m^8 + 4600554*(3*d + 8*e)*m^7 + 623198
94*(3*d + 8*e)*m^6 + 568863686*(3*d + 8*e)*m^5 + 3466775738*(3*d
+ 8*e)*m^4 + 13643071845*(3*d + 8*e)*m^3 + 32368407579*(3*d + 8*e
)*m^2 + 40283194455*(3*d + 8*e)*m + 55806025275*d + 148816067400*
e)*x^17 + 30*((4*d + 7*e)*m^11 + 129*(4*d + 7*e)*m^10 + 7283*(4*d
 + 7*e)*m^9 + 236595*(4*d + 7*e)*m^8 + 4890858*(4*d + 7*e)*m^7 +
67166442*(4*d + 7*e)*m^6 + 620805254*(4*d + 7*e)*m^5 + 3825379590
*(4*d + 7*e)*m^4 + 15197565541*(4*d + 7*e)*m^3 + 36337145829*(4*d
 + 7*e)*m^2 + 45488935863*(4*d + 7*e)*m + 84329104860*d + 1475759
33505*e)*x^15 + 42*((5*d + 6*e)*m^11 + 131*(5*d + 6*e)*m^10 + 751
5*(5*d + 6*e)*m^9 + 248145*(5*d + 6*e)*m^8 + 5213898*(5*d + 6*e)*
m^7 + 72748638*(5*d + 6*e)*m^6 + 682569590*(5*d + 6*e)*m^5 + 4264
053730*(5*d + 6*e)*m^4 + 17145560901*(5*d + 6*e)*m^3 + 4140833723
1*(5*d + 6*e)*m^2 + 52237739295*(5*d + 6*e)*m + 121628516625*d +
145954219950*e)*x^13 + 42*((6*d + 5*e)*m^11 + 133*(6*d + 5*e)*m^1
0 + 7755*(6*d + 5*e)*m^9 + 260535*(6*d + 5*e)*m^8 + 5573898*(6*d
+ 5*e)*m^7 + 79216434*(6*d + 5*e)*m^6 + 756921110*(6*d + 5*e)*m^5
 + 4811326190*(6*d + 5*e)*m^4 + 19653671301*(6*d + 5*e)*m^3 + 481
10244633*(6*d + 5*e)*m^2 + 61333432335*(6*d + 5*e)*m + 1724913508
50*d + 143742792375*e)*x^11 + 30*((7*d + 4*e)*m^11 + 135*(7*d + 4
*e)*m^10 + 8003*(7*d + 4*e)*m^9 + 273813*(7*d + 4*e)*m^8 + 597546
6*(7*d + 4*e)*m^7 + 86750118*(7*d + 4*e)*m^6 + 847550822*(7*d + 4
*e)*m^5 + 5509501002*(7*d + 4*e)*m^4 + 22992750373*(7*d + 4*e)*m^
3 + 57365875587*(7*d + 4*e)*m^2 + 74253243015*(7*d + 4*e)*m + 245
959889175*d + 140548508100*e)*x^9 + 15*((8*d + 3*e)*m^11 + 137*(8
*d + 3*e)*m^10 + 8259*(8*d + 3*e)*m^9 + 288027*(8*d + 3*e)*m^8 +
```

```
6423594*(8*d + 3*e)*m^7 + 95564154*(8*d + 3*e)*m^6 + 959352806*(8
*d + 3*e)*m^5 + 6421988758*(8*d + 3*e)*m^4 + 27624338085*(8*d + 3
*e)*m^3 + 70930262349*(8*d + 3*e)*m^2 + 94034286855*(8*d + 3*e)*m
 + 361410449400*d + 135528918525*e)*x^7 + 5*((9*d + 2*e)*m^11 + 1
39*(9*d + 2*e)*m^10 + 8523*(9*d + 2*e)*m^9 + 303225*(9*d + 2*e)*m
^8 + 6923658*(9*d + 2*e)*m^7 + 105911022*(9*d + 2*e)*m^6 + 109874
6774*(9*d + 2*e)*m^5 + 7643724530*(9*d + 2*e)*m^4 + 34359636741*(
9*d + 2*e)*m^3 + 92502445239*(9*d + 2*e)*m^2 + 128033897103*(9*d
+ 2*e)*m + 569221457805*d + 126493657290*e)*x^5 + ((10*d + e)*m^1
1 + 141*(10*d + e)*m^10 + 8795*(10*d + e)*m^9 + 319455*(10*d + e)
*m^8 + 7481418*(10*d + e)*m^7 + 118085058*(10*d + e)*m^6 + 127404
6710*(10*d + e)*m^5 + 9315318270*(10*d + e)*m^4 + 44632304581*(10
*d + e)*m^3 + 130403715201*(10*d + e)*m^2 + 199334977695*(10*d +
e)*m + 1054113810750*d + 105411381075*e)*x^3 + (d*m^11 + 143*d*m^
10 + 9075*d*m^9 + 336765*d*m^8 + 8103018*d*m^7 + 132426294*d*m^6
+ 1495875590*d*m^5 + 11641582810*d*m^4 + 60936676581*d*m^3 + 2033
63952363*d*m^2 + 387182170935*d*m + 316234143225*d)*x)*(f*x)^m/(m
^12 + 144*m^11 + 9218*m^10 + 345840*m^9 + 8439783*m^8 + 140529312
*m^7 + 1628301884*m^6 + 13137458400*m^5 + 72578259391*m^4 + 26430
0628944*m^3 + 590546123298*m^2 + 703416314160*m + 316234143225)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.31382, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)*(f*x)^m,x, algorithm="giac")
```

```
[Out] Done
```

3.56 $\int x^5 \left(d+e x^2\right) \left(1+2 x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=63

$$\frac{1}{26} \left(x^2+1\right)^{13} (d-3 e)-\frac{1}{24} \left(x^2+1\right)^{12} (2 d-3 e)+\frac{1}{22} \left(x^2+1\right)^{11} (d-e)+\frac{1}{28} e \left(x^2+1\right)^{14}$$

```
[Out] ((d - e)*(1 + x^2)^11)/22 - ((2*d - 3*e)*(1 + x^2)^12)/24 + ((d -
 3*e)*(1 + x^2)^13)/26 + (e*(1 + x^2)^14)/28
```

Rubi [A] time = 0.421653, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{1}{26} \left(x^2+1\right)^{13} (d-3 e)-\frac{1}{24} \left(x^2+1\right)^{12} (2 d-3 e)+\frac{1}{22} \left(x^2+1\right)^{11} (d-e)+\frac{1}{28} e \left(x^2+1\right)^{14}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] ((d - e)*(1 + x^2)^11)/22 - ((2*d - 3*e)*(1 + x^2)^12)/24 + ((d -
 3*e)*(1 + x^2)^13)/26 + (e*(1 + x^2)^14)/28
```

Rubi in Sympy [A] time = 22.3772, size = 51, normalized size = 0.81

$$\frac{e \left(x^2+1\right)^{14}}{28}+\left(\frac{d}{26}-\frac{3 e}{26}\right) \left(x^2+1\right)^{13}+\left(\frac{d}{22}-\frac{e}{22}\right) \left(x^2+1\right)^{11}-\left(\frac{d}{12}-\frac{e}{8}\right) \left(x^2+1\right)^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] e*(x**2 + 1)**14/28 + (d/26 - 3*e/26)*(x**2 + 1)**13 + (d/22 - e/
22)*(x**2 + 1)**11 - (d/12 - e/8)*(x**2 + 1)**12
```

Mathematica [B] time = 0.0415111, size = 153, normalized size = 2.43

$$\frac{1}{26} x^{26} (d+10 e)+\frac{5}{24} x^{24} (2 d+9 e)+\frac{15}{22} x^{22} (3 d+8 e)+\frac{3}{2} x^{20} (4 d+7 e)+\frac{7}{3} x^{18} (5 d+6 e)+\frac{21}{8} x^{16} (6 d+5 e)+\frac{15}{7} x^{14} (7 d+4 e)+\frac{5}{4} x^{12} (8 d+3 e)+\frac{1}{2} x^{10} (9 d+2 e)+\frac{1}{8} x^8 (10 d+e)+\frac{d x^6}{6}+\frac{e x^{28}}{28}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^6)/6 + ((10*d + e)*x^8)/8 + ((9*d + 2*e)*x^10)/2 + (5*(8*d +
 3*e)*x^12)/4 + (15*(7*d + 4*e)*x^14)/7 + (21*(6*d + 5*e)*x^16)/8
 + (7*(5*d + 6*e)*x^18)/3 + (3*(4*d + 7*e)*x^20)/2 + (15*(3*d + 8
*e)*x^22)/22 + (5*(2*d + 9*e)*x^24)/24 + ((d + 10*e)*x^26)/26 + (
e*x^28)/28
```

Maple [B] time = 0.001, size = 130, normalized size = 2.1

$$\frac{ex^{28}}{28}+\frac{(d+10\,e)\,x^{26}}{26}+\frac{(10\,d+45\,e)\,x^{24}}{24}+\frac{(45\,d+120\,e)\,x^{22}}{22}+\frac{(120\,d+210\,e)\,x^{20}}{20}+\frac{(210\,d+252\,e)\,x^{18}}{18}+\frac{(252\,d+210\,e)\,x^{16}}{16}+\frac{(210\,d+120\,e)\,x^{14}}{14}+\frac{(120\,d+45\,e)\,x^{12}}{12}+\frac{(45\,d+10\,e)\,x^{10}}{10}+\frac{(10\,d+e)\,x^{8}}{8}+\frac{dx^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/28*e*x^28+1/26*(d+10*e)*x^26+1/24*(10*d+45*e)*x^24+1/22*(45*d+1
20*e)*x^22+1/20*(120*d+210*e)*x^20+1/18*(210*d+252*e)*x^18+1/16*(
252*d+210*e)*x^16+1/14*(210*d+120*e)*x^14+1/12*(120*d+45*e)*x^12+
1/10*(45*d+10*e)*x^10+1/8*(10*d+e)*x^8+1/6*d*x^6
```

Maxima [A] time = 0.694938, size = 174, normalized size = 2.76

$$\frac{1}{28}\,ex^{28}+\frac{1}{26}\,(d+10\,e)x^{26}+\frac{5}{24}\,(2\,d+9\,e)x^{24}+\frac{15}{22}\,(3\,d+8\,e)x^{22}+\frac{3}{2}\,(4\,d+7\,e)x^{20}+\frac{7}{3}\,(5\,d+6\,e)x^{18}+\frac{21}{8}\,(6\,d+5\,e)x^{16}+\frac{15}{7}\,(7\,d+4\,e)x^{14}+\frac{5}{4}\,(8\,d+3\,e)x^{12}+\frac{1}{2}\,(9\,d+2\,e)x^{10}+\frac{1}{8}\,(10\,d+e)x^{8}+\frac{1}{6}\,dx^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)*x^5,x, algorithm="maxima")
```

```
[Out] 1/28*e*x^28 + 1/26*(d + 10*e)*x^26 + 5/24*(2*d + 9*e)*x^24 + 15/2
2*(3*d + 8*e)*x^22 + 3/2*(4*d + 7*e)*x^20 + 7/3*(5*d + 6*e)*x^18
+ 21/8*(6*d + 5*e)*x^16 + 15/7*(7*d + 4*e)*x^14 + 5/4*(8*d + 3*e)
*x^12 + 1/2*(9*d + 2*e)*x^10 + 1/8*(10*d + e)*x^8 + 1/6*d*x^6
```

Fricas [A] time = 0.24395, size = 1, normalized size = 0.02

$$\frac{1}{28}x^{28}e + \frac{5}{13}x^{26}e + \frac{1}{26}x^{26}d + \frac{15}{8}x^{24}e + \frac{5}{12}x^{24}d + \frac{60}{11}x^{22}e + \frac{45}{22}x^{22}d + \frac{21}{2}x^{20}e + 6x^{20}d + 14x^{18}e + \frac{35}{3}x^{18}d + \frac{105}{8}x^{16}e + \frac{63}{4}x^{16}d + \frac{60}{7}x^{14}e + 15x^{14}d + \frac{15}{4}x^{12}e + 10x^{12}d + x^{10}e + \frac{9}{2}x^{10}d + \frac{1}{8}x^{8}e + \frac{5}{4}x^{8}d + \frac{1}{6}x^{6}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)*x^5,x, algorithm="fricas")
```

```
[Out] 1/28*x^28*e + 5/13*x^26*e + 1/26*x^26*d + 15/8*x^24*e + 5/12*x^24
*d + 60/11*x^22*e + 45/22*x^22*d + 21/2*x^20*e + 6*x^20*d + 14*x^
18*e + 35/3*x^18*d + 105/8*x^16*e + 63/4*x^16*d + 60/7*x^14*e + 1
5*x^14*d + 15/4*x^12*e + 10*x^12*d + x^10*e + 9/2*x^10*d + 1/8*x^
8*e + 5/4*x^8*d + 1/6*x^6*d
```

Sympy [A] time = 0.178714, size = 134, normalized size = 2.13

$$\frac{dx^{6}}{6} + \frac{ex^{28}}{28} + x^{26}\left(\frac{d}{26} + \frac{5e}{13}\right) + x^{24}\left(\frac{5d}{12} + \frac{15e}{8}\right) + x^{22}\left(\frac{45d}{22} + \frac{60e}{11}\right) + x^{20}\left(6d + \frac{21e}{2}\right) + x^{18}\left(\frac{35d}{3} + 14e\right) + x^{16}\left(\frac{63d}{4} + \frac{105e}{8}\right) + x^{14}\left(15d + \frac{60e}{7}\right) + x^{12}\left(10d + \frac{15e}{4}\right) + x^{10}\left(\frac{9d}{2} + e\right) + x^{8}\left(\frac{5d}{4} + \frac{e}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**6/6 + e*x**28/28 + x**26*(d/26 + 5*e/13) + x**24*(5*d/12 + 1
5*e/8) + x**22*(45*d/22 + 60*e/11) + x**20*(6*d + 21*e/2) + x**18
*(35*d/3 + 14*e) + x**16*(63*d/4 + 105*e/8) + x**14*(15*d + 60*e/
7) + x**12*(10*d + 15*e/4) + x**10*(9*d/2 + e) + x**8*(5*d/4 + e/
8)
```

GIAC/XCAS [A] time = 0.260514, size = 193, normalized size = 3.06

$$\frac{1}{28}x^{28}e + \frac{1}{26}dx^{26} + \frac{5}{13}x^{26}e + \frac{5}{12}dx^{24} + \frac{15}{8}x^{24}e + \frac{45}{22}dx^{22} + \frac{60}{11}x^{22}e + 6\,dx^{20} + \frac{21}{2}x^{20}e + \frac{35}{3}dx^{18} + 14\,x^{18}e + \frac{63}{4}dx^{16} + \frac{105}{8}x^{16}e + 15\,dx^{14} + \frac{60}{7}x^{14}e + 10\,dx^{12} + \frac{15}{4}x^{12}e + \frac{9}{2}dx^{10} + x^{10}e + \frac{5}{4}dx^{8} + \frac{1}{8}x^{8}e + \frac{1}{6}dx^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)*x^5,x, algorithm="giac")
```

```
[Out] 1/28*x^28*e + 1/26*d*x^26 + 5/13*x^26*e + 5/12*d*x^24 + 15/8*x^24
*e + 45/22*d*x^22 + 60/11*x^22*e + 6*d*x^20 + 21/2*x^20*e + 35/3*
d*x^18 + 14*x^18*e + 63/4*d*x^16 + 105/8*x^16*e + 15*d*x^14 + 60/
7*x^14*e + 10*d*x^12 + 15/4*x^12*e + 9/2*d*x^10 + x^10*e + 5/4*d*
x^8 + 1/8*x^8*e + 1/6*d*x^6
```

3.57 $\int x^4 \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=153

$$\frac{1}{25}x^{25}(d+10e)+\frac{5}{23}x^{23}(2d+9e)+\frac{5}{7}x^{21}(3d+8e)+\frac{30}{19}x^{19}(4d+7e)+\frac{42}{17}x^{17}(5d+6e)+\frac{14}{5}x^{15}(6d+5e)+\frac{30}{13}x^{13}(7d+4e)+\frac{15}{11}x^{11}(8d+3e)+\frac{5}{9}x^9(9d+2e)+\frac{1}{7}x^7(10d+e)+\frac{dx^5}{5}+\frac{ex^{27}}{27}$$

```
[Out] (d*x^5)/5 + ((10*d + e)*x^7)/7 + (5*(9*d + 2*e)*x^9)/9 + (15*(8*d
 + 3*e)*x^11)/11 + (30*(7*d + 4*e)*x^13)/13 + (14*(6*d + 5*e)*x^1
5)/5 + (42*(5*d + 6*e)*x^17)/17 + (30*(4*d + 7*e)*x^19)/19 + (5*(
3*d + 8*e)*x^21)/7 + (5*(2*d + 9*e)*x^23)/23 + ((d + 10*e)*x^25)/
25 + (e*x^27)/27
```

Rubi [A] time = 0.276694, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{1}{25}x^{25}(d+10e)+\frac{5}{23}x^{23}(2d+9e)+\frac{5}{7}x^{21}(3d+8e)+\frac{30}{19}x^{19}(4d+7e)+\frac{42}{17}x^{17}(5d+6e)+\frac{14}{5}x^{15}(6d+5e)+\frac{30}{13}x^{13}(7d+4e)+\frac{15}{11}x^{11}(8d+3e)+\frac{5}{9}x^9(9d+2e)+\frac{1}{7}x^7(10d+e)+\frac{dx^5}{5}+\frac{ex^{27}}{27}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^5)/5 + ((10*d + e)*x^7)/7 + (5*(9*d + 2*e)*x^9)/9 + (15*(8*d
 + 3*e)*x^11)/11 + (30*(7*d + 4*e)*x^13)/13 + (14*(6*d + 5*e)*x^1
5)/5 + (42*(5*d + 6*e)*x^17)/17 + (30*(4*d + 7*e)*x^19)/19 + (5*(
3*d + 8*e)*x^21)/7 + (5*(2*d + 9*e)*x^23)/23 + ((d + 10*e)*x^25)/
25 + (e*x^27)/27
```

Rubi in Sympy [A] time = 25.8363, size = 141, normalized size = 0.92

$$\frac{dx^5}{5}+\frac{ex^{27}}{27}+x^{25}\left(\frac{d}{25}+\frac{2e}{5}\right)+x^{23}\left(\frac{10d}{23}+\frac{45e}{23}\right)+x^{21}\left(\frac{15d}{7}+\frac{40e}{7}\right)+x^{19}\left(\frac{120d}{19}+\frac{210e}{19}\right)+x^{17}\left(\frac{210d}{17}+\frac{252e}{17}\right)+x^{15}\left(\frac{84d}{5}+14e\right)+x^{13}\left(\frac{210d}{13}+\frac{120e}{13}\right)+x^{11}\left(\frac{120d}{11}+\frac{45e}{11}\right)+x^9\left(5d+\frac{10e}{9}\right)+x^7\left(\frac{10d}{7}+\frac{e}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**5/5 + e*x**27/27 + x**25*(d/25 + 2*e/5) + x**23*(10*d/23 + 4
5*e/23) + x**21*(15*d/7 + 40*e/7) + x**19*(120*d/19 + 210*e/19) +
 x**17*(210*d/17 + 252*e/17) + x**15*(84*d/5 + 14*e) + x**13*(210
*d/13 + 120*e/13) + x**11*(120*d/11 + 45*e/11) + x**9*(5*d + 10*e
/9) + x**7*(10*d/7 + e/7)
```

Mathematica [A] time = 0.0402823, size = 153, normalized size = 1.

$$\frac{1}{25}x^{25}(d+10e)+\frac{5}{23}x^{23}(2d+9e)+\frac{5}{7}x^{21}(3d+8e)+\frac{30}{19}x^{19}(4d+7e)+\frac{42}{17}x^{17}(5d+6e)+\frac{14}{5}x^{15}(6d+5e)+\frac{30}{13}x^{13}(7d+4e)+\frac{15}{11}x^{11}(8d+3e)+\frac{5}{9}x^9(9d+2e)+\frac{1}{7}x^7(10d+e)+\frac{dx^5}{5}+\frac{ex^{27}}{27}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^5)/5 + ((10*d + e)*x^7)/7 + (5*(9*d + 2*e)*x^9)/9 + (15*(8*d
 + 3*e)*x^11)/11 + (30*(7*d + 4*e)*x^13)/13 + (14*(6*d + 5*e)*x^1
5)/5 + (42*(5*d + 6*e)*x^17)/17 + (30*(4*d + 7*e)*x^19)/19 + (5*(
3*d + 8*e)*x^21)/7 + (5*(2*d + 9*e)*x^23)/23 + ((d + 10*e)*x^25)/
25 + (e*x^27)/27
```

Maple [A] time = 0.002, size = 130, normalized size = 0.9

$$\frac{ex^{27}}{27}+\frac{(d+10e)x^{25}}{25}+\frac{(10d+45e)x^{23}}{23}+\frac{(45d+120e)x^{21}}{21}+\frac{(120d+210e)x^{19}}{19}+\frac{(210d+252e)x^{17}}{17}+\frac{(252d+210e)x^{15}}{15}+\frac{(210d+120e)x^{13}}{13}+\frac{(120d+45e)x^{11}}{11}+\frac{(45d+10e)x^9}{9}+\frac{(10d+e)x^7}{7}+\frac{dx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/27*e*x^27+1/25*(d+10*e)*x^25+1/23*(10*d+45*e)*x^23+1/21*(45*d+1
20*e)*x^21+1/19*(120*d+210*e)*x^19+1/17*(210*d+252*e)*x^17+1/15*(
252*d+210*e)*x^15+1/13*(210*d+120*e)*x^13+1/11*(120*d+45*e)*x^11+
1/9*(45*d+10*e)*x^9+1/7*(10*d+e)*x^7+1/5*d*x^5
```

Maxima [A] time = 0.699084, size = 174, normalized size = 1.14

$$\frac{1}{27}\,ex^{27}+\frac{1}{25}\,(d+10\,e)x^{25}+\frac{5}{23}\,(2\,d+9\,e)x^{23}+\frac{5}{7}\,(3\,d+8\,e)x^{21}+\frac{30}{19}\,(4\,d+7\,e)x^{19}+\frac{42}{17}\,(5\,d+6\,e)x^{17}+\frac{14}{5}\,(6\,d+5\,e)x^{15}+\frac{30}{13}\,(7\,d+4\,e)x^{13}+\frac{15}{11}\,(8\,d+3\,e)x^{11}+\frac{5}{9}\,(9\,d+2\,e)x^{9}+\frac{1}{7}\,(10\,d+e)x^{7}+\frac{1}{5}\,dx^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)*x^4,x, algorithm="maxima")
```

```
[Out] 1/27*e*x^27 + 1/25*(d + 10*e)*x^25 + 5/23*(2*d + 9*e)*x^23 + 5/7*
(3*d + 8*e)*x^21 + 30/19*(4*d + 7*e)*x^19 + 42/17*(5*d + 6*e)*x^1
7 + 14/5*(6*d + 5*e)*x^15 + 30/13*(7*d + 4*e)*x^13 + 15/11*(8*d +
 3*e)*x^11 + 5/9*(9*d + 2*e)*x^9 + 1/7*(10*d + e)*x^7 + 1/5*d*x^5
```

Fricas [A] time = 0.23042, size = 1, normalized size = 0.01

$$\frac{1}{27}x^{27}e+\frac{2}{5}x^{25}e+\frac{1}{25}x^{25}d+\frac{45}{23}x^{23}e+\frac{10}{23}x^{23}d+\frac{40}{7}x^{21}e+\frac{15}{7}x^{21}d+\frac{210}{19}x^{19}e+\frac{120}{19}x^{19}d+\frac{252}{17}x^{17}e+\frac{210}{17}x^{17}d+14x^{15}e+\frac{84}{5}x^{15}d+\frac{120}{13}x^{13}e+\frac{210}{13}x^{13}d+\frac{45}{11}x^{11}e+\frac{120}{11}x^{11}d+\frac{10}{9}x^{9}e+5x^{9}d+\frac{1}{7}x^{7}e+\frac{10}{7}x^{7}d+\frac{1}{5}x^{5}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)*x^4,x, algorithm="fricas")
```

```
[Out] 1/27*x^27*e + 2/5*x^25*e + 1/25*x^25*d + 45/23*x^23*e + 10/23*x^2
3*d + 40/7*x^21*e + 15/7*x^21*d + 210/19*x^19*e + 120/19*x^19*d +
 252/17*x^17*e + 210/17*x^17*d + 14*x^15*e + 84/5*x^15*d + 120/13
*x^13*e + 210/13*x^13*d + 45/11*x^11*e + 120/11*x^11*d + 10/9*x^9
*e + 5*x^9*d + 1/7*x^7*e + 10/7*x^7*d + 1/5*x^5*d
```

Sympy [A] time = 0.177747, size = 141, normalized size = 0.92

$$\frac{dx^{5}}{5}+\frac{ex^{27}}{27}+x^{25}\left(\frac{d}{25}+\frac{2e}{5}\right)+x^{23}\left(\frac{10d}{23}+\frac{45e}{23}\right)+x^{21}\left(\frac{15d}{7}+\frac{40e}{7}\right)+x^{19}\left(\frac{120d}{19}+\frac{210e}{19}\right)+x^{17}\left(\frac{210d}{17}+\frac{252e}{17}\right)+x^{15}\left(\frac{84d}{5}+14e\right)+x^{13}\left(\frac{210d}{13}+\frac{120e}{13}\right)+x^{11}\left(\frac{120d}{11}+\frac{45e}{11}\right)+x^{9}\left(5d+\frac{10e}{9}\right)+x^{7}\left(\frac{10d}{7}+\frac{e}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**5/5 + e*x**27/27 + x**25*(d/25 + 2*e/5) + x**23*(10*d/23 + 4
5*e/23) + x**21*(15*d/7 + 40*e/7) + x**19*(120*d/19 + 210*e/19) +
 x**17*(210*d/17 + 252*e/17) + x**15*(84*d/5 + 14*e) + x**13*(210
*d/13 + 120*e/13) + x**11*(120*d/11 + 45*e/11) + x**9*(5*d + 10*e
/9) + x**7*(10*d/7 + e/7)
```

GIAC/XCAS [A] time = 0.261885, size = 194, normalized size = 1.27

$$\frac{1}{27}\,x^{27}e + \frac{1}{25}\,dx^{25} + \frac{2}{5}\,x^{25}e + \frac{10}{23}\,dx^{23} + \frac{45}{23}\,x^{23}e + \frac{15}{7}\,dx^{21} + \frac{40}{7}\,x^{21}e + \frac{120}{19}\,dx^{19} + \frac{210}{19}\,x^{19}e + \frac{210}{17}\,dx^{17} + \frac{252}{17}\,x^{17}e + \frac{84}{5}\,dx^{15} + 14\,x^{15}e + \frac{210}{13}\,dx^{13} + \frac{120}{13}\,x^{13}e + \frac{120}{11}\,dx^{11} + \frac{45}{11}\,x^{11}e + 5\,dx^{9} + \frac{10}{9}\,x^{9}e + \frac{10}{7}\,dx^{7} + \frac{1}{7}\,x^{7}e + \frac{1}{5}\,dx^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)*x^4,x, algorithm="giac")
```

```
[Out] 1/27*x^27*e + 1/25*d*x^25 + 2/5*x^25*e + 10/23*d*x^23 + 45/23*x^2
3*e + 15/7*d*x^21 + 40/7*x^21*e + 120/19*d*x^19 + 210/19*x^19*e +
 210/17*d*x^17 + 252/17*x^17*e + 84/5*d*x^15 + 14*x^15*e + 210/13
*d*x^13 + 120/13*x^13*e + 120/11*d*x^11 + 45/11*x^11*e + 5*d*x^9
+ 10/9*x^9*e + 10/7*d*x^7 + 1/7*x^7*e + 1/5*d*x^5
```

3.58 $\int x^3 \left(d+e x^2\right) \left(1+2 x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=45

$$\frac{1}{24} \left(x^2+1\right)^{12} (d-2 e)-\frac{1}{22} \left(x^2+1\right)^{11} (d-e)+\frac{1}{26} e \left(x^2+1\right)^{13}$$

```
[Out] -((d - e)*(1 + x^2)^11)/22 + ((d - 2*e)*(1 + x^2)^12)/24 + (e*(1
+ x^2)^13)/26
```

Rubi [A] time = 0.333815, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{1}{24} \left(x^2+1\right)^{12} (d-2 e)-\frac{1}{22} \left(x^2+1\right)^{11} (d-e)+\frac{1}{26} e \left(x^2+1\right)^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] -((d - e)*(1 + x^2)^11)/22 + ((d - 2*e)*(1 + x^2)^12)/24 + (e*(1
+ x^2)^13)/26
```

Rubi in Sympy [A] time = 19.2049, size = 36, normalized size = 0.8

$$\frac{e \left(x^2+1\right)^{13}}{26}+\left(\frac{d}{24}-\frac{e}{12}\right) \left(x^2+1\right)^{12}-\left(\frac{d}{22}-\frac{e}{22}\right) \left(x^2+1\right)^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] e*(x**2 + 1)**13/26 + (d/24 - e/12)*(x**2 + 1)**12 - (d/22 - e/22
)*(x**2 + 1)**11
```

Mathematica [B] time = 0.0381065, size = 151, normalized size = 3.36

$$\frac{1}{24} x^{24} (d+10 e)+\frac{5}{22} x^{22} (2 d+9 e)+\frac{3}{4} x^{20} (3 d+8 e)+\frac{5}{3} x^{18} (4 d+7 e)+\frac{21}{8} x^{16} (5 d+6 e)+3 x^{14} (6 d+5 e)+\frac{5}{2} x^{12} (7 d+4 e)+\frac{3}{2} x^{10} (8 d+3 e)+\frac{5}{8} x^8 (9 d+2 e)+\frac{1}{6} x^6 (10 d+e)+\frac{d x^4}{4}+\frac{e x^{26}}{26}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^4)/4 + ((10*d + e)*x^6)/6 + (5*(9*d + 2*e)*x^8)/8 + (3*(8*d
+ 3*e)*x^10)/2 + (5*(7*d + 4*e)*x^12)/2 + 3*(6*d + 5*e)*x^14 + (2
1*(5*d + 6*e)*x^16)/8 + (5*(4*d + 7*e)*x^18)/3 + (3*(3*d + 8*e)*x
^20)/4 + (5*(2*d + 9*e)*x^22)/22 + ((d + 10*e)*x^24)/24 + (e*x^26
)/26
```

Maple [B] time = 0.001, size = 130, normalized size = 2.9

$$\frac{ex^{26}}{26}+\frac{(d+10\,e)\,x^{24}}{24}+\frac{(10\,d+45\,e)\,x^{22}}{22}+\frac{(45\,d+120\,e)\,x^{20}}{20}+\frac{(120\,d+210\,e)\,x^{18}}{18}+\frac{(210\,d+252\,e)\,x^{16}}{16}+\frac{(252\,d+210\,e)\,x^{14}}{14}+\frac{(210\,d+120\,e)\,x^{12}}{12}+\frac{(120\,d+45\,e)\,x^{10}}{10}+\frac{(45\,d+10\,e)\,x^{8}}{8}+\frac{(10\,d+e)\,x^{6}}{6}+\frac{dx^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/26*e*x^26+1/24*(d+10*e)*x^24+1/22*(10*d+45*e)*x^22+1/20*(45*d+1
20*e)*x^20+1/18*(120*d+210*e)*x^18+1/16*(210*d+252*e)*x^16+1/14*(
252*d+210*e)*x^14+1/12*(210*d+120*e)*x^12+1/10*(120*d+45*e)*x^10+
1/8*(45*d+10*e)*x^8+1/6*(10*d+e)*x^6+1/4*d*x^4
```

Maxima [A] time = 0.70191, size = 174, normalized size = 3.87

$$\frac{1}{26}\,ex^{26}+\frac{1}{24}\,(d+10\,e)x^{24}+\frac{5}{22}\,(2\,d+9\,e)x^{22}+\frac{3}{4}\,(3\,d+8\,e)x^{20}+\frac{5}{3}\,(4\,d+7\,e)x^{18}+\frac{21}{8}\,(5\,d+6\,e)x^{16}+3\,(6\,d+5\,e)x^{14}+\frac{5}{2}\,(7\,d+4\,e)x^{12}+\frac{3}{2}\,(8\,d+3\,e)x^{10}+\frac{5}{8}\,(9\,d+2\,e)x^{8}+\frac{1}{6}\,(10\,d+e)x^{6}+\frac{1}{4}\,dx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)*x^3,x, algorithm="maxima")
```

```
[Out] 1/26*e*x^26 + 1/24*(d + 10*e)*x^24 + 5/22*(2*d + 9*e)*x^22 + 3/4*
(3*d + 8*e)*x^20 + 5/3*(4*d + 7*e)*x^18 + 21/8*(5*d + 6*e)*x^16 +
 3*(6*d + 5*e)*x^14 + 5/2*(7*d + 4*e)*x^12 + 3/2*(8*d + 3*e)*x^10
 + 5/8*(9*d + 2*e)*x^8 + 1/6*(10*d + e)*x^6 + 1/4*d*x^4
```

Fricas [A] time = 0.238737, size = 1, normalized size = 0.02

$$\frac{1}{26}x^{26}e+\frac{5}{12}x^{24}e+\frac{1}{24}x^{24}d+\frac{45}{22}x^{22}e+\frac{5}{11}x^{22}d+6x^{20}e+\frac{9}{4}x^{20}d+\frac{35}{3}x^{18}e+\frac{20}{3}x^{18}d+\frac{63}{4}x^{16}e+\frac{105}{8}x^{16}d+15x^{14}e+18x^{14}d+10x^{12}e+\frac{35}{2}x^{12}d+\frac{9}{2}x^{10}e+12x^{10}d+\frac{5}{4}x^{8}e+\frac{45}{8}x^{8}d+\frac{1}{6}x^{6}e+\frac{5}{3}x^{6}d+\frac{1}{4}x^{4}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)*x^3,x, algorithm="fricas")
```

```
[Out] 1/26*x^26*e + 5/12*x^24*e + 1/24*x^24*d + 45/22*x^22*e + 5/11*x^2
2*d + 6*x^20*e + 9/4*x^20*d + 35/3*x^18*e + 20/3*x^18*d + 63/4*x^
16*e + 105/8*x^16*d + 15*x^14*e + 18*x^14*d + 10*x^12*e + 35/2*x^
12*d + 9/2*x^10*e + 12*x^10*d + 5/4*x^8*e + 45/8*x^8*d + 1/6*x^6*
e + 5/3*x^6*d + 1/4*x^4*d
```

Sympy [A] time = 0.174029, size = 136, normalized size = 3.02

$$\frac{dx^4}{4}+\frac{ex^{26}}{26}+x^{24}\left(\frac{d}{24}+\frac{5e}{12}\right)+x^{22}\left(\frac{5d}{11}+\frac{45e}{22}\right)+x^{20}\left(\frac{9d}{4}+6e\right)+x^{18}\left(\frac{20d}{3}+\frac{35e}{3}\right)+x^{16}\left(\frac{105d}{8}+\frac{63e}{4}\right)+x^{14}(18d+15e)+x^{12}\left(\frac{35d}{2}+10e\right)+x^{10}\left(12d+\frac{9e}{2}\right)+x^{8}\left(\frac{45d}{8}+\frac{5e}{4}\right)+x^{6}\left(\frac{5d}{3}+\frac{e}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**4/4 + e*x**26/26 + x**24*(d/24 + 5*e/12) + x**22*(5*d/11 + 4
5*e/22) + x**20*(9*d/4 + 6*e) + x**18*(20*d/3 + 35*e/3) + x**16*(
105*d/8 + 63*e/4) + x**14*(18*d + 15*e) + x**12*(35*d/2 + 10*e) +
 x**10*(12*d + 9*e/2) + x**8*(45*d/8 + 5*e/4) + x**6*(5*d/3 + e/6
)
```

GIAC/XCAS [A] time = 0.262726, size = 194, normalized size = 4.31

$$\frac{1}{26}x^{26}e+\frac{1}{24}dx^{24}+\frac{5}{12}x^{24}e+\frac{5}{11}dx^{22}+\frac{45}{22}x^{22}e+\frac{9}{4}dx^{20}+6x^{20}e+\frac{20}{3}dx^{18}+\frac{35}{3}x^{18}e+\frac{105}{8}dx^{16}+\frac{63}{4}x^{16}e+18dx^{14}+15x^{14}e+\frac{35}{2}dx^{12}+10x^{12}e+12dx^{10}+\frac{9}{2}x^{10}e+\frac{45}{8}dx^{8}+\frac{5}{4}x^{8}e+\frac{5}{3}dx^{6}+\frac{1}{6}x^{6}e+\frac{1}{4}dx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)*x^3,x, algorithm="giac")
```

```
[Out] 1/26*x^26*e + 1/24*d*x^24 + 5/12*x^24*e + 5/11*d*x^22 + 45/22*x^2
2*e + 9/4*d*x^20 + 6*x^20*e + 20/3*d*x^18 + 35/3*x^18*e + 105/8*d
*x^16 + 63/4*x^16*e + 18*d*x^14 + 15*x^14*e + 35/2*d*x^12 + 10*x^
12*e + 12*d*x^10 + 9/2*x^10*e + 45/8*d*x^8 + 5/4*x^8*e + 5/3*d*x^
6 + 1/6*x^6*e + 1/4*d*x^4
```

3.59 $\int x^2 \left(d+e x^2\right) \left(1+2 x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=153

$$\frac{1}{23}x^{23}(d+10e)+\frac{5}{21}x^{21}(2d+9e)+\frac{15}{19}x^{19}(3d+8e)+\frac{30}{17}x^{17}(4d+7e)+\frac{14}{5}x^{15}(5d+6e)+\frac{42}{13}x^{13}(6d+5e)+\frac{30}{11}x^{11}(7d+4e)+\frac{5}{3}x^{9}(8d+3e)+\frac{5}{7}x^{7}(9d+2e)+\frac{1}{5}x^{5}(10d+e)+\frac{dx^3}{3}+\frac{ex^{25}}{25}$$

```
[Out] (d*x^3)/3 + ((10*d + e)*x^5)/5 + (5*(9*d + 2*e)*x^7)/7 + (5*(8*d
+ 3*e)*x^9)/3 + (30*(7*d + 4*e)*x^11)/11 + (42*(6*d + 5*e)*x^13)/
13 + (14*(5*d + 6*e)*x^15)/5 + (30*(4*d + 7*e)*x^17)/17 + (15*(3*
d + 8*e)*x^19)/19 + (5*(2*d + 9*e)*x^21)/21 + ((d + 10*e)*x^23)/2
3 + (e*x^25)/25
```

Rubi [A] time = 0.274109, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{1}{23}x^{23}(d+10e)+\frac{5}{21}x^{21}(2d+9e)+\frac{15}{19}x^{19}(3d+8e)+\frac{30}{17}x^{17}(4d+7e)+\frac{14}{5}x^{15}(5d+6e)+\frac{42}{13}x^{13}(6d+5e)+\frac{30}{11}x^{11}(7d+4e)+\frac{5}{3}x^{9}(8d+3e)+\frac{5}{7}x^{7}(9d+2e)+\frac{1}{5}x^{5}(10d+e)+\frac{dx^3}{3}+\frac{ex^{25}}{25}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^3)/3 + ((10*d + e)*x^5)/5 + (5*(9*d + 2*e)*x^7)/7 + (5*(8*d
+ 3*e)*x^9)/3 + (30*(7*d + 4*e)*x^11)/11 + (42*(6*d + 5*e)*x^13)/
13 + (14*(5*d + 6*e)*x^15)/5 + (30*(4*d + 7*e)*x^17)/17 + (15*(3*
d + 8*e)*x^19)/19 + (5*(2*d + 9*e)*x^21)/21 + ((d + 10*e)*x^23)/2
3 + (e*x^25)/25
```

Rubi in Sympy [A] time = 27.9112, size = 139, normalized size = 0.91

$$\frac{dx^3}{3}+\frac{ex^{25}}{25}+x^{23}\left(\frac{d}{23}+\frac{10e}{23}\right)+x^{21}\left(\frac{10d}{21}+\frac{15e}{7}\right)+x^{19}\left(\frac{45d}{19}+\frac{120e}{19}\right)+x^{17}\left(\frac{120d}{17}+\frac{210e}{17}\right)+x^{15}\left(14d+\frac{84e}{5}\right)+x^{13}\left(\frac{252d}{13}+\frac{210e}{13}\right)+x^{11}\left(\frac{210d}{11}+\frac{120e}{11}\right)+x^{9}\left(\frac{40d}{3}+5e\right)+x^{7}\left(\frac{45d}{7}+\frac{10e}{7}\right)+x^{5}\left(2d+\frac{e}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**3/3 + e*x**25/25 + x**23*(d/23 + 10*e/23) + x**21*(10*d/21 +
 15*e/7) + x**19*(45*d/19 + 120*e/19) + x**17*(120*d/17 + 210*e/1
7) + x**15*(14*d + 84*e/5) + x**13*(252*d/13 + 210*e/13) + x**11*
(210*d/11 + 120*e/11) + x**9*(40*d/3 + 5*e) + x**7*(45*d/7 + 10*e
/7) + x**5*(2*d + e/5)
```

Mathematica [A] time = 0.0392462, size = 153, normalized size = 1.

$$\frac{1}{23}x^{23}(d+10e)+\frac{5}{21}x^{21}(2d+9e)+\frac{15}{19}x^{19}(3d+8e)+\frac{30}{17}x^{17}(4d+7e)+\frac{14}{5}x^{15}(5d+6e)+\frac{42}{13}x^{13}(6d+5e)+\frac{30}{11}x^{11}(7d+4e)+\frac{5}{3}x^{9}(8d+3e)+\frac{5}{7}x^{7}(9d+2e)+\frac{1}{5}x^{5}(10d+e)+\frac{dx^3}{3}+\frac{ex^{25}}{25}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^3)/3 + ((10*d + e)*x^5)/5 + (5*(9*d + 2*e)*x^7)/7 + (5*(8*d
+ 3*e)*x^9)/3 + (30*(7*d + 4*e)*x^11)/11 + (42*(6*d + 5*e)*x^13)/
13 + (14*(5*d + 6*e)*x^15)/5 + (30*(4*d + 7*e)*x^17)/17 + (15*(3*
d + 8*e)*x^19)/19 + (5*(2*d + 9*e)*x^21)/21 + ((d + 10*e)*x^23)/2
3 + (e*x^25)/25
```

Maple [A] time = 0.002, size = 130, normalized size = 0.9

$$\frac{ex^{25}}{25}+\frac{(d+10e)x^{23}}{23}+\frac{(10d+45e)x^{21}}{21}+\frac{(45d+120e)x^{19}}{19}+\frac{(120d+210e)x^{17}}{17}+\frac{(210d+252e)x^{15}}{15}+\frac{(252d+210e)x^{13}}{13}+\frac{(210d+120e)x^{11}}{11}+\frac{(120d+45e)x^{9}}{9}+\frac{(45d+10e)x^{7}}{7}+\frac{(10d+e)x^{5}}{5}+\frac{dx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/25*e*x^25+1/23*(d+10*e)*x^23+1/21*(10*d+45*e)*x^21+1/19*(45*d+1
20*e)*x^19+1/17*(120*d+210*e)*x^17+1/15*(210*d+252*e)*x^15+1/13*(
252*d+210*e)*x^13+1/11*(210*d+120*e)*x^11+1/9*(120*d+45*e)*x^9+1/
7*(45*d+10*e)*x^7+1/5*(10*d+e)*x^5+1/3*d*x^3
```

Maxima [A] time = 0.69894, size = 174, normalized size = 1.14

$$\frac{1}{25}\,ex^{25}+\frac{1}{23}\,(d+10\,e)x^{23}+\frac{5}{21}\,(2\,d+9\,e)x^{21}+\frac{15}{19}\,(3\,d+8\,e)x^{19}+\frac{30}{17}\,(4\,d+7\,e)x^{17}+\frac{14}{5}\,(5\,d+6\,e)x^{15}+\frac{42}{13}\,(6\,d+5\,e)x^{13}+\frac{30}{11}\,(7\,d+4\,e)x^{11}+\frac{5}{3}\,(8\,d+3\,e)x^{9}+\frac{5}{7}\,(9\,d+2\,e)x^{7}+\frac{1}{5}\,(10\,d+e)x^{5}+\frac{1}{3}\,dx^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)*x^2,x, algorithm="maxima")
```

```
[Out] 1/25*e*x^25 + 1/23*(d + 10*e)*x^23 + 5/21*(2*d + 9*e)*x^21 + 15/1
9*(3*d + 8*e)*x^19 + 30/17*(4*d + 7*e)*x^17 + 14/5*(5*d + 6*e)*x^
15 + 42/13*(6*d + 5*e)*x^13 + 30/11*(7*d + 4*e)*x^11 + 5/3*(8*d +
 3*e)*x^9 + 5/7*(9*d + 2*e)*x^7 + 1/5*(10*d + e)*x^5 + 1/3*d*x^3
```

Fricas [A] time = 0.250639, size = 1, normalized size = 0.01

$$\frac{1}{25}x^{25}e+\frac{10}{23}x^{23}e+\frac{1}{23}x^{23}d+\frac{15}{7}x^{21}e+\frac{10}{21}x^{21}d+\frac{120}{19}x^{19}e+\frac{45}{19}x^{19}d+\frac{210}{17}x^{17}e+\frac{120}{17}x^{17}d+\frac{84}{5}x^{15}e+14x^{15}d+\frac{210}{13}x^{13}e+\frac{252}{13}x^{13}d+\frac{120}{11}x^{11}e+\frac{210}{11}x^{11}d+5x^{9}e+\frac{40}{3}x^{9}d+\frac{10}{7}x^{7}e+\frac{45}{7}x^{7}d+\frac{1}{5}x^{5}e+2x^{5}d+\frac{1}{3}x^{3}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)*x^2,x, algorithm="fricas")
```

```
[Out] 1/25*x^25*e + 10/23*x^23*e + 1/23*x^23*d + 15/7*x^21*e + 10/21*x^
21*d + 120/19*x^19*e + 45/19*x^19*d + 210/17*x^17*e + 120/17*x^17
*d + 84/5*x^15*e + 14*x^15*d + 210/13*x^13*e + 252/13*x^13*d + 12
0/11*x^11*e + 210/11*x^11*d + 5*x^9*e + 40/3*x^9*d + 10/7*x^7*e +
 45/7*x^7*d + 1/5*x^5*e + 2*x^5*d + 1/3*x^3*d
```

Sympy [A] time = 0.173969, size = 139, normalized size = 0.91

$$\frac{dx^{3}}{3}+\frac{ex^{25}}{25}+x^{23}\left(\frac{d}{23}+\frac{10e}{23}\right)+x^{21}\left(\frac{10d}{21}+\frac{15e}{7}\right)+x^{19}\left(\frac{45d}{19}+\frac{120e}{19}\right)+x^{17}\left(\frac{120d}{17}+\frac{210e}{17}\right)+x^{15}\left(14d+\frac{84e}{5}\right)+x^{13}\left(\frac{252d}{13}+\frac{210e}{13}\right)+x^{11}\left(\frac{210d}{11}+\frac{120e}{11}\right)+x^{9}\left(\frac{40d}{3}+5e\right)+x^{7}\left(\frac{45d}{7}+\frac{10e}{7}\right)+x^{5}\left(2d+\frac{e}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**3/3 + e*x**25/25 + x**23*(d/23 + 10*e/23) + x**21*(10*d/21 +
 15*e/7) + x**19*(45*d/19 + 120*e/19) + x**17*(120*d/17 + 210*e/1
7) + x**15*(14*d + 84*e/5) + x**13*(252*d/13 + 210*e/13) + x**11*
(210*d/11 + 120*e/11) + x**9*(40*d/3 + 5*e) + x**7*(45*d/7 + 10*e
/7) + x**5*(2*d + e/5)
```

GIAC/XCAS [A] time = 0.261024, size = 194, normalized size = 1.27

$$\frac{1}{25}\,x^{25}e+\frac{1}{23}\,dx^{23}+\frac{10}{23}\,x^{23}e+\frac{10}{21}\,dx^{21}+\frac{15}{7}\,x^{21}e+\frac{45}{19}\,dx^{19}+\frac{120}{19}\,x^{19}e+\frac{120}{17}\,dx^{17}+\frac{210}{17}\,x^{17}e+14\,dx^{15}+\frac{84}{5}\,x^{15}e+\frac{252}{13}\,dx^{13}+\frac{210}{13}\,x^{13}e+\frac{210}{11}\,dx^{11}+\frac{120}{11}\,x^{11}e+\frac{40}{3}\,dx^{9}+5\,x^{9}e+\frac{45}{7}\,dx^{7}+\frac{10}{7}\,x^{7}e+2\,dx^{5}+\frac{1}{5}\,x^{5}e+\frac{1}{3}\,dx^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)*x^2,x, algorithm="giac")
```

```
[Out] 1/25*x^25*e + 1/23*d*x^23 + 10/23*x^23*e + 10/21*d*x^21 + 15/7*x^
21*e + 45/19*d*x^19 + 120/19*x^19*e + 120/17*d*x^17 + 210/17*x^17
*e + 14*d*x^15 + 84/5*x^15*e + 252/13*d*x^13 + 210/13*x^13*e + 21
0/11*d*x^11 + 120/11*x^11*e + 40/3*d*x^9 + 5*x^9*e + 45/7*d*x^7 +
 10/7*x^7*e + 2*d*x^5 + 1/5*x^5*e + 1/3*d*x^3
```

3.60 $\int x \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=29

$$\frac{1}{22}\left(x^2+1\right)^{11}(d-e)+\frac{1}{24}e\left(x^2+1\right)^{12}$$

```
[Out] ((d - e)*(1 + x^2)^11)/22 + (e*(1 + x^2)^12)/24
```

Rubi [A] time = 0.129737, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{1}{22}\left(x^2+1\right)^{11}(d-e)+\frac{1}{24}e\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] ((d - e)*(1 + x^2)^11)/22 + (e*(1 + x^2)^12)/24
```

Rubi in Sympy [A] time = 16.1771, size = 22, normalized size = 0.76

$$\frac{e\left(x^2+1\right)^{12}}{24}+\left(\frac{d}{22}-\frac{e}{22}\right)\left(x^2+1\right)^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**2+d)*(x**4+2*x**2+1)**5,x)

[Out] e*(x**2 + 1)**12/24 + (d/22 - e/22)*(x**2 + 1)**11
```

Mathematica [B] time = 0.0256812, size = 149, normalized size = 5.14

$$\frac{1}{22}x^{22}(d+10e)+\frac{1}{4}x^{20}(2d+9e)+\frac{5}{6}x^{18}(3d+8e)+\frac{15}{8}x^{16}(4d+7e)+3x^{14}(5d+6e) +\frac{7}{2}x^{12}(6d+5e)+3x^{10}(7d+4e)+\frac{15}{8}x^8(8d+3e)+\frac{5}{6}x^6(9d+2e)+\frac{1}{4}x^4(10d+e)+\frac{dx^2}{2}+\frac{ex^{24}}{24}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^2)/2 + ((10*d + e)*x^4)/4 + (5*(9*d + 2*e)*x^6)/6 + (15*(8*d
 + 3*e)*x^8)/8 + 3*(7*d + 4*e)*x^10 + (7*(6*d + 5*e)*x^12)/2 + 3*
(5*d + 6*e)*x^14 + (15*(4*d + 7*e)*x^16)/8 + (5*(3*d + 8*e)*x^18)
/6 + ((2*d + 9*e)*x^20)/4 + ((d + 10*e)*x^22)/22 + (e*x^24)/24
```

Maple [B] time = 0.002, size = 130, normalized size = 4.5

$$\frac{ex^{24}}{24}+\frac{(d+10\,e)\,x^{22}}{22}+\frac{(10\,d+45\,e)\,x^{20}}{20}+\frac{(45\,d+120\,e)\,x^{18}}{18}+\frac{(120\,d+210\,e)\,x^{16}}{16}+\frac{(210\,d+252\,e)\,x^{14}}{14}+\frac{(252\,d+210\,e)\,x^{12}}{12}+\frac{(210\,d+120\,e)\,x^{10}}{10}+\frac{(120\,d+45\,e)\,x^{8}}{8}+\frac{(45\,d+10\,e)\,x^{6}}{6}+\frac{(10\,d+e)\,x^{4}}{4}+\frac{dx^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/24*e*x^24+1/22*(d+10*e)*x^22+1/20*(10*d+45*e)*x^20+1/18*(45*d+1
20*e)*x^18+1/16*(120*d+210*e)*x^16+1/14*(210*d+252*e)*x^14+1/12*(
252*d+210*e)*x^12+1/10*(210*d+120*e)*x^10+1/8*(120*d+45*e)*x^8+1/
6*(45*d+10*e)*x^6+1/4*(10*d+e)*x^4+1/2*d*x^2
```

Maxima [A] time = 0.693949, size = 174, normalized size = 6.

$$\frac{1}{24}\,ex^{24}+\frac{1}{22}\,(d+10\,e)x^{22}+\frac{1}{4}\,(2\,d+9\,e)x^{20}+\frac{5}{6}\,(3\,d+8\,e)x^{18}+\frac{15}{8}\,(4\,d+7\,e)x^{16}+3\,(5\,d+6\,e)x^{14}+\frac{7}{2}\,(6\,d+5\,e)x^{12}+3\,(7\,d+4\,e)x^{10}+\frac{15}{8}\,(8\,d+3\,e)x^{8}+\frac{5}{6}\,(9\,d+2\,e)x^{6}+\frac{1}{4}\,(10\,d+e)x^{4}+\frac{1}{2}\,dx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)*x,x, algorithm="maxima")
```

```
[Out] 1/24*e*x^24 + 1/22*(d + 10*e)*x^22 + 1/4*(2*d + 9*e)*x^20 + 5/6*(
3*d + 8*e)*x^18 + 15/8*(4*d + 7*e)*x^16 + 3*(5*d + 6*e)*x^14 + 7/
2*(6*d + 5*e)*x^12 + 3*(7*d + 4*e)*x^10 + 15/8*(8*d + 3*e)*x^8 +
5/6*(9*d + 2*e)*x^6 + 1/4*(10*d + e)*x^4 + 1/2*d*x^2
```

Fricas [A] time = 0.238505, size = 1, normalized size = 0.03

$$\frac{1}{24}x^{24}e + \frac{5}{11}x^{22}e + \frac{1}{22}x^{22}d + \frac{9}{4}x^{20}e + \frac{1}{2}x^{20}d + \frac{20}{3}x^{18}e + \frac{5}{2}x^{18}d + \frac{105}{8}x^{16}e + \frac{15}{2}x^{16}d + 18x^{14}e + 15x^{14}d + \frac{35}{2}x^{12}e + 21x^{12}d + 12x^{10}e + 21x^{10}d + \frac{45}{8}x^{8}e + 15x^{8}d + \frac{5}{3}x^{6}e + \frac{15}{2}x^{6}d + \frac{1}{4}x^{4}e + \frac{5}{2}x^{4}d + \frac{1}{2}x^{2}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)*x,x, algorithm="fricas")
```

```
[Out] 1/24*x^24*e + 5/11*x^22*e + 1/22*x^22*d + 9/4*x^20*e + 1/2*x^20*d
 + 20/3*x^18*e + 5/2*x^18*d + 105/8*x^16*e + 15/2*x^16*d + 18*x^1
4*e + 15*x^14*d + 35/2*x^12*e + 21*x^12*d + 12*x^10*e + 21*x^10*d
 + 45/8*x^8*e + 15*x^8*d + 5/3*x^6*e + 15/2*x^6*d + 1/4*x^4*e + 5
/2*x^4*d + 1/2*x^2*d
```

Sympy [A] time = 0.175594, size = 133, normalized size = 4.59

$$\frac{dx^2}{2} + \frac{ex^{24}}{24} + x^{22}\left(\frac{d}{22} + \frac{5e}{11}\right) + x^{20}\left(\frac{d}{2} + \frac{9e}{4}\right) + x^{18}\left(\frac{5d}{2} + \frac{20e}{3}\right) + x^{16}\left(\frac{15d}{2} + \frac{105e}{8}\right) + x^{14}(15d + 18e) + x^{12}\left(21d + \frac{35e}{2}\right) + x^{10}(21d + 12e) + x^{8}\left(15d + \frac{45e}{8}\right) + x^{6}\left(\frac{15d}{2} + \frac{5e}{3}\right) + x^{4}\left(\frac{5d}{2} + \frac{e}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**2/2 + e*x**24/24 + x**22*(d/22 + 5*e/11) + x**20*(d/2 + 9*e/
4) + x**18*(5*d/2 + 20*e/3) + x**16*(15*d/2 + 105*e/8) + x**14*(1
5*d + 18*e) + x**12*(21*d + 35*e/2) + x**10*(21*d + 12*e) + x**8*
(15*d + 45*e/8) + x**6*(15*d/2 + 5*e/3) + x**4*(5*d/2 + e/4)
```

GIAC/XCAS [A] time = 0.261238, size = 194, normalized size = 6.69

$$\frac{1}{24}x^{24}e + \frac{1}{22}dx^{22} + \frac{5}{11}x^{22}e + \frac{1}{2}dx^{20} + \frac{9}{4}x^{20}e + \frac{5}{2}dx^{18} + \frac{20}{3}x^{18}e + \frac{15}{2}dx^{16} + \frac{105}{8}x^{16}e + 15\,dx^{14} + 18\,x^{14}e + 21\,dx^{12} + \frac{35}{2}x^{12}e + 21\,dx^{10} + 12\,x^{10}e + 15\,dx^{8} + \frac{45}{8}x^{8}e + \frac{15}{2}dx^{6} + \frac{5}{3}x^{6}e + \frac{5}{2}dx^{4} + \frac{1}{4}x^{4}e + \frac{1}{2}dx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)*x,x, algorithm="giac")
```

```
[Out] 1/24*x^24*e + 1/22*d*x^22 + 5/11*x^22*e + 1/2*d*x^20 + 9/4*x^20*e
 + 5/2*d*x^18 + 20/3*x^18*e + 15/2*d*x^16 + 105/8*x^16*e + 15*d*x
^14 + 18*x^14*e + 21*d*x^12 + 35/2*x^12*e + 21*d*x^10 + 12*x^10*e
 + 15*d*x^8 + 45/8*x^8*e + 15/2*d*x^6 + 5/3*x^6*e + 5/2*d*x^4 + 1
/4*x^4*e + 1/2*d*x^2
```

3.61 $\int \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=143

$$\frac{1}{21}x^{21}(d+10e)+\frac{5}{19}x^{19}(2d+9e)+\frac{15}{17}x^{17}(3d+8e)+2x^{15}(4d+7e)+\frac{42}{13}x^{13}(5d+6e)+\frac{42}{11}x^{11}(6d+5e)+\frac{10}{3}x^9(7d+4e)+\frac{15}{7}x^7(8d+3e)+x^5(9d+2e)+\frac{1}{3}x^3(10d+e)+dx+\frac{ex^{23}}{23}$$

```
[Out] d*x + ((10*d + e)*x^3)/3 + (9*d + 2*e)*x^5 + (15*(8*d + 3*e)*x^7)
/7 + (10*(7*d + 4*e)*x^9)/3 + (42*(6*d + 5*e)*x^11)/11 + (42*(5*d
 + 6*e)*x^13)/13 + 2*(4*d + 7*e)*x^15 + (15*(3*d + 8*e)*x^17)/17
+ (5*(2*d + 9*e)*x^19)/19 + ((d + 10*e)*x^21)/21 + (e*x^23)/23
```

Rubi [A] time = 0.204778, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{1}{21}x^{21}(d+10e)+\frac{5}{19}x^{19}(2d+9e)+\frac{15}{17}x^{17}(3d+8e)+2x^{15}(4d+7e)+\frac{42}{13}x^{13}(5d+6e)+\frac{42}{11}x^{11}(6d+5e)+\frac{10}{3}x^9(7d+4e)+\frac{15}{7}x^7(8d+3e)+x^5(9d+2e)+\frac{1}{3}x^3(10d+e)+dx+\frac{ex^{23}}{23}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] d*x + ((10*d + e)*x^3)/3 + (9*d + 2*e)*x^5 + (15*(8*d + 3*e)*x^7)
/7 + (10*(7*d + 4*e)*x^9)/3 + (42*(6*d + 5*e)*x^11)/11 + (42*(5*d
 + 6*e)*x^13)/13 + 2*(4*d + 7*e)*x^15 + (15*(3*d + 8*e)*x^17)/17
+ (5*(2*d + 9*e)*x^19)/19 + ((d + 10*e)*x^21)/21 + (e*x^23)/23
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ex^{23}}{23}+x^{21}\left(\frac{d}{21}+\frac{10e}{21}\right)+x^{19}\left(\frac{10d}{19}+\frac{45e}{19}\right)+x^{17}\left(\frac{45d}{17}+\frac{120e}{17}\right)+x^{15}\left(8d+14e\right)+x^{13}\left(\frac{210d}{13}+\frac{252e}{13}\right)+x^{11}\left(\frac{252d}{11}+\frac{210e}{11}\right)+x^9\left(\frac{70d}{3}+\frac{40e}{3}\right)+x^7\left(\frac{120d}{7}+\frac{45e}{7}\right)+x^5\left(9d+2e\right)+x^3\left(\frac{10d}{3}+\frac{e}{3}\right)+\int d\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] e*x**23/23 + x**21*(d/21 + 10*e/21) + x**19*(10*d/19 + 45*e/19) +
 x**17*(45*d/17 + 120*e/17) + x**15*(8*d + 14*e) + x**13*(210*d/1
3 + 252*e/13) + x**11*(252*d/11 + 210*e/11) + x**9*(70*d/3 + 40*e
/3) + x**7*(120*d/7 + 45*e/7) + x**5*(9*d + 2*e) + x**3*(10*d/3 +
 e/3) + Integral(d, x)
```

Mathematica [A] time = 0.0379346, size = 143, normalized size = 1.

$$\frac{1}{21}x^{21}(d+10e)+\frac{5}{19}x^{19}(2d+9e)+\frac{15}{17}x^{17}(3d+8e)+2x^{15}(4d+7e)+\frac{42}{13}x^{13}(5d+6e)+\frac{42}{11}x^{11}(6d+5e)+\frac{10}{3}x^9(7d+4e)+\frac{15}{7}x^7(8d+3e)+x^5(9d+2e)+\frac{1}{3}x^3(10d+e)+dx+\frac{ex^{23}}{23}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] d*x + ((10*d + e)*x^3)/3 + (9*d + 2*e)*x^5 + (15*(8*d + 3*e)*x^7)
/7 + (10*(7*d + 4*e)*x^9)/3 + (42*(6*d + 5*e)*x^11)/11 + (42*(5*d
 + 6*e)*x^13)/13 + 2*(4*d + 7*e)*x^15 + (15*(3*d + 8*e)*x^17)/17
+ (5*(2*d + 9*e)*x^19)/19 + ((d + 10*e)*x^21)/21 + (e*x^23)/23
```

Maple [A] time = 0.001, size = 127, normalized size = 0.9

$$\frac{ex^{23}}{23}+\frac{(d+10\,e)\,x^{21}}{21}+\frac{(10\,d+45\,e)\,x^{19}}{19}+\frac{(45\,d+120\,e)\,x^{17}}{17}+\frac{(120\,d+210\,e)\,x^{15}}{15}+\frac{(210\,d+252\,e)\,x^{13}}{13}+\frac{(252\,d+210\,e)\,x^{11}}{11}+\frac{(210\,d+120\,e)\,x^{9}}{9}+\frac{(120\,d+45\,e)\,x^{7}}{7}+\frac{(45\,d+10\,e)\,x^{5}}{5}+\frac{(10\,d+e)\,x^{3}}{3}+dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/23*e*x^23+1/21*(d+10*e)*x^21+1/19*(10*d+45*e)*x^19+1/17*(45*d+1
20*e)*x^17+1/15*(120*d+210*e)*x^15+1/13*(210*d+252*e)*x^13+1/11*(
252*d+210*e)*x^11+1/9*(210*d+120*e)*x^9+1/7*(120*d+45*e)*x^7+1/5*
(45*d+10*e)*x^5+1/3*(10*d+e)*x^3+d*x
```

Maxima [A] time = 0.704461, size = 169, normalized size = 1.18

$$\frac{1}{23}\,ex^{23}+\frac{1}{21}\,(d+10\,e)x^{21}+\frac{5}{19}\,(2\,d+9\,e)x^{19}+\frac{15}{17}\,(3\,d+8\,e)x^{17}+2\,(4\,d+7\,e)x^{15}+\frac{42}{13}\,(5\,d+6\,e)x^{13}+\frac{42}{11}\,(6\,d+5\,e)x^{11}+\frac{10}{3}\,(7\,d+4\,e)x^{9}+\frac{15}{7}\,(8\,d+3\,e)x^{7}+(9\,d+2\,e)x^{5}+\frac{1}{3}\,(10\,d+e)x^{3}+dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d),x, algorithm="maxima")
```

```
[Out] 1/23*e*x^23 + 1/21*(d + 10*e)*x^21 + 5/19*(2*d + 9*e)*x^19 + 15/1
7*(3*d + 8*e)*x^17 + 2*(4*d + 7*e)*x^15 + 42/13*(5*d + 6*e)*x^13
+ 42/11*(6*d + 5*e)*x^11 + 10/3*(7*d + 4*e)*x^9 + 15/7*(8*d + 3*e
)*x^7 + (9*d + 2*e)*x^5 + 1/3*(10*d + e)*x^3 + d*x
```

Fricas [A] time = 0.23482, size = 1, normalized size = 0.01

$$\frac{1}{23}x^{23}e+\frac{10}{21}x^{21}e+\frac{1}{21}x^{21}d+\frac{45}{19}x^{19}e+\frac{10}{19}x^{19}d+\frac{120}{17}x^{17}e+\frac{45}{17}x^{17}d+14x^{15}e+8x^{15}d+\frac{252}{13}x^{13}e+\frac{210}{13}x^{13}d+\frac{210}{11}x^{11}e+\frac{252}{11}x^{11}d+\frac{40}{3}x^{9}e+\frac{70}{3}x^{9}d+\frac{45}{7}x^{7}e+\frac{120}{7}x^{7}d+2x^{5}e+9x^{5}d+\frac{1}{3}x^{3}e+\frac{10}{3}x^{3}d+xd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d),x, algorithm="fricas")
```

```
[Out] 1/23*x^23*e + 10/21*x^21*e + 1/21*x^21*d + 45/19*x^19*e + 10/19*x
^19*d + 120/17*x^17*e + 45/17*x^17*d + 14*x^15*e + 8*x^15*d + 252
/13*x^13*e + 210/13*x^13*d + 210/11*x^11*e + 252/11*x^11*d + 40/3
*x^9*e + 70/3*x^9*d + 45/7*x^7*e + 120/7*x^7*d + 2*x^5*e + 9*x^5*
d + 1/3*x^3*e + 10/3*x^3*d + x*d
```

Sympy [A] time = 0.183484, size = 134, normalized size = 0.94

$$dx+\frac{ex^{23}}{23}+x^{21}\left(\frac{d}{21}+\frac{10e}{21}\right)+x^{19}\left(\frac{10d}{19}+\frac{45e}{19}\right)+x^{17}\left(\frac{45d}{17}+\frac{120e}{17}\right)+x^{15}\,(8d+14e)+x^{13}\left(\frac{210d}{13}+\frac{252e}{13}\right)+x^{11}\left(\frac{252d}{11}+\frac{210e}{11}\right)+x^{9}\left(\frac{70d}{3}+\frac{40e}{3}\right)+x^{7}\left(\frac{120d}{7}+\frac{45e}{7}\right)+x^{5}\,(9d+2e)+x^{3}\left(\frac{10d}{3}+\frac{e}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x + e*x**23/23 + x**21*(d/21 + 10*e/21) + x**19*(10*d/19 + 45*e
/19) + x**17*(45*d/17 + 120*e/17) + x**15*(8*d + 14*e) + x**13*(2
10*d/13 + 252*e/13) + x**11*(252*d/11 + 210*e/11) + x**9*(70*d/3
+ 40*e/3) + x**7*(120*d/7 + 45*e/7) + x**5*(9*d + 2*e) + x**3*(10
*d/3 + e/3)
```

GIAC/XCAS [A] time = 0.271536, size = 190, normalized size = 1.33

$$\frac{1}{23}\,x^{23}e + \frac{1}{21}\,dx^{21} + \frac{10}{21}\,x^{21}e + \frac{10}{19}\,dx^{19} + \frac{45}{19}\,x^{19}e + \frac{45}{17}\,dx^{17} + \frac{120}{17}\,x^{17}e + 8\,dx^{15} + 14\,x^{15}e + \frac{210}{13}\,dx^{13} + \frac{252}{13}\,x^{13}e + \frac{252}{11}\,dx^{11} + \frac{210}{11}\,x^{11}e + \frac{70}{3}\,dx^{9} + \frac{40}{3}\,x^{9}e + \frac{120}{7}\,dx^{7} + \frac{45}{7}\,x^{7}e + 9\,dx^{5} + 2\,x^{5}e + \frac{10}{3}\,dx^{3} + \frac{1}{3}\,x^{3}e + dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d),x, algorithm="giac")
```

```
[Out] 1/23*x^23*e + 1/21*d*x^21 + 10/21*x^21*e + 10/19*d*x^19 + 45/19*x
^19*e + 45/17*d*x^17 + 120/17*x^17*e + 8*d*x^15 + 14*x^15*e + 210
/13*d*x^13 + 252/13*x^13*e + 252/11*d*x^11 + 210/11*x^11*e + 70/3
*d*x^9 + 40/3*x^9*e + 120/7*d*x^7 + 45/7*x^7*e + 9*d*x^5 + 2*x^5*
e + 10/3*d*x^3 + 1/3*x^3*e + d*x
```

3.62 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x} dx$

Optimal. Leaf size=93

$$\frac{dx^{20}}{20}+\frac{5dx^{18}}{9}+\frac{45dx^{16}}{16}+\frac{60dx^{14}}{7}+\frac{35dx^{12}}{2}+\frac{126dx^{10}}{5}+\frac{105dx^8}{4} + 20dx^6+\frac{45dx^4}{4}+5dx^2+d\log(x)+\frac{1}{22}e\left(x^2+1\right)^{11}$$

```
[Out] 5*d*x^2 + (45*d*x^4)/4 + 20*d*x^6 + (105*d*x^8)/4 + (126*d*x^10)/
5 + (35*d*x^12)/2 + (60*d*x^14)/7 + (45*d*x^16)/16 + (5*d*x^18)/9
 + (d*x^20)/20 + (e*(1 + x^2)^11)/22 + d*Log[x]
```

Rubi [A] time = 0.108661, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{dx^{20}}{20}+\frac{5dx^{18}}{9}+\frac{45dx^{16}}{16}+\frac{60dx^{14}}{7}+\frac{35dx^{12}}{2}+\frac{126dx^{10}}{5}+\frac{105dx^8}{4} + 20dx^6+\frac{45dx^4}{4}+5dx^2+d\log(x)+\frac{1}{22}e\left(x^2+1\right)^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] 5*d*x^2 + (45*d*x^4)/4 + 20*d*x^6 + (105*d*x^8)/4 + (126*d*x^10)/
5 + (35*d*x^12)/2 + (60*d*x^14)/7 + (45*d*x^16)/16 + (5*d*x^18)/9
 + (d*x^20)/20 + (e*(1 + x^2)^11)/22 + d*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{dx^{20}}{20}+\frac{5dx^{18}}{9}+\frac{45dx^{16}}{16}+\frac{60dx^{14}}{7}+\frac{35dx^{12}}{2}+\frac{126dx^{10}}{5}+\frac{105dx^8}{4} + 20dx^6+5dx^2+\frac{d\log\left(x^2\right)}{2}+\frac{45d\int^{x^2} x\,dx}{2}+\frac{e\left(x^2+1\right)^{11}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x,x)
```

```
[Out] d*x**20/20 + 5*d*x**18/9 + 45*d*x**16/16 + 60*d*x**14/7 + 35*d*x*
*12/2 + 126*d*x**10/5 + 105*d*x**8/4 + 20*d*x**6 + 5*d*x**2 + d*l
```

```
og(x**2)/2 + 45*d*Integral(x, (x, x**2))/2 + e*(x**2 + 1)**11/22
```

Mathematica [A] time = 0.0514577, size = 149, normalized size = 1.6

$$\frac{1}{20}x^{20}(d+10e)+\frac{5}{18}x^{18}(2d+9e)+\frac{15}{16}x^{16}(3d+8e)+\frac{15}{7}x^{14}(4d+7e)+\frac{7}{2}x^{12}(5d+6e)+\frac{21}{5}x^{10}(6d+5e)+\frac{15}{4}x^{8}(7d+4e)+\frac{5}{2}x^{6}(8d+3e)+\frac{5}{4}x^{4}(9d+2e)+\frac{1}{2}x^{2}(10d+e)+d\log(x)+\frac{ex^{22}}{22}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] ((10*d + e)*x^2)/2 + (5*(9*d + 2*e)*x^4)/4 + (5*(8*d + 3*e)*x^6)/
2 + (15*(7*d + 4*e)*x^8)/4 + (21*(6*d + 5*e)*x^10)/5 + (7*(5*d +
6*e)*x^12)/2 + (15*(4*d + 7*e)*x^14)/7 + (15*(3*d + 8*e)*x^16)/16
 + (5*(2*d + 9*e)*x^18)/18 + ((d + 10*e)*x^20)/20 + (e*x^22)/22 +
 d*Log[x]
```

Maple [A] time = 0.005, size = 132, normalized size = 1.4

$$\frac{ex^{22}}{22}+\frac{dx^{20}}{20}+\frac{ex^{20}}{2}+\frac{5\,dx^{18}}{9}+\frac{5\,x^{18}e}{2}+\frac{45\,dx^{16}}{16}+\frac{15\,x^{16}e}{2}+\frac{60\,dx^{14}}{7}+15\,x^{14}e+\frac{35\,dx^{12}}{2}+21\,x^{12}e+\frac{126\,dx^{10}}{5}+21\,x^{10}e+\frac{105\,dx^{8}}{4}+15\,x^{8}e+20\,dx^{6}+\frac{15\,x^{6}e}{2}+\frac{45\,dx^{4}}{4}+\frac{5\,x^{4}e}{2}+5\,dx^{2}+\frac{ex^{2}}{2}+d\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)*(x^4+2*x^2+1)^5/x,x)
```

```
[Out] 1/22*e*x^22+1/20*d*x^20+1/2*e*x^20+5/9*d*x^18+5/2*x^18*e+45/16*d*
x^16+15/2*x^16*e+60/7*d*x^14+15*x^14*e+35/2*d*x^12+21*x^12*e+126/
5*d*x^10+21*x^10*e+105/4*d*x^8+15*x^8*e+20*d*x^6+15/2*x^6*e+45/4*
d*x^4+5/2*x^4*e+5*d*x^2+1/2*e*x^2+d*ln(x)
```

Maxima [A] time = 0.687503, size = 176, normalized size = 1.89

$$\frac{1}{22}\,ex^{22}+\frac{1}{20}\,(d+10\,e)x^{20}+\frac{5}{18}\,(2\,d+9\,e)x^{18}+\frac{15}{16}\,(3\,d+8\,e)x^{16}+\frac{15}{7}\,(4\,d+7\,e)x^{14}+\frac{7}{2}\,(5\,d+6\,e)x^{12}+\frac{21}{5}\,(6\,d+5\,e)x^{10}+\frac{15}{4}\,(7\,d+4\,e)x^{8}+\frac{5}{2}\,(8\,d+3\,e)x^{6}+\frac{5}{4}\,(9\,d+2\,e)x^{4}+\frac{1}{2}\,(10\,d+e)x^{2}+\frac{1}{2}\,d\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)/x,x, algorithm="maxima")
```

```
[Out] 1/22*e*x^22 + 1/20*(d + 10*e)*x^20 + 5/18*(2*d + 9*e)*x^18 + 15/1
6*(3*d + 8*e)*x^16 + 15/7*(4*d + 7*e)*x^14 + 7/2*(5*d + 6*e)*x^12
 + 21/5*(6*d + 5*e)*x^10 + 15/4*(7*d + 4*e)*x^8 + 5/2*(8*d + 3*e)
*x^6 + 5/4*(9*d + 2*e)*x^4 + 1/2*(10*d + e)*x^2 + 1/2*d*log(x^2)
```

Fricas [A] time = 0.302196, size = 171, normalized size = 1.84

$$\frac{1}{22}\,ex^{22} + \frac{1}{20}\,(d + 10\,e)x^{20} + \frac{5}{18}\,(2\,d + 9\,e)x^{18} + \frac{15}{16}\,(3\,d + 8\,e)x^{16} + \frac{15}{7}\,(4\,d + 7\,e)x^{14} + \frac{7}{2}\,(5\,d + 6\,e)x^{12} + \frac{21}{5}\,(6\,d + 5\,e)x^{10} + \frac{15}{4}\,(7\,d + 4\,e)x^{8} + \frac{5}{2}\,(8\,d + 3\,e)x^{6} + \frac{5}{4}\,(9\,d + 2\,e)x^{4} + \frac{1}{2}\,(10\,d + e)x^{2} + d\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)/x,x, algorithm="fricas")
```

```
[Out] 1/22*e*x^22 + 1/20*(d + 10*e)*x^20 + 5/18*(2*d + 9*e)*x^18 + 15/1
6*(3*d + 8*e)*x^16 + 15/7*(4*d + 7*e)*x^14 + 7/2*(5*d + 6*e)*x^12
 + 21/5*(6*d + 5*e)*x^10 + 15/4*(7*d + 4*e)*x^8 + 5/2*(8*d + 3*e)
*x^6 + 5/4*(9*d + 2*e)*x^4 + 1/2*(10*d + e)*x^2 + d*log(x)
```

Sympy [A] time = 0.927214, size = 131, normalized size = 1.41

$$d\log(x) + \frac{ex^{22}}{22} + x^{20}\left(\frac{d}{20} + \frac{e}{2}\right) + x^{18}\left(\frac{5d}{9} + \frac{5e}{2}\right) + x^{16}\left(\frac{45d}{16} + \frac{15e}{2}\right) + x^{14}\left(\frac{60d}{7} + 15e\right) + x^{12}\left(\frac{35d}{2} + 21e\right) + x^{10}\left(\frac{126d}{5} + 21e\right) + x^{8}\left(\frac{105d}{4} + 15e\right) + x^{6}\left(20d + \frac{15e}{2}\right) + x^{4}\left(\frac{45d}{4} + \frac{5e}{2}\right) + x^{2}\left(5d + \frac{e}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x,x)
```

```
[Out] d*log(x) + e*x**22/22 + x**20*(d/20 + e/2) + x**18*(5*d/9 + 5*e/2
) + x**16*(45*d/16 + 15*e/2) + x**14*(60*d/7 + 15*e) + x**12*(35*
d/2 + 21*e) + x**10*(126*d/5 + 21*e) + x**8*(105*d/4 + 15*e) + x*
*6*(20*d + 15*e/2) + x**4*(45*d/4 + 5*e/2) + x**2*(5*d + e/2)
```

GIAC/XCAS [A] time = 0.272075, size = 196, normalized size = 2.11

$$\frac{1}{22}x^{22}e+\frac{1}{20}dx^{20}+\frac{1}{2}x^{20}e+\frac{5}{9}dx^{18}+\frac{5}{2}x^{18}e+\frac{45}{16}dx^{16}+\frac{15}{2}x^{16}e+\frac{60}{7}dx^{14}+15x^{14}e+\frac{35}{2}dx^{12}+21x^{12}e+\frac{126}{5}dx^{10}+21x^{10}e+\frac{105}{4}dx^{8}+15x^{8}e+20dx^{6}+\frac{15}{2}x^{6}e+\frac{45}{4}dx^{4}+\frac{5}{2}x^{4}e+5dx^{2}+\frac{1}{2}x^{2}e+\frac{1}{2}d\ln\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)/x,x, algorithm="giac")
```

```
[Out] 1/22*x^22*e + 1/20*d*x^20 + 1/2*x^20*e + 5/9*d*x^18 + 5/2*x^18*e
+ 45/16*d*x^16 + 15/2*x^16*e + 60/7*d*x^14 + 15*x^14*e + 35/2*d*x
^12 + 21*x^12*e + 126/5*d*x^10 + 21*x^10*e + 105/4*d*x^8 + 15*x^8
*e + 20*d*x^6 + 15/2*x^6*e + 45/4*d*x^4 + 5/2*x^4*e + 5*d*x^2 + 1
/2*x^2*e + 1/2*d*ln(x^2)
```

3.63 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2}\,dx$

Optimal. Leaf size=141

$$\frac{1}{19}x^{19}(d+10e)+\frac{5}{17}x^{17}(2d+9e)+x^{15}(3d+8e)+\frac{30}{13}x^{13}(4d+7e)+\frac{42}{11}x^{11}(5d+6e)+\frac{14}{3}x^9(6d+5e)+\frac{30}{7}x^7(7d+4e)+3x^5(8d+3e)+\frac{5}{3}x^3(9d+2e)+x(10d+e)-\frac{d}{x}+\frac{ex^{21}}{21}$$

```
[Out] -(d/x) + (10*d + e)*x + (5*(9*d + 2*e)*x^3)/3 + 3*(8*d + 3*e)*x^5
 + (30*(7*d + 4*e)*x^7)/7 + (14*(6*d + 5*e)*x^9)/3 + (42*(5*d + 6
*e)*x^11)/11 + (30*(4*d + 7*e)*x^13)/13 + (3*d + 8*e)*x^15 + (5*(
2*d + 9*e)*x^17)/17 + ((d + 10*e)*x^19)/19 + (e*x^21)/21
```

Rubi [A] time = 0.225111, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{1}{19}x^{19}(d+10e)+\frac{5}{17}x^{17}(2d+9e)+x^{15}(3d+8e)+\frac{30}{13}x^{13}(4d+7e)+\frac{42}{11}x^{11}(5d+6e)+\frac{14}{3}x^9(6d+5e)+\frac{30}{7}x^7(7d+4e)+3x^5(8d+3e)+\frac{5}{3}x^3(9d+2e)+x(10d+e)-\frac{d}{x}+\frac{ex^{21}}{21}$$

Antiderivative was successfully verified.

```
[In]  Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -(d/x) + (10*d + e)*x + (5*(9*d + 2*e)*x^3)/3 + 3*(8*d + 3*e)*x^5
 + (30*(7*d + 4*e)*x^7)/7 + (14*(6*d + 5*e)*x^9)/3 + (42*(5*d + 6
*e)*x^11)/11 + (30*(4*d + 7*e)*x^13)/13 + (3*d + 8*e)*x^15 + (5*(
2*d + 9*e)*x^17)/17 + ((d + 10*e)*x^19)/19 + (e*x^21)/21
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{d}{x}+\frac{ex^{21}}{21}+x^{19}\left(\frac{d}{19}+\frac{10e}{19}\right)+x^{17}\left(\frac{10d}{17}+\frac{45e}{17}\right)+x^{15}(3d+8e)+x^{13}\left(\frac{120d}{13}+\frac{210e}{13}\right)+x^{11}\left(\frac{210d}{11}+\frac{252e}{11}\right)+x^9\left(28d+\frac{70e}{3}\right)+x^7\left(30d+\frac{120e}{7}\right)+x^5(24d+9e)+x^3\left(15d+\frac{10e}{3}\right)+\frac{(10d+e)\int e\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x**2,x)
```

```
[Out] -d/x + e*x**21/21 + x**19*(d/19 + 10*e/19) + x**17*(10*d/17 + 45*
e/17) + x**15*(3*d + 8*e) + x**13*(120*d/13 + 210*e/13) + x**11*(
210*d/11 + 252*e/11) + x**9*(28*d + 70*e/3) + x**7*(30*d + 120*e/
7) + x**5*(24*d + 9*e) + x**3*(15*d + 10*e/3) + (10*d + e)*Integr
al(e, x)/e
```

Mathematica [A] time = 0.0568517, size = 141, normalized size = 1.

$$\frac{1}{19}x^{19}(d+10e)+\frac{5}{17}x^{17}(2d+9e)+x^{15}(3d+8e)+\frac{30}{13}x^{13}(4d+7e)+\frac{42}{11}x^{11}(5d+6e)+\frac{14}{3}x^9(6d+5e)+\frac{30}{7}x^7(7d+4e)+3x^5(8d+3e)+\frac{5}{3}x^3(9d+2e)+x(10d+e)-\frac{d}{x}+\frac{ex^{21}}{21}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -(d/x) + (10*d + e)*x + (5*(9*d + 2*e)*x^3)/3 + 3*(8*d + 3*e)*x^5
 + (30*(7*d + 4*e)*x^7)/7 + (14*(6*d + 5*e)*x^9)/3 + (42*(5*d + 6
*e)*x^11)/11 + (30*(4*d + 7*e)*x^13)/13 + (3*d + 8*e)*x^15 + (5*(
2*d + 9*e)*x^17)/17 + ((d + 10*e)*x^19)/19 + (e*x^21)/21
```

Maple [A] time = 0.007, size = 129, normalized size = 0.9

$$\frac{ex^{21}}{21}+\frac{x^{19}d}{19}+\frac{10\,x^{19}e}{19}+\frac{10\,x^{17}d}{17}+\frac{45\,x^{17}e}{17}+3\,x^{15}d+8\,x^{15}e+\frac{120\,x^{13}d}{13}+\frac{210\,x^{13}e}{13}+\frac{210\,x^{11}d}{11}+\frac{252\,x^{11}e}{11}+28\,x^9d+\frac{70\,x^9e}{3}+30\,x^7d+\frac{120\,x^7e}{7}+24\,dx^5+9\,x^5e+15\,dx^3+\frac{10\,ex^3}{3}+10\,dx+ex-\frac{d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x)
```

```
[Out] 1/21*e*x^21+1/19*x^19*d+10/19*x^19*e+10/17*x^17*d+45/17*x^17*e+3*
x^15*d+8*x^15*e+120/13*x^13*d+210/13*x^13*e+210/11*x^11*d+252/11*
x^11*e+28*x^9*d+70/3*x^9*e+30*x^7*d+120/7*x^7*e+24*d*x^5+9*x^5*e+
15*d*x^3+10/3*e*x^3+10*d*x+e*x-d/x
```

Maxima [A] time = 0.68844, size = 169, normalized size = 1.2

$$\frac{1}{21}\,ex^{21} + \frac{1}{19}\,(d + 10\,e)x^{19} + \frac{5}{17}\,(2\,d + 9\,e)x^{17} + (3\,d + 8\,e)x^{15} + \frac{30}{13}\,(4\,d + 7\,e)x^{13} + \frac{42}{11}\,(5\,d + 6\,e)x^{11} + \frac{14}{3}\,(6\,d + 5\,e)x^{9} + \frac{30}{7}\,(7\,d + 4\,e)x^{7} + 3\,(8\,d + 3\,e)x^{5} + \frac{5}{3}\,(9\,d + 2\,e)x^{3} + (10\,d + e)x - \frac{d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)/x^2,x, algorithm="maxima")
```

```
[Out] 1/21*e*x^21 + 1/19*(d + 10*e)*x^19 + 5/17*(2*d + 9*e)*x^17 + (3*d
 + 8*e)*x^15 + 30/13*(4*d + 7*e)*x^13 + 42/11*(5*d + 6*e)*x^11 +
14/3*(6*d + 5*e)*x^9 + 30/7*(7*d + 4*e)*x^7 + 3*(8*d + 3*e)*x^5 +
 5/3*(9*d + 2*e)*x^3 + (10*d + e)*x - d/x
```

Fricas [A] time = 0.250225, size = 177, normalized size = 1.26

$$46189\,ex^{22} + 51051\,(d + 10\,e)x^{20} + 285285\,(2\,d + 9\,e)x^{18} + 969969\,(3\,d + 8\,e)x^{16} + 2238390\,(4\,d + 7\,e)x^{14} + 3703518\,(5\,d + 6\,e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)/x^2,x, algorithm="fricas")
```

```
[Out] 1/969969*(46189*e*x^22 + 51051*(d + 10*e)*x^20 + 285285*(2*d + 9*
e)*x^18 + 969969*(3*d + 8*e)*x^16 + 2238390*(4*d + 7*e)*x^14 + 37
03518*(5*d + 6*e)*x^12 + 4526522*(6*d + 5*e)*x^10 + 4157010*(7*d
+ 4*e)*x^8 + 2909907*(8*d + 3*e)*x^6 + 1616615*(9*d + 2*e)*x^4 +
969969*(10*d + e)*x^2 - 969969*d)/x
```

Sympy [A] time = 1.68783, size = 124, normalized size = 0.88

$$-\frac{d}{x} + \frac{ex^{21}}{21} + x^{19}\left(\frac{d}{19} + \frac{10e}{19}\right) + x^{17}\left(\frac{10d}{17} + \frac{45e}{17}\right) + x^{15}\left(3d + 8e\right) + x^{13}\left(\frac{120d}{13} + \frac{210e}{13}\right) + x^{11}\left(\frac{210d}{11} + \frac{252e}{11}\right) + x^{9}\left(28d + \frac{70e}{3}\right) + x^{7}\left(30d + \frac{120e}{7}\right) + x^{5}\left(24d + 9e\right) + x^{3}\left(15d + \frac{10e}{3}\right) + x\left(10d + e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x**2,x)
```

```
[Out] -d/x + e*x**21/21 + x**19*(d/19 + 10*e/19) + x**17*(10*d/17 + 45*
e/17) + x**15*(3*d + 8*e) + x**13*(120*d/13 + 210*e/13) + x**11*(
210*d/11 + 252*e/11) + x**9*(28*d + 70*e/3) + x**7*(30*d + 120*e/
7) + x**5*(24*d + 9*e) + x**3*(15*d + 10*e/3) + x*(10*d + e)
```

GIAC/XCAS [A] time = 0.268551, size = 188, normalized size = 1.33

$$\frac{1}{21}\,x^{21}e + \frac{1}{19}\,dx^{19} + \frac{10}{19}\,x^{19}e + \frac{10}{17}\,dx^{17} + \frac{45}{17}\,x^{17}e + 3\,dx^{15} + 8\,x^{15}e + \frac{120}{13}\,dx^{13} + \frac{210}{13}\,x^{13}e + \frac{210}{11}\,dx^{11} + \frac{252}{11}\,x^{11}e + 28\,dx^{9} + \frac{70}{3}\,x^{9}e + 30\,dx^{7} + \frac{120}{7}\,x^{7}e + 24\,dx^{5} + 9\,x^{5}e + 15\,dx^{3} + \frac{10}{3}\,x^{3}e + 10\,dx + xe - \frac{d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)/x^2,x, algorithm="giac")
```

```
[Out] 1/21*x^21*e + 1/19*d*x^19 + 10/19*x^19*e + 10/17*d*x^17 + 45/17*x
^17*e + 3*d*x^15 + 8*x^15*e + 120/13*d*x^13 + 210/13*x^13*e + 210
/11*d*x^11 + 252/11*x^11*e + 28*d*x^9 + 70/3*x^9*e + 30*d*x^7 + 1
20/7*x^7*e + 24*d*x^5 + 9*x^5*e + 15*d*x^3 + 10/3*x^3*e + 10*d*x
+ x*e - d/x
```

3.64 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3}\,dx$

Optimal. Leaf size=147

$$\frac{1}{18}x^{18}(d+10e)+\frac{5}{16}x^{16}(2d+9e)+\frac{15}{14}x^{14}(3d+8e)+\frac{5}{2}x^{12}(4d+7e)+\frac{21}{5}x^{10}(5d+6e)+\frac{21}{4}x^8(6d+5e)+5x^6(7d+4e)+\frac{15}{4}x^4(8d+3e)+\frac{5}{2}x^2(9d+2e)+(10d+e)\log(x)-\frac{d}{2x^2}+\frac{ex^{20}}{20}$$

```
[Out] -d/(2*x^2) + (5*(9*d + 2*e)*x^2)/2 + (15*(8*d + 3*e)*x^4)/4 + 5*(
7*d + 4*e)*x^6 + (21*(6*d + 5*e)*x^8)/4 + (21*(5*d + 6*e)*x^10)/5
 + (5*(4*d + 7*e)*x^12)/2 + (15*(3*d + 8*e)*x^14)/14 + (5*(2*d +
9*e)*x^16)/16 + ((d + 10*e)*x^18)/18 + (e*x^20)/20 + (10*d + e)*L
og[x]
```

Rubi [A] time = 0.3463, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{1}{18}x^{18}(d+10e)+\frac{5}{16}x^{16}(2d+9e)+\frac{15}{14}x^{14}(3d+8e)+\frac{5}{2}x^{12}(4d+7e)+\frac{21}{5}x^{10}(5d+6e)+\frac{21}{4}x^8(6d+5e)+5x^6(7d+4e)+\frac{15}{4}x^4(8d+3e)+\frac{5}{2}x^2(9d+2e)+(10d+e)\log(x)-\frac{d}{2x^2}+\frac{ex^{20}}{20}$$

Antiderivative was successfully verified.

```
[In]  Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -d/(2*x^2) + (5*(9*d + 2*e)*x^2)/2 + (15*(8*d + 3*e)*x^4)/4 + 5*(
7*d + 4*e)*x^6 + (21*(6*d + 5*e)*x^8)/4 + (21*(5*d + 6*e)*x^10)/5
 + (5*(4*d + 7*e)*x^12)/2 + (15*(3*d + 8*e)*x^14)/14 + (5*(2*d +
9*e)*x^16)/16 + ((d + 10*e)*x^18)/18 + (e*x^20)/20 + (10*d + e)*L
og[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{d}{2x^2}+\frac{ex^{20}}{20}+x^{18}\left(\frac{d}{18}+\frac{5e}{9}\right)+x^{16}\left(\frac{5d}{8}+\frac{45e}{16}\right)+x^{14}\left(\frac{45d}{14}+\frac{60e}{7}\right)+x^{12}\left(10d+\frac{35e}{2}\right)+x^{10}\left(21d+\frac{126e}{5}\right)+x^8\left(\frac{63d}{2}+\frac{105e}{4}\right)+x^6\left(35d+20e\right)+x^2\left(\frac{45d}{2}+5e\right)+\left(5d+\frac{e}{2}\right)\log\left(x^2\right)+\left(60d+\frac{45e}{2}\right)\int^{x^2}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x**3,x)
```

```
[Out] -d/(2*x**2) + e*x**20/20 + x**18*(d/18 + 5*e/9) + x**16*(5*d/8 +
45*e/16) + x**14*(45*d/14 + 60*e/7) + x**12*(10*d + 35*e/2) + x**
10*(21*d + 126*e/5) + x**8*(63*d/2 + 105*e/4) + x**6*(35*d + 20*e
) + x**2*(45*d/2 + 5*e) + (5*d + e/2)*log(x**2) + (60*d + 45*e/2)
*Integral(x, (x, x**2))
```

Mathematica [A] time = 0.0690923, size = 147, normalized size = 1.

$$\frac{1}{18}x^{18}(d+10e)+\frac{5}{16}x^{16}(2d+9e)+\frac{15}{14}x^{14}(3d+8e)+\frac{5}{2}x^{12}(4d+7e)+\frac{21}{5}x^{10}(5d+6e)+\frac{21}{4}x^{8}(6d+5e)+5x^{6}(7d+4e)+\frac{15}{4}x^{4}(8d+3e)+\frac{5}{2}x^{2}(9d+2e)+(10d+e)\log(x)-\frac{d}{2x^2}+\frac{ex^{20}}{20}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -d/(2*x^2) + (5*(9*d + 2*e)*x^2)/2 + (15*(8*d + 3*e)*x^4)/4 + 5*(
7*d + 4*e)*x^6 + (21*(6*d + 5*e)*x^8)/4 + (21*(5*d + 6*e)*x^10)/5
 + (5*(4*d + 7*e)*x^12)/2 + (15*(3*d + 8*e)*x^14)/14 + (5*(2*d +
9*e)*x^16)/16 + ((d + 10*e)*x^18)/18 + (e*x^20)/20 + (10*d + e)*L
og[x]
```

Maple [A] time = 0.01, size = 131, normalized size = 0.9

$$\frac{ex^{20}}{20}+\frac{dx^{18}}{18}+\frac{5\,x^{18}e}{9}+\frac{5\,dx^{16}}{8}+\frac{45\,x^{16}e}{16}+\frac{45\,dx^{14}}{14}+\frac{60\,x^{14}e}{7}+10\,dx^{12}+\frac{35\,x^{12}e}{2}+21\,dx^{10}+\frac{126\,x^{10}e}{5}+\frac{63\,dx^{8}}{2}+\frac{105\,x^{8}e}{4}+35\,dx^{6}+20\,x^{6}e+30\,dx^{4}+\frac{45\,x^{4}e}{4}+\frac{45\,dx^{2}}{2}+5\,ex^{2}+10\,d\ln(x)+\ln(x)\,e-\frac{d}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x)
```

```
[Out] 1/20*e*x^20+1/18*d*x^18+5/9*x^18*e+5/8*d*x^16+45/16*x^16*e+45/14*
d*x^14+60/7*x^14*e+10*d*x^12+35/2*x^12*e+21*d*x^10+126/5*x^10*e+6
3/2*d*x^8+105/4*x^8*e+35*d*x^6+20*x^6*e+30*d*x^4+45/4*x^4*e+45/2*
d*x^2+5*e*x^2+10*d*ln(x)+ln(x)*e-1/2*d/x^2
```

Maxima [A] time = 0.709218, size = 176, normalized size = 1.2

$$\frac{1}{20}ex^{20}+\frac{1}{18}(d+10e)x^{18}+\frac{5}{16}(2d+9e)x^{16}+\frac{15}{14}(3d+8e)x^{14}+\frac{5}{2}(4d+7e)x^{12}+\frac{21}{5}(5d+6e)x^{10}+\frac{21}{4}(6d+5e)x^{8}+5(7d+4e)x^{6}+\frac{15}{4}(8d+3e)x^{4}+\frac{5}{2}(9d+2e)x^{2}+\frac{1}{2}(10d+e)\log\left(x^{2}\right)-\frac{d}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)/x^3,x, algorithm="maxima")
```

```
[Out] 1/20*e*x^20 + 1/18*(d + 10*e)*x^18 + 5/16*(2*d + 9*e)*x^16 + 15/1
4*(3*d + 8*e)*x^14 + 5/2*(4*d + 7*e)*x^12 + 21/5*(5*d + 6*e)*x^10
 + 21/4*(6*d + 5*e)*x^8 + 5*(7*d + 4*e)*x^6 + 15/4*(8*d + 3*e)*x^
4 + 5/2*(9*d + 2*e)*x^2 + 1/2*(10*d + e)*log(x^2) - 1/2*d/x^2
```

Fricas [A] time = 0.253607, size = 180, normalized size = 1.22

$$\frac{252\,ex^{22}+280\,(d+10\,e)x^{20}+1575\,(2\,d+9\,e)x^{18}+5400\,(3\,d+8\,e)x^{16}+12600\,(4\,d+7\,e)x^{14}+21168\,(5\,d+6\,e)x^{12}+26460\,(6}{5040\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)/x^3,x, algorithm="fricas")
```

```
[Out] 1/5040*(252*e*x^22 + 280*(d + 10*e)*x^20 + 1575*(2*d + 9*e)*x^18
+ 5400*(3*d + 8*e)*x^16 + 12600*(4*d + 7*e)*x^14 + 21168*(5*d + 6
*e)*x^12 + 26460*(6*d + 5*e)*x^10 + 25200*(7*d + 4*e)*x^8 + 18900
*(8*d + 3*e)*x^6 + 12600*(9*d + 2*e)*x^4 + 5040*(10*d + e)*x^2*lo
g(x) - 2520*d)/x^2
```

Sympy [A] time = 1.90219, size = 131, normalized size = 0.89

$$-\frac{d}{2x^{2}}+\frac{ex^{20}}{20}+x^{18}\left(\frac{d}{18}+\frac{5e}{9}\right)+x^{16}\left(\frac{5d}{8}+\frac{45e}{16}\right)+x^{14}\left(\frac{45d}{14}+\frac{60e}{7}\right)+x^{12}\left(10d+\frac{35e}{2}\right)+x^{10}\left(21d+\frac{126e}{5}\right)+x^{8}\left(\frac{63d}{2}+\frac{105e}{4}\right)+x^{6}(35d+20e)+x^{4}\left(30d+\frac{45e}{4}\right)+x^{2}\left(\frac{45d}{2}+5e\right)+(10d+e)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x**3,x)
```

```
[Out] -d/(2*x**2) + e*x**20/20 + x**18*(d/18 + 5*e/9) + x**16*(5*d/8 +
45*e/16) + x**14*(45*d/14 + 60*e/7) + x**12*(10*d + 35*e/2) + x**
10*(21*d + 126*e/5) + x**8*(63*d/2 + 105*e/4) + x**6*(35*d + 20*e
) + x**4*(30*d + 45*e/4) + x**2*(45*d/2 + 5*e) + (10*d + e)*log(x
)
```

GIAC/XCAS [A] time = 0.274031, size = 211, normalized size = 1.44

$$\frac{1}{20}x^{20}e+\frac{1}{18}dx^{18}+\frac{5}{9}x^{18}e+\frac{5}{8}dx^{16}+\frac{45}{16}x^{16}e+\frac{45}{14}dx^{14}+\frac{60}{7}x^{14}e+10\,dx^{12}+\frac{35}{2}x^{12}e+21\,dx^{10}+\frac{126}{5}x^{10}e+\frac{63}{2}dx^{8}+\frac{105}{4}x^{8}e+35\,dx^{6}+20\,x^{6}e+30\,dx^{4}+\frac{45}{4}x^{4}e+\frac{45}{2}dx^{2}+5\,x^{2}e+\frac{1}{2}(10\,d+e)\ln\left(x^{2}\right)-\frac{10\,dx^{2}+x^{2}e+d}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(e*x^2 + d)/x^3,x, algorithm="giac")
```

```
[Out] 1/20*x^20*e + 1/18*d*x^18 + 5/9*x^18*e + 5/8*d*x^16 + 45/16*x^16*
e + 45/14*d*x^14 + 60/7*x^14*e + 10*d*x^12 + 35/2*x^12*e + 21*d*x
^10 + 126/5*x^10*e + 63/2*d*x^8 + 105/4*x^8*e + 35*d*x^6 + 20*x^6
*e + 30*d*x^4 + 45/4*x^4*e + 45/2*d*x^2 + 5*x^2*e + 1/2*(10*d + e
)*ln(x^2) - 1/2*(10*d*x^2 + x^2*e + d)/x^2
```

3.65 $\int (fx)^m \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=203

$$\frac{(fx)^{m+23}}{f^{23}(m+23)}+\frac{11(fx)^{m+21}}{f^{21}(m+21)}+\frac{55(fx)^{m+19}}{f^{19}(m+19)}+\frac{165(fx)^{m+17}}{f^{17}(m+17)}+\frac{330(fx)^{m+15}}{f^{15}(m+15)}+\frac{462(fx)^{m+13}}{f^{13}(m+13)}+\frac{462(fx)^{m+11}}{f^{11}(m+11)}+\frac{330(fx)^{m+9}}{f^{9}(m+9)}+\frac{165(fx)^{m+7}}{f^{7}(m+7)}+\frac{55(fx)^{m+5}}{f^{5}(m+5)}+\frac{11(fx)^{m+3}}{f^{3}(m+3)}+\frac{(fx)^{m+1}}{f(m+1)}$$

```
[Out] (f*x)^(1 + m)/(f*(1 + m)) + (11*(f*x)^(3 + m))/(f^3*(3 + m)) + (5
5*(f*x)^(5 + m))/(f^5*(5 + m)) + (165*(f*x)^(7 + m))/(f^7*(7 + m)
) + (330*(f*x)^(9 + m))/(f^9*(9 + m)) + (462*(f*x)^(11 + m))/(f^1
1*(11 + m)) + (462*(f*x)^(13 + m))/(f^13*(13 + m)) + (330*(f*x)^(
15 + m))/(f^15*(15 + m)) + (165*(f*x)^(17 + m))/(f^17*(17 + m)) +
 (55*(f*x)^(19 + m))/(f^19*(19 + m)) + (11*(f*x)^(21 + m))/(f^21*
(21 + m)) + (f*x)^(23 + m)/(f^23*(23 + m))
```

Rubi [A] time = 0.169792, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{(fx)^{m+23}}{f^{23}(m+23)}+\frac{11(fx)^{m+21}}{f^{21}(m+21)}+\frac{55(fx)^{m+19}}{f^{19}(m+19)}+\frac{165(fx)^{m+17}}{f^{17}(m+17)}+\frac{330(fx)^{m+15}}{f^{15}(m+15)}+\frac{462(fx)^{m+13}}{f^{13}(m+13)}+\frac{462(fx)^{m+11}}{f^{11}(m+11)}+\frac{330(fx)^{m+9}}{f^{9}(m+9)}+\frac{165(fx)^{m+7}}{f^{7}(m+7)}+\frac{55(fx)^{m+5}}{f^{5}(m+5)}+\frac{11(fx)^{m+3}}{f^{3}(m+3)}+\frac{(fx)^{m+1}}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(f*x)^m*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (f*x)^(1 + m)/(f*(1 + m)) + (11*(f*x)^(3 + m))/(f^3*(3 + m)) + (5
5*(f*x)^(5 + m))/(f^5*(5 + m)) + (165*(f*x)^(7 + m))/(f^7*(7 + m)
) + (330*(f*x)^(9 + m))/(f^9*(9 + m)) + (462*(f*x)^(11 + m))/(f^1
1*(11 + m)) + (462*(f*x)^(13 + m))/(f^13*(13 + m)) + (330*(f*x)^(
15 + m))/(f^15*(15 + m)) + (165*(f*x)^(17 + m))/(f^17*(17 + m)) +
 (55*(f*x)^(19 + m))/(f^19*(19 + m)) + (11*(f*x)^(21 + m))/(f^21*
(21 + m)) + (f*x)^(23 + m)/(f^23*(23 + m))
```

Rubi in Sympy [A] time = 35.597, size = 177, normalized size = 0.87

$$\frac{(fx)^{m+1}}{f\,(m+1)}+\frac{11\,(fx)^{m+3}}{f^{3}\,(m+3)}+\frac{55\,(fx)^{m+5}}{f^{5}\,(m+5)}+\frac{165\,(fx)^{m+7}}{f^{7}\,(m+7)}+\frac{330\,(fx)^{m+9}}{f^{9}\,(m+9)}+\frac{462\,(fx)^{m+11}}{f^{11}\,(m+11)}+\frac{462\,(fx)^{m+13}}{f^{13}\,(m+13)}+\frac{330\,(fx)^{m+15}}{f^{15}\,(m+15)}+\frac{165\,(fx)^{m+17}}{f^{17}\,(m+17)}+\frac{55\,(fx)^{m+19}}{f^{19}\,(m+19)}+\frac{11\,(fx)^{m+21}}{f^{21}\,(m+21)}+\frac{(fx)^{m+23}}{f^{23}\,(m+23)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] (f*x)**(m + 1)/(f*(m + 1)) + 11*(f*x)**(m + 3)/(f**3*(m + 3)) + 5
5*(f*x)**(m + 5)/(f**5*(m + 5)) + 165*(f*x)**(m + 7)/(f**7*(m + 7
)) + 330*(f*x)**(m + 9)/(f**9*(m + 9)) + 462*(f*x)**(m + 11)/(f**
11*(m + 11)) + 462*(f*x)**(m + 13)/(f**13*(m + 13)) + 330*(f*x)**
(m + 15)/(f**15*(m + 15)) + 165*(f*x)**(m + 17)/(f**17*(m + 17))
+ 55*(f*x)**(m + 19)/(f**19*(m + 19)) + 11*(f*x)**(m + 21)/(f**21
*(m + 21)) + (f*x)**(m + 23)/(f**23*(m + 23))
```

Mathematica [A] time = 0.0569759, size = 123, normalized size = 0.61

$$\left(\frac{x^{23}}{m+23}+\frac{11x^{21}}{m+21}+\frac{55x^{19}}{m+19}+\frac{165x^{17}}{m+17}+\frac{330x^{15}}{m+15}+\frac{462x^{13}}{m+13}+\frac{462x^{11}}{m+11}+\frac{330x^9}{m+9}+\frac{165x^7}{m+7}+\frac{55x^5}{m+5}+\frac{11x^3}{m+3}+\frac{x}{m+1}\right)(fx)^m$$

Antiderivative was successfully verified.

```
[In]  Integrate[(f*x)^m*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (f*x)^m*(x/(1 + m) + (11*x^3)/(3 + m) + (55*x^5)/(5 + m) + (165*x
^7)/(7 + m) + (330*x^9)/(9 + m) + (462*x^11)/(11 + m) + (462*x^13
)/(13 + m) + (330*x^15)/(15 + m) + (165*x^17)/(17 + m) + (55*x^19
)/(19 + m) + (11*x^21)/(21 + m) + x^23/(23 + m))
```

Maple [B] time = 0.015, size = 1121, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] (f*x)^m*(m^11*x^22+121*m^10*x^22+11*m^11*x^20+6435*m^9*x^22+1353*
m^10*x^20+197835*m^8*x^22+55*m^11*x^18+72985*m^9*x^20+3889578*m^7
*x^22+6875*m^10*x^18+2271555*m^8*x^20+51069018*m^6*x^22+165*m^11*
x^16+376365*m^9*x^18+45134958*m^7*x^20+453714470*m^5*x^22+20955*m
^10*x^16+11870265*m^8*x^18+597988314*m^6*x^20+2702025590*m^4*x^22
+330*m^11*x^14+1164735*m^9*x^16+238653030*m^7*x^18+5353566130*m^5
*x^20+10431670821*m^3*x^22+42570*m^10*x^14+37263105*m^8*x^16+3194
```

```
704590*m^6*x^18+32087153670*m^4*x^20+24372200061*m^2*x^22+462*m^1
1*x^12+2403390*m^9*x^14+759091410*m^7*x^16+28857216410*m^5*x^18+1
24530626231*m^3*x^20+29985521895*m*x^22+60522*m^10*x^12+78076350*
m^8*x^14+10282782510*m^6*x^16+174273100210*m^4*x^18+292163767533*
m^2*x^20+13749310575*x^22+462*m^11*x^10+3471930*m^9*x^12+16139831
40*m^7*x^14+93862508190*m^5*x^16+680615362515*m^3*x^18+3605682380
85*m*x^20+61446*m^10*x^10+114642990*m^8*x^12+22164925860*m^6*x^14
+572017996770*m^4*x^16+1604842704135*m^2*x^18+165646455975*x^20+3
30*m^11*x^8+3582810*m^9*x^10+2408820876*m^7*x^12+204865733820*m^5
*x^14+2251106854425*m^3*x^16+1988025402825*m*x^18+44550*m^10*x^8+
120367170*m^8*x^10+33609870756*m^6*x^12+1262375264700*m^4*x^14+53
40787250535*m^2*x^16+915414625125*x^18+165*m^11*x^6+2640990*m^9*x
^8+2575140876*m^7*x^10+315347150580*m^5*x^12+5015196628530*m^3*x^
14+6646727085075*m*x^16+22605*m^10*x^6+90358290*m^8*x^8+365979925
08*m^6*x^10+1969992823260*m^4*x^12+11991258123570*m^2*x^14+306933
1390125*x^16+55*m^11*x^4+1362735*m^9*x^6+1971903780*m^7*x^8+34969
7552820*m^5*x^10+7921249136262*m^3*x^12+15011348834790*m*x^14+764
5*m^10*x^4+47524455*m^8*x^6+28627538940*m^6*x^8+2222832699780*m^4
*x^10+19130651800722*m^2*x^12+6957151150950*x^14+11*m^11*x^2+4687
65*m^9*x^4+1059893010*m^7*x^6+279691771260*m^5*x^8+9079996141062*
m^3*x^10+24133835554290*m*x^12+1551*m^10*x^2+16677375*m^8*x^4+157
68085410*m^6*x^6+1818135330660*m^4*x^8+22226933020446*m^2*x^10+11
238474936150*x^12+m^11+96745*m^9*x^2+380801190*m^7*x^4+1582932129
90*m^5*x^6+7587607623090*m^3*x^8+28336045738770*m*x^10+143*m^10+3
514005*m^8*x^2+5825106210*m^6*x^4+1059628145070*m^4*x^6+189307389
43710*m^2*x^8+13281834015450*x^10+9075*m^9+82295598*m^7*x^2+60431
072570*m^5*x^4+4558015784025*m^3*x^6+24503570194950*m*x^8+336765*
m^8+1298935638*m^6*x^2+420404849150*m^4*x^4+11703493287585*m^2*x^
6+11595251918250*x^8+8103018*m^7+14014513810*m^5*x^2+188978002075
5*m^3*x^4+15515657331075*m*x^6+132426294*m^6+102468500970*m^4*x^2
+5087634488145*m^2*x^4+7454090518875*x^6+1495875590*m^5+490955350
391*m^3*x^2+7041864340665*m*x^4+11641582810*m^4+1434440867211*m^2
*x^2+3478575575475*x^4+60936676581*m^3+2192684754645*m*x^2+203363
952363*m^2+1159525191825*x^2+387182170935*m+316234143225)*x/(1+m)
/(3+m)/(5+m)/(7+m)/(9+m)/(11+m)/(13+m)/(15+m)/(17+m)/(19+m)/(21+m
)/(23+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)*(f*x)^m,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.293353, size = 1025, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)*(f*x)^m,x, algorithm="fricas")
```

```
[Out] ((m^11 + 121*m^10 + 6435*m^9 + 197835*m^8 + 3889578*m^7 + 5106901
8*m^6 + 453714470*m^5 + 2702025590*m^4 + 10431670821*m^3 + 243722
00061*m^2 + 29985521895*m + 13749310575)*x^23 + 11*(m^11 + 123*m^
10 + 6635*m^9 + 206505*m^8 + 4103178*m^7 + 54362574*m^6 + 4866878
30*m^5 + 2917013970*m^4 + 11320966021*m^3 + 26560342503*m^2 + 327
78930735*m + 15058768725)*x^21 + 55*(m^11 + 125*m^10 + 6843*m^9 +
 215823*m^8 + 4339146*m^7 + 58085538*m^6 + 524676662*m^5 + 316860
1822*m^4 + 12374824773*m^3 + 29178958257*m^2 + 36145916415*m + 16
643902275)*x^19 + 165*(m^11 + 127*m^10 + 7059*m^9 + 225837*m^8 +
4600554*m^7 + 62319894*m^6 + 568863686*m^5 + 3466775738*m^4 + 136
43071845*m^3 + 32368407579*m^2 + 40283194455*m + 18602008425)*x^1
7 + 330*(m^11 + 129*m^10 + 7283*m^9 + 236595*m^8 + 4890858*m^7 +
67166442*m^6 + 620805254*m^5 + 3825379590*m^4 + 15197565541*m^3 +
 36337145829*m^2 + 45488935863*m + 21082276215)*x^15 + 462*(m^11
+ 131*m^10 + 7515*m^9 + 248145*m^8 + 5213898*m^7 + 72748638*m^6 +
 682569590*m^5 + 4264053730*m^4 + 17145560901*m^3 + 41408337231*m
^2 + 52237739295*m + 24325703325)*x^13 + 462*(m^11 + 133*m^10 + 7
755*m^9 + 260535*m^8 + 5573898*m^7 + 79216434*m^6 + 756921110*m^5
 + 4811326190*m^4 + 19653671301*m^3 + 48110244633*m^2 + 613334323
35*m + 28748558475)*x^11 + 330*(m^11 + 135*m^10 + 8003*m^9 + 2738
13*m^8 + 5975466*m^7 + 86750118*m^6 + 847550822*m^5 + 5509501002*
m^4 + 22992750373*m^3 + 57365875587*m^2 + 74253243015*m + 3513712
7025)*x^9 + 165*(m^11 + 137*m^10 + 8259*m^9 + 288027*m^8 + 642359
4*m^7 + 95564154*m^6 + 959352806*m^5 + 6421988758*m^4 + 276243380
85*m^3 + 70930262349*m^2 + 94034286855*m + 45176306175)*x^7 + 55*
(m^11 + 139*m^10 + 8523*m^9 + 303225*m^8 + 6923658*m^7 + 10591102
2*m^6 + 1098746774*m^5 + 7643724530*m^4 + 34359636741*m^3 + 92502
445239*m^2 + 128033897103*m + 63246828645)*x^5 + 11*(m^11 + 141*m
^10 + 8795*m^9 + 319455*m^8 + 7481418*m^7 + 118085058*m^6 + 12740
46710*m^5 + 9315318270*m^4 + 44632304581*m^3 + 130403715201*m^2 +
 199334977695*m + 105411381075)*x^3 + (m^11 + 143*m^10 + 9075*m^9
 + 336765*m^8 + 8103018*m^7 + 132426294*m^6 + 1495875590*m^5 + 11
641582810*m^4 + 60936676581*m^3 + 203363952363*m^2 + 387182170935
*m + 316234143225)*x)*(f*x)^m/(m^12 + 144*m^11 + 9218*m^10 + 3458
40*m^9 + 8439783*m^8 + 140529312*m^7 + 1628301884*m^6 + 131374584
00*m^5 + 72578259391*m^4 + 264300628944*m^3 + 590546123298*m^2 +
703416314160*m + 316234143225)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.296308, size = 1, normalized size = 0.

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)*(f*x)^m,x, algorithm="giac")
```

```
[Out] Done
```

3.66 $\int x^5 \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=34

$$\frac{1}{28}\left(x^2+1\right)^{14}-\frac{1}{13}\left(x^2+1\right)^{13}+\frac{1}{24}\left(x^2+1\right)^{12}$$

```
[Out] (1 + x^2)^12/24 - (1 + x^2)^13/13 + (1 + x^2)^14/28
```

Rubi [A] time = 0.0938088, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{1}{28}\left(x^2+1\right)^{14}-\frac{1}{13}\left(x^2+1\right)^{13}+\frac{1}{24}\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (1 + x^2)^12/24 - (1 + x^2)^13/13 + (1 + x^2)^14/28
```

Rubi in Sympy [A] time = 10.4658, size = 24, normalized size = 0.71

$$\frac{\left(x^2+1\right)^{14}}{28}-\frac{\left(x^2+1\right)^{13}}{13}+\frac{\left(x^2+1\right)^{12}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**5*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] (x**2 + 1)**14/28 - (x**2 + 1)**13/13 + (x**2 + 1)**12/24
```

Mathematica [B] time = 0.00350701, size = 85, normalized size = 2.5

$$\frac{x^{28}}{28}+\frac{11x^{26}}{26}+\frac{55x^{24}}{24}+\frac{15x^{22}}{2}+\frac{33x^{20}}{2}+\frac{77x^{18}}{3}+\frac{231x^{16}}{8}+\frac{165x^{14}}{7}+\frac{55x^{12}}{4}+\frac{11x^{10}}{2}+\frac{11x^8}{8}+\frac{x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^6/6 + (11*x^8)/8 + (11*x^10)/2 + (55*x^12)/4 + (165*x^14)/7 + (
231*x^16)/8 + (77*x^18)/3 + (33*x^20)/2 + (15*x^22)/2 + (55*x^24)
/24 + (11*x^26)/26 + x^28/28
```

Maple [B] time = 0.002, size = 62, normalized size = 1.8

$$\frac{x^{28}}{28}+\frac{11\,x^{26}}{26}+\frac{55\,x^{24}}{24}+\frac{15\,x^{22}}{2}+\frac{33\,x^{20}}{2}+\frac{77\,x^{18}}{3}+\frac{231\,x^{16}}{8}+\frac{165\,x^{14}}{7}+\frac{55\,x^{12}}{4}+\frac{11\,x^{10}}{2}+\frac{11\,x^{8}}{8}+\frac{x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/28*x^28+11/26*x^26+55/24*x^24+15/2*x^22+33/2*x^20+77/3*x^18+231
/8*x^16+165/7*x^14+55/4*x^12+11/2*x^10+11/8*x^8+1/6*x^6
```

Maxima [A] time = 0.699125, size = 82, normalized size = 2.41

$$\frac{1}{28}\,x^{28}+\frac{11}{26}\,x^{26}+\frac{55}{24}\,x^{24}+\frac{15}{2}\,x^{22}+\frac{33}{2}\,x^{20}+\frac{77}{3}\,x^{18}+\frac{231}{8}\,x^{16}+\frac{165}{7}\,x^{14}+\frac{55}{4}\,x^{12}+\frac{11}{2}\,x^{10}+\frac{11}{8}\,x^{8}+\frac{1}{6}\,x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)*x^5,x, algorithm="maxima")
```

```
[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/
3*x^18 + 231/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x
^8 + 1/6*x^6
```

Fricas [A] time = 0.258133, size = 1, normalized size = 0.03

$$\frac{1}{28}x^{28}+\frac{11}{26}x^{26}+\frac{55}{24}x^{24}+\frac{15}{2}x^{22}+\frac{33}{2}x^{20}+\frac{77}{3}x^{18}+\frac{231}{8}x^{16}+\frac{165}{7}x^{14}+\frac{55}{4}x^{12}+\frac{11}{2}x^{10}+\frac{11}{8}x^{8}+\frac{1}{6}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)*x^5,x, algorithm="fricas")
```

```
[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/
3*x^18 + 231/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x
```

```
^8 + 1/6*x^6
```

Sympy [A] time = 0.108114, size = 76, normalized size = 2.24

$$\frac{x^{28}}{28} + \frac{11x^{26}}{26} + \frac{55x^{24}}{24} + \frac{15x^{22}}{2} + \frac{33x^{20}}{2} + \frac{77x^{18}}{3} + \frac{231x^{16}}{8} + \frac{165x^{14}}{7} + \frac{55x^{12}}{4} + \frac{11x^{10}}{2} + \frac{11x^{8}}{8} + \frac{x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**28/28 + 11*x**26/26 + 55*x**24/24 + 15*x**22/2 + 33*x**20/2 +
77*x**18/3 + 231*x**16/8 + 165*x**14/7 + 55*x**12/4 + 11*x**10/2
+ 11*x**8/8 + x**6/6
```

GIAC/XCAS [A] time = 0.268875, size = 82, normalized size = 2.41

$$\frac{1}{28}x^{28} + \frac{11}{26}x^{26} + \frac{55}{24}x^{24} + \frac{15}{2}x^{22} + \frac{33}{2}x^{20} + \frac{77}{3}x^{18} + \frac{231}{8}x^{16} + \frac{165}{7}x^{14} + \frac{55}{4}x^{12} + \frac{11}{2}x^{10} + \frac{11}{8}x^{8} + \frac{1}{6}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)*x^5,x, algorithm="giac")
```

```
[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/
3*x^18 + 231/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x
^8 + 1/6*x^6
```

3.67 $\int x^4 (1+x^2) (1+2x^2+x^4)^5 \, dx$

Optimal. Leaf size=83

$$\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

```
[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*
x^15)/5 + (462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)
/23 + (11*x^25)/25 + x^27/27
```

Rubi [A] time = 0.0647895, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*
x^15)/5 + (462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)
/23 + (11*x^25)/25 + x^27/27
```

Rubi in Sympy [A] time = 11.0443, size = 75, normalized size = 0.9

$$\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**27/27 + 11*x**25/25 + 55*x**23/23 + 55*x**21/7 + 330*x**19/19
+ 462*x**17/17 + 154*x**15/5 + 330*x**13/13 + 15*x**11 + 55*x**9/
9 + 11*x**7/7 + x**5/5
```

Mathematica [A] time = 0.00256754, size = 83, normalized size = 1.

$$\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*
x^15)/5 + (462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)
/23 + (11*x^25)/25 + x^27/27
```

Maple [A] time = 0.002, size = 62, normalized size = 0.8

$$\frac{x^5}{5}+\frac{11\,x^7}{7}+\frac{55\,x^9}{9}+15\,x^{11}+\frac{330\,x^{13}}{13}+\frac{154\,x^{15}}{5}+\frac{462\,x^{17}}{17}+\frac{330\,x^{19}}{19}+\frac{55\,x^{21}}{7}+\frac{55\,x^{23}}{23}+\frac{11\,x^{25}}{25}+\frac{x^{27}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/5*x^5+11/7*x^7+55/9*x^9+15*x^11+330/13*x^13+154/5*x^15+462/17*x
^17+330/19*x^19+55/7*x^21+55/23*x^23+11/25*x^25+1/27*x^27
```

Maxima [A] time = 0.702282, size = 82, normalized size = 0.99

$$\frac{1}{27}\,x^{27}+\frac{11}{25}\,x^{25}+\frac{55}{23}\,x^{23}+\frac{55}{7}\,x^{21}+\frac{330}{19}\,x^{19}+\frac{462}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{330}{13}\,x^{13}+15\,x^{11}+\frac{55}{9}\,x^9+\frac{11}{7}\,x^7+\frac{1}{5}\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)*x^4,x, algorithm="maxima")
```

```
[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 4
62/17*x^17 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7
*x^7 + 1/5*x^5
```

Fricas [A] time = 0.250039, size = 1, normalized size = 0.01

$$\frac{1}{27}x^{27}+\frac{11}{25}x^{25}+\frac{55}{23}x^{23}+\frac{55}{7}x^{21}+\frac{330}{19}x^{19}+\frac{462}{17}x^{17}+\frac{154}{5}x^{15}+\frac{330}{13}x^{13}+15x^{11}+\frac{55}{9}x^9+\frac{11}{7}x^7+\frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)*x^4,x, algorithm="fricas")
```

```
[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 4
62/17*x^17 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7
*x^7 + 1/5*x^5
```

Sympy [A] time = 0.099508, size = 75, normalized size = 0.9

$$\frac{x^{27}}{27} + \frac{11x^{25}}{25} + \frac{55x^{23}}{23} + \frac{55x^{21}}{7} + \frac{330x^{19}}{19} + \frac{462x^{17}}{17} + \frac{154x^{15}}{5} + \frac{330x^{13}}{13} + 15x^{11} + \frac{55x^{9}}{9} + \frac{11x^{7}}{7} + \frac{x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**27/27 + 11*x**25/25 + 55*x**23/23 + 55*x**21/7 + 330*x**19/19
+ 462*x**17/17 + 154*x**15/5 + 330*x**13/13 + 15*x**11 + 55*x**9/
9 + 11*x**7/7 + x**5/5
```

GIAC/XCAS [A] time = 0.271786, size = 82, normalized size = 0.99

$$\frac{1}{27}\,x^{27} + \frac{11}{25}\,x^{25} + \frac{55}{23}\,x^{23} + \frac{55}{7}\,x^{21} + \frac{330}{19}\,x^{19} + \frac{462}{17}\,x^{17} + \frac{154}{5}\,x^{15} + \frac{330}{13}\,x^{13} + 15\,x^{11} + \frac{55}{9}\,x^{9} + \frac{11}{7}\,x^{7} + \frac{1}{5}\,x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)*x^4,x, algorithm="giac")
```

```
[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 4
62/17*x^17 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7
*x^7 + 1/5*x^5
```

3.68 $\int x^3 \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=23

$$\frac{1}{26}\left(x^2+1\right)^{13}-\frac{1}{24}\left(x^2+1\right)^{12}$$

```
[Out] -(1 + x^2)^12/24 + (1 + x^2)^13/26
```

Rubi [A] time = 0.0528938, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{1}{26}\left(x^2+1\right)^{13}-\frac{1}{24}\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] -(1 + x^2)^12/24 + (1 + x^2)^13/26
```

Rubi in Sympy [A] time = 9.5844, size = 15, normalized size = 0.65

$$\frac{\left(x^2+1\right)^{13}}{26}-\frac{\left(x^2+1\right)^{12}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(x**2+1)*(x**4+2*x**2+1)**5,x)

[Out] (x**2 + 1)**13/26 - (x**2 + 1)**12/24
```

Mathematica [B] time = 0.00263026, size = 83, normalized size = 3.61

$$\frac{x^{26}}{26}+\frac{11x^{24}}{24}+\frac{5x^{22}}{2}+\frac{33x^{20}}{4}+\frac{55x^{18}}{3}+\frac{231x^{16}}{8}+33x^{14}+\frac{55x^{12}}{2}+\frac{33x^{10}}{2}+\frac{55x^8}{8}+\frac{11x^6}{6}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^4/4 + (11*x^6)/6 + (55*x^8)/8 + (33*x^10)/2 + (55*x^12)/2 + 33*
x^14 + (231*x^16)/8 + (55*x^18)/3 + (33*x^20)/4 + (5*x^22)/2 + (1
1*x^24)/24 + x^26/26
```

Maple [B] time = 0.002, size = 62, normalized size = 2.7

$$\frac{x^{26}}{26}+\frac{11\,x^{24}}{24}+\frac{5\,x^{22}}{2}+\frac{33\,x^{20}}{4}+\frac{55\,x^{18}}{3}+\frac{231\,x^{16}}{8}+33\,x^{14}+\frac{55\,x^{12}}{2}+\frac{33\,x^{10}}{2}+\frac{55\,x^{8}}{8}+\frac{11\,x^{6}}{6}+\frac{x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/26*x^26+11/24*x^24+5/2*x^22+33/4*x^20+55/3*x^18+231/8*x^16+33*x
^14+55/2*x^12+33/2*x^10+55/8*x^8+11/6*x^6+1/4*x^4
```

Maxima [A] time = 0.700554, size = 82, normalized size = 3.57

$$\frac{1}{26}\,x^{26}+\frac{11}{24}\,x^{24}+\frac{5}{2}\,x^{22}+\frac{33}{4}\,x^{20}+\frac{55}{3}\,x^{18}+\frac{231}{8}\,x^{16}+33\,x^{14}+\frac{55}{2}\,x^{12}+\frac{33}{2}\,x^{10}+\frac{55}{8}\,x^{8}+\frac{11}{6}\,x^{6}+\frac{1}{4}\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)*x^3,x, algorithm="maxima")
```

```
[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8
*x^16 + 33*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1
/4*x^4
```

Fricas [A] time = 0.227383, size = 1, normalized size = 0.04

$$\frac{1}{26}x^{26}+\frac{11}{24}x^{24}+\frac{5}{2}x^{22}+\frac{33}{4}x^{20}+\frac{55}{3}x^{18}+\frac{231}{8}x^{16}+33x^{14}+\frac{55}{2}x^{12}+\frac{33}{2}x^{10}+\frac{55}{8}x^{8}+\frac{11}{6}x^{6}+\frac{1}{4}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)*x^3,x, algorithm="fricas")
```

```
[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8
*x^16 + 33*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1
```

```
/4*x^4
```

Sympy [A] time = 0.103432, size = 75, normalized size = 3.26

$$\frac{x^{26}}{26}+\frac{11x^{24}}{24}+\frac{5x^{22}}{2}+\frac{33x^{20}}{4}+\frac{55x^{18}}{3}+\frac{231x^{16}}{8}+33x^{14}+\frac{55x^{12}}{2}+\frac{33x^{10}}{2}+\frac{55x^{8}}{8}+\frac{11x^{6}}{6}+\frac{x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**26/26 + 11*x**24/24 + 5*x**22/2 + 33*x**20/4 + 55*x**18/3 + 23
1*x**16/8 + 33*x**14 + 55*x**12/2 + 33*x**10/2 + 55*x**8/8 + 11*x
**6/6 + x**4/4
```

GIAC/XCAS [A] time = 0.267673, size = 82, normalized size = 3.57

$$\frac{1}{26}\,x^{26}+\frac{11}{24}\,x^{24}+\frac{5}{2}\,x^{22}+\frac{33}{4}\,x^{20}+\frac{55}{3}\,x^{18}+\frac{231}{8}\,x^{16}+33\,x^{14}+\frac{55}{2}\,x^{12}+\frac{33}{2}\,x^{10}+\frac{55}{8}\,x^{8}+\frac{11}{6}\,x^{6}+\frac{1}{4}\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)*x^3,x, algorithm="giac")
```

```
[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8
*x^16 + 33*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1
/4*x^4
```

3.69 $\int x^2 \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=83

$$\frac{x^{25}}{25}+\frac{11x^{23}}{23}+\frac{55x^{21}}{21}+\frac{165x^{19}}{19}+\frac{330x^{17}}{17}+\frac{154x^{15}}{5}+\frac{462x^{13}}{13}+30x^{11}+\frac{55x^9}{3}+\frac{55x^7}{7}+\frac{11x^5}{5}+\frac{x^3}{3}$$

```
[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^1
3)/13 + (154*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/
21 + (11*x^23)/23 + x^25/25
```

Rubi [A] time = 0.0593104, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{x^{25}}{25}+\frac{11x^{23}}{23}+\frac{55x^{21}}{21}+\frac{165x^{19}}{19}+\frac{330x^{17}}{17}+\frac{154x^{15}}{5}+\frac{462x^{13}}{13}+30x^{11}+\frac{55x^9}{3}+\frac{55x^7}{7}+\frac{11x^5}{5}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^1
3)/13 + (154*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/
21 + (11*x^23)/23 + x^25/25
```

Rubi in Sympy [A] time = 11.2157, size = 75, normalized size = 0.9

$$\frac{x^{25}}{25}+\frac{11x^{23}}{23}+\frac{55x^{21}}{21}+\frac{165x^{19}}{19}+\frac{330x^{17}}{17}+\frac{154x^{15}}{5}+\frac{462x^{13}}{13}+30x^{11}+\frac{55x^9}{3}+\frac{55x^7}{7}+\frac{11x^5}{5}+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**25/25 + 11*x**23/23 + 55*x**21/21 + 165*x**19/19 + 330*x**17/1
7 + 154*x**15/5 + 462*x**13/13 + 30*x**11 + 55*x**9/3 + 55*x**7/7
 + 11*x**5/5 + x**3/3
```

Mathematica [A] time = 0.00232436, size = 83, normalized size = 1.

$$\frac{x^{25}}{25}+\frac{11x^{23}}{23}+\frac{55x^{21}}{21}+\frac{165x^{19}}{19}+\frac{330x^{17}}{17}+\frac{154x^{15}}{5}+\frac{462x^{13}}{13}+30x^{11}+\frac{55x^9}{3}+\frac{55x^7}{7}+\frac{11x^5}{5}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^1
3)/13 + (154*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/
21 + (11*x^23)/23 + x^25/25
```

Maple [A] time = 0.002, size = 62, normalized size = 0.8

$$\frac{x^3}{3}+\frac{11\,x^5}{5}+\frac{55\,x^7}{7}+\frac{55\,x^9}{3}+30\,x^{11}+\frac{462\,x^{13}}{13}+\frac{154\,x^{15}}{5}+\frac{330\,x^{17}}{17}+\frac{165\,x^{19}}{19}+\frac{55\,x^{21}}{21}+\frac{11\,x^{23}}{23}+\frac{x^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/3*x^3+11/5*x^5+55/7*x^7+55/3*x^9+30*x^11+462/13*x^13+154/5*x^15
+330/17*x^17+165/19*x^19+55/21*x^21+11/23*x^23+1/25*x^25
```

Maxima [A] time = 0.696201, size = 82, normalized size = 0.99

$$\frac{1}{25}\,x^{25}+\frac{11}{23}\,x^{23}+\frac{55}{21}\,x^{21}+\frac{165}{19}\,x^{19}+\frac{330}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{462}{13}\,x^{13}+30\,x^{11}+\frac{55}{3}\,x^9+\frac{55}{7}\,x^7+\frac{11}{5}\,x^5+\frac{1}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)*x^2,x, algorithm="maxima")
```

```
[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 +
 154/5*x^15 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*
x^5 + 1/3*x^3
```

Fricas [A] time = 0.226396, size = 1, normalized size = 0.01

$$\frac{1}{25}x^{25}+\frac{11}{23}x^{23}+\frac{55}{21}x^{21}+\frac{165}{19}x^{19}+\frac{330}{17}x^{17}+\frac{154}{5}x^{15}+\frac{462}{13}x^{13}+30x^{11}+\frac{55}{3}x^9+\frac{55}{7}x^7+\frac{11}{5}x^5+\frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)*x^2,x, algorithm="fricas")
```

```
[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 +
 154/5*x^15 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*
x^5 + 1/3*x^3
```

Sympy [A] time = 0.100533, size = 75, normalized size = 0.9

$$\frac{x^{25}}{25} + \frac{11x^{23}}{23} + \frac{55x^{21}}{21} + \frac{165x^{19}}{19} + \frac{330x^{17}}{17} + \frac{154x^{15}}{5} + \frac{462x^{13}}{13} + 30x^{11} + \frac{55x^{9}}{3} + \frac{55x^{7}}{7} + \frac{11x^{5}}{5} + \frac{x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**25/25 + 11*x**23/23 + 55*x**21/21 + 165*x**19/19 + 330*x**17/1
7 + 154*x**15/5 + 462*x**13/13 + 30*x**11 + 55*x**9/3 + 55*x**7/7
 + 11*x**5/5 + x**3/3
```

GIAC/XCAS [A] time = 0.267529, size = 82, normalized size = 0.99

$$\frac{1}{25}\,x^{25} + \frac{11}{23}\,x^{23} + \frac{55}{21}\,x^{21} + \frac{165}{19}\,x^{19} + \frac{330}{17}\,x^{17} + \frac{154}{5}\,x^{15} + \frac{462}{13}\,x^{13} + 30\,x^{11} + \frac{55}{3}\,x^{9} + \frac{55}{7}\,x^{7} + \frac{11}{5}\,x^{5} + \frac{1}{3}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)*x^2,x, algorithm="giac")
```

```
[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 +
 154/5*x^15 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*
x^5 + 1/3*x^3
```

3.70 $\int x \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=11

$$\frac{1}{24}\left(x^2+1\right)^{12}$$

```
[Out] (1 + x^2)^12/24
```

Rubi [A] time = 0.00867986, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{1}{24}\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (1 + x^2)^12/24
```

Rubi in Sympy [A] time = 3.54478, size = 7, normalized size = 0.64

$$\frac{\left(x^2+1\right)^{12}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] (x**2 + 1)**12/24
```

Mathematica [A] time = 0.00245139, size = 11, normalized size = 1.

$$\frac{1}{24}\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (1 + x^2)^12/24
```

Maple [B] time = 0.002, size = 62, normalized size = 5.6

$$\frac{x^{24}}{24}+\frac{x^{22}}{2}+\frac{11\,x^{20}}{4}+\frac{55\,x^{18}}{6}+\frac{165\,x^{16}}{8}+33\,x^{14}+\frac{77\,x^{12}}{2}+33\,x^{10}+\frac{165\,x^{8}}{8}+\frac{55\,x^{6}}{6}+\frac{11\,x^{4}}{4}+\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/24*x^24+1/2*x^22+11/4*x^20+55/6*x^18+165/8*x^16+33*x^14+77/2*x^
12+33*x^10+165/8*x^8+55/6*x^6+11/4*x^4+1/2*x^2
```

Maxima [A] time = 0.699148, size = 82, normalized size = 7.45

$$\frac{1}{24}\,x^{24}+\frac{1}{2}\,x^{22}+\frac{11}{4}\,x^{20}+\frac{55}{6}\,x^{18}+\frac{165}{8}\,x^{16}+33\,x^{14}+\frac{77}{2}\,x^{12}+33\,x^{10}+\frac{165}{8}\,x^{8}+\frac{55}{6}\,x^{6}+\frac{11}{4}\,x^{4}+\frac{1}{2}\,x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)*x,x, algorithm="maxima")
```

```
[Out] 1/24*x^24 + 1/2*x^22 + 11/4*x^20 + 55/6*x^18 + 165/8*x^16 + 33*x^
14 + 77/2*x^12 + 33*x^10 + 165/8*x^8 + 55/6*x^6 + 11/4*x^4 + 1/2*
x^2
```

Fricas [A] time = 0.229068, size = 1, normalized size = 0.09

$$\frac{1}{24}x^{24}+\frac{1}{2}x^{22}+\frac{11}{4}x^{20}+\frac{55}{6}x^{18}+\frac{165}{8}x^{16}+33x^{14}+\frac{77}{2}x^{12}+33x^{10}+\frac{165}{8}x^{8}+\frac{55}{6}x^{6}+\frac{11}{4}x^{4}+\frac{1}{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)*x,x, algorithm="fricas")
```

```
[Out] 1/24*x^24 + 1/2*x^22 + 11/4*x^20 + 55/6*x^18 + 165/8*x^16 + 33*x^
14 + 77/2*x^12 + 33*x^10 + 165/8*x^8 + 55/6*x^6 + 11/4*x^4 + 1/2*
x^2
```

Sympy [A] time = 0.107175, size = 71, normalized size = 6.45

$$\frac{x^{24}}{24}+\frac{x^{22}}{2}+\frac{11x^{20}}{4}+\frac{55x^{18}}{6}+\frac{165x^{16}}{8}+33x^{14}+\frac{77x^{12}}{2}+33x^{10}+\frac{165x^{8}}{8}+\frac{55x^{6}}{6}+\frac{11x^{4}}{4}+\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**24/24 + x**22/2 + 11*x**20/4 + 55*x**18/6 + 165*x**16/8 + 33*x
**14 + 77*x**12/2 + 33*x**10 + 165*x**8/8 + 55*x**6/6 + 11*x**4/4
 + x**2/2
```

GIAC/XCAS [A] time = 0.261091, size = 82, normalized size = 7.45

$$\frac{1}{24}\,x^{24}+\frac{1}{2}\,x^{22}+\frac{11}{4}\,x^{20}+\frac{55}{6}\,x^{18}+\frac{165}{8}\,x^{16}+33\,x^{14}+\frac{77}{2}\,x^{12}+33\,x^{10}+\frac{165}{8}\,x^{8}+\frac{55}{6}\,x^{6}+\frac{11}{4}\,x^{4}+\frac{1}{2}\,x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)*x,x, algorithm="giac")
```

```
[Out] 1/24*x^24 + 1/2*x^22 + 11/4*x^20 + 55/6*x^18 + 165/8*x^16 + 33*x^
14 + 77/2*x^12 + 33*x^10 + 165/8*x^8 + 55/6*x^6 + 11/4*x^4 + 1/2*
x^2
```

3.71 $\int \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=73

$$\frac{x^{23}}{23}+\frac{11x^{21}}{21}+\frac{55x^{19}}{19}+\frac{165x^{17}}{17}+22x^{15}+\frac{462x^{13}}{13}+42x^{11}+\frac{110x^9}{3}+\frac{165x^7}{7}+11x^5+\frac{11x^3}{3}+x$$

```
[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (
462*x^13)/13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)
/21 + x^23/23
```

Rubi [A] time = 0.0446194, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{x^{23}}{23}+\frac{11x^{21}}{21}+\frac{55x^{19}}{19}+\frac{165x^{17}}{17}+22x^{15}+\frac{462x^{13}}{13}+42x^{11}+\frac{110x^9}{3}+\frac{165x^7}{7}+11x^5+\frac{11x^3}{3}+x$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (
462*x^13)/13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)
/21 + x^23/23
```

Rubi in Sympy [A] time = 9.60154, size = 68, normalized size = 0.93

$$\frac{x^{23}}{23}+\frac{11x^{21}}{21}+\frac{55x^{19}}{19}+\frac{165x^{17}}{17}+22x^{15}+\frac{462x^{13}}{13}+42x^{11}+\frac{110x^9}{3}+\frac{165x^7}{7}+11x^5+\frac{11x^3}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**23/23 + 11*x**21/21 + 55*x**19/19 + 165*x**17/17 + 22*x**15 +
462*x**13/13 + 42*x**11 + 110*x**9/3 + 165*x**7/7 + 11*x**5 + 11*
x**3/3 + x
```

Mathematica [A] time = 0.0011577, size = 73, normalized size = 1.

$$\frac{x^{23}}{23}+\frac{11x^{21}}{21}+\frac{55x^{19}}{19}+\frac{165x^{17}}{17}+22x^{15}+\frac{462x^{13}}{13}+42x^{11}+\frac{110x^9}{3}+\frac{165x^7}{7}+11x^5+\frac{11x^3}{3}+x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (
462*x^13)/13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)
/21 + x^23/23
```

Maple [A] time = 0.002, size = 58, normalized size = 0.8

$$x+\frac{11\,x^{3}}{3}+11\,x^{5}+\frac{165\,x^{7}}{7}+\frac{110\,x^{9}}{3}+42\,x^{11}+\frac{462\,x^{13}}{13}+22\,x^{15}+\frac{165\,x^{17}}{17}+\frac{55\,x^{19}}{19}+\frac{11\,x^{21}}{21}+\frac{x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] x+11/3*x^3+11*x^5+165/7*x^7+110/3*x^9+42*x^11+462/13*x^13+22*x^15
+165/17*x^17+55/19*x^19+11/21*x^21+1/23*x^23
```

Maxima [A] time = 0.699206, size = 77, normalized size = 1.05

$$\frac{1}{23}\,x^{23}+\frac{11}{21}\,x^{21}+\frac{55}{19}\,x^{19}+\frac{165}{17}\,x^{17}+22\,x^{15}+\frac{462}{13}\,x^{13}+42\,x^{11}+\frac{110}{3}\,x^{9}+\frac{165}{7}\,x^{7}+11\,x^{5}+\frac{11}{3}\,x^{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462
/13*x^13 + 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 +
x
```

Fricas [A] time = 0.256262, size = 1, normalized size = 0.01

$$\frac{1}{23}x^{23}+\frac{11}{21}x^{21}+\frac{55}{19}x^{19}+\frac{165}{17}x^{17}+22x^{15}+\frac{462}{13}x^{13}+42x^{11}+\frac{110}{3}x^{9}+\frac{165}{7}x^{7}+11x^{5}+\frac{11}{3}x^{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462
/13*x^13 + 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 +
x
```

Sympy [A] time = 0.109544, size = 68, normalized size = 0.93

$$\frac{x^{23}}{23} + \frac{11x^{21}}{21} + \frac{55x^{19}}{19} + \frac{165x^{17}}{17} + 22x^{15} + \frac{462x^{13}}{13} + 42x^{11} + \frac{110x^{9}}{3} + \frac{165x^{7}}{7} + 11x^{5} + \frac{11x^{3}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**23/23 + 11*x**21/21 + 55*x**19/19 + 165*x**17/17 + 22*x**15 +
462*x**13/13 + 42*x**11 + 110*x**9/3 + 165*x**7/7 + 11*x**5 + 11*
x**3/3 + x
```

GIAC/XCAS [A] time = 0.261, size = 77, normalized size = 1.05

$$\frac{1}{23}\,x^{23} + \frac{11}{21}\,x^{21} + \frac{55}{19}\,x^{19} + \frac{165}{17}\,x^{17} + 22\,x^{15} + \frac{462}{13}\,x^{13} + 42\,x^{11} + \frac{110}{3}\,x^{9} + \frac{165}{7}\,x^{7} + 11\,x^{5} + \frac{11}{3}\,x^{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462
/13*x^13 + 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 +
x
```

3.72 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x} \, dx$

Optimal. Leaf size=80

$$\frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^8}{4}+\frac{55x^6}{2}+\frac{55x^4}{4}+\frac{11x^2}{2}+\log(x)$$

```
[Out] (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5
 + (77*x^12)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (1
1*x^20)/20 + x^22/22 + Log[x]
```

Rubi [A] time = 0.0614809, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^8}{4}+\frac{55x^6}{2}+\frac{55x^4}{4}+\frac{11x^2}{2}+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5
 + (77*x^12)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (1
1*x^20)/20 + x^22/22 + Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^8}{4}+\frac{55x^6}{2}+\frac{11x^2}{2}+\frac{\log(x^2)}{2}+\frac{55\int^{x^2} x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)*(x**4+2*x**2+1)**5/x,x)
```

```
[Out] x**22/22 + 11*x**20/20 + 55*x**18/18 + 165*x**16/16 + 165*x**14/7
 + 77*x**12/2 + 231*x**10/5 + 165*x**8/4 + 55*x**6/2 + 11*x**2/2
+ log(x**2)/2 + 55*Integral(x, (x, x**2))/2
```

Mathematica [A] time = 0.00520964, size = 80, normalized size = 1.

$$\frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^8}{4}+\frac{55x^6}{2}+\frac{55x^4}{4}+\frac{11x^2}{2}+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5
 + (77*x^12)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (1
1*x^20)/20 + x^22/22 + Log[x]
```

Maple [A] time = 0.004, size = 59, normalized size = 0.7

$$\frac{11\,x^2}{2}+\frac{55\,x^4}{4}+\frac{55\,x^6}{2}+\frac{165\,x^8}{4}+\frac{231\,x^{10}}{5}+\frac{77\,x^{12}}{2}+\frac{165\,x^{14}}{7}+\frac{165\,x^{16}}{16}+\frac{55\,x^{18}}{18}+\frac{11\,x^{20}}{20}+\frac{x^{22}}{22}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)*(x^4+2*x^2+1)^5/x,x)
```

```
[Out] 11/2*x^2+55/4*x^4+55/2*x^6+165/4*x^8+231/5*x^10+77/2*x^12+165/7*x
^14+165/16*x^16+55/18*x^18+11/20*x^20+1/22*x^22+ln(x)
```

Maxima [A] time = 0.698766, size = 84, normalized size = 1.05

$$\frac{1}{22}\,x^{22}+\frac{11}{20}\,x^{20}+\frac{55}{18}\,x^{18}+\frac{165}{16}\,x^{16}+\frac{165}{7}\,x^{14}+\frac{77}{2}\,x^{12}+\frac{231}{5}\,x^{10}+\frac{165}{4}\,x^8+\frac{55}{2}\,x^6+\frac{55}{4}\,x^4+\frac{11}{2}\,x^2+\frac{1}{2}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)/x,x, algorithm="maxima")
```

```
[Out] 1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 +
77/2*x^12 + 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x
^2 + 1/2*log(x^2)
```

Fricas [A] time = 0.249745, size = 78, normalized size = 0.98

$$\frac{1}{22}\,x^{22} + \frac{11}{20}\,x^{20} + \frac{55}{18}\,x^{18} + \frac{165}{16}\,x^{16} + \frac{165}{7}\,x^{14} + \frac{77}{2}\,x^{12} + \frac{231}{5}\,x^{10} + \frac{165}{4}\,x^{8} + \frac{55}{2}\,x^{6} + \frac{55}{4}\,x^{4} + \frac{11}{2}\,x^{2} + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)/x,x, algorithm="fricas")
```

```
[Out] 1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 +
77/2*x^12 + 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x
^2 + log(x)
```

Sympy [A] time = 0.204824, size = 75, normalized size = 0.94

$$\frac{x^{22}}{22} + \frac{11x^{20}}{20} + \frac{55x^{18}}{18} + \frac{165x^{16}}{16} + \frac{165x^{14}}{7} + \frac{77x^{12}}{2} + \frac{231x^{10}}{5} + \frac{165x^{8}}{4} + \frac{55x^{6}}{2} + \frac{55x^{4}}{4} + \frac{11x^{2}}{2} + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)*(x**4+2*x**2+1)**5/x,x)
```

```
[Out] x**22/22 + 11*x**20/20 + 55*x**18/18 + 165*x**16/16 + 165*x**14/7
 + 77*x**12/2 + 231*x**10/5 + 165*x**8/4 + 55*x**6/2 + 55*x**4/4
+ 11*x**2/2 + log(x)
```

GIAC/XCAS [A] time = 0.262361, size = 84, normalized size = 1.05

$$\frac{1}{22}\,x^{22} + \frac{11}{20}\,x^{20} + \frac{55}{18}\,x^{18} + \frac{165}{16}\,x^{16} + \frac{165}{7}\,x^{14} + \frac{77}{2}\,x^{12} + \frac{231}{5}\,x^{10} + \frac{165}{4}\,x^{8} + \frac{55}{2}\,x^{6} + \frac{55}{4}\,x^{4} + \frac{11}{2}\,x^{2} + \frac{1}{2}\,\ln\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)/x,x, algorithm="giac")
```

```
[Out] 1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 +
77/2*x^12 + 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x
^2 + 1/2*ln(x^2)
```

3.73 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2}\,dx$

Optimal. Leaf size=73

$$\frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^9}{3}+\frac{330x^7}{7}+33x^5+\frac{55x^3}{3}+11x-\frac{1}{x}$$

```
[Out] -x^(-1) + 11*x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3
+ 42*x^11 + (330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19
 + x^21/21
```

Rubi [A] time = 0.0517278, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^9}{3}+\frac{330x^7}{7}+33x^5+\frac{55x^3}{3}+11x-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -x^(-1) + 11*x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3
+ 42*x^11 + (330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19
 + x^21/21
```

Rubi in Sympy [A] time = 11.2089, size = 66, normalized size = 0.9

$$\frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^9}{3}+\frac{330x^7}{7}+33x^5+\frac{55x^3}{3}+11x-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)*(x**4+2*x**2+1)**5/x**2,x)
```

```
[Out] x**21/21 + 11*x**19/19 + 55*x**17/17 + 11*x**15 + 330*x**13/13 +
42*x**11 + 154*x**9/3 + 330*x**7/7 + 33*x**5 + 55*x**3/3 + 11*x -
 1/x
```

Mathematica [A] time = 0.00483174, size = 73, normalized size = 1.

$$\frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^9}{3}+\frac{330x^7}{7}+33x^5+\frac{55x^3}{3}+11x-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -x^(-1) + 11*x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3
+ 42*x^11 + (330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19
 + x^21/21
```

Maple [A] time = 0.005, size = 60, normalized size = 0.8

$$-x^{-1}+11\,x+\frac{55\,x^{3}}{3}+33\,x^{5}+\frac{330\,x^{7}}{7}+\frac{154\,x^{9}}{3}+42\,x^{11}+\frac{330\,x^{13}}{13}+11\,x^{15}+\frac{55\,x^{17}}{17}+\frac{11\,x^{19}}{19}+\frac{x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)*(x^4+2*x^2+1)^5/x^2,x)
```

```
[Out] -1/x+11*x+55/3*x^3+33*x^5+330/7*x^7+154/3*x^9+42*x^11+330/13*x^13
+11*x^15+55/17*x^17+11/19*x^19+1/21*x^21
```

Maxima [A] time = 0.697339, size = 80, normalized size = 1.1

$$\frac{1}{21}\,x^{21}+\frac{11}{19}\,x^{19}+\frac{55}{17}\,x^{17}+11\,x^{15}+\frac{330}{13}\,x^{13}+42\,x^{11}+\frac{154}{3}\,x^{9}+\frac{330}{7}\,x^{7}+33\,x^{5}+\frac{55}{3}\,x^{3}+11\,x-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)/x^2,x, algorithm="maxima")
```

```
[Out] 1/21*x^21 + 11/19*x^19 + 55/17*x^17 + 11*x^15 + 330/13*x^13 + 42*
x^11 + 154/3*x^9 + 330/7*x^7 + 33*x^5 + 55/3*x^3 + 11*x - 1/x
```

Fricas [A] time = 0.242135, size = 84, normalized size = 1.15

$$\frac{4199\,x^{22}+51051\,x^{20}+285285\,x^{18}+969969\,x^{16}+2238390\,x^{14}+3703518\,x^{12}+4526522\,x^{10}+4157010\,x^{8}+2909907\,x^{6}+16}{88179\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)/x^2,x, algorithm="fricas")
```

```
[Out] 1/88179*(4199*x^22 + 51051*x^20 + 285285*x^18 + 969969*x^16 + 223
8390*x^14 + 3703518*x^12 + 4526522*x^10 + 4157010*x^8 + 2909907*x
^6 + 1616615*x^4 + 969969*x^2 - 88179)/x
```

Sympy [A] time = 0.188915, size = 66, normalized size = 0.9

$$\frac{x^{21}}{21} + \frac{11x^{19}}{19} + \frac{55x^{17}}{17} + 11x^{15} + \frac{330x^{13}}{13} + 42x^{11} + \frac{154x^{9}}{3} + \frac{330x^{7}}{7} + 33x^{5} + \frac{55x^{3}}{3} + 11x - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)*(x**4+2*x**2+1)**5/x**2,x)
```

```
[Out] x**21/21 + 11*x**19/19 + 55*x**17/17 + 11*x**15 + 330*x**13/13 +
42*x**11 + 154*x**9/3 + 330*x**7/7 + 33*x**5 + 55*x**3/3 + 11*x -
 1/x
```

GIAC/XCAS [A] time = 0.264007, size = 80, normalized size = 1.1

$$\frac{1}{21}\,x^{21} + \frac{11}{19}\,x^{19} + \frac{55}{17}\,x^{17} + 11\,x^{15} + \frac{330}{13}\,x^{13} + 42\,x^{11} + \frac{154}{3}\,x^{9} + \frac{330}{7}\,x^{7} + 33\,x^{5} + \frac{55}{3}\,x^{3} + 11\,x - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)/x^2,x, algorithm="giac")
```

```
[Out] 1/21*x^21 + 11/19*x^19 + 55/17*x^17 + 11*x^15 + 330/13*x^13 + 42*
x^11 + 154/3*x^9 + 330/7*x^7 + 33*x^5 + 55/3*x^3 + 11*x - 1/x
```

3.74 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3}\,dx$

Optimal. Leaf size=80

$$\frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5}+\frac{231x^8}{4}+55x^6+\frac{165x^4}{4}+\frac{55x^2}{2}-\frac{1}{2x^2}+11\log(x)$$

```
[Out] -1/(2*x^2) + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (2
31*x^10)/5 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^1
8)/18 + x^20/20 + 11*Log[x]
```

Rubi [A] time = 0.0802229, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5}+\frac{231x^8}{4}+55x^6+\frac{165x^4}{4}+\frac{55x^2}{2}-\frac{1}{2x^2}+11\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -1/(2*x^2) + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (2
31*x^10)/5 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^1
8)/18 + x^20/20 + 11*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5}+\frac{231x^8}{4}+55x^6+\frac{55x^2}{2}+\frac{11\log\left(x^2\right)}{2}+\frac{165\int^{x^2}x\,dx}{2}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)*(x**4+2*x**2+1)**5/x**3,x)
```

```
[Out] x**20/20 + 11*x**18/18 + 55*x**16/16 + 165*x**14/14 + 55*x**12/2
+ 231*x**10/5 + 231*x**8/4 + 55*x**6 + 55*x**2/2 + 11*log(x**2)/2
 + 165*Integral(x, (x, x**2))/2 - 1/(2*x**2)
```

Mathematica [A] time = 0.00445, size = 80, normalized size = 1.

$$\frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5}+\frac{231x^8}{4}+55x^6+\frac{165x^4}{4}+\frac{55x^2}{2}-\frac{1}{2x^2}+11\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -1/(2*x^2) + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (2
31*x^10)/5 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^1
8)/18 + x^20/20 + 11*Log[x]
```

Maple [A] time = 0.008, size = 61, normalized size = 0.8

$$-\frac{1}{2\,x^2}+\frac{55\,x^2}{2}+\frac{165\,x^4}{4}+55\,x^6+\frac{231\,x^8}{4}+\frac{231\,x^{10}}{5}+\frac{55\,x^{12}}{2}+\frac{165\,x^{14}}{14}+\frac{55\,x^{16}}{16}+\frac{11\,x^{18}}{18}+\frac{x^{20}}{20}+11\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)*(x^4+2*x^2+1)^5/x^3,x)
```

```
[Out] -1/2/x^2+55/2*x^2+165/4*x^4+55*x^6+231/4*x^8+231/5*x^10+55/2*x^12
+165/14*x^14+55/16*x^16+11/18*x^18+1/20*x^20+11*ln(x)
```

Maxima [A] time = 0.69737, size = 84, normalized size = 1.05

$$\frac{1}{20}\,x^{20}+\frac{11}{18}\,x^{18}+\frac{55}{16}\,x^{16}+\frac{165}{14}\,x^{14}+\frac{55}{2}\,x^{12}+\frac{231}{5}\,x^{10}+\frac{231}{4}\,x^8+55\,x^6+\frac{165}{4}\,x^4+\frac{55}{2}\,x^2-\frac{1}{2\,x^2}+\frac{11}{2}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)/x^3,x, algorithm="maxima")
```

```
[Out] 1/20*x^20 + 11/18*x^18 + 55/16*x^16 + 165/14*x^14 + 55/2*x^12 + 2
31/5*x^10 + 231/4*x^8 + 55*x^6 + 165/4*x^4 + 55/2*x^2 - 1/2/x^2 +
 11/2*log(x^2)
```

Fricas [A] time = 0.249939, size = 86, normalized size = 1.08

$$\frac{252\,x^{22}+3080\,x^{20}+17325\,x^{18}+59400\,x^{16}+138600\,x^{14}+232848\,x^{12}+291060\,x^{10}+277200\,x^8+207900\,x^6+138600\,x^4+5}{5040\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)/x^3,x, algorithm="fricas")
```

```
[Out] 1/5040*(252*x^22 + 3080*x^20 + 17325*x^18 + 59400*x^16 + 138600*x
^14 + 232848*x^12 + 291060*x^10 + 277200*x^8 + 207900*x^6 + 13860
0*x^4 + 55440*x^2*log(x) - 2520)/x^2
```

Sympy [A] time = 0.228366, size = 75, normalized size = 0.94

$$\frac{x^{20}}{20} + \frac{11x^{18}}{18} + \frac{55x^{16}}{16} + \frac{165x^{14}}{14} + \frac{55x^{12}}{2} + \frac{231x^{10}}{5} + \frac{231x^{8}}{4} + 55x^{6} + \frac{165x^{4}}{4} + \frac{55x^{2}}{2} + 11\log(x) - \frac{1}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)*(x**4+2*x**2+1)**5/x**3,x)
```

```
[Out] x**20/20 + 11*x**18/18 + 55*x**16/16 + 165*x**14/14 + 55*x**12/2
+ 231*x**10/5 + 231*x**8/4 + 55*x**6 + 165*x**4/4 + 55*x**2/2 + 1
1*log(x) - 1/(2*x**2)
```

GIAC/XCAS [A] time = 0.263279, size = 93, normalized size = 1.16

$$\frac{1}{20}x^{20} + \frac{11}{18}x^{18} + \frac{55}{16}x^{16} + \frac{165}{14}x^{14} + \frac{55}{2}x^{12} + \frac{231}{5}x^{10} + \frac{231}{4}x^{8} + 55x^{6} + \frac{165}{4}x^{4} + \frac{55}{2}x^{2} - \frac{11x^{2}+1}{2x^{2}} + \frac{11}{2}\ln\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x^2 + 1)^5*(x^2 + 1)/x^3,x, algorithm="giac")
```

```
[Out] 1/20*x^20 + 11/18*x^18 + 55/16*x^16 + 165/14*x^14 + 55/2*x^12 + 2
31/5*x^10 + 231/4*x^8 + 55*x^6 + 165/4*x^4 + 55/2*x^2 - 1/2*(11*x
^2 + 1)/x^2 + 11/2*ln(x^2)
```

3.75 $\int \frac{x^2\left(d+ex^2\right)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=145

$$\frac{x\left(a+bx^2\right)(bd-ae)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((b*d - a*e)*x*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 + (e*x^3*(a + b*x^2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (S
qrt[a]*(b*d - a*e)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(5
/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.266099, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121

$$\frac{x\left(a+bx^2\right)(bd-ae)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((b*d - a*e)*x*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 + (e*x^3*(a + b*x^2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (S
qrt[a]*(b*d - a*e)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(5
/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0860841, size = 80, normalized size = 0.55

$$\frac{(a+bx^2)\left(\sqrt{b}x\left(-3ae+3bd+bex^2\right)+3\sqrt{a}(ae-bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{3b^{5/2}\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(Sqrt[b]*x*(3*b*d - 3*a*e + b*e*x^2) + 3*Sqrt[a]*(-(
b*d) + a*e)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(3*b^(5/2)*Sqrt[(a + b*
x^2)^2])
```

Maple [A] time = 0.013, size = 90, normalized size = 0.6

$$\frac{bx^2+a}{3\,b^2}\left(\sqrt{ab}x^3be-3\,\sqrt{ab}xae+3\,\sqrt{ab}xbd+3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2e-3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)abd\right)\frac{1}{\sqrt{(bx^2+a)^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)*((a*b)^(1/2)*x^3*b*e-3*(a*b)^(1/2)*x*a*e+3*(a*b)^(1
/2)*x*b*d+3*arctan(x*b/(a*b)^(1/2))*a^2*e-3*arctan(x*b/(a*b)^(1/2
))*a*b*d)/((b*x^2+a)^2)^(1/2)/b^2/(a*b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*x^2/sqrt((b*x^2 + a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.260384, size = 1, normalized size = 0.01

$$\left[\frac{2\,bex^3 - 3\,(bd - ae)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) + 6\,(bd - ae)x}{6\,b^2}, \frac{bex^3 - 3\,(bd - ae)\sqrt{\frac{a}{b}}\arctan\left(\frac{x}{\sqrt{\frac{a}{b}}}\right) + 3\,(bd - ae)x}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*x^2/sqrt((b*x^2 + a)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*e*x^3 - 3*(b*d - a*e)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqr
t(-a/b) - a)/(b*x^2 + a)) + 6*(b*d - a*e)*x)/b^2, 1/3*(b*e*x^3 -
3*(b*d - a*e)*sqrt(a/b)*arctan(x/sqrt(a/b)) + 3*(b*d - a*e)*x)/b^
2]
```

Sympy [A] time = 1.76036, size = 90, normalized size = 0.62

$$-\frac{\sqrt{-\frac{a}{b^5}}\,(ae - bd)\log\left(-b^2\sqrt{-\frac{a}{b^5}} + x\right)}{2} + \frac{\sqrt{-\frac{a}{b^5}}\,(ae - bd)\log\left(b^2\sqrt{-\frac{a}{b^5}} + x\right)}{2} + \frac{ex^3}{3b} - \frac{x\,(ae - bd)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -sqrt(-a/b**5)*(a*e - b*d)*log(-b**2*sqrt(-a/b**5) + x)/2 + sqrt(
-a/b**5)*(a*e - b*d)*log(b**2*sqrt(-a/b**5) + x)/2 + e*x**3/(3*b)
 - x*(a*e - b*d)/b**2
```

GIAC/XCAS [A] time = 0.265498, size = 136, normalized size = 0.94

$$-\frac{\left(abd\mathrm{sign}\left(bx^2 + a\right) - a^2e\mathrm{sign}\left(bx^2 + a\right)\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{b^2x^3e\mathrm{sign}\left(bx^2 + a\right) + 3\,b^2dx\mathrm{sign}\left(bx^2 + a\right) - 3\,abxe\mathrm{sign}\left(bx^2 + a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*x^2/sqrt((b*x^2 + a)^2),x, algorithm="giac")
```

```
[Out] -(a*b*d*sign(b*x^2 + a) - a^2*e*sign(b*x^2 + a))*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^2) + 1/3*(b^2*x^3*e*sign(b*x^2 + a) + 3*b^2*d*
x*sign(b*x^2 + a) - 3*a*b*x*e*sign(b*x^2 + a))/b^3
```

3.76 $\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=83

$$\frac{(a+bx^2)(bd-ae)\log(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}$$

```
[Out] (e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*b^2) + ((b*d - a*e)*(a + b
*x^2)*Log[a + b*x^2])/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.191917, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129

$$\frac{(a+bx^2)(bd-ae)\log(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*b^2) + ((b*d - a*e)*(a + b
*x^2)*Log[a + b*x^2])/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [A] time = 24.5963, size = 76, normalized size = 0.92

$$\frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}-\frac{(a+bx^2)(ae-bd)\log(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] e*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(2*b**2) - (a + b*x**2)*(a*
e - b*d)*log(a + b*x**2)/(2*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x*
*4))
```

Mathematica [A] time = 0.0390962, size = 51, normalized size = 0.61

$$\frac{\left(a+bx^2\right)\left((bd-ae)\log\left(a+bx^2\right)+bex^2\right)}{2b^2\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(b*e*x^2 + (b*d - a*e)*Log[a + b*x^2]))/(2*b^2*Sqrt[
(a + b*x^2)^2])
```

Maple [A] time = 0.01, size = 55, normalized size = 0.7

$$-\frac{\left(bx^2+a\right)\left(-x^2be+\ln\left(bx^2+a\right)ae-\ln\left(bx^2+a\right)bd\right)}{2\,b^2}\frac{1}{\sqrt{\left(bx^2+a\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/2*(b*x^2+a)*(-x^2*b*e+ln(b*x^2+a)*a*e-ln(b*x^2+a)*b*d)/((b*x^2
+a)^2)^(1/2)/b^2
```

Maxima [A] time = 0.71101, size = 92, normalized size = 1.11

$$\frac{1}{2}\sqrt{\frac{1}{b^2}}d\log\left(x^2+\frac{a}{b}\right)-\frac{1}{2}\left(\frac{a\sqrt{\frac{1}{b^2}}\log\left(x^2+\frac{a}{b}\right)}{b}-\frac{\sqrt{b^2x^4+2\,abx^2+a^2}}{b^2}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*x/sqrt((b*x^2 + a)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b^(-2))*d*log(x^2 + a/b) - 1/2*(a*sqrt(b^(-2))*log(x^2 +
 a/b)/b - sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)/b^2)*e
```

Fricas [A] time = 0.28958, size = 39, normalized size = 0.47

$$\frac{bex^2 + (bd - ae)\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*x/sqrt((b*x^2 + a)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*e*x^2 + (b*d - a*e)*log(b*x^2 + a))/b^2
```

Sympy [A] time = 1.51513, size = 27, normalized size = 0.33

$$\frac{ex^2}{2b} - \frac{(ae - bd)\log\left(a + bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] e*x**2/(2*b) - (a*e - b*d)*log(a + b*x**2)/(2*b**2)
```

GIAC/XCAS [A] time = 0.263942, size = 57, normalized size = 0.69

$$\frac{1}{2}\left(\frac{x^2e}{b} + \frac{(bd - ae)\ln\left(\left|bx^2 + a\right|\right)}{b^2}\right)\mathrm{sign}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*x/sqrt((b*x^2 + a)^2),x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e/b + (b*d - a*e)*ln(abs(b*x^2 + a))/b^2)*sign(b*x^2 + a
)
```

3.77 $\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}} \, dx$

Optimal. Leaf size=97

$$\frac{ex\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (e*x*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a
*e)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.122208, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{ex\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (e*x*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a
*e)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex^2}{\sqrt{\left(a+bx^2\right)^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt((a + b*x**2)**2), x)
```

Mathematica [A] time = 0.0513147, size = 69, normalized size = 0.71

$$-\frac{(a+bx^2)\left((ae-bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\sqrt{a}\sqrt{b}ex\right)}{\sqrt{a}b^{3/2}\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -(((a + b*x^2)*(-(Sqrt[a]*Sqrt[b]*e*x) + (-(b*d) + a*e)*ArcTan[(S
qrt[b]*x)/Sqrt[a]]))/(Sqrt[a]*b^(3/2)*Sqrt[(a + b*x^2)^2]))
```

Maple [A] time = 0.009, size = 62, normalized size = 0.6

$$\frac{bx^2+a}{b}\left(xe\sqrt{ab}-\arctan\left(bx\frac{1}{\sqrt{ab}}\right)ae+\arctan\left(bx\frac{1}{\sqrt{ab}}\right)bd\right)\frac{1}{\sqrt{(bx^2+a)^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] (b*x^2+a)*(x*e*(a*b)^(1/2)-arctan(x*b/(a*b)^(1/2))*a*e+arctan(x*b
/(a*b)^(1/2))*b*d)/((b*x^2+a)^2)^(1/2)/b/(a*b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt((b*x^2 + a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270035, size = 1, normalized size = 0.01

$$\left[\frac{2\sqrt{-ab}ex - (bd - ae)\log\left(-\frac{2abx - (bx^2 - a)\sqrt{-ab}}{bx^2 + a}\right)}{2\sqrt{-ab}b}, \frac{\sqrt{ab}ex + (bd - ae)\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{\sqrt{ab}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt((b*x^2 + a)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(-a*b)*e*x - (b*d - a*e)*log(-(2*a*b*x - (b*x^2 - a)*
sqrt(-a*b))/(b*x^2 + a)))/(sqrt(-a*b)*b), (sqrt(a*b)*e*x + (b*d -
 a*e)*arctan(sqrt(a*b)*x/a))/(sqrt(a*b)*b)]
```

Sympy [A] time = 1.62966, size = 82, normalized size = 0.85

$$\frac{\sqrt{-\frac{1}{ab^3}}(ae - bd)\log\left(-ab\sqrt{-\frac{1}{ab^3}} + x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^3}}(ae - bd)\log\left(ab\sqrt{-\frac{1}{ab^3}} + x\right)}{2} + \frac{ex}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] sqrt(-1/(a*b**3))*(a*e - b*d)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 -
 sqrt(-1/(a*b**3))*(a*e - b*d)*log(a*b*sqrt(-1/(a*b**3)) + x)/2 +
 e*x/b
```

GIAC/XCAS [A] time = 0.263035, size = 80, normalized size = 0.82

$$\frac{x\text{esign}\left(bx^2 + a\right)}{b} + \frac{\left(bd\text{sign}\left(bx^2 + a\right) - a\text{esign}\left(bx^2 + a\right)\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/sqrt((b*x^2 + a)^2),x, algorithm="giac")
```

```
[Out] x*e*sign(b*x^2 + a)/b + (b*d*sign(b*x^2 + a) - a*e*sign(b*x^2 + a
))*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

3.78 $\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{d\log(x)\left(a+bx^2\right)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)(bd-ae)\log\left(a+bx^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (d*(a + b*x^2)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*
d - a*e)*(a + b*x^2)*Log[a + b*x^2])/(2*a*b*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])
```

Rubi [A] time = 0.211254, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{d\log(x)\left(a+bx^2\right)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)(bd-ae)\log\left(a+bx^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (d*(a + b*x^2)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*
d - a*e)*(a + b*x^2)*Log[a + b*x^2])/(2*a*b*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/x/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0371235, size = 54, normalized size = 0.59

$$\frac{\left(a+bx^2\right)\left((ae-bd)\log\left(a+bx^2\right)+2bd\log(x)\right)}{2ab\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*(2*b*d*Log[x] + (-(b*d) + a*e)*Log[a + b*x^2]))/(2*a
*b*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.014, size = 57, normalized size = 0.6

$$\frac{\left(bx^2+a\right)\left(\ln\left(bx^2+a\right)ae-\ln\left(bx^2+a\right)bd+2d\ln\left(x\right)b\right)}{2ab}\frac{1}{\sqrt{\left(bx^2+a\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/2*(b*x^2+a)*(ln(b*x^2+a)*a*e-ln(b*x^2+a)*b*d+2*d*ln(x)*b)/((b*x
^2+a)^2)^(1/2)/a/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/(sqrt((b*x^2 + a)^2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.256468, size = 45, normalized size = 0.49

$$\frac{2\,bd\log\left(x\right)-\left(bd-ae\right)\log\left(bx^2+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/(sqrt((b*x^2 + a)^2)*x),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*log(x) - (b*d - a*e)*log(b*x^2 + a))/(a*b)
```

Sympy [A] time = 2.19091, size = 26, normalized size = 0.28

$$\frac{d\log(x)}{a} + \frac{(ae - bd)\log\left(\frac{a}{b} + x^2\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/x/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] d*log(x)/a + (a*e - b*d)*log(a/b + x**2)/(2*a*b)
```

GIAC/XCAS [A] time = 0.263778, size = 82, normalized size = 0.89

$$\frac{d\ln\left(x^2\right)\,\mathrm{sign}\left(bx^2 + a\right)}{2\,a} - \frac{\left(bd\mathrm{sign}\left(bx^2 + a\right) - ae\mathrm{sign}\left(bx^2 + a\right)\right)\ln\left(\left|bx^2 + a\right|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(sqrt((b*x^2 + a)^2)*x),x, algorithm="giac")
```

```
[Out] 1/2*d*ln(x^2)*sign(b*x^2 + a)/a - 1/2*(b*d*sign(b*x^2 + a) - a*e*
sign(b*x^2 + a))*ln(abs(b*x^2 + a))/(a*b)
```

3.79 $\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}} \, dx$

Optimal. Leaf size=101

$$-\frac{d\left(a+bx^2\right)}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -((d*(a + b*x^2))/(a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - ((b*d
- a*e)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.194222, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{d\left(a+bx^2\right)}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -((d*(a + b*x^2))/(a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - ((b*d
- a*e)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/x**2/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0583863, size = 72, normalized size = 0.71

$$\frac{(a+bx^2)\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(aex-bdx)-\sqrt{a}\sqrt{b}d\right)}{a^{3/2}\sqrt{b}x\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*(-(Sqrt[a]*Sqrt[b]*d) + (-(b*d*x) + a*e*x)*ArcTan[(S
qrt[b]*x)/Sqrt[a]]))/(a^(3/2)*Sqrt[b]*x*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.013, size = 67, normalized size = 0.7

$$-\frac{bx^2+a}{ax}\left(-\arctan\left(bx\frac{1}{\sqrt{ab}}\right)xae+\arctan\left(bx\frac{1}{\sqrt{ab}}\right)xbd+d\sqrt{ab}\right)\frac{1}{\sqrt{(bx^2+a)^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -(b*x^2+a)*(-arctan(x*b/(a*b)^(1/2))*x*a*e+arctan(x*b/(a*b)^(1/2)
)*x*b*d+d*(a*b)^(1/2))/((b*x^2+a)^2)^(1/2)/a/(a*b)^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/(sqrt((b*x^2 + a)^2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284924, size = 1, normalized size = 0.01

$$\left[-\frac{(bd-ae)x\log\left(\frac{2\,abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)+2\sqrt{-ab}d}{2\sqrt{-ab}ax},-\frac{(bd-ae)x\arctan\left(\frac{\sqrt{ab}x}{a}\right)+\sqrt{ab}d}{\sqrt{ab}ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/(sqrt((b*x^2 + a)^2)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b*d - a*e)*x*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^
2 + a)) + 2*sqrt(-a*b)*d)/(sqrt(-a*b)*a*x), -((b*d - a*e)*x*arcta
n(sqrt(a*b)*x/a) + sqrt(a*b)*d)/(sqrt(a*b)*a*x)]
```

Sympy [A] time = 1.8109, size = 82, normalized size = 0.81

$$-\frac{\sqrt{-\frac{1}{a^3b}}\,(ae-bd)\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^3b}}\,(ae-bd)\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}-\frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x**2/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -sqrt(-1/(a**3*b))*(a*e - b*d)*log(-a**2*sqrt(-1/(a**3*b)) + x)/2
 + sqrt(-1/(a**3*b))*(a*e - b*d)*log(a**2*sqrt(-1/(a**3*b)) + x)/
2 - d/(a*x)
```

GIAC/XCAS [A] time = 0.265662, size = 84, normalized size = 0.83

$$-\frac{\left(bd\mathrm{sign}\left(bx^2+a\right)-ae\mathrm{sign}\left(bx^2+a\right)\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}-\frac{d\mathrm{sign}\left(bx^2+a\right)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/(sqrt((b*x^2 + a)^2)*x^2),x, algorithm="giac")
```

```
[Out] -(b*d*sign(b*x^2 + a) - a*e*sign(b*x^2 + a))*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*a) - d*sign(b*x^2 + a)/(a*x)
```

3.80 $\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=137

$$-\frac{\log(x)\left(a+bx^2\right)(bd-ae)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(bd-ae)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -(d*(a + b*x^2))/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*
d - a*e)*(a + b*x^2)*Log[x])/(a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + ((b*d - a*e)*(a + b*x^2)*Log[a + b*x^2])/(2*a^2*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.262672, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{\log(x)\left(a+bx^2\right)(bd-ae)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(bd-ae)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(d*(a + b*x^2))/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*
d - a*e)*(a + b*x^2)*Log[x])/(a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + ((b*d - a*e)*(a + b*x^2)*Log[a + b*x^2])/(2*a^2*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/x**3/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0560914, size = 70, normalized size = 0.51

$$\frac{(a+bx^2)\left(2x^2\log(x)(ae-bd)+x^2(bd-ae)\log\left(a+bx^2\right)-ad\right)}{2a^2x^2\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*(-(a*d) + 2*(-(b*d) + a*e)*x^2*Log[x] + (b*d - a*e)*
x^2*Log[a + b*x^2]))/(2*a^2*x^2*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.017, size = 78, normalized size = 0.6

$$-\frac{(bx^2+a)\left(\ln\left(bx^2+a\right)x^2ae-\ln\left(bx^2+a\right)x^2bd-2\ln(x)x^2ae+2\ln(x)x^2bd+ad\right)}{2x^2a^2}\frac{1}{\sqrt{(bx^2+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/2*(b*x^2+a)*(ln(b*x^2+a)*x^2*a*e-ln(b*x^2+a)*x^2*b*d-2*ln(x)*x
^2*a*e+2*ln(x)*x^2*b*d+a*d)/((b*x^2+a)^2)^(1/2)/x^2/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/(sqrt((b*x^2 + a)^2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266636, size = 65, normalized size = 0.47

$$\frac{(bd-ae)x^2\log\left(bx^2+a\right)-2(bd-ae)x^2\log(x)-ad}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(sqrt((b*x^2 + a)^2)*x^3),x, algorithm="fricas")
```

```
[Out] 1/2*((b*d - a*e)*x^2*log(b*x^2 + a) - 2*(b*d - a*e)*x^2*log(x) -
a*d)/(a^2*x^2)
```

Sympy [A]   time = 2.9685, size = 41, normalized size = 0.3

$$-\frac{d}{2ax^2} + \frac{(ae-bd)\log(x)}{a^2} - \frac{(ae-bd)\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/x**3/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -d/(2*a*x**2) + (a*e - b*d)*log(x)/a**2 - (a*e - b*d)*log(a/b + x
**2)/(2*a**2)
```

GIAC/XCAS [A]   time = 0.265249, size = 177, normalized size = 1.29

$$-\frac{\left(bd\mathrm{sign}\left(bx^2+a\right) - ae\mathrm{sign}\left(bx^2+a\right)\right)\ln\left(x^2\right)}{2\,a^2} + \frac{\left(b^2d\mathrm{sign}\left(bx^2+a\right) - abe\mathrm{sign}\left(bx^2+a\right)\right)\ln\left(\left|bx^2+a\right|\right)}{2\,a^2b} + \frac{bdx^2\mathrm{sign}\left(bx^2+a\right) - ax^2e\mathrm{sign}\left(bx^2+a\right) - ad\mathrm{sign}\left(bx^2+a\right)}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(sqrt((b*x^2 + a)^2)*x^3),x, algorithm="giac")
```

```
[Out] -1/2*(b*d*sign(b*x^2 + a) - a*e*sign(b*x^2 + a))*ln(x^2)/a^2 + 1/
2*(b^2*d*sign(b*x^2 + a) - a*b*e*sign(b*x^2 + a))*ln(abs(b*x^2 +
a))/(a^2*b) + 1/2*(b*d*x^2*sign(b*x^2 + a) - a*x^2*e*sign(b*x^2 +
 a) - a*d*sign(b*x^2 + a))/(a^2*x^2)
```

3.81 $\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=153

$$\frac{x(bd-5ae)}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x(bd-ae)}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)(3ae+bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((b*d - 5*a*e)*x)/(8*a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b
*d - a*e)*x)/(4*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
+ ((b*d + 3*a*e)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3
/2)*b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.32588, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$

$$\frac{x(bd-5ae)}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x(bd-ae)}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)(3ae+bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```


```
[Out] ((b*d - 5*a*e)*x)/(8*a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b
*d - a*e)*x)/(4*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
+ ((b*d + 3*a*e)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3
/2)*b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.108796, size = 108, normalized size = 0.71

$$\frac{\left(a+bx^2\right)^2(3ae+bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\sqrt{a}\sqrt{b}x\left(3a^2e+ab\left(d+5ex^2\right)-b^2dx^2\right)}{8a^{3/2}b^{5/2}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-(Sqrt[a]*Sqrt[b]*x*(3*a^2*e - b^2*d*x^2 + a*b*(d + 5*e*x^2))) +
 (b*d + 3*a*e)*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3
/2)*b^(5/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.024, size = 188, normalized size = 1.2

$$-\frac{bx^2+a}{8\,ab^2}\left(-3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^4ab^2e-\arctan\left(bx\frac{1}{\sqrt{ab}}\right)x^4b^3d+5\,\sqrt{ab}x^3abe-\sqrt{ab}x^3b^2d-6\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^2a^2be-2\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/8*(-3*arctan(x*b/(a*b)^(1/2))*x^4*a*b^2*e-arctan(x*b/(a*b)^(1/
2))*x^4*b^3*d+5*(a*b)^(1/2)*x^3*a*b*e-(a*b)^(1/2)*x^3*b^2*d-6*arc
tan(x*b/(a*b)^(1/2))*x^2*a^2*b*e-2*arctan(x*b/(a*b)^(1/2))*x^2*a*
b^2*d+3*(a*b)^(1/2)*x*a^2*e+(a*b)^(1/2)*x*a*b*d-3*arctan(x*b/(a*b
)^(1/2))*a^3*e-arctan(x*b/(a*b)^(1/2))*a^2*b*d)*(b*x^2+a)/(a*b)^(
1/2)/b^2/a/((b*x^2+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="maxima
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.267169, size = 1, normalized size = 0.01

$$\left[\frac{\left(\left(b^3d+3\,ab^2e\right)x^4+a^2bd+3\,a^3e+2\,\left(ab^2d+3\,a^2be\right)x^2\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)+2\,\left(\left(b^2d-5\,abe\right)x^3-\left(abd+3\,a^2e\right)x\right)}{16\,\left(ab^4x^4+2\,a^2b^3x^2+a^3b^2\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fricas
```

```
[Out] [1/16*(((b^3*d + 3*a*b^2*e)*x^4 + a^2*b*d + 3*a^3*e + 2*(a*b^2*d
+ 3*a^2*b*e)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 +
 a)) + 2*((b^2*d - 5*a*b*e)*x^3 - (a*b*d + 3*a^2*e)*x)*sqrt(-a*b)
)/((a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*sqrt(-a*b)), 1/8*(((b^3*
d + 3*a*b^2*e)*x^4 + a^2*b*d + 3*a^3*e + 2*(a*b^2*d + 3*a^2*b*e)*
x^2)*arctan(sqrt(a*b)*x/a) + ((b^2*d - 5*a*b*e)*x^3 - (a*b*d + 3*
a^2*e)*x)*sqrt(a*b))/((a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*sqrt(
a*b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(d+ex^2\right)}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**2*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.616807, size = 4, normalized size = 0.03

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.82 $\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{bd-ae}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -e/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*d - a*e)/(4*b^2*(
a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.180047, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$-\frac{bd-ae}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -e/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*d - a*e)/(4*b^2*(
a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [A] time = 18.0144, size = 48, normalized size = 0.62

$$\frac{(2a+2bx^2)(d+ex^2)^2}{8(ae-bd)(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] (2*a + 2*b*x**2)*(d + e*x**2)**2/(8*(a*e - b*d)*(a**2 + 2*a*b*x**
2 + b**2*x**4)**(3/2))
```

Mathematica [A] time = 0.0415956, size = 45, normalized size = 0.58

$$\frac{-ae-b(d+2ex^2)}{4b^2(a+bx^2)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-(a*e) - b*(d + 2*e*x^2))/(4*b^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2]
)
```

Maple [A]   time = 0.008, size = 38, normalized size = 0.5

$$-\frac{(bx^2+a)\,(2\,x^2be+ae+bd)}{4\,b^2}\left((bx^2+a)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/4*(b*x^2+a)*(2*b*e*x^2+a*e+b*d)/b^2/((b*x^2+a)^2)^(3/2)
```

Maxima [A]   time = 0.709258, size = 96, normalized size = 1.25

$$-\frac{1}{4}\,e\left(\frac{2}{\sqrt{b^2x^4+2\,abx^2+a^2}b^2}-\frac{a}{(b^2)^{\frac{3}{2}}\left(x^2+\frac{a}{b}\right)^2b}\right)-\frac{d}{4\,(b^2)^{\frac{3}{2}}\left(x^2+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*x/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*e*(2/(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*b^2) - a/((b^2)^(3/2)*
(x^2 + a/b)^2*b)) - 1/4*d/((b^2)^(3/2)*(x^2 + a/b)^2)
```

Fricas [A]   time = 0.258708, size = 57, normalized size = 0.74

$$-\frac{2\,bex^2+bd+ae}{4\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*x/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*e*x^2 + b*d + a*e)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(d+ex^2\right)}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.629919, size = 4, normalized size = 0.05

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*x/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.83 $\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=156

$$\frac{x(ae+3bd)}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x(bd-ae)}{4ab\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\,(ae+3bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((3*b*d + a*e)*x)/(8*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b
*d - a*e)*x)/(4*a*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
+ ((3*b*d + a*e)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5
/2)*b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.201955, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{x(ae+3bd)}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x(bd-ae)}{4ab\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\,(ae+3bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((3*b*d + a*e)*x)/(8*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b
*d - a*e)*x)/(4*a*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
+ ((3*b*d + a*e)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5
/2)*b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.0971276, size = 108, normalized size = 0.69

$$\frac{\sqrt{a}\sqrt{b}x\left(a^2(-e)+ab\left(5d+ex^2\right)+3b^2dx^2\right)+\left(a+bx^2\right)^2(ae+3bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-(a^2*e) + 3*b^2*d*x^2 + a*b*(5*d + e*x^2)) +
 (3*b*d + a*e)*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5
/2)*b^(3/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.02, size = 186, normalized size = 1.2

$$\frac{bx^2+a}{8\,a^2b}\left(\arctan\left(bx\frac{1}{\sqrt{ab}}\right)x^4ab^2e+3\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^4b^3d+\sqrt{ab}x^3abe+3\sqrt{ab}x^3b^2d+2\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^2a^2be+6\ \mathrm{arctan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*(arctan(x*b/(a*b)^(1/2))*x^4*a*b^2*e+3*arctan(x*b/(a*b)^(1/2)
)*x^4*b^3*d+(a*b)^(1/2)*x^3*a*b*e+3*(a*b)^(1/2)*x^3*b^2*d+2*arcta
n(x*b/(a*b)^(1/2))*x^2*a^2*b*e+6*arctan(x*b/(a*b)^(1/2))*x^2*a*b^
2*d-(a*b)^(1/2)*x*a^2*e+5*(a*b)^(1/2)*x*a*b*d+arctan(x*b/(a*b)^(1
/2))*a^3*e+3*arctan(x*b/(a*b)^(1/2))*a^2*b*d)*(b*x^2+a)/(a*b)^(1/
2)/b/a^2/((b*x^2+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272678, size = 1, normalized size = 0.01

$$\left[\frac{\left(\left(3\,b^3d + ab^2e\right)x^4 + 3\,a^2bd + a^3e + 2\left(3\,ab^2d + a^2be\right)x^2\right)\log\left(\frac{2\,abx + \left(bx^2 - a\right)\sqrt{-ab}}{bx^2 + a}\right) + 2\left(\left(3\,b^2d + abe\right)x^3 + \left(5\,abd - a^2e\right)x\right)\sqrt{}}{16\left(a^2b^3x^4 + 2\,a^3b^2x^2 + a^4b\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((3*b^3*d + a*b^2*e)*x^4 + 3*a^2*b*d + a^3*e + 2*(3*a*b^2*
d + a^2*b*e)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 +
 a)) + 2*((3*b^2*d + a*b*e)*x^3 + (5*a*b*d - a^2*e)*x)*sqrt(-a*b)
)/((a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*sqrt(-a*b)), 1/8*(((3*b^
3*d + a*b^2*e)*x^4 + 3*a^2*b*d + a^3*e + 2*(3*a*b^2*d + a^2*b*e)*
x^2)*arctan(sqrt(a*b)*x/a) + ((3*b^2*d + a*b*e)*x^3 + (5*a*b*d -
a^2*e)*x)*sqrt(a*b))/((a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*sqrt(
a*b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^2}{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.627621, size = 4, normalized size = 0.03

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.84 $\int \frac{d+ex^2}{x(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=161

$$\frac{bd-ae}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\log(x)\left(a+bx^2\right)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] d/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*d - a*e)/(4*a*b*(a
 + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*(a + b*x^2)*Log[x
])/(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2)*Log[a +
 b*x^2])/(2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.31255, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{bd-ae}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\log(x)\left(a+bx^2\right)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] d/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*d - a*e)/(4*a*b*(a
 + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*(a + b*x^2)*Log[x
])/(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2)*Log[a +
 b*x^2])/(2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [A] time = 43.4842, size = 163, normalized size = 1.01

$$-\frac{(ae-bd)\sqrt{a^2+2abx^2+b^2x^4}}{4ab\left(a+bx^2\right)^3}+\frac{d\sqrt{a^2+2abx^2+b^2x^4}}{2a^2\left(a+bx^2\right)^2}+\frac{d\sqrt{a^2+2abx^2+b^2x^4}\log\left(x^2\right)}{2a^3\left(a+bx^2\right)}-\frac{d\sqrt{a^2+2abx^2+b^2x^4}\log\left(a+bx^2\right)}{2a^3\left(a+bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] -(a*e - b*d)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(4*a*b*(a + b*x*
*2)**3) + d*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(2*a**2*(a + b*x*
*2)**2) + d*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)*log(x**2)/(2*a**3
*(a + b*x**2)) - d*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)*log(a + b*
x**2)/(2*a**3*(a + b*x**2))
```

Mathematica [A] time = 0.075948, size = 92, normalized size = 0.57

$$\frac{a\left(a^2(-e)+3abd+2b^2dx^2\right)+4bd\log(x)\left(a+bx^2\right)^2-2bd\left(a+bx^2\right)^2\log\left(a+bx^2\right)}{4a^3b\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (a*(3*a*b*d - a^2*e + 2*b^2*d*x^2) + 4*b*d*(a + b*x^2)^2*Log[x] -
 2*b*d*(a + b*x^2)^2*Log[a + b*x^2])/(4*a^3*b*(a + b*x^2)*Sqrt[(a
 + b*x^2)^2])
```

Maple [A] time = 0.029, size = 132, normalized size = 0.8

$$-\frac{\left(2\ln\left(bx^2+a\right)x^4b^3d-4\ln(x)x^4b^3d+4\ln\left(bx^2+a\right)x^2ab^2d-8\ln(x)x^2ab^2d-2b^2dx^2a+2\ln\left(bx^2+a\right)a^2bd-4\ln(\right.}{4a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/4*(2*ln(b*x^2+a)*x^4*b^3*d-4*ln(x)*x^4*b^3*d+4*ln(b*x^2+a)*x^2
*a*b^2*d-8*ln(x)*x^2*a*b^2*d-2*b^2*d*x^2*a+2*ln(b*x^2+a)*a^2*b*d-
4*ln(x)*a^2*b*d+a^3*e-3*a^2*b*d)*(b*x^2+a)/b/a^3/((b*x^2+a)^2)^(3
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x),x, algorithm="maxima
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266362, size = 161, normalized size = 1.

$$\frac{2\,ab^2dx^2+3\,a^2bd-a^3e-2\left(b^3dx^4+2\,ab^2dx^2+a^2bd\right)\log\left(bx^2+a\right)+4\left(b^3dx^4+2\,ab^2dx^2+a^2bd\right)\log\left(x\right)}{4\left(a^3b^3x^4+2\,a^4b^2x^2+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x),x, algorithm="fricas
```

```
[Out] 1/4*(2*a*b^2*d*x^2 + 3*a^2*b*d - a^3*e - 2*(b^3*d*x^4 + 2*a*b^2*d
*x^2 + a^2*b*d)*log(b*x^2 + a) + 4*(b^3*d*x^4 + 2*a*b^2*d*x^2 + a
^2*b*d)*log(x))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{d+ex^2}{x\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(x*((a + b*x**2)**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.626014, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.85 $\int \frac{d+ex^2}{x^2(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=190

$$-\frac{x(bd-ae)}{4a^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3(a+bx^2)(5bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x(7bd-3ae)}{8a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(a+bx^2)}{a^3x\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -((7*b*d - 3*a*e)*x)/(8*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((
b*d - a*e)*x)/(4*a^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 - (d*(a + b*x^2))/(a^3*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*(
5*b*d - a*e)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*
Sqrt[b]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.472031, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$

$$-\frac{x(bd-ae)}{4a^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3(a+bx^2)(5bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x(7bd-3ae)}{8a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(a+bx^2)}{a^3x\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] -((7*b*d - 3*a*e)*x)/(8*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((
b*d - a*e)*x)/(4*a^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 - (d*(a + b*x^2))/(a^3*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*(
5*b*d - a*e)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*
Sqrt[b]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((e*x**2+d)/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.13916, size = 124, normalized size = 0.65

$$\frac{\sqrt{a}\sqrt{b}\left(a^2\left(5ex^2-8d\right)+ab\left(3ex^4-25dx^2\right)-15b^2dx^4\right)+3x\left(a+bx^2\right)^2\left(ae-5bd\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}x\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*(-15*b^2*d*x^4 + a^2*(-8*d + 5*e*x^2) + a*b*(-25
*d*x^2 + 3*e*x^4)) + 3*(-5*b*d + a*e)*x*(a + b*x^2)^2*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]*x*(a + b*x^2)*Sqrt[(a + b*x^
2)^2])
```

Maple [A] time = 0.027, size = 206, normalized size = 1.1

$$\frac{bx^2+a}{8\,xa^3}\left(3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^5ab^2e-15\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^5b^3d+6\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^3a^2be-30\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^3ab^2d+3\sqrt{ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*(3*arctan(x*b/(a*b)^(1/2))*x^5*a*b^2*e-15*arctan(x*b/(a*b)^(1
/2))*x^5*b^3*d+6*arctan(x*b/(a*b)^(1/2))*x^3*a^2*b*e-30*arctan(x*
b/(a*b)^(1/2))*x^3*a*b^2*d+3*(a*b)^(1/2)*x^4*a*b*e-15*(a*b)^(1/2)
*x^4*b^2*d+3*arctan(x*b/(a*b)^(1/2))*x*a^3*e-15*arctan(x*b/(a*b)^
(1/2))*x*a^2*b*d+5*(a*b)^(1/2)*x^2*a^2*e-25*(a*b)^(1/2)*x^2*a*b*d
-8*(a*b)^(1/2)*a^2*d)*(b*x^2+a)/x/(a*b)^(1/2)/a^3/((b*x^2+a)^2)^(
3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^2),x, algorithm="maxi
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.281941, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(\left(5\,b^3d-ab^2e\right)x^5+2\left(5\,ab^2d-a^2be\right)x^3+\left(5\,a^2bd-a^3e\right)x\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)+2\left(3\left(5\,b^2d-abe\right)x^4+8\,a^2d+\right.}{16\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)\sqrt{-ab}}\right.$$

$$-\frac{3\left(\left(5\,b^3d-ab^2e\right)x^5+2\left(5\,ab^2d-a^2be\right)x^3+\left(5\,a^2bd-a^3e\right)x\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right)+\left(3\left(5\,b^2d-abe\right)x^4+8\,a^2d+5\left(5\,abd-a\right.\right.}{8\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^2),x, algorithm="fric
```

```
[Out] [-1/16*(3*((5*b^3*d - a*b^2*e)*x^5 + 2*(5*a*b^2*d - a^2*b*e)*x^3
+ (5*a^2*b*d - a^3*e)*x)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(
b*x^2 + a)) + 2*(3*(5*b^2*d - a*b*e)*x^4 + 8*a^2*d + 5*(5*a*b*d -
 a^2*e)*x^2)*sqrt(-a*b))/((a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*sqr
t(-a*b)), -1/8*(3*((5*b^3*d - a*b^2*e)*x^5 + 2*(5*a*b^2*d - a^2*b
*e)*x^3 + (5*a^2*b*d - a^3*e)*x)*arctan(sqrt(a*b)*x/a) + (3*(5*b^
2*d - a*b*e)*x^4 + 8*a^2*d + 5*(5*a*b*d - a^2*e)*x^2)*sqrt(a*b))/
((a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*sqrt(a*b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex^2}{x^2\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(x**2*((a + b*x**2)**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.636847, size = 4, normalized size = 0.02

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^2),x, algorithm="giac
```

```
[Out] sage0*x
```

3.86 $\int \frac{d+ex^2}{x^3(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=223

$$-\frac{bd-ae}{4a^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\log(x)\left(a+bx^2\right)(3bd-ae)}{a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(3bd-ae)\log\left(a+bx^2\right)}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2bd-ae}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{2a^3x^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -(2*b*d - a*e)/(2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*d - a
*e)/(4*a^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(a +
 b*x^2))/(2*a^3*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((3*b*d -
a*e)*(a + b*x^2)*Log[x])/(a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) +
((3*b*d - a*e)*(a + b*x^2)*Log[a + b*x^2])/(2*a^4*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.429396, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{bd-ae}{4a^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\log(x)\left(a+bx^2\right)(3bd-ae)}{a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(3bd-ae)\log\left(a+bx^2\right)}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2bd-ae}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{2a^3x^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] -(2*b*d - a*e)/(2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*d - a
*e)/(4*a^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(a +
 b*x^2))/(2*a^3*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((3*b*d -
a*e)*(a + b*x^2)*Log[x])/(a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) +
((3*b*d - a*e)*(a + b*x^2)*Log[a + b*x^2])/(2*a^4*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])
```

Rubi in Sympy [A] time = 59.0125, size = 223, normalized size = 1.

$$\frac{(ae-bd)\sqrt{a^2+2abx^2+b^2x^4}}{4a^2\left(a+bx^2\right)^3}-\frac{d\sqrt{a^2+2abx^2+b^2x^4}}{2a^3x^2\left(a+bx^2\right)}+\frac{(ae-2bd)\sqrt{a^2+2abx^2+b^2x^4}}{2a^3\left(a+bx^2\right)^2}+\frac{(ae-3bd)\sqrt{a^2+2abx^2+b^2x^4}\log\left(x^2\right)}{2a^4\left(a+bx^2\right)}-\frac{(ae-3bd)\sqrt{a^2+2abx^2+b^2x^4}\log\left(a+bx^2\right)}{2a^4\left(a+bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((e*x**2+d)/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] (a*e - b*d)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(4*a**2*(a + b*x*
*2)**3) - d*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(2*a**3*x**2*(a +
 b*x**2)) + (a*e - 2*b*d)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(2*
a**3*(a + b*x**2)**2) + (a*e - 3*b*d)*sqrt(a**2 + 2*a*b*x**2 + b*
*2*x**4)*log(x**2)/(2*a**4*(a + b*x**2)) - (a*e - 3*b*d)*sqrt(a**
2 + 2*a*b*x**2 + b**2*x**4)*log(a + b*x**2)/(2*a**4*(a + b*x**2))
```

Mathematica [A] time = 0.129722, size = 130, normalized size = 0.58

$$\frac{a\left(a^2\left(3ex^2-2d\right)+ab\left(2ex^4-9dx^2\right)-6b^2dx^4\right)+4x^2\log(x)\left(a+bx^2\right)^2(ae-3bd)+2x^2\left(a+bx^2\right)^2(3bd-ae)\log\left(a+bx^2\right)}{4a^4x^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (a*(-6*b^2*d*x^4 + a^2*(-2*d + 3*e*x^2) + a*b*(-9*d*x^2 + 2*e*x^4
)) + 4*(-3*b*d + a*e)*x^2*(a + b*x^2)^2*Log[x] + 2*(3*b*d - a*e)*
x^2*(a + b*x^2)^2*Log[a + b*x^2])/(4*a^4*x^2*(a + b*x^2)*Sqrt[(a
+ b*x^2)^2])
```

Maple [A] time = 0.033, size = 249, normalized size = 1.1

$$-\frac{\left(2\ln\left(bx^2+a\right)x^6ab^2e-6\ln\left(bx^2+a\right)x^6b^3d-4\ln(x)x^6ab^2e+12\ln(x)x^6b^3d+4\ln\left(bx^2+a\right)x^4a^2be-12\ln\left(bx^2+a\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/4*(2*ln(b*x^2+a)*x^6*a*b^2*e-6*ln(b*x^2+a)*x^6*b^3*d-4*ln(x)*x
^6*a*b^2*e+12*ln(x)*x^6*b^3*d+4*ln(b*x^2+a)*x^4*a^2*b*e-12*ln(b*x
^2+a)*x^4*a*b^2*d-8*ln(x)*x^4*a^2*b*e+24*ln(x)*x^4*a*b^2*d-2*x^4*
a^2*b*e+6*x^4*a*b^2*d+2*ln(b*x^2+a)*x^2*a^3*e-6*ln(b*x^2+a)*x^2*a
^2*b*d-4*ln(x)*x^2*a^3*e+12*ln(x)*x^2*a^2*b*d-3*x^2*a^3*e+9*x^2*a
^2*b*d+2*a^3*d)*(b*x^2+a)/x^2/a^4/((b*x^2+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^3),x, algorithm="maxi
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274007, size = 277, normalized size = 1.24

$$-\frac{2\left(3\,ab^2d-a^2be\right)x^4+2\,a^3d+3\left(3\,a^2bd-a^3e\right)x^2-2\left(\left(3\,b^3d-ab^2e\right)x^6+2\left(3\,ab^2d-a^2be\right)x^4+\left(3\,a^2bd-a^3e\right)x^2\right)\log\left(b\right.}{4\left(a^4b^2x^6+2\,a^5bx^4+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^3),x, algorithm="fric
```

```
[Out] -1/4*(2*(3*a*b^2*d - a^2*b*e)*x^4 + 2*a^3*d + 3*(3*a^2*b*d - a^3*
e)*x^2 - 2*((3*b^3*d - a*b^2*e)*x^6 + 2*(3*a*b^2*d - a^2*b*e)*x^4
 + (3*a^2*b*d - a^3*e)*x^2)*log(b*x^2 + a) + 4*((3*b^3*d - a*b^2*
e)*x^6 + 2*(3*a*b^2*d - a^2*b*e)*x^4 + (3*a^2*b*d - a^3*e)*x^2)*l
og(x))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex^2}{x^3\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(x**3*((a + b*x**2)**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.604119, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^3),x, algorithm="giac
```

```
[Out] sage0*x
```

3.87 $\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=400

$$\frac{10a^2b^2\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+7}(ae+bd)}{f^7(m+7)(a+bx^2)}+\frac{b^5e\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+13}}{f^{13}(m+13)(a+bx^2)}+\frac{b^4\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+11}(5ae+bd)}{f^{11}(m+11)(a+bx^2)}+\frac{5ab^3\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+9}(2ae+bd)}{f^9(m+9)(a+bx^2)}+\frac{a^5d\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+1}}{f(m+1)(a+bx^2)}+\frac{a^4\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+3}(ae+5bd)}{f^3(m+3)(a+bx^2)}+\frac{5a^3b\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+5}(ae+2bd)}{f^5(m+5)(a+bx^2)}$$

```
[Out] (a^5*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*
(a + b*x^2)) + (a^4*(5*b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(f^3*(3 + m)*(a + b*x^2)) + (5*a^3*b*(2*b*d + a*e
)*(f*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^5*(5 + m)*(a
+ b*x^2)) + (10*a^2*b^2*(b*d + a*e)*(f*x)^(7 + m)*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(f^7*(7 + m)*(a + b*x^2)) + (5*a*b^3*(b*d + 2*a
*e)*(f*x)^(9 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^9*(9 + m)*(
a + b*x^2)) + (b^4*(b*d + 5*a*e)*(f*x)^(11 + m)*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(f^11*(11 + m)*(a + b*x^2)) + (b^5*e*(f*x)^(13 +
m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^13*(13 + m)*(a + b*x^2))
```

Rubi [A] time = 0.587138, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$

$$\frac{10a^2b^2\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+7}(ae+bd)}{f^7(m+7)(a+bx^2)}+\frac{b^5e\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+13}}{f^{13}(m+13)(a+bx^2)}+\frac{b^4\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+11}(5ae+bd)}{f^{11}(m+11)(a+bx^2)}+\frac{5ab^3\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+9}(2ae+bd)}{f^9(m+9)(a+bx^2)}+\frac{a^5d\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+1}}{f(m+1)(a+bx^2)}+\frac{a^4\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+3}(ae+5bd)}{f^3(m+3)(a+bx^2)}+\frac{5a^3b\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+5}(ae+2bd)}{f^5(m+5)(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*
(a + b*x^2)) + (a^4*(5*b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(f^3*(3 + m)*(a + b*x^2)) + (5*a^3*b*(2*b*d + a*e
```

```
)*(f*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^5*(5 + m)*(a
+ b*x^2)) + (10*a^2*b^2*(b*d + a*e)*(f*x)^(7 + m)*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(f^7*(7 + m)*(a + b*x^2)) + (5*a*b^3*(b*d + 2*a
*e)*(f*x)^(9 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^9*(9 + m)*(
a + b*x^2)) + (b^4*(b*d + 5*a*e)*(f*x)^(11 + m)*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(f^11*(11 + m)*(a + b*x^2)) + (b^5*e*(f*x)^(13 +
m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^13*(13 + m)*(a + b*x^2))
```

Rubi in Sympy [A] time = 85.5961, size = 374, normalized size = 0.94

$$\frac{a^5 d (fx)^{m+1} \sqrt{a^2 + 2abx^2 + b^2x^4}}{f(a + bx^2)(m+1)} + \frac{a^4 (fx)^{m+3} (ae + 5bd) \sqrt{a^2 + 2abx^2 + b^2x^4}}{f^3 (a + bx^2)(m+3)} + \frac{5a^3 b (fx)^{m+5} (ae + 2bd) \sqrt{a^2 + 2abx^2 + b^2x^4}}{f^5 (a + bx^2)(m+5)} + \frac{10a^2 b^2 (fx)^{m+7} (ae + bd) \sqrt{a^2 + 2abx^2 + b^2x^4}}{f^7 (a + bx^2)(m+7)} + \frac{5ab^3 (fx)^{m+9} (2ae + bd) \sqrt{a^2 + 2abx^2 + b^2x^4}}{f^9 (a + bx^2)(m+9)} + \frac{b^5 e (fx)^{m+13} \sqrt{a^2 + 2abx^2 + b^2x^4}}{f^{13} (a + bx^2)(m+13)} + \frac{b^4 (fx)^{m+11} (5ae + bd) \sqrt{a^2 + 2abx^2 + b^2x^4}}{f^{11} (a + bx^2)(m+11)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] a**5*d*(f*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(f*(a +
 b*x**2)*(m + 1)) + a**4*(f*x)**(m + 3)*(a*e + 5*b*d)*sqrt(a**2 +
 2*a*b*x**2 + b**2*x**4)/(f**3*(a + b*x**2)*(m + 3)) + 5*a**3*b*(
f*x)**(m + 5)*(a*e + 2*b*d)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(
f**5*(a + b*x**2)*(m + 5)) + 10*a**2*b**2*(f*x)**(m + 7)*(a*e + b
*d)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(f**7*(a + b*x**2)*(m + 7
)) + 5*a*b**3*(f*x)**(m + 9)*(2*a*e + b*d)*sqrt(a**2 + 2*a*b*x**2
 + b**2*x**4)/(f**9*(a + b*x**2)*(m + 9)) + b**5*e*(f*x)**(m + 13
)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(f**13*(a + b*x**2)*(m + 13
)) + b**4*(f*x)**(m + 11)*(5*a*e + b*d)*sqrt(a**2 + 2*a*b*x**2 +
b**2*x**4)/(f**11*(a + b*x**2)*(m + 11))
```

Mathematica [A] time = 0.302714, size = 160, normalized size = 0.4

$$\frac{\sqrt{(a+bx^2)^2}(fx)^m \left(\frac{a^5 dx}{m+1} + \frac{a^4 x^3 (ae+5bd)}{m+3} + \frac{5a^3 bx^5 (ae+2bd)}{m+5} + \frac{10a^2 b^2 x^7 (ae+bd)}{m+7} + \frac{b^4 x^{11} (5ae+bd)}{m+11} + \frac{5ab^3 x^9 (2ae+bd)}{m+9} + \frac{b^5 e x^{13}}{m+13}\right)}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((f*x)^m*Sqrt[(a + b*x^2)^2]*((a^5*d*x)/(1 + m) + (a^4*(5*b*d + a
*e)*x^3)/(3 + m) + (5*a^3*b*(2*b*d + a*e)*x^5)/(5 + m) + (10*a^2*
b^2*(b*d + a*e)*x^7)/(7 + m) + (5*a*b^3*(b*d + 2*a*e)*x^9)/(9 + m
) + (b^4*(b*d + 5*a*e)*x^11)/(11 + m) + (b^5*e*x^13)/(13 + m)))/(
a + b*x^2)
```

Maple [B] time = 0.014, size = 1099, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] x*(b^5*e*m^6*x^12+36*b^5*e*m^5*x^12+5*a*b^4*e*m^6*x^10+b^5*d*m^6*
x^10+505*b^5*e*m^4*x^12+190*a*b^4*e*m^5*x^10+38*b^5*d*m^5*x^10+34
80*b^5*e*m^3*x^12+10*a^2*b^3*e*m^6*x^8+5*a*b^4*d*m^6*x^8+2775*a*b
^4*e*m^4*x^10+555*b^5*d*m^4*x^10+12139*b^5*e*m^2*x^12+400*a^2*b^3
*e*m^5*x^8+200*a*b^4*d*m^5*x^8+19700*a*b^4*e*m^3*x^10+3940*b^5*d*
m^3*x^10+19524*b^5*e*m*x^12+10*a^3*b^2*e*m^6*x^6+10*a^2*b^3*d*m^6
*x^6+6130*a^2*b^3*e*m^4*x^8+3065*a*b^4*d*m^4*x^8+70195*a*b^4*e*m^
2*x^10+14039*b^5*d*m^2*x^10+10395*b^5*e*x^12+420*a^3*b^2*e*m^5*x^
6+420*a^2*b^3*d*m^5*x^6+45280*a^2*b^3*e*m^3*x^8+22640*a*b^4*d*m^3
*x^8+114510*a*b^4*e*m*x^10+22902*b^5*d*m*x^10+5*a^4*b*e*m^6*x^4+1
0*a^3*b^2*d*m^6*x^4+6790*a^3*b^2*e*m^4*x^6+6790*a^2*b^3*d*m^4*x^6
+166270*a^2*b^3*e*m^2*x^8+83135*a*b^4*d*m^2*x^8+61425*a*b^4*e*x^1
0+12285*b^5*d*x^10+220*a^4*b*e*m^5*x^4+440*a^3*b^2*d*m^5*x^4+5292
0*a^3*b^2*e*m^3*x^6+52920*a^2*b^3*d*m^3*x^6+276880*a^2*b^3*e*m*x^
8+138440*a*b^4*d*m*x^8+a^5*e*m^6*x^2+5*a^4*b*d*m^6*x^2+3765*a^4*b
*e*m^4*x^4+7530*a^3*b^2*d*m^4*x^4+203350*a^3*b^2*e*m^2*x^6+203350
*a^2*b^3*d*m^2*x^6+150150*a^2*b^3*e*x^8+75075*a*b^4*d*x^8+46*a^5*
e*m^5*x^2+230*a^4*b*d*m^5*x^2+31400*a^4*b*e*m^3*x^4+62800*a^3*b^2
*d*m^3*x^4+349860*a^3*b^2*e*m*x^6+349860*a^2*b^3*d*m*x^6+a^5*d*m^
6+835*a^5*e*m^4*x^2+4175*a^4*b*d*m^4*x^2+129895*a^4*b*e*m^2*x^4+2
59790*a^3*b^2*d*m^2*x^4+193050*a^3*b^2*e*x^6+193050*a^2*b^3*d*x^6
+48*a^5*d*m^5+7540*a^5*e*m^3*x^2+37700*a^4*b*d*m^3*x^2+237180*a^4
*b*e*m*x^4+474360*a^3*b^2*d*m*x^4+925*a^5*d*m^4+34759*a^5*e*m^2*x
^2+173795*a^4*b*d*m^2*x^2+135135*a^4*b*e*x^4+270270*a^3*b^2*d*x^4
+9120*a^5*d*m^3+73054*a^5*e*m*x^2+365270*a^4*b*d*m*x^2+48259*a^5*
d*m^2+45045*a^5*e*x^2+225225*a^4*b*d*x^2+129072*a^5*d*m+135135*a^
5*d)*(f*x)^m*((b*x^2+a)^2)^(5/2)/(13+m)/(11+m)/(9+m)/(7+m)/(5+m)/
(3+m)/(1+m)/(b*x^2+a)^5
```

Maxima [A] time = 0.736163, size = 663, normalized size = 1.66

$$\frac{\left(\left(m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945\right)b^5f^mx^{11}+5\left(m^5+27\,m^4+262\,m^3+1122\,m^2+2041\,m+1155\right)ab^4f^mx^9+\right.}{}$$
$$+\frac{\left(\left(m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395\right)b^5f^mx^{13}+5\left(m^5+37\,m^4+518\,m^3+3422\,m^2+10617\,m+12285\right)ab^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(e*x^2 + d)*(f*x)^m,x, algorithm="ma
```

```
[Out] ((m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)*b^5*f^m*x^11 +
 5*(m^5 + 27*m^4 + 262*m^3 + 1122*m^2 + 2041*m + 1155)*a*b^4*f^m*
x^9 + 10*(m^5 + 29*m^4 + 302*m^3 + 1366*m^2 + 2577*m + 1485)*a^2*
b^3*f^m*x^7 + 10*(m^5 + 31*m^4 + 350*m^3 + 1730*m^2 + 3489*m + 20
79)*a^3*b^2*f^m*x^5 + 5*(m^5 + 33*m^4 + 406*m^3 + 2262*m^2 + 5353
*m + 3465)*a^4*b*f^m*x^3 + (m^5 + 35*m^4 + 470*m^3 + 3010*m^2 + 9
129*m + 10395)*a^5*f^m*x)*d*x^m/(m^6 + 36*m^5 + 505*m^4 + 3480*m^
3 + 12139*m^2 + 19524*m + 10395) + ((m^5 + 35*m^4 + 470*m^3 + 301
0*m^2 + 9129*m + 10395)*b^5*f^m*x^13 + 5*(m^5 + 37*m^4 + 518*m^3
+ 3422*m^2 + 10617*m + 12285)*a*b^4*f^m*x^11 + 10*(m^5 + 39*m^4 +
 574*m^3 + 3954*m^2 + 12673*m + 15015)*a^2*b^3*f^m*x^9 + 10*(m^5
+ 41*m^4 + 638*m^3 + 4654*m^2 + 15681*m + 19305)*a^3*b^2*f^m*x^7
+ 5*(m^5 + 43*m^4 + 710*m^3 + 5570*m^2 + 20409*m + 27027)*a^4*b*f
^m*x^5 + (m^5 + 45*m^4 + 790*m^3 + 6750*m^2 + 28009*m + 45045)*a^
5*f^m*x^3)*e*x^m/(m^6 + 48*m^5 + 925*m^4 + 9120*m^3 + 48259*m^2 +
 129072*m + 135135)
```

Fricas [A] time = 0.287381, size = 1152, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(e*x^2 + d)*(f*x)^m,x, algorithm="fr
```

```
[Out] ((b^5*e*m^6 + 36*b^5*e*m^5 + 505*b^5*e*m^4 + 3480*b^5*e*m^3 + 121
39*b^5*e*m^2 + 19524*b^5*e*m + 10395*b^5*e)*x^13 + ((b^5*d + 5*a*
b^4*e)*m^6 + 12285*b^5*d + 61425*a*b^4*e + 38*(b^5*d + 5*a*b^4*e)
*m^5 + 555*(b^5*d + 5*a*b^4*e)*m^4 + 3940*(b^5*d + 5*a*b^4*e)*m^3
 + 14039*(b^5*d + 5*a*b^4*e)*m^2 + 22902*(b^5*d + 5*a*b^4*e)*m)*x
^11 + 5*((a*b^4*d + 2*a^2*b^3*e)*m^6 + 15015*a*b^4*d + 30030*a^2*
b^3*e + 40*(a*b^4*d + 2*a^2*b^3*e)*m^5 + 613*(a*b^4*d + 2*a^2*b^3
*e)*m^4 + 4528*(a*b^4*d + 2*a^2*b^3*e)*m^3 + 16627*(a*b^4*d + 2*a
^2*b^3*e)*m^2 + 27688*(a*b^4*d + 2*a^2*b^3*e)*m)*x^9 + 10*((a^2*b
^3*d + a^3*b^2*e)*m^6 + 19305*a^2*b^3*d + 19305*a^3*b^2*e + 42*(a
^2*b^3*d + a^3*b^2*e)*m^5 + 679*(a^2*b^3*d + a^3*b^2*e)*m^4 + 529
```

```
2*(a^2*b^3*d + a^3*b^2*e)*m^3 + 20335*(a^2*b^3*d + a^3*b^2*e)*m^2
 + 34986*(a^2*b^3*d + a^3*b^2*e)*m)*x^7 + 5*((2*a^3*b^2*d + a^4*b
*e)*m^6 + 54054*a^3*b^2*d + 27027*a^4*b*e + 44*(2*a^3*b^2*d + a^4
*b*e)*m^5 + 753*(2*a^3*b^2*d + a^4*b*e)*m^4 + 6280*(2*a^3*b^2*d +
 a^4*b*e)*m^3 + 25979*(2*a^3*b^2*d + a^4*b*e)*m^2 + 47436*(2*a^3*
b^2*d + a^4*b*e)*m)*x^5 + ((5*a^4*b*d + a^5*e)*m^6 + 225225*a^4*b
*d + 45045*a^5*e + 46*(5*a^4*b*d + a^5*e)*m^5 + 835*(5*a^4*b*d +
a^5*e)*m^4 + 7540*(5*a^4*b*d + a^5*e)*m^3 + 34759*(5*a^4*b*d + a^
5*e)*m^2 + 73054*(5*a^4*b*d + a^5*e)*m)*x^3 + (a^5*d*m^6 + 48*a^5
*d*m^5 + 925*a^5*d*m^4 + 9120*a^5*d*m^3 + 48259*a^5*d*m^2 + 12907
2*a^5*d*m + 135135*a^5*d)*x)*(f*x)^m/(m^7 + 49*m^6 + 973*m^5 + 10
045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.308387, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(e*x^2 + d)*(f*x)^m,x, algorithm="gi

[Out] Done
```

3.88 $\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=276

$$\frac{b^2\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+7}(3ae+bd)}{f^7(m+7)\,(a+bx^2)}+\frac{3ab\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+5}(ae+bd)}{f^5(m+5)\,(a+bx^2)}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+3}(ae+3bd)}{f^3(m+3)\,(a+bx^2)}+\frac{b^3e\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+9}}{f^9(m+9)\,(a+bx^2)}+\frac{a^3d\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+1}}{f(m+1)\,(a+bx^2)}$$

```
[Out] (a^3*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*
(a + b*x^2)) + (a^2*(3*b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(f^3*(3 + m)*(a + b*x^2)) + (3*a*b*(b*d + a*e)*(f
*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^5*(5 + m)*(a + b*
x^2)) + (b^2*(b*d + 3*a*e)*(f*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(f^7*(7 + m)*(a + b*x^2)) + (b^3*e*(f*x)^(9 + m)*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(f^9*(9 + m)*(a + b*x^2))
```

Rubi [A] time = 0.398662, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$

$$\frac{b^2\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+7}(3ae+bd)}{f^7(m+7)\,(a+bx^2)}+\frac{3ab\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+5}(ae+bd)}{f^5(m+5)\,(a+bx^2)}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+3}(ae+3bd)}{f^3(m+3)\,(a+bx^2)}+\frac{b^3e\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+9}}{f^9(m+9)\,(a+bx^2)}+\frac{a^3d\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+1}}{f(m+1)\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*
(a + b*x^2)) + (a^2*(3*b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(f^3*(3 + m)*(a + b*x^2)) + (3*a*b*(b*d + a*e)*(f
*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^5*(5 + m)*(a + b*
x^2)) + (b^2*(b*d + 3*a*e)*(f*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(f^7*(7 + m)*(a + b*x^2)) + (b^3*e*(f*x)^(9 + m)*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(f^9*(9 + m)*(a + b*x^2))
```

Rubi in Sympy [A] time = 61.7697, size = 255, normalized size = 0.92

$$\frac{a^3d(fx)^{m+1}\sqrt{a^2+2abx^2+b^2x^4}}{f(a+bx^2)(m+1)}+\frac{a^2(fx)^{m+3}(ae+3bd)\sqrt{a^2+2abx^2+b^2x^4}}{f^3(a+bx^2)(m+3)}+\frac{3ab(fx)^{m+5}(ae+bd)\sqrt{a^2+2abx^2+b^2x^4}}{f^5(a+bx^2)(m+5)}+\frac{b^3e(fx)^{m+9}\sqrt{a^2+2abx^2+b^2x^4}}{f^9(a+bx^2)(m+9)}+\frac{b^2(fx)^{m+7}(3ae+bd)\sqrt{a^2+2abx^2+b^2x^4}}{f^7(a+bx^2)(m+7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] a**3*d*(f*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(f*(a +
 b*x**2)*(m + 1)) + a**2*(f*x)**(m + 3)*(a*e + 3*b*d)*sqrt(a**2 +
 2*a*b*x**2 + b**2*x**4)/(f**3*(a + b*x**2)*(m + 3)) + 3*a*b*(f*x
)**(m + 5)*(a*e + b*d)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(f**5*
(a + b*x**2)*(m + 5)) + b**3*e*(f*x)**(m + 9)*sqrt(a**2 + 2*a*b*x
**2 + b**2*x**4)/(f**9*(a + b*x**2)*(m + 9)) + b**2*(f*x)**(m + 7
)*(3*a*e + b*d)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(f**7*(a + b*
x**2)*(m + 7))
```

Mathematica [A] time = 0.179688, size = 112, normalized size = 0.41

$$\frac{\left(\left(a+bx^2\right)^2\right)^{3/2}(fx)^m\left(\frac{a^3dx}{m+1}+\frac{a^2x^3(ae+3bd)}{m+3}+\frac{b^2x^7(3ae+bd)}{m+7}+\frac{3abx^5(ae+bd)}{m+5}+\frac{b^3ex^9}{m+9}\right)}{(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((f*x)^m*((a + b*x^2)^2)^(3/2)*((a^3*d*x)/(1 + m) + (a^2*(3*b*d +
 a*e)*x^3)/(3 + m) + (3*a*b*(b*d + a*e)*x^5)/(5 + m) + (b^2*(b*d
+ 3*a*e)*x^7)/(7 + m) + (b^3*e*x^9)/(9 + m)))/(a + b*x^2)^3
```

Maple [B] time = 0.012, size = 495, normalized size = 1.8

$$\frac{(b^3em^4x^8+16\,b^3em^3x^8+3\,ab^2em^4x^6+b^3dm^4x^6+86\,b^3em^2x^8+54\,ab^2em^3x^6+18\,b^3dm^3x^6+176\,b^3emx^8+3\,a^2bem^4x^4+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] x*(b^3*e*m^4*x^8+16*b^3*e*m^3*x^8+3*a*b^2*e*m^4*x^6+b^3*d*m^4*x^6
+86*b^3*e*m^2*x^8+54*a*b^2*e*m^3*x^6+18*b^3*d*m^3*x^6+176*b^3*e*m
*x^8+3*a^2*b*e*m^4*x^4+3*a*b^2*d*m^4*x^4+312*a*b^2*e*m^2*x^6+104*
b^3*d*m^2*x^6+105*b^3*e*x^8+60*a^2*b*e*m^3*x^4+60*a*b^2*d*m^3*x^4
+666*a*b^2*e*m*x^6+222*b^3*d*m*x^6+a^3*e*m^4*x^2+3*a^2*b*d*m^4*x^
2+390*a^2*b*e*m^2*x^4+390*a*b^2*d*m^2*x^4+405*a*b^2*e*x^6+135*b^3
*d*x^6+22*a^3*e*m^3*x^2+66*a^2*b*d*m^3*x^2+900*a^2*b*e*m*x^4+900*
a*b^2*d*m*x^4+a^3*d*m^4+164*a^3*e*m^2*x^2+492*a^2*b*d*m^2*x^2+567
*a^2*b*e*x^4+567*a*b^2*d*x^4+24*a^3*d*m^3+458*a^3*e*m*x^2+1374*a^
2*b*d*m*x^2+206*a^3*d*m^2+315*a^3*e*x^2+945*a^2*b*d*x^2+744*a^3*d
*m+945*a^3*d)*(f*x)^m*((b*x^2+a)^2)^(3/2)/(9+m)/(7+m)/(5+m)/(3+m)
/(1+m)/(b*x^2+a)^3
```

Maxima [A] time = 0.713054, size = 328, normalized size = 1.19

$$\frac{\left(\left(m^{3}+9\,m^{2}+23\,m+15\right)b^{3}f^{m}x^{7}+3\,\left(m^{3}+11\,m^{2}+31\,m+21\right)ab^{2}f^{m}x^{5}+3\,\left(m^{3}+13\,m^{2}+47\,m+35\right)a^{2}bf^{m}x^{3}+\left(m^{3}+15\right.\right.}{m^{4}+16\,m^{3}+86\,m^{2}+176\,m+105}+\frac{\left(\left(m^{3}+15\,m^{2}+71\,m+105\right)b^{3}f^{m}x^{9}+3\,\left(m^{3}+17\,m^{2}+87\,m+135\right)ab^{2}f^{m}x^{7}+3\,\left(m^{3}+19\,m^{2}+111\,m+189\right)a^{2}bf^{m}x^{5}+\left(n\right.\right.}{m^{4}+24\,m^{3}+206\,m^{2}+744\,m+945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(e*x^2 + d)*(f*x)^m,x, algorithm="ma
```

```
[Out] ((m^3 + 9*m^2 + 23*m + 15)*b^3*f^m*x^7 + 3*(m^3 + 11*m^2 + 31*m +
 21)*a*b^2*f^m*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*a^2*b*f^m*x^3 +
 (m^3 + 15*m^2 + 71*m + 105)*a^3*f^m*x)*d*x^m/(m^4 + 16*m^3 + 86*
m^2 + 176*m + 105) + ((m^3 + 15*m^2 + 71*m + 105)*b^3*f^m*x^9 + 3
*(m^3 + 17*m^2 + 87*m + 135)*a*b^2*f^m*x^7 + 3*(m^3 + 19*m^2 + 11
1*m + 189)*a^2*b*f^m*x^5 + (m^3 + 21*m^2 + 143*m + 315)*a^3*f^m*x
^3)*e*x^m/(m^4 + 24*m^3 + 206*m^2 + 744*m + 945)
```

Fricas [A] time = 0.284129, size = 514, normalized size = 1.86

$$\frac{\left(\left(b^{3}em^{4}+16\,b^{3}em^{3}+86\,b^{3}em^{2}+176\,b^{3}em+105\,b^{3}e\right)x^{9}+\left(\left(b^{3}d+3\,ab^{2}e\right)m^{4}+135\,b^{3}d+405\,ab^{2}e+18\,\left(b^{3}d+3\,ab^{2}e\right)m^{3}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(e*x^2 + d)*(f*x)^m,x, algorithm="fr
```

```
[Out] ((b^3*e*m^4 + 16*b^3*e*m^3 + 86*b^3*e*m^2 + 176*b^3*e*m + 105*b^3
*e)*x^9 + ((b^3*d + 3*a*b^2*e)*m^4 + 135*b^3*d + 405*a*b^2*e + 18
```

```
*(b^3*d + 3*a*b^2*e)*m^3 + 104*(b^3*d + 3*a*b^2*e)*m^2 + 222*(b^3
*d + 3*a*b^2*e)*m)*x^7 + 3*((a*b^2*d + a^2*b*e)*m^4 + 189*a*b^2*d
 + 189*a^2*b*e + 20*(a*b^2*d + a^2*b*e)*m^3 + 130*(a*b^2*d + a^2*
b*e)*m^2 + 300*(a*b^2*d + a^2*b*e)*m)*x^5 + ((3*a^2*b*d + a^3*e)*
m^4 + 945*a^2*b*d + 315*a^3*e + 22*(3*a^2*b*d + a^3*e)*m^3 + 164*
(3*a^2*b*d + a^3*e)*m^2 + 458*(3*a^2*b*d + a^3*e)*m)*x^3 + (a^3*d
*m^4 + 24*a^3*d*m^3 + 206*a^3*d*m^2 + 744*a^3*d*m + 945*a^3*d)*x)
*(f*x)^m/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.284723, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(e*x^2 + d)*(f*x)^m,x, algorithm="gia

[Out] Done
```

3.89 $\int (fx)^m \left(d+ex^2\right) \sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+3}(ae+bd)}{f^3(m+3)\left(a+bx^2\right)}+\frac{ad\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+1}}{f(m+1)\left(a+bx^2\right)}+\frac{be\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+5}}{f^5(m+5)\left(a+bx^2\right)}$$

```
[Out] (a*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a
 + b*x^2)) + ((b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(f^3*(3 + m)*(a + b*x^2)) + (b*e*(f*x)^(5 + m)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(f^5*(5 + m)*(a + b*x^2))
```

Rubi [A] time = 0.221173, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+3}(ae+bd)}{f^3(m+3)\left(a+bx^2\right)}+\frac{ad\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+1}}{f(m+1)\left(a+bx^2\right)}+\frac{be\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+5}}{f^5(m+5)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a
 + b*x^2)) + ((b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(f^3*(3 + m)*(a + b*x^2)) + (b*e*(f*x)^(5 + m)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(f^5*(5 + m)*(a + b*x^2))
```

Rubi in Sympy [A] time = 34.6397, size = 138, normalized size = 0.9

$$\frac{ad\,(fx)^{m+1}\sqrt{a^2+2abx^2+b^2x^4}}{f\left(a+bx^2\right)(m+1)}+\frac{be\,(fx)^{m+5}\sqrt{a^2+2abx^2+b^2x^4}}{f^5\left(a+bx^2\right)(m+5)}+\frac{(fx)^{m+3}\,(ae+bd)\sqrt{a^2+2abx^2+b^2x^4}}{f^3\left(a+bx^2\right)(m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] a*d*(f*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(f*(a + b*
x**2)*(m + 1)) + b*e*(f*x)**(m + 5)*sqrt(a**2 + 2*a*b*x**2 + b**2
*x**4)/(f**5*(a + b*x**2)*(m + 5)) + (f*x)**(m + 3)*(a*e + b*d)*s
qrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(f**3*(a + b*x**2)*(m + 3))
```

Mathematica [A] time = 0.10254, size = 65, normalized size = 0.42

$$\frac{\sqrt{(a+bx^2)^2}(fx)^m\left(\frac{x^3(ae+bd)}{m+3}+\frac{adx}{m+1}+\frac{bex^5}{m+5}\right)}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((f*x)^m*Sqrt[(a + b*x^2)^2]*((a*d*x)/(1 + m) + ((b*d + a*e)*x^3)
/(3 + m) + (b*e*x^5)/(5 + m)))/(a + b*x^2)
```

Maple [A] time = 0.007, size = 131, normalized size = 0.9

$$\frac{\left(bem^2x^4+4\,bemx^4+aem^2x^2+bdm^2x^2+3\,bex^4+6\,aemx^2+6\,bdmx^2+adm^2+5\,aex^2+5\,bdx^2+8\,adm+15\,ad\right)x\,(fx)^m}{(5+m)\,(3+m)\,(1+m)\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

```
[Out] x*(b*e*m^2*x^4+4*b*e*m*x^4+a*e*m^2*x^2+b*d*m^2*x^2+3*b*e*x^4+6*a*
e*m*x^2+6*b*d*m*x^2+a*d*m^2+5*a*e*x^2+5*b*d*x^2+8*a*d*m+15*a*d)*(
f*x)^m*((b*x^2+a)^2)^(1/2)/(5+m)/(3+m)/(1+m)/(b*x^2+a)
```

Maxima [A] time = 0.719245, size = 101, normalized size = 0.66

$$\frac{\left(bf^m(m+1)x^3+af^m(m+3)x\right)dx^m}{m^2+4\,m+3}+\frac{\left(bf^m(m+3)x^5+af^m(m+5)x^3\right)ex^m}{m^2+8\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*(e*x^2 + d)*(f*x)^m,x, algorithm="maxi
```

```
[Out] (b*f^m*(m + 1)*x^3 + a*f^m*(m + 3)*x)*d*x^m/(m^2 + 4*m + 3) + (b*
f^m*(m + 3)*x^5 + a*f^m*(m + 5)*x^3)*e*x^m/(m^2 + 8*m + 15)
```

Fricas [A] time = 0.285681, size = 127, normalized size = 0.83

$$\frac{\left(\left(bem^2 + 4\,bem + 3\,be\right)x^5 + \left((bd + ae)m^2 + 5\,bd + 5\,ae + 6\,(bd + ae)m\right)x^3 + \left(adm^2 + 8\,adm + 15\,ad\right)x\right)(fx)^m}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*(e*x^2 + d)*(f*x)^m,x, algorithm="frica
```

```
[Out] ((b*e*m^2 + 4*b*e*m + 3*b*e)*x^5 + ((b*d + a*e)*m^2 + 5*b*d + 5*a
*e + 6*(b*d + a*e)*m)*x^3 + (a*d*m^2 + 8*a*d*m + 15*a*d)*x)*(f*x)
^m/(m^3 + 9*m^2 + 23*m + 15)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (fx)^m\left(d + ex^2\right)\sqrt{\left(a + bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)*sqrt((a + b*x**2)**2), x)
```

GIAC/XCAS [A] time = 0.270117, size = 396, normalized size = 2.59

$$bm^2x^5e^{(m\ln(fx)+1)}\mathrm{sign}\left(bx^2 + a\right) + 4\,bmx^5e^{(m\ln(fx)+1)}\mathrm{sign}\left(bx^2 + a\right) + bdm^2x^3e^{(m\ln(fx))}\mathrm{sign}\left(bx^2 + a\right) + am^2x^3e^{(m\ln(fx)+1)}\mathrm{sig}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*(e*x^2 + d)*(f*x)^m,x, algorithm="giac
```

```
[Out] (b*m^2*x^5*e^(m*ln(f*x) + 1)*sign(b*x^2 + a) + 4*b*m*x^5*e^(m*ln(
f*x) + 1)*sign(b*x^2 + a) + b*d*m^2*x^3*e^(m*ln(f*x))*sign(b*x^2
+ a) + a*m^2*x^3*e^(m*ln(f*x) + 1)*sign(b*x^2 + a) + 3*b*x^5*e^(m
*ln(f*x) + 1)*sign(b*x^2 + a) + 6*b*d*m*x^3*e^(m*ln(f*x))*sign(b*
x^2 + a) + 6*a*m*x^3*e^(m*ln(f*x) + 1)*sign(b*x^2 + a) + a*d*m^2*
x*e^(m*ln(f*x))*sign(b*x^2 + a) + 5*b*d*x^3*e^(m*ln(f*x))*sign(b*
x^2 + a) + 5*a*x^3*e^(m*ln(f*x) + 1)*sign(b*x^2 + a) + 8*a*d*m*x*
e^(m*ln(f*x))*sign(b*x^2 + a) + 15*a*d*x*e^(m*ln(f*x))*sign(b*x^2
 + a))/(m^3 + 9*m^2 + 23*m + 15)
```

3.90 $\int \frac{(fx)^m(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=134

$$\frac{(a+bx^2)(fx)^{m+1}(bd-ae)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{e(a+bx^2)(fx)^{m+1}}{bf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (e*(f*x)^(1 + m)*(a + b*x^2))/(b*f*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) + ((b*d - a*e)*(f*x)^(1 + m)*(a + b*x^2)*Hypergeometri
c2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*f*(1 + m)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.246658, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086

$$\frac{(a+bx^2)(fx)^{m+1}(bd-ae)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{e(a+bx^2)(fx)^{m+1}}{bf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[((f*x)^m*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (e*(f*x)^(1 + m)*(a + b*x^2))/(b*f*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) + ((b*d - a*e)*(f*x)^(1 + m)*(a + b*x^2)*Hypergeometri
c2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*f*(1 + m)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [A] time = 34.6009, size = 114, normalized size = 0.85

$$\frac{e(fx)^{m+1}\sqrt{a^2+2abx^2+b^2x^4}}{bf(a+bx^2)(m+1)}-\frac{(fx)^{m+1}(ae-bd)\sqrt{a^2+2abx^2+b^2x^4}\,{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{abf(a+bx^2)(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((f*x)**m*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] e*(f*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(b*f*(a + b*
x**2)*(m + 1)) - (f*x)**(m + 1)*(a*e - b*d)*sqrt(a**2 + 2*a*b*x**
```

```
2 + b**2*x**4)*hyper((1, m/2 + 1/2), (m/2 + 3/2,), -b*x**2/a)/(a*
b*f*(a + b*x**2)*(m + 1))
```

Mathematica [A] time = 0.10266, size = 78, normalized size = 0.58

$$-\frac{x\left(a+bx^2\right)(fx)^m\left((ae-bd)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)-ae\right)}{ab(m+1)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((f*x)^m*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((x*(f*x)^m*(a + b*x^2)*(-(a*e) + (-(b*d) + a*e)*Hypergeometric2
F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a*b*(1 + m)*Sqrt[(a
+ b*x^2)^2]))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int (fx)^m\left(ex^2+d\right)\frac{1}{\sqrt{b^2x^4+2\,abx^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)(fx)^m}{\sqrt{b^2x^4+2\,abx^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxi
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex^2+d\right)(fx)^m}{\sqrt{b^2x^4+2\,abx^2+a^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="frica
```

```
[Out] integral((e*x^2 + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(fx)^m\left(d+ex^2\right)}{\sqrt{\left(a+bx^2\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)/sqrt((a + b*x**2)**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^2+d\right)(fx)^m}{\sqrt{b^2x^4+2\,abx^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

3.91 $\int \frac{(fx)^m(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=154

$$\frac{(fx)^{m+1}(bd-ae)}{4abf\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\,(fx)^{m+1}(ae(m+1)+bd(3-m))\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{4a^3bf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((b*d - a*e)*(f*x)^(1 + m))/(4*a*b*f*(a + b*x^2)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) + ((b*d*(3 - m) + a*e*(1 + m))*(f*x)^(1 + m)*(a
+ b*x^2)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]
)/(4*a^3*b*f*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.312783, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086

$$\frac{(fx)^{m+1}(bd-ae)}{4abf\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\,(fx)^{m+1}(ae(m+1)+bd(3-m))\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{4a^3bf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[((f*x)^m*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((b*d - a*e)*(f*x)^(1 + m))/(4*a*b*f*(a + b*x^2)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) + ((b*d*(3 - m) + a*e*(1 + m))*(f*x)^(1 + m)*(a
+ b*x^2)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]
)/(4*a^3*b*f*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [A] time = 36.5839, size = 131, normalized size = 0.85

$$-\frac{(fx)^{m+1}\,(ae-bd)\sqrt{a^2+2abx^2+b^2x^4}}{4abf\,(a+bx^2)^3}+\frac{(fx)^{m+1}\,(ae\,(m+1)+bd\,(-m+3))\sqrt{a^2+2abx^2+b^2x^4}\,{}_2F_1\left(\begin{matrix}2,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{4a^3bf\,(a+bx^2)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((f*x)**m*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] -(f*x)**(m + 1)*(a*e - b*d)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(
4*a*b*f*(a + b*x**2)**3) + (f*x)**(m + 1)*(a*e*(m + 1) + b*d*(-m
+ 3))*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)*hyper((2, m/2 + 1/2), (
m/2 + 3/2,), -b*x**2/a)/(4*a**3*b*f*(a + b*x**2)*(m + 1))
```

Mathematica [A] time = 0.127806, size = 101, normalized size = 0.66

$$\frac{x\left(a+bx^2\right)(fx)^m\left((bd-ae)\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+ae\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)\right)}{a^3b(m+1)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((f*x)^m*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x*(f*x)^m*(a + b*x^2)*(a*e*Hypergeometric2F1[2, (1 + m)/2, (3 +
m)/2, -((b*x^2)/a)] + (b*d - a*e)*Hypergeometric2F1[3, (1 + m)/2,
 (3 + m)/2, -((b*x^2)/a)]))/(a^3*b*(1 + m)*Sqrt[(a + b*x^2)^2])
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int (fx)^m\left(ex^2+d\right)\left(b^2x^4+2\,abx^2+a^2\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)(fx)^m}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*(f*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="ma
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex^2+d\right)\left(fx\right)^m}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*(f*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fr
```

```
[Out] integral((e*x^2 + d)*(f*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^2+d\right)\left(fx\right)^m}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*(f*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="gi
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),
x)
```

3.92 $\int x \left(a+bx^2\right) \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=34

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{p+1}}{4b(p+1)}$$

```
[Out] (a^2 + 2*a*b*x^2 + b^2*x^4)^(1 + p)/(4*b*(1 + p))
```

Rubi [A] time = 0.0215157, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{p+1}}{4b(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]

[Out] (a^2 + 2*a*b*x^2 + b^2*x^4)^(1 + p)/(4*b*(1 + p))
```

Rubi in Sympy [A] time = 7.22144, size = 27, normalized size = 0.79

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{p+1}}{4b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)

[Out] (a**2 + 2*a*b*x**2 + b**2*x**4)**(p + 1)/(4*b*(p + 1))
```

Mathematica [A] time = 0.0148955, size = 25, normalized size = 0.74

$$\frac{\left(\left(a+bx^2\right)^2\right)^{p+1}}{4b(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)^2)^(1 + p)/(4*b*(1 + p))
```

Maple [A] time = 0.003, size = 40, normalized size = 1.2

$$\frac{\left(bx^2+a\right)^2\left(b^2x^4+2\,abx^2+a^2\right)^p}{4\,b\,(1+p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] 1/4*(b*x^2+a)^2/b/(1+p)*(b^2*x^4+2*a*b*x^2+a^2)^p
```

Maxima [A] time = 0.717644, size = 116, normalized size = 3.41

$$\frac{\left(bx^2+a\right)\left(bx^2+a\right)^{2\,p}a}{2\,b(2\,p+1)}+\frac{\left(b^2(2\,p+1)x^4+2\,abpx^2-a^2\right)\left(bx^2+a\right)^{2\,p}}{4\,(2\,p^2+3\,p+1)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*x,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + a)*(b*x^2 + a)^(2*p)*a/(b*(2*p + 1)) + 1/4*(b^2*(2*p
 + 1)*x^4 + 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)/((2*p^2 + 3*p +
1)*b)
```

Fricas [A] time = 0.286097, size = 63, normalized size = 1.85

$$\frac{\left(b^2x^4+2\,abx^2+a^2\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{4\,(bp+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*x,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^4 + 2*a*b*x^2 + a^2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(b*
p + b)
```

Sympy [A] time = 37.524, size = 165, normalized size = 4.85

$$\begin{cases} \frac{x^2}{2a} & \text{for } b = 0 \wedge p = -1 \\ \frac{ax^2(a^2)^p}{2} & \text{for } b = 0 \\ \frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b} + \frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b} & \text{for } p = -1 \\ \frac{a^2(a^2+2abx^2+b^2x^4)^p}{4bp+4b} + \frac{2abx^2(a^2+2abx^2+b^2x^4)^p}{4bp+4b} + \frac{b^2x^4(a^2+2abx^2+b^2x^4)^p}{4bp+4b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((x**2/(2*a), Eq(b, 0) & Eq(p, -1)), (a*x**2*(a**2)**p/2
, Eq(b, 0)), (log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b) + log(I*sqrt(a)
*sqrt(1/b) + x)/(2*b), Eq(p, -1)), (a**2*(a**2 + 2*a*b*x**2 + b**
2*x**4)**p/(4*b*p + 4*b) + 2*a*b*x**2*(a**2 + 2*a*b*x**2 + b**2*x
**4)**p/(4*b*p + 4*b) + b**2*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)
**p/(4*b*p + 4*b), True))
```

GIAC/XCAS [A] time = 0.270911, size = 127, normalized size = 3.74

$$\frac{b^2x^4e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)} + 2\,abx^2e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)} + a^2e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)}}{4\,(bp+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*x,x, algorithm="giac")
```

```
[Out] 1/4*(b^2*x^4*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*a*b*x^2*e^(p
*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) + a^2*e^(p*ln(b^2*x^4 + 2*a*b*x^2
 + a^2)))/(b*p + b)
```

3.93 $\int x^3 \left(a+bx^2\right) \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=86

$$\frac{\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(2p+3)}-\frac{a\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(p+1)}$$

```
[Out] -(a*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^2*(1 + p))
+ ((a + b*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^2*(3 + 2*p))
```

Rubi [A] time = 0.211069, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129

$$\frac{\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(2p+3)}-\frac{a\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] -(a*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^2*(1 + p))
+ ((a + b*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^2*(3 + 2*p))
```

Rubi in Sympy [A] time = 27.9409, size = 70, normalized size = 0.81

$$-\frac{a\left(a^2+2abx^2+b^2x^4\right)^{p+1}}{4b^2\left(p+1\right)\left(2p+3\right)}+\frac{x^2\left(a^2+2abx^2+b^2x^4\right)^{p+1}}{2b\left(2p+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] -a*(a**2 + 2*a*b*x**2 + b**2*x**4)**(p + 1)/(4*b**2*(p + 1)*(2*p
+ 3)) + x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(p + 1)/(2*b*(2*p +
 3))
```

Mathematica [A] time = 0.0348295, size = 45, normalized size = 0.52

$$\frac{\left(\left(a+bx^2\right)^2\right)^{p+1} \left(2b(p+1)x^2-a\right)}{4b^2(p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (((a + b*x^2)^2)^(1 + p)*(-a + 2*b*(1 + p)*x^2))/(4*b^2*(1 + p)*(
3 + 2*p))
```

Maple [A] time = 0.007, size = 62, normalized size = 0.7

$$-\frac{\left(b^2x^4+2\,abx^2+a^2\right)^p\left(-2\,x^2pb-2\,bx^2+a\right)\left(bx^2+a\right)^2}{4\,b^2\left(2\,p^2+5\,p+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] -1/4*(b^2*x^4+2*a*b*x^2+a^2)^p*(-2*b*p*x^2-2*b*x^2+a)*(b*x^2+a)^2
/b^2/(2*p^2+5*p+3)
```

Maxima [A] time = 0.721337, size = 182, normalized size = 2.12

$$\frac{\left(b^2(2\,p+1)x^4+2\,abpx^2-a^2\right)\left(bx^2+a\right)^{2\,p}a}{4\left(2\,p^2+3\,p+1\right)b^2} + \frac{\left(\left(2\,p^2+3\,p+1\right)b^3x^6+\left(2\,p^2+p\right)ab^2x^4-2\,a^2bpx^2+a^3\right)\left(bx^2+a\right)^{2\,p}}{2\left(4\,p^3+12\,p^2+11\,p+3\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*(2*p + 1)*x^4 + 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)*a/(
(2*p^2 + 3*p + 1)*b^2) + 1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2
+ p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + a^3)*(b*x^2 + a)^(2*p)/((4*p^3 +
 12*p^2 + 11*p + 3)*b^2)
```

Fricas [A] time = 0.284246, size = 124, normalized size = 1.44

$$\frac{\left(2\left(b^3p+b^3\right)x^6+2\,a^2bpx^2+\left(4\,ab^2p+3\,ab^2\right)x^4-a^3\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{4\left(2\,b^2p^2+5\,b^2p+3\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^3*p + b^3)*x^6 + 2*a^2*b*p*x^2 + (4*a*b^2*p + 3*a*b^2)*
x^4 - a^3)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(2*b^2*p^2 + 5*b^2*p + 3
*b^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [A] time = 0.270793, size = 281, normalized size = 3.27

$$\frac{2\,b^3px^6e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)} + 2\,b^3x^6e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)} + 4\,ab^2px^4e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)} + 3\,ab^2x^4e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)} + 2\,a}{4\,(2\,b^2p^2 + 5\,b^2p + 3\,b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*b^3*p*x^6*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*b^3*x^6*
e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*a*b^2*p*x^4*e^(p*ln(b^2*x
^4 + 2*a*b*x^2 + a^2)) + 3*a*b^2*x^4*e^(p*ln(b^2*x^4 + 2*a*b*x^2
+ a^2)) + 2*a^2*b*p*x^2*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) - a^3
*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)))/(2*b^2*p^2 + 5*b^2*p + 3*b^
2)
```

3.94 $\int x^5 \left(a+bx^2\right) \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=128

$$\frac{\left(a+bx^2\right)^4 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+2)}-\frac{a\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{b^3(2p+3)}+\frac{a^2\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+1)}$$

```
[Out] (a^2*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^3*(1 + p))
 - (a*(a + b*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(b^3*(3 + 2*p)
) + ((a + b*x^2)^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^3*(2 + p))
```

Rubi [A] time = 0.288279, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$

$$\frac{\left(a+bx^2\right)^4 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+2)}-\frac{a\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{b^3(2p+3)}+\frac{a^2\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (a^2*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^3*(1 + p))
 - (a*(a + b*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(b^3*(3 + 2*p)
) + ((a + b*x^2)^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^3*(2 + p))
```

Rubi in Sympy [A] time = 43.1691, size = 114, normalized size = 0.89

$$\frac{a^2\left(a^2+2abx^2+b^2x^4\right)^{p+1}}{4b^3\left(p+1\right)\left(p+2\right)\left(2p+3\right)}-\frac{ax^2\left(a^2+2abx^2+b^2x^4\right)^{p+1}}{2b^2\left(p+2\right)\left(2p+3\right)}+\frac{x^4\left(a^2+2abx^2+b^2x^4\right)^{p+1}}{4b\left(p+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] a**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(p + 1)/(4*b**3*(p + 1)*(p
+ 2)*(2*p + 3)) - a*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(p + 1)
/(2*b**2*(p + 2)*(2*p + 3)) + x**4*(a**2 + 2*a*b*x**2 + b**2*x**4
)**(p + 1)/(4*b*(p + 2))
```

Mathematica [A] time = 0.0516488, size = 68, normalized size = 0.53

$$\frac{\left(\left(a+bx^2\right)^2\right)^{p+1}\left(a^2-2ab(p+1)x^2+b^2\left(2p^2+5p+3\right)x^4\right)}{4b^3(p+1)(p+2)(2p+3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (((a + b*x^2)^2)^(1 + p)*(a^2 - 2*a*b*(1 + p)*x^2 + b^2*(3 + 5*p
+ 2*p^2)*x^4))/(4*b^3*(1 + p)*(2 + p)*(3 + 2*p))
```

Maple [A] time = 0.009, size = 99, normalized size = 0.8

$$\frac{\left(2\,b^2p^2x^4+5\,b^2px^4+3\,b^2x^4-2\,abpx^2-2\,abx^2+a^2\right)\left(bx^2+a\right)^2\left(b^2x^4+2\,abx^2+a^2\right)^p}{4\,b^3\left(2\,p^3+9\,p^2+13\,p+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] 1/4*(b*x^2+a)^2*(2*b^2*p^2*x^4+5*b^2*p*x^4+3*b^2*x^4-2*a*b*p*x^2-
2*a*b*x^2+a^2)*(b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(2*p^3+9*p^2+13*p+6)
```

Maxima [A] time = 0.717756, size = 265, normalized size = 2.07

$$\frac{\left(\left(2\,p^2+3\,p+1\right)b^3x^6+\left(2\,p^2+p\right)ab^2x^4-2\,a^2bpx^2+a^3\right)\left(bx^2+a\right)^{2\,p}a}{2\,(4\,p^3+12\,p^2+11\,p+3)b^3}+\frac{\left(\left(4\,p^3+12\,p^2+11\,p+3\right)b^4x^8+2\left(2\,p^3+3\,p^2+p\right)ab^3x^6-3\left(2\,p^2+p\right)a^2b^2x^4+6\,a^3bpx^2-3\,a^4\right)\left(bx^2+a\right)^{2\,p}}{4\,(4\,p^4+20\,p^3+35\,p^2+25\,p+6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^5,x, algorithm="maxima")
```

```
[Out] 1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2 + p)*a*b^2*x^4 - 2*a^2*b*
p*x^2 + a^3)*(b*x^2 + a)^(2*p)*a/((4*p^3 + 12*p^2 + 11*p + 3)*b^3
) + 1/4*((4*p^3 + 12*p^2 + 11*p + 3)*b^4*x^8 + 2*(2*p^3 + 3*p^2 +
 p)*a*b^3*x^6 - 3*(2*p^2 + p)*a^2*b^2*x^4 + 6*a^3*b*p*x^2 - 3*a^4
)*(b*x^2 + a)^(2*p)/((4*p^4 + 20*p^3 + 35*p^2 + 25*p + 6)*b^3)
```

Fricas [A] time = 0.277798, size = 189, normalized size = 1.48

$$\frac{\left(\left(2\,b^4p^2 + 5\,b^4p + 3\,b^4\right)x^8 - 2\,a^3bpx^2 + 4\,\left(ab^3p^2 + 2\,ab^3p + ab^3\right)x^6 + \left(2\,a^2b^2p^2 + a^2b^2p\right)x^4 + a^4\right)\left(b^2x^4 + 2\,abx^2 + a^2\right)^p}{4\,(2\,b^3p^3 + 9\,b^3p^2 + 13\,b^3p + 6\,b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^5,x, algorithm="fricas")
```

```
[Out] 1/4*((2*b^4*p^2 + 5*b^4*p + 3*b^4)*x^8 - 2*a^3*b*p*x^2 + 4*(a*b^3
*p^2 + 2*a*b^3*p + a*b^3)*x^6 + (2*a^2*b^2*p^2 + a^2*b^2*p)*x^4 +
 a^4)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(2*b^3*p^3 + 9*b^3*p^2 + 13*b
^3*p + 6*b^3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [A] time = 0.270142, size = 474, normalized size = 3.7

$$\frac{2\,b^4p^2x^8e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)} + 5\,b^4px^8e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)} + 4\,ab^3p^2x^6e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)} + 3\,b^4x^8e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)} + \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^5,x, algorithm="giac")
```

```
[Out] 1/4*(2*b^4*p^2*x^8*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) + 5*b^4*p*
x^8*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*a*b^3*p^2*x^6*e^(p*ln
(b^2*x^4 + 2*a*b*x^2 + a^2)) + 3*b^4*x^8*e^(p*ln(b^2*x^4 + 2*a*b*
x^2 + a^2)) + 8*a*b^3*p*x^6*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) +
 2*a^2*b^2*p^2*x^4*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*a*b^3*
x^6*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) + a^2*b^2*p*x^4*e^(p*ln(b
^2*x^4 + 2*a*b*x^2 + a^2)) - 2*a^3*b*p*x^2*e^(p*ln(b^2*x^4 + 2*a*
```

```
b*x^2 + a^2)) + a^4*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)))/(2*b^3*p
^3 + 9*b^3*p^2 + 13*b^3*p + 6*b^3)
```

3.95 $\int x^3 \left(A+Bx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=166

$$\frac{1}{4}a^3Ax^4+\frac{1}{6}a^2x^6(aB+3Ab)+\frac{3}{14}cx^{14}\left(aBc+Abc+b^2B\right)+\frac{3}{8}ax^8\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{12}x^{12}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{10}x^{10}\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{16}c^2x^{16}(Ac+3bB)+\frac{1}{18}Bc^3x^{18}$$

```
[Out] (a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^6)/6 + (3*a*(a*b*B + A*(b^2
+ a*c))*x^8)/8 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^10)/1
0 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^12)/12 + (3*c*
(b^2*B + A*b*c + a*B*c)*x^14)/14 + (c^2*(3*b*B + A*c)*x^16)/16 +
(B*c^3*x^18)/18
```

Rubi [A] time = 0.775258, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{1}{4}a^3Ax^4+\frac{1}{6}a^2x^6(aB+3Ab)+\frac{3}{14}cx^{14}\left(aBc+Abc+b^2B\right)+\frac{3}{8}ax^8\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{12}x^{12}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{10}x^{10}\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{16}c^2x^{16}(Ac+3bB)+\frac{1}{18}Bc^3x^{18}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^6)/6 + (3*a*(a*b*B + A*(b^2
+ a*c))*x^8)/8 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^10)/1
0 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^12)/12 + (3*c*
(b^2*B + A*b*c + a*B*c)*x^14)/14 + (c^2*(3*b*B + A*c)*x^16)/16 +
(B*c^3*x^18)/18
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Aa^3\int^{x^2} x\,dx}{2}+\frac{Bc^3x^{18}}{18}+\frac{a^2x^6\left(3Ab+Ba\right)}{6}+\frac{3ax^8\left(Aac+Ab^2+Bab\right)}{8}+\frac{c^2x^{16}\left(Ac+3Bb\right)}{16}+\frac{3cx^{14}\left(Abc+Bac+Bb^2\right)}{14}+x^{12}\left(\frac{Aac^2}{4}+\frac{Ab^2c}{4}+\frac{Babc}{2}+\frac{Bb^3}{12}\right)+x^{10}\left(\frac{3Aabc}{5}+\frac{Ab^3}{10}+\frac{3Ba^2c}{10}+\frac{3Bab^2}{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*Integral(x, (x, x**2))/2 + B*c**3*x**18/18 + a**2*x**6*(3*
A*b + B*a)/6 + 3*a*x**8*(A*a*c + A*b**2 + B*a*b)/8 + c**2*x**16*(
A*c + 3*B*b)/16 + 3*c*x**14*(A*b*c + B*a*c + B*b**2)/14 + x**12*(
A*a*c**2/4 + A*b**2*c/4 + B*a*b*c/2 + B*b**3/12) + x**10*(3*A*a*b
*c/5 + A*b**3/10 + 3*B*a**2*c/10 + 3*B*a*b**2/10)
```

Mathematica [A] time = 0.0839754, size = 166, normalized size = 1.

$$\frac{1}{4}a^3Ax^4+\frac{1}{6}a^2x^6(aB+3Ab)+\frac{3}{14}cx^{14}\left(aBc+Abc+b^2B\right)+\frac{3}{8}ax^8\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{12}x^{12}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{10}x^{10}\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{16}c^2x^{16}(Ac+3bB)+\frac{1}{18}Bc^3x^{18}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^6)/6 + (3*a*(a*b*B + A*(b^2
+ a*c))*x^8)/8 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^10)/1
0 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^12)/12 + (3*c*
(b^2*B + A*b*c + a*B*c)*x^14)/14 + (c^2*(3*b*B + A*c)*x^16)/16 +
(B*c^3*x^18)/18
```

Maple [A] time = 0.001, size = 226, normalized size = 1.4

$$\frac{Bc^3x^{18}}{18}+\frac{\left(Ac^3+3\,Bc^2b\right)x^{16}}{16}+\frac{\left(3\,Ac^2b+B\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)\right)x^{14}}{14}+\frac{\left(A\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)+B\left(4\,abc+b\left(2\,ac+b^2\right)\right)\right)x^{12}}{12}+\frac{\left(A\left(4\,abc+b\left(2\,ac+b^2\right)\right)+B\left(a\left(2\,ac+b^2\right)+2\,ab^2+a^2c\right)\right)x^{10}}{10}+\frac{\left(A\left(a\left(2\,ac+b^2\right)+2\,ab^2+a^2c\right)+3\,Ba^2b\right)x^8}{8}+\frac{\left(3\,Aa^2b+Ba^3\right)x^6}{6}+\frac{a^3Ax^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/18*B*c^3*x^18+1/16*(A*c^3+3*B*b*c^2)*x^16+1/14*(3*A*c^2*b+B*(a*
c^2+2*b^2*c+c*(2*a*c+b^2)))*x^14+1/12*(A*(a*c^2+2*b^2*c+c*(2*a*c+
b^2))+B*(4*a*b*c+b*(2*a*c+b^2)))*x^12+1/10*(A*(4*a*b*c+b*(2*a*c+b
^2))+B*(a*(2*a*c+b^2)+2*a*b^2+a^2*c))*x^10+1/8*(A*(a*(2*a*c+b^2)+
2*a*b^2+a^2*c)+3*B*a^2*b)*x^8+1/6*(3*A*a^2*b+B*a^3)*x^6+1/4*a^3*A
*x^4
```

Maxima [A] time = 0.70335, size = 224, normalized size = 1.35

$$\frac{1}{18}Bc^3x^{18}+\frac{1}{16}\left(3\,Bbc^2+Ac^3\right)x^{16}+\frac{3}{14}\left(Bb^2c+(Ba+Ab)c^2\right)x^{14}+\frac{1}{12}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^{12}+\frac{1}{10}\left(3\,Bab^2+Ab^3+3\left(Ba^2+2\,Aab\right)c\right)x^{10}+\frac{3}{8}\left(Ba^2b+Aab^2+Aa^2c\right)x^8+\frac{1}{4}Aa^3x^4+\frac{1}{6}\left(Ba^3+3\,Aa^2b\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A)*x^3,x, algorithm="maxima")
```

```
[Out] 1/18*B*c^3*x^18 + 1/16*(3*B*b*c^2 + A*c^3)*x^16 + 3/14*(B*b^2*c +
 (B*a + A*b)*c^2)*x^14 + 1/12*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A
*b^2)*c)*x^12 + 1/10*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*
x^10 + 3/8*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^8 + 1/4*A*a^3*x^4 + 1/
6*(B*a^3 + 3*A*a^2*b)*x^6
```

Fricas [A] time = 0.24504, size = 1, normalized size = 0.01

$$\frac{1}{18}x^{18}c^3B+\frac{3}{16}x^{16}c^2bB+\frac{1}{16}x^{16}c^3A+\frac{3}{14}x^{14}cb^2B+\frac{3}{14}x^{14}c^2aB+\frac{3}{14}x^{14}c^2bA+\frac{1}{12}x^{12}b^3B+\frac{1}{2}x^{12}cbaB+\frac{1}{4}x^{12}cb^2A+\frac{1}{4}x^{12}c^2aA+\frac{3}{10}x^{10}b^2aB+\frac{3}{10}x^{10}ca^2B+\frac{1}{10}x^{10}b^3A+\frac{3}{5}x^{10}cbaA+\frac{3}{8}x^8ba^2B+\frac{3}{8}x^8b^2aA+\frac{3}{8}x^8ca^2A+\frac{1}{6}x^6a^3B+\frac{1}{2}x^6ba^2A+\frac{1}{4}x^4a^3A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A)*x^3,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*c^3*B + 3/16*x^16*c^2*b*B + 1/16*x^16*c^3*A + 3/14*x^14
*c*b^2*B + 3/14*x^14*c^2*a*B + 3/14*x^14*c^2*b*A + 1/12*x^12*b^3*
B + 1/2*x^12*c*b*a*B + 1/4*x^12*c*b^2*A + 1/4*x^12*c^2*a*A + 3/10
*x^10*b^2*a*B + 3/10*x^10*c*a^2*B + 1/10*x^10*b^3*A + 3/5*x^10*c*
b*a*A + 3/8*x^8*b*a^2*B + 3/8*x^8*b^2*a*A + 3/8*x^8*c*a^2*A + 1/6
```

```
*x^6*a^3*B + 1/2*x^6*b*a^2*A + 1/4*x^4*a^3*A
```

Sympy [A] time = 0.194115, size = 202, normalized size = 1.22

$$\frac{Aa^3x^4}{4} + \frac{Bc^3x^{18}}{18} + x^{16}\left(\frac{Ac^3}{16} + \frac{3Bbc^2}{16}\right) + x^{14}\left(\frac{3Abc^2}{14} + \frac{3Bac^2}{14} + \frac{3Bb^2c}{14}\right) + x^{12}\left(\frac{Aac^2}{4} + \frac{Ab^2c}{4} + \frac{Babc}{2} + \frac{Bb^3}{12}\right) + x^{10}\left(\frac{3Aabc}{5} + \frac{Ab^3}{10} + \frac{3Ba^2c}{10} + \frac{3Bab^2}{10}\right) + x^8\left(\frac{3Aa^2c}{8} + \frac{3Aab^2}{8} + \frac{3Ba^2b}{8}\right) + x^6\left(\frac{Aa^2b}{2} + \frac{Ba^3}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x**4/4 + B*c**3*x**18/18 + x**16*(A*c**3/16 + 3*B*b*c**2/1
6) + x**14*(3*A*b*c**2/14 + 3*B*a*c**2/14 + 3*B*b**2*c/14) + x**1
2*(A*a*c**2/4 + A*b**2*c/4 + B*a*b*c/2 + B*b**3/12) + x**10*(3*A*
a*b*c/5 + A*b**3/10 + 3*B*a**2*c/10 + 3*B*a*b**2/10) + x**8*(3*A*
a**2*c/8 + 3*A*a*b**2/8 + 3*B*a**2*b/8) + x**6*(A*a**2*b/2 + B*a*
*3/6)
```

GIAC/XCAS [A] time = 0.263473, size = 261, normalized size = 1.57

$$\frac{1}{18}Bc^3x^{18} + \frac{3}{16}Bbc^2x^{16} + \frac{1}{16}Ac^3x^{16} + \frac{3}{14}Bb^2cx^{14} + \frac{3}{14}Bac^2x^{14} + \frac{3}{14}Abc^2x^{14} + \frac{1}{12}Bb^3x^{12} + \frac{1}{2}Babcx^{12} + \frac{1}{4}Ab^2cx^{12} + \frac{1}{4}Aac^2x^{12} + \frac{3}{10}Bab^2x^{10} + \frac{1}{10}Ab^3x^{10} + \frac{3}{10}Ba^2cx^{10} + \frac{3}{5}Aabcx^{10} + \frac{3}{8}Ba^2bx^8 + \frac{3}{8}Aab^2x^8 + \frac{3}{8}Aa^2cx^8 + \frac{1}{6}Ba^3x^6 + \frac{1}{2}Aa^2bx^6 + \frac{1}{4}Aa^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A)*x^3,x, algorithm="giac")
```

```
[Out] 1/18*B*c^3*x^18 + 3/16*B*b*c^2*x^16 + 1/16*A*c^3*x^16 + 3/14*B*b^
2*c*x^14 + 3/14*B*a*c^2*x^14 + 3/14*A*b*c^2*x^14 + 1/12*B*b^3*x^1
2 + 1/2*B*a*b*c*x^12 + 1/4*A*b^2*c*x^12 + 1/4*A*a*c^2*x^12 + 3/10
*B*a*b^2*x^10 + 1/10*A*b^3*x^10 + 3/10*B*a^2*c*x^10 + 3/5*A*a*b*c
*x^10 + 3/8*B*a^2*b*x^8 + 3/8*A*a*b^2*x^8 + 3/8*A*a^2*c*x^8 + 1/6
*B*a^3*x^6 + 1/2*A*a^2*b*x^6 + 1/4*A*a^3*x^4
```

3.96 $\int x^2 \left(A+Bx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=166

$$\frac{1}{3}a^3Ax^3+\frac{1}{5}a^2x^5(aB+3Ab)+\frac{3}{13}cx^{13}\left(aBc+Abc+b^2B\right)+\frac{3}{7}ax^7\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{11}x^{11}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{9}x^9\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{15}c^2x^{15}(Ac+3bB)+\frac{1}{17}Bc^3x^{17}$$

```
[Out] (a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^5)/5 + (3*a*(a*b*B + A*(b^2
+ a*c))*x^7)/7 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^9)/9
+ ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^11)/11 + (3*c*(b
^2*B + A*b*c + a*B*c)*x^13)/13 + (c^2*(3*b*B + A*c)*x^15)/15 + (B
*c^3*x^17)/17
```

Rubi [A] time = 0.367125, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04

$$\frac{1}{3}a^3Ax^3+\frac{1}{5}a^2x^5(aB+3Ab)+\frac{3}{13}cx^{13}\left(aBc+Abc+b^2B\right)+\frac{3}{7}ax^7\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{11}x^{11}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{9}x^9\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{15}c^2x^{15}(Ac+3bB)+\frac{1}{17}Bc^3x^{17}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^5)/5 + (3*a*(a*b*B + A*(b^2
+ a*c))*x^7)/7 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^9)/9
+ ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^11)/11 + (3*c*(b
^2*B + A*b*c + a*B*c)*x^13)/13 + (c^2*(3*b*B + A*c)*x^15)/15 + (B
*c^3*x^17)/17
```

Rubi in Sympy [A] time = 51.4368, size = 178, normalized size = 1.07

$$\frac{Aa^3x^3}{3}+\frac{Bc^3x^{17}}{17}+\frac{a^2x^5\left(3Ab+Ba\right)}{5}+\frac{3ax^7\left(Aac+Ab^2+Bab\right)}{7}+\frac{c^2x^{15}\left(Ac+3Bb\right)}{15}+\frac{3cx^{13}\left(Abc+Bac+Bb^2\right)}{13}+x^{11}\left(\frac{3Aac^2}{11}+\frac{3Ab^2c}{11}+\frac{6Babc}{11}+\frac{Bb^3}{11}\right)+x^9\left(\frac{2Aabc}{3}+\frac{Ab^3}{9}+\frac{Ba^2c}{3}+\frac{Bab^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x**3/3 + B*c**3*x**17/17 + a**2*x**5*(3*A*b + B*a)/5 + 3*a
*x**7*(A*a*c + A*b**2 + B*a*b)/7 + c**2*x**15*(A*c + 3*B*b)/15 +
3*c*x**13*(A*b*c + B*a*c + B*b**2)/13 + x**11*(3*A*a*c**2/11 + 3*
A*b**2*c/11 + 6*B*a*b*c/11 + B*b**3/11) + x**9*(2*A*a*b*c/3 + A*b
**3/9 + B*a**2*c/3 + B*a*b**2/3)
```

Mathematica [A]   time = 0.094003, size = 166, normalized size = 1.

$$\frac{1}{3}a^3Ax^3+\frac{1}{5}a^2x^5(aB+3Ab)+\frac{3}{13}cx^{13}\left(aBc+Abc+b^2B\right)+\frac{3}{7}ax^7\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{11}x^{11}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{9}x^9\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{15}c^2x^{15}(Ac+3bB)+\frac{1}{17}Bc^3x^{17}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^5)/5 + (3*a*(a*b*B + A*(b^2
+ a*c))*x^7)/7 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^9)/9
+ ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^11)/11 + (3*c*(b
^2*B + A*b*c + a*B*c)*x^13)/13 + (c^2*(3*b*B + A*c)*x^15)/15 + (B
*c^3*x^17)/17
```

Maple [A]   time = 0.002, size = 226, normalized size = 1.4

$$\frac{Bc^3x^{17}}{17}+\frac{\left(Ac^3+3\,Bc^2b\right)x^{15}}{15}+\frac{\left(3\,Ac^2b+B\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)\right)x^{13}}{13}+\frac{\left(A\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)+B\left(4\,abc+b\left(2\,ac+b^2\right)\right)\right)x^{11}}{11}+\frac{\left(A\left(4\,abc+b\left(2\,ac+b^2\right)\right)+B\left(a\left(2\,ac+b^2\right)+2\,ab^2+a^2c\right)\right)x^9}{9}+\frac{\left(A\left(a\left(2\,ac+b^2\right)+2\,ab^2+a^2c\right)+3\,Ba^2b\right)x^7}{7}+\frac{\left(3\,Aa^2b+Ba^3\right)x^5}{5}+\frac{a^3Ax^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/17*B*c^3*x^17+1/15*(A*c^3+3*B*b*c^2)*x^15+1/13*(3*A*c^2*b+B*(a*
c^2+2*b^2*c+c*(2*a*c+b^2)))*x^13+1/11*(A*(a*c^2+2*b^2*c+c*(2*a*c+
b^2))+B*(4*a*b*c+b*(2*a*c+b^2)))*x^11+1/9*(A*(4*a*b*c+b*(2*a*c+b^
2))+B*(a*(2*a*c+b^2)+2*a*b^2+a^2*c))*x^9+1/7*(A*(a*(2*a*c+b^2)+2*
a*b^2+a^2*c)+3*B*a^2*b)*x^7+1/5*(3*A*a^2*b+B*a^3)*x^5+1/3*a^3*A*x
^3
```

Maxima [A] time = 0.695521, size = 224, normalized size = 1.35

$$\frac{1}{17}Bc^3x^{17} + \frac{1}{15}\left(3Bbc^2 + Ac^3\right)x^{15} + \frac{3}{13}\left(Bb^2c + (Ba + Ab)c^2\right)x^{13} + \frac{1}{11}\left(Bb^3 + 3Aac^2 + 3\left(2Bab + Ab^2\right)c\right)x^{11} + \frac{1}{9}\left(3Bab^2 + Ab^3 + 3\left(Ba^2 + 2Aab\right)c\right)x^9 + \frac{3}{7}\left(Ba^2b + Aab^2 + Aa^2c\right)x^7 + \frac{1}{3}Aa^3x^3 + \frac{1}{5}\left(Ba^3 + 3Aa^2b\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A)*x^2,x, algorithm="maxima")
```

```
[Out] 1/17*B*c^3*x^17 + 1/15*(3*B*b*c^2 + A*c^3)*x^15 + 3/13*(B*b^2*c +
 (B*a + A*b)*c^2)*x^13 + 1/11*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A
*b^2)*c)*x^11 + 1/9*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x
^9 + 3/7*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^7 + 1/3*A*a^3*x^3 + 1/5*
(B*a^3 + 3*A*a^2*b)*x^5
```

Fricas [A] time = 0.23316, size = 1, normalized size = 0.01

$$\frac{1}{17}x^{17}c^3B + \frac{1}{5}x^{15}c^2bB + \frac{1}{15}x^{15}c^3A + \frac{3}{13}x^{13}cb^2B + \frac{3}{13}x^{13}c^2aB + \frac{3}{13}x^{13}c^2bA + \frac{1}{11}x^{11}b^3B + \frac{6}{11}x^{11}cbaB + \frac{3}{11}x^{11}cb^2A + \frac{3}{11}x^{11}c^2aA + \frac{1}{3}x^9b^2aB + \frac{1}{3}x^9ca^2B + \frac{1}{9}x^9b^3A + \frac{2}{3}x^9cbaA + \frac{3}{7}x^7ba^2B + \frac{3}{7}x^7b^2aA + \frac{3}{7}x^7ca^2A + \frac{1}{5}x^5a^3B + \frac{3}{5}x^5ba^2A + \frac{1}{3}x^3a^3A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A)*x^2,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*c^3*B + 1/5*x^15*c^2*b*B + 1/15*x^15*c^3*A + 3/13*x^13*
c*b^2*B + 3/13*x^13*c^2*a*B + 3/13*x^13*c^2*b*A + 1/11*x^11*b^3*B
 + 6/11*x^11*c*b*a*B + 3/11*x^11*c*b^2*A + 3/11*x^11*c^2*a*A + 1/
3*x^9*b^2*a*B + 1/3*x^9*c*a^2*B + 1/9*x^9*b^3*A + 2/3*x^9*c*b*a*A
 + 3/7*x^7*b*a^2*B + 3/7*x^7*b^2*a*A + 3/7*x^7*c*a^2*A + 1/5*x^5*
```

```
a^3*B + 3/5*x^5*b*a^2*A + 1/3*x^3*a^3*A
```

Sympy [A] time = 0.201057, size = 204, normalized size = 1.23

$$\frac{Aa^3x^3}{3} + \frac{Bc^3x^{17}}{17} + x^{15}\left(\frac{Ac^3}{15} + \frac{Bbc^2}{5}\right) + x^{13}\left(\frac{3Abc^2}{13} + \frac{3Bac^2}{13} + \frac{3Bb^2c}{13}\right) + x^{11}\left(\frac{3Aac^2}{11} + \frac{3Ab^2c}{11} + \frac{6Babc}{11} + \frac{Bb^3}{11}\right) + x^9\left(\frac{2Aabc}{3} + \frac{Ab^3}{9} + \frac{Ba^2c}{3} + \frac{Bab^2}{3}\right) + x^7\left(\frac{3Aa^2c}{7} + \frac{3Aab^2}{7} + \frac{3Ba^2b}{7}\right) + x^5\left(\frac{3Aa^2b}{5} + \frac{Ba^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x**3/3 + B*c**3*x**17/17 + x**15*(A*c**3/15 + B*b*c**2/5)
+ x**13*(3*A*b*c**2/13 + 3*B*a*c**2/13 + 3*B*b**2*c/13) + x**11*(
3*A*a*c**2/11 + 3*A*b**2*c/11 + 6*B*a*b*c/11 + B*b**3/11) + x**9*
(2*A*a*b*c/3 + A*b**3/9 + B*a**2*c/3 + B*a*b**2/3) + x**7*(3*A*a*
*2*c/7 + 3*A*a*b**2/7 + 3*B*a**2*b/7) + x**5*(3*A*a**2*b/5 + B*a*
*3/5)
```

GIAC/XCAS [A] time = 0.262897, size = 261, normalized size = 1.57

$$\frac{1}{17}\,Bc^3x^{17} + \frac{1}{5}\,Bbc^2x^{15} + \frac{1}{15}\,Ac^3x^{15} + \frac{3}{13}\,Bb^2cx^{13} + \frac{3}{13}\,Bac^2x^{13} + \frac{3}{13}\,Abc^2x^{13} + \frac{1}{11}\,Bb^3x^{11} + \frac{6}{11}\,Babcx^{11} + \frac{3}{11}\,Ab^2cx^{11} + \frac{3}{11}\,Aac^2x^{11} + \frac{1}{3}\,Bab^2x^9 + \frac{1}{9}\,Ab^3x^9 + \frac{1}{3}\,Ba^2cx^9 + \frac{2}{3}\,Aabcx^9 + \frac{3}{7}\,Ba^2bx^7 + \frac{3}{7}\,Aab^2x^7 + \frac{3}{7}\,Aa^2cx^7 + \frac{1}{5}\,Ba^3x^5 + \frac{3}{5}\,Aa^2bx^5 + \frac{1}{3}\,Aa^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A)*x^2,x, algorithm="giac")
```

```
[Out] 1/17*B*c^3*x^17 + 1/5*B*b*c^2*x^15 + 1/15*A*c^3*x^15 + 3/13*B*b^2
*c*x^13 + 3/13*B*a*c^2*x^13 + 3/13*A*b*c^2*x^13 + 1/11*B*b^3*x^11
 + 6/11*B*a*b*c*x^11 + 3/11*A*b^2*c*x^11 + 3/11*A*a*c^2*x^11 + 1/
3*B*a*b^2*x^9 + 1/9*A*b^3*x^9 + 1/3*B*a^2*c*x^9 + 2/3*A*a*b*c*x^9
 + 3/7*B*a^2*b*x^7 + 3/7*A*a*b^2*x^7 + 3/7*A*a^2*c*x^7 + 1/5*B*a^
3*x^5 + 3/5*A*a^2*b*x^5 + 1/3*A*a^3*x^3
```

3.97 $\int x\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=166

$$\frac{1}{2}a^3Ax^2+\frac{1}{4}a^2x^4(aB+3Ab)+\frac{1}{4}cx^{12}\left(aBc+Abc+b^2B\right)+\frac{1}{2}ax^6\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{10}x^{10}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{8}x^8\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{14}c^2x^{14}(Ac+3bB)+\frac{1}{16}Bc^3x^{16}$$

```
[Out] (a^3*A*x^2)/2 + (a^2*(3*A*b + a*B)*x^4)/4 + (a*(a*b*B + A*(b^2 +
a*c))*x^6)/2 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^8)/8 +
((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^10)/10 + (c*(b^2*B
 + A*b*c + a*B*c)*x^12)/4 + (c^2*(3*b*B + A*c)*x^14)/14 + (B*c^3*
x^16)/16
```

Rubi [A] time = 0.649906, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{1}{2}a^3Ax^2+\frac{1}{4}a^2x^4(aB+3Ab)+\frac{1}{4}cx^{12}\left(aBc+Abc+b^2B\right)+\frac{1}{2}ax^6\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{10}x^{10}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{8}x^8\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{14}c^2x^{14}(Ac+3bB)+\frac{1}{16}Bc^3x^{16}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^2)/2 + (a^2*(3*A*b + a*B)*x^4)/4 + (a*(a*b*B + A*(b^2 +
a*c))*x^6)/2 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^8)/8 +
((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^10)/10 + (c*(b^2*B
 + A*b*c + a*B*c)*x^12)/4 + (c^2*(3*b*B + A*c)*x^14)/14 + (B*c^3*
x^16)/16
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bc^3x^{16}}{16}+\frac{a^3\int^{x^2}A\,dx}{2}+\frac{a^2\left(3Ab+Ba\right)\int^{x^2}x\,dx}{2}+\frac{ax^6\left(Aac+Ab^2+Bab\right)}{2}+\frac{c^2x^{14}\left(Ac+3Bb\right)}{14}+\frac{cx^{12}\left(Abc+Bac+Bb^2\right)}{4}+x^{10}\left(\frac{3Aac^2}{10}+\frac{3Ab^2c}{10}+\frac{3Babc}{5}+\frac{Bb^3}{10}\right)+x^8\left(\frac{3Aabc}{4}+\frac{Ab^3}{8}+\frac{3Ba^2c}{8}+\frac{3Bab^2}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] B*c**3*x**16/16 + a**3*Integral(A, (x, x**2))/2 + a**2*(3*A*b + B
*a)*Integral(x, (x, x**2))/2 + a*x**6*(A*a*c + A*b**2 + B*a*b)/2
+ c**2*x**14*(A*c + 3*B*b)/14 + c*x**12*(A*b*c + B*a*c + B*b**2)/
4 + x**10*(3*A*a*c**2/10 + 3*A*b**2*c/10 + 3*B*a*b*c/5 + B*b**3/1
0) + x**8*(3*A*a*b*c/4 + A*b**3/8 + 3*B*a**2*c/8 + 3*B*a*b**2/8)
```

Mathematica [A] time = 0.135843, size = 154, normalized size = 0.93

$$\frac{1}{560}x^2\left(280a^3A+140a^2x^2(aB+3Ab)+140cx^{10}\left(aBc+Abc+b^2B\right)+280ax^4\left(A\left(ac+b^2\right)+abB\right)+56x^8\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+70x^6\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+40c^2x^{12}(Ac+3bB)+35Bc^3x^{14}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^2*(280*a^3*A + 140*a^2*(3*A*b + a*B)*x^2 + 280*a*(a*b*B + A*(b
^2 + a*c))*x^4 + 70*(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6 +
 56*(b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^8 + 140*c*(b^2*
B + A*b*c + a*B*c)*x^10 + 40*c^2*(3*b*B + A*c)*x^12 + 35*B*c^3*x^
14))/560
```

Maple [A] time = 0.001, size = 226, normalized size = 1.4

$$\frac{Bc^3x^{16}}{16}+\frac{\left(Ac^3+3\,Bc^2b\right)x^{14}}{14}+\frac{\left(3\,Ac^2b+B\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)\right)x^{12}}{12}+\frac{\left(A\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)+B\left(4\,abc+b\left(2\,ac+b^2\right)\right)\right)x^{10}}{10}+\frac{\left(A\left(4\,abc+b\left(2\,ac+b^2\right)\right)+B\left(a\left(2\,ac+b^2\right)+2\,ab^2+a^2c\right)\right)x^8}{8}+\frac{\left(A\left(a\left(2\,ac+b^2\right)+2\,ab^2+a^2c\right)+3\,Ba^2b\right)x^6}{6}+\frac{\left(3\,Aa^2b+Ba^3\right)x^4}{4}+\frac{a^3Ax^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/16*B*c^3*x^16+1/14*(A*c^3+3*B*b*c^2)*x^14+1/12*(3*A*c^2*b+B*(a*
c^2+2*b^2*c+c*(2*a*c+b^2)))*x^12+1/10*(A*(a*c^2+2*b^2*c+c*(2*a*c+
b^2))+B*(4*a*b*c+b*(2*a*c+b^2)))*x^10+1/8*(A*(4*a*b*c+b*(2*a*c+b^
2))+B*(a*(2*a*c+b^2)+2*a*b^2+a^2*c))*x^8+1/6*(A*(a*(2*a*c+b^2)+2*
a*b^2+a^2*c)+3*B*a^2*b)*x^6+1/4*(3*A*a^2*b+B*a^3)*x^4+1/2*a^3*A*x
^2
```

Maxima [A] time = 0.695589, size = 224, normalized size = 1.35

$$\frac{1}{16}Bc^3x^{16} + \frac{1}{14}\left(3\,Bbc^2 + Ac^3\right)x^{14} + \frac{1}{4}\left(Bb^2c + (Ba + Ab)c^2\right)x^{12} + \frac{1}{10}\left(Bb^3 + 3\,Aac^2 + 3\left(2\,Bab + Ab^2\right)c\right)x^{10} + \frac{1}{8}\left(3\,Bab^2 + Ab^3 + 3\left(Ba^2 + 2\,Aab\right)c\right)x^8 + \frac{1}{2}\left(Ba^2b + Aab^2 + Aa^2c\right)x^6 + \frac{1}{2}Aa^3x^2 + \frac{1}{4}\left(Ba^3 + 3\,Aa^2b\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A)*x,x, algorithm="maxima")
```

```
[Out] 1/16*B*c^3*x^16 + 1/14*(3*B*b*c^2 + A*c^3)*x^14 + 1/4*(B*b^2*c +
(B*a + A*b)*c^2)*x^12 + 1/10*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*
b^2)*c)*x^10 + 1/8*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^
8 + 1/2*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^6 + 1/2*A*a^3*x^2 + 1/4*(
B*a^3 + 3*A*a^2*b)*x^4
```

Fricas [A] time = 0.228335, size = 1, normalized size = 0.01

$$\frac{1}{16}x^{16}c^3B + \frac{3}{14}x^{14}c^2bB + \frac{1}{14}x^{14}c^3A + \frac{1}{4}x^{12}cb^2B + \frac{1}{4}x^{12}c^2aB + \frac{1}{4}x^{12}c^2bA + \frac{1}{10}x^{10}b^3B + \frac{3}{5}x^{10}cbaB + \frac{3}{10}x^{10}cb^2A + \frac{3}{10}x^{10}c^2aA + \frac{3}{8}x^8b^2aB + \frac{3}{8}x^8ca^2B + \frac{1}{8}x^8b^3A + \frac{3}{4}x^8cbaA + \frac{1}{2}x^6ba^2B + \frac{1}{2}x^6b^2aA + \frac{1}{2}x^6ca^2A + \frac{1}{4}x^4a^3B + \frac{3}{4}x^4ba^2A + \frac{1}{2}x^2a^3A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A)*x,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*c^3*B + 3/14*x^14*c^2*b*B + 1/14*x^14*c^3*A + 1/4*x^12*
c*b^2*B + 1/4*x^12*c^2*a*B + 1/4*x^12*c^2*b*A + 1/10*x^10*b^3*B +
 3/5*x^10*c*b*a*B + 3/10*x^10*c*b^2*A + 3/10*x^10*c^2*a*A + 3/8*x
^8*b^2*a*B + 3/8*x^8*c*a^2*B + 1/8*x^8*b^3*A + 3/4*x^8*c*b*a*A +
1/2*x^6*b*a^2*B + 1/2*x^6*b^2*a*A + 1/2*x^6*c*a^2*A + 1/4*x^4*a^3
```

```
*B + 3/4*x^4*b*a^2*A + 1/2*x^2*a^3*A
```

Sympy [A] time = 0.187363, size = 199, normalized size = 1.2

$$\frac{Aa^3x^2}{2}+\frac{Bc^3x^{16}}{16}+x^{14}\left(\frac{Ac^3}{14}+\frac{3Bbc^2}{14}\right)+x^{12}\left(\frac{Abc^2}{4}+\frac{Bac^2}{4}+\frac{Bb^2c}{4}\right)+x^{10}\left(\frac{3Aac^2}{10}+\frac{3Ab^2c}{10}+\frac{3Babc}{5}+\frac{Bb^3}{10}\right)+x^8\left(\frac{3Aabc}{4}+\frac{Ab^3}{8}+\frac{3Ba^2c}{8}+\frac{3Bab^2}{8}\right)+x^6\left(\frac{Aa^2c}{2}+\frac{Aab^2}{2}+\frac{Ba^2b}{2}\right)+x^4\left(\frac{3Aa^2b}{4}+\frac{Ba^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x**2/2 + B*c**3*x**16/16 + x**14*(A*c**3/14 + 3*B*b*c**2/1
4) + x**12*(A*b*c**2/4 + B*a*c**2/4 + B*b**2*c/4) + x**10*(3*A*a*
c**2/10 + 3*A*b**2*c/10 + 3*B*a*b*c/5 + B*b**3/10) + x**8*(3*A*a*
b*c/4 + A*b**3/8 + 3*B*a**2*c/8 + 3*B*a*b**2/8) + x**6*(A*a**2*c/
2 + A*a*b**2/2 + B*a**2*b/2) + x**4*(3*A*a**2*b/4 + B*a**3/4)
```

GIAC/XCAS [A] time = 0.262962, size = 261, normalized size = 1.57

$$\frac{1}{16}Bc^3x^{16}+\frac{3}{14}Bbc^2x^{14}+\frac{1}{14}Ac^3x^{14}+\frac{1}{4}Bb^2cx^{12}+\frac{1}{4}Bac^2x^{12}+\frac{1}{4}Abc^2x^{12}+\frac{1}{10}Bb^3x^{10}+\frac{3}{5}Babcx^{10}+\frac{3}{10}Ab^2cx^{10}+\frac{3}{10}Aac^2x^{10}+\frac{3}{8}Bab^2x^8+\frac{1}{8}Ab^3x^8+\frac{3}{8}Ba^2cx^8+\frac{3}{4}Aabcx^8+\frac{1}{2}Ba^2bx^6+\frac{1}{2}Aab^2x^6+\frac{1}{2}Aa^2cx^6+\frac{1}{4}Ba^3x^4+\frac{3}{4}Aa^2bx^4+\frac{1}{2}Aa^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A)*x,x, algorithm="giac")
```

```
[Out] 1/16*B*c^3*x^16 + 3/14*B*b*c^2*x^14 + 1/14*A*c^3*x^14 + 1/4*B*b^2
*c*x^12 + 1/4*B*a*c^2*x^12 + 1/4*A*b*c^2*x^12 + 1/10*B*b^3*x^10 +
 3/5*B*a*b*c*x^10 + 3/10*A*b^2*c*x^10 + 3/10*A*a*c^2*x^10 + 3/8*B
*a*b^2*x^8 + 1/8*A*b^3*x^8 + 3/8*B*a^2*c*x^8 + 3/4*A*a*b*c*x^8 +
1/2*B*a^2*b*x^6 + 1/2*A*a*b^2*x^6 + 1/2*A*a^2*c*x^6 + 1/4*B*a^3*x
^4 + 3/4*A*a^2*b*x^4 + 1/2*A*a^3*x^2
```

3.98 $\int \left(A+Bx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=161

$$a^3Ax+\frac{1}{3}a^2x^3(aB+3Ab)+\frac{3}{11}cx^{11}\left(aBc+Abc+b^2B\right)+\frac{3}{5}ax^5\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{9}x^9\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{7}x^7\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{13}c^2x^{13}(Ac+3bB)+\frac{1}{15}Bc^3x^{15}$$

```
[Out] a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*(a*b*B + A*(b^2 + a*c)
)*x^5)/5 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^7)/7 + ((b^
3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^9)/9 + (3*c*(b^2*B + A
*b*c + a*B*c)*x^11)/11 + (c^2*(3*b*B + A*c)*x^13)/13 + (B*c^3*x^1
5)/15
```

Rubi [A] time = 0.302797, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$a^3Ax+\frac{1}{3}a^2x^3(aB+3Ab)+\frac{3}{11}cx^{11}\left(aBc+Abc+b^2B\right)+\frac{3}{5}ax^5\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{9}x^9\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{7}x^7\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{13}c^2x^{13}(Ac+3bB)+\frac{1}{15}Bc^3x^{15}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*(a*b*B + A*(b^2 + a*c)
)*x^5)/5 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^7)/7 + ((b^
3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^9)/9 + (3*c*(b^2*B + A
*b*c + a*B*c)*x^11)/11 + (c^2*(3*b*B + A*c)*x^13)/13 + (B*c^3*x^1
5)/15
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bc^3x^{15}}{15}+a^3\int A\,dx+\frac{a^2x^3\left(3Ab+Ba\right)}{3}+\frac{3ax^5\left(Aac+Ab^2+Bab\right)}{5}+\frac{c^2x^{13}\left(Ac+3Bb\right)}{13}+\frac{3cx^{11}\left(Abc+Bac+Bb^2\right)}{11}+x^9\left(\frac{Aac^2}{3}+\frac{Ab^2c}{3}+\frac{2Babc}{3}+\frac{Bb^3}{9}\right)+x^7\left(\frac{6Aabc}{7}+\frac{Ab^3}{7}+\frac{3Ba^2c}{7}+\frac{3Bab^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] B*c**3*x**15/15 + a**3*Integral(A, x) + a**2*x**3*(3*A*b + B*a)/3
 + 3*a*x**5*(A*a*c + A*b**2 + B*a*b)/5 + c**2*x**13*(A*c + 3*B*b)
/13 + 3*c*x**11*(A*b*c + B*a*c + B*b**2)/11 + x**9*(A*a*c**2/3 +
A*b**2*c/3 + 2*B*a*b*c/3 + B*b**3/9) + x**7*(6*A*a*b*c/7 + A*b**3
/7 + 3*B*a**2*c/7 + 3*B*a*b**2/7)
```

Mathematica [A] time = 0.0896112, size = 161, normalized size = 1.

$$a^3Ax+\frac{1}{3}a^2x^3(aB+3Ab)+\frac{3}{11}cx^{11}\left(aBc+Abc+b^2B\right)+\frac{3}{5}ax^5\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{9}x^9\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{7}x^7\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{13}c^2x^{13}(Ac+3bB)+\frac{1}{15}Bc^3x^{15}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*(a*b*B + A*(b^2 + a*c)
)*x^5)/5 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^7)/7 + ((b^
3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^9)/9 + (3*c*(b^2*B + A
*b*c + a*B*c)*x^11)/11 + (c^2*(3*b*B + A*c)*x^13)/13 + (B*c^3*x^1
5)/15
```

Maple [A] time = 0.001, size = 223, normalized size = 1.4

$$\frac{Bc^3x^{15}}{15}+\frac{\left(Ac^3+3\,Bc^2b\right)x^{13}}{13}+\frac{\left(3\,Ac^2b+B\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)\right)x^{11}}{11}+\frac{\left(A\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)+B\left(4\,abc+b\left(2\,ac+b^2\right)\right)\right)x^9}{9}+\frac{\left(A\left(4\,abc+b\left(2\,ac+b^2\right)\right)+B\left(a\left(2\,ac+b^2\right)+2\,ab^2+a^2c\right)\right)x^7}{7}+\frac{\left(A\left(a\left(2\,ac+b^2\right)+2\,ab^2+a^2c\right)+3\,Ba^2b\right)x^5}{5}+\frac{\left(3\,Aa^2b+Ba^3\right)x^3}{3}+a^3Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/15*B*c^3*x^15+1/13*(A*c^3+3*B*b*c^2)*x^13+1/11*(3*A*c^2*b+B*(a*
c^2+2*b^2*c+c*(2*a*c+b^2)))*x^11+1/9*(A*(a*c^2+2*b^2*c+c*(2*a*c+b
^2))+B*(4*a*b*c+b*(2*a*c+b^2)))*x^9+1/7*(A*(4*a*b*c+b*(2*a*c+b^2)
)+B*(a*(2*a*c+b^2)+2*a*b^2+a^2*c))*x^7+1/5*(A*(a*(2*a*c+b^2)+2*a*
b^2+a^2*c)+3*B*a^2*b)*x^5+1/3*(3*A*a^2*b+B*a^3)*x^3+a^3*A*x
```

Maxima [A] time = 0.712528, size = 220, normalized size = 1.37

$$\frac{1}{15}\,Bc^3x^{15} + \frac{1}{13}\left(3\,Bbc^2 + Ac^3\right)x^{13} + \frac{3}{11}\left(Bb^2c + (Ba + Ab)c^2\right)x^{11} + \frac{1}{9}\left(Bb^3 + 3\,Aac^2 + 3\left(2\,Bab + Ab^2\right)c\right)x^9 + \frac{1}{7}\left(3\,Bab^2 + Ab^3 + 3\left(Ba^2 + 2\,Aab\right)c\right)x^7 + \frac{3}{5}\left(Ba^2b + Aab^2 + Aa^2c\right)x^5 + Aa^3x + \frac{1}{3}\left(Ba^3 + 3\,Aa^2b\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A),x, algorithm="maxima")
```

```
[Out] 1/15*B*c^3*x^15 + 1/13*(3*B*b*c^2 + A*c^3)*x^13 + 3/11*(B*b^2*c +
 (B*a + A*b)*c^2)*x^11 + 1/9*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*
b^2)*c)*x^9 + 1/7*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^7
 + 3/5*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^5 + A*a^3*x + 1/3*(B*a^3 +
 3*A*a^2*b)*x^3
```

Fricas [A] time = 0.239825, size = 1, normalized size = 0.01

$$\frac{1}{15}x^{15}c^3B + \frac{3}{13}x^{13}c^2bB + \frac{1}{13}x^{13}c^3A + \frac{3}{11}x^{11}cb^2B + \frac{3}{11}x^{11}c^2aB + \frac{3}{11}x^{11}c^2bA + \frac{1}{9}x^9b^3B + \frac{2}{3}x^9cbaB + \frac{1}{3}x^9cb^2A + \frac{1}{3}x^9c^2aA + \frac{3}{7}x^7b^2aB + \frac{3}{7}x^7ca^2B + \frac{1}{7}x^7b^3A + \frac{6}{7}x^7cbaA + \frac{3}{5}x^5ba^2B + \frac{3}{5}x^5b^2aA + \frac{3}{5}x^5ca^2A + \frac{1}{3}x^3a^3B + x^3ba^2A + xa^3A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A),x, algorithm="fricas")
```

```
[Out] 1/15*x^15*c^3*B + 3/13*x^13*c^2*b*B + 1/13*x^13*c^3*A + 3/11*x^11
*c*b^2*B + 3/11*x^11*c^2*a*B + 3/11*x^11*c^2*b*A + 1/9*x^9*b^3*B
+ 2/3*x^9*c*b*a*B + 1/3*x^9*c*b^2*A + 1/3*x^9*c^2*a*A + 3/7*x^7*b
^2*a*B + 3/7*x^7*c*a^2*B + 1/7*x^7*b^3*A + 6/7*x^7*c*b*a*A + 3/5*
x^5*b*a^2*B + 3/5*x^5*b^2*a*A + 3/5*x^5*c*a^2*A + 1/3*x^3*a^3*B +
 x^3*b*a^2*A + x*a^3*A
```

Sympy [A] time = 0.186605, size = 199, normalized size = 1.24

$$Aa^3x + \frac{Bc^3x^{15}}{15} + x^{13}\left(\frac{Ac^3}{13} + \frac{3Bbc^2}{13}\right) + x^{11}\left(\frac{3Abc^2}{11} + \frac{3Bac^2}{11} + \frac{3Bb^2c}{11}\right) + x^9\left(\frac{Aac^2}{3} + \frac{Ab^2c}{3} + \frac{2Babc}{3} + \frac{Bb^3}{9}\right) + x^7\left(\frac{6Aabc}{7} + \frac{Ab^3}{7} + \frac{3Ba^2c}{7} + \frac{3Bab^2}{7}\right) + x^5\left(\frac{3Aa^2c}{5} + \frac{3Aab^2}{5} + \frac{3Ba^2b}{5}\right) + x^3\left(Aa^2b + \frac{Ba^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x + B*c**3*x**15/15 + x**13*(A*c**3/13 + 3*B*b*c**2/13) +
x**11*(3*A*b*c**2/11 + 3*B*a*c**2/11 + 3*B*b**2*c/11) + x**9*(A*a
*c**2/3 + A*b**2*c/3 + 2*B*a*b*c/3 + B*b**3/9) + x**7*(6*A*a*b*c/
7 + A*b**3/7 + 3*B*a**2*c/7 + 3*B*a*b**2/7) + x**5*(3*A*a**2*c/5
+ 3*A*a*b**2/5 + 3*B*a**2*b/5) + x**3*(A*a**2*b + B*a**3/3)
```

GIAC/XCAS [A] time = 0.262109, size = 255, normalized size = 1.58

$$\frac{1}{15}Bc^3x^{15} + \frac{3}{13}Bbc^2x^{13} + \frac{1}{13}Ac^3x^{13} + \frac{3}{11}Bb^2cx^{11} + \frac{3}{11}Bac^2x^{11} + \frac{3}{11}Abc^2x^{11} + \frac{1}{9}Bb^3x^9 + \frac{2}{3}Babcx^9 + \frac{1}{3}Ab^2cx^9 + \frac{1}{3}Aac^2x^9 + \frac{3}{7}Bab^2x^7 + \frac{1}{7}Ab^3x^7 + \frac{3}{7}Ba^2cx^7 + \frac{6}{7}Aabcx^7 + \frac{3}{5}Ba^2bx^5 + \frac{3}{5}Aab^2x^5 + \frac{3}{5}Aa^2cx^5 + \frac{1}{3}Ba^3x^3 + Aa^2bx^3 + Aa^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A),x, algorithm="giac")
```

```
[Out] 1/15*B*c^3*x^15 + 3/13*B*b*c^2*x^13 + 1/13*A*c^3*x^13 + 3/11*B*b^
2*c*x^11 + 3/11*B*a*c^2*x^11 + 3/11*A*b*c^2*x^11 + 1/9*B*b^3*x^9
+ 2/3*B*a*b*c*x^9 + 1/3*A*b^2*c*x^9 + 1/3*A*a*c^2*x^9 + 3/7*B*a*b
^2*x^7 + 1/7*A*b^3*x^7 + 3/7*B*a^2*c*x^7 + 6/7*A*a*b*c*x^7 + 3/5*
B*a^2*b*x^5 + 3/5*A*a*b^2*x^5 + 3/5*A*a^2*c*x^5 + 1/3*B*a^3*x^3 +
 A*a^2*b*x^3 + A*a^3*x
```

3.99 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x}\,dx$

Optimal. Leaf size=162

$$a^3A\log(x)+\frac{1}{2}a^2x^2(aB+3Ab)+\frac{3}{10}cx^{10}\left(aBc+Abc+b^2B\right)+\frac{3}{4}ax^4\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{8}x^8\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{6}x^6\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{12}c^2x^{12}(Ac+3bB)+\frac{1}{14}Bc^3x^{14}$$

```
[Out] (a^2*(3*A*b + a*B)*x^2)/2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^4)/4 +
 ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/6 + ((b^3*B + 3*A*
b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^8)/8 + (3*c*(b^2*B + A*b*c + a*B
*c)*x^10)/10 + (c^2*(3*b*B + A*c)*x^12)/12 + (B*c^3*x^14)/14 + a^
3*A*Log[x]
```

Rubi [A] time = 0.439926, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$a^3A\log(x)+\frac{1}{2}a^2x^2(aB+3Ab)+\frac{3}{10}cx^{10}\left(aBc+Abc+b^2B\right)+\frac{3}{4}ax^4\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{8}x^8\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{6}x^6\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{12}c^2x^{12}(Ac+3bB)+\frac{1}{14}Bc^3x^{14}$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x]
```

```
[Out] (a^2*(3*A*b + a*B)*x^2)/2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^4)/4 +
 ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/6 + ((b^3*B + 3*A*
b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^8)/8 + (3*c*(b^2*B + A*b*c + a*B
*c)*x^10)/10 + (c^2*(3*b*B + A*c)*x^12)/12 + (B*c^3*x^14)/14 + a^
3*A*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{Aa^3\log\left(x^2\right)}{2}+\frac{Bc^3x^{14}}{14}+\frac{3a\left(Aac+Ab^2+Bab\right)\int^{x^2}x\,dx}{2}+\frac{c^2x^{12}\left(Ac+3Bb\right)}{12}+\frac{3cx^{10}\left(Abc+Bac+Bb^2\right)}{10}+x^8\left(\frac{3Aac^2}{8}+\frac{3Ab^2c}{8}+\frac{3Babc}{4}+\frac{Bb^3}{8}\right)+x^6\left(Aabc+\frac{Ab^3}{6}+\frac{Ba^2c}{2}+\frac{Bab^2}{2}\right)+\left(\frac{3Ab}{2}+\frac{Ba}{2}\right)\int^{x^2}a^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x,x)
```

```
[Out] A*a**3*log(x**2)/2 + B*c**3*x**14/14 + 3*a*(A*a*c + A*b**2 + B*a*
b)*Integral(x, (x, x**2))/2 + c**2*x**12*(A*c + 3*B*b)/12 + 3*c*x
**10*(A*b*c + B*a*c + B*b**2)/10 + x**8*(3*A*a*c**2/8 + 3*A*b**2*
c/8 + 3*B*a*b*c/4 + B*b**3/8) + x**6*(A*a*b*c + A*b**3/6 + B*a**2
*c/2 + B*a*b**2/2) + (3*A*b/2 + B*a/2)*Integral(a**2, (x, x**2))
```

Mathematica [A] time = 0.123168, size = 162, normalized size = 1.

$$a^3A\log(x)+\frac{1}{2}a^2x^2(aB+3Ab)+\frac{3}{10}cx^{10}\left(aBc+Abc+b^2B\right)+\frac{3}{4}ax^4\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{8}x^8\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{6}x^6\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{12}c^2x^{12}(Ac+3bB)+\frac{1}{14}Bc^3x^{14}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x]
```

```
[Out] (a^2*(3*A*b + a*B)*x^2)/2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^4)/4 +
 ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/6 + ((b^3*B + 3*A*
b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^8)/8 + (3*c*(b^2*B + A*b*c + a*B
*c)*x^10)/10 + (c^2*(3*b*B + A*c)*x^12)/12 + (B*c^3*x^14)/14 + a^
3*A*Log[x]
```

Maple [A] time = 0.004, size = 191, normalized size = 1.2

$$\frac{Bc^3x^{14}}{14}+\frac{Ax^{12}c^3}{12}+\frac{Bx^{12}bc^2}{4}+\frac{3\,Ax^{10}bc^2}{10}+\frac{3\,Bx^{10}ac^2}{10}+\frac{3\,Bx^{10}b^2c}{10}+\frac{3\,Ax^8ac^2}{8}+\frac{3\,Ax^8b^2c}{8}+\frac{3\,Bx^8abc}{4}+\frac{Bx^8b^3}{8}+Ax^6abc+\frac{Ax^6b^3}{6}+\frac{Bx^6a^2c}{2}+\frac{Bx^6ab^2}{2}+\frac{3\,Ax^4a^2c}{4}+\frac{3\,Ax^4ab^2}{4}+\frac{3\,Bx^4a^2b}{4}+\frac{3\,Ax^2a^2b}{2}+\frac{Bx^2a^3}{2}+a^3A\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x)
```

```
[Out] 1/14*B*c^3*x^14+1/12*A*x^12*c^3+1/4*B*x^12*b*c^2+3/10*A*x^10*b*c^
2+3/10*B*x^10*a*c^2+3/10*B*x^10*b^2*c+3/8*A*x^8*a*c^2+3/8*A*x^8*b
^2*c+3/4*B*x^8*a*b*c+1/8*B*x^8*b^3+A*x^6*a*b*c+1/6*A*x^6*b^3+1/2*
B*x^6*a^2*c+1/2*B*x^6*a*b^2+3/4*A*x^4*a^2*c+3/4*A*x^4*a*b^2+3/4*B
*x^4*a^2*b+3/2*A*x^2*a^2*b+1/2*B*x^2*a^3+a^3*A*ln(x)
```

Maxima [A] time = 0.703261, size = 225, normalized size = 1.39

$$\frac{1}{14}\,Bc^3x^{14}+\frac{1}{12}\left(3\,Bbc^2+Ac^3\right)x^{12}+\frac{3}{10}\left(Bb^2c+(Ba+Ab)c^2\right)x^{10}+\frac{1}{8}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^8+\frac{1}{6}\left(3\,Bab^2+Ab^3+3\left(Ba^2+2\,Aab\right)c\right)x^6+\frac{3}{4}\left(Ba^2b+Aab^2+Aa^2c\right)x^4+\frac{1}{2}\,Aa^3\log\left(x^2\right)+\frac{1}{2}\left(Ba^3+3\,Aa^2b\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A)/x,x, algorithm="maxima")
```

```
[Out] 1/14*B*c^3*x^14 + 1/12*(3*B*b*c^2 + A*c^3)*x^12 + 3/10*(B*b^2*c +
 (B*a + A*b)*c^2)*x^10 + 1/8*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*
b^2)*c)*x^8 + 1/6*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6
 + 3/4*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^4 + 1/2*A*a^3*log(x^2) + 1
/2*(B*a^3 + 3*A*a^2*b)*x^2
```

Fricas [A] time = 0.255229, size = 221, normalized size = 1.36

$$\frac{1}{14}\,Bc^3x^{14}+\frac{1}{12}\left(3\,Bbc^2+Ac^3\right)x^{12}+\frac{3}{10}\left(Bb^2c+(Ba+Ab)c^2\right)x^{10}+\frac{1}{8}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^8+\frac{1}{6}\left(3\,Bab^2+Ab^3+3\left(Ba^2+2\,Aab\right)c\right)x^6+\frac{3}{4}\left(Ba^2b+Aab^2+Aa^2c\right)x^4+Aa^3\log(x)+\frac{1}{2}\left(Ba^3+3\,Aa^2b\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A)/x,x, algorithm="fricas")
```

```
[Out] 1/14*B*c^3*x^14 + 1/12*(3*B*b*c^2 + A*c^3)*x^12 + 3/10*(B*b^2*c +
 (B*a + A*b)*c^2)*x^10 + 1/8*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*
b^2)*c)*x^8 + 1/6*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6
 + 3/4*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^4 + A*a^3*log(x) + 1/2*(B*
a^3 + 3*A*a^2*b)*x^2
```

Sympy [A] time = 1.73224, size = 199, normalized size = 1.23

$$Aa^3\log(x) + \frac{Bc^3x^{14}}{14} + x^{12}\left(\frac{Ac^3}{12} + \frac{Bbc^2}{4}\right) + x^{10}\left(\frac{3Abc^2}{10} + \frac{3Bac^2}{10} + \frac{3Bb^2c}{10}\right) + x^8\left(\frac{3Aac^2}{8} + \frac{3Ab^2c}{8} + \frac{3Babc}{4} + \frac{Bb^3}{8}\right) + x^6\left(Aabc + \frac{Ab^3}{6} + \frac{Ba^2c}{2} + \frac{Bab^2}{2}\right) + x^4\left(\frac{3Aa^2c}{4} + \frac{3Aab^2}{4} + \frac{3Ba^2b}{4}\right) + x^2\left(\frac{3Aa^2b}{2} + \frac{Ba^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x,x)
```

```
[Out] A*a**3*log(x) + B*c**3*x**14/14 + x**12*(A*c**3/12 + B*b*c**2/4)
+ x**10*(3*A*b*c**2/10 + 3*B*a*c**2/10 + 3*B*b**2*c/10) + x**8*(3
*A*a*c**2/8 + 3*A*b**2*c/8 + 3*B*a*b*c/4 + B*b**3/8) + x**6*(A*a*
b*c + A*b**3/6 + B*a**2*c/2 + B*a*b**2/2) + x**4*(3*A*a**2*c/4 +
3*A*a*b**2/4 + 3*B*a**2*b/4) + x**2*(3*A*a**2*b/2 + B*a**3/2)
```

GIAC/XCAS [A] time = 0.264959, size = 261, normalized size = 1.61

$$\frac{1}{14}Bc^3x^{14} + \frac{1}{4}Bbc^2x^{12} + \frac{1}{12}Ac^3x^{12} + \frac{3}{10}Bb^2cx^{10} + \frac{3}{10}Bac^2x^{10} + \frac{3}{10}Abc^2x^{10} + \frac{1}{8}Bb^3x^8 + \frac{3}{4}Babcx^8 + \frac{3}{8}Ab^2cx^8 + \frac{3}{8}Aac^2x^8 + \frac{1}{2}Bab^2x^6 + \frac{1}{6}Ab^3x^6 + \frac{1}{2}Ba^2cx^6 + Aabcx^6 + \frac{3}{4}Ba^2bx^4 + \frac{3}{4}Aab^2x^4 + \frac{3}{4}Aa^2cx^4 + \frac{1}{2}Ba^3x^2 + \frac{3}{2}Aa^2bx^2 + \frac{1}{2}Aa^3\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A)/x,x, algorithm="giac")
```

```
[Out] 1/14*B*c^3*x^14 + 1/4*B*b*c^2*x^12 + 1/12*A*c^3*x^12 + 3/10*B*b^2
*c*x^10 + 3/10*B*a*c^2*x^10 + 3/10*A*b*c^2*x^10 + 1/8*B*b^3*x^8 +
 3/4*B*a*b*c*x^8 + 3/8*A*b^2*c*x^8 + 3/8*A*a*c^2*x^8 + 1/2*B*a*b^
2*x^6 + 1/6*A*b^3*x^6 + 1/2*B*a^2*c*x^6 + A*a*b*c*x^6 + 3/4*B*a^2
*b*x^4 + 3/4*A*a*b^2*x^4 + 3/4*A*a^2*c*x^4 + 1/2*B*a^3*x^2 + 3/2*
A*a^2*b*x^2 + 1/2*A*a^3*ln(x^2)
```

3.100 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2} dx$

Optimal. Leaf size=156

$$-\frac{a^3A}{x}+a^2x(aB+3Ab)+\frac{1}{3}cx^9\left(aBc+Abc+b^2B\right)+ax^3\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{7}x^7\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{5}x^5\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{11}c^2x^{11}(Ac+3bB)+\frac{1}{13}Bc^3x^{13}$$

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*B)*x + a*(a*b*B + A*(b^2 + a*c))*x^
3 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^5)/5 + ((b^3*B + 3
*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^7)/7 + (c*(b^2*B + A*b*c + a*
B*c)*x^9)/3 + (c^2*(3*b*B + A*c)*x^11)/11 + (B*c^3*x^13)/13
```

Rubi [A] time = 0.27807, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04

$$-\frac{a^3A}{x}+a^2x(aB+3Ab)+\frac{1}{3}cx^9\left(aBc+Abc+b^2B\right)+ax^3\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{7}x^7\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{5}x^5\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{11}c^2x^{11}(Ac+3bB)+\frac{1}{13}Bc^3x^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*B)*x + a*(a*b*B + A*(b^2 + a*c))*x^
3 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^5)/5 + ((b^3*B + 3
*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^7)/7 + (c*(b^2*B + A*b*c + a*
B*c)*x^9)/3 + (c^2*(3*b*B + A*c)*x^11)/11 + (B*c^3*x^13)/13
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^3}{x}+\frac{Bc^3x^{13}}{13}+ax^3\left(Aac+Ab^2+Bab\right)+\frac{c^2x^{11}\left(Ac+3Bb\right)}{11}+\frac{cx^9\left(Abc+Bac+Bb^2\right)}{3}+x^7\left(\frac{3Aac^2}{7}+\frac{3Ab^2c}{7}+\frac{6Babc}{7}+\frac{Bb^3}{7}\right)+x^5\left(\frac{6Aabc}{5}+\frac{Ab^3}{5}+\frac{3Ba^2c}{5}+\frac{3Bab^2}{5}\right)+\frac{a^2\left(3Ab+Ba\right)\int B\,dx}{B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x**2,x)
```

```
[Out] -A*a**3/x + B*c**3*x**13/13 + a*x**3*(A*a*c + A*b**2 + B*a*b) + c
**2*x**11*(A*c + 3*B*b)/11 + c*x**9*(A*b*c + B*a*c + B*b**2)/3 +
x**7*(3*A*a*c**2/7 + 3*A*b**2*c/7 + 6*B*a*b*c/7 + B*b**3/7) + x**
5*(6*A*a*b*c/5 + A*b**3/5 + 3*B*a**2*c/5 + 3*B*a*b**2/5) + a**2*(
3*A*b + B*a)*Integral(B, x)/B
```

Mathematica [A] time = 0.16053, size = 156, normalized size = 1.

$$-\frac{a^3A}{x}+a^2x(aB+3Ab)+\frac{1}{3}cx^9\left(aBc+Abc+b^2B\right)+ax^3\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{7}x^7\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{5}x^5\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{11}c^2x^{11}(Ac+3bB)+\frac{1}{13}Bc^3x^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*B)*x + a*(a*b*B + A*(b^2 + a*c))*x^
3 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^5)/5 + ((b^3*B + 3
*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^7)/7 + (c*(b^2*B + A*b*c + a*
B*c)*x^9)/3 + (c^2*(3*b*B + A*c)*x^11)/11 + (B*c^3*x^13)/13
```

Maple [A] time = 0.006, size = 186, normalized size = 1.2

$$\frac{Bc^3x^{13}}{13}+\frac{Ax^{11}c^3}{11}+\frac{3\,Bx^{11}bc^2}{11}+\frac{Ax^9bc^2}{3}+\frac{Bx^9ac^2}{3}+\frac{Bx^9b^2c}{3}+\frac{3\,Ax^7ac^2}{7}+\frac{3\,Ax^7b^2c}{7}+\frac{6\,Bx^7abc}{7}+\frac{Bx^7b^3}{7}+\frac{6\,Ax^5abc}{5}+\frac{Ax^5b^3}{5}+\frac{3\,Bx^5a^2c}{5}+\frac{3\,Bx^5ab^2}{5}+Ax^3a^2c+Ax^3ab^2+Bx^3a^2b+3\,Axa^2b+Bxa^3-\frac{Aa^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x)
```

```
[Out] 1/13*B*c^3*x^13+1/11*A*x^11*c^3+3/11*B*x^11*b*c^2+1/3*A*x^9*b*c^2
+1/3*B*x^9*a*c^2+1/3*B*x^9*b^2*c+3/7*A*x^7*a*c^2+3/7*A*x^7*b^2*c+
6/7*B*x^7*a*b*c+1/7*B*x^7*b^3+6/5*A*x^5*a*b*c+1/5*A*x^5*b^3+3/5*B
*x^5*a^2*c+3/5*B*x^5*a*b^2+A*x^3*a^2*c+A*x^3*a*b^2+B*x^3*a^2*b+3*
A*x*a^2*b+B*x*a^3-a^3*A/x
```

Maxima [A] time = 0.699918, size = 219, normalized size = 1.4

$$\frac{1}{13} Bc^3x^{13} + \frac{1}{11}\left(3\,Bbc^2 + Ac^3\right)x^{11} + \frac{1}{3}\left(Bb^2c + (Ba + Ab)c^2\right)x^9 + \frac{1}{7}\left(Bb^3 + 3\,Aac^2 + 3\left(2\,Bab + Ab^2\right)c\right)x^7 + \frac{1}{5}\left(3\,Bab^2 + Ab^3 + 3\left(Ba^2 + 2\,Aab\right)c\right)x^5 + \left(Ba^2b + Aab^2 + Aa^2c\right)x^3 - \frac{Aa^3}{x} + \left(Ba^3 + 3\,Aa^2b\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/13*B*c^3*x^13 + 1/11*(3*B*b*c^2 + A*c^3)*x^11 + 1/3*(B*b^2*c +
(B*a + A*b)*c^2)*x^9 + 1/7*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^
2)*c)*x^7 + 1/5*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^5 +
 (B*a^2*b + A*a*b^2 + A*a^2*c)*x^3 - A*a^3/x + (B*a^3 + 3*A*a^2*b
)*x
```

Fricas [A] time = 0.241027, size = 227, normalized size = 1.46

$$\frac{1155\,Bc^3x^{14} + 1365\left(3\,Bbc^2 + Ac^3\right)x^{12} + 5005\left(Bb^2c + (Ba + Ab)c^2\right)x^{10} + 2145\left(Bb^3 + 3\,Aac^2 + 3\left(2\,Bab + Ab^2\right)c\right)x^8 + 3003\,(\ldots}{15015\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*B*c^3*x^14 + 1365*(3*B*b*c^2 + A*c^3)*x^12 + 5005*(
B*b^2*c + (B*a + A*b)*c^2)*x^10 + 2145*(B*b^3 + 3*A*a*c^2 + 3*(2*
B*a*b + A*b^2)*c)*x^8 + 3003*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*
a*b)*c)*x^6 + 15015*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^4 - 15015*A*a
^3 + 15015*(B*a^3 + 3*A*a^2*b)*x^2)/x
```

Sympy [A] time = 1.73799, size = 185, normalized size = 1.19

$$-\frac{Aa^3}{x} + \frac{Bc^3x^{13}}{13} + x^{11}\left(\frac{Ac^3}{11} + \frac{3Bbc^2}{11}\right) + x^9\left(\frac{Abc^2}{3} + \frac{Bac^2}{3} + \frac{Bb^2c}{3}\right) + x^7\left(\frac{3Aac^2}{7} + \frac{3Ab^2c}{7} + \frac{6Babc}{7} + \frac{Bb^3}{7}\right) + x^5\left(\frac{6Aabc}{5} + \frac{Ab^3}{5} + \frac{3Ba^2c}{5} + \frac{3Bab^2}{5}\right) + x^3\left(Aa^2c + Aab^2 + Ba^2b\right) + x\left(3Aa^2b + Ba^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x**2,x)
```

```
[Out] -A*a**3/x + B*c**3*x**13/13 + x**11*(A*c**3/11 + 3*B*b*c**2/11) +
 x**9*(A*b*c**2/3 + B*a*c**2/3 + B*b**2*c/3) + x**7*(3*A*a*c**2/7
 + 3*A*b**2*c/7 + 6*B*a*b*c/7 + B*b**3/7) + x**5*(6*A*a*b*c/5 + A
*b**3/5 + 3*B*a**2*c/5 + 3*B*a*b**2/5) + x**3*(A*a**2*c + A*a*b**
2 + B*a**2*b) + x*(3*A*a**2*b + B*a**3)
```

GIAC/XCAS [A] time = 0.262282, size = 250, normalized size = 1.6

$$\frac{1}{13}Bc^3x^{13} + \frac{3}{11}Bbc^2x^{11} + \frac{1}{11}Ac^3x^{11} + \frac{1}{3}Bb^2cx^9 + \frac{1}{3}Bac^2x^9 + \frac{1}{3}Abc^2x^9 + \frac{1}{7}Bb^3x^7 + \frac{6}{7}Babcx^7 + \frac{3}{7}Ab^2cx^7 + \frac{3}{7}Aac^2x^7 + \frac{3}{5}Bab^2x^5 + \frac{1}{5}Ab^3x^5 + \frac{3}{5}Ba^2cx^5 + \frac{6}{5}Aabcx^5 + Ba^2bx^3 + Aab^2x^3 + Aa^2cx^3 + Ba^3x + 3\,Aa^2bx - \frac{Aa^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A)/x^2,x, algorithm="giac")
```

```
[Out] 1/13*B*c^3*x^13 + 3/11*B*b*c^2*x^11 + 1/11*A*c^3*x^11 + 1/3*B*b^2
*c*x^9 + 1/3*B*a*c^2*x^9 + 1/3*A*b*c^2*x^9 + 1/7*B*b^3*x^7 + 6/7*
B*a*b*c*x^7 + 3/7*A*b^2*c*x^7 + 3/7*A*a*c^2*x^7 + 3/5*B*a*b^2*x^5
 + 1/5*A*b^3*x^5 + 3/5*B*a^2*c*x^5 + 6/5*A*a*b*c*x^5 + B*a^2*b*x^
3 + A*a*b^2*x^3 + A*a^2*c*x^3 + B*a^3*x + 3*A*a^2*b*x - A*a^3/x
```

3.101 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3}\,dx$

Optimal. Leaf size=162

$$-\frac{a^3A}{2x^2}+a^2\log(x)(aB+3Ab)+\frac{3}{8}cx^8\left(aBc+Abc+b^2B\right)+\frac{3}{2}ax^2\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{6}x^6\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{4}x^4\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{10}c^2x^{10}(Ac+3bB)+\frac{1}{12}Bc^3x^{12}$$

```
[Out] -(a^3*A)/(2*x^2) + (3*a*(a*b*B + A*(b^2 + a*c))*x^2)/2 + ((3*a*B*
(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^4)/4 + ((b^3*B + 3*A*b^2*c + 6
*a*b*B*c + 3*a*A*c^2)*x^6)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/
8 + (c^2*(3*b*B + A*c)*x^10)/10 + (B*c^3*x^12)/12 + a^2*(3*A*b +
a*B)*Log[x]
```

Rubi [A] time = 0.507832, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$-\frac{a^3A}{2x^2}+a^2\log(x)(aB+3Ab)+\frac{3}{8}cx^8\left(aBc+Abc+b^2B\right)+\frac{3}{2}ax^2\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{6}x^6\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{4}x^4\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{10}c^2x^{10}(Ac+3bB)+\frac{1}{12}Bc^3x^{12}$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x]
```

```
[Out] -(a^3*A)/(2*x^2) + (3*a*(a*b*B + A*(b^2 + a*c))*x^2)/2 + ((3*a*B*
(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^4)/4 + ((b^3*B + 3*A*b^2*c + 6
*a*b*B*c + 3*a*A*c^2)*x^6)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/
8 + (c^2*(3*b*B + A*c)*x^10)/10 + (B*c^3*x^12)/12 + a^2*(3*A*b +
a*B)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{Aa^3}{2x^2}+\frac{Bc^3x^{12}}{12}+\frac{a^2(3Ab+Ba)\log\left(x^2\right)}{2}+\frac{3ax^2\left(Aac+Ab^2+Bab\right)}{2}+\frac{c^2x^{10}(Ac+3Bb)}{10}+\frac{3cx^8\left(Abc+Bac+Bb^2\right)}{8}+x^6\left(\frac{Aac^2}{2}+\frac{Ab^2c}{2}+Babc+\frac{Bb^3}{6}\right)+\left(3Aabc+\frac{Ab^3}{2}+\frac{3Ba^2c}{2}+\frac{3Bab^2}{2}\right)\int^{x^2}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x**3,x)
```

```
[Out] -A*a**3/(2*x**2) + B*c**3*x**12/12 + a**2*(3*A*b + B*a)*log(x**2)
/2 + 3*a*x**2*(A*a*c + A*b**2 + B*a*b)/2 + c**2*x**10*(A*c + 3*B*
b)/10 + 3*c*x**8*(A*b*c + B*a*c + B*b**2)/8 + x**6*(A*a*c**2/2 +
A*b**2*c/2 + B*a*b*c + B*b**3/6) + (3*A*a*b*c + A*b**3/2 + 3*B*a*
*2*c/2 + 3*B*a*b**2/2)*Integral(x, (x, x**2))
```

Mathematica [A] time = 0.178136, size = 162, normalized size = 1.

$$-\frac{a^3A}{2x^2}+a^2\log(x)(aB+3Ab)+\frac{3}{8}cx^8\left(aBc+Abc+b^2B\right)+\frac{3}{2}ax^2\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{6}x^6\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{4}x^4\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{10}c^2x^{10}(Ac+3bB)+\frac{1}{12}Bc^3x^{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x]
```

```
[Out] -(a^3*A)/(2*x^2) + (3*a*(a*b*B + A*(b^2 + a*c))*x^2)/2 + ((3*a*B*
(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^4)/4 + ((b^3*B + 3*A*b^2*c + 6
*a*b*B*c + 3*a*A*c^2)*x^6)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/
8 + (c^2*(3*b*B + A*c)*x^10)/10 + (B*c^3*x^12)/12 + a^2*(3*A*b +
a*B)*Log[x]
```

Maple [A] time = 0.01, size = 190, normalized size = 1.2

$$\frac{Bc^3x^{12}}{12}+\frac{Ax^{10}c^3}{10}+\frac{3\,Bx^{10}bc^2}{10}+\frac{3\,Ax^8bc^2}{8}+\frac{3\,Bx^8ac^2}{8}+\frac{3\,Bx^8b^2c}{8}+\frac{Ax^6ac^2}{2}+\frac{Ax^6b^2c}{2}+Bx^6abc+\frac{Bx^6b^3}{6}+\frac{3\,Ax^4abc}{2}+\frac{Ax^4b^3}{4}+\frac{3\,Bx^4a^2c}{4}+\frac{3\,Bx^4ab^2}{4}+\frac{3\,Ax^2a^2c}{2}+\frac{3\,Ax^2ab^2}{2}+\frac{3\,Bx^2a^2b}{2}+3\,A\ln(x)\,a^2b+B\ln(x)\,a^3-\frac{Aa^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x)
```

```
[Out] 1/12*B*c^3*x^12+1/10*A*x^10*c^3+3/10*B*x^10*b*c^2+3/8*A*x^8*b*c^2
+3/8*B*x^8*a*c^2+3/8*B*x^8*b^2*c+1/2*A*x^6*a*c^2+1/2*A*x^6*b^2*c+
```

```
B*x^6*a*b*c+1/6*B*x^6*b^3+3/2*A*x^4*a*b*c+1/4*A*x^4*b^3+3/4*B*x^4
*a^2*c+3/4*B*x^4*a*b^2+3/2*A*x^2*a^2*c+3/2*A*x^2*a*b^2+3/2*B*x^2*
a^2*b+3*A*ln(x)*a^2*b+B*ln(x)*a^3-1/2*a^3*A/x^2
```

Maxima [A] time = 0.704833, size = 225, normalized size = 1.39

$$\frac{1}{12}Bc^3x^{12} + \frac{1}{10}\left(3Bbc^2 + Ac^3\right)x^{10} + \frac{3}{8}\left(Bb^2c + (Ba + Ab)c^2\right)x^8 + \frac{1}{6}\left(Bb^3 + 3Aac^2 + 3\left(2Bab + Ab^2\right)c\right)x^6 + \frac{1}{4}\left(3Bab^2 + Ab^3 + 3\left(Ba^2 + 2Aab\right)c\right)x^4 + \frac{3}{2}\left(Ba^2b + Aab^2 + Aa^2c\right)x^2 - \frac{Aa^3}{2x^2} + \frac{1}{2}\left(Ba^3 + 3Aa^2b\right)\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*B*c^3*x^12 + 1/10*(3*B*b*c^2 + A*c^3)*x^10 + 3/8*(B*b^2*c +
(B*a + A*b)*c^2)*x^8 + 1/6*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^
2)*c)*x^6 + 1/4*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^4 +
 3/2*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^2 - 1/2*A*a^3/x^2 + 1/2*(B*a
^3 + 3*A*a^2*b)*log(x^2)
```

Fricas [A] time = 0.246103, size = 230, normalized size = 1.42

$$\frac{10Bc^3x^{14} + 12\left(3Bbc^2 + Ac^3\right)x^{12} + 45\left(Bb^2c + (Ba + Ab)c^2\right)x^{10} + 20\left(Bb^3 + 3Aac^2 + 3\left(2Bab + Ab^2\right)c\right)x^8 + 30\left(3Bab^2 + Ab\right.}{120x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/120*(10*B*c^3*x^14 + 12*(3*B*b*c^2 + A*c^3)*x^12 + 45*(B*b^2*c
+ (B*a + A*b)*c^2)*x^10 + 20*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*
b^2)*c)*x^8 + 30*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6
+ 180*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^4 - 60*A*a^3 + 120*(B*a^3 +
 3*A*a^2*b)*x^2*log(x))/x^2
```

Sympy [A] time = 2.09926, size = 197, normalized size = 1.22

$$-\frac{Aa^3}{2x^2}+\frac{Bc^3x^{12}}{12}+a^2(3Ab+Ba)\log(x)+x^{10}\left(\frac{Ac^3}{10}+\frac{3Bbc^2}{10}\right)+x^8\left(\frac{3Abc^2}{8}+\frac{3Bac^2}{8}+\frac{3Bb^2c}{8}\right)$$
$$+x^6\left(\frac{Aac^2}{2}+\frac{Ab^2c}{2}+Babc+\frac{Bb^3}{6}\right)+x^4\left(\frac{3Aabc}{2}+\frac{Ab^3}{4}+\frac{3Ba^2c}{4}+\frac{3Bab^2}{4}\right)+x^2\left(\frac{3Aa^2c}{2}+\frac{3Aab^2}{2}+\frac{3Ba^2b}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x**3,x)
```

```
[Out] -A*a**3/(2*x**2) + B*c**3*x**12/12 + a**2*(3*A*b + B*a)*log(x) +
x**10*(A*c**3/10 + 3*B*b*c**2/10) + x**8*(3*A*b*c**2/8 + 3*B*a*c*
*2/8 + 3*B*b**2*c/8) + x**6*(A*a*c**2/2 + A*b**2*c/2 + B*a*b*c +
B*b**3/6) + x**4*(3*A*a*b*c/2 + A*b**3/4 + 3*B*a**2*c/4 + 3*B*a*b
**2/4) + x**2*(3*A*a**2*c/2 + 3*A*a*b**2/2 + 3*B*a**2*b/2)
```

GIAC/XCAS [A] time = 0.267513, size = 286, normalized size = 1.77

$$\frac{1}{12}Bc^3x^{12}+\frac{3}{10}Bbc^2x^{10}+\frac{1}{10}Ac^3x^{10}+\frac{3}{8}Bb^2cx^8+\frac{3}{8}Bac^2x^8+\frac{3}{8}Abc^2x^8+\frac{1}{6}Bb^3x^6$$
$$+Babcx^6+\frac{1}{2}Ab^2cx^6+\frac{1}{2}Aac^2x^6+\frac{3}{4}Bab^2x^4+\frac{1}{4}Ab^3x^4+\frac{3}{4}Ba^2cx^4+\frac{3}{2}Aabcx^4$$
$$+\frac{3}{2}Ba^2bx^2+\frac{3}{2}Aab^2x^2+\frac{3}{2}Aa^2cx^2+\frac{1}{2}\left(Ba^3+3\,Aa^2b\right)\ln\left(x^2\right)-\frac{Ba^3x^2+3\,Aa^2bx^2+Aa^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*(B*x^2 + A)/x^3,x, algorithm="giac")
```

```
[Out] 1/12*B*c^3*x^12 + 3/10*B*b*c^2*x^10 + 1/10*A*c^3*x^10 + 3/8*B*b^2
*c*x^8 + 3/8*B*a*c^2*x^8 + 3/8*A*b*c^2*x^8 + 1/6*B*b^3*x^6 + B*a*
b*c*x^6 + 1/2*A*b^2*c*x^6 + 1/2*A*a*c^2*x^6 + 3/4*B*a*b^2*x^4 + 1
/4*A*b^3*x^4 + 3/4*B*a^2*c*x^4 + 3/2*A*a*b*c*x^4 + 3/2*B*a^2*b*x^
2 + 3/2*A*a*b^2*x^2 + 3/2*A*a^2*c*x^2 + 1/2*(B*a^3 + 3*A*a^2*b)*l
n(x^2) - 1/2*(B*a^3*x^2 + 3*A*a^2*b*x^2 + A*a^3)/x^2
```

3.102 $\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4} dx$

Optimal. Leaf size=133

$$\frac{(-aBc - Abc + b^2B)\log(a+bx^2+cx^4)}{4c^3} + \frac{(2aAc^2 - 3abBc - Ab^2c + b^3B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} - \frac{x^2(bB-Ac)}{2c^2} + \frac{Bx^4}{4c}$$

```
[Out] -((b*B - A*c)*x^2)/(2*c^2) + (B*x^4)/(4*c) + ((b^3*B - A*b^2*c -
3*a*b*B*c + 2*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2*B - A*b*c - a*B*c)*Log[a + b*x^
2 + c*x^4])/(4*c^3)
```

Rubi [A] time = 0.432134, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{(-aBc - Abc + b^2B)\log(a+bx^2+cx^4)}{4c^3} + \frac{(2aAc^2 - 3abBc - Ab^2c + b^3B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} - \frac{x^2(bB-Ac)}{2c^2} + \frac{Bx^4}{4c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*B - A*c)*x^2)/(2*c^2) + (B*x^4)/(4*c) + ((b^3*B - A*b^2*c -
3*a*b*B*c + 2*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2*B - A*b*c - a*B*c)*Log[a + b*x^
2 + c*x^4])/(4*c^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{B\int^{x^2} x\,dx}{2c} + \left(\frac{Ac}{2} - \frac{Bb}{2}\right)\int^{x^2} \frac{1}{c^2}\,dx + \frac{(-Abc - Bac + Bb^2)\log(a+bx^2+cx^4)}{4c^3} + \frac{(2Aac^2 - Ab^2c - 3Babc + Bb^3)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^3\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*Integral(x, (x, x**2))/(2*c) + (A*c/2 - B*b/2)*Integral(c**(-2)
, (x, x**2)) + (-A*b*c - B*a*c + B*b**2)*log(a + b*x**2 + c*x**4)
/(4*c**3) + (2*A*a*c**2 - A*b**2*c - 3*B*a*b*c + B*b**3)*atanh((b
 + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c**3*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.101535, size = 126, normalized size = 0.95

$$\frac{\left(-aBc-Abc+b^2B\right)\log\left(a+bx^2+cx^4\right)+\frac{2\left(-2aAc^2+3abBc+Ab^2c+b^3(-B)\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+2cx^2(Ac-bB)+Bc^2x^4}{4c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(-(b*B) + A*c)*x^2 + B*c^2*x^4 + (2*(-(b^3*B) + A*b^2*c + 3*
a*b*B*c - 2*a*A*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sq
rt[-b^2 + 4*a*c] + (b^2*B - A*b*c - a*B*c)*Log[a + b*x^2 + c*x^4]
)/(4*c^3)
```

Maple [B] time = 0.008, size = 261, normalized size = 2.

$$\frac{Bx^4}{4\,c}+\frac{Ax^2}{2\,c}-\frac{bBx^2}{2\,c^2}-\frac{\ln\left(cx^4+bx^2+a\right)Ab}{4\,c^2}-\frac{\ln\left(cx^4+bx^2+a\right)aB}{4\,c^2}+\frac{\ln\left(cx^4+bx^2+a\right)b^2B}{4\,c^3}-\frac{aA}{c}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{3\,abB}{2\,c^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{Ab^2}{2\,c^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{b^3B}{2\,c^3}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*B*x^4/c+1/2/c*A*x^2-1/2/c^2*b*B*x^2-1/4/c^2*ln(c*x^4+b*x^2+a)
*A*b-1/4/c^2*ln(c*x^4+b*x^2+a)*a*B+1/4/c^3*ln(c*x^4+b*x^2+a)*b^2*
B-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*A
+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*
a*b*B+1/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1
```

```
/2))*A*b^2-1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^
2)^(1/2))*b^3*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^5/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.279729, size = 1, normalized size = 0.01

$$\left[\frac{\left(Bb^3 + 2\,Aac^2 - \left(3\,Bab + Ab^2\right)c\right)\log\left(\frac{b^3 - 4\,abc + 2\left(b^2c - 4\,ac^2\right)x^2 + \left(2\,c^2x^4 + 2\,bcx^2 + b^2 - 2\,ac\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right) + \left(Bc^2x^4 - 2\left(Bbc - Ac^2\right)x^2 + \right.}{4\sqrt{b^2 - 4\,ac}c^3}\right.$$

$$-\frac{2\left(Bb^3 + 2\,Aac^2 - \left(3\,Bab + Ab^2\right)c\right)\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right) - \left(Bc^2x^4 - 2\left(Bbc - Ac^2\right)x^2 + \left(Bb^2 - (Ba + Ab)c\right)\log\left(cx\right.\right.}{4\sqrt{-b^2 + 4\,ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^5/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/4*((B*b^3 + 2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*log((b^3 - 4*a*b*
c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*
c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (B*c^2*x^4 - 2*(B*b*
c - A*c^2)*x^2 + (B*b^2 - (B*a + A*b)*c)*log(c*x^4 + b*x^2 + a))*
sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*c^3), -1/4*(2*(B*b^3 + 2*A*
a*c^2 - (3*B*a*b + A*b^2)*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*
a*c)/(b^2 - 4*a*c)) - (B*c^2*x^4 - 2*(B*b*c - A*c^2)*x^2 + (B*b^2
 - (B*a + A*b)*c)*log(c*x^4 + b*x^2 + a))*sqrt(-b^2 + 4*a*c))/(sq
rt(-b^2 + 4*a*c)*c^3)]
```

Sympy [A] time = 24.859, size = 619, normalized size = 4.65

$$\frac{Bx^4}{4c} + \left(-\frac{\sqrt{-4ac+b^2}\left(-2Aac^2+Ab^2c+3Babc-Bb^3\right)}{4c^3\left(4ac-b^2\right)} - \frac{Abc+Bac-Bb^2}{4c^3}\right)\log\left(x^2 + \frac{Aabc+2Ba^2c-Bab^2+8ac^3\left(-\frac{\sqrt{-4ac+b^2}(-2Aac^2+Ab^2c+3Babc-Bb^3)}{4c^3(4ac-b^2)} - \frac{Abc+Bac-Bb^2}{4c^3}\right) - 2b^2c^2\left(-\right.}{-2Aac^2+Ab^2c+3Babc-Bb^3}\right) + \left(\frac{\sqrt{-4ac+b^2}\left(-2Aac^2+Ab^2c+3Babc-Bb^3\right)}{4c^3\left(4ac-b^2\right)} - \frac{Abc+Bac-Bb^2}{4c^3}\right)\log\left(x^2 + \frac{Aabc+2Ba^2c-Bab^2+8ac^3\left(\frac{\sqrt{-4ac+b^2}(-2Aac^2+Ab^2c+3Babc-Bb^3)}{4c^3(4ac-b^2)} - \frac{Abc+Bac-Bb^2}{4c^3}\right) - 2b^2c^2\left(\sqrt{-}\right.}{-2Aac^2+Ab^2c+3Babc-Bb^3}\right) - \frac{x^2(-Ac+Bb)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*x**4/(4*c) + (-sqrt(-4*a*c + b**2)*(-2*A*a*c**2 + A*b**2*c + 3*
B*a*b*c - B*b**3)/(4*c**3*(4*a*c - b**2)) - (A*b*c + B*a*c - B*b*
*2)/(4*c**3))*log(x**2 + (A*a*b*c + 2*B*a**2*c - B*a*b**2 + 8*a*c
**3*(-sqrt(-4*a*c + b**2)*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B
*b**3)/(4*c**3*(4*a*c - b**2)) - (A*b*c + B*a*c - B*b**2)/(4*c**3
)) - 2*b**2*c**2*(-sqrt(-4*a*c + b**2)*(-2*A*a*c**2 + A*b**2*c +
3*B*a*b*c - B*b**3)/(4*c**3*(4*a*c - b**2)) - (A*b*c + B*a*c - B*
b**2)/(4*c**3)))/(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)) +
 (sqrt(-4*a*c + b**2)*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**
3)/(4*c**3*(4*a*c - b**2)) - (A*b*c + B*a*c - B*b**2)/(4*c**3))*l
og(x**2 + (A*a*b*c + 2*B*a**2*c - B*a*b**2 + 8*a*c**3*(sqrt(-4*a*
c + b**2)*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)/(4*c**3*(
4*a*c - b**2)) - (A*b*c + B*a*c - B*b**2)/(4*c**3)) - 2*b**2*c**2
*(sqrt(-4*a*c + b**2)*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**
3)/(4*c**3*(4*a*c - b**2)) - (A*b*c + B*a*c - B*b**2)/(4*c**3)))/
(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)) - x**2*(-A*c + B*b
)/(2*c**2)
```

GIAC/XCAS [A] time = 0.294702, size = 170, normalized size = 1.28

$$\frac{Bcx^4 - 2\,Bbx^2 + 2\,Acx^2}{4\,c^2} + \frac{\left(Bb^2 - Bac - Abc\right)\ln\left(cx^4 + bx^2 + a\right)}{4\,c^3} - \frac{\left(Bb^3 - 3\,Babc - Ab^2c + 2\,Aac^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/4*(B*c*x^4 - 2*B*b*x^2 + 2*A*c*x^2)/c^2 + 1/4*(B*b^2 - B*a*c -
A*b*c)*ln(c*x^4 + b*x^2 + a)/c^3 - 1/2*(B*b^3 - 3*B*a*b*c - A*b^2
*c + 2*A*a*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b
^2 + 4*a*c)*c^3)
```

3.103 $\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=97

$$-\frac{(-2aBc-Abc+b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{Bx^2}{2c}$$

```
[Out] (B*x^2)/(2*c) - ((b^2*B - A*b*c - 2*a*B*c)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) - ((b*B - A*c)*Log[
a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi [A] time = 0.259455, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$-\frac{(-2aBc-Abc+b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{Bx^2}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x^2)/(2*c) - ((b^2*B - A*b*c - 2*a*B*c)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) - ((b*B - A*c)*Log[
a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{x^2} B\,dx}{2c}+\frac{(Ac-Bb)\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{(2Bac+b\,(Ac-Bb))\,\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(B, (x, x**2))/(2*c) + (A*c - B*b)*log(a + b*x**2 + c*x**
4)/(4*c**2) + (2*B*a*c + b*(A*c - B*b))*atanh((b + 2*c*x**2)/sqrt
(-4*a*c + b**2))/(2*c**2*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.123189, size = 93, normalized size = 0.96

$$\frac{\frac{2\left(-2aBc-Abc+b^2B\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+(Ac-bB)\log\left(a+bx^2+cx^4\right)+2Bcx^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*B*c*x^2 + (2*(b^2*B - A*b*c - 2*a*B*c)*ArcTan[(b + 2*c*x^2)/Sq
rt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-(b*B) + A*c)*Log[a + b*
x^2 + c*x^4])/(4*c^2)
```

Maple [A] time = 0.005, size = 175, normalized size = 1.8

$$\frac{Bx^2}{2c}+\frac{\ln\left(cx^4+bx^2+a\right)A}{4c}-\frac{\ln\left(cx^4+bx^2+a\right)bB}{4c^2}-\frac{Ba}{c}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-\frac{Ab}{2c}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}+\frac{b^2B}{2c^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*B*x^2/c+1/4/c*ln(c*x^4+b*x^2+a)*A-1/4/c^2*ln(c*x^4+b*x^2+a)*b
*B-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B*
a-1/2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A
*b+1/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)
)*b^2*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273215, size = 1, normalized size = 0.01

$$\left[-\frac{\left(Bb^2-(2\,Ba+Ab)c\right)\log\left(\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x^2+(2\,c^2x^4+2\,bcx^2+b^2-2\,ac)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(2\,Bcx^2-(Bb-Ac)\log\left(cx^4+bx^2+a\right)\right.}{4\,\sqrt{b^2-4\,ac}c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [-1/4*((B*b^2 - (2*B*a + A*b)*c)*log((b^3 - 4*a*b*c + 2*(b^2*c -
4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4
*a*c))/(c*x^4 + b*x^2 + a)) - (2*B*c*x^2 - (B*b - A*c)*log(c*x^4
+ b*x^2 + a))*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*c^2), 1/4*(2*
(B*b^2 - (2*B*a + A*b)*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c
)/(b^2 - 4*a*c)) + (2*B*c*x^2 - (B*b - A*c)*log(c*x^4 + b*x^2 + a
))*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)]
```

Sympy [A] time = 15.7278, size = 434, normalized size = 4.47

$$\frac{Bx^2}{2c}+\left(-\frac{-Ac+Bb}{4c^2}-\frac{\sqrt{-4ac+b^2}\left(Abc+2Bac-Bb^2\right)}{4c^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{2Aac-Bab-8ac^2\left(-\frac{-Ac+Bb}{4c^2}-\frac{\sqrt{-4ac+b^2}(Abc+2Bac-Bb^2)}{4c^2(4ac-b^2)}\right)+2b^2c\left(-\frac{-Ac+Bb}{4c^2}-\right.}{Abc+2Bac-Bb^2}\right.$$
$$+\left(-\frac{-Ac+Bb}{4c^2}+\frac{\sqrt{-4ac+b^2}\left(Abc+2Bac-Bb^2\right)}{4c^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{2Aac-Bab-8ac^2\left(-\frac{-Ac+Bb}{4c^2}+\frac{\sqrt{-4ac+b^2}(Abc+2Bac-Bb^2)}{4c^2(4ac-b^2)}\right)+2b^2c\left(-\frac{-Ac+Bb}{4c^2}+\right.}{Abc+2Bac-Bb^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*x**2/(2*c) + (-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c + b**2)*(A*b
*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (2*A*a
*c - B*a*b - 8*a*c**2*(-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c + b**
```

```
2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c
*(-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c -
 B*b**2)/(4*c**2*(4*a*c - b**2))))/(A*b*c + 2*B*a*c - B*b**2)) +
(-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c -
B*b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (2*A*a*c - B*a*b - 8*
a*c**2*(-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c + b**2)*(A*b*c + 2*B
*a*c - B*b**2)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-(-A*c + B*b)
/(4*c**2) + sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**
2*(4*a*c - b**2))))/(A*b*c + 2*B*a*c - B*b**2))
```

GIAC/XCAS [A] time = 0.291799, size = 123, normalized size = 1.27

$$\frac{Bx^2}{2\,c} - \frac{(Bb - Ac)\ln\left(cx^4 + bx^2 + a\right)}{4\,c^2} + \frac{\left(Bb^2 - 2\,Bac - Abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c - 1/4*(B*b - A*c)*ln(c*x^4 + b*x^2 + a)/c^2 + 1/2*(B*
b^2 - 2*B*a*c - A*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(
sqrt(-b^2 + 4*a*c)*c^2)
```

3.104 $\int \frac{x(A+Bx^2)}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=71

$$\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c}$$

```
[Out] ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqr
t[b^2 - 4*a*c]) + (B*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rubi [A] time = 0.159537, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqr
t[b^2 - 4*a*c]) + (B*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rubi in Sympy [A] time = 23.2393, size = 63, normalized size = 0.89

$$\frac{B\log\left(a+bx^2+cx^4\right)}{4c}-\frac{(2Ac-Bb)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*log(a + b*x**2 + c*x**4)/(4*c) - (2*A*c - B*b)*atanh((b + 2*c*x
**2)/sqrt(-4*a*c + b**2))/(2*c*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.0797695, size = 71, normalized size = 1.

$$\frac{B\log\left(a+bx^2+cx^4\right)-\frac{2(bB-2Ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-2*(b*B - 2*A*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] + B*Log[a + b*x^2 + c*x^4])/(4*c)
```

Maple [A] time = 0.004, size = 98, normalized size = 1.4

$$\frac{B\ln\left(cx^4+bx^2+a\right)}{4\,c}+A\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{bB}{2\,c}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*B*ln(c*x^4+b*x^2+a)/c+1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/
(4*a*c-b^2)^(1/2))*A-1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*
a*c-b^2)^(1/2))*B*b/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.271636, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{b^2-4\,ac}B\log\left(cx^4+bx^2+a\right)-(Bb-2\,Ac)\log\left(-\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x^2-\left(2\,c^2x^4+2\,bcx^2+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{4\sqrt{b^2-4\,ac}c}, \frac{\sqrt{-b^2+4\,ac}B\log\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*B*log(c*x^4 + b*x^2 + a) - (B*b - 2*A*c)*
log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - (2*c^2*x^4 + 2*b*
c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(sq
rt(b^2 - 4*a*c)*c), 1/4*(sqrt(-b^2 + 4*a*c)*B*log(c*x^4 + b*x^2 +
 a) - 2*(B*b - 2*A*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b
^2 - 4*a*c)))/(sqrt(-b^2 + 4*a*c)*c)]
```

Sympy [A] time = 7.68165, size = 287, normalized size = 4.04

$$\left(\frac{B}{4c} - \frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}\right)\log\left(x^2+\frac{-Ab+2Ba-8ac\left(\frac{B}{4c}-\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)+2b^2\left(\frac{B}{4c}-\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)}{-2Ac+Bb}\right) + \left(\frac{B}{4c} + \frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}\right)\log\left(x^2+\frac{-Ab+2Ba-8ac\left(\frac{B}{4c}+\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)+2b^2\left(\frac{B}{4c}+\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)}{-2Ac+Bb}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] (B/(4*c) - (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)
))*log(x**2 + (-A*b + 2*B*a - 8*a*c*(B/(4*c) - (-2*A*c + B*b)*sqr
t(-4*a*c + b**2)/(4*c*(4*a*c - b**2))) + 2*b**2*(B/(4*c) - (-2*A*
c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2))))/(-2*A*c + B*b
)) + (B/(4*c) + (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c -
b**2)))*log(x**2 + (-A*b + 2*B*a - 8*a*c*(B/(4*c) + (-2*A*c + B*b
)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2))) + 2*b**2*(B/(4*c) + (
-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2))))/(-2*A*c
```

```
+ B*b))
```

GIAC/XCAS [A] time = 0.292196, size = 90, normalized size = 1.27

$$\frac{B\ln\left(cx^4+bx^2+a\right)}{4\,c}-\frac{(Bb-2\,Ac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/4*B*ln(c*x^4 + b*x^2 + a)/c - 1/2*(B*b - 2*A*c)*arctan((2*c*x^2
 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

3.105 $\int \frac{A+Bx^2}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=78

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}+\frac{A\log(x)}{a}$$

```
[Out] ((A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqr
t[b^2 - 4*a*c]) + (A*Log[x])/a - (A*Log[a + b*x^2 + c*x^4])/(4*a)
```

Rubi [A] time = 0.277219, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}+\frac{A\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqr
t[b^2 - 4*a*c]) + (A*Log[x])/a - (A*Log[a + b*x^2 + c*x^4])/(4*a)
```

Rubi in Sympy [A] time = 35.5782, size = 73, normalized size = 0.94

$$\frac{A\log\left(x^2\right)}{2a}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}+\frac{(Ab-2Ba)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] A*log(x**2)/(2*a) - A*log(a + b*x**2 + c*x**4)/(4*a) + (A*b - 2*B
*a)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*a*sqrt(-4*a*c +
b**2))
```

Mathematica [A] time = 0.180405, size = 128, normalized size = 1.64

$$\frac{-\left(A\left(\sqrt{b^2-4ac}+b\right)-2aB\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)+\left(A\left(b-\sqrt{b^2-4ac}\right)-2aB\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)+4A}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*A*Sqrt[b^2 - 4*a*c]*Log[x] - (-2*a*B + A*(b + Sqrt[b^2 - 4*a*c
]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2] + (-2*a*B + A*(b - Sqrt[
b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*Sqrt[b^
2 - 4*a*c])
```

Maple [A] time = 0.009, size = 105, normalized size = 1.4

$$-\frac{A\ln\left(cx^4+bx^2+a\right)}{4\,a}-\frac{Ab}{2\,a}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+B\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{A\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*A*ln(c*x^4+b*x^2+a)/a-1/2/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))*A*b+1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b
)/(4*a*c-b^2)^(1/2))*B+A*ln(x)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.318328, size = 1, normalized size = 0.01

$$\left[-\frac{(2\,Ba-Ab)\log\left(\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x^2+(2\,c^2x^4+2\,bcx^2+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\sqrt{b^2-4\,ac}\left(A\log\left(cx^4+bx^2+a\right)-4\,A\log\left(x\right)\right)}{4\,\sqrt{b^2-4\,ac}a},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*B*a - A*b)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2
 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^
4 + b*x^2 + a)) + sqrt(b^2 - 4*a*c)*(A*log(c*x^4 + b*x^2 + a) - 4
*A*log(x)))/(sqrt(b^2 - 4*a*c)*a), 1/4*(2*(2*B*a - A*b)*arctan(-(
2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - sqrt(-b^2 + 4*a*
c)*(A*log(c*x^4 + b*x^2 + a) - 4*A*log(x)))/(sqrt(-b^2 + 4*a*c)*a
)]
```

Sympy [A] time = 133.358, size = 330, normalized size = 4.23

$$\frac{A\log(x)}{a}+\left(-\frac{A}{4a}-\frac{(-Ab+2Ba)\sqrt{-4ac+b^2}}{4a\,(4ac-b^2)}\right)\log\left(x^2+\frac{2Aac-Ab^2+Bab+8a^2c\left(-\frac{A}{4a}-\frac{(-Ab+2Ba)\sqrt{-4ac+b^2}}{4a(4ac-b^2)}\right)-2ab^2\left(-\frac{A}{4a}-\frac{(-Ab+2Ba)\sqrt{-4ac+}}{4a(4ac-b^2)}\right.}{-Abc+2Bac}\right.$$
$$+\left(-\frac{A}{4a}+\frac{(-Ab+2Ba)\sqrt{-4ac+b^2}}{4a\,(4ac-b^2)}\right)\log\left(x^2+\frac{2Aac-Ab^2+Bab+8a^2c\left(-\frac{A}{4a}+\frac{(-Ab+2Ba)\sqrt{-4ac+b^2}}{4a(4ac-b^2)}\right)-2ab^2\left(-\frac{A}{4a}+\frac{(-Ab+2Ba)\sqrt{-4ac+b}}{4a(4ac-b^2)}\right.}{-Abc+2Bac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] A*log(x)/a + (-A/(4*a) - (-A*b + 2*B*a)*sqrt(-4*a*c + b**2)/(4*a*
(4*a*c - b**2)))*log(x**2 + (2*A*a*c - A*b**2 + B*a*b + 8*a**2*c*
(-A/(4*a) - (-A*b + 2*B*a)*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2
))) - 2*a*b**2*(-A/(4*a) - (-A*b + 2*B*a)*sqrt(-4*a*c + b**2)/(4*
a*(4*a*c - b**2))))/(-A*b*c + 2*B*a*c)) + (-A/(4*a) + (-A*b + 2*B
*a)*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)))*log(x**2 + (2*A*a*c
 - A*b**2 + B*a*b + 8*a**2*c*(-A/(4*a) + (-A*b + 2*B*a)*sqrt(-4*a
```

```
*c + b**2)/(4*a*(4*a*c - b**2))) - 2*a*b**2*(-A/(4*a) + (-A*b + 2
*B*a)*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2))))/(-A*b*c + 2*B*a*
c))
```

GIAC/XCAS [A] time = 0.289236, size = 105, normalized size = 1.35

$$-\frac{A\ln\left(cx^4+bx^2+a\right)}{4\,a}+\frac{A\ln\left(x^2\right)}{2\,a}+\frac{(2\,Ba-Ab)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] -1/4*A*ln(c*x^4 + b*x^2 + a)/a + 1/2*A*ln(x^2)/a + 1/2*(2*B*a - A
*b)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*
a)
```

3.106 $\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=112

$$-\frac{(-2aAc-abB+Ab^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{2ax^2}$$

```
[Out] -A/(2*a*x^2) - ((A*b^2 - a*b*B - 2*a*A*c)*ArcTanh[(b + 2*c*x^2)/S
qrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - ((A*b - a*B)*Log[x
])/a^2 + ((A*b - a*B)*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rubi [A] time = 0.456796, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{\left(abB-A\left(b^2-2ac\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -A/(2*a*x^2) + ((a*b*B - A*(b^2 - 2*a*c))*ArcTanh[(b + 2*c*x^2)/S
qrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - ((A*b - a*B)*Log[x
])/a^2 + ((A*b - a*B)*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rubi in Sympy [A] time = 51.7108, size = 105, normalized size = 0.94

$$-\frac{A}{2ax^2}-\frac{(Ab-Ba)\log\left(x^2\right)}{2a^2}+\frac{(Ab-Ba)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\left(-2Aac+Ab^2-Bab\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] -A/(2*a*x**2) - (A*b - B*a)*log(x**2)/(2*a**2) + (A*b - B*a)*log(
a + b*x**2 + c*x**4)/(4*a**2) - (-2*A*a*c + A*b**2 - B*a*b)*atanh
((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*a**2*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.423775, size = 186, normalized size = 1.66

$$\frac{\frac{\left(A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)-aB\left(\sqrt{b^2-4ac}+b\right)\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(A\left(b\sqrt{b^2-4ac}+2ac-b^2\right)+aB\left(b-\sqrt{b^2-4ac}\right)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+4\log(x}{4a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*A)/x^2 + 4*(-(A*b) + a*B)*Log[x] + ((-(a*B*(b + Sqrt[b^2 -
 4*a*c])) + A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b
^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((a*B*(b - Sqrt[b^2 -
 4*a*c]) + A*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Maple [A] time = 0.013, size = 191, normalized size = 1.7

$$\frac{\ln\left(cx^4+bx^2+a\right)Ab}{4a^2}-\frac{\ln\left(cx^4+bx^2+a\right)B}{4a}-\frac{Ac}{a}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}+\frac{Ab^2}{2a^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-\frac{bB}{2a}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-\frac{A}{2ax^2}-\frac{\ln(x)Ab}{a^2}+\frac{\ln(x)B}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/a^2*ln(c*x^4+b*x^2+a)*A*b-1/4/a*ln(c*x^4+b*x^2+a)*B-1/a/(4*a*
c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*c+1/2/a^2/(4
*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^2-1/2/a
/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*B-1/2*
A/a/x^2-1/a^2*ln(x)*A*b+1/a*ln(x)*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.471726, size = 1, normalized size = 0.01

$$\left[\frac{\left(Bab - Ab^2 + 2\,Aac\right)x^2 \log\left(\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x^2+(2\,c^2x^4+2\,bcx^2+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right) - \left((Ba - Ab)x^2 \log\left(cx^4 + bx^2 + a\right) - 4\,(B\right.}{4\,\sqrt{b^2 - 4\,ac}a^2x^2}\right.$$

$$-\frac{2\left(Bab - Ab^2 + 2\,Aac\right)x^2 \arctan\left(-\frac{(2\,cx^2+b)\,\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right) + \left((Ba - Ab)x^2 \log\left(cx^4 + bx^2 + a\right) - 4\,(Ba - Ab)x^2 \log\left(x\right) + 2\,Aa\right)}{4\,\sqrt{-b^2 + 4\,ac}a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] [1/4*((B*a*b - A*b^2 + 2*A*a*c)*x^2*log((b^3 - 4*a*b*c + 2*(b^2*c
 - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2
- 4*a*c))/(c*x^4 + b*x^2 + a)) - ((B*a - A*b)*x^2*log(c*x^4 + b*x
^2 + a) - 4*(B*a - A*b)*x^2*log(x) + 2*A*a)*sqrt(b^2 - 4*a*c))/(s
qrt(b^2 - 4*a*c)*a^2*x^2), -1/4*(2*(B*a*b - A*b^2 + 2*A*a*c)*x^2*
arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((B*a -
 A*b)*x^2*log(c*x^4 + b*x^2 + a) - 4*(B*a - A*b)*x^2*log(x) + 2*A
*a)*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.289932, size = 167, normalized size = 1.49

$$-\frac{(Ba-Ab)\ln\left(cx^4+bx^2+a\right)}{4\,a^2}+\frac{(Ba-Ab)\ln\left(x^2\right)}{2\,a^2}-\frac{\left(Bab-Ab^2+2\,Aac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^2}-\frac{Bax^2-Abx^2+Aa}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] -1/4*(B*a - A*b)*ln(c*x^4 + b*x^2 + a)/a^2 + 1/2*(B*a - A*b)*ln(x
^2)/a^2 - 1/2*(B*a*b - A*b^2 + 2*A*a*c)*arctan((2*c*x^2 + b)/sqrt
(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/2*(B*a*x^2 - A*b*x^2
 + A*a)/(a^2*x^2)
```

3.107 $\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=261

$$\frac{\left(-\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x(bB-Ac)}{c^2}+\frac{Bx^3}{3c}$$

```
[Out] -(((b*B - A*c)*x)/c^2) + (B*x^3)/(3*c) + ((b^2*B - A*b*c - a*B*c
- (b^3*B - A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*B - A*b*c - a*B*c + (
b^3*B - A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5
/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 2.70497, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{\left(-\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x(bB-Ac)}{c^2}+\frac{Bx^3}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(((b*B - A*c)*x)/c^2) + (B*x^3)/(3*c) + ((b^2*B - A*b*c - a*B*c
- (b^3*B - A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*B - A*b*c - a*B*c + (
b^3*B - A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5
/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 119.699, size = 267, normalized size = 1.02

$$\frac{Bx^3}{3c}+\frac{x\left(Ac-Bb\right)}{c^2}-\frac{\sqrt{2}\left(-2ac\left(Ac-Bb\right)+b\left(Bac+b\left(Ac-Bb\right)\right)+\sqrt{-4ac+b^2}\left(Bac+b\left(Ac-Bb\right)\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{5}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\left(-2ac\left(Ac-Bb\right)+b\left(Bac+b\left(Ac-Bb\right)\right)-\sqrt{-4ac+b^2}\left(Bac+b\left(Ac-Bb\right)\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{5}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*x**3/(3*c) + x*(A*c - B*b)/c**2 - sqrt(2)*(-2*a*c*(A*c - B*b) +
 b*(B*a*c + b*(A*c - B*b)) + sqrt(-4*a*c + b**2)*(B*a*c + b*(A*c
- B*b)))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2
*c**(5/2)*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) + sq
rt(2)*(-2*a*c*(A*c - B*b) + b*(B*a*c + b*(A*c - B*b)) - sqrt(-4*a
*c + b**2)*(B*a*c + b*(A*c - B*b)))*atan(sqrt(2)*sqrt(c)*x/sqrt(b
 - sqrt(-4*a*c + b**2)))/(2*c**(5/2)*sqrt(b - sqrt(-4*a*c + b**2)
)*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.797184, size = 327, normalized size = 1.25

$$\frac{\left(-Abc\sqrt{b^2-4ac}-2aAc^2+b^2B\sqrt{b^2-4ac}-aBc\sqrt{b^2-4ac}+3abBc+Ab^2c+b^3(-B)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(-Abc\sqrt{b^2-4ac}+2aAc^2+b^2B\sqrt{b^2-4ac}-aBc\sqrt{b^2-4ac}-3abBc-Ab^2c+b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(Ac-bB)}{c^2}+\frac{Bx^3}{3c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-(b*B) + A*c)*x)/c^2 + (B*x^3)/(3*c) + ((-(b^3*B) + A*b^2*c + 3
*a*b*B*c - 2*a*A*c^2 + b^2*B*Sqrt[b^2 - 4*a*c] - A*b*c*Sqrt[b^2 -
 4*a*c] - a*B*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqr
```

```
t[b - Sqrt[b^2 - 4*a*c]]) + ((b^3*B - A*b^2*c - 3*a*b*B*c + 2*a*A
*c^2 + b^2*B*Sqrt[b^2 - 4*a*c] - A*b*c*Sqrt[b^2 - 4*a*c] - a*B*c*
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 -
 4*a*c]])
```

Maple [B] time = 0.038, size = 825, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*B*x^3/c+1/c*A*x-1/c^2*b*B*x-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*A*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*A-1/2/c
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2-1/2/c*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*a*B+1/2/c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*b^2*B-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*
B+1/2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*B+1
/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b+1/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*a*A-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*A*b^2+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*a*B-1/2/c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*B-3/2/c/(-4*a
*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c
*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*B+1/2/c^2/(-4*a
*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c
*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bcx^3 - 3\,(Bb - Ac)x}{3\,c^2} - \frac{-\int \frac{Bab - Aac + \left(Bb^2 - (Ba + Ab)c\right)x^2}{cx^4 + bx^2 + a}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^4/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/3*(B*c*x^3 - 3*(B*b - A*c)*x)/c^2 - integrate(-(B*a*b - A*a*c +
 (B*b^2 - (B*a + A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/c^2
```

Fricas [A] time = 1.36959, size = 6939, normalized size = 26.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^4/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*B*c*x^3 + 3*sqrt(1/2)*c^2*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*
A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2
*a*b^3 + 2*A*B*b^4)*c + (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a
^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a
^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^
4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 +
 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*
b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)
))/(b^2*c^5 - 4*a*c^6))*log(-2*(B^4*a^2*b^4 - A*B^3*a*b^5 - A^4*a
^2*c^4 + (5*A^3*B*a^2*b + A^4*a*b^2)*c^3 + (B^4*a^4 + 3*A*B^3*a^3
*b - 6*A^2*B^2*a^2*b^2 - 3*A^3*B*a*b^3)*c^2 - (3*B^4*a^3*b^2 - A*
B^3*a^2*b^3 - 3*A^2*B^2*a*b^4)*c)*x + sqrt(1/2)*(B^3*b^7 - 4*A^3*
a^2*c^5 + (4*A*B^2*a^3 + 20*A^2*B*a^2*b + 5*A^3*a*b^2)*c^4 - (4*B
^3*a^3*b + 29*A*B^2*a^2*b^2 + 17*A^2*B*a*b^3 + A^3*b^4)*c^3 + (13
*B^3*a^2*b^3 + 19*A*B^2*a*b^4 + 3*A^2*B*b^5)*c^2 - (7*B^3*a*b^5 +
 3*A*B^2*b^6)*c - (B*b^4*c^5 + 4*(2*B*a^2 + A*a*b)*c^7 - (6*B*a*b
^2 + A*b^3)*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4
*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2
*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 +
 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4
*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 +
 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(B^2*b^5 - (4*A*B*
a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2
- (5*B^2*a*b^3 + 2*A*B*b^4)*c + (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8
 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5
+ (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3
+ A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2
*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*
A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4
*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(1/2)*c^2*sqrt(-(B^2*b^5
 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2
*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*c + (b^2*c^5 - 4*a*c^6)*sqr
t((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a
*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^
3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 +
12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*
```

```
a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^
2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-2*(B^4*a^2*b^4 - A
*B^3*a*b^5 - A^4*a^2*c^4 + (5*A^3*B*a^2*b + A^4*a*b^2)*c^3 + (B^4
*a^4 + 3*A*B^3*a^3*b - 6*A^2*B^2*a^2*b^2 - 3*A^3*B*a*b^3)*c^2 - (
3*B^4*a^3*b^2 - A*B^3*a^2*b^3 - 3*A^2*B^2*a*b^4)*c)*x - sqrt(1/2)
*(B^3*b^7 - 4*A^3*a^2*c^5 + (4*A*B^2*a^3 + 20*A^2*B*a^2*b + 5*A^3
*a*b^2)*c^4 - (4*B^3*a^3*b + 29*A*B^2*a^2*b^2 + 17*A^2*B*a*b^3 +
A^3*b^4)*c^3 + (13*B^3*a^2*b^3 + 19*A*B^2*a*b^4 + 3*A^2*B*b^5)*c^
2 - (7*B^3*a*b^5 + 3*A*B^2*b^6)*c - (B*b^4*c^5 + 4*(2*B*a^2 + A*a
*b)*c^7 - (6*B*a*b^2 + A*b^3)*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 -
2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*
B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 -
2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*
b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2
- 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))*sqrt(-
(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b
^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*c + (b^2*c^5 - 4*a*
c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b
 + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2
 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^
2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 2
0*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7
)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 3*sqrt(1/2)*
c^2*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b +
 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*c - (b^2*
c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A
^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B
^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 1
4*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a
^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2
*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-
2*(B^4*a^2*b^4 - A*B^3*a*b^5 - A^4*a^2*c^4 + (5*A^3*B*a^2*b + A^4
*a*b^2)*c^3 + (B^4*a^4 + 3*A*B^3*a^3*b - 6*A^2*B^2*a^2*b^2 - 3*A^
3*B*a*b^3)*c^2 - (3*B^4*a^3*b^2 - A*B^3*a^2*b^3 - 3*A^2*B^2*a*b^4
)*c)*x + sqrt(1/2)*(B^3*b^7 - 4*A^3*a^2*c^5 + (4*A*B^2*a^3 + 20*A
^2*B*a^2*b + 5*A^3*a*b^2)*c^4 - (4*B^3*a^3*b + 29*A*B^2*a^2*b^2 +
 17*A^2*B*a*b^3 + A^3*b^4)*c^3 + (13*B^3*a^2*b^3 + 19*A*B^2*a*b^4
 + 3*A^2*B*b^5)*c^2 - (7*B^3*a*b^5 + 3*A*B^2*b^6)*c + (B*b^4*c^5
+ 4*(2*B*a^2 + A*a*b)*c^7 - (6*B*a*b^2 + A*b^3)*c^6)*sqrt((B^4*b^
8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5
 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3
 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^
2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6
*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 -
4*a*c^11)))*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2
*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*c
 - (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a
^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b +
24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3
*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (
11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a
*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6
))) - 3*sqrt(1/2)*c^2*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c^
3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*
```

```
A*B*b^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*
(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^
3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*
(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^
5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 -
2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5
 - 4*a*c^6))*log(-2*(B^4*a^2*b^4 - A*B^3*a*b^5 - A^4*a^2*c^4 + (5
*A^3*B*a^2*b + A^4*a*b^2)*c^3 + (B^4*a^4 + 3*A*B^3*a^3*b - 6*A^2*
B^2*a^2*b^2 - 3*A^3*B*a*b^3)*c^2 - (3*B^4*a^3*b^2 - A*B^3*a^2*b^3
 - 3*A^2*B^2*a*b^4)*c)*x - sqrt(1/2)*(B^3*b^7 - 4*A^3*a^2*c^5 + (
4*A*B^2*a^3 + 20*A^2*B*a^2*b + 5*A^3*a*b^2)*c^4 - (4*B^3*a^3*b +
29*A*B^2*a^2*b^2 + 17*A^2*B*a*b^3 + A^3*b^4)*c^3 + (13*B^3*a^2*b^
3 + 19*A*B^2*a*b^4 + 3*A^2*B*b^5)*c^2 - (7*B^3*a*b^5 + 3*A*B^2*b^
6)*c + (B*b^4*c^5 + 4*(2*B*a^2 + A*a*b)*c^7 - (6*B*a*b^2 + A*b^3)
*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*
b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^
2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a
^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 +
20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^
7)*c)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2
*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*
b^3 + 2*A*B*b^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2*
c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4
+ 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*
c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*
A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6
)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))/
(b^2*c^5 - 4*a*c^6))) - 6*(B*b - A*c)*x)/c^2
```

Sympy [A] time = 81.8879, size = 709, normalized size = 2.72

$$\frac{Bx^3}{3c} + \operatorname{RootSum}\left(t^4\left(256a^2c^7 - 128ab^2c^6 + 16b^4c^5\right) + t^2\left(48A^2a^2bc^4 - 28A^2ab^3c^3 + 4A^2b^5c^2 + 64ABa^3c^4 - 144ABa^2b^2c^3 + 64ABa\right.\right. - \frac{x\left(-Ac + Bb\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*x**3/(3*c) + RootSum(_t**4*(256*a**2*c**7 - 128*a*b**2*c**6 + 1
6*b**4*c**5) + _t**2*(48*A**2*a**2*b*c**4 - 28*A**2*a*b**3*c**3 +
 4*A**2*b**5*c**2 + 64*A*B*a**3*c**4 - 144*A*B*a**2*b**2*c**3 + 6
4*A*B*a*b**4*c**2 - 8*A*B*b**6*c - 80*B**2*a**3*b*c**3 + 100*B**2
*a**2*b**3*c**2 - 36*B**2*a*b**5*c + 4*B**2*b**7) + A**4*a**3*c**
2 - 2*A**3*B*a**3*b*c + 2*A**2*B**2*a**4*c + A**2*B**2*a**3*b**2
- 2*A*B**3*a**4*b + B**4*a**5, Lambda(_t, _t*log(x + (-32*_t**3*A
```

```
*a*b*c**7 + 8*_t**3*A*b**3*c**6 - 64*_t**3*B*a**2*c**7 + 48*_t**3
*B*a*b**2*c**6 - 8*_t**3*B*b**4*c**5 + 4*_t*A**3*a**2*c**5 - 8*_t
*A**3*a*b**2*c**4 + 2*_t*A**3*b**4*c**3 - 30*_t*A**2*B*a**2*b*c**
4 + 30*_t*A**2*B*a*b**3*c**3 - 6*_t*A**2*B*b**5*c**2 - 12*_t*A*B*
*2*a**3*c**4 + 54*_t*A*B**2*a**2*b**2*c**3 - 36*_t*A*B**2*a*b**4*
c**2 + 6*_t*A*B**2*b**6*c + 14*_t*B**3*a**3*b*c**3 - 28*_t*B**3*a
**2*b**3*c**2 + 14*_t*B**3*a*b**5*c - 2*_t*B**3*b**7)/(-A**4*a**2
*c**4 + A**4*a*b**2*c**3 + 5*A**3*B*a**2*b*c**3 - 3*A**3*B*a*b**3
*c**2 - 6*A**2*B**2*a**2*b**2*c**2 + 3*A**2*B**2*a*b**4*c + 3*A*B
**3*a**3*b*c**2 + A*B**3*a**2*b**3*c - A*B**3*a*b**5 + B**4*a**4*
c**2 - 3*B**4*a**3*b**2*c + B**4*a**2*b**4)))) - x*(-A*c + B*b)/c
**2
```

GIAC/XCAS [A] time = 1.30199, size = 1, normalized size = 0.

$$\text{Done}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^4/(c*x^4 + b*x^2 + a),x, algorithm="giac")

[Out] Done
```

3.108 $\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=208

$$-\frac{\left(-\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Bx}{c}$$

```
[Out] (B*x)/c - ((b*B - A*c - (b^2*B - A*b*c - 2*a*B*c)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b*B - A*c + (b^2*B
- A*b*c - 2*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2
- 4*a*c]])
```

Rubi [A] time = 1.09481, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$-\frac{\left(-\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Bx}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c - ((b*B - A*c - (b^2*B - A*b*c - 2*a*B*c)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b*B - A*c + (b^2*B
- A*b*c - 2*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2
- 4*a*c]])
```

Rubi in Sympy [A] time = 67.239, size = 214, normalized size = 1.03

$$\frac{Bx}{c}+\frac{\sqrt{2}\left(2Bac+b\left(Ac-Bb\right)+\left(Ac-Bb\right)\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\left(2Bac+b\left(Ac-Bb\right)-\left(Ac-Bb\right)\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*x/c + sqrt(2)*(2*B*a*c + b*(A*c - B*b) + (A*c - B*b)*sqrt(-4*a*
c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/
(2*c**(3/2)*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) -
sqrt(2)*(2*B*a*c + b*(A*c - B*b) - (A*c - B*b)*sqrt(-4*a*c + b**2
))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*c**(3
/2)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.310108, size = 251, normalized size = 1.21

$$-\frac{\left(-Ac\sqrt{b^2-4ac}+bB\sqrt{b^2-4ac}+2aBc+Abc+b^2(-B)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(-Ac\sqrt{b^2-4ac}+bB\sqrt{b^2-4ac}-2aBc-Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Bx}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c - ((-(b^2*B) + A*b*c + 2*a*B*c + b*B*Sqrt[b^2 - 4*a*c] -
A*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) - ((b^2*B - A*b*c - 2*a*B*c + b*B*Sqrt[b^2 - 4*a*c]
 - A*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqr
t[b^2 - 4*a*c]])
```

Maple [B] time = 0.03, size = 560, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] B*x/c+1/2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A+1/2/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*A*b-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*b*B+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*B-1/2/c
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*B-1/2*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*A+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*A*b+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*B+1
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*B-1/2/c/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bx}{c} + \frac{-\int \frac{(Bb-Ac)x^2+Ba}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] B*x/c + integrate(-((B*b - A*c)*x^2 + B*a)/(c*x^4 + b*x^2 + a), x
)/c
```

Fricas [A] time = 0.454011, size = 3553, normalized size = 17.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^2/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*
a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4
- 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*
B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)
))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A*B^3*b^3 - 3*A^3*B*b*
c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*c)*x + sqrt
(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B
*b^2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c - (B*b^3*c^3 + 8*A*a*c^
5 - 2*(2*B*a*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2
*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2
 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^
2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c + (b^2*
c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b
)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^
2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) -
 sqrt(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b
+ 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*
(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*
b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(
b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A*B^3*b^3 - 3*A^3*B*b*c^2
+ A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*c)*x - sqrt(1/2
)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^2
)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c - (B*b^3*c^3 + 8*A*a*c^5 -
2*(2*B*a*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a +
 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2
*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^
3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c + (b^2*c^3
- 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^
3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 +
2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqr
t(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*
A*B*b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2
*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)
*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*
c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A*B^3*b^3 - 3*A^3*B*b*c^2 + A^
4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*c)*x + sqrt(1/2)*(B
^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^2)*c^
2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c + (B*b^3*c^3 + 8*A*a*c^5 - 2*(2
*B*a*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A
^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^
4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 +
(4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c - (b^2*c^3 - 4*
a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 +
(B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*
B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/
2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*
b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2
*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2
 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3
- 4*a*c^4))*log(2*(B^4*a*b^2 - A*B^3*b^3 - 3*A^3*B*b*c^2 + A^4*c^
3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*c)*x - sqrt(1/2)*(B^3*b
```

```
^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^2)*c^2 -
(5*B^3*a*b^2 + 2*A*B^2*b^3)*c + (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*B*a
*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B
*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*
b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A
*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c - (b^2*c^3 - 4*a*c^
4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4
*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*
b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + 2*B*x)/c
```

Sympy [A] time = 32.1715, size = 428, normalized size = 2.06

$$\frac{Bx}{c} + \operatorname{RootSum}\left(t^4\left(256a^2c^5 - 128ab^2c^4 + 16b^4c^3\right) + t^2\left(-16A^2abc^3 + 4A^2b^3c^2 - 64ABa^2c^3 + 48ABab^2c^2 - 8ABb^4c + 48B^2a^2bc^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*x/c + RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*
c**3) + _t**2*(-16*A**2*a*b*c**3 + 4*A**2*b**3*c**2 - 64*A*B*a**2
*c**3 + 48*A*B*a*b**2*c**2 - 8*A*B*b**4*c + 48*B**2*a**2*b*c**2 -
 28*B**2*a*b**3*c + 4*B**2*b**5) + A**4*a*c**2 - 2*A**3*B*a*b*c +
 2*A**2*B**2*a**2*c + A**2*B**2*a*b**2 - 2*A*B**3*a**2*b + B**4*a
**3, Lambda(_t, _t*log(x + (-64*_t**3*A*a*c**5 + 16*_t**3*A*b**2*
c**4 + 32*_t**3*B*a*b*c**4 - 8*_t**3*B*b**3*c**3 + 2*_t*A**3*b*c*
*3 + 12*_t*A**2*B*a*c**3 - 6*_t*A**2*B*b**2*c**2 - 18*_t*A*B**2*a
*b*c**2 + 6*_t*A*B**2*b**3*c - 4*_t*B**3*a**2*c**2 + 8*_t*B**3*a*
b**2*c - 2*_t*B**3*b**4)/(-A**4*c**3 + 3*A**3*B*b*c**2 - 3*A**2*B
**2*b**2*c + A*B**3*a*b*c + A*B**3*b**3 + B**4*a**2*c - B**4*a*b*
*2))))
```

GIAC/XCAS [A] time = 1.20981, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.109 $\int \frac{A+Bx^2}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=172

$$\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] ((B - (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + ((B + (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqr
t[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.415627, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((B - (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + ((B + (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqr
t[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 36.1828, size = 185, normalized size = 1.08

$$-\frac{\sqrt{2}\left(2Ac-Bb-B\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\left(2Ac-Bb+B\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] -sqrt(2)*(2*A*c - B*b - B*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(
c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*sqrt(c)*sqrt(b + sqrt(-4*a
*c + b**2))*sqrt(-4*a*c + b**2)) + sqrt(2)*(2*A*c - B*b + B*sqrt(
-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**
2)))/(2*sqrt(c)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)
)
```

Mathematica [A] time = 0.168077, size = 173, normalized size = 1.01

$$\frac{\frac{\left(B\sqrt{b^2-4ac}+2Ac-bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(B\sqrt{b^2-4ac}-2Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((
b*B - 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*
Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.024, size = 328, normalized size = 1.9

$$-c\sqrt{2}A\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{\sqrt{2}B}{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{b\sqrt{2}B}{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$-c\sqrt{2}A\mathit{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$-\frac{\sqrt{2}B}{2}\mathit{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{b\sqrt{2}B}{2}\mathit{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] -c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A+1/2*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*B+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*b*B-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*A-1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B+1/2/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{cx^4 + bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 0.373776, size = 2118, normalized size = 12.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4
*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 -
 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(B^4*a^2 - A*B^3*a*b
+ A^3*B*b*c - A^4*c^2)*x + sqrt(1/2)*(A*B^2*a*b^2 + 4*A^3*a*c^2 -
 (4*A*B^2*a^2 + A^3*b^2)*c + (4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B*a^
2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2
*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a
*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2
*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*s
qrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*sqrt(
(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(
a*b^2*c - 4*a^2*c^2))*log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A
^4*c^2)*x - sqrt(1/2)*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 +
 A^3*b^2)*c + (4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3
)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^
3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*
c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^
3*c^3)))/(a*b^2*c - 4*a^2*c^2))) + 1/2*sqrt(1/2)*sqrt(-(B^2*a*b -
 (4*A*B*a - A^2*b)*c - (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^
2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2
*c^2))*log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x + sqr
t(1/2)*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c - (
4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*
a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-
(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*
a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2
*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*sqrt(-(B^2*a*b - (4*A*B*a - A^2
*b)*c - (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4
```

```
*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(
B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x - sqrt(1/2)*(A*B^2*a
*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c - (4*(2*B*a^3 - A*
a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2
*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A
*B*a - A^2*b)*c - (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2
*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2)
))
```

Sympy [A] time = 15.3781, size = 314, normalized size = 1.83

$$\mathrm{RootSum}\left(t^4\left(256a^3c^3 - 128a^2b^2c^2 + 16ab^4c\right) + t^2\left(-16A^2abc^2 + 4A^2b^3c + 64ABa^2c^2 - 16ABab^2c - 16B^2a^2bc + 4B^2ab^3\right) +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**3 - 128*a**2*b**2*c**2 + 16*a*b**4*c)
+ _t**2*(-16*A**2*a*b*c**2 + 4*A**2*b**3*c + 64*A*B*a**2*c**2 - 1
6*A*B*a*b**2*c - 16*B**2*a**2*b*c + 4*B**2*a*b**3) + A**4*c**2 -
2*A**3*B*b*c + 2*A**2*B**2*a*c + A**2*B**2*b**2 - 2*A*B**3*a*b +
B**4*a**2, Lambda(_t, _t*log(x + (-32*_t**3*A*a**2*b*c**2 + 8*_t*
*3*A*a*b**3*c + 64*_t**3*B*a**3*c**2 - 16*_t**3*B*a**2*b**2*c - 4
*_t*A**3*a*c**2 + 2*_t*A**3*b**2*c - 6*_t*A**2*B*a*b*c + 12*_t*A*
B**2*a**2*c - 2*_t*B**3*a**2*b)/(-A**4*c**2 + A**3*B*b*c - A*B**3
*a*b + B**4*a**2))))
```

GIAC/XCAS [A] time = 0.814709, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.110 $\int \frac{A+Bx^2}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=189

$$-\frac{\sqrt{c}\left(\frac{Ab-2aB}{\sqrt{b^2-4ac}}+A\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{A}{ax}$$

```
[Out] -(A/(a*x)) - (Sqrt[c]*(A + (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(A - (A*b - 2*a*B)/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.761438, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12

$$-\frac{\sqrt{c}\left(\frac{Ab-2aB}{\sqrt{b^2-4ac}}+A\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{A}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(A/(a*x)) - (Sqrt[c]*(A + (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(A - (A*b - 2*a*B)/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 64.0168, size = 194, normalized size = 1.03

$$-\frac{A}{ax}+\frac{\sqrt{2}\sqrt{c}\left(Ab-A\sqrt{-4ac+b^2}-2Ba\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2a\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\sqrt{c}\left(Ab+A\sqrt{-4ac+b^2}-2Ba\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2a\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] -A/(a*x) + sqrt(2)*sqrt(c)*(A*b - A*sqrt(-4*a*c + b**2) - 2*B*a)*
atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*a*sqrt(b
 + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) - sqrt(2)*sqrt(c)*(A
*b + A*sqrt(-4*a*c + b**2) - 2*B*a)*atan(sqrt(2)*sqrt(c)*x/sqrt(b
 - sqrt(-4*a*c + b**2)))/(2*a*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(
-4*a*c + b**2))
```

Mathematica [A] time = 0.521094, size = 206, normalized size = 1.09

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(A\left(\sqrt{b^2-4ac}+b\right)-2aB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(A\left(\sqrt{b^2-4ac}-b\right)+2aB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2A}{x}}{2a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((2*A)/x + (Sqrt[2]*Sqrt[c]*(-2*a*B + A*(b + Sqrt[b^2 - 4*a*c]))
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b
^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*a*
B + A*(-b + Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*
a*c]]))/(2*a)
```

Maple [B] time = 0.028, size = 353, normalized size = 1.9

$$-\frac{c\sqrt{2}A}{2a}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}Ab}{2a}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$-c\sqrt{2}B\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}A}{2a}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}Ab}{2a}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$
$$-c\sqrt{2}BArtanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*c/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A+1/2*c/a/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*B+1/2*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A+1
/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b-c/(
-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B-A/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{Acx^2-Ba+Ab}{cx^4+bx^2+a}\,dx}{a}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((A*c*x^2 - B*a + A*b)/(c*x^4 + b*x^2 + a), x)/a - A/(a
*x)
```

Fricas [A] time = 0.50651, size = 3934, normalized size = 20.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*x*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*
A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*
B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2
*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 -
4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(2*(A^4*a*c^3 + (A^3*B*a*b - A
^4*b^2)*c^2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3*A^2*B^2*a*b^2 - A^3*B*
b^3)*c)*x + sqrt(1/2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3 + 3*A^2*B*a*
b^4 - A^3*b^5 + 4*(A^2*B*a^3 - A^3*a^2*b)*c^2 - (4*B^3*a^4 - 12*A
*B^2*a^3*b + 13*A^2*B*a^2*b^2 - 5*A^3*a*b^3)*c - (B*a^4*b^3 - A*a
^3*b^4 - 8*A*a^5*c^2 - 2*(2*B*a^5*b - 3*A*a^4*b^2)*c)*sqrt((B^4*a
^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4
+ A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a
^6*b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4
*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A
*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^
2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(B^2*a^2*
b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2
- 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*
A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^
2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*lo
g(2*(A^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*c^2 - (B^4*a^3 - 3*A*B^3*a
^2*b + 3*A^2*B^2*a*b^2 - A^3*B*b^3)*c)*x - sqrt(1/2)*(B^3*a^3*b^2
 - 3*A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A^3*b^5 + 4*(A^2*B*a^3 - A^3
*a^2*b)*c^2 - (4*B^3*a^4 - 12*A*B^2*a^3*b + 13*A^2*B*a^2*b^2 - 5*
A^3*a*b^3)*c - (B*a^4*b^3 - A*a^3*b^4 - 8*A*a^5*c^2 - 2*(2*B*a^5*
b - 3*A*a^4*b^2)*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2
```

```
*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2
*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2
*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2
 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4
*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a
^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))
+ sqrt(1/2)*a*x*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B
*a^2 - 3*A^2*a*b)*c - (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3
*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^
2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c))*log(2*(A^4*a*c^3 + (A^3*B*a*b - A^4*
b^2)*c^2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3*A^2*B^2*a*b^2 - A^3*B*b^3
)*c)*x + sqrt(1/2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3 + 3*A^2*B*a*b^4
 - A^3*b^5 + 4*(A^2*B*a^3 - A^3*a^2*b)*c^2 - (4*B^3*a^4 - 12*A*B^
2*a^3*b + 13*A^2*B*a^2*b^2 - 5*A^3*a*b^3)*c + (B*a^4*b^3 - A*a^3*
b^4 - 8*A*a^5*c^2 - 2*(2*B*a^5*b - 3*A*a^4*b^2)*c)*sqrt((B^4*a^4
- 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A
^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*
b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*
B*a^2 - 3*A^2*a*b)*c - (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^
3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c
^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*
a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(B^2*a^2*b -
 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c - (a^3*b^2 - 4
*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3
*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b
 + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(2
*(A^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*c^2 - (B^4*a^3 - 3*A*B^3*a^2*
b + 3*A^2*B^2*a*b^2 - A^3*B*b^3)*c)*x - sqrt(1/2)*(B^3*a^3*b^2 -
3*A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A^3*b^5 + 4*(A^2*B*a^3 - A^3*a^
2*b)*c^2 - (4*B^3*a^4 - 12*A*B^2*a^3*b + 13*A^2*B*a^2*b^2 - 5*A^3
*a*b^3)*c + (B*a^4*b^3 - A*a^3*b^4 - 8*A*a^5*c^2 - 2*(2*B*a^5*b -
 3*A*a^4*b^2)*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^
2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^
3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2*b
- 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c - (a^3*b^2 -
4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^
3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*
b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - 2
*A)/(a*x)
```

Sympy [A] time = 36.2761, size = 490, normalized size = 2.59

$$-\frac{A}{ax} + \operatorname{RootSum}\left(t^4\left(256a^5c^2 - 128a^4b^2c + 16a^3b^4\right) + t^2\left(48A^2a^2bc^2 - 28A^2ab^3c + 4A^2b^5 - 64ABa^3c^2 + 48ABa^2b^2c - 8ABab^4 - 1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] -A/(a*x) + RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a*
*3*b**4) + _t**2*(48*A**2*a**2*b*c**2 - 28*A**2*a*b**3*c + 4*A**2
*b**5 - 64*A*B*a**3*c**2 + 48*A*B*a**2*b**2*c - 8*A*B*a*b**4 - 16
*B**2*a**3*b*c + 4*B**2*a**2*b**3) + A**4*c**3 - 2*A**3*B*b*c**2
+ 2*A**2*B**2*a*c**2 + A**2*B**2*b**2*c - 2*A*B**3*a*b*c + B**4*a
**2*c, Lambda(_t, _t*log(x + (64*_t**3*A*a**5*c**2 - 48*_t**3*A*a
**4*b**2*c + 8*_t**3*A*a**3*b**4 + 32*_t**3*B*a**5*b*c - 8*_t**3*
B*a**4*b**3 + 10*_t*A**3*a**2*b*c**2 - 10*_t*A**3*a*b**3*c + 2*_t
*A**3*b**5 - 12*_t*A**2*B*a**3*c**2 + 24*_t*A**2*B*a**2*b**2*c -
6*_t*A**2*B*a*b**4 - 18*_t*A*B**2*a**3*b*c + 6*_t*A*B**2*a**2*b**
3 + 4*_t*B**3*a**4*c - 2*_t*B**3*a**3*b**2)/(-A**4*a*c**3 + A**4*
b**2*c**2 - A**3*B*a*b*c**2 - A**3*B*b**3*c + 3*A**2*B**2*a*b**2*
c - 3*A*B**3*a**2*b*c + B**4*a**3*c))))
```

GIAC/XCAS [A] time = 0.883985, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] Done
```

3.111 $\int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=271

$$-\frac{\sqrt{c}\left(aB\left(\sqrt{b^2-4ac}+b\right)-A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(aB\left(b-\sqrt{b^2-4ac}\right)-A\left(-b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Ab-aB}{a^2x}-\frac{A}{3ax^3}$$

```
[Out] -A/(3*a*x^3) + (A*b - a*B)/(a^2*x) - (Sqrt[c]*(a*B*(b + Sqrt[b^2
- 4*a*c]) - A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2
- 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(a*B*(b - Sqrt[b
^2 - 4*a*c]) - A*(b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b
^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.32346, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$-\frac{\sqrt{c}\left(aB\left(\sqrt{b^2-4ac}+b\right)-A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(aB\left(b-\sqrt{b^2-4ac}\right)-A\left(-b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Ab-aB}{a^2x}-\frac{A}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -A/(3*a*x^3) + (A*b - a*B)/(a^2*x) - (Sqrt[c]*(a*B*(b + Sqrt[b^2
- 4*a*c]) - A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2
- 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(a*B*(b - Sqrt[b
^2 - 4*a*c]) - A*(b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b
^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 111.36, size = 236, normalized size = 0.87

$$-\frac{A}{3ax^3} - \frac{\sqrt{2}\sqrt{c}\left(-2Aac + b(Ab - Ba) - (Ab - Ba)\sqrt{-4ac + b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2a^2\sqrt{b + \sqrt{-4ac + b^2}}\sqrt{-4ac + b^2}} + \frac{\sqrt{2}\sqrt{c}\left(-2Aac + b(Ab - Ba) + (Ab - Ba)\sqrt{-4ac + b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2a^2\sqrt{b - \sqrt{-4ac + b^2}}\sqrt{-4ac + b^2}} + \frac{Ab - Ba}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] -A/(3*a*x**3) - sqrt(2)*sqrt(c)*(-2*A*a*c + b*(A*b - B*a) - (A*b
- B*a)*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(
-4*a*c + b**2)))/(2*a**2*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*
c + b**2)) + sqrt(2)*sqrt(c)*(-2*A*a*c + b*(A*b - B*a) + (A*b - B
*a)*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*
a*c + b**2)))/(2*a**2*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c +
 b**2)) + (A*b - B*a)/(a**2*x)
```

Mathematica [A] time = 0.673657, size = 267, normalized size = 0.99

$$\frac{-\frac{3\sqrt{2}\sqrt{c}\left(aB\left(\sqrt{b^2-4ac}+b\right)-A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(A\left(b\sqrt{b^2-4ac}+2ac-b^2\right)+aB\left(b-\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+ 6\ldots}{6a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*A)/x^3 + (6*A*b - 6*a*B)/x - (3*Sqrt[2]*Sqrt[c]*(a*B*(b +
Sqrt[b^2 - 4*a*c]) - A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4
*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(a*B*(b -
 Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c]))*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*a^2)
```

Maple [B] time = 0.034, size = 611, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*c/a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b+c^2/a/(-4*a*c+b^2)^(1/
2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A-1/2*c/a^2/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2-1/2*c/a*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*B+1/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*
B-1/2*c/a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b+c^2/a/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A-1/2*c/a^2/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2+1/2*c/a*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*B+1/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*b*B-1/3*A/a/x^3+1/a^2/x*A*b-1/a/x*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{-\int \frac{(Ba-Ab)cx^2+Bab-Ab^2+Aac}{cx^4+bx^2+a}\,dx}{a^2}-\frac{3\,(Ba-Ab)x^2+Aa}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(-((B*a - A*b)*c*x^2 + B*a*b - A*b^2 + A*a*c)/(c*x^4 + b
*x^2 + a), x)/a^2 - 1/3*(3*(B*a - A*b)*x^2 + A*a)/(a^2*x^3)
```

Fricas [A] time = 1.40414, size = 7347, normalized size = 27.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)*x^4),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(1/2)*a^2*x^3*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b
^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2
 + 5*A^2*a*b^3)*c + (a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B
^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^
4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^
4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 1
1*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^
2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11
*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(A^4*a^2*c^5 + 3*(A^3*B*a^2*b -
A^4*a*b^2)*c^4 - (B^4*a^4 - 5*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - A
^3*B*a*b^3 - A^4*b^4)*c^3 + (B^4*a^3*b^2 - 3*A*B^3*a^2*b^3 + 3*A^
2*B^2*a*b^4 - A^3*B*b^5)*c^2)*x + sqrt(1/2)*(B^3*a^3*b^5 - 3*A*B^
2*a^2*b^6 + 3*A^2*B*a*b^7 - A^3*b^8 - 4*A^3*a^4*c^4 + (4*A*B^2*a^
5 - 20*A^2*B*a^4*b + 17*A^3*a^3*b^2)*c^3 + (4*B^3*a^5*b - 25*A*B^
2*a^4*b^2 + 41*A^2*B*a^3*b^3 - 20*A^3*a^2*b^4)*c^2 - (5*B^3*a^4*b
^3 - 18*A*B^2*a^3*b^4 + 21*A^2*B*a^2*b^5 - 8*A^3*a*b^6)*c - (B*a^
6*b^4 - A*a^5*b^5 + 4*(2*B*a^8 - 3*A*a^7*b)*c^2 - (6*B*a^7*b^2 -
7*A*a^6*b^3)*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a
^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 -
 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b +
24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(
B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2
*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))*sqrt(-(B^2*a^2*b^3
 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2
*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c + (a^5*b^2 - 4*a^6*c)*sqr
t((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*
b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*
A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2
- 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B
^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)
*c)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 3*sqrt(1/2)*a
^2*x^3*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 -
5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c
+ (a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2
*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2
*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^
5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2
 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3
*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4
*a^6*c))*log(2*(A^4*a^2*c^5 + 3*(A^3*B*a^2*b - A^4*a*b^2)*c^4 - (
B^4*a^4 - 5*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - A^3*B*a*b^3 - A^4*b
^4)*c^3 + (B^4*a^3*b^2 - 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 - A^3*
B*b^5)*c^2)*x - sqrt(1/2)*(B^3*a^3*b^5 - 3*A*B^2*a^2*b^6 + 3*A^2*
B*a*b^7 - A^3*b^8 - 4*A^3*a^4*c^4 + (4*A*B^2*a^5 - 20*A^2*B*a^4*b
 + 17*A^3*a^3*b^2)*c^3 + (4*B^3*a^5*b - 25*A*B^2*a^4*b^2 + 41*A^2
*B*a^3*b^3 - 20*A^3*a^2*b^4)*c^2 - (5*B^3*a^4*b^3 - 18*A*B^2*a^3*
b^4 + 21*A^2*B*a^2*b^5 - 8*A^3*a*b^6)*c - (B*a^6*b^4 - A*a^5*b^5
+ 4*(2*B*a^8 - 3*A*a^7*b)*c^2 - (6*B*a^7*b^2 - 7*A*a^6*b^3)*c)*sq
rt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a
*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3
*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2
 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*
B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6
)*c)/(a^10*b^2 - 4*a^11*c)))*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A
```

```
^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2
*b^2 + 5*A^2*a*b^3)*c + (a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4
*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^
4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 +
 (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3
 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^
2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*
a^11*c)))/(a^5*b^2 - 4*a^6*c))) + 3*sqrt(1/2)*a^2*x^3*sqrt(-(B^2*
a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 -
 (3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c - (a^5*b^2 - 4*a^6
*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A
^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4
*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a
^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2
- 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4
*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(A^
4*a^2*c^5 + 3*(A^3*B*a^2*b - A^4*a*b^2)*c^4 - (B^4*a^4 - 5*A*B^3*
a^3*b + 6*A^2*B^2*a^2*b^2 - A^3*B*a*b^3 - A^4*b^4)*c^3 + (B^4*a^3
*b^2 - 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 - A^3*B*b^5)*c^2)*x + sq
rt(1/2)*(B^3*a^3*b^5 - 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 - A^3*b^8
- 4*A^3*a^4*c^4 + (4*A*B^2*a^5 - 20*A^2*B*a^4*b + 17*A^3*a^3*b^2)
*c^3 + (4*B^3*a^5*b - 25*A*B^2*a^4*b^2 + 41*A^2*B*a^3*b^3 - 20*A^
3*a^2*b^4)*c^2 - (5*B^3*a^4*b^3 - 18*A*B^2*a^3*b^4 + 21*A^2*B*a^2
*b^5 - 8*A^3*a*b^6)*c + (B*a^6*b^4 - A*a^5*b^5 + 4*(2*B*a^8 - 3*A
*a^7*b)*c^2 - (6*B*a^7*b^2 - 7*A*a^6*b^3)*c)*sqrt((B^4*a^4*b^4 -
4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A
^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3
+ (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^
3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A
^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4
*a^11*c)))*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B*a^
3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3
)*c - (a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6
*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2
*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^
3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)
*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10
*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2
 - 4*a^6*c))) - 3*sqrt(1/2)*a^2*x^3*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*
b^4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*
A*B*a^2*b^2 + 5*A^2*a*b^3)*c - (a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*
b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b
^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2
)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*
a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3
+ 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b
^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(A^4*a^2*c^5 + 3*(A^3
*B*a^2*b - A^4*a*b^2)*c^4 - (B^4*a^4 - 5*A*B^3*a^3*b + 6*A^2*B^2*
a^2*b^2 - A^3*B*a*b^3 - A^4*b^4)*c^3 + (B^4*a^3*b^2 - 3*A*B^3*a^2
*b^3 + 3*A^2*B^2*a*b^4 - A^3*B*b^5)*c^2)*x - sqrt(1/2)*(B^3*a^3*b
^5 - 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 - A^3*b^8 - 4*A^3*a^4*c^4 +
(4*A*B^2*a^5 - 20*A^2*B*a^4*b + 17*A^3*a^3*b^2)*c^3 + (4*B^3*a^5*
b - 25*A*B^2*a^4*b^2 + 41*A^2*B*a^3*b^3 - 20*A^3*a^2*b^4)*c^2 - (
5*B^3*a^4*b^3 - 18*A*B^2*a^3*b^4 + 21*A^2*B*a^2*b^5 - 8*A^3*a*b^6
```

```
)*c + (B*a^6*b^4 - A*a^5*b^5 + 4*(2*B*a^8 - 3*A*a^7*b)*c^2 - (6*B
*a^7*b^2 - 7*A*a^6*b^3)*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 +
6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^
2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B
^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4
)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 1
0*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))*sqrt(-(
B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c
^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c - (a^5*b^2 - 4
*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 -
 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B
*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B
^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*
b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3
*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 6*
(B*a - A*b)*x^2 - 2*A*a)/(a^2*x^3)
```

Sympy [A] time = 90.0553, size = 774, normalized size = 2.86

$$\operatorname{RootSum}\left(t^4\left(256a^7c^2 - 128a^6b^2c + 16a^5b^4\right) + t^2\left(-80A^2a^3bc^3 + 100A^2a^2b^3c^2 - 36A^2ab^5c + 4A^2b^7 + 64ABa^4c^3 - 144ABa^3\right.\right.$$

$$- \frac{Aa + x^2(-3Ab + 3Ba)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**7*c**2 - 128*a**6*b**2*c + 16*a**5*b**4) +
_t**2*(-80*A**2*a**3*b*c**3 + 100*A**2*a**2*b**3*c**2 - 36*A**2*a
*b**5*c + 4*A**2*b**7 + 64*A*B*a**4*c**3 - 144*A*B*a**3*b**2*c**2
 + 64*A*B*a**2*b**4*c - 8*A*B*a*b**6 + 48*B**2*a**4*b*c**2 - 28*B
**2*a**3*b**3*c + 4*B**2*a**2*b**5) + A**4*c**5 - 2*A**3*B*b*c**4
 + 2*A**2*B**2*a*c**4 + A**2*B**2*b**2*c**3 - 2*A*B**3*a*b*c**3 +
 B**4*a**2*c**3, Lambda(_t, _t*log(x + (96*_t**3*A*a**7*b*c**2 -
56*_t**3*A*a**6*b**3*c + 8*_t**3*A*a**5*b**5 - 64*_t**3*B*a**8*c*
*2 + 48*_t**3*B*a**7*b**2*c - 8*_t**3*B*a**6*b**4 + 4*_t*A**3*a**
4*c**4 - 32*_t*A**3*a**3*b**2*c**3 + 40*_t*A**3*a**2*b**4*c**2 -
16*_t*A**3*a*b**6*c + 2*_t*A**3*b**8 + 42*_t*A**2*B*a**4*b*c**3 -
 84*_t*A**2*B*a**3*b**3*c**2 + 42*_t*A**2*B*a**2*b**5*c - 6*_t*A*
*2*B*a*b**7 - 12*_t*A*B**2*a**5*c**3 + 54*_t*A*B**2*a**4*b**2*c**
2 - 36*_t*A*B**2*a**3*b**4*c + 6*_t*A*B**2*a**2*b**6 - 10*_t*B**3
*a**5*b*c**2 + 10*_t*B**3*a**4*b**3*c - 2*_t*B**3*a**3*b**5)/(-A*
*4*a**2*c**5 + 3*A**4*a*b**2*c**4 - A**4*b**4*c**3 - 3*A**3*B*a**
2*b*c**4 - A**3*B*a*b**3*c**3 + A**3*B*b**5*c**2 + 6*A**2*B**2*a*
*2*b**2*c**3 - 3*A**2*B**2*a*b**4*c**2 - 5*A*B**3*a**3*b*c**3 + 3
*A*B**3*a**2*b**3*c**2 + B**4*a**4*c**3 - B**4*a**3*b**2*c**2))))
 - (A*a + x**2*(-3*A*b + 3*B*a))/(3*a**2*x**3)
```

GIAC/XCAS [A] time = 1.25934, size = 1, normalized size = 0.

$$\text{Done}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)*x^4),x, algorithm="giac")

[Out] Done
```

3.112 $\int \frac{x^7 (A+Bx^2)}{(a+bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=212

$$-\frac{(12a^2Bc^2 + 6aAbc^2 - 12ab^2Bc - Ab^3c + 2b^4B) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3 (b^2-4ac)^{3/2}} + \frac{x^2 (-6aBc - Abc + 2b^2B)}{2c^2 (b^2-4ac)} - \frac{x^4 (x^2 (-2aBc - Abc + b^2B) + a(bB - 2Ac))}{2c (b^2-4ac) (a+bx^2+cx^4)} - \frac{(2bB - Ac) \log(a+bx^2+cx^4)}{4c^3}$$

```
[Out] ((2*b^2*B - A*b*c - 6*a*B*c)*x^2)/(2*c^2*(b^2 - 4*a*c)) - (x^4*(a
*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(2*c*(b^2 - 4*a*
c)*(a + b*x^2 + c*x^4)) - ((2*b^4*B - A*b^3*c - 12*a*b^2*B*c + 6*
a*A*b*c^2 + 12*a^2*B*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]
])/(2*c^3*(b^2 - 4*a*c)^(3/2)) - ((2*b*B - A*c)*Log[a + b*x^2 + c
*x^4])/(4*c^3)
```

Rubi [A] time = 0.761874, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$-\frac{(12a^2Bc^2 + 6aAbc^2 - 12ab^2Bc - Ab^3c + 2b^4B) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3 (b^2-4ac)^{3/2}} + \frac{x^2 (-6aBc - Abc + 2b^2B)}{2c^2 (b^2-4ac)} - \frac{x^4 (x^2 (-2aBc - Abc + b^2B) + a(bB - 2Ac))}{2c (b^2-4ac) (a+bx^2+cx^4)} - \frac{(2bB - Ac) \log(a+bx^2+cx^4)}{4c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*b^2*B - A*b*c - 6*a*B*c)*x^2)/(2*c^2*(b^2 - 4*a*c)) - (x^4*(a
*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(2*c*(b^2 - 4*a*
c)*(a + b*x^2 + c*x^4)) - ((2*b^4*B - A*b^3*c - 12*a*b^2*B*c + 6*
a*A*b*c^2 + 12*a^2*B*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]
])/(2*c^3*(b^2 - 4*a*c)^(3/2)) - ((2*b*B - A*c)*Log[a + b*x^2 + c
*x^4])/(4*c^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3Bax^2}{c\,(-4ac+b^2)}+\frac{Bb^2x^2}{c^2\,(-4ac+b^2)}-\frac{b\int^{x^2}A\,dx}{2c\,(-4ac+b^2)}+\frac{x^4\,(a\,(2Ac-Bb)-x^2\,(-Abc-2Bac+Bb^2))}{2c\,(-4ac+b^2)\,(a+bx^2+cx^4)}+\frac{(Ac-2Bb)\log\,(a+bx^2+cx^4)}{4c^3}-\frac{(6Aabc^2-Ab^3c+12Ba^2c^2-12Bab^2c+2Bb^4)\,\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^3\,(-4ac+b^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -3*B*a*x**2/(c*(-4*a*c + b**2)) + B*b**2*x**2/(c**2*(-4*a*c + b**
2)) - b*Integral(A, (x, x**2))/(2*c*(-4*a*c + b**2)) + x**4*(a*(2
*A*c - B*b) - x**2*(-A*b*c - 2*B*a*c + B*b**2))/(2*c*(-4*a*c + b*
*2)*(a + b*x**2 + c*x**4)) + (A*c - 2*B*b)*log(a + b*x**2 + c*x**
4)/(4*c**3) - (6*A*a*b*c**2 - A*b**3*c + 12*B*a**2*c**2 - 12*B*a*
b**2*c + 2*B*b**4)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c
**3*(-4*a*c + b**2)**(3/2))
```

Mathematica [A] time = 0.52633, size = 208, normalized size = 0.98

$$\frac{-\frac{2(a^2c(2c(A+Bx^2)-3bB)+ab(-bc(A+4Bx^2)+3Ac^2x^2+b^2B)+b^3x^2(bB-Ac))}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2(12a^2Bc^2+6aAbc^2-12ab^2Bc-Ab^3c+2b^4B)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}+(Ac-2}{4c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*B*c*x^2 - (2*(b^3*(b*B - A*c)*x^2 + a^2*c*(-3*b*B + 2*c*(A + B
*x^2)) + a*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*B*x^2))))/((b^2 -
4*a*c)*(a + b*x^2 + c*x^4)) - (2*(2*b^4*B - A*b^3*c - 12*a*b^2*B*
c + 6*a*A*b*c^2 + 12*a^2*B*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 +
4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (-2*b*B + A*c)*Log[a + b*x^2 + c*
x^4])/(4*c^3)
```

Maple [B] time = 0.026, size = 1058, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*B*x^2/c^2+3/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*A*b-1/2/c^2
/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*A*b^3+1/c/(c*x^4+b*x^2+a)/(4*a*c
-b^2)*x^2*a^2*B-2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*b^2*B+1/2
/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b^4*B+1/c/(c*x^4+b*x^2+a)*a^
2/(4*a*c-b^2)*A-1/2/c^2/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*A*b^2-3/2/c
^2/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*b*B+1/2/c^3/(c*x^4+b*x^2+a)*a/
(4*a*c-b^2)*b^3*B+1/c/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))
*a*A-1/4/c^2/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*A*b^2-2/
c^2/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*a*b*B+1/2/c^3/(4*
a*c-b^2)*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*b^3*B-3/c/(64*a^3*c^3-48
*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4
*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*A*
a*b-6/c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((
2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*
a*b^4*c-b^6)^(1/2))*a^2*B+6/c^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b
^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^
3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*B*a*b^2+1/2/c^2/(64*a
^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)
*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^
(1/2))*b^3*A-1/c^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/
2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*
b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^4*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^7/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.350885, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^7/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((2*B*a*b^4 + (2*B*b^4*c + 6*(2*B*a^2 + A*a*b)*c^3 - (12*B*
a*b^2 + A*b^3)*c^2)*x^4 + 6*(2*B*a^3 + A*a^2*b)*c^2 + (2*B*b^5 +
```

```
6*(2*B*a^2*b + A*a*b^2)*c^2 - (12*B*a*b^3 + A*b^4)*c)*x^2 - (12*B
*a^2*b^2 + A*a*b^3)*c)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x
^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*
x^4 + b*x^2 + a)) - (2*(B*b^2*c^2 - 4*B*a*c^3)*x^6 - 2*B*a*b^3 -
4*A*a^2*c^2 + 2*(B*b^3*c - 4*B*a*b*c^2)*x^4 - 2*(B*b^4 + 3*(2*B*a
^2 + A*a*b)*c^2 - (5*B*a*b^2 + A*b^3)*c)*x^2 + 2*(3*B*a^2*b + A*a
*b^2)*c - (2*B*a*b^3 + 4*A*a^2*c^2 + (2*B*b^3*c + 4*A*a*c^3 - (8*
B*a*b + A*b^2)*c^2)*x^4 + (2*B*b^4 + 4*A*a*b*c^2 - (8*B*a*b^2 + A
*b^3)*c)*x^2 - (8*B*a^2*b + A*a*b^2)*c)*log(c*x^4 + b*x^2 + a))*s
qrt(b^2 - 4*a*c))/((a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x
^4 + (b^3*c^3 - 4*a*b*c^4)*x^2)*sqrt(b^2 - 4*a*c)), 1/4*(2*(2*B*a
*b^4 + (2*B*b^4*c + 6*(2*B*a^2 + A*a*b)*c^3 - (12*B*a*b^2 + A*b^3
)*c^2)*x^4 + 6*(2*B*a^3 + A*a^2*b)*c^2 + (2*B*b^5 + 6*(2*B*a^2*b
+ A*a*b^2)*c^2 - (12*B*a*b^3 + A*b^4)*c)*x^2 - (12*B*a^2*b^2 + A*
a*b^3)*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))
 + (2*(B*b^2*c^2 - 4*B*a*c^3)*x^6 - 2*B*a*b^3 - 4*A*a^2*c^2 + 2*(
B*b^3*c - 4*B*a*b*c^2)*x^4 - 2*(B*b^4 + 3*(2*B*a^2 + A*a*b)*c^2 -
 (5*B*a*b^2 + A*b^3)*c)*x^2 + 2*(3*B*a^2*b + A*a*b^2)*c - (2*B*a*
b^3 + 4*A*a^2*c^2 + (2*B*b^3*c + 4*A*a*c^3 - (8*B*a*b + A*b^2)*c^
2)*x^4 + (2*B*b^4 + 4*A*a*b*c^2 - (8*B*a*b^2 + A*b^3)*c)*x^2 - (8
*B*a^2*b + A*a*b^2)*c)*log(c*x^4 + b*x^2 + a))*sqrt(-b^2 + 4*a*c)
)/((a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 -
4*a*b*c^4)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 103.608, size = 1266, normalized size = 5.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] B*x**2/(2*c**2) + (-sqrt(-(4*a*c - b**2)**3)*(6*A*a*b*c**2 - A*b*
*3*c + 12*B*a**2*c**2 - 12*B*a*b**2*c + 2*B*b**4)/(4*c**3*(64*a**
3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (-A*c + 2*B*b
)/(4*c**3))*log(x**2 + (8*A*a**2*c**2 - A*a*b**2*c - 10*B*a**2*b*
c + 2*B*a*b**3 - 32*a**2*c**4*(-sqrt(-(4*a*c - b**2)**3)*(6*A*a*b
*c**2 - A*b**3*c + 12*B*a**2*c**2 - 12*B*a*b**2*c + 2*B*b**4)/(4*
c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (
-A*c + 2*B*b)/(4*c**3)) + 16*a*b**2*c**3*(-sqrt(-(4*a*c - b**2)**
3)*(6*A*a*b*c**2 - A*b**3*c + 12*B*a**2*c**2 - 12*B*a*b**2*c + 2*
B*b**4)/(4*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6)) - (-A*c + 2*B*b)/(4*c**3)) - 2*b**4*c**2*(-sqrt(-(4*a*c -
 b**2)**3)*(6*A*a*b*c**2 - A*b**3*c + 12*B*a**2*c**2 - 12*B*a*b**
2*c + 2*B*b**4)/(4*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*
b**4*c - b**6)) - (-A*c + 2*B*b)/(4*c**3)))/(6*A*a*b*c**2 - A*b**
3*c + 12*B*a**2*c**2 - 12*B*a*b**2*c + 2*B*b**4)) + (sqrt(-(4*a*c
 - b**2)**3)*(6*A*a*b*c**2 - A*b**3*c + 12*B*a**2*c**2 - 12*B*a*b
**2*c + 2*B*b**4)/(4*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
```

```
a*b**4*c - b**6)) - (-A*c + 2*B*b)/(4*c**3))*log(x**2 + (8*A*a**2
*c**2 - A*a*b**2*c - 10*B*a**2*b*c + 2*B*a*b**3 - 32*a**2*c**4*(s
qrt(-(4*a*c - b**2)**3)*(6*A*a*b*c**2 - A*b**3*c + 12*B*a**2*c**2
 - 12*B*a*b**2*c + 2*B*b**4)/(4*c**3*(64*a**3*c**3 - 48*a**2*b**2
*c**2 + 12*a*b**4*c - b**6)) - (-A*c + 2*B*b)/(4*c**3)) + 16*a*b*
*2*c**3*(sqrt(-(4*a*c - b**2)**3)*(6*A*a*b*c**2 - A*b**3*c + 12*B
*a**2*c**2 - 12*B*a*b**2*c + 2*B*b**4)/(4*c**3*(64*a**3*c**3 - 48
*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (-A*c + 2*B*b)/(4*c**3))
 - 2*b**4*c**2*(sqrt(-(4*a*c - b**2)**3)*(6*A*a*b*c**2 - A*b**3*c
 + 12*B*a**2*c**2 - 12*B*a*b**2*c + 2*B*b**4)/(4*c**3*(64*a**3*c*
*3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (-A*c + 2*B*b)/(4
*c**3)))/(6*A*a*b*c**2 - A*b**3*c + 12*B*a**2*c**2 - 12*B*a*b**2*
c + 2*B*b**4)) + (2*A*a**2*c**2 - A*a*b**2*c - 3*B*a**2*b*c + B*a
*b**3 + x**2*(3*A*a*b*c**2 - A*b**3*c + 2*B*a**2*c**2 - 4*B*a*b**
2*c + B*b**4))/(8*a**2*c**4 - 2*a*b**2*c**3 + x**4*(8*a*c**5 - 2*
b**2*c**4) + x**2*(8*a*b*c**4 - 2*b**3*c**3))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^7/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.113 $\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{x^2\left(x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(4aAc^2-6abBc+b^3B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[Out] -(x^2*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(2*c*(b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^3*B - 6*a*b*B*c + 4*a*A*c^2
)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^
(3/2)) + (B*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi [A] time = 0.371795, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$-\frac{x^2\left(x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(4aAc^2-6abBc+b^3B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(x^2*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(2*c*(b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^3*B - 6*a*b*B*c + 4*a*A*c^2
)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^
(3/2)) + (B*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi in Sympy [A] time = 49.5501, size = 138, normalized size = 0.94

$$\frac{B\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x^2\left(a\left(2Ac-Bb\right)-x^2\left(-Abc-2Bac+Bb^2\right)\right)}{2c\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}+\frac{\left(4Aac^2-6Babc+Bb^3\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^2\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] B*log(a + b*x**2 + c*x**4)/(4*c**2) + x**2*(a*(2*A*c - B*b) - x**
2*(-A*b*c - 2*B*a*c + B*b**2))/(2*c*(-4*a*c + b**2)*(a + b*x**2 +
 c*x**4)) + (4*A*a*c**2 - 6*B*a*b*c + B*b**3)*atanh((b + 2*c*x**2
)/sqrt(-4*a*c + b**2))/(2*c**2*(-4*a*c + b**2)**(3/2))
```

Mathematica [A]   time = 0.337398, size = 160, normalized size = 1.09

$$\frac{-\frac{2\left(2a^2Bc+a\left(bc\left(A+3Bx^2\right)-2Ac^2x^2+b^2(-B)\right)+b^2x^2(Ac-bB)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2\left(4aAc^2-6abBc+b^3B\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+B\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*(2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A*c^2*
x^2 + b*c*(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
(2*(b^3*B - 6*a*b*B*c + 4*a*A*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2
 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + B*Log[a + b*x^2 + c*x^4])/(4*c
^2)
```

Maple [B]   time = 0.022, size = 542, normalized size = 3.7

$$\frac{1}{2\,cx^4+2\,bx^2+2\,a}\left(-\frac{\left(2\,aAc^2-Ab^2c-3\,abBc+b^3B\right)x^2}{\left(4\,ac-b^2\right)c^2}+\frac{a\left(Abc+2\,aBc-b^2B\right)}{\left(4\,ac-b^2\right)c^2}\right)$$
$$+\frac{\ln\left(c\left(4\,ac-b^2\right)\left(cx^4+bx^2+a\right)\right)aB}{\left(4\,ac-b^2\right)c}-\frac{\ln\left(c\left(4\,ac-b^2\right)\left(cx^4+bx^2+a\right)\right)b^2B}{\left(16\,ac-4\,b^2\right)c^2}$$
$$+2\,\frac{aAc}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\arctan\left(\frac{2\,c^2\left(4\,ac-b^2\right)x^2+c\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\right)$$
$$-3\,\frac{abB}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\arctan\left(\frac{2\,c^2\left(4\,ac-b^2\right)x^2+c\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\right)$$
$$+\frac{b^3B}{2\,c}\arctan\left(\left(2\,c^2\left(4\,ac-b^2\right)x^2+c\left(4\,ac-b^2\right)b\right)\frac{1}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\right)\frac{1}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(-1/c^2*(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)/(4*a*c-b^2)*x^2+a
*(A*b*c+2*B*a*c-B*b^2)/(4*a*c-b^2)/c^2)/(c*x^4+b*x^2+a)+1/(4*a*c-
```

```
b^2)/c*ln(c*(4*a*c-b^2)*(c*x^4+b*x^2+a))*a*B-1/4/(4*a*c-b^2)/c^2*
ln(c*(4*a*c-b^2)*(c*x^4+b*x^2+a))*b^2*B+2/(64*a^3*c^5-48*a^2*b^2*
c^4+12*a*b^4*c^3-b^6*c^2)^(1/2)*arctan((2*c^2*(4*a*c-b^2)*x^2+c*(
4*a*c-b^2)*b)/(64*a^3*c^5-48*a^2*b^2*c^4+12*a*b^4*c^3-b^6*c^2)^(1
/2))*a*A*c-3/(64*a^3*c^5-48*a^2*b^2*c^4+12*a*b^4*c^3-b^6*c^2)^(1/
2)*arctan((2*c^2*(4*a*c-b^2)*x^2+c*(4*a*c-b^2)*b)/(64*a^3*c^5-48*
a^2*b^2*c^4+12*a*b^4*c^3-b^6*c^2)^(1/2))*a*b*B+1/2/(64*a^3*c^5-48
*a^2*b^2*c^4+12*a*b^4*c^3-b^6*c^2)^(1/2)*arctan((2*c^2*(4*a*c-b^2
)*x^2+c*(4*a*c-b^2)*b)/(64*a^3*c^5-48*a^2*b^2*c^4+12*a*b^4*c^3-b^
6*c^2)^(1/2))*b^3/c*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^5/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284112, size = 1, normalized size = 0.01

$$\left[-\frac{\left(Bab^3 - 6\,Ba^2bc + 4\,Aa^2c^2 + \left(Bb^3c - 6\,Babc^2 + 4\,Aac^3\right)x^4 + \left(Bb^4 - 6\,Bab^2c + 4\,Aabc^2\right)x^2\right)\log\left(-\frac{b^3 - 4\,abc + 2\,\left(b^2c - 4\,ac^2\right)x^2 -}{cx}\right.}{}\right.$$

$$-\frac{2\left(Bab^3 - 6\,Ba^2bc + 4\,Aa^2c^2 + \left(Bb^3c - 6\,Babc^2 + 4\,Aac^3\right)x^4 + \left(Bb^4 - 6\,Bab^2c + 4\,Aabc^2\right)x^2\right)\arctan\left(-\frac{\left(2\,cx^2 + b\right)\sqrt{-b^2 + 4\,ac}}{b^2 - 4\,ac}\right)}{4\left(ab^2c^2 - 4\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^5/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((B*a*b^3 - 6*B*a^2*b*c + 4*A*a^2*c^2 + (B*b^3*c - 6*B*a*b*
c^2 + 4*A*a*c^3)*x^4 + (B*b^4 - 6*B*a*b^2*c + 4*A*a*b*c^2)*x^2)*l
og(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - (2*c^2*x^4 + 2*b*c
*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (2*
B*a*b^2 + 2*(B*b^3 + 2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*x^2 - 2*(2*
B*a^2 + A*a*b)*c + ((B*b^2*c - 4*B*a*c^2)*x^4 + B*a*b^2 - 4*B*a^2
*c + (B*b^3 - 4*B*a*b*c)*x^2)*log(c*x^4 + b*x^2 + a))*sqrt(b^2 -
```

```
4*a*c))/((a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*
c^2 - 4*a*b*c^3)*x^2)*sqrt(b^2 - 4*a*c)), -1/4*(2*(B*a*b^3 - 6*B*
a^2*b*c + 4*A*a^2*c^2 + (B*b^3*c - 6*B*a*b*c^2 + 4*A*a*c^3)*x^4 +
 (B*b^4 - 6*B*a*b^2*c + 4*A*a*b*c^2)*x^2)*arctan(-(2*c*x^2 + b)*s
qrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (2*B*a*b^2 + 2*(B*b^3 + 2*A*a*
c^2 - (3*B*a*b + A*b^2)*c)*x^2 - 2*(2*B*a^2 + A*a*b)*c + ((B*b^2*
c - 4*B*a*c^2)*x^4 + B*a*b^2 - 4*B*a^2*c + (B*b^3 - 4*B*a*b*c)*x^
2)*log(c*x^4 + b*x^2 + a))*sqrt(-b^2 + 4*a*c))/((a*b^2*c^2 - 4*a^
2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt
(-b^2 + 4*a*c))]
```

Sympy [A] time = 54.662, size = 916, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (B/(4*c**2) - sqrt(-(4*a*c - b**2)**3)*(-4*A*a*c**2 + 6*B*a*b*c -
 B*b**3)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6)))*log(x**2 + (-2*A*a*b*c + 8*B*a**2*c - B*a*b**2 - 32*a**
2*c**3*(B/(4*c**2) - sqrt(-(4*a*c - b**2)**3)*(-4*A*a*c**2 + 6*B*
a*b*c - B*b**3)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*
b**4*c - b**6))) + 16*a*b**2*c**2*(B/(4*c**2) - sqrt(-(4*a*c - b*
*2)**3)*(-4*A*a*c**2 + 6*B*a*b*c - B*b**3)/(4*c**2*(64*a**3*c**3
- 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 2*b**4*c*(B/(4*c**2
) - sqrt(-(4*a*c - b**2)**3)*(-4*A*a*c**2 + 6*B*a*b*c - B*b**3)/(
4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))))
/(-4*A*a*c**2 + 6*B*a*b*c - B*b**3)) + (B/(4*c**2) + sqrt(-(4*a*c
 - b**2)**3)*(-4*A*a*c**2 + 6*B*a*b*c - B*b**3)/(4*c**2*(64*a**3*
c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x**2 + (-2*A
*a*b*c + 8*B*a**2*c - B*a*b**2 - 32*a**2*c**3*(B/(4*c**2) + sqrt(
-(4*a*c - b**2)**3)*(-4*A*a*c**2 + 6*B*a*b*c - B*b**3)/(4*c**2*(6
4*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 16*a*b*
*2*c**2*(B/(4*c**2) + sqrt(-(4*a*c - b**2)**3)*(-4*A*a*c**2 + 6*B
*a*b*c - B*b**3)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a
*b**4*c - b**6))) - 2*b**4*c*(B/(4*c**2) + sqrt(-(4*a*c - b**2)**
3)*(-4*A*a*c**2 + 6*B*a*b*c - B*b**3)/(4*c**2*(64*a**3*c**3 - 48*
a**2*b**2*c**2 + 12*a*b**4*c - b**6))))/(-4*A*a*c**2 + 6*B*a*b*c
- B*b**3)) + (A*a*b*c + 2*B*a**2*c - B*a*b**2 + x**2*(-2*A*a*c**2
 + A*b**2*c + 3*B*a*b*c - B*b**3))/(8*a**2*c**3 - 2*a*b**2*c**2 +
 x**4*(8*a*c**4 - 2*b**2*c**3) + x**2*(8*a*b*c**3 - 2*b**3*c**2))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.114 $\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=107

$$-\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

```
[Out] -(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(2*c*(b^2 - 4*
a*c)*(a + b*x^2 + c*x^4)) - ((A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.261731, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$-\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(2*c*(b^2 - 4*
a*c)*(a + b*x^2 + c*x^4)) - ((A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [A] time = 27.1527, size = 78, normalized size = 0.73

$$\frac{\left(A+Bx^2\right)\left(2a+bx^2\right)}{2\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}-\frac{(Ab-2Ba)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (A + B*x**2)*(2*a + b*x**2)/(2*(-4*a*c + b**2)*(a + b*x**2 + c*x*
*4)) - (A*b - 2*B*a)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(-
4*a*c + b**2)**(3/2)
```

Mathematica [A] time = 0.139666, size = 111, normalized size = 1.04

$$\frac{-2ac\left(A+Bx^2\right)+abB+bx^2(bB-Ac)}{2c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}-\frac{(Ab-2aB)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a*b*B + b*(b*B - A*c)*x^2 - 2*a*c*(A + B*x^2))/(2*c*(-b^2 + 4*a*
c)*(a + b*x^2 + c*x^4)) - ((A*b - 2*a*B)*ArcTan[(b + 2*c*x^2)/Sqr
t[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] time = 0.014, size = 158, normalized size = 1.5

$$\frac{1}{2\,cx^4+2\,bx^2+2\,a}\left(-\frac{\left(Abc+2\,aBc-b^2B\right)x^2}{\left(4\,ac-b^2\right)c}-\frac{a\left(2\,Ac-bB\right)}{\left(4\,ac-b^2\right)c}\right)-Ab\arctan\left(\left(2\,cx^2+b\right)\frac{1}{\sqrt{4\,ac-b^2}}\right)\left(4\,ac-b^2\right)^{-\frac{3}{2}}+2\,\frac{Ba}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(-(A*b*c+2*B*a*c-B*b^2)/(4*a*c-b^2)/c*x^2-a*(2*A*c-B*b)/c/(4*
a*c-b^2))/(c*x^4+b*x^2+a)-1/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/
(4*a*c-b^2)^(1/2))*A*b+2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*
a*c-b^2)^(1/2))*B*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^3/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.28607, size = 1, normalized size = 0.01

$$\left[\frac{\left((2\,Ba-Ab)c^2x^4+\left(2\,Bab-Ab^2\right)cx^2+\left(2\,Ba^2-Aab\right)c\right)\log\left(\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x^2+\left(2\,c^2x^4+2\,bcx^2+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(Bab\right.}{2\left((b^2c^2-4\,ac^3)x^4+ab^2c-4\,a^2c^2+(b^3c-4\,abc^2)x^2\right)\sqrt{b^2-4\,ac}}\right.$$

$$-\frac{2\left((2\,Ba-Ab)c^2x^4+\left(2\,Bab-Ab^2\right)cx^2+\left(2\,Ba^2-Aab\right)c\right)\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+\left(Bab-2\,Aac+\left(Bb^2-(2\,Ba+\right.\right.}{2\left((b^2c^2-4\,ac^3)x^4+ab^2c-4\,a^2c^2+(b^3c-4\,abc^2)x^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(((2*B*a - A*b)*c^2*x^4 + (2*B*a*b - A*b^2)*c*x^2 + (2*B*a^2
 - A*a*b)*c)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^
2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^
2 + a)) - (B*a*b - 2*A*a*c + (B*b^2 - (2*B*a + A*b)*c)*x^2)*sqrt(
b^2 - 4*a*c))/(((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (
b^3*c - 4*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)), -1/2*(2*((2*B*a - A*b
)*c^2*x^4 + (2*B*a*b - A*b^2)*c*x^2 + (2*B*a^2 - A*a*b)*c)*arctan
(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (B*a*b - 2*A*
a*c + (B*b^2 - (2*B*a + A*b)*c)*x^2)*sqrt(-b^2 + 4*a*c))/(((b^2*c
^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2
)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 20.5855, size = 394, normalized size = 3.68

$$-\frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(-Ab+2Ba)\log\left(x^2+\frac{-Ab^2+2Bab-16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}(-Ab+2Ba)+8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}(-Ab+2Ba)-b^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}(-Ab+2Ba)}{-2Abc+4Bac}\right)}{2}$$
$$+\frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(-Ab+2Ba)\log\left(x^2+\frac{-Ab^2+2Bab+16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}(-Ab+2Ba)-8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}(-Ab+2Ba)+b^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}(-Ab+2Ba)}{-2Abc+4Bac}\right)}{2}$$
$$-\frac{2Aac-Bab+x^2\left(Abc+2Bac-Bb^2\right)}{8a^2c^2-2ab^2c+x^4\left(8ac^3-2b^2c^2\right)+x^2\left(8abc^2-2b^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a)*log(x**2 + (-A*b**2 +
2*B*a*b - 16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a)
+ 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a) - b**4*sqr
t(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a))/(-2*A*b*c + 4*B*a*c))/2 +
 sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a)*log(x**2 + (-A*b**2 +
2*B*a*b + 16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a)
- 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a) + b**4*sqr
t(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a))/(-2*A*b*c + 4*B*a*c))/2 -
 (2*A*a*c - B*a*b + x**2*(A*b*c + 2*B*a*c - B*b**2))/(8*a**2*c**2
 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x**2*(8*a*b*c**2
- 2*b**3*c))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^3/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.115 $\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=94

$$-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{-2aB+x^2(-(bB-2Ac))+Ab}{2(b^2-4ac)(a+bx^2+cx^4)}$$

```
[Out] -(A*b - 2*a*B - (b*B - 2*A*c)*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)) - ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]
)/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.183677, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{-2aB+x^2(-(bB-2Ac))+Ab}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(A*b - 2*a*B - (b*B - 2*A*c)*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)) - ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]
)/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [A] time = 22.5939, size = 83, normalized size = 0.88

$$\frac{(2Ac-Bb)\,\mathrm{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{Ab-2Ba+x^2(2Ac-Bb)}{2(-4ac+b^2)(a+bx^2+cx^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (2*A*c - B*b)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(-4*a*c +
 b**2)**(3/2) - (A*b - 2*B*a + x**2*(2*A*c - B*b))/(2*(-4*a*c + b
**2)*(a + b*x**2 + c*x**4))
```

Mathematica [A] time = 0.143549, size = 101, normalized size = 1.07

$$\frac{\frac{2(bB-2Ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{B(2a+bx^2)-A(b+2cx^2)}{a+bx^2+cx^4}}{2\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((B*(2*a + b*x^2) - A*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + (2*(b*
B - 2*A*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
4*a*c])/(2*(b^2 - 4*a*c))
```

Maple [A] time = 0.008, size = 127, normalized size = 1.4

$$\frac{(2\,Ac-bB)\,x^2+Ab-2\,Ba}{(8\,ac-2\,b^2)\,(cx^4+bx^2+a)}+2\,\frac{Ac}{(4\,ac-b^2)^{3/2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$
$$-bB\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\left(4\,ac-b^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*((2*A*c-B*b)*x^2+A*b-2*B*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+2/(4*
a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*c-1/(4*a*c
-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.290959, size = 1, normalized size = 0.01

$$\left[\frac{\left(\left(Bbc-2\,Ac^{2}\right)x^{4}+Bab-2\,Aac+\left(Bb^{2}-2\,Abc\right)x^{2}\right)\log\left(-\frac{b^{3}-4\,abc+2\,\left(b^{2}c-4\,ac^{2}\right)x^{2}-\left(2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)+\left((Bb-\right.}{2\left((b^{2}c-4\,ac^{2})x^{4}+ab^{2}-4\,a^{2}c+(b^{3}-4\,abc)x^{2}\right)\sqrt{b^{2}-4\,ac}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(((B*b*c - 2*A*c^2)*x^4 + B*a*b - 2*A*a*c + (B*b^2 - 2*A*b*c
)*x^2)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - (2*c^2*x^4
 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a
)) + ((B*b - 2*A*c)*x^2 + 2*B*a - A*b)*sqrt(b^2 - 4*a*c))/(((b^2*
c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(b^
2 - 4*a*c)), 1/2*(2*((B*b*c - 2*A*c^2)*x^4 + B*a*b - 2*A*a*c + (B
*b^2 - 2*A*b*c)*x^2)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^
2 - 4*a*c)) + ((B*b - 2*A*c)*x^2 + 2*B*a - A*b)*sqrt(-b^2 + 4*a*c
))/(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^
2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 12.2112, size = 374, normalized size = 3.98

$$\frac{\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}\,(-2Ac+Bb)\log\left(x^{2}+\frac{-2Abc+Bb^{2}-16a^{2}c^{2}\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}(-2Ac+Bb)+8ab^{2}c\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}(-2Ac+Bb)-b^{4}\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}(-2Ac+Bb)}{-4Ac^{2}+2Bbc}\right)}{2}$$
$$-\frac{\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}\,(-2Ac+Bb)\log\left(x^{2}+\frac{-2Abc+Bb^{2}+16a^{2}c^{2}\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}(-2Ac+Bb)-8ab^{2}c\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}(-2Ac+Bb)+b^{4}\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}(-2Ac+Bb)}{-4Ac^{2}+2Bbc}\right)}{2}$$
$$-\frac{-Ab+2Ba+x^{2}\,(-2Ac+Bb)}{8a^{2}c-2ab^{2}+x^{4}\,(8ac^{2}-2b^{2}c)+x^{2}\,(8abc-2b^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b)*log(x**2 + (-2*A*b*c +
B*b**2 - 16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b) +
 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b) - b**4*sqrt
(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b))/(-4*A*c**2 + 2*B*b*c))/2 -
```

```
 sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b)*log(x**2 + (-2*A*b*c +
 B*b**2 + 16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b)
- 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b) + b**4*sqr
t(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b))/(-4*A*c**2 + 2*B*b*c))/2
- (-A*b + 2*B*a + x**2*(-2*A*c + B*b))/(8*a**2*c - 2*a*b**2 + x**
4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.116 $\int \frac{A+Bx^2}{x(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=150

$$\frac{\left(4a^2Bc+A\left(b^3-6abc\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{A\log(x)}{a^2}-\frac{-A\left(b^2-2ac\right)+cx^2(-(Ab-2aB))+abB}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*
c)*(a + b*x^2 + c*x^4)) + ((4*a^2*B*c + A*(b^3 - 6*a*b*c))*ArcTan
h[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) +
 (A*Log[x])/a^2 - (A*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rubi [A] time = 0.648672, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\frac{\left(4a^2Bc+A\left(b^3-6abc\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{A\log(x)}{a^2}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c
)*(a + b*x^2 + c*x^4)) + ((4*a^2*B*c + A*(b^3 - 6*a*b*c))*ArcTanh
[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) +
(A*Log[x])/a^2 - (A*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rubi in Sympy [A] time = 71.3186, size = 146, normalized size = 0.97

$$\frac{A\log\left(x^2\right)}{2a^2}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{-2Aac+Ab^2-Bab+cx^2\left(Ab-2Ba\right)}{2a\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}+\frac{\left(-6Aabc+Ab^3+4Ba^2c\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^2\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*log(x**2)/(2*a**2) - A*log(a + b*x**2 + c*x**4)/(4*a**2) + (-2*
A*a*c + A*b**2 - B*a*b + c*x**2*(A*b - 2*B*a))/(2*a*(-4*a*c + b**
2)*(a + b*x**2 + c*x**4)) + (-6*A*a*b*c + A*b**3 + 4*B*a**2*c)*at
anh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*a**2*(-4*a*c + b**2)**
(3/2))
```

Mathematica [A] time = 0.624461, size = 243, normalized size = 1.62

$$\frac{-\frac{\left(4a^2Bc+A\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{(b^2-4ac)^{3/2}}+\frac{\left(4a^2Bc+A\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3\right)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right.}{(b^2-4ac)^{3/2}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*(a*B*(b + 2*c*x^2) - A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) + 4*A*Log[x] - ((4*a^2*B*c + A*(b^3 -
6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c]))*Log[b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((4*a^2*B*
c + A*(b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4
*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2)
)/(4*a^2)
```

Maple [B] time = 0.024, size = 578, normalized size = 3.9

$$-\frac{cx^2Ab}{2a(cx^4+bx^2+a)(4ac-b^2)}+\frac{cx^2B}{(4ac-b^2)(cx^4+bx^2+a)}+\frac{Ac}{(4ac-b^2)(cx^4+bx^2+a)}$$
$$-\frac{Ab^2}{2a(cx^4+bx^2+a)(4ac-b^2)}+\frac{bB}{(2cx^4+2bx^2+2a)(4ac-b^2)}$$
$$-\frac{c\ln\left(\left(4ac-b^2\right)\left(cx^4+bx^2+a\right)\right)A}{a(4ac-b^2)}+\frac{\ln\left(\left(4ac-b^2\right)\left(cx^4+bx^2+a\right)\right)Ab^2}{4\,(4ac-b^2)a^2}$$
$$-3\frac{Abc}{a\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\arctan\left(\frac{2\left(4ac-b^2\right)cx^2+\left(4ac-b^2\right)b}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)$$
$$+\frac{Ab^3}{2a^2}\arctan\left(\left(2\left(4ac-b^2\right)cx^2+\left(4ac-b^2\right)b\right)\frac{1}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)\frac{1}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}$$
$$+2\frac{Bc}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\arctan\left(\frac{2\left(4ac-b^2\right)cx^2+\left(4ac-b^2\right)b}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)+\frac{A\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*A*b+1/(c*x^4+b*x^2+a)*c/
(4*a*c-b^2)*x^2*B+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A*c-1/2/a/(c*x^4+
b*x^2+a)/(4*a*c-b^2)*A*b^2+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b*B-1/
a/(4*a*c-b^2)*c*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*A+1/4/a^2/(4*a*c-
b^2)*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*A*b^2-3/a/(64*a^3*c^3-48*a^2
*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c
-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*A*b*c+
1/2/a^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((
2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*
a*b^4*c-b^6)^(1/2))*A*b^3+2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c
-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^
3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*B*c+A*ln(x)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^2*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.764086, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^2*x),x, algorithm="fricas")
```

```
[Out] [1/4*((A*a*b^3 + (A*b^3*c + 2*(2*B*a^2 - 3*A*a*b)*c^2)*x^4 + (A*b
^4 + 2*(2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + 2*(2*B*a^3 - 3*A*a^2*b)*c
)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b
*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (
2*B*a^2*b - 2*A*a*b^2 + 4*A*a^2*c + 2*(2*B*a^2 - A*a*b)*c*x^2 + (
(A*b^2*c - 4*A*a*c^2)*x^4 + A*a*b^2 - 4*A*a^2*c + (A*b^3 - 4*A*a*
b*c)*x^2)*log(c*x^4 + b*x^2 + a) - 4*((A*b^2*c - 4*A*a*c^2)*x^4 +
 A*a*b^2 - 4*A*a^2*c + (A*b^3 - 4*A*a*b*c)*x^2)*log(x))*sqrt(b^2
- 4*a*c))/((a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^
2*b^3 - 4*a^3*b*c)*x^2)*sqrt(b^2 - 4*a*c)), -1/4*(2*(A*a*b^3 + (A
*b^3*c + 2*(2*B*a^2 - 3*A*a*b)*c^2)*x^4 + (A*b^4 + 2*(2*B*a^2*b -
```

```
 3*A*a*b^2)*c)*x^2 + 2*(2*B*a^3 - 3*A*a^2*b)*c)*arctan(-(2*c*x^2
+ b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (2*B*a^2*b - 2*A*a*b^2 +
 4*A*a^2*c + 2*(2*B*a^2 - A*a*b)*c*x^2 + ((A*b^2*c - 4*A*a*c^2)*x
^4 + A*a*b^2 - 4*A*a^2*c + (A*b^3 - 4*A*a*b*c)*x^2)*log(c*x^4 + b
*x^2 + a) - 4*((A*b^2*c - 4*A*a*c^2)*x^4 + A*a*b^2 - 4*A*a^2*c +
(A*b^3 - 4*A*a*b*c)*x^2)*log(x))*sqrt(-b^2 + 4*a*c))/((a^3*b^2 -
4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2
)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^2*x),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.117 $\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=223

$$\frac{(2Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{-6aAc-abB+2Ab^2}{2a^2x^2\left(b^2-4ac\right)}+\frac{\left(abB\left(b^2-6ac\right)-2A\left(6a^2c^2-6ab^2c+b^4\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}}-\frac{-A\left(b^2-2ac\right)+cx^2(-(Ab-2aB))+abB}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -(2*A*b^2 - a*b*B - 6*a*A*c)/(2*a^2*(b^2 - 4*a*c)*x^2) - (a*b*B -
 A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x^2*(a
 + b*x^2 + c*x^4)) + ((a*b*B*(b^2 - 6*a*c) - 2*A*(b^4 - 6*a*b^2*c
 + 6*a^2*c^2))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(
b^2 - 4*a*c)^(3/2)) - ((2*A*b - a*B)*Log[x])/a^3 + ((2*A*b - a*B)
*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rubi [A] time = 0.856827, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28

$$\frac{(2Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{-6aAc-abB+2Ab^2}{2a^2x^2\left(b^2-4ac\right)}+\frac{\left(abB\left(b^2-6ac\right)-2A\left(6a^2c^2-6ab^2c+b^4\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(2*A*b^2 - a*b*B - 6*a*A*c)/(2*a^2*(b^2 - 4*a*c)*x^2) + (A*b^2 -
 a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x^2*(a
 + b*x^2 + c*x^4)) + ((a*b*B*(b^2 - 6*a*c) - 2*A*(b^4 - 6*a*b^2*c
 + 6*a^2*c^2))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(
b^2 - 4*a*c)^(3/2)) - ((2*A*b - a*B)*Log[x])/a^3 + ((2*A*b - a*B)
*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rubi in Sympy [A] time = 169.834, size = 221, normalized size = 0.99

$$\frac{-2Aac+Ab^2-Bab+cx^2(Ab-2Ba)}{2ax^2(-4ac+b^2)(a+bx^2+cx^4)}-\frac{-6Aac+2Ab^2-Bab}{2a^2x^2(-4ac+b^2)}-\frac{(2Ab-Ba)\log(x^2)}{2a^3}+\frac{(2Ab-Ba)\log(a+bx^2+cx^4)}{4a^3}-\frac{(12Aa^2c^2-12Aab^2c+2Ab^4+6Ba^2bc-Bab^3)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^3(-4ac+b^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (-2*A*a*c + A*b**2 - B*a*b + c*x**2*(A*b - 2*B*a))/(2*a*x**2*(-4*
a*c + b**2)*(a + b*x**2 + c*x**4)) - (-6*A*a*c + 2*A*b**2 - B*a*b
)/(2*a**2*x**2*(-4*a*c + b**2)) - (2*A*b - B*a)*log(x**2)/(2*a**3
) + (2*A*b - B*a)*log(a + b*x**2 + c*x**4)/(4*a**3) - (12*A*a**2*
c**2 - 12*A*a*b**2*c + 2*A*b**4 + 6*B*a**2*b*c - B*a*b**3)*atanh(
(b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*a**3*(-4*a*c + b**2)**(3/2
))
```

Mathematica [A] time = 1.06577, size = 379, normalized size = 1.7

$$\frac{\frac{\left(2A\left(6a^2c^2-6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)+aB\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}+6abc-b^3\right)\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{(b^2-4ac)^{3/2}}+\frac{\left(2A\left(-6a^2c^2+6ab^2c-4abc\sqrt{b\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*A)/x^2 - (2*a*(a*B*(-b^2 + 2*a*c - b*c*x^2) + A*(b^3 - 3*a
*b*c + b^2*c*x^2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + 4*(-2*A*b + a*B)*Log[x] + ((a*B*(-b^3 + 6*a*b*c - b^2*Sqrt
[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c]) + 2*A*(b^4 - 6*a*b^2*c +
 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c]))*
Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((a*B
*(b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c]
) + 2*A*(-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4
*a*b*c*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/
(b^2 - 4*a*c)^(3/2))/(4*a^3)
```

Maple [B] time = 0.031, size = 991, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^2*A+1/2/a^2/(c*x^4+b*x^2+a
)*c/(4*a*c-b^2)*x^2*A*b^2-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2
*b*B-3/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A*b*c+1/2/a^2/(c*x^4+b*x^2
+a)/(4*a*c-b^2)*A*b^3+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*B*c-1/2/a/(c*
x^4+b*x^2+a)/(4*a*c-b^2)*B*b^2+2/a^2/(4*a*c-b^2)*c*ln((4*a*c-b^2)
*(c*x^4+b*x^2+a))*A*b-1/2/a^3/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^4+b
*x^2+a))*A*b^3-1/a/(4*a*c-b^2)*c*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*
B+1/4/a^2/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*b^2*B-6/a/(
64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-
b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b
^6)^(1/2))*A*c^2+6/a^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)
^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*
a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*A*b^2*c-1/a^3/(64*a^3*c^3-48*a
^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a
*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*A*b^
4-3/a/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*
(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*
b^4*c-b^6)^(1/2))*b*B*c+1/2/a^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b
^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^
3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*B*b^3-1/2*A/a^2/x^2-2
/a^3*ln(x)*A*b+1/a^2*ln(x)*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^2*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66542, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^2*x^3),x, algorithm="fricas")
```

```
[Out] [-1/4*(((12*A*a^2*c^3 + 6*(B*a^2*b - 2*A*a*b^2)*c^2 - (B*a*b^3 -
2*A*b^4)*c)*x^6 - (B*a*b^4 - 2*A*b^5 - 12*A*a^2*b*c^2 - 6*(B*a^2*
b^2 - 2*A*a*b^3)*c)*x^4 - (B*a^2*b^3 - 2*A*a*b^4 - 12*A*a^3*c^2 -
 6*(B*a^3*b - 2*A*a^2*b^2)*c)*x^2)*log((b^3 - 4*a*b*c + 2*(b^2*c
- 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 -
 4*a*c))/(c*x^4 + b*x^2 + a)) + (2*A*a^2*b^2 - 8*A*a^3*c - 2*(6*A
*a^2*c^2 + (B*a^2*b - 2*A*a*b^2)*c)*x^4 - 2*(B*a^2*b^2 - 2*A*a*b^
3 - (2*B*a^3 - 7*A*a^2*b)*c)*x^2 - ((4*(B*a^2 - 2*A*a*b)*c^2 - (B
*a*b^2 - 2*A*b^3)*c)*x^6 - (B*a*b^3 - 2*A*b^4 - 4*(B*a^2*b - 2*A*
a*b^2)*c)*x^4 - (B*a^2*b^2 - 2*A*a*b^3 - 4*(B*a^3 - 2*A*a^2*b)*c)
*x^2)*log(c*x^4 + b*x^2 + a) + 4*((4*(B*a^2 - 2*A*a*b)*c^2 - (B*a
*b^2 - 2*A*b^3)*c)*x^6 - (B*a*b^3 - 2*A*b^4 - 4*(B*a^2*b - 2*A*a*
b^2)*c)*x^4 - (B*a^2*b^2 - 2*A*a*b^3 - 4*(B*a^3 - 2*A*a^2*b)*c)*x
^2)*log(x))*sqrt(b^2 - 4*a*c))/(((a^3*b^2*c - 4*a^4*c^2)*x^6 + (a
^3*b^3 - 4*a^4*b*c)*x^4 + (a^4*b^2 - 4*a^5*c)*x^2)*sqrt(b^2 - 4*a
*c)), 1/4*(2*((12*A*a^2*c^3 + 6*(B*a^2*b - 2*A*a*b^2)*c^2 - (B*a*
b^3 - 2*A*b^4)*c)*x^6 - (B*a*b^4 - 2*A*b^5 - 12*A*a^2*b*c^2 - 6*(
B*a^2*b^2 - 2*A*a*b^3)*c)*x^4 - (B*a^2*b^3 - 2*A*a*b^4 - 12*A*a^3
*c^2 - 6*(B*a^3*b - 2*A*a^2*b^2)*c)*x^2)*arctan(-(2*c*x^2 + b)*sq
rt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (2*A*a^2*b^2 - 8*A*a^3*c - 2*(6
*A*a^2*c^2 + (B*a^2*b - 2*A*a*b^2)*c)*x^4 - 2*(B*a^2*b^2 - 2*A*a*
b^3 - (2*B*a^3 - 7*A*a^2*b)*c)*x^2 - ((4*(B*a^2 - 2*A*a*b)*c^2 -
(B*a*b^2 - 2*A*b^3)*c)*x^6 - (B*a*b^3 - 2*A*b^4 - 4*(B*a^2*b - 2*
A*a*b^2)*c)*x^4 - (B*a^2*b^2 - 2*A*a*b^3 - 4*(B*a^3 - 2*A*a^2*b)*
c)*x^2)*log(c*x^4 + b*x^2 + a) + 4*((4*(B*a^2 - 2*A*a*b)*c^2 - (B
*a*b^2 - 2*A*b^3)*c)*x^6 - (B*a*b^3 - 2*A*b^4 - 4*(B*a^2*b - 2*A*
a*b^2)*c)*x^4 - (B*a^2*b^2 - 2*A*a*b^3 - 4*(B*a^3 - 2*A*a^2*b)*c)
*x^2)*log(x))*sqrt(-b^2 + 4*a*c))/(((a^3*b^2*c - 4*a^4*c^2)*x^6 +
 (a^3*b^3 - 4*a^4*b*c)*x^4 + (a^4*b^2 - 4*a^5*c)*x^2)*sqrt(-b^2 +
 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^2*x^3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.118 $\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=425

$$-\frac{\left(-\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}}+6aAc^2-13abBc-Ab^2c+3b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}}+6aAc^2-13abBc-Ab^2c+3b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(-10aBc-Abc+3b^2B\right)}{2c^2(b^2-4ac)}-\frac{x^3(bB-2Ac)}{2c(b^2-4ac)}-\frac{x^5\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

```
[Out] ((3*b^2*B - A*b*c - 10*a*B*c)*x)/(2*c^2*(b^2 - 4*a*c)) - ((b*B -
2*A*c)*x^3)/(2*c*(b^2 - 4*a*c)) - (x^5*(A*b - 2*a*B - (b*B - 2*A*
c)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3*B - A*b^
2*c - 13*a*b*B*c + 6*a*A*c^2 - (3*b^4*B - A*b^3*c - 19*a*b^2*B*c
+ 8*a*A*b*c^2 + 20*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2
- 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3*B - A*b^2*c - 13*
a*b*B*c + 6*a*A*c^2 + (3*b^4*B - A*b^3*c - 19*a*b^2*B*c + 8*a*A*b
*c^2 + 20*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 7.93909, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$-\frac{\left(-\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}}+6aAc^2-13abBc-Ab^2c+3b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}}+6aAc^2-13abBc-Ab^2c+3b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(-10aBc-Abc+3b^2B\right)}{2c^2(b^2-4ac)}-\frac{x^3(bB-2Ac)}{2c(b^2-4ac)}-\frac{x^5\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((3*b^2*B - A*b*c - 10*a*B*c)*x)/(2*c^2*(b^2 - 4*a*c)) - ((b*B -
2*A*c)*x^3)/(2*c*(b^2 - 4*a*c)) - (x^5*(A*b - 2*a*B - (b*B - 2*A*
```

```
c)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3*B - A*b^
2*c - 13*a*b*B*c + 6*a*A*c^2 - (3*b^4*B - A*b^3*c - 19*a*b^2*B*c
+ 8*a*A*b*c^2 + 20*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2
- 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3*B - A*b^2*c - 13*
a*b*B*c + 6*a*A*c^2 + (3*b^4*B - A*b^3*c - 19*a*b^2*B*c + 8*a*A*b
*c^2 + 20*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.37943, size = 455, normalized size = 1.07

$$\frac{\frac{2\sqrt{c}x\left(-2a^2Bc+a\left(-bc\left(A+3Bx^2\right)+2Ac^2x^2+b^2B\right)+b^2x^2(bB-Ac)\right)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{\sqrt{2}\left(2ac^2\left(3A\sqrt{b^2-4ac}-10aB\right)+b^2c\left(19aB-A\sqrt{b^2-4ac}\right)-abc\left(13B\sqrt{b^2-4ac}+8Ac\right)+b^3\left(3B\right.\right.}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*B*Sqrt[c]*x + (2*Sqrt[c]*x*(-2*a^2*B*c + b^2*(b*B - A*c)*x^2 +
 a*(b^2*B + 2*A*c^2*x^2 - b*c*(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4*B + b^2*c*(19*a*B - A*Sqrt[b
^2 - 4*a*c]) + 2*a*c^2*(-10*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + b^3*(A
*c + 3*B*Sqrt[b^2 - 4*a*c]) - a*b*c*(8*A*c + 13*B*Sqrt[b^2 - 4*a*
c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b
^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3*b^4*
B - b^2*c*(19*a*B + A*Sqrt[b^2 - 4*a*c]) + 2*a*c^2*(10*a*B + 3*A*
Sqrt[b^2 - 4*a*c]) + a*b*c*(8*A*c - 13*B*Sqrt[b^2 - 4*a*c]) + b^3
*(-(A*c) + 3*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
 - 4*a*c]]))/(4*c^(5/2))
```

Maple [B] time = 0.087, size = 4263, normalized size = 10.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -3/4/c^2/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c
-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4
*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*b^5-1/
2/c^2/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*b^2*B+3/2/c/(c*x^4+b*x^2+a)
/(4*a*c-b^2)*x^3*a*b*B+1/2/c/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*A*b+
1/4/c/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^
2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*
c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*b^4+43/4/
c/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*
a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b
^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2
)*c)^(1/2))*B*a*b^6-43/4/c/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(
1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*a
rctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3
+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*a*b^6-116*c/(-(4*a*c-b^2)^3)^(
1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2
)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c
-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*a^3*b^2+20
*c/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4
*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*
b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^
2)*c)^(1/2))*A*a^2*b^3+116*c/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2
^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2
)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*
c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*B*a^3*b^2+32*c^2/(-(4*a*c-
b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(
4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2
)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*a
^3*b-20*c/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)
*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^
2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)
^(1/2)))^(1/2))*A*a^2*b^3-32*c^2/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^
2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^
(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(
4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*a^3*b-55/(-(4*a*c-b^
2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(
1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/
2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*B
*a^2*b^4+4/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2
)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c
^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3
)^(1/2)))^(1/2))*A*a*b^5+55/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^
```

```
(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*
arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^
3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*a^2*b^4+6*c/(4*a*c-b^2)*2^(1/
2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*ar
ctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^
2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*a^2-3/4/c^2/(4*a*c-b^2)*2^(1
/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*a
rctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b
^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*B*b^5+1/4/c/(4*a*c-b^2)*2^(1/
2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*ar
ctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^
2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*b^4-13/(4*a*c-b^2)*2^(1/2)/(
(4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(
1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*
a*c-b^2)^3)^(1/2)))^(1/2))*B*a^2*b-5/2/(4*a*c-b^2)*2^(1/2)/((-4*a
*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2
*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/
2))*(4*a*c-b^2)*c)^(1/2))*A*a*b^2-13/(4*a*c-b^2)*2^(1/2)/((-4*a*b
*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(
-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2)
)*(4*a*c-b^2)*c)^(1/2))*B*a^2*b-5/2/(4*a*c-b^2)*2^(1/2)/((4*a*c-b
^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a
*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)
^3)^(1/2)))^(1/2))*A*a*b^2+6*c/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-
2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(
1/2)))^(1/2))*A*a^2+B*x/c^2-1/4/c/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b
^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^
(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*
b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*b^7+25/4/c/(4*a*c-b^2)*
2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/
2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a
*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*B*a*b^3-80*c^2/(-(4*a*c-b
^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^
(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1
/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*
B*a^4-3/4/c^2/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b
*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(
-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2)
)*(4*a*c-b^2)*c)^(1/2))*B*b^8+80*c^2/(-(4*a*c-b^2)^3)^(1/2)/(4*a*
c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)
))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4
*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*a^4+1/4/c/(-(4*a*c-b
^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^
(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1
/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*
A*b^7+3/4/c^2/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-
b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*
a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2
)^3)^(1/2)))^(1/2))*B*b^8-4/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^
(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)
*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c
-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*a*b^5+25/4/c/(4*a*c-b^2)*
2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2
```

```
)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-
b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*a*b^3-1/2/c^2/(c*x^4+b*x^2+
a)/(4*a*c-b^2)*x^3*b^3*B+1/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*A*
b^2+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*x*B-1/(c*x^4+b*x^2+a)/(4*
a*c-b^2)*x^3*a*A
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(Bb^3+2\,Aac^2-\left(3\,Bab+Ab^2\right)c\right)x^3+\left(Bab^2-\left(2\,Ba^2+Aab\right)c\right)x}{2\left(ab^2c^2-4\,a^2c^3+(b^2c^3-4\,ac^4)x^4+(b^3c^2-4\,abc^3)x^2\right)}+\frac{Bx}{c^2}-\frac{\int\frac{3\,Bab^2+\left(3\,Bb^3+6\,Aac^2-\left(13\,Bab+Ab^2\right)c\right)x^2-\left(10\,Ba^2+Aab\right)c}{cx^4+bx^2+a}\,dx}{2\left(b^2c^2-4\,ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^6/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b^3 + 2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*x^3 + (B*a*b^2 - (
2*B*a^2 + A*a*b)*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^
4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + B*x/c^2 - 1/2*integrate((3*
B*a*b^2 + (3*B*b^3 + 6*A*a*c^2 - (13*B*a*b + A*b^2)*c)*x^2 - (10*
B*a^2 + A*a*b)*c)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4*a*c^3)
```

Fricas [A] time = 3.70885, size = 9790, normalized size = 23.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^6/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(B*b^2*c - 4*B*a*c^2)*x^5 + 2*(3*B*b^3 + 2*A*a*c^2 - (11*B
*a*b + A*b^2)*c)*x^3 - sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^
3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-(9*B^2*b^7 +
60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^
2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c
^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5 - 12*a*b^4*c^6 + 4
8*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 1
8*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^
4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A
^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^
2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5
 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c
^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 -
```

12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log((189*B^4*a^2*b^6 - 135*A*B^3*a*b^7 + 324*A^4*a^3*c^5 - 81*(28*A^3*B*a^3*b + A^4*a^2*b^2)*c^4 - (2500*B^4*a^5 + 2500*A*B^3*a^4*b - 5016*A^2*B^2*a^3*b^2 - 647*A^3*B*a^2*b^3 - 5*A^4*a*b^4)*c^3 + 9*(625*B^4*a^4*b^2 - 303*A*B^3*a^3*b^3 - 186*A^2*B^2*a^2*b^4 - 5*A^3*B*a*b^5)*c^2 - 27*(73*B^4*a^3*b^4 - 49*A*B^3*a^2*b^5 - 5*A^2*B^2*a*b^6)*c)*x + 1/2*sqrt(1/2)*(27*B^3*b^10 + 144*(10*A^2*B*a^4 + A^3*a^3*b)*c^6 - 8*(500*B^3*a^5 + 930*A*B^2*a^4*b + 252*A^2*B*a^3*b^2 + 11*A^3*a^2*b^3)*c^5 + (11360*B^3*a^4*b^2 + 7608*A*B^2*a^3*b^3 + 882*A^2*B*a^2*b^4 + 17*A^3*a*b^5)*c^4 - (8818*B^3*a^3*b^4 + 2841*A*B^2*a^2*b^5 + 153*A^2*B*a*b^6 + A^3*b^7)*c^3 + 9*(329*B^3*a^2*b^6 + 51*A*B^2*a*b^7 + A^2*B*b^8)*c^2 - 27*(17*B^3*a*b^8 + A*B^2*b^9)*c - (3*B*b^9*c^5 - 768*A*a^4*c^10 + 128*(8*B*a^4*b + 5*A*a^3*b^2)*c^9 - 192*(5*B*a^3*b^3 + A*a^2*b^4)*c^8 + 24*(14*B*a^2*b^5 + A*a*b^6)*c^7 - (52*B*a*b^7 + A*b^8)*c^6)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log((189*B^4*a^2*b^6 - 135*A*B^3*a*b^7 + 324*A^4*a^3*c^5 - 81*(28*A^3*B*a^3*b + A^4*a^2*b^2)*c^4 - (2500*B^4*a^5 + 2500*A*B^3*a^4*b - 5016*A^2*B^2*a^3*b^2 - 647*A^3*B*a^2*b^3 - 5*A^4*a*b^4)*c^3 + 9*(625*B^4*a^4*b^2 - 303*A*B^3*a^3*b^3 - 186*A^2*B^2*a^2*b^4 - 5*A^3*B*a*b^5)*c^2 - 27*(73*B^4*a^3*b^4 - 49*A*B^3*a^2*b^5 - 5*A^2*B^2*a*b^6)*c)*x - 1/2*sqrt(1/2)*(27*B^3*b^10 + 144*(10*A^2*B*a^4 + A^3*a^3*b)*c^6 - 8*(500*B^3*a^5 + 930*A*B^2*a^4*b + 252*A^2*B*a^3*b^2 + 11*A^3*a^2*b^3)*c^5 + (11360*B^3*a^4*b^2 + 7608*A*B^2*a^3*b^3 + 882*A^2*B*a^2*b^4 + 17*A^3*a*b^5)*c^4 - (8818*B^3*a^3*b^4 + 2841*A*B^2*a^2*b^5 + 153*A^2*B*a*b^6 + A^

3*b^7)*c^3 + 9*(329*B^3*a^2*b^6 + 51*A*B^2*a*b^7 + A^2*B*b^8)*c^2
 - 27*(17*B^3*a*b^8 + A*B^2*b^9)*c - (3*B*b^9*c^5 - 768*A*a^4*c^1
0 + 128*(8*B*a^4*b + 5*A*a^3*b^2)*c^9 - 192*(5*B*a^3*b^3 + A*a^2*
b^4)*c^8 + 24*(14*B*a^2*b^5 + A*a*b^6)*c^7 - (52*B*a*b^7 + A*b^8)
*c^6)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44
*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b +
 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B
^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)
c^3 + 27(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2
- 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3
 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3
)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^
2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7
 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2
*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^
3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 -
 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A
^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*
b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^
4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6
 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2)*(a*b^2*c^2 - 4*a^2*
c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-
(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b +
20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4
 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c - (b^6*c^5 - 12*
a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^
4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5
+ (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^
3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3
 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 5
2*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b
^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log((1
89*B^4*a^2*b^6 - 135*A*B^3*a*b^7 + 324*A^4*a^3*c^5 - 81*(28*A^3*B
*a^3*b + A^4*a^2*b^2)*c^4 - (2500*B^4*a^5 + 2500*A*B^3*a^4*b - 50
16*A^2*B^2*a^3*b^2 - 647*A^3*B*a^2*b^3 - 5*A^4*a*b^4)*c^3 + 9*(62
5*B^4*a^4*b^2 - 303*A*B^3*a^3*b^3 - 186*A^2*B^2*a^2*b^4 - 5*A^3*B
*a*b^5)*c^2 - 27*(73*B^4*a^3*b^4 - 49*A*B^3*a^2*b^5 - 5*A^2*B^2*a
*b^6)*c)*x + 1/2*sqrt(1/2)*(27*B^3*b^10 + 144*(10*A^2*B*a^4 + A^3
*a^3*b)*c^6 - 8*(500*B^3*a^5 + 930*A*B^2*a^4*b + 252*A^2*B*a^3*b^
2 + 11*A^3*a^2*b^3)*c^5 + (11360*B^3*a^4*b^2 + 7608*A*B^2*a^3*b^3
 + 882*A^2*B*a^2*b^4 + 17*A^3*a*b^5)*c^4 - (8818*B^3*a^3*b^4 + 28
41*A*B^2*a^2*b^5 + 153*A^2*B*a*b^6 + A^3*b^7)*c^3 + 9*(329*B^3*a^
2*b^6 + 51*A*B^2*a*b^7 + A^2*B*b^8)*c^2 - 27*(17*B^3*a*b^8 + A*B^
2*b^9)*c + (3*B*b^9*c^5 - 768*A*a^4*c^10 + 128*(8*B*a^4*b + 5*A*a
^3*b^2)*c^9 - 192*(5*B*a^3*b^3 + A*a^2*b^4)*c^8 + 24*(14*B*a^2*b^
5 + A*a*b^6)*c^7 - (52*B*a*b^7 + A*b^8)*c^6)*sqrt((81*B^4*b^8 + 8
1*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*
c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 19
6*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2
*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4
 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B
^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c

^13)))*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28
*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 +
 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c - (
b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^
4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^
4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2
*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*
A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^
4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^
6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 -
 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3
c^8))) + sqrt(1/2)(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*
x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3
 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3
)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^
2*a*b^5 + 2*A*B*b^6)*c - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7
 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2
*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^
3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 -
 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A
^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*
b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^
4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6
 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log((189*B^4*a^2*b^6 - 135*A*B^3
*a*b^7 + 324*A^4*a^3*c^5 - 81*(28*A^3*B*a^3*b + A^4*a^2*b^2)*c^4
 - (2500*B^4*a^5 + 2500*A*B^3*a^4*b - 5016*A^2*B^2*a^3*b^2 - 647*A
^3*B*a^2*b^3 - 5*A^4*a*b^4)*c^3 + 9*(625*B^4*a^4*b^2 - 303*A*B^3*
a^3*b^3 - 186*A^2*B^2*a^2*b^4 - 5*A^3*B*a*b^5)*c^2 - 27*(73*B^4*a
^3*b^4 - 49*A*B^3*a^2*b^5 - 5*A^2*B^2*a*b^6)*c)*x - 1/2*sqrt(1/2)
*(27*B^3*b^10 + 144*(10*A^2*B*a^4 + A^3*a^3*b)*c^6 - 8*(500*B^3*a
^5 + 930*A*B^2*a^4*b + 252*A^2*B*a^3*b^2 + 11*A^3*a^2*b^3)*c^5 +
(11360*B^3*a^4*b^2 + 7608*A*B^2*a^3*b^3 + 882*A^2*B*a^2*b^4 + 17*
A^3*a*b^5)*c^4 - (8818*B^3*a^3*b^4 + 2841*A*B^2*a^2*b^5 + 153*A^2
*B*a*b^6 + A^3*b^7)*c^3 + 9*(329*B^3*a^2*b^6 + 51*A*B^2*a*b^7 + A
^2*B*b^8)*c^2 - 27*(17*B^3*a*b^8 + A*B^2*b^9)*c + (3*B*b^9*c^5 -
768*A*a^4*c^10 + 128*(8*B*a^4*b + 5*A*a^3*b^2)*c^9 - 192*(5*B*a^3
*b^3 + A*a^2*b^4)*c^8 + 24*(14*B*a^2*b^5 + A*a*b^6)*c^7 - (52*B*a
*b^7 + A*b^8)*c^6)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2
*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*
A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c
^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 +
 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*
B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*
a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*B^2*b^7 +
60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^
2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c
^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c - (b^6*c^5 - 12*a*b^4*c^6 + 4
8*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 1
8*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^
4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A
^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^
2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5
 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c
^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 -

```
 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + 2*(3*B*a*b^2 - (
10*B*a^2 + A*a*b)*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c
^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^6/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.119 $\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=336

$$-\frac{x^3\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{x(bB-2Ac)}{2c\left(b^2-4ac\right)}+\frac{\left(-\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -((b*B - 2*A*c)*x)/(2*c*(b^2 - 4*a*c)) - (x^3*(A*b - 2*a*B - (b*B
 - 2*A*c)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^2*B +
 A*b*c - 6*a*B*c - (b^3*B + A*b^2*c - 8*a*b*B*c + 4*a*A*c^2)/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) + ((b^2*B + A*b*c - 6*a*B*c + (b^3*B + A*b^2*c - 8*a*b*B*c +
4*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + S
qrt[b^2 - 4*a*c]])
```

Rubi [A] time = 3.34295, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$-\frac{x^3\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{x(bB-2Ac)}{2c\left(b^2-4ac\right)}+\frac{\left(-\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b*B - 2*A*c)*x)/(2*c*(b^2 - 4*a*c)) - (x^3*(A*b - 2*a*B - (b*B
 - 2*A*c)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^2*B +
 A*b*c - 6*a*B*c - (b^3*B + A*b^2*c - 8*a*b*B*c + 4*a*A*c^2)/Sqrt
```

```
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) + ((b^2*B + A*b*c - 6*a*B*c + (b^3*B + A*b^2*c - 8*a*b*B*c +
4*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + S
qrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 127.181, size = 330, normalized size = 0.98

$$-\frac{x^{3}\left(Ab-2Ba+x^{2}\left(2Ac-Bb\right)\right)}{2\left(-4ac+b^{2}\right)\left(a+bx^{2}+cx^{4}\right)}+\frac{x\left(Ac-\frac{Bb}{2}\right)}{c\left(-4ac+b^{2}\right)}$$
$$+\frac{\sqrt{2}\left(2ac\left(2Ac-Bb\right)+b\left(Abc-6Bac+Bb^{2}\right)+\sqrt{-4ac+b^{2}}\left(Abc-6Bac+Bb^{2}\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^{2}}}}\right)}{4c^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^{2}}}\left(-4ac+b^{2}\right)^{\frac{3}{2}}}$$
$$-\frac{\sqrt{2}\left(2ac\left(2Ac-Bb\right)+b\left(Abc-6Bac+Bb^{2}\right)-\sqrt{-4ac+b^{2}}\left(Abc-6Bac+Bb^{2}\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^{2}}}}\right)}{4c^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^{2}}}\left(-4ac+b^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -x**3*(A*b - 2*B*a + x**2*(2*A*c - B*b))/(2*(-4*a*c + b**2)*(a +
b*x**2 + c*x**4)) + x*(A*c - B*b/2)/(c*(-4*a*c + b**2)) + sqrt(2)
*(2*a*c*(2*A*c - B*b) + b*(A*b*c - 6*B*a*c + B*b**2) + sqrt(-4*a*
c + b**2)*(A*b*c - 6*B*a*c + B*b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt
(b + sqrt(-4*a*c + b**2)))/(4*c**(3/2)*sqrt(b + sqrt(-4*a*c + b**
2))*(-4*a*c + b**2)**(3/2)) - sqrt(2)*(2*a*c*(2*A*c - B*b) + b*(A
*b*c - 6*B*a*c + B*b**2) - sqrt(-4*a*c + b**2)*(A*b*c - 6*B*a*c +
 B*b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(
4*c**(3/2)*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2))
```

Mathematica [A] time = 1.68845, size = 362, normalized size = 1.08

$$\frac{\frac{2\sqrt{c}\left(2acx\left(A+Bx^{2}\right)-abBx+bx^{3}(Ac-bB)\right)}{(b^{2}-4ac)(a+bx^{2}+cx^{4})}+\frac{\sqrt{2}\left(b^{2}\left(B\sqrt{b^{2}-4ac}-Ac\right)+bc\left(A\sqrt{b^{2}-4ac}+8aB\right)-2ac\left(3B\sqrt{b^{2}-4ac}+2Ac\right)+b^{3}(-B)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^{2}-4ac}}}\right)}{(b^{2}-4ac)^{3/2}\sqrt{b-\sqrt{b^{2}-4ac}}}+\frac{\sqrt{2}(b}{}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*Sqrt[c]*(-(a*b*B*x) + b*(-(b*B) + A*c)*x^3 + 2*a*c*x*(A + B*x
^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(b^3*B) +
b*c*(8*a*B + A*Sqrt[b^2 - 4*a*c]) + b^2*(-(A*c) + B*Sqrt[b^2 - 4*
a*c]) - 2*a*c*(2*A*c + 3*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3*B + 2*a*c*(2*A*c - 3*B*Sq
rt[b^2 - 4*a*c]) + b^2*(A*c + B*Sqrt[b^2 - 4*a*c]) + b*(-8*a*B*c
+ A*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]
]))/(4*c^(3/2))
```

Maple [B] time = 0.082, size = 4009, normalized size = 11.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] c^2/(-c^2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*(
4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*
a*c^3-2*b^2*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*
(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*b^4*a+32*c^3/(-c^2*(4*a*c-b^2)^3)
^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4
*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^3-2*b^2*c^2)*x*2^(1/
2)/c/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(
1/2))*B*a^3*b+4*c/(-c^2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/
((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)*
arctan(1/2*(8*a*c^3-2*b^2*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c^
2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*a*b^5-1/4*c/(-c^2*(
4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*(4*a*b*c^2-b
^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^3-2*b^2
*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)
^3)^(1/2)))^(1/2))*A*b^6+1/4/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*(4*
a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*
c^3-2*b^2*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4
*a*c-b^2)^3)^(1/2)))^(1/2))*A*b^3+1/4/(4*a*c-b^2)*2^(1/2)/((-4*a*
b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2)*arctan
h(1/2*(-8*a*c^3+2*b^2*c^2)*x*2^(1/2)/c/((-4*a*b*c^2+b^3*c+(-c^2*(
4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2))*A*b^3+(-1/2*(A*b*c+2*B*a
*c-B*b^2)/(4*a*c-b^2)/c*x^3-1/2*a*(2*A*c-B*b)/c/(4*a*c-b^2)*x)/(c
*x^4+b*x^2+a)-c/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*(4*a*b*c^2-b^3*c
+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^3-2*b^2*c^2
)*x*2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^
(1/2)))^(1/2))*A*b*a+16*c^4/(-c^2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2
)*2^(1/2)/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b
^2))^(1/2)*arctanh(1/2*(-8*a*c^3+2*b^2*c^2)*x*2^(1/2)/c/((-4*a*b*
c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2))*A*a^3-4
*c^3/(-c^2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c^2+
```

```
b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2)*arctanh(1/2*
(-8*a*c^3+2*b^2*c^2)*x*2^(1/2)/c/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2))*A*a^2*b^2-c^2/(-c^2*(4*a*c-b^2
)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^
2)^3)^(1/2))*(4*a*c-b^2))^(1/2)*arctanh(1/2*(-8*a*c^3+2*b^2*c^2)*
x*2^(1/2)/c/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c
-b^2))^(1/2))*A*b^4*a-32*c^3/(-c^2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^
2)*2^(1/2)/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-
b^2))^(1/2)*arctanh(1/2*(-8*a*c^3+2*b^2*c^2)*x*2^(1/2)/c/((-4*a*b
*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2))*B*a^3*
b-4*c/(-c^2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c^2
+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2)*arctanh(1/2
*(-8*a*c^3+2*b^2*c^2)*x*2^(1/2)/c/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c
-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2))*B*a*b^5+4*c^3/(-c^2*(4*a*c-b^
2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c
^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^3-2*b^2*c^2)*x*
2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2
)))^(1/2))*A*a^2*b^2+1/4*c/(-c^2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)
*2^(1/2)/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^
2))^(1/2)*arctanh(1/2*(-8*a*c^3+2*b^2*c^2)*x*2^(1/2)/c/((-4*a*b*c
^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2))*A*b^6-c/
(4*a*c-b^2)*2^(1/2)/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)
)*(4*a*c-b^2))^(1/2)*arctanh(1/2*(-8*a*c^3+2*b^2*c^2)*x*2^(1/2)/c
/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2
))*A*b*a-16*c^4/(-c^2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((
4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)*ar
ctan(1/2*(8*a*c^3-2*b^2*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c^2-
b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*a^3+20*c^2/(-c^2*(4*a
*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c^2+b^3*c+(-c^2*(4*
a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2)*arctanh(1/2*(-8*a*c^3+2*b^2
*c^2)*x*2^(1/2)/c/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*
(4*a*c-b^2))^(1/2))*B*a^2*b^3-20*c^2/(-c^2*(4*a*c-b^2)^3)^(1/2)/(
4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2
)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^3-2*b^2*c^2)*x*2^(1/2)/c/((4
*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*
a^2*b^3-5/2/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b
^2)^3)^(1/2))*(4*a*c-b^2))^(1/2)*arctanh(1/2*(-8*a*c^3+2*b^2*c^2)
*x*2^(1/2)/c/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*
c-b^2))^(1/2))*B*a*b^2+6*c/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*(4*a*
b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^
3-2*b^2*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a
*c-b^2)^3)^(1/2)))^(1/2))*B*a^2+1/4/c/(4*a*c-b^2)*2^(1/2)/((4*a*c
-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(
1/2*(8*a*c^3-2*b^2*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c^2-b^3*c
+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*b^4+6*c/(4*a*c-b^2)*2^(1/2
)/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/
2)*arctanh(1/2*(-8*a*c^3+2*b^2*c^2)*x*2^(1/2)/c/((-4*a*b*c^2+b^3*
c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2))*B*a^2+1/4/c/(4*
a*c-b^2)*2^(1/2)/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(
4*a*c-b^2))^(1/2)*arctanh(1/2*(-8*a*c^3+2*b^2*c^2)*x*2^(1/2)/c/((
-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2))*
B*b^4-5/2/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2
*(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^3-2*b^2*c^2)*x*2^
(1/2)/c/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))
```

```
)^(1/2))*B*a*b^2+1/4/(-c^2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/
2)/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1
/2)*arctanh(1/2*(-8*a*c^3+2*b^2*c^2)*x*2^(1/2)/c/((-4*a*b*c^2+b^3
*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2))*B*b^7-1/4/(-c^
2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*(4*a*b*c^
2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^3-2*
b^2*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b
^2)^3)^(1/2)))^(1/2))*B*b^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(Bb^2-(2\,Ba+Ab)c\right)x^3+(Bab-2\,Aac)x}{2\left((b^2c^2-4\,ac^3)x^4+ab^2c-4\,a^2c^2+(b^3c-4\,abc^2)x^2\right)}+\frac{\int\frac{Bab-2\,Aac+\left(Bb^2-(6\,Ba-Ab)c\right)x^2}{cx^4+bx^2+a}\,dx}{2\left(b^2c-4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^4/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((B*b^2 - (2*B*a + A*b)*c)*x^3 + (B*a*b - 2*A*a*c)*x)/((b^2*
c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^
2) + 1/2*integrate((B*a*b - 2*A*a*c + (B*b^2 - (6*B*a - A*b)*c)*x
^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Fricas [A] time = 1.18604, size = 6288, normalized size = 18.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^4/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(B*b^2 - (2*B*a + A*b)*c)*x^3 + sqrt(1/2)*((b^2*c^2 - 4*a
*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-
(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*
a*b^2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 -
12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c
^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a
*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*
a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A
*B^3*b^5 - 4*A^4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108*
B^4*a^3 - 108*A*B^3*a^2*b + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 -
 (81*B^4*a^2*b^2 - 65*A*B^3*a*b^3 + 9*A^2*B^2*b^4)*c)*x + 1/2*sqr
t(1/2)*(B^3*b^7 - 17*B^3*a*b^5*c - 32*A^3*a^2*c^5 + 16*(18*A*B^2*
```

a^3 - 3*A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3*b + 72*A*B^2
*a^2*b^2 - 12*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 18*A
*B^2*a*b^4 - 3*A^2*B*b^5)*c^2 - (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3
*b)*c^7 - 64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 -
A*a*b^5)*c^5 - 4*(6*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^
4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*
b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^
6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(B^2*b^5
- 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A
^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*
c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9
*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^
2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a
*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4
+ 48*a^2*b^2*c^5 - 64*a^3*c^6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c^3
)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(B^2
b^5 - 12(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^
2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a
*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 -
2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b +
2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 -
12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^
4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A*B^3
*b^5 - 4*A^4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108*B^4*
a^3 - 108*A*B^3*a^2*b + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81
*B^4*a^2*b^2 - 65*A*B^3*a*b^3 + 9*A^2*B^2*b^4)*c)*x - 1/2*sqrt(1/
2)*(B^3*b^7 - 17*B^3*a*b^5*c - 32*A^3*a^2*c^5 + 16*(18*A*B^2*a^3
- 3*A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3*b + 72*A*B^2*a^2
*b^2 - 12*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 18*A*B^2
*a*b^4 - 3*A^2*B*b^5)*c^2 - (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3*b)*
c^7 - 64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a
*b^5)*c^5 - 4*(6*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^4 -
2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b +
2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 -
12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(B^2*b^5 - 12
*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b
^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4
+ 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2
*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^
2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4
*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 4
8*a^2*b^2*c^5 - 64*a^3*c^6))) + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^
4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(B^2*b^5
- 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 +
A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c - (b^6*c^3 - 12*a*b^4
*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(
9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A
^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*
a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^
4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A*B^3*b^5
- 4*A^4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108*B^4*a^3
- 108*A*B^3*a^2*b + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81*B^4
*a^2*b^2 - 65*A*B^3*a*b^3 + 9*A^2*B^2*b^4)*c)*x + 1/2*sqrt(1/2)*(
B^3*b^7 - 17*B^3*a*b^5*c - 32*A^3*a^2*c^5 + 16*(18*A*B^2*a^3 - 3*

```
A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3*b + 72*A*B^2*a^2*b^2
 - 12*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 18*A*B^2*a*b
^4 - 3*A^2*B*b^5)*c^2 + (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3*b)*c^7
- 64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a*b^5
)*c^5 - 4*(6*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9
*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^
2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a
*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(B^2*b^5 - 12*(4*
A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*
c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c - (b^6*c^3 - 12*a*b^4*c^4 + 48
*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2
*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^
2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7
 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^
2*b^2*c^5 - 64*a^3*c^6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 +
a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(B^2*b^5 - 1
2*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*
b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c - (b^6*c^3 - 12*a*b^4*c^4
 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^
2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B
^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^
4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 +
48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A*B^3*b^5 - 4
*A^4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108*B^4*a^3 - 10
8*A*B^3*a^2*b + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81*B^4*a^2
*b^2 - 65*A*B^3*a*b^3 + 9*A^2*B^2*b^4)*c)*x - 1/2*sqrt(1/2)*(B^3*
b^7 - 17*B^3*a*b^5*c - 32*A^3*a^2*c^5 + 16*(18*A*B^2*a^3 - 3*A^2*
B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3*b + 72*A*B^2*a^2*b^2 - 1
2*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 18*A*B^2*a*b^4 -
 3*A^2*B*b^5)*c^2 + (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3*b)*c^7 - 64
*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a*b^5)*c^
5 - 4*(6*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2
*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^
2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4
*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(B^2*b^5 - 12*(4*A*B*
a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2
- (15*B^2*a*b^3 - 2*A*B*b^4)*c - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2
*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a -
 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c
^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 4
8*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^
2*c^5 - 64*a^3*c^6))) + 2*(B*a*b - 2*A*a*c)*x)/((b^2*c^2 - 4*a*c^
3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
```

Sympy [A] time = 151.981, size = 1129, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -(x**3*(A*b*c + 2*B*a*c - B*b**2) + x*(2*A*a*c - B*a*b))/(8*a**2*
c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x**2*(8*a*b*c
**2 - 2*b**3*c)) + RootSum(_t**4*(1048576*a**6*c**9 - 1572864*a**
5*b**2*c**8 + 983040*a**4*b**4*c**7 - 327680*a**3*b**6*c**6 + 614
40*a**2*b**8*c**5 - 6144*a*b**10*c**4 + 256*b**12*c**3) + _t**2*(
-12288*A**2*a**4*b*c**6 + 8192*A**2*a**3*b**3*c**5 - 1536*A**2*a*
*2*b**5*c**4 + 16*A**2*b**9*c**2 + 49152*A*B*a**5*c**6 - 24576*A*
B*a**4*b**2*c**5 - 2048*A*B*a**3*b**4*c**4 + 3072*A*B*a**2*b**6*c
**3 - 576*A*B*a*b**8*c**2 + 32*A*B*b**10*c - 61440*B**2*a**5*b*c*
*5 + 61440*B**2*a**4*b**3*c**4 - 24064*B**2*a**3*b**5*c**3 + 4608
*B**2*a**2*b**7*c**2 - 432*B**2*a*b**9*c + 16*B**2*b**11) + 16*A*
*4*a**3*c**4 + 24*A**4*a**2*b**2*c**3 + 9*A**4*a*b**4*c**2 - 224*
A**3*B*a**3*b*c**3 - 144*A**3*B*a**2*b**3*c**2 + 18*A**3*B*a*b**5
*c + 288*A**2*B**2*a**4*c**3 + 960*A**2*B**2*a**3*b**2*c**2 - 198
*A**2*B**2*a**2*b**4*c + 9*A**2*B**2*a*b**6 - 2016*A*B**3*a**4*b*
c**2 + 496*A*B**3*a**3*b**3*c - 30*A*B**3*a**2*b**5 + 1296*B**4*a
**5*c**2 - 360*B**4*a**4*b**2*c + 25*B**4*a**3*b**4, Lambda(_t, _
t*log(x + (-16384*_t**3*A*a**3*b*c**7 + 12288*_t**3*A*a**2*b**3*c
**6 - 3072*_t**3*A*a*b**5*c**5 + 256*_t**3*A*b**7*c**4 + 49152*_t
**3*B*a**4*c**7 - 40960*_t**3*B*a**3*b**2*c**6 + 12288*_t**3*B*a*
*2*b**4*c**5 - 1536*_t**3*B*a*b**6*c**4 + 64*_t**3*B*b**8*c**3 -
64*_t*A**3*a**2*c**5 + 128*_t*A**3*a*b**2*c**4 + 4*_t*A**3*b**4*c
**3 - 768*_t*A**2*B*a**2*b*c**4 - 48*_t*A**2*B*a*b**3*c**3 + 12*_
t*A**2*B*b**5*c**2 + 1728*_t*A*B**2*a**3*c**4 + 384*_t*A*B**2*a**
2*b**2*c**3 - 156*_t*A*B**2*a*b**4*c**2 + 12*_t*A*B**2*b**6*c - 1
728*_t*B**3*a**3*b*c**3 + 656*_t*B**3*a**2*b**3*c**2 - 88*_t*B**3
*a*b**5*c + 4*_t*B**3*b**7)/(-4*A**4*a*c**4 - 3*A**4*b**2*c**3 +
20*A**3*B*a*b*c**3 - 9*A**3*B*b**3*c**2 + 84*A**2*B**2*a*b**2*c**
2 - 9*A**2*B**2*b**4*c - 324*A*B**3*a**2*b*c**2 + 65*A*B**3*a*b**
3*c - 3*A*B**3*b**5 + 324*B**4*a**3*c**2 - 81*B**4*a**2*b**2*c +
5*B**4*a*b**4))))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^4/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.120 $\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=276

$$-\frac{x\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(-\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)) + ((b*B - 2*A*c - (b^2*B - 4*A*b*c + 4*a*B*c)/Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + ((b*B - 2*A*c + (b^2*B - 4*A*b*c + 4*a*B*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt
[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.06507, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12

$$-\frac{x\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(-\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)) + ((b*B - 2*A*c - (b^2*B - 4*A*b*c + 4*a*B*c)/Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + ((b*B - 2*A*c + (b^2*B - 4*A*b*c + 4*a*B*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt
[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 172.384, size = 264, normalized size = 0.96

$$-\frac{x\left(Ab-2Ba+x^{2}\left(2Ac-Bb\right)\right)}{2\left(-4ac+b^{2}\right)\left(a+bx^{2}+cx^{4}\right)}+\frac{\sqrt{2}\left(-4Abc+4Bac+Bb^{2}-\left(2Ac-Bb\right)\sqrt{-4ac+b^{2}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^{2}}}}\right)}{4\sqrt{c}\sqrt{b+\sqrt{-4ac+b^{2}}}\left(-4ac+b^{2}\right)^{\frac{3}{2}}}-\frac{\sqrt{2}\left(-4Abc+4Bac+Bb^{2}+\left(2Ac-Bb\right)\sqrt{-4ac+b^{2}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^{2}}}}\right)}{4\sqrt{c}\sqrt{b-\sqrt{-4ac+b^{2}}}\left(-4ac+b^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -x*(A*b - 2*B*a + x**2*(2*A*c - B*b))/(2*(-4*a*c + b**2)*(a + b*x
**2 + c*x**4)) + sqrt(2)*(-4*A*b*c + 4*B*a*c + B*b**2 - (2*A*c -
B*b)*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4
*a*c + b**2)))/(4*sqrt(c)*sqrt(b + sqrt(-4*a*c + b**2))*(-4*a*c +
 b**2)**(3/2)) - sqrt(2)*(-4*A*b*c + 4*B*a*c + B*b**2 + (2*A*c -
B*b)*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4
*a*c + b**2)))/(4*sqrt(c)*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c +
 b**2)**(3/2))
```

Mathematica [A] time = 1.20225, size = 298, normalized size = 1.08

$$\frac{1}{4}\left(\frac{2x\left(B\left(2a+bx^{2}\right)-A\left(b+2cx^{2}\right)\right)}{\left(b^{2}-4ac\right)\left(a+bx^{2}+cx^{4}\right)}+\frac{\sqrt{2}\left(-2Ac\sqrt{b^{2}-4ac}+bB\sqrt{b^{2}-4ac}-4aBc+4Abc+b^{2}(-B)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^{2}-4ac}}}\right)}{\sqrt{c}\left(b^{2}-4ac\right)^{3/2}\sqrt{b-\sqrt{b^{2}-4ac}}}+\frac{\sqrt{2}\left(-2Ac\sqrt{b^{2}-4ac}+bB\sqrt{b^{2}-4ac}+4aBc-4Abc+b^{2}B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^{2}-4ac}+b}}\right)}{\sqrt{c}\left(b^{2}-4ac\right)^{3/2}\sqrt{\sqrt{b^{2}-4ac}+b}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(B*(2*a + b*x^2) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)) + (Sqrt[2]*(-(b^2*B) + 4*A*b*c - 4*a*B*c + b*B*Sqrt
[b^2 - 4*a*c] - 2*A*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2*B - 4*A*b*c + 4*a*B*c +
 b*B*Sqrt[b^2 - 4*a*c] - 2*A*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^
(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/4
```

Maple [B] time = 0.059, size = 2995, normalized size = 10.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (1/2*(2*A*c-B*b)/(4*a*c-b^2)*x^3+1/2*(A*b-2*B*a)/(4*a*c-b^2)*x)/(
c*x^4+b*x^2+a)+16*c^3/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/
((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan
(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4
*a*c-b^2)^3)^(1/2)))^(1/2))*a^2*A*b-8*c^2/(-(4*a*c-b^2)^3)^(1/2)/
(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^
(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)
*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*a*A*b^3+c/(-(4*a*
c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(
-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1
/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A
*b^5-16*c^3/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^
2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*
c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^
3)^(1/2)))^(1/2))*a^3*B+4*c^2/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*
2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2
)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-
b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*a^2*B*b^2+c/(-(4*a*c-b^2)^3)^
(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^
2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*
c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*b^4*B*a-1/4
/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b
*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c
)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^
(1/2))*b^6*B+2/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-
(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/
2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*
a*c^2-1/2*c/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*
a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/
((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*b^2
-c/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^
3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b
```

```
^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*b*a+1/4/(4*a
*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2
)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(
4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*b^3-16/(-(4*a*c-b^2
)^3)^(1/2)/(4*a*c-b^2)*c^3*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3
)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^
(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)
)*a^2*A*b+8/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*c^2*2^(1/2)/((-4*a
*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2
*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/
2))*(4*a*c-b^2)*c)^(1/2))*a*A*b^3-c/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c
-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*
c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+
(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*b^5+16/(-(4*a*c-b
^2)^3)^(1/2)/(4*a*c-b^2)*c^3*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)
^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*
2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/
2))*a^3*B-4/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*c^2*2^(1/2)/((-4*a
*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2
*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/
2))*(4*a*c-b^2)*c)^(1/2))*a^2*B*b^2-c/(-(4*a*c-b^2)^3)^(1/2)/(4*a
*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2
)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^
3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*b^4*B*a+1/4/(-(4*
a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2
)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x
*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1
/2))*b^6*B+2/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^
(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1
/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*
A*a*c^2-1/2*c/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)
^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(
1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))
*A*b^2-c/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2
))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/
((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*B*b*
a+1/4/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*
(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-
4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*B*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(Bb-2\,Ac)x^3+(2\,Ba-Ab)x}{2\,((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2)}-\frac{-\int\frac{(Bb-2\,Ac)x^2-2\,Ba+Ab}{cx^4+bx^2+a}\,dx}{2\,(b^2-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b - 2*A*c)*x^3 + (2*B*a - A*b)*x)/((b^2*c - 4*a*c^2)*x^4
+ a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-((B*b -
 2*A*c)*x^2 - 2*B*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [A] time = 0.671491, size = 4680, normalized size = 16.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*b - 2*A*c)*x^3 - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b
^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a
^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c +
 (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B
^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 +
 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^
3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^2*b^2 - A*B^3*a*b^3 - 4*A^
4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 12*A*B^3*a
^2*b + A^3*B*b^3)*c)*x + 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B^2*a*b
^5 - 16*(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^
3*b + 2*A^2*B*a^2*b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^
2*a^2*b^3 + 2*A^2*B*a*b^4 - A^3*b^5)*c + (192*B*a^4*b^3*c^3 + 256
*A*a^5*c^5 - 128*(2*B*a^5*b + A*a^4*b^2)*c^4 - 8*(6*B*a^3*b^5 - A
*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c)*sqrt((B^4*a^2 - 2*A^2*
B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4
 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2
 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*
b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*
a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 6
4*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*
c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^
3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^
2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2
*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2
*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 -
64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4
*c^4))*log(-(3*B^4*a^2*b^2 - A*B^3*a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3
*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 12*A*B^3*a^2*b + A^3*B*b^3)*
c)*x - 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B^2*a*b^5 - 16*(2*A^2*B*a
^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2*B*a^2*
b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*
B*a*b^4 - A^3*b^5)*c + (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(
2*B*a^5*b + A*a^4*b^2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4
*B*a^2*b^7 - A*a*b^8)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)
/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))*s
qrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b -
 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b
^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^
```

```
2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^
6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))) - sqrt(1/2)
*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*
sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b
- 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*
b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a
^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b
^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4
*a^2*b^2 - A*B^3*a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*
c^2 + (4*B^4*a^3 - 12*A*B^3*a^2*b + A^3*B*b^3)*c)*x + 1/2*sqrt(1/
2)*(2*B^3*a^2*b^4 - A*B^2*a*b^5 - 16*(2*A^2*B*a^3 - A^3*a^2*b)*c^
3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2*B*a^2*b^2 - A^3*a*b^3)*c
^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a*b^4 - A^3*b^5)
*c - (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A*a^4*
b^2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b
^8)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12
*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 -
4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^
2*b^3)*c - (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^
4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3
*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*c
^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2
)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 -
 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A
^2*b^3)*c - (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c
^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^
3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*
c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^2*b^2 - A*B^3*a
*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 -
 12*A*B^3*a^2*b + A^3*B*b^3)*c)*x - 1/2*sqrt(1/2)*(2*B^3*a^2*b^4
- A*B^2*a*b^5 - 16*(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 -
 2*A*B^2*a^3*b + 2*A^2*B*a^2*b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b
^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a*b^4 - A^3*b^5)*c - (192*B*a^4*b^
3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A*a^4*b^2)*c^4 - 8*(6*B*
a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c)*sqrt((B^4*a
^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*
a^4*b^2*c^4 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A
^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*
c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 -
 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*
b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^
3 - 64*a^4*c^4))) + 2*(2*B*a - A*b)*x)/((b^2*c - 4*a*c^2)*x^4 + a
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [A] time = 79.6696, size = 923, normalized size = 3.34

$$-\frac{x^3(-2Ac+Bb)+x(-Ab+2Ba)}{8a^2c-2ab^2+x^4(8ac^2-2b^2c)+x^2(8abc-2b^3)}$$

$$+\operatorname{RootSum}\Big(t^4\left(1048576a^7c^7-1572864a^6b^2c^6+983040a^5b^4c^5-327680a^4b^6c^4+61440a^3b^8c^3-6144a^2b^{10}c^2+256ab^{12}c\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -(x**3*(-2*A*c + B*b) + x*(-A*b + 2*B*a))/(8*a**2*c - 2*a*b**2 +
x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3)) + RootSum(_
t**4*(1048576*a**7*c**7 - 1572864*a**6*b**2*c**6 + 983040*a**5*b*
*4*c**5 - 327680*a**4*b**6*c**4 + 61440*a**3*b**8*c**3 - 6144*a**
2*b**10*c**2 + 256*a*b**12*c) + _t**2*(-12288*A**2*a**4*b*c**5 +
8192*A**2*a**3*b**3*c**4 - 1536*A**2*a**2*b**5*c**3 + 16*A**2*b**
9*c + 16384*A*B*a**5*c**5 - 6144*A*B*a**3*b**4*c**3 + 2048*A*B*a*
*2*b**6*c**2 - 192*A*B*a*b**8*c - 12288*B**2*a**5*b*c**4 + 8192*B
**2*a**4*b**3*c**3 - 1536*B**2*a**3*b**5*c**2 + 16*B**2*a*b**9) +
 16*A**4*a**2*c**4 + 24*A**4*a*b**2*c**3 + 9*A**4*b**4*c**2 - 96*
A**3*B*a**2*b*c**3 - 80*A**3*B*a*b**3*c**2 - 6*A**3*B*b**5*c + 32
*A**2*B**2*a**3*c**3 + 192*A**2*B**2*a**2*b**2*c**2 + 42*A**2*B**
2*a*b**4*c + A**2*B**2*b**6 - 96*A*B**3*a**3*b*c**2 - 80*A*B**3*a
**2*b**3*c - 6*A*B**3*a*b**5 + 16*B**4*a**4*c**2 + 24*B**4*a**3*b
**2*c + 9*B**4*a**2*b**4, Lambda(_t, _t*log(x + (-16384*_t**3*A*a
**5*c**5 + 8192*_t**3*A*a**4*b**2*c**4 - 512*_t**3*A*a**2*b**6*c*
*2 + 64*_t**3*A*a*b**8*c + 16384*_t**3*B*a**5*b*c**4 - 12288*_t**
3*B*a**4*b**3*c**3 + 3072*_t**3*B*a**3*b**5*c**2 - 256*_t**3*B*a*
*2*b**7*c + 128*_t*A**3*a**2*b*c**3 + 16*_t*A**3*a*b**3*c**2 + 4*
_t*A**3*b**5*c - 192*_t*A**2*B*a**3*c**3 - 192*_t*A**2*B*a**2*b**
2*c**2 - 36*_t*A**2*B*a*b**4*c + 192*_t*A*B**2*a**3*b*c**2 + 144*
_t*A*B**2*a**2*b**3*c + 64*_t*B**3*a**4*c**2 - 128*_t*B**3*a**3*b
**2*c - 4*_t*B**3*a**2*b**4)/(-4*A**4*a*c**3 - 3*A**4*b**2*c**2 +
 12*A**3*B*a*b*c**2 + A**3*B*b**3*c - 12*A*B**3*a**2*b*c - A*B**3
*a*b**3 + 4*B**4*a**3*c + 3*B**4*a**2*b**2))))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.121 $\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=294

$$-\frac{x\left(-A\left(b^2-2ac\right)+cx^2(-(Ab-2aB))+abB\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{A\left(b^2-12ac\right)+4abB}{\sqrt{b^2-4ac}}-2aB+Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4abB+Ab^2}{\sqrt{b^2-4ac}}-2aB+Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -(x*(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2))/(2*a*(b^2 -
4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(A*b - 2*a*B + (4*a*b*B +
A*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a*B - (A*b^2 + 4*a*b*B -
12*a*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2
 - 4*a*c]])
```

Rubi [A] time = 1.72935, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{A\left(b^2-12ac\right)+4abB}{\sqrt{b^2-4ac}}-2aB+Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4abB+Ab^2}{\sqrt{b^2-4ac}}-2aB+Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(2*a*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(A*b - 2*a*B + (4*a*b*B + A
*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a*B - (A*b^2 + 4*a*b*B - 1
2*a*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
```

```
qrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2
- 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.47236, size = 304, normalized size = 1.03

$$\frac{\frac{2x\left(A\left(-2ac+b^2+bcx^2\right)-aB\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(A\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-2aB\left(\sqrt{b^2-4ac}-2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(A\left(b\sqrt{b^2-4ac}+12ac-b^2\right)-2a\cdots}{\left(b^2-4ac\right)^{3/2}\cdots}}{4a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(-(a*B*(b + 2*c*x^2)) + A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 -
 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-2*a*B*(-2*b + S
qrt[b^2 - 4*a*c]) + A*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-2*a*B*(
2*b + Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c]
))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2
 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [B] time = 0.095, size = 1761, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/4/(4*a*c-b^2)*(-4*a*c+b^2)^(1/2)/a*x/(x^2+1/2/c*(-4*a*c+b^2)^(1
/2)+1/2*b/c)*A-1/4/(4*a*c-b^2)/a*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+
1/2*b/c)*A*b+1/2/(4*a*c-b^2)*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*
b/c)*B-12*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*A*a-8*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)/(
4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2+3/4*c/(4*a*c-b^2)
/(-4*a*c+b^2)^(1/2)/a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
A*b^4-c^2/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
A*b-3/4*c/(4*a*c-b^2)/a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*A*b^3+2*c^2/(4*a*c-b^2)*a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))*B+3/2*c/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*B*b^2+4*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*a/(4*a*c+3*b^2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*b*B+3*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)
/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*b^3-1/4/(4*a*c-b^2)
*(-4*a*c+b^2)^(1/2)/a*x/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*A-
1/4/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*A*b+1/
2/(4*a*c-b^2)*x/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*B-12*c^3/(
4*a*c-b^2)/(-4*a*c+b^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*A*a-8*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)/(4*a*c+3*b^2)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2+3/4*c/(4*a*c-b^2)/(-4*a*c+
b^2)^(1/2)/a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^4+
c^2/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b
+3/4*c/(4*a*c-b^2)/a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*A*b^3-2*c^2/(4*a*c-b^2)*a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*B-3/2*c/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*B*b^2+4*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*a/(4*a*c+3*b
^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*B+3*c/(4*a*c-b^2)/(-4*a*c+b
^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(2\,Ba-Ab)cx^3+\left(Bab-Ab^2+2\,Aac\right)x}{2\,((ab^2c-4\,a^2c^2)x^4+a^2b^2-4\,a^3c+(ab^3-4\,a^2bc)x^2)}+\frac{-\int\frac{(2\,Ba-Ab)cx^2-Bab-Ab^2+6\,Aac}{cx^4+bx^2+a}\,dx}{2\,(ab^2-4\,a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*B*a - A*b)*c*x^3 + (B*a*b - A*b^2 + 2*A*a*c)*x)/((a*b^2*
c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
 + 1/2*integrate(-((2*B*a - A*b)*c*x^2 - B*a*b - A*b^2 + 6*A*a*c)
/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [A] time = 1.59919, size = 6595, normalized size = 22.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*B*a - A*b)*c*x^3 - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^
4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(B^2*a^2*b
^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3
*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^
4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^
3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^
2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12
*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4
*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((324*A^4*a^2*c^4 - 81*(4*A
^3*B*a^2*b + A^4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84*A^
2*B^2*a^2*b^2 - 65*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2
+ 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 + A^3*B*b^5)*c)*x + 1/2*sqrt(
1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 + A^3*b^8 + 8
64*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*a^4*b + 14*A^3*a^3*b^2
)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*B*a^3*b^3 + 9
5*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A^2*B
*a^2*b^5 + 23*A^3*a*b^6)*c - (B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b
^5*c^2 - 256*(B*a^8 - 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*
b^3)*c^3 - 4*(2*B*a^5*b^6 + 5*A*a^4*b^7)*c)*sqrt((B^4*a^4 + 4*A*B
^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a
^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(B^2*a^2*b
^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3
*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^
4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^
3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^
2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12
*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4
*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c - 4*a^2
*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(B
^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)
*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^3*b^6
```

- 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((324*A^4*a^2*c^4 - 81*(4*A^3*B*a^2*b + A^4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 - 65*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 + A^3*B*b^5)*c)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*a^4*b + 14*A^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*B*a^3*b^3 + 95*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A^2*B*a^2*b^5 + 23*A^3*a*b^6)*c - (B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b^5*c^2 - 256*(B*a^8 - 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(2*B*a^5*b^6 + 5*A*a^4*b^7)*c)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((324*A^4*a^2*c^4 - 81*(4*A^3*B*a^2*b + A^4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 - 65*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 + A^3*B*b^5)*c)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*a^4*b + 14*A^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*B*a^3*b^3 + 95*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A^2*B*a^2*b^5 + 23*A^3*a*b^6)*c + (B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b^5*c^2 - 256*(B*a^8 - 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(2*B*a^5*b^6 + 5*A*a^4*b^7)*c)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*

```
c)*x^2)*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^
3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b
^3)*c - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sq
rt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 +
 A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4
*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)
))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((3
24*A^4*a^2*c^4 - 81*(4*A^3*B*a^2*b + A^4*a*b^2)*c^3 - (4*B^4*a^4
- 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 - 65*A^3*B*a*b^3 - 5*A^4*b^
4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 + A^3
*B*b^5)*c)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6 + 3*A
^2*B*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*
B*a^4*b + 14*A^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2
 + 108*A^2*B*a^3*b^3 + 95*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*
A*B^2*a^3*b^4 + 45*A^2*B*a^2*b^5 + 23*A^3*a*b^6)*c + (B*a^4*b^8 +
 A*a^3*b^9 + 144*A*a^5*b^5*c^2 - 256*(B*a^8 - 2*A*a^7*b)*c^4 + 64
*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(2*B*a^5*b^6 + 5*A*a^4*b^7)*
c)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*
b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b
+ A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9
*c^3)))*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^
3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b
^3)*c - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sq
rt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 +
 A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4
*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)
))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + 2*(
B*a*b - A*b^2 + 2*A*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2
- 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

Sympy [A] time = 142.926, size = 1180, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (x**3*(-A*b*c + 2*B*a*c) + x*(2*A*a*c - A*b**2 + B*a*b))/(8*a**3*
c - 2*a**2*b**2 + x**4*(8*a**2*c**2 - 2*a*b**2*c) + x**2*(8*a**2*
b*c - 2*a*b**3)) + RootSum(_t**4*(1048576*a**9*c**6 - 1572864*a**
8*b**2*c**5 + 983040*a**7*b**4*c**4 - 327680*a**6*b**6*c**3 + 614
40*a**5*b**8*c**2 - 6144*a**4*b**10*c + 256*a**3*b**12) + _t**2*(
-61440*A**2*a**5*b*c**5 + 61440*A**2*a**4*b**3*c**4 - 24064*A**2*
a**3*b**5*c**3 + 4608*A**2*a**2*b**7*c**2 - 432*A**2*a*b**9*c + 1
6*A**2*b**11 + 49152*A*B*a**6*c**5 - 24576*A*B*a**5*b**2*c**4 - 2
048*A*B*a**4*b**4*c**3 + 3072*A*B*a**3*b**6*c**2 - 576*A*B*a**2*b
**8*c + 32*A*B*a*b**10 - 12288*B**2*a**6*b*c**4 + 8192*B**2*a**5*
b**3*c**3 - 1536*B**2*a**4*b**5*c**2 + 16*B**2*a**2*b**9) + 1296*
```

```
A**4*a**2*c**5 - 360*A**4*a*b**2*c**4 + 25*A**4*b**4*c**3 - 2016*
A**3*B*a**2*b*c**4 + 496*A**3*B*a*b**3*c**3 - 30*A**3*B*b**5*c**2
 + 288*A**2*B**2*a**3*c**4 + 960*A**2*B**2*a**2*b**2*c**3 - 198*A
**2*B**2*a*b**4*c**2 + 9*A**2*B**2*b**6*c - 224*A*B**3*a**3*b*c**
3 - 144*A*B**3*a**2*b**3*c**2 + 18*A*B**3*a*b**5*c + 16*B**4*a**4
*c**3 + 24*B**4*a**3*b**2*c**2 + 9*B**4*a**2*b**4*c, Lambda(_t, _
t*log(x + (-32768*_t**3*A*a**7*b*c**4 + 28672*_t**3*A*a**6*b**3*c
**3 - 9216*_t**3*A*a**5*b**5*c**2 + 1280*_t**3*A*a**4*b**7*c - 64
*_t**3*A*a**3*b**9 + 16384*_t**3*B*a**8*c**4 - 8192*_t**3*B*a**7*
b**2*c**3 + 512*_t**3*B*a**5*b**6*c - 64*_t**3*B*a**4*b**8 - 1728
*_t*A**3*a**4*c**4 + 2304*_t*A**3*a**3*b**2*c**3 - 740*_t*A**3*a*
*2*b**4*c**2 + 92*_t*A**3*a*b**6*c - 4*_t*A**3*b**8 - 576*_t*A**2
*B*a**4*b*c**3 - 528*_t*A**2*B*a**3*b**3*c**2 + 168*_t*A**2*B*a**
2*b**5*c - 12*_t*A**2*B*a*b**7 + 576*_t*A*B**2*a**5*c**3 + 192*_t
*A*B**2*a**4*b**2*c**2 + 60*_t*A*B**2*a**3*b**4*c - 12*_t*A*B**2*
a**2*b**6 - 128*_t*B**3*a**5*b*c**2 - 16*_t*B**3*a**4*b**3*c - 4*
_t*B**3*a**3*b**5)/(-324*A**4*a**2*c**4 + 81*A**4*a*b**2*c**3 - 5
*A**4*b**4*c**2 + 324*A**3*B*a**2*b*c**3 - 65*A**3*B*a*b**3*c**2
+ 3*A**3*B*b**5*c - 84*A**2*B**2*a**2*b**2*c**2 + 9*A**2*B**2*a*b
**4*c - 20*A*B**3*a**3*b*c**2 + 9*A*B**3*a**2*b**3*c + 4*B**4*a**
4*c**2 + 3*B**4*a**3*b**2*c))))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.122 $\int \frac{A+Bx^2}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=389

$$-\frac{-10aAc-abB+3Ab^2}{2a^2x\left(b^2-4ac\right)}+\frac{\sqrt{c}\left(aB\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\frac{aB\left(b^2-12ac\right)-A\left(3b^3-16abc\right)}{\sqrt{b^2-4ac}}-10aAc-abB+3Ab^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{-A\left(b^2-2ac\right)+cx^2(-(Ab-2aB))+abB}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -(3*A*b^2 - a*b*B - 10*a*A*c)/(2*a^2*(b^2 - 4*a*c)*x) - (a*b*B -
A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x*(a +
b*x^2 + c*x^4)) + (Sqrt[c]*(a*B*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*
c]) - A*(3*b^3 - 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt
[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) - (Sqrt[c]*(3*A*b^2 - a*b*B - 10*a*A*c + (a*B*(b^2 - 12*a*
c) - A*(3*b^3 - 16*a*b*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c
)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 2.44306, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$-\frac{-10aAc-abB+3Ab^2}{2a^2x\left(b^2-4ac\right)}+\frac{\sqrt{c}\left(aB\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\frac{aB\left(b^2-12ac\right)-A\left(3b^3-16abc\right)}{\sqrt{b^2-4ac}}-10aAc-abB+3Ab^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(3*A*b^2 - a*b*B - 10*a*A*c)/(2*a^2*(b^2 - 4*a*c)*x) + (A*b^2 -
a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x*(a +
b*x^2 + c*x^4)) + (Sqrt[c]*(a*B*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*
c]) - A*(3*b^3 - 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt
[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) - (Sqrt[c]*(3*A*b^2 - a*b*B - 10*a*A*c + (a*B*(b^2 - 12*a*
c) - A*(3*b^3 - 16*a*b*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c
)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.03667, size = 382, normalized size = 0.98

$$\frac{\frac{2x\left(aB\left(-2ac+b^2+bcx^2\right)-A\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(A\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)+aB\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{b-\sqrt{b}}}\right.}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*A)/x + (2*x*(a*B*(b^2 - 2*a*c + b*c*x^2) - A*(b^3 - 3*a*b*c
+ b^2*c*x^2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
+ (Sqrt[2]*Sqrt[c]*(a*B*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]) + A*
(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 -
4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqr
t[c]*(a*B*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c]) + A*(3*b^3 - 16*a
*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
```

```
)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

Maple [B] time = 0.073, size = 3752, normalized size = 9.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 11/2/a/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b
^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a
*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*c^2*b^2-
1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*B*b^2+1/2/a^2/(c*x^4+b*x^2+a)
/(4*a*c-b^2)*x*A*b^3-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*A-10
/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)
^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2
)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*c^3-10/(4*a*c-
b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c
)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(
-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*c^3+1/4/a*c/(4*a*c
-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))
)^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*
a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*b^3+1/4/a*c/(4*a*c-b^
2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^
(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(
4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*B*b^3+44/(-(4*a*c-b^2)
^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/
2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)
/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*b
^3*c^3+5/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^
3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*
c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*
a*c-b^2)*c)^(1/2))*B*b^4*c^2-44/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2
)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1
/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*
c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*b^3*c^3-48*a^2/(-(4*a*c-b
^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4
*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)
/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*c^
4-3/4/a^2*c/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*
a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/
((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*b^4
-3/4/a^2*c/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1
/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2
)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*
b^4+48*a^2/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+
b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*
a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(
```

```
4*a*c-b^2)*c)^(1/2))*B*c^4+11/2/a/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+
b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*
a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(
4*a*c-b^2)*c)^(1/2))*A*c^2*b^2-5/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^
2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(
1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b
*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*b^4*c^2+10/a/(-(4*a*c-b^
2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*
a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/
((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*b^5
*c^2+28*a/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)
*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^
2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)
^(1/2)))^(1/2))*B*c^3*b^2-64*a/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)
*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1
/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*
a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*b*c^4-10/a/(-(4*a*c-b^
2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(
1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/
2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A
*b^5*c^2-28*a/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b
*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(
-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2)
)*(4*a*c-b^2)*c)^(1/2))*B*c^3*b^2+64*a/(-(4*a*c-b^2)^3)^(1/2)/(4*
a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/
2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*
(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*b*c^4-1/4/a*c/(-(4
*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^
2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*
x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(
1/2))*B*b^6+1/4/a*c/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((
4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1
/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a
*c-b^2)^3)^(1/2)))^(1/2))*B*b^6-3/4/a^2*c/(-(4*a*c-b^2)^3)^(1/2)/
(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^
(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)
*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*b^7+3/4/a^2*c/(
-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c
-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*
c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c
)^(1/2))*A*b^7+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*B*c-3/2/a/(c*x^4+b
*x^2+a)/(4*a*c-b^2)*x*A*b*c+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)
*x^3*A*b^2-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*b*B-1/(4*a*c-b
^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^
(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*
b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*b*c^2-1/(4*a*c-b^2)*2^(
1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*
arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*B*b*c^2-A/a^2/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(10\,Aac^2+\left(Bab-3\,Ab^2\right)c\right)x^4-2\,Aab^2+8\,Aa^2c+\left(Bab^2-3\,Ab^3-\left(2\,Ba^2-11\,Aab\right)c\right)x^2}{2\left((a^2b^2c-4\,a^3c^2)x^5+(a^2b^3-4\,a^3bc)x^3+(a^3b^2-4\,a^4c)x\right)}+\frac{\int\frac{Bab^2-3\,Ab^3+\left(10\,Aac^2+\left(Bab-3\,Ab^2\right)c\right)x^2-\left(6\,Ba^2-13\,Aab\right)c}{cx^4+bx^2+a}\,dx}{2\left(a^2b^2-4\,a^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^2*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*((10*A*a*c^2 + (B*a*b - 3*A*b^2)*c)*x^4 - 2*A*a*b^2 + 8*A*a^2
*c + (B*a*b^2 - 3*A*b^3 - (2*B*a^2 - 11*A*a*b)*c)*x^2)/((a^2*b^2*
c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4
*c)*x) + 1/2*integrate((B*a*b^2 - 3*A*b^3 + (10*A*a*c^2 + (B*a*b
- 3*A*b^2)*c)*x^2 - (6*B*a^2 - 13*A*a*b)*c)/(c*x^4 + b*x^2 + a),
x)/(a^2*b^2 - 4*a^3*c)
```

Fricas [A] time = 4.25768, size = 10237, normalized size = 26.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^2*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(10*A*a*c^2 + (B*a*b - 3*A*b^2)*c)*x^4 - 4*A*a*b^2 + 16*A*
a^2*c + 2*(B*a*b^2 - 3*A*b^3 - (2*B*a^2 - 11*A*a*b)*c)*x^2 - sqrt
(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 +
(a^3*b^2 - 4*a^4*c)*x)*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b
^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*
a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4
 + 21*A^2*a*b^5)*c + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 6
4*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*
b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*
B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 -
264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017
*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2
*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^
6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log((2500*A^4*a^3*c^6 + 6
25*(4*A^3*B*a^3*b - 9*A^4*a^2*b^2)*c^5 - 3*(108*B^4*a^5 - 756*A*B
^3*a^4*b + 1672*A^2*B^2*a^3*b^2 - 909*A^3*B*a^2*b^3 - 657*A^4*a*b
^4)*c^4 + (81*B^4*a^4*b^2 - 647*A*B^3*a^3*b^3 + 1674*A^2*B^2*a^2*
b^4 - 1323*A^3*B*a*b^5 - 189*A^4*b^6)*c^3 - 5*(B^4*a^3*b^4 - 9*A*
B^3*a^2*b^5 + 27*A^2*B^2*a*b^6 - 27*A^3*B*b^7)*c^2)*x + 1/2*sqrt(
```

```
1/2)*(B^3*a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2*B*a*b^10 - 27*A^3*b^
11 - 400*(6*A^2*B*a^6 - 13*A^3*a^5*b)*c^5 + 8*(108*B^3*a^7 - 762*
A*B^2*a^6*b + 1956*A^2*B*a^5*b^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B
^3*a^6*b^2 - 4968*A*B^2*a^5*b^3 + 12414*A^2*B*a^4*b^4 - 10549*A^3
*a^3*b^5)*c^3 + 5*(38*B^3*a^5*b^4 - 297*A*B^2*a^4*b^5 + 771*A^2*B
*a^3*b^6 - 666*A^3*a^2*b^7)*c^2 - (23*B^3*a^4*b^6 - 192*A*B^2*a^3
*b^7 + 531*A^2*B*a^2*b^8 - 486*A^3*a*b^9)*c - (B*a^6*b^9 - 3*A*a^
5*b^10 + 1280*A*a^10*c^5 + 128*(4*B*a^10*b - 17*A*a^9*b^2)*c^4 -
448*(B*a^9*b^3 - 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^5 - 49*A*a^7*b^
6)*c^2 - 5*(4*B*a^7*b^7 - 11*A*a^6*b^8)*c)*sqrt((B^4*a^4*b^4 - 12
*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^
8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4
*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4
*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*
b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5
+ 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 -
 64*a^13*c^3)))*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60
*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2
 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A
^2*a*b^5)*c + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c
^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 1
08*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5
 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B
^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^
2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^
3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11
*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c
 + 48*a^7*b^2*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3
*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sq
rt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^
2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)
*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c + (a^5
*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*
b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 8
1*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b
+ 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2
*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*
B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*
a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b
^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
- 64*a^8*c^3))*log((2500*A^4*a^3*c^6 + 625*(4*A^3*B*a^3*b - 9*A^4
*a^2*b^2)*c^5 - 3*(108*B^4*a^5 - 756*A*B^3*a^4*b + 1672*A^2*B^2*a
^3*b^2 - 909*A^3*B*a^2*b^3 - 657*A^4*a*b^4)*c^4 + (81*B^4*a^4*b^2
 - 647*A*B^3*a^3*b^3 + 1674*A^2*B^2*a^2*b^4 - 1323*A^3*B*a*b^5 -
189*A^4*b^6)*c^3 - 5*(B^4*a^3*b^4 - 9*A*B^3*a^2*b^5 + 27*A^2*B^2*
a*b^6 - 27*A^3*B*b^7)*c^2)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^8 - 9*A*B
^2*a^2*b^9 + 27*A^2*B*a*b^10 - 27*A^3*b^11 - 400*(6*A^2*B*a^6 - 1
3*A^3*a^5*b)*c^5 + 8*(108*B^3*a^7 - 762*A*B^2*a^6*b + 1956*A^2*B*
a^5*b^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B^3*a^6*b^2 - 4968*A*B^2*a
^5*b^3 + 12414*A^2*B*a^4*b^4 - 10549*A^3*a^3*b^5)*c^3 + 5*(38*B^3
*a^5*b^4 - 297*A*B^2*a^4*b^5 + 771*A^2*B*a^3*b^6 - 666*A^3*a^2*b^
7)*c^2 - (23*B^3*a^4*b^6 - 192*A*B^2*a^3*b^7 + 531*A^2*B*a^2*b^8
- 486*A^3*a*b^9)*c - (B*a^6*b^9 - 3*A*a^5*b^10 + 1280*A*a^10*c^5
+ 128*(4*B*a^10*b - 17*A*a^9*b^2)*c^4 - 448*(B*a^9*b^3 - 3*A*a^8*
```

```
b^4)*c^3 + 8*(18*B*a^8*b^5 - 49*A*a^7*b^6)*c^2 - 5*(4*B*a^7*b^7 -
 11*A*a^6*b^8)*c)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B
^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*
(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4
*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3
 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 +
396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10
*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(B^
2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b
)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 -
5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c + (a^5*b^6 -
12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 1
2*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b
^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^
4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^
4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5
*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5
 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2
- 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^
8*c^3))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*
a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(B^2*a^2*b^5 - 6*A*B*
a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*
a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 -
 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c - (a^5*b^6 - 12*a^6*b^4*c + 48*
a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 +
54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c
^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3
*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B
*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^
4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*
c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log((2500*
A^4*a^3*c^6 + 625*(4*A^3*B*a^3*b - 9*A^4*a^2*b^2)*c^5 - 3*(108*B^
4*a^5 - 756*A*B^3*a^4*b + 1672*A^2*B^2*a^3*b^2 - 909*A^3*B*a^2*b^
3 - 657*A^4*a*b^4)*c^4 + (81*B^4*a^4*b^2 - 647*A*B^3*a^3*b^3 + 16
74*A^2*B^2*a^2*b^4 - 1323*A^3*B*a*b^5 - 189*A^4*b^6)*c^3 - 5*(B^4
*a^3*b^4 - 9*A*B^3*a^2*b^5 + 27*A^2*B^2*a*b^6 - 27*A^3*B*b^7)*c^2
)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2*B*a*b
^10 - 27*A^3*b^11 - 400*(6*A^2*B*a^6 - 13*A^3*a^5*b)*c^5 + 8*(108
*B^3*a^7 - 762*A*B^2*a^6*b + 1956*A^2*B*a^5*b^2 - 1801*A^3*a^4*b^
3)*c^4 - (672*B^3*a^6*b^2 - 4968*A*B^2*a^5*b^3 + 12414*A^2*B*a^4*
b^4 - 10549*A^3*a^3*b^5)*c^3 + 5*(38*B^3*a^5*b^4 - 297*A*B^2*a^4*
b^5 + 771*A^2*B*a^3*b^6 - 666*A^3*a^2*b^7)*c^2 - (23*B^3*a^4*b^6
- 192*A*B^2*a^3*b^7 + 531*A^2*B*a^2*b^8 - 486*A^3*a*b^9)*c + (B*a
^6*b^9 - 3*A*a^5*b^10 + 1280*A*a^10*c^5 + 128*(4*B*a^10*b - 17*A*
a^9*b^2)*c^4 - 448*(B*a^9*b^3 - 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^
5 - 49*A*a^7*b^6)*c^2 - 5*(4*B*a^7*b^7 - 11*A*a^6*b^8)*c)*sqrt((B
^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*
b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B
*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b +
968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2
- 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702
*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48
*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 +
```

```
 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b -
 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B
*a^2*b^4 + 21*A^2*a*b^5)*c - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*
B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50
*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^
4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^
3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 +
 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^1
0*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6
 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^
2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 -
 4*a^4*c)*x)*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4
*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 +
77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*
a*b^5)*c - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)
*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*
A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 -
44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*
a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b
^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b
^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^
4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c +
48*a^7*b^2*c^2 - 64*a^8*c^3))*log((2500*A^4*a^3*c^6 + 625*(4*A^3*
B*a^3*b - 9*A^4*a^2*b^2)*c^5 - 3*(108*B^4*a^5 - 756*A*B^3*a^4*b +
 1672*A^2*B^2*a^3*b^2 - 909*A^3*B*a^2*b^3 - 657*A^4*a*b^4)*c^4 +
(81*B^4*a^4*b^2 - 647*A*B^3*a^3*b^3 + 1674*A^2*B^2*a^2*b^4 - 1323
*A^3*B*a*b^5 - 189*A^4*b^6)*c^3 - 5*(B^4*a^3*b^4 - 9*A*B^3*a^2*b^
5 + 27*A^2*B^2*a*b^6 - 27*A^3*B*b^7)*c^2)*x - 1/2*sqrt(1/2)*(B^3*
a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2*B*a*b^10 - 27*A^3*b^11 - 400*(
6*A^2*B*a^6 - 13*A^3*a^5*b)*c^5 + 8*(108*B^3*a^7 - 762*A*B^2*a^6*
b + 1956*A^2*B*a^5*b^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B^3*a^6*b^2
 - 4968*A*B^2*a^5*b^3 + 12414*A^2*B*a^4*b^4 - 10549*A^3*a^3*b^5)*
c^3 + 5*(38*B^3*a^5*b^4 - 297*A*B^2*a^4*b^5 + 771*A^2*B*a^3*b^6 -
 666*A^3*a^2*b^7)*c^2 - (23*B^3*a^4*b^6 - 192*A*B^2*a^3*b^7 + 531
*A^2*B*a^2*b^8 - 486*A^3*a*b^9)*c + (B*a^6*b^9 - 3*A*a^5*b^10 + 1
280*A*a^10*c^5 + 128*(4*B*a^10*b - 17*A*a^9*b^2)*c^4 - 448*(B*a^9
*b^3 - 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^5 - 49*A*a^7*b^6)*c^2 - 5
*(4*B*a^7*b^7 - 11*A*a^6*b^8)*c)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3
*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^
4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*
c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 159
6*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A
*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*
a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c
^3)))*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^
4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*
a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*
c - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((
B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a
*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*
B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b +
 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2
 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 70
```

```
2*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 4
8*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*
b^2*c^2 - 64*a^8*c^3))))/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3
- 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^2*x^2),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.123 $\int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=522

$$-\frac{aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)}{2a^3x\left(b^2-4ac\right)}-\frac{-14aAc-3abB+5Ab^2}{6a^2x^3\left(b^2-4ac\right)}$$
$$-\frac{\sqrt{c}\left(aB\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-A\left(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\left(aB\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-A\left(28a^2c^2-29ab^2c+19abc\sqrt{b^2-4ac}-5b^3\sqrt{b^2-4ac}+5b^4\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$
$$-\frac{-A\left(b^2-2ac\right)+cx^2(-(Ab-2aB))+abB}{2ax^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -(5*A*b^2 - 3*a*b*B - 14*a*A*c)/(6*a^2*(b^2 - 4*a*c)*x^3) - (a*B*
(3*b^2 - 10*a*c) - A*(5*b^3 - 19*a*b*c))/(2*a^3*(b^2 - 4*a*c)*x)
- (a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a
*c)*x^3*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(a*B*(3*b^3 - 16*a*b*c +
3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]) - A*(5*b^4 -
29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt
[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) + (Sqrt[c]*(a*B*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c
] + 10*a*c*Sqrt[b^2 - 4*a*c]) - A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^
2 - 5*b^3*Sqrt[b^2 - 4*a*c] + 19*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3
*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 2.92715, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$-\frac{aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)}{2a^3x\left(b^2-4ac\right)}-\frac{-14aAc-3abB+5Ab^2}{6a^2x^3\left(b^2-4ac\right)}$$
$$-\frac{\sqrt{c}\left(aB\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-A\left(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\left(aB\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-A\left(28a^2c^2-29ab^2c+19abc\sqrt{b^2-4ac}-5b^3\sqrt{b^2-4ac}+5b^4\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{2ax^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(5*A*b^2 - 3*a*b*B - 14*a*A*c)/(6*a^2*(b^2 - 4*a*c)*x^3) - (a*B*
(3*b^2 - 10*a*c) - A*(5*b^3 - 19*a*b*c))/(2*a^3*(b^2 - 4*a*c)*x)
+ (A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a
*c)*x^3*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(a*B*(3*b^3 - 16*a*b*c +
3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]) - A*(5*b^4 -
29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt
[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) + (Sqrt[c]*(a*B*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c
] + 10*a*c*Sqrt[b^2 - 4*a*c]) - A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^
2 - 5*b^3*Sqrt[b^2 - 4*a*c] + 19*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3
*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.40301, size = 487, normalized size = 0.93

$$\frac{\frac{6x\left(A\left(2a^2c^2-4ab^2c-3abc^2x^2+b^4+b^3cx^2\right)+aB\left(3abc+2ac^2x^2-b^3-b^2cx^2\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{3\sqrt{2}\sqrt{c}\left(A\left(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4\right)+aB\left(-3b^2\sqrt{b^2\cdots}\right.\right.}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4\cdots}}}}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*a*A)/x^3 + (24*A*b - 12*a*B)/x + (6*x*(a*B*(-b^3 + 3*a*b*c -
 b^2*c*x^2 + 2*a*c^2*x^2) + A*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*
c*x^2 - 3*a*b*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3
*Sqrt[2]*Sqrt[c]*(a*B*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c
] + 10*a*c*Sqrt[b^2 - 4*a*c]) + A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^
```

```
2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)
^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(a*B*(-3
*b^3 + 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a
*c]) + A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - 5*b^3*Sqrt[b^2 - 4*a*
c] + 19*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
- 4*a*c]]))/(12*a^3)
```

Maple [B] time = 0.083, size = 4401, normalized size = 8.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -3/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b*B*c+3/2/a^2/(c*x^4+b*x^2+a
)*c^2/(4*a*c-b^2)*x^3*A*b+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x
^3*B*b^2+2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*A*b^2*c+44/(-(4*a*c-
b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)
^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(
1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))
*B*b^3*c^3+172/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*
b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*
(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2
))*(4*a*c-b^2)*c)^(1/2))*A*c^4*b^2-172/(-(4*a*c-b^2)^3)^(1/2)/(4*
a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/
2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*
(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*c^4*b^2-44/(-(4*a*
c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(
-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1
/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B
*b^3*c^3-39/4/a^2/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3
+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^
(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))
*A*b^3*c^2-3/4/a^2*c/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-
b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x
*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/
2))*B*b^4+11/2/a/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+
(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(
1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*
B*c^2*b^2-112*a/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a
*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2
*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/
2))*(4*a*c-b^2)*c)^(1/2))*A*c^5+19/a/(4*a*c-b^2)*2^(1/2)/((-4*a*b
*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(
-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2)
)*(4*a*c-b^2)*c)^(1/2))*A*b*c^3-39/4/a^2/(4*a*c-b^2)*2^(1/2)/((-4
```

*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1
/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(
1/2))*(4*a*c-b^2)*c)^(1/2))*A*b^3*c^2+19/a/(4*a*c-b^2)*2^(1/2)/((
4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1
/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a
*c-b^2)^3)^(1/2)))^(1/2))*A*b*c^3+112*a/(-(4*a*c-b^2)^3)^(1/2)/(4
*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1
/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*c^5+5/4/a^3*c/(4*
a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/
2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*
(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*b^5+5/4/a^3*c/(4*a
*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2
)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^
3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*A*b^5+11/2/a/(4*a
*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2
)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^
3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*B*c^2*b^2-3/4/a^2
*c/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*
a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a
*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*B*b^4-85/a
/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a
c-b^2)^3)^(1/2))(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^
2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)
*c)^(1/2))*A*b^4*c^3+85/a/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1
/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*ar
ctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+
(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*b^4*c^3-69/4/a^2/(-(4*a*c-b^2)^
3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c
-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4
*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*b^6*c^
2+64*a/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*
(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2
*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1
/2)))^(1/2))*B*b*c^4+10/a/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1
/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*ar
ctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+
(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*b^5*c^2-3/4/a^2*c/(-(4*a*c-b^2)
^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*
c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((
4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*B*b^7+3
/4/a^2*c/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^
3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*
c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*
a*c-b^2)*c)^(1/2))*B*b^7-10/a/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*
2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/
2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a
c-b^2)^3)^(1/2))(4*a*c-b^2)*c)^(1/2))*B*b^5*c^2+69/4/a^2/(-(4*a
*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)
^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*
2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/
2))*A*b^6*c^2-64*a/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-
4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(
1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^

```
(1/2))*(4*a*c-b^2)*c)^(1/2))*B*b*c^4+5/4/a^3*c/(-(4*a*c-b^2)^3)^(
1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2
)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c
-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*A*b^8-5/4/a^
3*c/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(
4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2
*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b
^2)*c)^(1/2))*A*b^8-1/2/a^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*A*b^4-1
/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*B-1/a/(c*x^4+b*x^2+a)/(4*a
*c-b^2)*x*A*c^2+1/2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*B*b^3-1/2/a
^3/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*A*b^3+2/a^3/x*A*b-1/3*A/a^2/
x^3-1/a^2/x*B-10/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+
(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(
1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*
B*c^3-10/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2
))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/
((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*B*c^
3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(\left(10\,Ba^2 - 19\,Aab\right)c^2 - \left(3\,Bab^2 - 5\,Ab^3\right)c\right)x^6 - 2\,Aa^2b^2 + 8\,Aa^3c - \left(9\,Bab^3 - 15\,Ab^4 - 14\,Aa^2c^2 - \left(33\,Ba^2b - 62\,Aab^2\right)c}{6\left((a^3b^2c - 4\,a^4c^2)x^7 + (a^3b^3 - 4\,a^4bc)x^5 + (a^4b^2 - 4\,a^5c)x^3\right)}$$
$$- \frac{\int \frac{3\,Bab^3 - 5\,Ab^4 - 14\,Aa^2c^2 - \left(\left(10\,Ba^2 - 19\,Aab\right)c^2 - \left(3\,Bab^2 - 5\,Ab^3\right)c\right)x^2 - \left(13\,Ba^2b - 24\,Aab^2\right)c}{cx^4 + bx^2 + a}\,dx}{2\left(a^3b^2 - 4\,a^4c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^2*x^4),x, algorithm="maxima")
```

```
[Out] 1/6*(3*((10*B*a^2 - 19*A*a*b)*c^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^6
- 2*A*a^2*b^2 + 8*A*a^3*c - (9*B*a*b^3 - 15*A*b^4 - 14*A*a^2*c^2
- (33*B*a^2*b - 62*A*a*b^2)*c)*x^4 - 2*(3*B*a^2*b^2 - 5*A*a*b^3 -
 4*(3*B*a^3 - 5*A*a^2*b)*c)*x^2)/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (
a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) - 1/2*integra
te((3*B*a*b^3 - 5*A*b^4 - 14*A*a^2*c^2 - ((10*B*a^2 - 19*A*a*b)*c
^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^2 - (13*B*a^2*b - 24*A*a*b^2)*c)/
(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)
```

Fricas [A] time = 10.3564, size = 13757, normalized size = 26.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^2*x^4),x, algorithm="fricas")
```

```
[Out] 1/12*(6*((10*B*a^2 - 19*A*a*b)*c^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^6
 - 4*A*a^2*b^2 + 16*A*a^3*c - 2*(9*B*a*b^3 - 15*A*b^4 - 14*A*a^2*
c^2 - (33*B*a^2*b - 62*A*a*b^2)*c)*x^4 - 4*(3*B*a^2*b^2 - 5*A*a*b
^3 - 4*(3*B*a^3 - 5*A*a^2*b)*c)*x^2 - 3*sqrt(1/2)*((a^3*b^2*c - 4
*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x
^3)*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B
*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*
A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*
a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)
*c + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt
((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 15
00*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^
2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9
300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 +
76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^
3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b
^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^
2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459
*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A
^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48
*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b
^2*c^2 - 64*a^10*c^3))*log((9604*A^4*a^4*c^8 + 7203*(4*A^3*B*a^4*
b - 7*A^4*a^3*b^2)*c^7 - (2500*B^4*a^6 - 22500*A*B^3*a^5*b + 4352
4*A^2*B^2*a^4*b^2 + 4343*A^3*B*a^3*b^3 - 43410*A^4*a^2*b^4)*c^6 +
 (5625*B^4*a^5*b^2 - 31137*A*B^3*a^4*b^3 + 52821*A^2*B^2*a^3*b^4
- 20190*A^3*B*a^2*b^5 - 12325*A^4*a*b^6)*c^5 - 3*(657*B^4*a^4*b^4
 - 3351*A*B^3*a^3*b^5 + 5560*A^2*B^2*a^2*b^6 - 2775*A^3*B*a*b^7 -
 375*A^4*b^8)*c^4 + 7*(27*B^4*a^3*b^6 - 135*A*B^3*a^2*b^7 + 225*A
^2*B^2*a*b^8 - 125*A^3*B*b^9)*c^3)*x + 1/2*sqrt(1/2)*(27*B^3*a^3*
b^11 - 135*A*B^2*a^2*b^12 + 225*A^2*B*a*b^13 - 125*A^3*b^14 + 109
76*A^3*a^7*c^7 - 112*(50*A*B^2*a^8 - 463*A^2*B*a^7*b + 709*A^3*a^
6*b^2)*c^6 - 2*(2600*B^3*a^8*b - 31256*A*B^2*a^7*b^2 + 96044*A^2*
B*a^6*b^3 - 86495*A^3*a^5*b^4)*c^5 + (14408*B^3*a^7*b^3 - 101006*
A*B^2*a^6*b^4 + 224705*A^2*B*a^5*b^5 - 160932*A^3*a^4*b^6)*c^4 -
7*(1507*B^3*a^6*b^5 - 8820*A*B^2*a^5*b^6 + 16991*A^2*B*a^4*b^7 -
10797*A^3*a^3*b^8)*c^3 + (3330*B^3*a^5*b^7 - 17889*A*B^2*a^4*b^8
+ 31929*A^2*B*a^3*b^9 - 18940*A^3*a^2*b^10)*c^2 - (486*B^3*a^4*b^
9 - 2493*A*B^2*a^3*b^10 + 4260*A^2*B*a^2*b^11 - 2425*A^3*a*b^12)*
c - (3*B*a^8*b^10 - 5*A*a^7*b^11 - 256*(5*B*a^13 - 13*A*a^12*b)*c
^5 + 64*(34*B*a^12*b^2 - 73*A*a^11*b^3)*c^4 - 112*(12*B*a^11*b^4
- 23*A*a^10*b^5)*c^3 + 28*(14*B*a^10*b^6 - 25*A*a^9*b^7)*c^2 - (5
5*B*a^9*b^8 - 94*A*a^8*b^9)*c)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a
^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12
 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*
A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^
2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(12
75*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77
424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4
- 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^
7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*
b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10
```

```
)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))
*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^
5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2
*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2
*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c
+ (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((8
1*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*
A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a
^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300
*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 766
86*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 +
 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)
*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a
^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^
4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*
B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^
16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*
c^2 - 64*a^10*c^3))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 +
 (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^
2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^
4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^
3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 -
7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c + (a^7*b^6 -
 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^
8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11
 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3
*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b
 + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b
^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B
^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(10
17*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 282
60*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 -
3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 +
4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 -
 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^1
0*c^3))*log((9604*A^4*a^4*c^8 + 7203*(4*A^3*B*a^4*b - 7*A^4*a^3*b
^2)*c^7 - (2500*B^4*a^6 - 22500*A*B^3*a^5*b + 43524*A^2*B^2*a^4*b
^2 + 4343*A^3*B*a^3*b^3 - 43410*A^4*a^2*b^4)*c^6 + (5625*B^4*a^5*
b^2 - 31137*A*B^3*a^4*b^3 + 52821*A^2*B^2*a^3*b^4 - 20190*A^3*B*a
^2*b^5 - 12325*A^4*a*b^6)*c^5 - 3*(657*B^4*a^4*b^4 - 3351*A*B^3*a
^3*b^5 + 5560*A^2*B^2*a^2*b^6 - 2775*A^3*B*a*b^7 - 375*A^4*b^8)*c
^4 + 7*(27*B^4*a^3*b^6 - 135*A*B^3*a^2*b^7 + 225*A^2*B^2*a*b^8 -
125*A^3*B*b^9)*c^3)*x - 1/2*sqrt(1/2)*(27*B^3*a^3*b^11 - 135*A*B^
2*a^2*b^12 + 225*A^2*B*a*b^13 - 125*A^3*b^14 + 10976*A^3*a^7*c^7
- 112*(50*A*B^2*a^8 - 463*A^2*B*a^7*b + 709*A^3*a^6*b^2)*c^6 - 2*
(2600*B^3*a^8*b - 31256*A*B^2*a^7*b^2 + 96044*A^2*B*a^6*b^3 - 864
95*A^3*a^5*b^4)*c^5 + (14408*B^3*a^7*b^3 - 101006*A*B^2*a^6*b^4 +
 224705*A^2*B*a^5*b^5 - 160932*A^3*a^4*b^6)*c^4 - 7*(1507*B^3*a^6
*b^5 - 8820*A*B^2*a^5*b^6 + 16991*A^2*B*a^4*b^7 - 10797*A^3*a^3*b
^8)*c^3 + (3330*B^3*a^5*b^7 - 17889*A*B^2*a^4*b^8 + 31929*A^2*B*a
^3*b^9 - 18940*A^3*a^2*b^10)*c^2 - (486*B^3*a^4*b^9 - 2493*A*B^2*
a^3*b^10 + 4260*A^2*B*a^2*b^11 - 2425*A^3*a*b^12)*c - (3*B*a^8*b^
10 - 5*A*a^7*b^11 - 256*(5*B*a^13 - 13*A*a^12*b)*c^5 + 64*(34*B*a
^12*b^2 - 73*A*a^11*b^3)*c^4 - 112*(12*B*a^11*b^4 - 23*A*a^10*b^5
```

)*c^3 + 28*(14*B*a^10*b^6 - 25*A*a^9*b^7)*c^2 - (55*B*a^9*b^8 - 9
4*A*a^8*b^9)*c)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A
^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6
c^6 - 98(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^
5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109
544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2
- 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b
^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^
5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a
^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*
B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6
- 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(9*B^2*a
^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b
)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 +
7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(
15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c + (a^7*b^6 - 12
*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 -
540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 +
625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*
a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b +
51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)
c^4 - 2(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*
a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*
B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*
A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 318
6*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 412
5*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64
*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c
^3))) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a
^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^2*a^2*b^7 - 30*
A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*
(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a
^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b
^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c - (a^7*b^6 - 12*a^8*b^4*c +
48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a
^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12
+ 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*
A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^
2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(12
75*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77
424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4
- 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^
7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*
b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10
)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))
/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log((96
04*A^4*a^4*c^8 + 7203*(4*A^3*B*a^4*b - 7*A^4*a^3*b^2)*c^7 - (2500
*B^4*a^6 - 22500*A*B^3*a^5*b + 43524*A^2*B^2*a^4*b^2 + 4343*A^3*B
*a^3*b^3 - 43410*A^4*a^2*b^4)*c^6 + (5625*B^4*a^5*b^2 - 31137*A*B
^3*a^4*b^3 + 52821*A^2*B^2*a^3*b^4 - 20190*A^3*B*a^2*b^5 - 12325*
A^4*a*b^6)*c^5 - 3*(657*B^4*a^4*b^4 - 3351*A*B^3*a^3*b^5 + 5560*A
^2*B^2*a^2*b^6 - 2775*A^3*B*a*b^7 - 375*A^4*b^8)*c^4 + 7*(27*B^4*
a^3*b^6 - 135*A*B^3*a^2*b^7 + 225*A^2*B^2*a*b^8 - 125*A^3*B*b^9)*
c^3)*x + 1/2*sqrt(1/2)*(27*B^3*a^3*b^11 - 135*A*B^2*a^2*b^12 + 22

5*A^2*B*a*b^13 - 125*A^3*b^14 + 10976*A^3*a^7*c^7 - 112*(50*A*B^2
*a^8 - 463*A^2*B*a^7*b + 709*A^3*a^6*b^2)*c^6 - 2*(2600*B^3*a^8*b
 - 31256*A*B^2*a^7*b^2 + 96044*A^2*B*a^6*b^3 - 86495*A^3*a^5*b^4)
*c^5 + (14408*B^3*a^7*b^3 - 101006*A*B^2*a^6*b^4 + 224705*A^2*B*a
^5*b^5 - 160932*A^3*a^4*b^6)*c^4 - 7*(1507*B^3*a^6*b^5 - 8820*A*B
^2*a^5*b^6 + 16991*A^2*B*a^4*b^7 - 10797*A^3*a^3*b^8)*c^3 + (3330
*B^3*a^5*b^7 - 17889*A*B^2*a^4*b^8 + 31929*A^2*B*a^3*b^9 - 18940*
A^3*a^2*b^10)*c^2 - (486*B^3*a^4*b^9 - 2493*A*B^2*a^3*b^10 + 4260
*A^2*B*a^2*b^11 - 2425*A^3*a*b^12)*c + (3*B*a^8*b^10 - 5*A*a^7*b^
11 - 256*(5*B*a^13 - 13*A*a^12*b)*c^5 + 64*(34*B*a^12*b^2 - 73*A*
a^11*b^3)*c^4 - 112*(12*B*a^11*b^4 - 23*A*a^10*b^5)*c^3 + 28*(14*
B*a^10*b^6 - 25*A*a^9*b^7)*c^2 - (55*B*a^9*b^8 - 94*A*a^8*b^9)*c)
*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10
 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A
^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^
8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b
^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a
^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*
a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A
^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2
*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9
550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c
 + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(9*B^2*a^2*b^7 - 30*A*B
*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*
B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*
b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5
- 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c - (a^7*b^6 - 12*a^8*b^4*c + 48
*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*
b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 +
2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4
*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a
^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*
B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424
*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7
872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 +
 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7
 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c
)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a
^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) + 3*sqrt(
1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (
a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*
A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b -
20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*
B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2
*b^6 + 45*A^2*a*b^7)*c - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2
 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A
^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6
c^6 - 98(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^
5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109
544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2
- 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b
^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^
5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a
^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*

```
B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6
- 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*
a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log((9604*A^4*a^4*c^8
+ 7203*(4*A^3*B*a^4*b - 7*A^4*a^3*b^2)*c^7 - (2500*B^4*a^6 - 2250
0*A*B^3*a^5*b + 43524*A^2*B^2*a^4*b^2 + 4343*A^3*B*a^3*b^3 - 4341
0*A^4*a^2*b^4)*c^6 + (5625*B^4*a^5*b^2 - 31137*A*B^3*a^4*b^3 + 52
821*A^2*B^2*a^3*b^4 - 20190*A^3*B*a^2*b^5 - 12325*A^4*a*b^6)*c^5
- 3*(657*B^4*a^4*b^4 - 3351*A*B^3*a^3*b^5 + 5560*A^2*B^2*a^2*b^6
- 2775*A^3*B*a*b^7 - 375*A^4*b^8)*c^4 + 7*(27*B^4*a^3*b^6 - 135*A
*B^3*a^2*b^7 + 225*A^2*B^2*a*b^8 - 125*A^3*B*b^9)*c^3)*x - 1/2*sq
rt(1/2)*(27*B^3*a^3*b^11 - 135*A*B^2*a^2*b^12 + 225*A^2*B*a*b^13
- 125*A^3*b^14 + 10976*A^3*a^7*c^7 - 112*(50*A*B^2*a^8 - 463*A^2*
B*a^7*b + 709*A^3*a^6*b^2)*c^6 - 2*(2600*B^3*a^8*b - 31256*A*B^2*
a^7*b^2 + 96044*A^2*B*a^6*b^3 - 86495*A^3*a^5*b^4)*c^5 + (14408*B
^3*a^7*b^3 - 101006*A*B^2*a^6*b^4 + 224705*A^2*B*a^5*b^5 - 160932
*A^3*a^4*b^6)*c^4 - 7*(1507*B^3*a^6*b^5 - 8820*A*B^2*a^5*b^6 + 16
991*A^2*B*a^4*b^7 - 10797*A^3*a^3*b^8)*c^3 + (3330*B^3*a^5*b^7 -
17889*A*B^2*a^4*b^8 + 31929*A^2*B*a^3*b^9 - 18940*A^3*a^2*b^10)*c
^2 - (486*B^3*a^4*b^9 - 2493*A*B^2*a^3*b^10 + 4260*A^2*B*a^2*b^11
 - 2425*A^3*a*b^12)*c + (3*B*a^8*b^10 - 5*A*a^7*b^11 - 256*(5*B*a
^13 - 13*A*a^12*b)*c^5 + 64*(34*B*a^12*b^2 - 73*A*a^11*b^3)*c^4 -
 112*(12*B*a^11*b^4 - 23*A*a^10*b^5)*c^3 + 28*(14*B*a^10*b^6 - 25
*A*a^9*b^7)*c^2 - (55*B*a^9*b^8 - 94*A*a^8*b^9)*c)*sqrt((81*B^4*a
^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a
*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 18
6*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*
a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*
a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*
A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 +
3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6
- 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b
^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b
^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*
c^2 - 64*a^17*c^3)))*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2
*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*
A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a
^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^
6 + 45*A^2*a*b^7)*c - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 -
64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*
B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^
6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 +
 (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544
*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 1
4086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5
+ 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b
^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*
b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2
*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 1
2*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8
*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))))/((a^3*b^2*c - 4*a^4*c^2
)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^2*x^4),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.124 $\int \frac{x^{11}\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=365

$$-\frac{x^4\left(x^2\left(20a^2Bc^2+10aAbc^2-20ab^2Bc-Ab^3c+3b^4B\right)+a\left(16aAc^2-18abBc-Ab^2c+3b^3B\right)\right)}{4c^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^2\left(30a^2Bc^2+7aAbc^2-21ab^2Bc-Ab^3c+3b^4B\right)}{2c^3\left(b^2-4ac\right)^2}-\frac{\left(-60a^3Bc^3-30a^2Abc^3+90a^2b^2Bc^2+10aAb^3c^2-30ab^4Bc-Ab^5c+3b^6B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^4\left(b^2-4ac\right)^{5/2}}-\frac{x^8\left(x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{(3bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^4}$$

```
[Out] ((3*b^4*B - A*b^3*c - 21*a*b^2*B*c + 7*a*A*b*c^2 + 30*a^2*B*c^2)*
x^2)/(2*c^3*(b^2 - 4*a*c)^2) - (x^8*(a*(b*B - 2*A*c) + (b^2*B - A
*b*c - 2*a*B*c)*x^2))/(4*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) -
 (x^4*(a*(3*b^3*B - A*b^2*c - 18*a*b*B*c + 16*a*A*c^2) + (3*b^4*B
 - A*b^3*c - 20*a*b^2*B*c + 10*a*A*b*c^2 + 20*a^2*B*c^2)*x^2))/(4
*c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((3*b^6*B - A*b^5*c -
 30*a*b^4*B*c + 10*a*A*b^3*c^2 + 90*a^2*b^2*B*c^2 - 30*a^2*A*b*c^
3 - 60*a^3*B*c^3)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^
4*(b^2 - 4*a*c)^(5/2)) - ((3*b*B - A*c)*Log[a + b*x^2 + c*x^4])/(
4*c^4)
```

Rubi [A] time = 2.7144, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28

$$-\frac{x^4\left(x^2\left(20a^2Bc^2+10aAbc^2-20ab^2Bc-Ab^3c+3b^4B\right)+a\left(16aAc^2-18abBc-Ab^2c+3b^3B\right)\right)}{4c^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^2\left(30a^2Bc^2+7aAbc^2-21ab^2Bc-Ab^3c+3b^4B\right)}{2c^3\left(b^2-4ac\right)^2}-\frac{\left(-60a^3Bc^3-30a^2Abc^3+90a^2b^2Bc^2+10aAb^3c^2-30ab^4Bc-Ab^5c+3b^6B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^4\left(b^2-4ac\right)^{5/2}}-\frac{x^8\left(x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{(3bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((3*b^4*B - A*b^3*c - 21*a*b^2*B*c + 7*a*A*b*c^2 + 30*a^2*B*c^2)*
x^2)/(2*c^3*(b^2 - 4*a*c)^2) - (x^8*(a*(b*B - 2*A*c) + (b^2*B - A
*b*c - 2*a*B*c)*x^2))/(4*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) -
 (x^4*(a*(3*b^3*B - A*b^2*c - 18*a*b*B*c + 16*a*A*c^2) + (3*b^4*B
 - A*b^3*c - 20*a*b^2*B*c + 10*a*A*b*c^2 + 20*a^2*B*c^2)*x^2))/(4
*c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((3*b^6*B - A*b^5*c -
 30*a*b^4*B*c + 10*a*A*b^3*c^2 + 90*a^2*b^2*B*c^2 - 30*a^2*A*b*c^
3 - 60*a^3*B*c^3)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^
4*(b^2 - 4*a*c)^(5/2)) - ((3*b*B - A*c)*Log[a + b*x^2 + c*x^4])/(
4*c^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.38408, size = 435, normalized size = 1.19

$$\frac{\frac{a^3c^2\left(2c\left(A+Bx^2\right)-5bB\right)+a^2bc\left(-bc\left(4A+9Bx^2\right)+5Ac^2x^2+5b^2B\right)+ab^3\left(bc\left(A+6Bx^2\right)-5Ac^2x^2+b^2(-B)\right)+b^5x^2(Ac-bB)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{2c\left(60a^3Bc^3+30a^2Abc^3-90a^2b^2Bc^2-\cdots\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*B*c^2*x^2 + (b^7*B - b^6*c*(A + 6*B*x^2) + 4*a^3*c^4*(8*A + 9*
B*x^2) - 3*a^2*b^2*c^3*(13*A + 34*B*x^2) + a*b^4*c^2*(11*A + 48*B
*x^2) + a*b^3*c^2*(61*a*B - 30*A*c*x^2) + 2*b^5*c*(-7*a*B + 2*A*c
*x^2) + 2*a^2*b*c^3*(-39*a*B + 25*A*c*x^2))/((b^2 - 4*a*c)^2*(a +
 b*x^2 + c*x^4)) + (b^5*(-(b*B) + A*c)*x^2 + a^3*c^2*(-5*b*B + 2*
c*(A + B*x^2)) + a*b^3*(-(b^2*B) - 5*A*c^2*x^2 + b*c*(A + 6*B*x^2
)) + a^2*b*c*(5*b^2*B + 5*A*c^2*x^2 - b*c*(4*A + 9*B*x^2)))/((b^2
 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (2*c*(-3*b^6*B + A*b^5*c + 30*
a*b^4*B*c - 10*a*A*b^3*c^2 - 90*a^2*b^2*B*c^2 + 30*a^2*A*b*c^3 +
60*a^3*B*c^3)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4
*a*c)^(5/2) + c*(-3*b*B + A*c)*Log[a + b*x^2 + c*x^4])/(4*c^5)
```

Maple [B] time = 0.034, size = 2916, normalized size = 8.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 7/c/(c*x^4+b*x^2+a)^2*a^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B-21/4/c
^2/(c*x^4+b*x^2+a)^2*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^2-29/2/c^
2/(c*x^4+b*x^2+a)^2*a^4/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b-5/4/c^4/(c
*x^4+b*x^2+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b^5-3/2/c^3/(c*x
^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*B*b^6+9/(c*x^4+b*x^2
+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*B*a^3+8/(c*x^4+b*x^2+a)^2/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*a^3-51/2/c/(c*x^4+b*x^2+a)^2/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^6*B*a^2*b^2+12/c^2/(c*x^4+b*x^2+a)^2/(16*
a^2*c^2-8*a*b^2*c+b^4)*x^6*B*a*b^4+1/c^2/(c*x^4+b*x^2+a)^2/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^6*A*b^5-5/4/c^4/(c*x^4+b*x^2+a)^2/(16*a^2*
c^2-8*a*b^2*c+b^4)*x^4*B*b^7+11/4/c/(c*x^4+b*x^2+a)^2/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^4*A*a^2*b^2-19/4/c^2/(c*x^4+b*x^2+a)^2/(16*a^2*
c^2-8*a*b^2*c+b^4)*x^4*A*a*b^4-21/2/c/(c*x^4+b*x^2+a)^2/(16*a^2*c
^2-8*a*b^2*c+b^4)*x^4*B*a^3*b-41/4/c^2/(c*x^4+b*x^2+a)^2/(16*a^2*
c^2-8*a*b^2*c+b^4)*x^4*B*a^2*b^3+31/2/c/(c*x^4+b*x^2+a)^2*a^3/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b-11/c^2/(c*x^4+b*x^2+a)^2*a^2/(16*
a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^3-71/2/c^2/(c*x^4+b*x^2+a)^2*a^3/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b^2-5/2/c^4/(c*x^4+b*x^2+a)^2*a/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b^6+17/2/c^3/(c*x^4+b*x^2+a)^2/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^4*B*a*b^5+3/2/c^3/(c*x^4+b*x^2+a)^2*a/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^5+3/4/c^3/(c*x^4+b*x^2+a)^2/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b^6+3/4/c^3/(c*x^4+b*x^2+a)^2*a^2/
(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^4+9/c^3/(c*x^4+b*x^2+a)^2*a^3/(16*
a^2*c^2-8*a*b^2*c+b^4)*B*b^3-15/c/(1024*a^5*c^5-1280*a^4*b^2*c^4+
640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2)*arctan((2*
c*x^2*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1
024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a
*b^8*c-b^10)^(1/2))*A*a^2*b+5/c^2/(1024*a^5*c^5-1280*a^4*b^2*c^4+
640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2)*arctan((2*
c*x^2*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1
024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a
*b^8*c-b^10)^(1/2))*A*a*b^3+45/c^2/(1024*a^5*c^5-1280*a^4*b^2*c^4
+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2)*arctan((2
*c*x^2*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(
1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*
a*b^8*c-b^10)^(1/2))*B*a^2*b^2-2/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*l
n((16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^4+b*x^2+a))*A*a*b^2-12/c^2/(16*
a^2*c^2-8*a*b^2*c+b^4)*ln((16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^4+b*x^2
+a))*B*a^2*b+6/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln((16*a^2*c^2-8*a*
b^2*c+b^4)*(c*x^4+b*x^2+a))*B*a*b^3-15/c^3/(1024*a^5*c^5-1280*a^4
*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2)*a
rctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^2*c+b
^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6
*c^2+20*a*b^8*c-b^10)^(1/2))*B*a*b^4+25/2/(c*x^4+b*x^2+a)^2/(16*a
```

```
^2*c^2-8*a*b^2*c+b^4)*x^6*A*a^2*b+1/2*B*x^2/c^3+19/c^3/(c*x^4+b*x
^2+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b^4-15/2/c/(c*x^4+b*
x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*a*b^3+6/c/(c*x^4+b*x^2+
a)^2*a^4/(16*a^2*c^2-8*a*b^2*c+b^4)*A-1/2/c^3/(1024*a^5*c^5-1280*
a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2
)*arctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^2*
c+b^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*
b^6*c^2+20*a*b^8*c-b^10)^(1/2))*A*b^5-30/c/(1024*a^5*c^5-1280*a^4
*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2)*a
rctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^2*c+b
^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6
*c^2+20*a*b^8*c-b^10)^(1/2))*a^3*B+3/2/c^4/(1024*a^5*c^5-1280*a^4
*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2)*a
rctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^2*c+b
^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6
*c^2+20*a*b^8*c-b^10)^(1/2))*b^6*B+4/c/(16*a^2*c^2-8*a*b^2*c+b^4)
*ln((16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^4+b*x^2+a))*A*a^2+1/4/c^3/(16
*a^2*c^2-8*a*b^2*c+b^4)*ln((16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^4+b*x^
2+a))*A*b^4-3/4/c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*ln((16*a^2*c^2-8*a
*b^2*c+b^4)*(c*x^4+b*x^2+a))*B*b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^11/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.605216, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^11/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*((3*B*a^2*b^6 + (3*B*b^6*c^2 - 30*(2*B*a^3 + A*a^2*b)*c^5 +
 10*(9*B*a^2*b^2 + A*a*b^3)*c^4 - (30*B*a*b^4 + A*b^5)*c^3)*x^8 +
 2*(3*B*b^7*c - 30*(2*B*a^3*b + A*a^2*b^2)*c^4 + 10*(9*B*a^2*b^3
+ A*a*b^4)*c^3 - (30*B*a*b^5 + A*b^6)*c^2)*x^6 + (3*B*b^8 - 60*(2
*B*a^4 + A*a^3*b)*c^4 + 10*(12*B*a^3*b^2 - A*a^2*b^3)*c^3 + 2*(15
*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (24*B*a*b^6 + A*b^7)*c)*x^4 - 30*(2
```

*B*a^5 + A*a^4*b)*c^3 + 10*(9*B*a^4*b^2 + A*a^3*b^3)*c^2 + 2*(3*B
*a*b^7 - 30*(2*B*a^4*b + A*a^3*b^2)*c^3 + 10*(9*B*a^3*b^3 + A*a^2
*b^4)*c^2 - (30*B*a^2*b^5 + A*a*b^6)*c)*x^2 - (30*B*a^3*b^4 + A*a
^2*b^5)*c)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*
x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
+ a)) - (2*(B*b^4*c^3 - 8*B*a*b^2*c^4 + 16*B*a^2*c^5)*x^10 + 4*(B
*b^5*c^2 - 8*B*a*b^3*c^3 + 16*B*a^2*b*c^4)*x^8 - 5*B*a^2*b^5 + 24
*A*a^4*c^3 - 2*(2*B*b^6*c - 25*(2*B*a^3 + A*a^2*b)*c^4 + 3*(17*B*
a^2*b^2 + 5*A*a*b^3)*c^3 - 2*(9*B*a*b^4 + A*b^5)*c^2)*x^6 - (5*B*
b^7 - 32*A*a^3*c^4 - 11*(2*B*a^3*b + A*a^2*b^2)*c^3 + (73*B*a^2*b
^3 + 19*A*a*b^4)*c^2 - (38*B*a*b^5 + 3*A*b^6)*c)*x^4 - (58*B*a^4*
b + 21*A*a^3*b^2)*c^2 - 2*(5*B*a*b^6 - (30*B*a^4 + 31*A*a^3*b)*c^
3 + (79*B*a^3*b^2 + 22*A*a^2*b^3)*c^2 - 3*(13*B*a^2*b^4 + A*a*b^5
)*c)*x^2 + 3*(12*B*a^3*b^3 + A*a^2*b^4)*c - ((3*B*b^5*c^2 - 16*A*
a^2*c^5 + 8*(6*B*a^2*b + A*a*b^2)*c^4 - (24*B*a*b^3 + A*b^4)*c^3)
*x^8 + 3*B*a^2*b^5 - 16*A*a^4*c^3 + 2*(3*B*b^6*c - 16*A*a^2*b*c^4
 + 8*(6*B*a^2*b^2 + A*a*b^3)*c^3 - (24*B*a*b^4 + A*b^5)*c^2)*x^6
+ (3*B*b^7 + 6*A*a*b^4*c^2 + 96*B*a^3*b*c^3 - 32*A*a^3*c^4 - (18*
B*a*b^5 + A*b^6)*c)*x^4 + 8*(6*B*a^4*b + A*a^3*b^2)*c^2 + 2*(3*B*
a*b^6 - 16*A*a^3*b*c^3 + 8*(6*B*a^3*b^2 + A*a^2*b^3)*c^2 - (24*B*
a^2*b^4 + A*a*b^5)*c)*x^2 - (24*B*a^3*b^3 + A*a^2*b^4)*c)*log(c*x
^4 + b*x^2 + a))*sqrt(b^2 - 4*a*c))/((a^2*b^4*c^4 - 8*a^3*b^2*c^5
 + 16*a^4*c^6 + (b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*x^8 + 2*(b^5
*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*x^6 + (b^6*c^4 - 6*a*b^4*c^5 +
 32*a^3*c^7)*x^4 + 2*(a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x
^2)*sqrt(b^2 - 4*a*c)), 1/4*(2*(3*B*a^2*b^6 + (3*B*b^6*c^2 - 30*(
2*B*a^3 + A*a^2*b)*c^5 + 10*(9*B*a^2*b^2 + A*a*b^3)*c^4 - (30*B*a
*b^4 + A*b^5)*c^3)*x^8 + 2*(3*B*b^7*c - 30*(2*B*a^3*b + A*a^2*b^2
)*c^4 + 10*(9*B*a^2*b^3 + A*a*b^4)*c^3 - (30*B*a*b^5 + A*b^6)*c^2
)*x^6 + (3*B*b^8 - 60*(2*B*a^4 + A*a^3*b)*c^4 + 10*(12*B*a^3*b^2
- A*a^2*b^3)*c^3 + 2*(15*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (24*B*a*b^6
 + A*b^7)*c)*x^4 - 30*(2*B*a^5 + A*a^4*b)*c^3 + 10*(9*B*a^4*b^2 +
 A*a^3*b^3)*c^2 + 2*(3*B*a*b^7 - 30*(2*B*a^4*b + A*a^3*b^2)*c^3 +
 10*(9*B*a^3*b^3 + A*a^2*b^4)*c^2 - (30*B*a^2*b^5 + A*a*b^6)*c)*x
^2 - (30*B*a^3*b^4 + A*a^2*b^5)*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^
2 + 4*a*c)/(b^2 - 4*a*c)) + (2*(B*b^4*c^3 - 8*B*a*b^2*c^4 + 16*B*
a^2*c^5)*x^10 + 4*(B*b^5*c^2 - 8*B*a*b^3*c^3 + 16*B*a^2*b*c^4)*x^
8 - 5*B*a^2*b^5 + 24*A*a^4*c^3 - 2*(2*B*b^6*c - 25*(2*B*a^3 + A*a
^2*b)*c^4 + 3*(17*B*a^2*b^2 + 5*A*a*b^3)*c^3 - 2*(9*B*a*b^4 + A*b
^5)*c^2)*x^6 - (5*B*b^7 - 32*A*a^3*c^4 - 11*(2*B*a^3*b + A*a^2*b^
2)*c^3 + (73*B*a^2*b^3 + 19*A*a*b^4)*c^2 - (38*B*a*b^5 + 3*A*b^6)
*c)*x^4 - (58*B*a^4*b + 21*A*a^3*b^2)*c^2 - 2*(5*B*a*b^6 - (30*B*
a^4 + 31*A*a^3*b)*c^3 + (79*B*a^3*b^2 + 22*A*a^2*b^3)*c^2 - 3*(13
*B*a^2*b^4 + A*a*b^5)*c)*x^2 + 3*(12*B*a^3*b^3 + A*a^2*b^4)*c - (
(3*B*b^5*c^2 - 16*A*a^2*c^5 + 8*(6*B*a^2*b + A*a*b^2)*c^4 - (24*B
*a*b^3 + A*b^4)*c^3)*x^8 + 3*B*a^2*b^5 - 16*A*a^4*c^3 + 2*(3*B*b^
6*c - 16*A*a^2*b*c^4 + 8*(6*B*a^2*b^2 + A*a*b^3)*c^3 - (24*B*a*b^
4 + A*b^5)*c^2)*x^6 + (3*B*b^7 + 6*A*a*b^4*c^2 + 96*B*a^3*b*c^3 -
 32*A*a^3*c^4 - (18*B*a*b^5 + A*b^6)*c)*x^4 + 8*(6*B*a^4*b + A*a^
3*b^2)*c^2 + 2*(3*B*a*b^6 - 16*A*a^3*b*c^3 + 8*(6*B*a^3*b^2 + A*a
^2*b^3)*c^2 - (24*B*a^2*b^4 + A*a*b^5)*c)*x^2 - (24*B*a^3*b^3 + A
*a^2*b^4)*c)*log(c*x^4 + b*x^2 + a))*sqrt(-b^2 + 4*a*c))/((a^2*b^
4*c^4 - 8*a^3*b^2*c^5 + 16*a^4*c^6 + (b^4*c^6 - 8*a*b^2*c^7 + 16*
a^2*c^8)*x^8 + 2*(b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*x^6 + (b^

```
6*c^4 - 6*a*b^4*c^5 + 32*a^3*c^7)*x^4 + 2*(a*b^5*c^4 - 8*a^2*b^3*
c^5 + 16*a^3*b*c^6)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 15.9764, size = 807, normalized size = 2.21

$$\frac{\left(3\,Bb^6-30\,Bab^4c-Ab^5c+90\,Ba^2b^2c^2+10\,Aab^3c^2-60\,Ba^3c^3-30\,Aa^2bc^3\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(b^4c^4-8\,ab^2c^5+16\,a^2c^6\right)\sqrt{-b^2+4\,ac}}+\frac{Bx^2}{2\,c^3}$$
$$+\frac{9\,Bb^5c^2x^8-72\,Bab^3c^3x^8-3\,Ab^4c^3x^8+144\,Ba^2bc^4x^8+24\,Aab^2c^4x^8-48\,Aa^2c^5x^8+6\,Bb^6cx^6-48\,Bab^4c^2x^6+2\,Ab^5c^2x^6+}{}$$
$$-\frac{(3\,Bb-Ac)\ln\left(cx^4+bx^2+a\right)}{4\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^11/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*B*b^6 - 30*B*a*b^4*c - A*b^5*c + 90*B*a^2*b^2*c^2 + 10*A*a
*b^3*c^2 - 60*B*a^3*c^3 - 30*A*a^2*b*c^3)*arctan((2*c*x^2 + b)/sq
rt(-b^2 + 4*a*c))/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*sqrt(-b^2
 + 4*a*c)) + 1/2*B*x^2/c^3 + 1/8*(9*B*b^5*c^2*x^8 - 72*B*a*b^3*c^
3*x^8 - 3*A*b^4*c^3*x^8 + 144*B*a^2*b*c^4*x^8 + 24*A*a*b^2*c^4*x^
8 - 48*A*a^2*c^5*x^8 + 6*B*b^6*c*x^6 - 48*B*a*b^4*c^2*x^6 + 2*A*b
^5*c^2*x^6 + 84*B*a^2*b^2*c^3*x^6 - 12*A*a*b^3*c^3*x^6 + 72*B*a^3
*c^4*x^6 + 4*A*a^2*b*c^4*x^6 - B*b^7*x^4 + 14*B*a*b^5*c*x^4 + 3*A
*b^6*c*x^4 - 82*B*a^2*b^3*c^2*x^4 - 20*A*a*b^4*c^2*x^4 + 204*B*a^
3*b*c^3*x^4 + 22*A*a^2*b^2*c^3*x^4 - 32*A*a^3*c^4*x^4 - 2*B*a*b^6
*x^2 + 8*B*a^2*b^4*c*x^2 + 6*A*a*b^5*c*x^2 + 4*B*a^3*b^2*c^2*x^2
- 40*A*a^2*b^3*c^2*x^2 + 56*B*a^4*c^3*x^2 + 28*A*a^3*b*c^3*x^2 -
B*a^2*b^5 + 3*A*a^2*b^4*c + 28*B*a^4*b*c^2 - 18*A*a^3*b^2*c^2)/((
b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*(c*x^4 + b*x^2 + a)^2) - 1/4*
(3*B*b - A*c)*ln(c*x^4 + b*x^2 + a)/c^4
```

3.125 $\int \frac{x^9\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=254

$$\frac{\left(-12a^2Ac^3+30a^2bBc^2-10ab^3Bc+b^5B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\left(b^2-4ac\right)^{5/2}}-\frac{x^2\left(x^2\left(16a^2Bc^2+6aAbc^2-15ab^2Bc+2b^4B\right)+2a\left(6aAc^2-7abBc+b^3B\right)\right)}{4c^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^6\left(x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c^3}$$

```
[Out] -(x^6*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(4*c*(b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^2*(2*a*(b^3*B - 7*a*b*B*c
+ 6*a*A*c^2) + (2*b^4*B - 15*a*b^2*B*c + 6*a*A*b*c^2 + 16*a^2*B*c
^2)*x^2))/(4*c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((b^5*B -
 10*a*b^3*B*c + 30*a^2*b*B*c^2 - 12*a^2*A*c^3)*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(5/2)) + (B*Log[a +
b*x^2 + c*x^4])/(4*c^3)
```

Rubi [A] time = 0.802426, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{\left(-12a^2Ac^3+30a^2bBc^2-10ab^3Bc+b^5B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\left(b^2-4ac\right)^{5/2}}-\frac{x^2\left(x^2\left(16a^2Bc^2+6aAbc^2-15ab^2Bc+2b^4B\right)+2a\left(6aAc^2-7abBc+b^3B\right)\right)}{4c^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^6\left(x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^6*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(4*c*(b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^2*(2*a*(b^3*B - 7*a*b*B*c
+ 6*a*A*c^2) + (2*b^4*B - 15*a*b^2*B*c + 6*a*A*b*c^2 + 16*a^2*B*c
^2)*x^2))/(4*c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((b^5*B -
 10*a*b^3*B*c + 30*a^2*b*B*c^2 - 12*a^2*A*c^3)*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(5/2)) + (B*Log[a +
b*x^2 + c*x^4])/(4*c^3)
```

Rubi in Sympy [A] time = 94.6042, size = 252, normalized size = 0.99

$$\frac{B\log\left(a+bx^2+cx^4\right)}{4c^3}+\frac{x^6\left(a\left(2Ac-Bb\right)-x^2\left(-Abc-2Bac+Bb^2\right)\right)}{4c\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^2\left(2a\left(6Aac^2-7Babc+Bb^3\right)+x^2\left(6Aabc^2+16Ba^2c^2-15Bab^2c+2Bb^4\right)\right)}{4c^2\left(-4ac+b^2\right)^2\left(a+bx^2+cx^4\right)}+\frac{\left(-12Aa^2c^3+30Ba^2bc^2-10Bab^3c+Bb^5\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^3\left(-4ac+b^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] B*log(a + b*x**2 + c*x**4)/(4*c**3) + x**6*(a*(2*A*c - B*b) - x**
2*(-A*b*c - 2*B*a*c + B*b**2))/(4*c*(-4*a*c + b**2)*(a + b*x**2 +
 c*x**4)**2) - x**2*(2*a*(6*A*a*c**2 - 7*B*a*b*c + B*b**3) + x**2
*(6*A*a*b*c**2 + 16*B*a**2*c**2 - 15*B*a*b**2*c + 2*B*b**4))/(4*c
**2*(-4*a*c + b**2)**2*(a + b*x**2 + c*x**4)) + (-12*A*a**2*c**3
+ 30*B*a**2*b*c**2 - 10*B*a*b**3*c + B*b**5)*atanh((b + 2*c*x**2)
/sqrt(-4*a*c + b**2))/(2*c**3*(-4*a*c + b**2)**(5/2))
```

Mathematica [A] time = 0.865887, size = 354, normalized size = 1.39

$$-\frac{2c\left(-12a^2Ac^3+30a^2bBc^2-10ab^3Bc+b^5B\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}+\frac{2a^2bc^3\left(11A+25Bx^2\right)+4a^2c^3\left(8aB-5Acx^2\right)+b^4c\left(11aB-2Acx^2\right)-2ab^3c^2\left(4A+15Bx^2\right)+ab^2c^2(1}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-(b^6*B) + b^5*c*(A + 4*B*x^2) - 2*a*b^3*c^2*(4*A + 15*B*x^2) +
 2*a^2*b*c^3*(11*A + 25*B*x^2) + 4*a^2*c^3*(8*a*B - 5*A*c*x^2) +
b^4*c*(11*a*B - 2*A*c*x^2) + a*b^2*c^2*(-39*a*B + 16*A*c*x^2))/((
b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (2*a^3*B*c^2 + b^4*(b*B - A
*c)*x^2 + a*b^2*(b^2*B + 4*A*c^2*x^2 - b*c*(A + 5*B*x^2)) + a^2*c
*(-4*b^2*B - 2*A*c^2*x^2 + b*c*(3*A + 5*B*x^2)))/((b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)^2) - (2*c*(b^5*B - 10*a*b^3*B*c + 30*a^2*b*B*c
^2 - 12*a^2*A*c^3)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^
2 + 4*a*c)^(5/2) + B*c*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Maple [B] time = 0.034, size = 1274, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-1/c^2*(10*A*a^2*c^3-8*A*a*b^2*c^2+A*b^4*c-25*B*a^2*b*c^2+15
*B*a*b^3*c-2*B*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*(2*A*a^2*b
*c^3+8*A*a*b^3*c^2-A*b^5*c+32*B*a^3*c^3+11*B*a^2*b^2*c^2-19*B*a*b
^4*c+3*B*b^6)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-a*(6*A*a^2*c^3-1
0*A*a*b^2*c^2+A*b^4*c-31*B*a^2*b*c^2+22*B*a*b^3*c-3*B*b^5)/c^3/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/2*a^2*(10*A*a*b*c^2-A*b^3*c+24*B*a
^2*c^2-21*B*a*b^2*c+3*B*b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x
^4+b*x^2+a)^2+4/c/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c^2*(16*a^2*c^2-8
*a*b^2*c+b^4)*(c*x^4+b*x^2+a))*a^2*B-2/c^2/(16*a^2*c^2-8*a*b^2*c+
b^4)*ln(c^2*(16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^4+b*x^2+a))*a*b^2*B+1
/4/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c^2*(16*a^2*c^2-8*a*b^2*c+b^
4)*(c*x^4+b*x^2+a))*b^4*B+6/(1024*a^5*c^9-1280*a^4*b^2*c^8+640*a^
3*b^4*c^7-160*a^2*b^6*c^6+20*a*b^8*c^5-b^10*c^4)^(1/2)*arctan((2*
(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*x^2+c^2*(16*a^2*c^2-8*a*b^2*c+b^4)
*b)/(1024*a^5*c^9-1280*a^4*b^2*c^8+640*a^3*b^4*c^7-160*a^2*b^6*c^
6+20*a*b^8*c^5-b^10*c^4)^(1/2))*A*a^2*c^2-15/(1024*a^5*c^9-1280*a
^4*b^2*c^8+640*a^3*b^4*c^7-160*a^2*b^6*c^6+20*a*b^8*c^5-b^10*c^4)
^(1/2)*arctan((2*(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*x^2+c^2*(16*a^2*c
^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^9-1280*a^4*b^2*c^8+640*a^3*b^4*c
^7-160*a^2*b^6*c^6+20*a*b^8*c^5-b^10*c^4)^(1/2))*a^2*b*B*c+5/(102
4*a^5*c^9-1280*a^4*b^2*c^8+640*a^3*b^4*c^7-160*a^2*b^6*c^6+20*a*b
^8*c^5-b^10*c^4)^(1/2)*arctan((2*(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*x
^2+c^2*(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^9-1280*a^4*b^2*c
^8+640*a^3*b^4*c^7-160*a^2*b^6*c^6+20*a*b^8*c^5-b^10*c^4)^(1/2))*
B*a*b^3-1/2/(1024*a^5*c^9-1280*a^4*b^2*c^8+640*a^3*b^4*c^7-160*a^
2*b^6*c^6+20*a*b^8*c^5-b^10*c^4)^(1/2)*arctan((2*(16*a^2*c^2-8*a*
b^2*c+b^4)*c^3*x^2+c^2*(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^
9-1280*a^4*b^2*c^8+640*a^3*b^4*c^7-160*a^2*b^6*c^6+20*a*b^8*c^5-b
^10*c^4)^(1/2))*b^5/c*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^9/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.432666, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^9/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(((B*b^5*c^2 - 10*B*a*b^3*c^3 + 30*B*a^2*b*c^4 - 12*A*a^2*c
^5)*x^8 + B*a^2*b^5 - 10*B*a^3*b^3*c + 30*B*a^4*b*c^2 - 12*A*a^4*
c^3 + 2*(B*b^6*c - 10*B*a*b^4*c^2 + 30*B*a^2*b^2*c^3 - 12*A*a^2*b
*c^4)*x^6 + (B*b^7 - 8*B*a*b^5*c + 10*B*a^2*b^3*c^2 - 24*A*a^3*c^
4 + 12*(5*B*a^3*b - A*a^2*b^2)*c^3)*x^4 + 2*(B*a*b^6 - 10*B*a^2*b
^4*c + 30*B*a^3*b^2*c^2 - 12*A*a^3*b*c^3)*x^2)*log(-(b^3 - 4*a*b*
c + 2*(b^2*c - 4*a*c^2)*x^2 - (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*
c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (3*B*a^2*b^4 + 2*(2*
B*b^5*c - 10*A*a^2*c^4 + (25*B*a^2*b + 8*A*a*b^2)*c^3 - (15*B*a*b
^3 + A*b^4)*c^2)*x^6 + (3*B*b^6 + 2*(16*B*a^3 + A*a^2*b)*c^3 + (1
1*B*a^2*b^2 + 8*A*a*b^3)*c^2 - (19*B*a*b^4 + A*b^5)*c)*x^4 + 2*(1
2*B*a^4 + 5*A*a^3*b)*c^2 + 2*(3*B*a*b^5 - 6*A*a^3*c^3 + (31*B*a^3
*b + 10*A*a^2*b^2)*c^2 - (22*B*a^2*b^3 + A*a*b^4)*c)*x^2 - (21*B*
a^3*b^2 + A*a^2*b^3)*c + ((B*b^4*c^2 - 8*B*a*b^2*c^3 + 16*B*a^2*c
^4)*x^8 + B*a^2*b^4 - 8*B*a^3*b^2*c + 16*B*a^4*c^2 + 2*(B*b^5*c -
 8*B*a*b^3*c^2 + 16*B*a^2*b*c^3)*x^6 + (B*b^6 - 6*B*a*b^4*c + 32*
B*a^3*c^3)*x^4 + 2*(B*a*b^5 - 8*B*a^2*b^3*c + 16*B*a^3*b*c^2)*x^2
)*log(c*x^4 + b*x^2 + a))*sqrt(b^2 - 4*a*c))/((a^2*b^4*c^3 - 8*a^
3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^8
 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^6 + (b^6*c^3 - 6*a*
b^4*c^4 + 32*a^3*c^6)*x^4 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3
*b*c^5)*x^2)*sqrt(b^2 - 4*a*c)), -1/4*(2*((B*b^5*c^2 - 10*B*a*b^3
*c^3 + 30*B*a^2*b*c^4 - 12*A*a^2*c^5)*x^8 + B*a^2*b^5 - 10*B*a^3*
b^3*c + 30*B*a^4*b*c^2 - 12*A*a^4*c^3 + 2*(B*b^6*c - 10*B*a*b^4*c
^2 + 30*B*a^2*b^2*c^3 - 12*A*a^2*b*c^4)*x^6 + (B*b^7 - 8*B*a*b^5*
c + 10*B*a^2*b^3*c^2 - 24*A*a^3*c^4 + 12*(5*B*a^3*b - A*a^2*b^2)*
c^3)*x^4 + 2*(B*a*b^6 - 10*B*a^2*b^4*c + 30*B*a^3*b^2*c^2 - 12*A*
a^3*b*c^3)*x^2)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c)) - (3*B*a^2*b^4 + 2*(2*B*b^5*c - 10*A*a^2*c^4 + (25*B*a^2*b
 + 8*A*a*b^2)*c^3 - (15*B*a*b^3 + A*b^4)*c^2)*x^6 + (3*B*b^6 + 2*
(16*B*a^3 + A*a^2*b)*c^3 + (11*B*a^2*b^2 + 8*A*a*b^3)*c^2 - (19*B
*a*b^4 + A*b^5)*c)*x^4 + 2*(12*B*a^4 + 5*A*a^3*b)*c^2 + 2*(3*B*a*
b^5 - 6*A*a^3*c^3 + (31*B*a^3*b + 10*A*a^2*b^2)*c^2 - (22*B*a^2*b
^3 + A*a*b^4)*c)*x^2 - (21*B*a^3*b^2 + A*a^2*b^3)*c + ((B*b^4*c^2
 - 8*B*a*b^2*c^3 + 16*B*a^2*c^4)*x^8 + B*a^2*b^4 - 8*B*a^3*b^2*c
+ 16*B*a^4*c^2 + 2*(B*b^5*c - 8*B*a*b^3*c^2 + 16*B*a^2*b*c^3)*x^6
 + (B*b^6 - 6*B*a*b^4*c + 32*B*a^3*c^3)*x^4 + 2*(B*a*b^5 - 8*B*a^
2*b^3*c + 16*B*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a))*sqrt(-b^2
+ 4*a*c))/((a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 -
 8*a*b^2*c^6 + 16*a^2*c^7)*x^8 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^
2*b*c^6)*x^6 + (b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^4 + 2*(a*b^
5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 15.7499, size = 629, normalized size = 2.48

$$-\frac{\left(Bb^{5}-10\,Bab^{3}c+30\,Ba^{2}bc^{2}-12\,Aa^{2}c^{3}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\,\left(b^{4}c^{3}-8\,ab^{2}c^{4}+16\,a^{2}c^{5}\right)\sqrt{-b^{2}+4\,ac}}+\frac{B\ln\left(cx^{4}+bx^{2}+a\right)}{4\,c^{3}}$$
$$-\frac{3\,Bb^{4}c^{2}x^{8}-24\,Bab^{2}c^{3}x^{8}+48\,Ba^{2}c^{4}x^{8}-2\,Bb^{5}cx^{6}+12\,Bab^{3}c^{2}x^{6}+4\,Ab^{4}c^{2}x^{6}-4\,Ba^{2}bc^{3}x^{6}-32\,Aab^{2}c^{3}x^{6}+40\,Aa^{2}c^{4}x^{6}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^9/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(B*b^5 - 10*B*a*b^3*c + 30*B*a^2*b*c^2 - 12*A*a^2*c^3)*arcta
n((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4*c^3 - 8*a*b^2*c^4 + 16*
a^2*c^5)*sqrt(-b^2 + 4*a*c)) + 1/4*B*ln(c*x^4 + b*x^2 + a)/c^3 -
1/8*(3*B*b^4*c^2*x^8 - 24*B*a*b^2*c^3*x^8 + 48*B*a^2*c^4*x^8 - 2*
B*b^5*c*x^6 + 12*B*a*b^3*c^2*x^6 + 4*A*b^4*c^2*x^6 - 4*B*a^2*b*c^
3*x^6 - 32*A*a*b^2*c^3*x^6 + 40*A*a^2*c^4*x^6 - 3*B*b^6*x^4 + 20*
B*a*b^4*c*x^4 + 2*A*b^5*c*x^4 - 22*B*a^2*b^2*c^2*x^4 - 16*A*a*b^3
*c^2*x^4 + 32*B*a^3*c^3*x^4 - 4*A*a^2*b*c^3*x^4 - 6*B*a*b^5*x^2 +
 40*B*a^2*b^3*c*x^2 + 4*A*a*b^4*c*x^2 - 28*B*a^3*b*c^2*x^2 - 40*A
*a^2*b^2*c^2*x^2 + 24*A*a^3*c^3*x^2 - 3*B*a^2*b^4 + 18*B*a^3*b^2*
c + 2*A*a^2*b^3*c - 20*A*a^3*b*c^2)/((b^4*c^3 - 8*a*b^2*c^4 + 16*
a^2*c^5)*(c*x^4 + b*x^2 + a)^2)
```

3.126 $\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=146

$$\frac{3a(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{3x^2\left(2a+bx^2\right)(Ab-2aB)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^6\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

```
[Out] -(x^6*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)^2) + (3*(A*b - 2*a*B)*x^2*(2*a + b*x^2))/(4*(b^2 - 4
*a*c)^2*(a + b*x^2 + c*x^4)) + (3*a*(A*b - 2*a*B)*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.30925, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{3a(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{3x^2\left(2a+bx^2\right)(Ab-2aB)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^6\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^6*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)^2) + (3*(A*b - 2*a*B)*x^2*(2*a + b*x^2))/(4*(b^2 - 4
*a*c)^2*(a + b*x^2 + c*x^4)) + (3*a*(A*b - 2*a*B)*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi in Sympy [A] time = 38.4934, size = 138, normalized size = 0.95

$$\frac{3a\left(Ab-2Ba\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{\left(-4ac+b^2\right)^{\frac{5}{2}}}-\frac{x^6\left(Ab-2Ba+x^2\left(2Ac-Bb\right)\right)}{4\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x^2\left(2a+bx^2\right)\left(\frac{Ab}{2}-Ba\right)}{2\left(-4ac+b^2\right)^2\left(a+bx^2+cx^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 3*a*(A*b - 2*B*a)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(-4*a
*c + b**2)**(5/2) - x**6*(A*b - 2*B*a + x**2*(2*A*c - B*b))/(4*(-
```

```
4*a*c + b**2)*(a + b*x**2 + c*x**4)**2) + 3*x**2*(2*a + b*x**2)*(
A*b/2 - B*a)/(2*(-4*a*c + b**2)**2*(a + b*x**2 + c*x**4))
```

Mathematica [A] time = 0.474767, size = 261, normalized size = 1.79

$$\frac{1}{4}\left(\frac{a^2c\left(2c\left(A+Bx^2\right)-3bB\right)+ab\left(-bc\left(A+4Bx^2\right)+3Ac^2x^2+b^2B\right)+b^3x^2(bB-Ac)}{c^3\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2}+\frac{-4a^2c^3\left(4A+5Bx^2\right)+ab^2c^2\left(5A+16Bx^2\right)+2abc^2\left(11aB-3Acx^2\right)-8ab^3Bc-b^4c\left(A+2Bx^2\right)+b^5B}{c^3\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{12a(Ab-2aB)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b^5*B - 8*a*b^3*B*c - b^4*c*(A + 2*B*x^2) - 4*a^2*c^3*(4*A + 5*
B*x^2) + a*b^2*c^2*(5*A + 16*B*x^2) + 2*a*b*c^2*(11*a*B - 3*A*c*x
^2))/(c^3*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b^3*(b*B - A*c)
*x^2 + a^2*c*(-3*b*B + 2*c*(A + B*x^2)) + a*b*(b^2*B + 3*A*c^2*x^
2 - b*c*(A + 4*B*x^2)))/(c^3*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2
) - (12*a*(A*b - 2*a*B)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])
/(-b^2 + 4*a*c)^(5/2))/4
```

Maple [B] time = 0.022, size = 398, normalized size = 2.7

$$\frac{1}{2\left(cx^4+bx^2+a\right)^2}\left(-\frac{\left(3\,aAbc^2+10\,a^2Bc^2-8\,ab^2Bc+b^4B\right)x^6}{\left(16\,a^2c^2-8\,ab^2c+b^4\right)c}-\frac{\left(16\,Aa^2c^3+Aab^2c^2+Ab^4c-2\,Ba^2bc^2-8\,Bab^3c+Bb^5\right)x}{\left(32\,a^2c^2-16\,ab^2c+2\,b^4\right)c^2}\right.$$
$$-3\,\frac{abA}{\left(16\,a^2c^2-8\,ab^2c+b^4\right)\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$
$$+6\,\frac{Ba^2}{\left(16\,a^2c^2-8\,ab^2c+b^4\right)\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-(3*A*a*b*c^2+10*B*a^2*c^2-8*B*a*b^2*c+B*b^4)/(16*a^2*c^2-8*
a*b^2*c+b^4)/c*x^6-1/2*(16*A*a^2*c^3+A*a*b^2*c^2+A*b^4*c-2*B*a^2*
```

```
b*c^2-8*B*a*b^3*c+B*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^4-a*(5*
A*a*b*c^2+A*b^3*c+6*B*a^2*c^2-10*B*a*b^2*c+B*b^4)/c^2/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^2-1/2*a^2/c^2*(8*A*a*c^2+A*b^2*c-10*B*a*b*c+B*b
^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2-3*a/(16*a^2*c^2
-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^
(1/2))*A*b+6*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^7/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.265707, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^7/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*((2*B*a^2 - A*a*b)*c^4*x^8 + 2*(2*B*a^2*b - A*a*b^2)*c^3
*x^6 + 2*(2*B*a^3*b - A*a^2*b^2)*c^2*x^2 + (2*(2*B*a^3 - A*a^2*b)
*c^3 + (2*B*a^2*b^2 - A*a*b^3)*c^2)*x^4 + (2*B*a^4 - A*a^3*b)*c^2
)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b
*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (
2*(B*b^4*c - 8*B*a*b^2*c^2 + (10*B*a^2 + 3*A*a*b)*c^3)*x^6 + B*a^
2*b^3 + 8*A*a^3*c^2 + (B*b^5 + 16*A*a^2*c^3 - (2*B*a^2*b - A*a*b^
2)*c^2 - (8*B*a*b^3 - A*b^4)*c)*x^4 + 2*(B*a*b^4 + (6*B*a^3 + 5*A
*a^2*b)*c^2 - (10*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (10*B*a^3*b - A*a
^2*b^2)*c)*sqrt(b^2 - 4*a*c))/(((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c
^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 -
 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^
3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sq
rt(b^2 - 4*a*c)), 1/4*(12*((2*B*a^2 - A*a*b)*c^4*x^8 + 2*(2*B*a^2
*b - A*a*b^2)*c^3*x^6 + 2*(2*B*a^3*b - A*a^2*b^2)*c^2*x^2 + (2*(2
*B*a^3 - A*a^2*b)*c^3 + (2*B*a^2*b^2 - A*a*b^3)*c^2)*x^4 + (2*B*a
^4 - A*a^3*b)*c^2)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
- 4*a*c)) - (2*(B*b^4*c - 8*B*a*b^2*c^2 + (10*B*a^2 + 3*A*a*b)*c^
3)*x^6 + B*a^2*b^3 + 8*A*a^3*c^2 + (B*b^5 + 16*A*a^2*c^3 - (2*B*a
```

```
^2*b - A*a*b^2)*c^2 - (8*B*a*b^3 - A*b^4)*c)*x^4 + 2*(B*a*b^4 + (
6*B*a^3 + 5*A*a^2*b)*c^2 - (10*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (10*
B*a^3*b - A*a^2*b^2)*c)*sqrt(-b^2 + 4*a*c))/(((b^4*c^4 - 8*a*b^2*
c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4
+ 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b
^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*
b*c^4)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 15.9766, size = 429, normalized size = 2.94

$$\frac{3\left(2\,Ba^2 - Aab\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4 - 8\,ab^2c + 16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{2\,Bb^4cx^6 - 16\,Bab^2c^2x^6 + 20\,Ba^2c^3x^6 + 6\,Aabc^3x^6 + Bb^5x^4 - 8\,Bab^3cx^4 + Ab^4cx^4 - 2\,Ba^2bc^2x^4 + Aab^2c^2x^4 + 16\,Aa^2c^3x^4}{4\,(b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)(c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^7/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 3*(2*B*a^2 - A*a*b)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^
4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(2*B*b^4*c*
x^6 - 16*B*a*b^2*c^2*x^6 + 20*B*a^2*c^3*x^6 + 6*A*a*b*c^3*x^6 + B
*b^5*x^4 - 8*B*a*b^3*c*x^4 + A*b^4*c*x^4 - 2*B*a^2*b*c^2*x^4 + A*
a*b^2*c^2*x^4 + 16*A*a^2*c^3*x^4 + 2*B*a*b^4*x^2 - 20*B*a^2*b^2*c
*x^2 + 2*A*a*b^3*c*x^2 + 12*B*a^3*c^2*x^2 + 10*A*a^2*b*c^2*x^2 +
B*a^2*b^3 - 10*B*a^3*b*c + A*a^2*b^2*c + 8*A*a^3*c^2)/((b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*(c*x^4 + b*x^2 + a)^2)
```

3.127 $\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=185

$$\frac{\left(3abB-A\left(2ac+b^2\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}-\frac{x^4\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{a\left(8aBc-6Abc+b^2B\right)+x^2\left(4aAc^2+2abBc-4Ab^2c+b^3B\right)}{4c(b^2-4ac)^2(a+bx^2+cx^4)}$$

```
[Out] -(x^4*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)^2) - (a*(b^2*B - 6*A*b*c + 8*a*B*c) + (b^3*B - 4*A*b
^2*c + 2*a*b*B*c + 4*a*A*c^2)*x^2)/(4*c*(b^2 - 4*a*c)^2*(a + b*x^
2 + c*x^4)) + ((3*a*b*B - A*(b^2 + 2*a*c))*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.543225, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\left(3abB-A\left(2ac+b^2\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}-\frac{x^4\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{a\left(8aBc-6Abc+b^2B\right)+x^2\left(4aAc^2+2abBc-4Ab^2c+b^3B\right)}{4c(b^2-4ac)^2(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^4*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)^2) - (a*(b^2*B - 6*A*b*c + 8*a*B*c) + (b^3*B - 4*A*b
^2*c + 2*a*b*B*c + 4*a*A*c^2)*x^2)/(4*c*(b^2 - 4*a*c)^2*(a + b*x^
2 + c*x^4)) + ((3*a*b*B - A*(b^2 + 2*a*c))*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi in Sympy [A] time = 49.8733, size = 153, normalized size = 0.83

$$-\frac{x^4\left(Ab-2Ba+x^2(2Ac-Bb)\right)}{4(-4ac+b^2)(a+bx^2+cx^4)^2}+\frac{\left(2a+bx^2\right)\left(2Ab-4Ba-x^2(2Ac-Bb)\right)}{4(-4ac+b^2)^2(a+bx^2+cx^4)}-\frac{\left(2Aac+Ab^2-3Bab\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -x**4*(A*b - 2*B*a + x**2*(2*A*c - B*b))/(4*(-4*a*c + b**2)*(a +
b*x**2 + c*x**4)**2) + (2*a + b*x**2)*(2*A*b - 4*B*a - x**2*(2*A*
c - B*b))/(4*(-4*a*c + b**2)**2*(a + b*x**2 + c*x**4)) - (2*A*a*c
 + A*b**2 - 3*B*a*b)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(-
4*a*c + b**2)**(5/2)
```

Mathematica [A] time = 0.429168, size = 233, normalized size = 1.26

$$\frac{1}{4}\left(\frac{2a^2Bc+a\left(bc\left(A+3Bx^2\right)-2Ac^2x^2+b^2(-B)\right)+b^2x^2(Ac-bB)}{c^2\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2}+\frac{4\left(A\left(2ac+b^2\right)-3abB\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}+\frac{b^2c\left(5aB+2Acx^2\right)+2abc^2\left(A-3Bx^2\right)+4ac^2\left(Acx^2-4aB\right)+Ab^3c+b^4(-B)}{c^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-(b^4*B) + A*b^3*c + 2*a*b*c^2*(A - 3*B*x^2) + 4*a*c^2*(-4*a*B
+ A*c*x^2) + b^2*c*(5*a*B + 2*A*c*x^2))/(c^2*(b^2 - 4*a*c)^2*(a +
 b*x^2 + c*x^4)) + (2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2
*B) - 2*A*c^2*x^2 + b*c*(A + 3*B*x^2)))/(c^2*(-b^2 + 4*a*c)*(a +
b*x^2 + c*x^4)^2) + (4*(-3*a*b*B + A*(b^2 + 2*a*c))*ArcTan[(b + 2
*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/4
```

Maple [B] time = 0.022, size = 411, normalized size = 2.2

$$\frac{1}{2\,(cx^4+bx^2+a)^2}\left(\frac{c\left(2\,aAc+Ab^2-3\,abB\right)x^6}{16\,a^2c^2-8\,ab^2c+b^4}+\frac{\left(6\,aAbc^2+3\,Ab^3c-16\,a^2Bc^2-ab^2Bc-b^4B\right)x^4}{(32\,a^2c^2-16\,ab^2c+2\,b^4)\,c}-\frac{a\left(2\,aAc^2-5\,Ab^2c+\right.}{(16\,a^2c^2-8\,ab}\right.$$
$$+2\,\frac{aAc}{(16\,a^2c^2-8\,ab^2c+b^4)\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$
$$+\frac{Ab^2}{16\,a^2c^2-8\,ab^2c+b^4}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$-3\,\frac{abB}{(16\,a^2c^2-8\,ab^2c+b^4)\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(c*(2*A*a*c+A*b^2-3*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2
*(6*A*a*b*c^2+3*A*b^3*c-16*B*a^2*c^2-B*a*b^2*c-B*b^4)/(16*a^2*c^2
-8*a*b^2*c+b^4)/c*x^4-a/c*(2*A*a*c^2-5*A*b^2*c+5*B*a*b*c+B*b^3)/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/2*a^2*(6*A*b*c-8*B*a*c-B*b^2)/(16
*a^2*c^2-8*a*b^2*c+b^4)/c)/(c*x^4+b*x^2+a)^2+2/(16*a^2*c^2-8*a*b^
2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*
a*A*c+1/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x^2+b)/(4*a*c-b^2)^(1/2))*A*b^2-3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a
*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^5/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.306413, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^5/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*((2*A*a*c^4 - (3*B*a*b - A*b^2)*c^3)*x^8 + 2*(2*A*a*b*c^3
 - (3*B*a*b^2 - A*b^3)*c^2)*x^6 + 2*A*a^3*c^2 + (4*A*a^2*c^3 - 2*
(3*B*a^2*b - 2*A*a*b^2)*c^2 - (3*B*a*b^3 - A*b^4)*c)*x^4 + 2*(2*A
*a^2*b*c^2 - (3*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (3*B*a^3*b - A*a^2*
b^2)*c)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - (2*c^2*x^
4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 +
a)) + (2*(2*A*a*c^3 - (3*B*a*b - A*b^2)*c^2)*x^6 - B*a^2*b^2 - (B
*b^4 + 2*(8*B*a^2 - 3*A*a*b)*c^2 + (B*a*b^2 - 3*A*b^3)*c)*x^4 - 2
*(B*a*b^3 + 2*A*a^2*c^2 + 5*(B*a^2*b - A*a*b^2)*c)*x^2 - 2*(4*B*a
^3 - 3*A*a^2*b)*c)*sqrt(b^2 - 4*a*c))/(((b^4*c^3 - 8*a*b^2*c^4 +
16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5
*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 3
2*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*
sqrt(b^2 - 4*a*c)), 1/4*(4*((2*A*a*c^4 - (3*B*a*b - A*b^2)*c^3)*x
^8 + 2*(2*A*a*b*c^3 - (3*B*a*b^2 - A*b^3)*c^2)*x^6 + 2*A*a^3*c^2
+ (4*A*a^2*c^3 - 2*(3*B*a^2*b - 2*A*a*b^2)*c^2 - (3*B*a*b^3 - A*b
^4)*c)*x^4 + 2*(2*A*a^2*b*c^2 - (3*B*a^2*b^2 - A*a*b^3)*c)*x^2 -
(3*B*a^3*b - A*a^2*b^2)*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*
c)/(b^2 - 4*a*c)) + (2*(2*A*a*c^3 - (3*B*a*b - A*b^2)*c^2)*x^6 -
B*a^2*b^2 - (B*b^4 + 2*(8*B*a^2 - 3*A*a*b)*c^2 + (B*a*b^2 - 3*A*b
^3)*c)*x^4 - 2*(B*a*b^3 + 2*A*a^2*c^2 + 5*(B*a^2*b - A*a*b^2)*c)*
x^2 - 2*(4*B*a^3 - 3*A*a^2*b)*c)*sqrt(-b^2 + 4*a*c))/(((b^4*c^3 -
 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a
^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c -
6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a
^3*b*c^3)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 172.415, size = 833, normalized size = 4.5

$$\frac{\sqrt{-\frac{1}{(4ac-b^2)^5}}\left(-2Aac - Ab^2 + 3Bab\right)\log\left(x^2 + \frac{-2Aabc - Ab^3 + 3Bab^2 - 64a^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-2Aac - Ab^2 + 3Bab\right) + 48a^2b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-2Aac \cdots}{-4Aac^2 - 2A \cdots}\right.}{2} - \frac{\sqrt{-\frac{1}{(4ac-b^2)^5}}\left(-2Aac - Ab^2 + 3Bab\right)\log\left(x^2 + \frac{-2Aabc - Ab^3 + 3Bab^2 + 64a^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-2Aac - Ab^2 + 3Bab\right) - 48a^2b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-2A \cdots}{-4Aac^2 - 2 \cdots}\right.}{2} - \frac{-6Aa^2bc + 8Ba^3c + Ba^2b^2 + x^6\left(-4Aac^3 - 2Ab^2c^2 + 6Babc^2\right) + x^4\left(-6Aabc^2 - 3Ab^3c + 16Ba^2c^2 + Bab^2c + Bb^4\right) + x^2\left(\cdots\right.}{64a^4c^3 - 32a^3b^2c^2 + 4a^2b^4c + x^8\left(64a^2c^5 - 32ab^2c^4 + 4b^4c^3\right) + x^6\left(128a^2bc^4 - 64ab^3c^3 + 8b^5c^2\right) + x^4\left(128a^3c^4 - 24ab^4c^2 + \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*B*a*b)*log(x**2
 + (-2*A*a*b*c - A*b**3 + 3*B*a*b**2 - 64*a**3*c**3*sqrt(-1/(4*a*
```

```
c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*B*a*b) + 48*a**2*b**2*c**2*s
qrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*B*a*b) - 12*a*b*
*4*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*B*a*b) + b
**6*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*B*a*b))/(-4
*A*a*c**2 - 2*A*b**2*c + 6*B*a*b*c))/2 - sqrt(-1/(4*a*c - b**2)**
5)*(-2*A*a*c - A*b**2 + 3*B*a*b)*log(x**2 + (-2*A*a*b*c - A*b**3
+ 3*B*a*b**2 + 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c
- A*b**2 + 3*B*a*b) - 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5
)*(-2*A*a*c - A*b**2 + 3*B*a*b) + 12*a*b**4*c*sqrt(-1/(4*a*c - b*
*2)**5)*(-2*A*a*c - A*b**2 + 3*B*a*b) - b**6*sqrt(-1/(4*a*c - b**
2)**5)*(-2*A*a*c - A*b**2 + 3*B*a*b))/(-4*A*a*c**2 - 2*A*b**2*c +
 6*B*a*b*c))/2 - (-6*A*a**2*b*c + 8*B*a**3*c + B*a**2*b**2 + x**6
*(-4*A*a*c**3 - 2*A*b**2*c**2 + 6*B*a*b*c**2) + x**4*(-6*A*a*b*c*
*2 - 3*A*b**3*c + 16*B*a**2*c**2 + B*a*b**2*c + B*b**4) + x**2*(4
*A*a**2*c**2 - 10*A*a*b**2*c + 10*B*a**2*b*c + 2*B*a*b**3))/(64*a
**4*c**3 - 32*a**3*b**2*c**2 + 4*a**2*b**4*c + x**8*(64*a**2*c**5
 - 32*a*b**2*c**4 + 4*b**4*c**3) + x**6*(128*a**2*b*c**4 - 64*a*b
**3*c**3 + 8*b**5*c**2) + x**4*(128*a**3*c**4 - 24*a*b**4*c**2 +
4*b**6*c) + x**2*(128*a**3*b*c**3 - 64*a**2*b**3*c**2 + 8*a*b**5*
c))
```

GIAC/XCAS [A] time = 15.973, size = 362, normalized size = 1.96

$$-\frac{\left(3\,Bab - Ab^2 - 2\,Aac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4 - 8\,ab^2c + 16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{6\,Babc^2x^6 - 2\,Ab^2c^2x^6 - 4\,Aac^3x^6 + Bb^4x^4 + Bab^2cx^4 - 3\,Ab^3cx^4 + 16\,Ba^2c^2x^4 - 6\,Aabc^2x^4 + 2\,Bab^3x^2 + 10\,Ba^2bcx^2 - 1}{4\,(b^4c - 8\,ab^2c^2 + 16\,a^2c^3)(cx^4 + bx^2 + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^5/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] -(3*B*a*b - A*b^2 - 2*A*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a
*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6
*B*a*b*c^2*x^6 - 2*A*b^2*c^2*x^6 - 4*A*a*c^3*x^6 + B*b^4*x^4 + B*
a*b^2*c*x^4 - 3*A*b^3*c*x^4 + 16*B*a^2*c^2*x^4 - 6*A*a*b*c^2*x^4
+ 2*B*a*b^3*x^2 + 10*B*a^2*b*c*x^2 - 10*A*a*b^2*c*x^2 + 4*A*a^2*c
^2*x^2 + B*a^2*b^2 + 8*B*a^3*c - 6*A*a^2*b*c)/((b^4*c - 8*a*b^2*c
^2 + 16*a^2*c^3)*(c*x^4 + b*x^2 + a)^2)
```

3.128 $\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=170

$$-\frac{(2aBc-3Abc+b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{(b+2cx^2)\,(2aBc-3Abc+b^2B)}{4c\,(b^2-4ac)^2\,(a+bx^2+cx^4)}-\frac{x^2\,(-2aBc-Abc+b^2B)+a(bB-2Ac)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2}$$

```
[Out] -(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(4*c*(b^2 - 4*
a*c)*(a + b*x^2 + c*x^4)^2) + ((b^2*B - 3*A*b*c + 2*a*B*c)*(b + 2
*c*x^2))/(4*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((b^2*B - 3*
A*b*c + 2*a*B*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 -
 4*a*c)^(5/2)
```

Rubi [A] time = 0.361918, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{(2aBc-3Abc+b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{(b+2cx^2)\,(2aBc-3Abc+b^2B)}{4c\,(b^2-4ac)^2\,(a+bx^2+cx^4)}-\frac{x^2\,(-2aBc-Abc+b^2B)+a(bB-2Ac)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(4*c*(b^2 - 4*
a*c)*(a + b*x^2 + c*x^4)^2) + ((b^2*B - 3*A*b*c + 2*a*B*c)*(b + 2
*c*x^2))/(4*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((b^2*B - 3*
A*b*c + 2*a*B*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 -
 4*a*c)^(5/2)
```

Rubi in Sympy [A] time = 46.9016, size = 160, normalized size = 0.94

$$-\frac{(-3Abc+2Bac+Bb^2)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{5}{2}}}+\frac{(b+2cx^2)\,(-3Abc+2Bac+Bb^2)}{4c\,(-4ac+b^2)^2\,(a+bx^2+cx^4)}+\frac{a\,(2Ac-Bb)-x^2\,(-Abc-2Bac+Bb^2)}{4c\,(-4ac+b^2)\,(a+bx^2+cx^4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -(-3*A*b*c + 2*B*a*c + B*b**2)*atanh((b + 2*c*x**2)/sqrt(-4*a*c +
 b**2))/(-4*a*c + b**2)**(5/2) + (b + 2*c*x**2)*(-3*A*b*c + 2*B*a
*c + B*b**2)/(4*c*(-4*a*c + b**2)**2*(a + b*x**2 + c*x**4)) + (a*
(2*A*c - B*b) - x**2*(-A*b*c - 2*B*a*c + B*b**2))/(4*c*(-4*a*c +
b**2)*(a + b*x**2 + c*x**4)**2)
```

Mathematica [A] time = 0.406292, size = 172, normalized size = 1.01

$$\frac{1}{4}\left(\frac{4\left(2aBc-3Abc+b^2B\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}}+\frac{\left(b+2cx^2\right)\left(2aBc-3Abc+b^2B\right)}{c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{-2ac\left(A+Bx^2\right)+abB+bx^2(bB-Ac)}{c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (((b^2*B - 3*A*b*c + 2*a*B*c)*(b + 2*c*x^2))/(c*(b^2 - 4*a*c)^2*(
a + b*x^2 + c*x^4)) + (a*b*B + b*(b*B - A*c)*x^2 - 2*a*c*(A + B*x
^2))/(c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (4*(b^2*B - 3*A*b
*c + 2*a*B*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4
*a*c)^(5/2))/4
```

Maple [B] time = 0.021, size = 379, normalized size = 2.2

$$\frac{1}{2\,(cx^4+bx^2+a)^2}\left(-\frac{c\left(3\,Abc-2\,aBc-b^2B\right)x^6}{16\,a^2c^2-8\,ab^2c+b^4}-\frac{3\,b\left(3\,Abc-2\,aBc-b^2B\right)x^4}{32\,a^2c^2-16\,ab^2c+2\,b^4}-\frac{\left(5\,Aabc+Ab^3+2\,a^2Bc-5\,Bab^2\right)x^2}{16\,a^2c^2-8\,ab^2c+b^4}-\right.$$
$$-3\,\frac{Abc}{(16\,a^2c^2-8\,ab^2c+b^4)\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$
$$+2\,\frac{aBc}{(16\,a^2c^2-8\,ab^2c+b^4)\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$
$$+\frac{b^2B}{16\,a^2c^2-8\,ab^2c+b^4}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-c*(3*A*b*c-2*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-3/
2*b*(3*A*b*c-2*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-(5*A*a
*b*c+A*b^3+2*B*a^2*c-5*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/
2*a*(8*A*a*c+A*b^2-6*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*
x^2+a)^2-3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b*c+2/(16*a^2*c^2-8*a*b^2*c+b^4)/(
4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*B*c+1/(1
6*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*
a*c-b^2)^(1/2))*b^2*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^3/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.298419, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^3/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*((B*b^2*c^2 + (2*B*a - 3*A*b)*c^3)*x^8 + 2*(B*b^3*c + (2
*B*a*b - 3*A*b^2)*c^2)*x^6 + B*a^2*b^2 + (B*b^4 + 2*(2*B*a^2 - 3*
A*a*b)*c^2 + (4*B*a*b^2 - 3*A*b^3)*c)*x^4 + 2*(B*a*b^3 + (2*B*a^2
*b - 3*A*a*b^2)*c)*x^2 + (2*B*a^3 - 3*A*a^2*b)*c)*log((b^3 - 4*a*
b*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*
a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (2*(B*b^2*c + (2*B
*a - 3*A*b)*c^2)*x^6 + 3*(B*b^3 + (2*B*a*b - 3*A*b^2)*c)*x^4 + 6*
B*a^2*b - A*a*b^2 - 8*A*a^2*c + 2*(5*B*a*b^2 - A*b^3 - (2*B*a^2 +
 5*A*a*b)*c)*x^2)*sqrt(b^2 - 4*a*c))/(((b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2
*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*
x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(b^2 - 4*a*
c)), 1/4*(4*((B*b^2*c^2 + (2*B*a - 3*A*b)*c^3)*x^8 + 2*(B*b^3*c +
 (2*B*a*b - 3*A*b^2)*c^2)*x^6 + B*a^2*b^2 + (B*b^4 + 2*(2*B*a^2 -
```

```
 3*A*a*b)*c^2 + (4*B*a*b^2 - 3*A*b^3)*c)*x^4 + 2*(B*a*b^3 + (2*B*
a^2*b - 3*A*a*b^2)*c)*x^2 + (2*B*a^3 - 3*A*a^2*b)*c)*arctan(-(2*c
*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (2*(B*b^2*c + (2*B*
a - 3*A*b)*c^2)*x^6 + 3*(B*b^3 + (2*B*a*b - 3*A*b^2)*c)*x^4 + 6*B
*a^2*b - A*a*b^2 - 8*A*a^2*c + 2*(5*B*a*b^2 - A*b^3 - (2*B*a^2 +
5*A*a*b)*c)*x^2)*sqrt(-b^2 + 4*a*c))/(((b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2
*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*
x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-b^2 + 4*a
*c))]
```

Sympy [A] time = 85.1589, size = 789, normalized size = 4.64

$$-\frac{\sqrt{-\frac{1}{(4ac-b^2)^5}}\left(-3Abc+2Bac+Bb^2\right)\log\left(x^2+\frac{-3Ab^2c+2Babc+Bb^3-64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\left(-3Abc+2Bac+Bb^2\right)+48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\left(-3Abc\cdots\right.}{-6Abc^2+4B\cdots}\right.}{2}$$
$$+\frac{\sqrt{-\frac{1}{(4ac-b^2)^5}}\left(-3Abc+2Bac+Bb^2\right)\log\left(x^2+\frac{-3Ab^2c+2Babc+Bb^3+64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\left(-3Abc+2Bac+Bb^2\right)-48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\left(-3Abc\cdots\right.}{-6Abc^2+4B\cdots}\right.}{2}$$
$$+\frac{-8Aa^2c-Aab^2+6Ba^2b+x^6\left(-6Abc^2+4Bac^2+2Bb^2c\right)+x^4\left(-9Ab^2c+6Babc+3Bb^3\right)+x^2\left(-10Aabc-2A\cdots\right.}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8\left(64a^2c^4-32ab^2c^3+4b^4c^2\right)+x^6\left(128a^2bc^3-64ab^3c^2+8b^5c\right)+x^4\left(128a^3c^3-24ab^4c+4b^6\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2)*log(x**
2 + (-3*A*b**2*c + 2*B*a*b*c + B*b**3 - 64*a**3*c**3*sqrt(-1/(4*a
*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2) + 48*a**2*b**2*c**2*
sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2) - 12*a*b
**4*c*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2) +
b**6*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2))/(-
6*A*b*c**2 + 4*B*a*c**2 + 2*B*b**2*c))/2 + sqrt(-1/(4*a*c - b**2)
**5)*(-3*A*b*c + 2*B*a*c + B*b**2)*log(x**2 + (-3*A*b**2*c + 2*B*
a*b*c + B*b**3 + 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*
c + 2*B*a*c + B*b**2) - 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)*
*5)*(-3*A*b*c + 2*B*a*c + B*b**2) + 12*a*b**4*c*sqrt(-1/(4*a*c -
b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2) - b**6*sqrt(-1/(4*a*c - b
**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2))/(-6*A*b*c**2 + 4*B*a*c**2
 + 2*B*b**2*c))/2 + (-8*A*a**2*c - A*a*b**2 + 6*B*a**2*b + x**6*(
-6*A*b*c**2 + 4*B*a*c**2 + 2*B*b**2*c) + x**4*(-9*A*b**2*c + 6*B*
a*b*c + 3*B*b**3) + x**2*(-10*A*a*b*c - 2*A*b**3 - 4*B*a**2*c + 1
0*B*a*b**2))/(64*a**4*c**2 - 32*a**3*b**2*c + 4*a**2*b**4 + x**8*
(64*a**2*c**4 - 32*a*b**2*c**3 + 4*b**4*c**2) + x**6*(128*a**2*b*
c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(128*a**3*c**3 - 24*a*b*
*4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**3*c + 8*a*b**
```

```
5))
```

GIAC/XCAS [A] time = 15.7279, size = 308, normalized size = 1.81

$$\frac{\left(Bb^2 + 2\,Bac - 3\,Abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4 - 8\,ab^2c + 16\,a^2c^2\right)\sqrt{-b^2+4\,ac}} + \frac{2\,Bb^2cx^6 + 4\,Bac^2x^6 - 6\,Abc^2x^6 + 3\,Bb^3x^4 + 6\,Babcx^4 - 9\,Ab^2cx^4 + 10\,Bab^2x^2 - 2\,Ab^3x^2 - 4\,Ba^2cx^2 - 10\,Aabcx^2 + 6\,Ba^2b}{4\left(cx^4 + bx^2 + a\right)^2\left(b^4 - 8\,ab^2c + 16\,a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^3/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] (B*b^2 + 2*B*a*c - 3*A*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*
c))/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(2*
B*b^2*c*x^6 + 4*B*a*c^2*x^6 - 6*A*b*c^2*x^6 + 3*B*b^3*x^4 + 6*B*a
*b*c*x^4 - 9*A*b^2*c*x^4 + 10*B*a*b^2*x^2 - 2*A*b^3*x^2 - 4*B*a^2
*c*x^2 - 10*A*a*b*c*x^2 + 6*B*a^2*b - A*a*b^2 - 8*A*a^2*c)/((c*x^
4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

3.129 $\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$

Optimal. Leaf size=139

$$\frac{3c(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}-\frac{3\left(b+2cx^2\right)(bB-2Ac)}{4(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{-2aB+x^2(-(bB-2Ac))+Ab}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

```
[Out] -(A*b - 2*a*B - (b*B - 2*A*c)*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)^2) - (3*(b*B - 2*A*c)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a
 + b*x^2 + c*x^4)) + (3*c*(b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.243689, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{3c(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}-\frac{3\left(b+2cx^2\right)(bB-2Ac)}{4(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{-2aB+x^2(-(bB-2Ac))+Ab}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(A*b - 2*a*B - (b*B - 2*A*c)*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)^2) - (3*(b*B - 2*A*c)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a
 + b*x^2 + c*x^4)) + (3*c*(b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi in Sympy [A] time = 30.0505, size = 131, normalized size = 0.94

$$-\frac{3c(2Ac-Bb)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{5}{2}}}+\frac{3\left(b+2cx^2\right)\left(Ac-\frac{Bb}{2}\right)}{2(-4ac+b^2)^2(a+bx^2+cx^4)}-\frac{Ab-2Ba+x^2(2Ac-Bb)}{4(-4ac+b^2)(a+bx^2+cx^4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -3*c*(2*A*c - B*b)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(-4*
a*c + b**2)**(5/2) + 3*(b + 2*c*x**2)*(A*c - B*b/2)/(2*(-4*a*c +
```

```
b**2)**2*(a + b*x**2 + c*x**4)) - (A*b - 2*B*a + x**2*(2*A*c - B*
b))/(4*(-4*a*c + b**2)*(a + b*x**2 + c*x**4)**2)
```

Mathematica [A] time = 0.235279, size = 142, normalized size = 1.02

$$\frac{-\frac{12c(bB-2Ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{(b^2-4ac)\left(B(2a+bx^2)-A(b+2cx^2)\right)}{(a+bx^2+cx^4)^2}-\frac{3(b+2cx^2)(bB-2Ac)}{a+bx^2+cx^4}}{4\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-3*(b*B - 2*A*c)*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + ((b^2 - 4
*a*c)*(B*(2*a + b*x^2) - A*(b + 2*c*x^2)))/(a + b*x^2 + c*x^4)^2
- (12*c*(b*B - 2*A*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/S
qrt[-b^2 + 4*a*c])/(4*(b^2 - 4*a*c)^2)
```

Maple [A] time = 0.012, size = 262, normalized size = 1.9

$$\frac{(2\,Ac-bB)\,x^2+Ab-2\,Ba}{(16\,ac-4\,b^2)\,(cx^4+bx^2+a)^2}+3\,\frac{c^2x^2A}{(4\,ac-b^2)^2\,(cx^4+bx^2+a)}-\frac{3\,bcx^2B}{2\,(4\,ac-b^2)^2\,(cx^4+bx^2+a)}$$
$$+\frac{3\,Abc}{2\,(4\,ac-b^2)^2\,(cx^4+bx^2+a)}-\frac{3\,b^2B}{4\,(4\,ac-b^2)^2\,(cx^4+bx^2+a)}$$
$$+6\,\frac{Ac^2}{(4\,ac-b^2)^{5/2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)-3\,\frac{Bbc}{(4\,ac-b^2)^{5/2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/4*((2*A*c-B*b)*x^2+A*b-2*B*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2+3/(
4*a*c-b^2)^2/(c*x^4+b*x^2+a)*x^2*c^2*A-3/2/(4*a*c-b^2)^2/(c*x^4+b
*x^2+a)*x^2*c*b*B+3/2/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)*b*A*c-3/4/(4*
a*c-b^2)^2/(c*x^4+b*x^2+a)*b^2*B+6/(4*a*c-b^2)^(5/2)*c^2*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A-3/(4*a*c-b^2)^(5/2)*c*arctan((2*c
*x^2+b)/(4*a*c-b^2)^(1/2))*b*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.267678, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 - 2*A*b*c^3)*x^6
 + B*a^2*b*c - 2*A*a^2*c^2 + (B*b^3*c - 4*A*a*c^3 + 2*(B*a*b - A*
b^2)*c^2)*x^4 + 2*(B*a*b^2*c - 2*A*a*b*c^2)*x^2)*log(-(b^3 - 4*a*
b*c + 2*(b^2*c - 4*a*c^2)*x^2 - (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*
a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (6*(B*b*c^2 - 2*A*
c^3)*x^6 + 9*(B*b^2*c - 2*A*b*c^2)*x^4 + B*a*b^2 + A*b^3 + 2*(B*b
^3 - 10*A*a*c^2 + (5*B*a*b - 2*A*b^2)*c)*x^2 + 2*(4*B*a^2 - 5*A*a
*b)*c)*sqrt(b^2 - 4*a*c))/(((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*
x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^
3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*
b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)), -1/4*(
12*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 - 2*A*b*c^3)*x^6 + B*a
^2*b*c - 2*A*a^2*c^2 + (B*b^3*c - 4*A*a*c^3 + 2*(B*a*b - A*b^2)*c
^2)*x^4 + 2*(B*a*b^2*c - 2*A*a*b*c^2)*x^2)*arctan(-(2*c*x^2 + b)*
sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (6*(B*b*c^2 - 2*A*c^3)*x^6 +
9*(B*b^2*c - 2*A*b*c^2)*x^4 + B*a*b^2 + A*b^3 + 2*(B*b^3 - 10*A*a
*c^2 + (5*B*a*b - 2*A*b^2)*c)*x^2 + 2*(4*B*a^2 - 5*A*a*b)*c)*sqrt
(-b^2 + 4*a*c))/(((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c +
16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^
2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 50.1615, size = 661, normalized size = 4.76

$$\frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(-2Ac+Bb)\log\left(x^2+\frac{-6Abc^2+3Bb^2c-192a^3c^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}(-2Ac+Bb)+144a^2b^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}(-2Ac+Bb)-36ab^4c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)}}}{-12Ac^3+6Bbc^2}\right.}{2}$$

$$-\frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(-2Ac+Bb)\log\left(x^2+\frac{-6Abc^2+3Bb^2c+192a^3c^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}(-2Ac+Bb)-144a^2b^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}(-2Ac+Bb)+36ab^4c^2\sqrt{-\frac{1}{\left(4ac-b\right.}}}{-12Ac^3+6Bbc^2}\right.}{2}$$

$$-\frac{-10Aabc+Ab^3+8Ba^2c+Bab^2+x^6\left(-12Ac^3+6Bbc^2\right)+x^4\left(-18Abc^2+9Bb^2c\right)+x^2\left(-20Aac^2-4Ab^2c\right.}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8\left(64a^2c^4-32ab^2c^3+4b^4c^2\right)+x^6\left(128a^2bc^3-64ab^3c^2+8b^5c\right)+x^4\left(128a^3c^3-24ab^4c+4b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 3*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b)*log(x**2 + (-6*A*b*
c**2 + 3*B*b**2*c - 192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(-2*
A*c + B*b) + 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*
c + B*b) - 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*
b) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b))/(-12*A*c
**3 + 6*B*b*c**2))/2 - 3*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B
*b)*log(x**2 + (-6*A*b*c**2 + 3*B*b**2*c + 192*a**3*c**4*sqrt(-1/
(4*a*c - b**2)**5)*(-2*A*c + B*b) - 144*a**2*b**2*c**3*sqrt(-1/(4
*a*c - b**2)**5)*(-2*A*c + B*b) + 36*a*b**4*c**2*sqrt(-1/(4*a*c -
 b**2)**5)*(-2*A*c + B*b) - 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(
-2*A*c + B*b))/(-12*A*c**3 + 6*B*b*c**2))/2 - (-10*A*a*b*c + A*b*
*3 + 8*B*a**2*c + B*a*b**2 + x**6*(-12*A*c**3 + 6*B*b*c**2) + x**
4*(-18*A*b*c**2 + 9*B*b**2*c) + x**2*(-20*A*a*c**2 - 4*A*b**2*c +
 10*B*a*b*c + 2*B*b**3))/(64*a**4*c**2 - 32*a**3*b**2*c + 4*a**2*
b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 + 4*b**4*c**2) + x**6*
(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(128*a**3*c*
*3 - 24*a*b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**3
*c + 8*a*b**5))
```

GIAC/XCAS [A] time = 15.9525, size = 281, normalized size = 2.02

$$-\frac{3\left(Bbc-2\,Ac^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{6\,Bbc^2x^6-12\,Ac^3x^6+9\,Bb^2cx^4-18\,Abc^2x^4+2\,Bb^3x^2+10\,Babcx^2-4\,Ab^2cx^2-20\,Aac^2x^2+Bab^2+Ab^3+8\,Ba^2c-10\,A}{4\left(cx^4+bx^2+a\right)^2\left(b^4-8\,ab^2c+16\,a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] -3*(B*b*c - 2*A*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b
^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*B*b*c^2
*x^6 - 12*A*c^3*x^6 + 9*B*b^2*c*x^4 - 18*A*b*c^2*x^4 + 2*B*b^3*x^
2 + 10*B*a*b*c*x^2 - 4*A*b^2*c*x^2 - 20*A*a*c^2*x^2 + B*a*b^2 + A
*b^3 + 8*B*a^2*c - 10*A*a*b*c)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*
b^2*c + 16*a^2*c^2))
```

3.130 $\int \frac{A+Bx^2}{x(a+bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=252

$$-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{A\log(x)}{a^3}+\frac{2cx^2\left(6a^2Bc+A\left(b^3-7abc\right)\right)+A\left(16a^2c^2-15ab^2c+2b^4\right)+6a^2bBc}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\left(12a^3Bc^2-A\left(30a^2bc^2-10ab^3c+b^5\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{5/2}}-\frac{-A\left(b^2-2ac\right)+cx^2(-(Ab-2aB))+abB}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

```
[Out] -(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(4*a*(b^2 - 4*a*
c)*(a + b*x^2 + c*x^4)^2) + (6*a^2*b*B*c + A*(2*b^4 - 15*a*b^2*c
+ 16*a^2*c^2) + 2*c*(6*a^2*B*c + A*(b^3 - 7*a*b*c))*x^2)/(4*a^2*(
b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((12*a^3*B*c^2 - A*(b^5 - 1
0*a*b^3*c + 30*a^2*b*c^2))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c
]])/(2*a^3*(b^2 - 4*a*c)^(5/2)) + (A*Log[x])/a^3 - (A*Log[a + b*x
^2 + c*x^4])/(4*a^3)
```

Rubi [A] time = 1.06121, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{A\log(x)}{a^3}+\frac{2cx^2\left(6a^2Bc+A\left(b^3-7abc\right)\right)+A\left(16a^2c^2-15ab^2c+2b^4\right)+6a^2bBc}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\left(12a^3Bc^2-A\left(30a^2bc^2-10ab^3c+b^5\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{5/2}}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2)/(4*a*(b^2 - 4*a*c
)*(a + b*x^2 + c*x^4)^2) + (6*a^2*b*B*c + A*(2*b^4 - 15*a*b^2*c +
 16*a^2*c^2) + 2*c*(6*a^2*B*c + A*(b^3 - 7*a*b*c))*x^2)/(4*a^2*(b
^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((12*a^3*B*c^2 - A*(b^5 - 10
*a*b^3*c + 30*a^2*b*c^2))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]
])/(2*a^3*(b^2 - 4*a*c)^(5/2)) + (A*Log[x])/a^3 - (A*Log[a + b*x^
2 + c*x^4])/(4*a^3)
```

Rubi in Sympy [A] time = 137.877, size = 257, normalized size = 1.02

$$\frac{A\log\left(x^2\right)}{2a^3}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{-2Aac+Ab^2-Bab+cx^2\left(Ab-2Ba\right)}{4a\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)^2}+\frac{16Aa^2c^2-15Aab^2c+2Ab^4+6Ba^2bc+2cx^2\left(-7Aabc+Ab^3+6Ba^2c\right)}{4a^2\left(-4ac+b^2\right)^2\left(a+bx^2+cx^4\right)}+\frac{\left(30Aa^2bc^2-10Aab^3c+Ab^5-12Ba^3c^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^3\left(-4ac+b^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*log(x**2)/(2*a**3) - A*log(a + b*x**2 + c*x**4)/(4*a**3) + (-2*
A*a*c + A*b**2 - B*a*b + c*x**2*(A*b - 2*B*a))/(4*a*(-4*a*c + b**
2)*(a + b*x**2 + c*x**4)**2) + (16*A*a**2*c**2 - 15*A*a*b**2*c +
2*A*b**4 + 6*B*a**2*b*c + 2*c*x**2*(-7*A*a*b*c + A*b**3 + 6*B*a**
2*c))/(4*a**2*(-4*a*c + b**2)**2*(a + b*x**2 + c*x**4)) + (30*A*a
**2*b*c**2 - 10*A*a*b**3*c + A*b**5 - 12*B*a**3*c**2)*atanh((b +
2*c*x**2)/sqrt(-4*a*c + b**2))/(2*a**3*(-4*a*c + b**2)**(5/2))
```

Mathematica [A] time = 1.16134, size = 396, normalized size = 1.57

$$\frac{\frac{a^2\left(A\left(-2ac+b^2+bcx^2\right)-aB\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{a\left(2a^2c\left(8Ac+3bB+6Bcx^2\right)-aAbc\left(15b+14cx^2\right)+2Ab^3\left(b+cx^2\right)\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\left(A\left(16a^2c^2\sqrt{b^2-4ac}+30a^2bc^2-10ab^3c-8ab^2c\sqrt{\cdots}\right.\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((a^2*(-(a*B*(b + 2*c*x^2)) + A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 -
 4*a*c)*(a + b*x^2 + c*x^4)^2) + (a*(2*A*b^3*(b + c*x^2) - a*A*b*
c*(15*b + 14*c*x^2) + 2*a^2*c*(3*b*B + 8*A*c + 6*B*c*x^2)))/((b^2
 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + 4*A*Log[x] - ((-12*a^3*B*c^2 +
 A*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 8*a
*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]))*Log[b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) - ((12*a^3*B*c
^2 + A*(-b^5 + 10*a*b^3*c - 30*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c]
- 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]))*Lo
g[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3)
```

Maple [B] time = 0.033, size = 1645, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 6/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+
20*a*b^8*c-b^10)^(1/2)*arctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+b^4)
+(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+640
*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2))*B*c^2-1/4/(c
*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b^3+1/2/a^2/(c*x^4+b
*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^5+5*a/(c*x^4+b*x^2+a
)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*c^2+5/2*a/(c*x^4+b*x^2+a)^2/
(16*a^2*c^2-8*a*b^2*c+b^4)*b*B*c+2/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)
*c*ln((16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^4+b*x^2+a))*A*b^2-15/a/(102
4*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b
^8*c-b^10)^(1/2)*arctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a
^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b
^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2))*A*b*c^2+5/a^2/(102
4*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b
^8*c-b^10)^(1/2)*arctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a
^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b
^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2))*A*b^3*c+9/2/(c*x^4
+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b*B-1/2/(c*x^4+b*x
^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b*c^2+1/(c*x^4+b*x^2+a)^
2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b^2*c+A*ln(x)/a^3+4/(c*x^4+b*x
^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A+6*a/(c*x^4+b*x^2+a)^
2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*c^2+3/4/a/(c*x^4+b*x^2+a)^2/(16*a^
2*c^2-8*a*b^2*c+b^4)*A*b^4-4/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln(
(16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^4+b*x^2+a))*A-1/4/a^3/(16*a^2*c^2
-8*a*b^2*c+b^4)*ln((16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^4+b*x^2+a))*A*
b^4-1/2/a^3/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^
2*b^6*c^2+20*a*b^8*c-b^10)^(1/2)*arctan((2*c*x^2*(16*a^2*c^2-8*a*
b^2*c+b^4)+(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^5-1280*a^4*b
^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2))*A*
b^5-21/4/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^2*c+3/(
c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*B-3/a/(c*x^4+
b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^3*c+1/a^2/(c*x^4+b*
x^2+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b^4-7/2/a/(c*x^4+b*x^
2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b+1/2/a^2/(c*x^4+b*x^
2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b^3-29/4/a/(c*x^4+b*x
^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^3*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.71744, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^3*x),x, algorithm="fricas")
```

```
[Out] [1/4*(((A*b^5*c^2 - 10*A*a*b^3*c^3 - 6*(2*B*a^3 - 5*A*a^2*b)*c^4)
*x^8 + A*a^2*b^5 - 10*A*a^3*b^3*c + 2*(A*b^6*c - 10*A*a*b^4*c^2 -
 6*(2*B*a^3*b - 5*A*a^2*b^2)*c^3)*x^6 + (A*b^7 - 8*A*a*b^5*c - 12
*(2*B*a^4 - 5*A*a^3*b)*c^3 - 2*(6*B*a^3*b^2 - 5*A*a^2*b^3)*c^2)*x
^4 - 6*(2*B*a^5 - 5*A*a^4*b)*c^2 + 2*(A*a*b^6 - 10*A*a^2*b^4*c -
6*(2*B*a^4*b - 5*A*a^3*b^2)*c^2)*x^2)*log((b^3 - 4*a*b*c + 2*(b^2
*c - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^
2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (B*a^3*b^3 - 3*A*a^2*b^4 - 24*
A*a^4*c^2 - 2*(A*a*b^3*c^2 + (6*B*a^3 - 7*A*a^2*b)*c^3)*x^6 - (4*
A*a*b^4*c + 16*A*a^3*c^3 + (18*B*a^3*b - 29*A*a^2*b^2)*c^2)*x^4 -
 2*(A*a*b^5 + (10*B*a^4 - A*a^3*b)*c^2 + 2*(B*a^3*b^2 - 3*A*a^2*b
^3)*c)*x^2 - (10*B*a^4*b - 21*A*a^3*b^2)*c + ((A*b^4*c^2 - 8*A*a*
b^2*c^3 + 16*A*a^2*c^4)*x^8 + A*a^2*b^4 - 8*A*a^3*b^2*c + 16*A*a^
4*c^2 + 2*(A*b^5*c - 8*A*a*b^3*c^2 + 16*A*a^2*b*c^3)*x^6 + (A*b^6
 - 6*A*a*b^4*c + 32*A*a^3*c^3)*x^4 + 2*(A*a*b^5 - 8*A*a^2*b^3*c +
 16*A*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) - 4*((A*b^4*c^2 - 8*
A*a*b^2*c^3 + 16*A*a^2*c^4)*x^8 + A*a^2*b^4 - 8*A*a^3*b^2*c + 16*
A*a^4*c^2 + 2*(A*b^5*c - 8*A*a*b^3*c^2 + 16*A*a^2*b*c^3)*x^6 + (A
*b^6 - 6*A*a*b^4*c + 32*A*a^3*c^3)*x^4 + 2*(A*a*b^5 - 8*A*a^2*b^3
*c + 16*A*a^3*b*c^2)*x^2)*log(x))*sqrt(b^2 - 4*a*c))/((a^5*b^4 -
8*a^6*b^2*c + 16*a^7*c^2 + (a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*
c^4)*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^6 + (a^
3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c
+ 16*a^6*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)), -1/4*(2*((A*b^5*c^2 - 10
*A*a*b^3*c^3 - 6*(2*B*a^3 - 5*A*a^2*b)*c^4)*x^8 + A*a^2*b^5 - 10*
A*a^3*b^3*c + 2*(A*b^6*c - 10*A*a*b^4*c^2 - 6*(2*B*a^3*b - 5*A*a^
2*b^2)*c^3)*x^6 + (A*b^7 - 8*A*a*b^5*c - 12*(2*B*a^4 - 5*A*a^3*b)
*c^3 - 2*(6*B*a^3*b^2 - 5*A*a^2*b^3)*c^2)*x^4 - 6*(2*B*a^5 - 5*A*
a^4*b)*c^2 + 2*(A*a*b^6 - 10*A*a^2*b^4*c - 6*(2*B*a^4*b - 5*A*a^3
*b^2)*c^2)*x^2)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c)) + (B*a^3*b^3 - 3*A*a^2*b^4 - 24*A*a^4*c^2 - 2*(A*a*b^3*c^2
 + (6*B*a^3 - 7*A*a^2*b)*c^3)*x^6 - (4*A*a*b^4*c + 16*A*a^3*c^3 +
 (18*B*a^3*b - 29*A*a^2*b^2)*c^2)*x^4 - 2*(A*a*b^5 + (10*B*a^4 -
A*a^3*b)*c^2 + 2*(B*a^3*b^2 - 3*A*a^2*b^3)*c)*x^2 - (10*B*a^4*b -
 21*A*a^3*b^2)*c + ((A*b^4*c^2 - 8*A*a*b^2*c^3 + 16*A*a^2*c^4)*x^
```

```
8 + A*a^2*b^4 - 8*A*a^3*b^2*c + 16*A*a^4*c^2 + 2*(A*b^5*c - 8*A*a
*b^3*c^2 + 16*A*a^2*b*c^3)*x^6 + (A*b^6 - 6*A*a*b^4*c + 32*A*a^3*
c^3)*x^4 + 2*(A*a*b^5 - 8*A*a^2*b^3*c + 16*A*a^3*b*c^2)*x^2)*log(
c*x^4 + b*x^2 + a) - 4*((A*b^4*c^2 - 8*A*a*b^2*c^3 + 16*A*a^2*c^4
)*x^8 + A*a^2*b^4 - 8*A*a^3*b^2*c + 16*A*a^4*c^2 + 2*(A*b^5*c - 8
*A*a*b^3*c^2 + 16*A*a^2*b*c^3)*x^6 + (A*b^6 - 6*A*a*b^4*c + 32*A*
a^3*c^3)*x^4 + 2*(A*a*b^5 - 8*A*a^2*b^3*c + 16*A*a^3*b*c^2)*x^2)*
log(x))*sqrt(-b^2 + 4*a*c))/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
+ (a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^8 + 2*(a^3*b^5*c -
 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*
a^6*c^3)*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^2)*sqrt
(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 15.9649, size = 568, normalized size = 2.25

$$-\frac{\left(Ab^5 - 10\,Aab^3c - 12\,Ba^3c^2 + 30\,Aa^2bc^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^3b^4 - 8\,a^4b^2c + 16\,a^5c^2\right)\sqrt{-b^2+4\,ac}} - \frac{A\ln\left(cx^4+bx^2+a\right)}{4\,a^3} + \frac{A\ln\left(x^2\right)}{2\,a^3}$$

$$+ \frac{3\,Ab^4c^2x^8 - 24\,Aab^2c^3x^8 + 48\,Aa^2c^4x^8 + 6\,Ab^5cx^6 - 44\,Aab^3c^2x^6 + 24\,Ba^3c^3x^6 + 68\,Aa^2bc^3x^6 + 3\,Ab^6x^4 - 10\,Aab^4cx^4 + 3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^3*x),x, algorithm="giac")
```

```
[Out] -1/2*(A*b^5 - 10*A*a*b^3*c - 12*B*a^3*c^2 + 30*A*a^2*b*c^2)*arcta
n((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^4 - 8*a^4*b^2*c + 16*
a^5*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*A*ln(c*x^4 + b*x^2 + a)/a^3 +
1/2*A*ln(x^2)/a^3 + 1/8*(3*A*b^4*c^2*x^8 - 24*A*a*b^2*c^3*x^8 + 4
8*A*a^2*c^4*x^8 + 6*A*b^5*c*x^6 - 44*A*a*b^3*c^2*x^6 + 24*B*a^3*c
^3*x^6 + 68*A*a^2*b*c^3*x^6 + 3*A*b^6*x^4 - 10*A*a*b^4*c*x^4 + 36
*B*a^3*b*c^2*x^4 - 58*A*a^2*b^2*c^2*x^4 + 128*A*a^3*c^3*x^4 + 10*
A*a*b^5*x^2 + 8*B*a^3*b^2*c*x^2 - 72*A*a^2*b^3*c*x^2 + 40*B*a^4*c
^2*x^2 + 92*A*a^3*b*c^2*x^2 - 2*B*a^3*b^3 + 9*A*a^2*b^4 + 20*B*a^
```

```
4*b*c - 66*A*a^3*b^2*c + 96*A*a^4*c^2)/((a^3*b^4 - 8*a^4*b^2*c +
16*a^5*c^2)*(c*x^4 + b*x^2 + a)^2)
```

3.131 $\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=363

$$\frac{(3Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{-A\left(20a^2c^2-20ab^2c+3b^4\right)+cx^2\left(aB\left(b^2-16ac\right)-3A\left(b^3-6abc\right)\right)+abB\left(b^2-10ac\right)}{4a^2x^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{abB\left(b^2-7ac\right)-3A\left(10a^2c^2-7ab^2c+b^4\right)}{2a^3x^2\left(b^2-4ac\right)^2}+\frac{\left(abB\left(30a^2c^2-10ab^2c+b^4\right)-3A\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{5/2}}-\frac{-A\left(b^2-2ac\right)+cx^2(-(Ab-2aB))+abB}{4ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

```
[Out] (a*b*B*(b^2 - 7*a*c) - 3*A*(b^4 - 7*a*b^2*c + 10*a^2*c^2))/(2*a^3
*(b^2 - 4*a*c)^2*x^2) - (a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*
c*x^2)/(4*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)^2) - (a*b*B*(b^
2 - 10*a*c) - A*(3*b^4 - 20*a*b^2*c + 20*a^2*c^2) + c*(a*B*(b^2 -
 16*a*c) - 3*A*(b^3 - 6*a*b*c))*x^2)/(4*a^2*(b^2 - 4*a*c)^2*x^2*(
a + b*x^2 + c*x^4)) + ((a*b*B*(b^4 - 10*a*b^2*c + 30*a^2*c^2) - 3
*A*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3))*ArcTanh[(b +
 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2)) - ((3*A
*b - a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^2 + c*x^4])/(4
*a^4)
```

Rubi [A] time = 1.51923, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\frac{(3Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{-A\left(20a^2c^2-20ab^2c+3b^4\right)+cx^2\left(aB\left(b^2-16ac\right)-3A\left(b^3-6abc\right)\right)+abB\left(b^2-10ac\right)}{4a^2x^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{abB\left(b^2-7ac\right)-3A\left(10a^2c^2-7ab^2c+b^4\right)}{2a^3x^2\left(b^2-4ac\right)^2}+\frac{\left(abB\left(30a^2c^2-10ab^2c+b^4\right)-3A\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{5/2}}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{4ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (a*b*B*(b^2 - 7*a*c) - 3*A*(b^4 - 7*a*b^2*c + 10*a^2*c^2))/(2*a^3
*(b^2 - 4*a*c)^2*x^2) + (A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*
c*x^2)/(4*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)^2) - (a*b*B*(b^
2 - 10*a*c) - A*(3*b^4 - 20*a*b^2*c + 20*a^2*c^2) + c*(a*B*(b^2 -
 16*a*c) - 3*A*(b^3 - 6*a*b*c))*x^2)/(4*a^2*(b^2 - 4*a*c)^2*x^2*(
a + b*x^2 + c*x^4)) + ((a*b*B*(b^4 - 10*a*b^2*c + 30*a^2*c^2) - 3
*A*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3))*ArcTanh[(b +
 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2)) - ((3*A
*b - a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^2 + c*x^4])/(4
*a^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.97907, size = 642, normalized size = 1.77

$$-\frac{a^2\left(A\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)+aB\left(2ac-b^2-bcx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{a\left(aB\left(16a^2c^2-15ab^2c-14abc^2x^2+2b^4+2b^3cx^2\right)-A\left(46a^2bc^2+28a^2c^3x^2-29ab^3c-26ab^2c^2x^2+4b\right.\right.}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-2*a*A)/x^2 - (a^2*(a*B*(-b^2 + 2*a*c - b*c*x^2) + A*(b^3 - 3*a
*b*c + b^2*c*x^2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)^2) + (a*(a*B*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x^2 -
 14*a*b*c^2*x^2) - A*(4*b^5 - 29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c
*x^2 - 26*a*b^2*c^2*x^2 + 28*a^2*c^3*x^2)))/((b^2 - 4*a*c)^2*(a +
 b*x^2 + c*x^4)) + 4*(-3*A*b + a*B)*Log[x] + ((-(a*B*(b^5 - 10*a*
b^3*c + 30*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2
 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c])) + 3*A*(b^6 - 10*a*b^4*
c + 30*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3
*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c]))*Log[b - S
qrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) + ((a*B*(b^5 - 1
```

```
0*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt
[b^2 - 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c]) + 3*A*(-b^6 + 10*a*
b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a
*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c]))*Log[b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^4)
```

Maple [B] time = 0.045, size = 2724, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] -30/a/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*
c^2+20*a*b^8*c-b^10)^(1/2)*arctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+
b^4)+(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4
+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2))*A*c^3-1/
2/a^3/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*
c^2+20*a*b^8*c-b^10)^(1/2)*arctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+
b^4)+(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4
+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2))*B*b^5+3/
4/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)*ln((16*a^2*c^2-8*a*b^2*c+b^4)*(c
*x^4+b*x^2+a))*A*b^5-4/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln((16*a^
2*c^2-8*a*b^2*c+b^4)*(c*x^4+b*x^2+a))*B+3/2/a^4/(1024*a^5*c^5-128
0*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1
/2)*arctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^
2*c+b^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^
2*b^6*c^2+20*a*b^8*c-b^10)^(1/2))*A*b^6-1/4/a^3/(16*a^2*c^2-8*a*b
^2*c+b^4)*ln((16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^4+b*x^2+a))*B*b^4-5/
4/a^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^5+6*a/(c*x
^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*c^2+3/4/a/(c*x^4+b*x^2
+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b^4-3/a^4*ln(x)*A*b-29/4/a/(c*
x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B*b^2+1/a^2/(c*
x^4+b*x^2+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B*b^4-7/2/a/(c*x^
4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^2*c^2+6/a^2/(c*x^
4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^4*c-3/a/(c*x^4+b*
x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b^3*c+13/2/a^2/(c*x^4+b
*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b^2-1/a^3/(c*x^4+b
*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b^4-7/2/a/(c*x^4+b
*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*b*B+5/a^2/(1024*a^5*
c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b
^10)^(1/2)*arctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2
-8*a*b^2*c+b^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3
-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2))*B*b^3*c-1/2/(c*x^4+b*x^2
+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b*c^2+1/2/a^2/(c*x^4+b*x^2
+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b^5+9/a/(c*x^4+b*x^2+a)^2/
(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^3*c-6/a^3/(16*a^2*c^2-8*a*b^2*c+b^
4)*c*ln((16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^4+b*x^2+a))*A*b^3+2/a^2/(
```

```
16*a^2*c^2-8*a*b^2*c+b^4)*c*ln((16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^4+
b*x^2+a))*B*b^2+45/a^2/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4
*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2)*arctan((2*c*x^2*(16*a
^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^5
-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10
)^(1/2))*A*b^2*c^2+12/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln((16*a
^2*c^2-8*a*b^2*c+b^4)*(c*x^4+b*x^2+a))*A*b-15/a^3/(1024*a^5*c^5-1
280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^
(1/2)*arctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*
b^2*c+b^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*
a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2))*A*b^4*c-15/a/(1024*a^5*c^5-12
80*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(
1/2)*arctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*a^2*c^2-8*a*b
^2*c+b^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a
^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2))*B*b*c^2-7/a/(c*x^4+b*x^2+a)^2*
c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A-1/a^3/(c*x^4+b*x^2+a)^2/(16*
a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^6+1/2/a^2/(c*x^4+b*x^2+a)^2*c^2/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^6*B*b^3-37/2/a/(c*x^4+b*x^2+a)^2*c^3/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b+55/4/a^2/(c*x^4+b*x^2+a)^2*c^2/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b^3-2/a^3/(c*x^4+b*x^2+a)^2*c/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b^5-29/2/(c*x^4+b*x^2+a)^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*A*b*c^2-21/4/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*
a*b^2*c+b^4)*B*b^2*c+4/(c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*
c+b^4)*x^4*B-9/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A
*c^3-1/2*A/a^3/x^2+1/a^3*ln(x)*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^3*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.15119, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^3*x^3),x, algorithm="fricas")
```

```
[Out] [1/4*(((60*A*a^3*c^5 + 30*(B*a^3*b - 3*A*a^2*b^2)*c^4 - 10*(B*a^2
*b^3 - 3*A*a*b^4)*c^3 + (B*a*b^5 - 3*A*b^6)*c^2)*x^10 + 2*(60*A*a
```

^3*b*c^4 + 30*(B*a^3*b^2 - 3*A*a^2*b^3)*c^3 - 10*(B*a^2*b^4 - 3*A
*a*b^5)*c^2 + (B*a*b^6 - 3*A*b^7)*c)*x^8 + (B*a*b^7 - 3*A*b^8 + 1
20*A*a^4*c^4 + 60*(B*a^4*b - 2*A*a^3*b^2)*c^3 + 10*(B*a^3*b^3 - 3
*A*a^2*b^4)*c^2 - 8*(B*a^2*b^5 - 3*A*a*b^6)*c)*x^6 + 2*(B*a^2*b^6
 - 3*A*a*b^7 + 60*A*a^4*b*c^3 + 30*(B*a^4*b^2 - 3*A*a^3*b^3)*c^2
- 10*(B*a^3*b^4 - 3*A*a^2*b^5)*c)*x^4 + (B*a^3*b^5 - 3*A*a^2*b^6
+ 60*A*a^5*c^3 + 30*(B*a^5*b - 3*A*a^4*b^2)*c^2 - 10*(B*a^4*b^3 -
 3*A*a^3*b^4)*c)*x^2)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^
2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x
^4 + b*x^2 + a)) - (2*(30*A*a^3*c^4 + 7*(B*a^3*b - 3*A*a^2*b^2)*c
^3 - (B*a^2*b^3 - 3*A*a*b^4)*c^2)*x^8 + 2*A*a^3*b^4 - 16*A*a^4*b^
2*c + 32*A*a^5*c^2 - (2*(8*B*a^4 - 69*A*a^3*b)*c^3 - 29*(B*a^3*b^
2 - 3*A*a^2*b^3)*c^2 + 4*(B*a^2*b^4 - 3*A*a*b^5)*c)*x^6 - 2*(B*a^
2*b^5 - 3*A*a*b^6 - 50*A*a^4*c^3 - (B*a^4*b + 7*A*a^3*b^2)*c^2 -
6*(B*a^3*b^3 - 3*A*a^2*b^4)*c)*x^4 - (3*B*a^3*b^4 - 9*A*a^2*b^5 +
 2*(12*B*a^5 - 61*A*a^4*b)*c^2 - (21*B*a^4*b^2 - 68*A*a^3*b^3)*c)
x^2 + ((16(B*a^3 - 3*A*a^2*b)*c^4 - 8*(B*a^2*b^2 - 3*A*a*b^3)*c
^3 + (B*a*b^4 - 3*A*b^5)*c^2)*x^10 + 2*(16*(B*a^3*b - 3*A*a^2*b^2
)*c^3 - 8*(B*a^2*b^3 - 3*A*a*b^4)*c^2 + (B*a*b^5 - 3*A*b^6)*c)*x^
8 + (B*a*b^6 - 3*A*b^7 + 32*(B*a^4 - 3*A*a^3*b)*c^3 - 6*(B*a^2*b^
4 - 3*A*a*b^5)*c)*x^6 + 2*(B*a^2*b^5 - 3*A*a*b^6 + 16*(B*a^4*b -
3*A*a^3*b^2)*c^2 - 8*(B*a^3*b^3 - 3*A*a^2*b^4)*c)*x^4 + (B*a^3*b^
4 - 3*A*a^2*b^5 + 16*(B*a^5 - 3*A*a^4*b)*c^2 - 8*(B*a^4*b^2 - 3*A
*a^3*b^3)*c)*x^2)*log(c*x^4 + b*x^2 + a) - 4*((16*(B*a^3 - 3*A*a^
2*b)*c^4 - 8*(B*a^2*b^2 - 3*A*a*b^3)*c^3 + (B*a*b^4 - 3*A*b^5)*c^
2)*x^10 + 2*(16*(B*a^3*b - 3*A*a^2*b^2)*c^3 - 8*(B*a^2*b^3 - 3*A*
a*b^4)*c^2 + (B*a*b^5 - 3*A*b^6)*c)*x^8 + (B*a*b^6 - 3*A*b^7 + 32
*(B*a^4 - 3*A*a^3*b)*c^3 - 6*(B*a^2*b^4 - 3*A*a*b^5)*c)*x^6 + 2*(
B*a^2*b^5 - 3*A*a*b^6 + 16*(B*a^4*b - 3*A*a^3*b^2)*c^2 - 8*(B*a^3
*b^3 - 3*A*a^2*b^4)*c)*x^4 + (B*a^3*b^4 - 3*A*a^2*b^5 + 16*(B*a^5
 - 3*A*a^4*b)*c^2 - 8*(B*a^4*b^2 - 3*A*a^3*b^3)*c)*x^2)*log(x))*s
qrt(b^2 - 4*a*c))/(((a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*x^
10 + 2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*x^8 + (a^4*b^6
- 6*a^5*b^4*c + 32*a^7*c^3)*x^6 + 2*(a^5*b^5 - 8*a^6*b^3*c + 16*a
^7*b*c^2)*x^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^2)*sqrt(b^
2 - 4*a*c)), -1/4*(2*((60*A*a^3*c^5 + 30*(B*a^3*b - 3*A*a^2*b^2)*
c^4 - 10*(B*a^2*b^3 - 3*A*a*b^4)*c^3 + (B*a*b^5 - 3*A*b^6)*c^2)*x
^10 + 2*(60*A*a^3*b*c^4 + 30*(B*a^3*b^2 - 3*A*a^2*b^3)*c^3 - 10*(
B*a^2*b^4 - 3*A*a*b^5)*c^2 + (B*a*b^6 - 3*A*b^7)*c)*x^8 + (B*a*b^
7 - 3*A*b^8 + 120*A*a^4*c^4 + 60*(B*a^4*b - 2*A*a^3*b^2)*c^3 + 10
*(B*a^3*b^3 - 3*A*a^2*b^4)*c^2 - 8*(B*a^2*b^5 - 3*A*a*b^6)*c)*x^6
 + 2*(B*a^2*b^6 - 3*A*a*b^7 + 60*A*a^4*b*c^3 + 30*(B*a^4*b^2 - 3*
A*a^3*b^3)*c^2 - 10*(B*a^3*b^4 - 3*A*a^2*b^5)*c)*x^4 + (B*a^3*b^5
 - 3*A*a^2*b^6 + 60*A*a^5*c^3 + 30*(B*a^5*b - 3*A*a^4*b^2)*c^2 -
10*(B*a^4*b^3 - 3*A*a^3*b^4)*c)*x^2)*arctan(-(2*c*x^2 + b)*sqrt(-
b^2 + 4*a*c)/(b^2 - 4*a*c)) + (2*(30*A*a^3*c^4 + 7*(B*a^3*b - 3*A
*a^2*b^2)*c^3 - (B*a^2*b^3 - 3*A*a*b^4)*c^2)*x^8 + 2*A*a^3*b^4 -
16*A*a^4*b^2*c + 32*A*a^5*c^2 - (2*(8*B*a^4 - 69*A*a^3*b)*c^3 - 2
9*(B*a^3*b^2 - 3*A*a^2*b^3)*c^2 + 4*(B*a^2*b^4 - 3*A*a*b^5)*c)*x^
6 - 2*(B*a^2*b^5 - 3*A*a*b^6 - 50*A*a^4*c^3 - (B*a^4*b + 7*A*a^3*
b^2)*c^2 - 6*(B*a^3*b^3 - 3*A*a^2*b^4)*c)*x^4 - (3*B*a^3*b^4 - 9*
A*a^2*b^5 + 2*(12*B*a^5 - 61*A*a^4*b)*c^2 - (21*B*a^4*b^2 - 68*A*
a^3*b^3)*c)*x^2 + ((16*(B*a^3 - 3*A*a^2*b)*c^4 - 8*(B*a^2*b^2 - 3
*A*a*b^3)*c^3 + (B*a*b^4 - 3*A*b^5)*c^2)*x^10 + 2*(16*(B*a^3*b -

```
3*A*a^2*b^2)*c^3 - 8*(B*a^2*b^3 - 3*A*a*b^4)*c^2 + (B*a*b^5 - 3*A
*b^6)*c)*x^8 + (B*a*b^6 - 3*A*b^7 + 32*(B*a^4 - 3*A*a^3*b)*c^3 -
6*(B*a^2*b^4 - 3*A*a*b^5)*c)*x^6 + 2*(B*a^2*b^5 - 3*A*a*b^6 + 16*
(B*a^4*b - 3*A*a^3*b^2)*c^2 - 8*(B*a^3*b^3 - 3*A*a^2*b^4)*c)*x^4
+ (B*a^3*b^4 - 3*A*a^2*b^5 + 16*(B*a^5 - 3*A*a^4*b)*c^2 - 8*(B*a^
4*b^2 - 3*A*a^3*b^3)*c)*x^2)*log(c*x^4 + b*x^2 + a) - 4*((16*(B*a
^3 - 3*A*a^2*b)*c^4 - 8*(B*a^2*b^2 - 3*A*a*b^3)*c^3 + (B*a*b^4 -
3*A*b^5)*c^2)*x^10 + 2*(16*(B*a^3*b - 3*A*a^2*b^2)*c^3 - 8*(B*a^2
*b^3 - 3*A*a*b^4)*c^2 + (B*a*b^5 - 3*A*b^6)*c)*x^8 + (B*a*b^6 - 3
*A*b^7 + 32*(B*a^4 - 3*A*a^3*b)*c^3 - 6*(B*a^2*b^4 - 3*A*a*b^5)*c
)*x^6 + 2*(B*a^2*b^5 - 3*A*a*b^6 + 16*(B*a^4*b - 3*A*a^3*b^2)*c^2
 - 8*(B*a^3*b^3 - 3*A*a^2*b^4)*c)*x^4 + (B*a^3*b^4 - 3*A*a^2*b^5
+ 16*(B*a^5 - 3*A*a^4*b)*c^2 - 8*(B*a^4*b^2 - 3*A*a^3*b^3)*c)*x^2
)*log(x))*sqrt(-b^2 + 4*a*c))/(((a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16
*a^6*c^4)*x^10 + 2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*x^8
 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*x^6 + 2*(a^5*b^5 - 8*a^6*
b^3*c + 16*a^7*b*c^2)*x^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*
x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 15.7322, size = 875, normalized size = 2.41

$$-\frac{\left(Bab^5 - 3\,Ab^6 - 10\,Ba^2b^3c + 30\,Aab^4c + 30\,Ba^3bc^2 - 90\,Aa^2b^2c^2 + 60\,Aa^3c^3\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(a^4b^4 - 8\,a^5b^2c + 16\,a^6c^2)\sqrt{-b^2+4\,ac}} + \frac{3\,Bab^4c^2x^8 - 9\,Ab^5c^2x^8 - 24\,Ba^2b^2c^3x^8 + 72\,Aab^3c^3x^8 + 48\,Ba^3c^4x^8 - 144\,Aa^2bc^4x^8 + 6\,Bab^5cx^6 - 18\,Ab^6cx^6 - 44\,Ba^2b^3c}{}$$

$$-\frac{(Ba-3\,Ab)\ln\left(cx^4+bx^2+a\right)}{4\,a^4} + \frac{(Ba-3\,Ab)\ln\left(x^2\right)}{2\,a^4} - \frac{Bax^2 - 3\,Abx^2 + Aa}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^3*x^3),x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b^5 - 3*A*b^6 - 10*B*a^2*b^3*c + 30*A*a*b^4*c + 30*B*a^
3*b*c^2 - 90*A*a^2*b^2*c^2 + 60*A*a^3*c^3)*arctan((2*c*x^2 + b)/s
```

```
qrt(-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^
2 + 4*a*c)) + 1/8*(3*B*a*b^4*c^2*x^8 - 9*A*b^5*c^2*x^8 - 24*B*a^2
*b^2*c^3*x^8 + 72*A*a*b^3*c^3*x^8 + 48*B*a^3*c^4*x^8 - 144*A*a^2*
b*c^4*x^8 + 6*B*a*b^5*c*x^6 - 18*A*b^6*c*x^6 - 44*B*a^2*b^3*c^2*x
^6 + 136*A*a*b^4*c^2*x^6 + 68*B*a^3*b*c^3*x^6 - 236*A*a^2*b^2*c^3
*x^6 - 56*A*a^3*c^4*x^6 + 3*B*a*b^6*x^4 - 9*A*b^7*x^4 - 10*B*a^2*
b^4*c*x^4 + 38*A*a*b^5*c*x^4 - 58*B*a^3*b^2*c^2*x^4 + 110*A*a^2*b
^3*c^2*x^4 + 128*B*a^4*c^3*x^4 - 436*A*a^3*b*c^3*x^4 + 10*B*a^2*b
^5*x^2 - 26*A*a*b^6*x^2 - 72*B*a^3*b^3*c*x^2 + 192*A*a^2*b^4*c*x^
2 + 92*B*a^4*b*c^2*x^2 - 316*A*a^3*b^2*c^2*x^2 - 72*A*a^4*c^3*x^2
 + 9*B*a^3*b^4 - 19*A*a^2*b^5 - 66*B*a^4*b^2*c + 144*A*a^3*b^3*c
+ 96*B*a^5*c^2 - 260*A*a^4*b*c^2)/((a^4*b^4 - 8*a^5*b^2*c + 16*a^
6*c^2)*(c*x^4 + b*x^2 + a)^2) - 1/4*(B*a - 3*A*b)*ln(c*x^4 + b*x^
2 + a)/a^4 + 1/2*(B*a - 3*A*b)*ln(x^2)/a^4 - 1/2*(B*a*x^2 - 3*A*b
*x^2 + A*a)/(a^4*x^2)
```

3.132 $\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=554

$$\frac{\left(-\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}}+84a^2Bc^2-16aAbc^2-27ab^2Bc+Ab^3c+3b^4B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}}+84a^2Bc^2-16aAbc^2-27ab^2Bc+Ab^3c+3b^4B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3\left(-28aBc+12Abc+b^2B\right)}{8c\left(b^2-4ac\right)^2}-\frac{x^7\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^5\left(x^2\left(-28aBc+12Abc+b^2B\right)-4aAc-12abB+7Ab^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x\left(20aAc^2-24abBc+Ab^2c+3b^3B\right)}{8c^2\left(b^2-4ac\right)^2}$$

```
[Out] -((3*b^3*B + A*b^2*c - 24*a*b*B*c + 20*a*A*c^2)*x)/(8*c^2*(b^2 -
4*a*c)^2) + ((b^2*B + 12*A*b*c - 28*a*B*c)*x^3)/(8*c*(b^2 - 4*a*c
)^2) - (x^7*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)^2) - (x^5*(7*A*b^2 - 12*a*b*B - 4*a*A*c + (b^2
*B + 12*A*b*c - 28*a*B*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c
*x^4)) + ((3*b^4*B + A*b^3*c - 27*a*b^2*B*c - 16*a*A*b*c^2 + 84*a
^2*B*c^2 - (3*b^5*B + A*b^4*c - 33*a*b^3*B*c - 18*a*A*b^2*c^2 + 1
32*a^2*b*B*c^2 - 40*a^2*A*c^3)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(5/2)*(b^2
 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((3*b^4*B + A*b^3*c -
27*a*b^2*B*c - 16*a*A*b*c^2 + 84*a^2*B*c^2 + (3*b^5*B + A*b^4*c -
 33*a*b^3*B*c - 18*a*A*b^2*c^2 + 132*a^2*b*B*c^2 - 40*a^2*A*c^3)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 -
 4*a*c]])
```

Rubi [A] time = 21.581, antiderivative size = 554, normalized size of antiderivative = 1., number of

steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{\left(-\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}}+84a^2Bc^2-16aAbc^2-27ab^2Bc+Ab^3c+3b^4B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}}+84a^2Bc^2-16aAbc^2-27ab^2Bc+Ab^3c+3b^4B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3\left(-28aBc+12Abc+b^2B\right)}{8c\left(b^2-4ac\right)^2}-\frac{x^7\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^5\left(x^2\left(-28aBc+12Abc+b^2B\right)-4aAc-12abB+7Ab^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x\left(20aAc^2-24abBc+Ab^2c+3b^3B\right)}{8c^2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -((3*b^3*B + A*b^2*c - 24*a*b*B*c + 20*a*A*c^2)*x)/(8*c^2*(b^2 -
4*a*c)^2) + ((b^2*B + 12*A*b*c - 28*a*B*c)*x^3)/(8*c*(b^2 - 4*a*c
)^2) - (x^7*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)^2) - (x^5*(7*A*b^2 - 12*a*b*B - 4*a*A*c + (b^2
*B + 12*A*b*c - 28*a*B*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c
*x^4)) + ((3*b^4*B + A*b^3*c - 27*a*b^2*B*c - 16*a*A*b*c^2 + 84*a
^2*B*c^2 - (3*b^5*B + A*b^4*c - 33*a*b^3*B*c - 18*a*A*b^2*c^2 + 1
32*a^2*b*B*c^2 - 40*a^2*A*c^3)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(5/2)*(b^2
 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((3*b^4*B + A*b^3*c -
27*a*b^2*B*c - 16*a*A*b*c^2 + 84*a^2*B*c^2 + (3*b^5*B + A*b^4*c -
 33*a*b^3*B*c - 18*a*A*b^2*c^2 + 132*a^2*b*B*c^2 - 40*a^2*A*c^3)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 -
 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 5.2301, size = 644, normalized size = 1.16

$$-\frac{4x\left(a^2c\left(2c\left(A+Bx^2\right)-3bB\right)+ab\left(-bc\left(A+4Bx^2\right)+3Ac^2x^2+b^2B\right)+b^3x^2(bB-Ac)\right)}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{\sqrt{2}\sqrt{c}\left(4a^2c^2\left(21B\sqrt{b^2-4ac}+10Ac\right)-4abc^2\left(4A\sqrt{b^2-4ac}+33aB\right)+9ab^2c\left(2\ldots\right.\right.}{(b^2\ldots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(2*b^5*B - b^4*c*(2*A + 5*B*x^2) - 4*a^2*c^3*(9*A + 11*B*x^
2) + a*b^2*c^2*(11*A + 37*B*x^2) + 16*a*b*c^2*(3*a*B - A*c*x^2) +
 b^3*c*(-17*a*B + A*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)
) - (4*x*(b^3*(b*B - A*c)*x^2 + a^2*c*(-3*b*B + 2*c*(A + B*x^2))
+ a*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*B*x^2))))/((b^2 - 4*a*c)*
(a + b*x^2 + c*x^4)^2) + (Sqrt[2]*Sqrt[c]*(-3*b^5*B + b^3*c*(33*a
*B + A*Sqrt[b^2 - 4*a*c]) - 4*a*b*c^2*(33*a*B + 4*A*Sqrt[b^2 - 4*
a*c]) + 9*a*b^2*c*(2*A*c - 3*B*Sqrt[b^2 - 4*a*c]) + b^4*(-(A*c) +
 3*B*Sqrt[b^2 - 4*a*c]) + 4*a^2*c^2*(10*A*c + 21*B*Sqrt[b^2 - 4*a
*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((
b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c
]*(3*b^5*B + 4*a*b*c^2*(33*a*B - 4*A*Sqrt[b^2 - 4*a*c]) + b^4*(A*
c + 3*B*Sqrt[b^2 - 4*a*c]) - 9*a*b^2*c*(2*A*c + 3*B*Sqrt[b^2 - 4*
a*c]) + 4*a^2*c^2*(-10*A*c + 21*B*Sqrt[b^2 - 4*a*c]) + b^3*(-33*a
*B*c + A*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4
*a*c]]))/(16*c^3)
```

Maple [B] time = 0.128, size = 10352, normalized size = 18.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(5\,Bb^4c+4\left(11\,Ba^2+4\,Aab\right)c^3-\left(37\,Bab^2+Ab^3\right)c^2\right)x^7+\left(3\,Bb^5+36\,Aa^2c^3-\left(4\,Ba^2b-5\,Aab^2\right)c^2-\left(20\,Bab^3-Ab^4\right)c\right)x\ldots}{8\left(\left(b^4c^4-8\,ab^2c^5+16\,a^2c^6\right)x^8+a^2b^4c^2-8\,a^3b^2c^3+16\,a^4c^4+2\left(b^5c^3-8\,ab^3c^4+16\,a^2\ldots\right.\right.}$$
$$-\frac{-\int\frac{3\,Bab^3+20\,Aa^2c^2+\left(3\,Bb^4+4\left(21\,Ba^2-4\,Aab\right)c^2-\left(27\,Bab^2-Ab^3\right)c\right)x^2-\left(24\,Ba^2b-Aab^2\right)c}{cx^4+bx^2+a}\,dx}{8\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^8/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*B*b^4*c + 4*(11*B*a^2 + 4*A*a*b)*c^3 - (37*B*a*b^2 + A*b
^3)*c^2)*x^7 + (3*B*b^5 + 36*A*a^2*c^3 - (4*B*a^2*b - 5*A*a*b^2)*
c^2 - (20*B*a*b^3 - A*b^4)*c)*x^5 + (6*B*a*b^4 + 28*(B*a^3 + A*a^
2*b)*c^2 - (49*B*a^2*b^2 - 2*A*a*b^3)*c)*x^3 + (3*B*a^2*b^3 + 20*
A*a^3*c^2 - (24*B*a^3*b - A*a^2*b^2)*c)*x)/((b^4*c^4 - 8*a*b^2*c^
5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 +
2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4
*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*
c^4)*x^2) - 1/8*integrate(-(3*B*a*b^3 + 20*A*a^2*c^2 + (3*B*b^4 +
 4*(21*B*a^2 - 4*A*a*b)*c^2 - (27*B*a*b^2 - A*b^3)*c)*x^2 - (24*B
*a^2*b - A*a*b^2)*c)/(c*x^4 + b*x^2 + a), x)/(b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4)
```

Fricas [A] time = 9.19933, size = 13009, normalized size = 23.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^8/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*(5*B*b^4*c + 4*(11*B*a^2 + 4*A*a*b)*c^3 - (37*B*a*b^2 +
A*b^3)*c^2)*x^7 + 2*(3*B*b^5 + 36*A*a^2*c^3 - (4*B*a^2*b - 5*A*a*
b^2)*c^2 - (20*B*a*b^3 - A*b^4)*c)*x^5 + 2*(6*B*a*b^4 + 28*(B*a^3
 + A*a^2*b)*c^2 - (49*B*a^2*b^2 - 2*A*a*b^3)*c)*x^3 - sqrt(1/2)*(
(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^
2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6
 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^
2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4
 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*
b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3
+ (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*
b^7 - 2*A*B*b^8)*c + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 -
 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((81*B^4
*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b +
A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*
B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b
^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 +
27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(
33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a
^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^1
5)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8
 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(-(1701*B^4*a^2*b^8 - 94
5*A*B^3*a*b^9 - 10000*A^4*a^4*c^6 + 15000*(6*A^3*B*a^4*b - A^4*a^
3*b^2)*c^5 + 3*(1037232*B^4*a^6 - 1037232*A*B^3*a^5*b + 287712*A^
```

2*B^2*a^4*b^2 - 32952*A^3*B*a^3*b^3 + 497*A^4*a^2*b^4)*c^4 - (155
5848*B^4*a^5*b^2 - 1298376*A*B^3*a^4*b^3 + 238464*A^2*B^2*a^3*b^4
 - 11277*A^3*B*a^2*b^5 + 35*A^4*a*b^6)*c^3 + 9*(37701*B^4*a^4*b^4
 - 26973*A*B^3*a^3*b^5 + 3066*A^2*B^2*a^2*b^6 - 35*A^3*B*a*b^7)*c
^2 - 27*(1341*B^4*a^3*b^6 - 819*A*B^3*a^2*b^7 + 35*A^2*B^2*a*b^8)
*c)*x + 1/2*sqrt(1/2)*(27*B^3*b^13 + 32000*A^3*a^5*c^8 - 640*(882
*A*B^2*a^6 - 156*A^2*B*a^5*b + 37*A^3*a^4*b^2)*c^7 + 64*(10584*B^
3*a^6*b + 5562*A*B^2*a^5*b^2 - 1083*A^2*B*a^4*b^3 + 89*A^3*a^3*b^
4)*c^6 - 8*(93096*B^3*a^5*b^3 + 3816*A*B^2*a^4*b^4 - 1746*A^2*B*a
^3*b^5 + 49*A^3*a^2*b^6)*c^5 + (337392*B^3*a^4*b^5 - 24120*A*B^2*
a^3*b^6 - 84*A^2*B*a^2*b^7 - 17*A^3*a*b^8)*c^4 - (81324*B^3*a^3*b
^7 - 6993*A*B^2*a^2*b^8 + 195*A^2*B*a*b^9 - A^3*b^10)*c^3 + 9*(12
39*B^3*a^2*b^9 - 79*A*B^2*a*b^10 + A^2*B*b^11)*c^2 - 27*(31*B^3*a
*b^11 - A*B^2*b^12)*c - (3*B*b^14*c^5 - 4096*(42*B*a^7 - 13*A*a^6
*b)*c^12 + 6144*(40*B*a^6*b^2 - 11*A*a^5*b^3)*c^11 - 768*(194*B*a
^5*b^4 - 45*A*a^4*b^5)*c^10 + 1280*(39*B*a^4*b^6 - 7*A*a^3*b^7)*c
^9 - 240*(42*B*a^3*b^8 - 5*A*a^2*b^9)*c^8 + 24*(52*B*a^2*b^10 - 3
*A*a*b^11)*c^7 - (90*B*a*b^12 - A*b^13)*c^6)*sqrt((81*B^4*b^8 + 6
25*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^
2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*
b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 444
6*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*
B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a
*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c
^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqr
t(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^
4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 3
6*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168*A*B*a*b^
6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c + (b^10*c^5 - 2
0*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^
9 - 1024*a^5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A
^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 -
 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^
4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*
B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*
b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^
10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1
280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 16
0*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^1
0))) + sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*
b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 +
 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2
*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(9*B^2*b^9
 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*
a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4
 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^7)*
c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c + (b^10*c^5 - 20*a*b^8*c^6 +
 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*
c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 -
108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*
a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 -
6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2
*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B
^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*

a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c
^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7
 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(-(170
1*B^4*a^2*b^8 - 945*A*B^3*a*b^9 - 10000*A^4*a^4*c^6 + 15000*(6*A^
3*B*a^4*b - A^4*a^3*b^2)*c^5 + 3*(1037232*B^4*a^6 - 1037232*A*B^3
*a^5*b + 287712*A^2*B^2*a^4*b^2 - 32952*A^3*B*a^3*b^3 + 497*A^4*a
^2*b^4)*c^4 - (1555848*B^4*a^5*b^2 - 1298376*A*B^3*a^4*b^3 + 2384
64*A^2*B^2*a^3*b^4 - 11277*A^3*B*a^2*b^5 + 35*A^4*a*b^6)*c^3 + 9*
(37701*B^4*a^4*b^4 - 26973*A*B^3*a^3*b^5 + 3066*A^2*B^2*a^2*b^6 -
 35*A^3*B*a*b^7)*c^2 - 27*(1341*B^4*a^3*b^6 - 819*A*B^3*a^2*b^7 +
 35*A^2*B^2*a*b^8)*c)*x - 1/2*sqrt(1/2)*(27*B^3*b^13 + 32000*A^3*
a^5*c^8 - 640*(882*A*B^2*a^6 - 156*A^2*B*a^5*b + 37*A^3*a^4*b^2)*
c^7 + 64*(10584*B^3*a^6*b + 5562*A*B^2*a^5*b^2 - 1083*A^2*B*a^4*b
^3 + 89*A^3*a^3*b^4)*c^6 - 8*(93096*B^3*a^5*b^3 + 3816*A*B^2*a^4*
b^4 - 1746*A^2*B*a^3*b^5 + 49*A^3*a^2*b^6)*c^5 + (337392*B^3*a^4*
b^5 - 24120*A*B^2*a^3*b^6 - 84*A^2*B*a^2*b^7 - 17*A^3*a*b^8)*c^4
 - (81324*B^3*a^3*b^7 - 6993*A*B^2*a^2*b^8 + 195*A^2*B*a*b^9 - A^3
*b^10)*c^3 + 9*(1239*B^3*a^2*b^9 - 79*A*B^2*a*b^10 + A^2*B*b^11)*
c^2 - 27*(31*B^3*a*b^11 - A*B^2*b^12)*c - (3*B*b^14*c^5 - 4096*(4
2*B*a^7 - 13*A*a^6*b)*c^12 + 6144*(40*B*a^6*b^2 - 11*A*a^5*b^3)*c
^11 - 768*(194*B*a^5*b^4 - 45*A*a^4*b^5)*c^10 + 1280*(39*B*a^4*b^
6 - 7*A*a^3*b^7)*c^9 - 240*(42*B*a^3*b^8 - 5*A*a^2*b^9)*c^8 + 24*
(52*B*a^2*b^10 - 3*A*a*b^11)*c^7 - (90*B*a*b^12 - A*b^13)*c^6)*sq
rt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*
B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b +
17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553
*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b
^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*
c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^
11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 10
24*a^5*c^15)))*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^
5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(2
16*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*
b^5 - 168*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)
*c + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8
 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a
^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 +
 (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 51
6*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*
a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*
b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2
*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 64
0*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 -
 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*
c^9 - 1024*a^5*c^10))) - sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a
^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c
^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 3
2*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2
)*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^5 + 280*(54*B
^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(216*B^2*a^3*b^
3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168*A*B
*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c - (b^10*c^
5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b
^2*c^9 - 1024*a^5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(

```
441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*
a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3
 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324
*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B
^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c
)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^1
3 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6
 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^
5*c^10))*log(-(1701*B^4*a^2*b^8 - 945*A*B^3*a*b^9 - 10000*A^4*a^4
*c^6 + 15000*(6*A^3*B*a^4*b - A^4*a^3*b^2)*c^5 + 3*(1037232*B^4*a
^6 - 1037232*A*B^3*a^5*b + 287712*A^2*B^2*a^4*b^2 - 32952*A^3*B*a
^3*b^3 + 497*A^4*a^2*b^4)*c^4 - (1555848*B^4*a^5*b^2 - 1298376*A*
B^3*a^4*b^3 + 238464*A^2*B^2*a^3*b^4 - 11277*A^3*B*a^2*b^5 + 35*A
^4*a*b^6)*c^3 + 9*(37701*B^4*a^4*b^4 - 26973*A*B^3*a^3*b^5 + 3066
*A^2*B^2*a^2*b^6 - 35*A^3*B*a*b^7)*c^2 - 27*(1341*B^4*a^3*b^6 - 8
19*A*B^3*a^2*b^7 + 35*A^2*B^2*a*b^8)*c)*x + 1/2*sqrt(1/2)*(27*B^3
*b^13 + 32000*A^3*a^5*c^8 - 640*(882*A*B^2*a^6 - 156*A^2*B*a^5*b
+ 37*A^3*a^4*b^2)*c^7 + 64*(10584*B^3*a^6*b + 5562*A*B^2*a^5*b^2
- 1083*A^2*B*a^4*b^3 + 89*A^3*a^3*b^4)*c^6 - 8*(93096*B^3*a^5*b^3
 + 3816*A*B^2*a^4*b^4 - 1746*A^2*B*a^3*b^5 + 49*A^3*a^2*b^6)*c^5
+ (337392*B^3*a^4*b^5 - 24120*A*B^2*a^3*b^6 - 84*A^2*B*a^2*b^7 -
17*A^3*a*b^8)*c^4 - (81324*B^3*a^3*b^7 - 6993*A*B^2*a^2*b^8 + 195
*A^2*B*a*b^9 - A^3*b^10)*c^3 + 9*(1239*B^3*a^2*b^9 - 79*A*B^2*a*b
^10 + A^2*B*b^11)*c^2 - 27*(31*B^3*a*b^11 - A*B^2*b^12)*c + (3*B*
b^14*c^5 - 4096*(42*B*a^7 - 13*A*a^6*b)*c^12 + 6144*(40*B*a^6*b^2
 - 11*A*a^5*b^3)*c^11 - 768*(194*B*a^5*b^4 - 45*A*a^4*b^5)*c^10 +
 1280*(39*B*a^4*b^6 - 7*A*a^3*b^7)*c^9 - 240*(42*B*a^3*b^8 - 5*A*
a^2*b^9)*c^8 + 24*(52*B*a^2*b^10 - 3*A*a*b^11)*c^7 - (90*B*a*b^12
 - A*b^13)*c^6)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*
B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95
256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b
^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2
*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5
 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*
c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280
*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a
^4 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^
2*b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^
3 + (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*
a*b^7 - 2*A*B*b^8)*c - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7
 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((81*B
^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b
+ A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^
2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3
*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3
+ 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54
*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160
*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c
^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c
^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) + sqrt(1/2)*((b^4*c^4 -
8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*
a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2
 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 +
 16*a^3*b*c^4)*x^2)*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^2*a^3*
```

```
b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 -
35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2
*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B
*b^8)*c - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^
4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((81*B^4*b^8 + 625*
A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*
c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2
 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A
*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4
*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^
6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12
 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*
c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4
*b^2*c^9 - 1024*a^5*c^10))*log(-(1701*B^4*a^2*b^8 - 945*A*B^3*a*b
^9 - 10000*A^4*a^4*c^6 + 15000*(6*A^3*B*a^4*b - A^4*a^3*b^2)*c^5
+ 3*(1037232*B^4*a^6 - 1037232*A*B^3*a^5*b + 287712*A^2*B^2*a^4*b
^2 - 32952*A^3*B*a^3*b^3 + 497*A^4*a^2*b^4)*c^4 - (1555848*B^4*a^
5*b^2 - 1298376*A*B^3*a^4*b^3 + 238464*A^2*B^2*a^3*b^4 - 11277*A^
3*B*a^2*b^5 + 35*A^4*a*b^6)*c^3 + 9*(37701*B^4*a^4*b^4 - 26973*A*
B^3*a^3*b^5 + 3066*A^2*B^2*a^2*b^6 - 35*A^3*B*a*b^7)*c^2 - 27*(13
41*B^4*a^3*b^6 - 819*A*B^3*a^2*b^7 + 35*A^2*B^2*a*b^8)*c)*x - 1/2
*sqrt(1/2)*(27*B^3*b^13 + 32000*A^3*a^5*c^8 - 640*(882*A*B^2*a^6
- 156*A^2*B*a^5*b + 37*A^3*a^4*b^2)*c^7 + 64*(10584*B^3*a^6*b + 5
562*A*B^2*a^5*b^2 - 1083*A^2*B*a^4*b^3 + 89*A^3*a^3*b^4)*c^6 - 8*
(93096*B^3*a^5*b^3 + 3816*A*B^2*a^4*b^4 - 1746*A^2*B*a^3*b^5 + 49
*A^3*a^2*b^6)*c^5 + (337392*B^3*a^4*b^5 - 24120*A*B^2*a^3*b^6 - 8
4*A^2*B*a^2*b^7 - 17*A^3*a*b^8)*c^4 - (81324*B^3*a^3*b^7 - 6993*A
*B^2*a^2*b^8 + 195*A^2*B*a*b^9 - A^3*b^10)*c^3 + 9*(1239*B^3*a^2*
b^9 - 79*A*B^2*a*b^10 + A^2*B*b^11)*c^2 - 27*(31*B^3*a*b^11 - A*B
^2*b^12)*c + (3*B*b^14*c^5 - 4096*(42*B*a^7 - 13*A*a^6*b)*c^12 +
6144*(40*B*a^6*b^2 - 11*A*a^5*b^3)*c^11 - 768*(194*B*a^5*b^4 - 45
*A*a^4*b^5)*c^10 + 1280*(39*B*a^4*b^6 - 7*A*a^3*b^7)*c^9 - 240*(4
2*B*a^3*b^8 - 5*A*a^2*b^9)*c^8 + 24*(52*B*a^2*b^10 - 3*A*a*b^11)*
c^7 - (90*B*a*b^12 - A*b^13)*c^6)*sqrt((81*B^4*b^8 + 625*A^4*a^2*
c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (1
94481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A
^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2
*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4
 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*
B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a
^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(9*B^2*b
^9 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*
B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b
^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^7
)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c - (b^10*c^5 - 20*a*b^8*c^6
 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^
5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3
- 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^
3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4
- 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 -
 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2
*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 2
0*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2
*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c
```

```
^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) + 2*(3
*B*a^2*b^3 + 20*A*a^3*c^2 - (24*B*a^3*b - A*a^2*b^2)*c)*x)/((b^4*
c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3
 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b
^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3
*c^3 + 16*a^3*b*c^4)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 44.9131, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^8/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] Done
```

3.133 $\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=461

$$\frac{\left(-\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+12aAc^2-16abBc+3Ab^2c+b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{3/2}(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+12aAc^2-16abBc+3Ab^2c+b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}c^{3/2}(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x^5\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{x^3\left(-x^2\left(20aBc-12Abc+b^2B\right)+4aAc-12abB+5Ab^2\right)}{8(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{x\left(20aBc-12Abc+b^2B\right)}{8c(b^2-4ac)^2}$$

```
[Out] -((b^2*B - 12*A*b*c + 20*a*B*c)*x)/(8*c*(b^2 - 4*a*c)^2) - (x^5*(
A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)^2) - (x^3*(5*A*b^2 - 12*a*b*B + 4*a*A*c - (b^2*B - 12*A*b*c
 + 20*a*B*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((b^
3*B + 3*A*b^2*c - 16*a*b*B*c + 12*a*A*c^2 - (b^4*B + 3*A*b^3*c -
18*a*b^2*B*c + 36*a*A*b*c^2 - 40*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]
*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3*B +
 3*A*b^2*c - 16*a*b*B*c + 12*a*A*c^2 + (b^4*B + 3*A*b^3*c - 18*a*
b^2*B*c + 36*a*A*b*c^2 - 40*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(3
/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 9.64625, antiderivative size = 461, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$\frac{\left(-\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+12aAc^2-16abBc+3Ab^2c+b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{3/2}(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+12aAc^2-16abBc+3Ab^2c+b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}c^{3/2}(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x^5\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{x^3\left(-x^2\left(20aBc-12Abc+b^2B\right)+4aAc-12abB+5Ab^2\right)}{8(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{x\left(20aBc-12Abc+b^2B\right)}{8c(b^2-4ac)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -((b^2*B - 12*A*b*c + 20*a*B*c)*x)/(8*c*(b^2 - 4*a*c)^2) - (x^5*(
A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)^2) - (x^3*(5*A*b^2 - 12*a*b*B + 4*a*A*c - (b^2*B - 12*A*b*c
 + 20*a*B*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((b^
3*B + 3*A*b^2*c - 16*a*b*B*c + 12*a*A*c^2 - (b^4*B + 3*A*b^3*c -
18*a*b^2*B*c + 36*a*A*b*c^2 - 40*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]
*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3*B +
 3*A*b^2*c - 16*a*b*B*c + 12*a*A*c^2 + (b^4*B + 3*A*b^3*c - 18*a*
b^2*B*c + 36*a*A*b*c^2 - 40*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(3
/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 4.39109, size = 543, normalized size = 1.18

$$-\frac{4x\left(2a^2Bc+a\left(bc\left(A+3Bx^2\right)-2Ac^2x^2+b^2(-B)\right)+b^2x^2(Ac-bB)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{2x\left(b^2c\left(11aB+3Acx^2\right)+4abc^2\left(A-4Bx^2\right)+12ac^2\left(Acx^2-3aB\right)+b^3c\left(2A+Bx^2\right)-2b^4B\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+ -$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(-2*b^4*B + 4*a*b*c^2*(A - 4*B*x^2) + b^3*c*(2*A + B*x^2) +
 12*a*c^2*(-3*a*B + A*c*x^2) + b^2*c*(11*a*B + 3*A*c*x^2)))/((b^2
 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (4*x*(2*a^2*B*c + b^2*(-(b*B)
+ A*c)*x^2 + a*(-(b^2*B) - 2*A*c^2*x^2 + b*c*(A + 3*B*x^2))))/((b
^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[2]*Sqrt[c]*(-(b^4*B) +
 3*b^2*c*(6*a*B + A*Sqrt[b^2 - 4*a*c]) + 4*a*c^2*(10*a*B + 3*A*Sq
```

```
rt[b^2 - 4*a*c]) + b^3*(-3*A*c + B*Sqrt[b^2 - 4*a*c]) - 4*a*b*c*(
9*A*c + 4*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 -
4*a*c]]) + (Sqrt[2]*Sqrt[c]*(b^4*B + 3*b^2*c*(-6*a*B + A*Sqrt[b^2
 - 4*a*c]) + 4*a*c^2*(-10*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + 4*a*b*c*
(9*A*c - 4*B*Sqrt[b^2 - 4*a*c]) + b^3*(3*A*c + B*Sqrt[b^2 - 4*a*c
]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^
2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c^2)
```

Maple [B] time = 0.104, size = 9168, normalized size = 19.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(Bb^3c + 12\,Aac^3 - \left(16\,Bab - 3\,Ab^2\right)c^2\right)x^7 - \left(Bb^4 + 4\left(9\,Ba^2 - 4\,Aab\right)c^2 + 5\left(Bab^2 - Ab^3\right)c\right)x^5 - \left(2\,Bab^3 + 4\,Aa^2c^2 + (28\,B\right.}{8\left((b^4c^3 - 8\,ab^2c^4 + 16\,a^2c^5)x^8 + a^2b^4c - 8\,a^3b^2c^2 + 16\,a^4c^3 + 2\,(b^5c^2 - 8\,ab^3c^3 + 16\,a^2bc^4)x^6 + (b^6c - 6\,ab^4c^2 + 32\right.}$$
$$+ \frac{\int \frac{Bab^2 + \left(Bb^3 + 12\,Aac^2 - \left(16\,Bab - 3\,Ab^2\right)c\right)x^2 + 4\left(5\,Ba^2 - 3\,Aab\right)c}{cx^4 + bx^2 + a}\,dx}{8\left(b^4c - 8\,ab^2c^2 + 16\,a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^6/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((B*b^3*c + 12*A*a*c^3 - (16*B*a*b - 3*A*b^2)*c^2)*x^7 - (B*b
^4 + 4*(9*B*a^2 - 4*A*a*b)*c^2 + 5*(B*a*b^2 - A*b^3)*c)*x^5 - (2*
B*a*b^3 + 4*A*a^2*c^2 + (28*B*a^2*b - 19*A*a*b^2)*c)*x^3 - (B*a^2
*b^2 + 4*(5*B*a^3 - 3*A*a^2*b)*c)*x)/((b^4*c^3 - 8*a*b^2*c^4 + 16
*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c
^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*
a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2) +
1/8*integrate((B*a*b^2 + (B*b^3 + 12*A*a*c^2 - (16*B*a*b - 3*A*b^
2)*c)*x^2 + 4*(5*B*a^2 - 3*A*a*b)*c)/(c*x^4 + b*x^2 + a), x)/(b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)
```

Fricas [A] time = 3.00413, size = 9531, normalized size = 20.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^6/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(B*b^3*c + 12*A*a*c^3 - (16*B*a*b - 3*A*b^2)*c^2)*x^7 - 2
*(B*b^4 + 4*(9*B*a^2 - 4*A*a*b)*c^2 + 5*(B*a*b^2 - A*b^3)*c)*x^5
- 2*(2*B*a*b^3 + 4*A*a^2*c^2 + (28*B*a^2*b - 19*A*a*b^2)*c)*x^3 -
 sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c
- 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*
b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c
- 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(B^2*b^7 - 240*(4*A*B*
a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A
^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2
- (35*B^2*a*b^5 - 6*A*B*b^6)*c + (b^10*c^3 - 20*a*b^8*c^4 + 160*a
^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*s
qrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (
625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b
^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 -
 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3
 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^
2*c^7 - 1024*a^5*c^8))*log(-(35*B^4*a*b^6 - 15*A*B^3*b^7 - 1296*A
^4*a^2*c^5 + 648*(14*A^3*B*a^2*b - 5*A^4*a*b^2)*c^4 + (10000*B^4*
a^4 - 30000*A*B^3*a^3*b + 9936*A^2*B^2*a^2*b^2 + 1080*A^3*B*a*b^3
 - 405*A^4*b^4)*c^3 + 3*(5000*B^4*a^3*b^2 - 3864*A*B^3*a^2*b^3 +
1080*A^2*B^2*a*b^4 - 135*A^3*B*b^5)*c^2 - 3*(497*B^4*a^2*b^4 - 31
5*A*B^3*a*b^5 + 45*A^2*B^2*b^6)*c)*x + 1/2*sqrt(1/2)*(B^3*b^10 -
2304*(5*A^2*B*a^4 - 3*A^3*a^3*b)*c^6 + 64*(500*B^3*a^5 - 420*A*B^
2*a^4*b + 198*A^2*B*a^3*b^2 - 81*A^3*a^2*b^3)*c^5 - 16*(1480*B^3*
a^4*b^2 - 1284*A*B^2*a^3*b^3 + 324*A^2*B*a^2*b^4 - 81*A^3*a*b^5)*
c^4 + 4*(1424*B^3*a^3*b^4 - 1332*A*B^2*a^2*b^5 + 234*A^2*B*a*b^6
- 27*A^3*b^7)*c^3 - (392*B^3*a^2*b^6 - 492*A*B^2*a*b^7 + 63*A^2*B
*b^8)*c^2 - (17*B^3*a*b^8 + 6*A*B^2*b^9)*c - (B*b^13*c^3 - 24576*
A*a^6*c^10 + 4096*(13*B*a^6*b + 3*A*a^5*b^2)*c^9 - 1536*(44*B*a^5
*b^3 - 5*A*a^4*b^4)*c^8 + 3840*(9*B*a^4*b^5 - 2*A*a^3*b^6)*c^7 -
160*(56*B*a^3*b^7 - 15*A*a^2*b^8)*c^6 + 48*(25*B*a^2*b^9 - 7*A*a*
b^10)*c^5 - 18*(4*B*a*b^11 - A*b^12)*c^4)*sqrt((B^4*b^4 + 81*A^4*
c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^
3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(
b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 128
0*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(B^2*b^7 - 240*(4*A*B*a^3
 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*
a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (
35*B^2*a*b^5 - 6*A*B*b^6)*c + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*
b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt
((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625
*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2
- 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 64
0*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 -
```

20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c
^7 - 1024*a^5*c^8))) + sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2
*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 -
 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*
c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-
(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b
- 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a
*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c + (b^10*c^3
- 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2
*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*
a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^
2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c
^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024
*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3
*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log(-(35*B^4*a*b^6 -
 15*A*B^3*b^7 - 1296*A^4*a^2*c^5 + 648*(14*A^3*B*a^2*b - 5*A^4*a*
b^2)*c^4 + (10000*B^4*a^4 - 30000*A*B^3*a^3*b + 9936*A^2*B^2*a^2*
b^2 + 1080*A^3*B*a*b^3 - 405*A^4*b^4)*c^3 + 3*(5000*B^4*a^3*b^2 -
 3864*A*B^3*a^2*b^3 + 1080*A^2*B^2*a*b^4 - 135*A^3*B*b^5)*c^2 - 3
*(497*B^4*a^2*b^4 - 315*A*B^3*a*b^5 + 45*A^2*B^2*b^6)*c)*x - 1/2*
sqrt(1/2)*(B^3*b^10 - 2304*(5*A^2*B*a^4 - 3*A^3*a^3*b)*c^6 + 64*(
500*B^3*a^5 - 420*A*B^2*a^4*b + 198*A^2*B*a^3*b^2 - 81*A^3*a^2*b^
3)*c^5 - 16*(1480*B^3*a^4*b^2 - 1284*A*B^2*a^3*b^3 + 324*A^2*B*a^
2*b^4 - 81*A^3*a*b^5)*c^4 + 4*(1424*B^3*a^3*b^4 - 1332*A*B^2*a^2*
b^5 + 234*A^2*B*a*b^6 - 27*A^3*b^7)*c^3 - (392*B^3*a^2*b^6 - 492*
A*B^2*a*b^7 + 63*A^2*B*b^8)*c^2 - (17*B^3*a*b^8 + 6*A*B^2*b^9)*c
- (B*b^13*c^3 - 24576*A*a^6*c^10 + 4096*(13*B*a^6*b + 3*A*a^5*b^2
)*c^9 - 1536*(44*B*a^5*b^3 - 5*A*a^4*b^4)*c^8 + 3840*(9*B*a^4*b^5
 - 2*A*a^3*b^6)*c^7 - 160*(56*B*a^3*b^7 - 15*A*a^2*b^8)*c^6 + 48*
(25*B*a^2*b^9 - 7*A*a*b^10)*c^5 - 18*(4*B*a*b^11 - A*b^12)*c^4)*s
qrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (
625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b
^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 -
 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(B^
2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 1
6*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^
4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c + (b^10*c^3 - 2
0*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^
7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a -
 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*
c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7
+ 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^
5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^
4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) - sqrt(1/2)*((b^4*c^3
- 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*
a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c -
 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*
a^3*b*c^3)*x^2)*sqrt(-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^
4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*
B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A
*B*b^6)*c - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*
b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4
c^4 - 18(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B
^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/

(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 12
80*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160
*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)
)*log(-(35*B^4*a*b^6 - 15*A*B^3*b^7 - 1296*A^4*a^2*c^5 + 648*(14*
A^3*B*a^2*b - 5*A^4*a*b^2)*c^4 + (10000*B^4*a^4 - 30000*A*B^3*a^3
*b + 9936*A^2*B^2*a^2*b^2 + 1080*A^3*B*a*b^3 - 405*A^4*b^4)*c^3 +
3*(5000*B^4*a^3*b^2 - 3864*A*B^3*a^2*b^3 + 1080*A^2*B^2*a*b^4 -
135*A^3*B*b^5)*c^2 - 3*(497*B^4*a^2*b^4 - 315*A*B^3*a*b^5 + 45*A^
2*B^2*b^6)*c)*x + 1/2*sqrt(1/2)*(B^3*b^10 - 2304*(5*A^2*B*a^4 - 3
*A^3*a^3*b)*c^6 + 64*(500*B^3*a^5 - 420*A*B^2*a^4*b + 198*A^2*B*a
^3*b^2 - 81*A^3*a^2*b^3)*c^5 - 16*(1480*B^3*a^4*b^2 - 1284*A*B^2*
a^3*b^3 + 324*A^2*B*a^2*b^4 - 81*A^3*a*b^5)*c^4 + 4*(1424*B^3*a^3
*b^4 - 1332*A*B^2*a^2*b^5 + 234*A^2*B*a*b^6 - 27*A^3*b^7)*c^3 - (
392*B^3*a^2*b^6 - 492*A*B^2*a*b^7 + 63*A^2*B*b^8)*c^2 - (17*B^3*a
*b^8 + 6*A*B^2*b^9)*c + (B*b^13*c^3 - 24576*A*a^6*c^10 + 4096*(13
*B*a^6*b + 3*A*a^5*b^2)*c^9 - 1536*(44*B*a^5*b^3 - 5*A*a^4*b^4)*c
^8 + 3840*(9*B*a^4*b^5 - 2*A*a^3*b^6)*c^7 - 160*(56*B*a^3*b^7 - 1
5*A*a^2*b^8)*c^6 + 48*(25*B*a^2*b^9 - 7*A*a*b^10)*c^5 - 18*(4*B*a
*b^11 - A*b^12)*c^4)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*
a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^
2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c
^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024
*a^5*c^11)))*sqrt(-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 +
120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*B^2
*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*
b^6)*c - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4
*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^
4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*
a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^
10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*
a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^
2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)))
+ sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c
- 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2
*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c
- 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(B^2*b^7 - 240*(4*A*B
*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*
A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2
- (35*B^2*a*b^5 - 6*A*B*b^6)*c - (b^10*c^3 - 20*a*b^8*c^4 + 160*
a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*
sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 +
(625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*
b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8
- 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^
3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b
^2*c^7 - 1024*a^5*c^8))*log(-(35*B^4*a*b^6 - 15*A*B^3*b^7 - 1296*
A^4*a^2*c^5 + 648*(14*A^3*B*a^2*b - 5*A^4*a*b^2)*c^4 + (10000*B^4
*a^4 - 30000*A*B^3*a^3*b + 9936*A^2*B^2*a^2*b^2 + 1080*A^3*B*a*b^
3 - 405*A^4*b^4)*c^3 + 3*(5000*B^4*a^3*b^2 - 3864*A*B^3*a^2*b^3 +
1080*A^2*B^2*a*b^4 - 135*A^3*B*b^5)*c^2 - 3*(497*B^4*a^2*b^4 - 3
15*A*B^3*a*b^5 + 45*A^2*B^2*b^6)*c)*x - 1/2*sqrt(1/2)*(B^3*b^10 -
2304*(5*A^2*B*a^4 - 3*A^3*a^3*b)*c^6 + 64*(500*B^3*a^5 - 420*A*B
^2*a^4*b + 198*A^2*B*a^3*b^2 - 81*A^3*a^2*b^3)*c^5 - 16*(1480*B^3
*a^4*b^2 - 1284*A*B^2*a^3*b^3 + 324*A^2*B*a^2*b^4 - 81*A^3*a*b^5)

```
*c^4 + 4*(1424*B^3*a^3*b^4 - 1332*A*B^2*a^2*b^5 + 234*A^2*B*a*b^6
 - 27*A^3*b^7)*c^3 - (392*B^3*a^2*b^6 - 492*A*B^2*a*b^7 + 63*A^2*
B*b^8)*c^2 - (17*B^3*a*b^8 + 6*A*B^2*b^9)*c + (B*b^13*c^3 - 24576
*A*a^6*c^10 + 4096*(13*B*a^6*b + 3*A*a^5*b^2)*c^9 - 1536*(44*B*a^
5*b^3 - 5*A*a^4*b^4)*c^8 + 3840*(9*B*a^4*b^5 - 2*A*a^3*b^6)*c^7 -
 160*(56*B*a^3*b^7 - 15*A*a^2*b^8)*c^6 + 48*(25*B*a^2*b^9 - 7*A*a
*b^10)*c^5 - 18*(4*B*a*b^11 - A*b^12)*c^4)*sqrt((B^4*b^4 + 81*A^4
*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B
^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/
(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 12
80*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(B^2*b^7 - 240*(4*A*B*a^
3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2
*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 -
(35*B^2*a*b^5 - 6*A*B*b^6)*c - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2
*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqr
t((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (62
5*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2
 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 6
40*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 -
 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*
c^7 - 1024*a^5*c^8))) - 2*(B*a^2*b^2 + 4*(5*B*a^3 - 3*A*a^2*b)*c)
*x)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3
*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*
x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2
*b^3*c^2 + 16*a^3*b*c^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^6/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.134 $\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=380

$$\frac{3x\left(x^2\left(4aBc-4Abc+b^2B\right)-A\left(4ac+b^2\right)+4abB\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^3\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3\left(-\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -(x^3*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)^2) + (3*x*(4*a*b*B - A*(b^2 + 4*a*c) + (b^2*B - 4*A*
b*c + 4*a*B*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3
*(b^2*B - 4*A*b*c + 4*a*B*c - (b^3*B - 6*A*b^2*c + 12*a*b*B*c - 8
*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^2*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) + (3*(b^2*B - 4*A*b*c + 4*a*B*c + (b^3*B - 6*
A*b^2*c + 12*a*b*B*c - 8*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(
b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 2.6797, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{3x\left(x^2\left(4aBc-4Abc+b^2B\right)-A\left(4ac+b^2\right)+4abB\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^3\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3\left(-\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^3*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)^2) + (3*x*(4*a*b*B - A*(b^2 + 4*a*c) + (b^2*B - 4*A*
```

```
b*c + 4*a*B*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3
*(b^2*B - 4*A*b*c + 4*a*B*c - (b^3*B - 6*A*b^2*c + 12*a*b*B*c - 8
*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^2*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) + (3*(b^2*B - 4*A*b*c + 4*a*B*c + (b^3*B - 6*
A*b^2*c + 12*a*b*B*c - 8*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(
b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 153.902, size = 408, normalized size = 1.07

$$-\frac{x^3\left(Ab-2Ba+x^2\left(2Ac-Bb\right)\right)}{4\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(12Aac+3Ab^2-12Bab-x^2\left(-12Abc+12Bac+3Bb^2\right)\right)}{8\left(-4ac+b^2\right)^2\left(a+bx^2+cx^4\right)}$$
$$+\frac{3\sqrt{2}\left(b\left(-4Abc+4Bac+Bb^2\right)-2c\left(4Aac+Ab^2-4Bab\right)+\sqrt{-4ac+b^2}\left(-4Abc+4Bac+Bb^2\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{16\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{5}{2}}}$$
$$-\frac{3\sqrt{2}\left(b\left(-4Abc+4Bac+Bb^2\right)-2c\left(4Aac+Ab^2-4Bab\right)-\sqrt{-4ac+b^2}\left(-4Abc+4Bac+Bb^2\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{16\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -x**3*(A*b - 2*B*a + x**2*(2*A*c - B*b))/(4*(-4*a*c + b**2)*(a +
b*x**2 + c*x**4)**2) - x*(12*A*a*c + 3*A*b**2 - 12*B*a*b - x**2*(
-12*A*b*c + 12*B*a*c + 3*B*b**2))/(8*(-4*a*c + b**2)**2*(a + b*x*
*2 + c*x**4)) + 3*sqrt(2)*(b*(-4*A*b*c + 4*B*a*c + B*b**2) - 2*c*
(4*A*a*c + A*b**2 - 4*B*a*b) + sqrt(-4*a*c + b**2)*(-4*A*b*c + 4*
B*a*c + B*b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b*
*2)))/(16*sqrt(c)*sqrt(b + sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**
(5/2)) - 3*sqrt(2)*(b*(-4*A*b*c + 4*B*a*c + B*b**2) - 2*c*(4*A*a*
c + A*b**2 - 4*B*a*b) - sqrt(-4*a*c + b**2)*(-4*A*b*c + 4*B*a*c +
 B*b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(
16*sqrt(c)*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(5/2))
```

Mathematica [A] time = 3.37491, size = 447, normalized size = 1.18

$$\frac{\frac{8acx\left(A+Bx^2\right)-4abBx+4bx^3(Ac-bB)}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{2x\left(4bc\left(aB-3Acx^2\right)+4ac^2\left(A+3Bx^2\right)+b^2\left(3Bcx^2-7Ac\right)+2b^3B\right)}{(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{3\sqrt{2}\sqrt{c}\left(b^2\left(B\sqrt{b^2-4ac}+6Ac\right)-4bc\left(A\sqrt{b^2-4ac}+3\ldots\right.\right.}{(b^2-4ac\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*a*b*B*x + 4*b*(-(b*B) + A*c)*x^3 + 8*a*c*x*(A + B*x^2))/((b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*x*(2*b^3*B + 4*a*c^2*(A +
3*B*x^2) + 4*b*c*(a*B - 3*A*c*x^2) + b^2*(-7*A*c + 3*B*c*x^2)))/(
(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(-(b^3*
B) - 4*b*c*(3*a*B + A*Sqrt[b^2 - 4*a*c]) + 4*a*c*(2*A*c + B*Sqrt[
b^2 - 4*a*c]) + b^2*(6*A*c + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(b^3*B + 4*b*c*(
3*a*B - A*Sqrt[b^2 - 4*a*c]) + b^2*(-6*A*c + B*Sqrt[b^2 - 4*a*c])
 + 4*a*c*(-2*A*c + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sq
rt[b^2 - 4*a*c]]))/(16*c)
```

Maple [B] time = 0.102, size = 7611, normalized size = 20.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(Bb^{2}c+4\left(Ba-Ab\right)c^{2}\right)x^{7}+\left(5\,Bb^{3}+4\,Aac^{2}+\left(16\,Bab-19\,Ab^{2}\right)c\right)x^{5}+\left(19\,Bab^{2}-5\,Ab^{3}-4\left(Ba^{2}+4\,Aab\right)c\right)x^{3}+3\left(4\,B\cdots\right.}{8\left((b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4})x^{8}+2\,(b^{5}c-8\,ab^{3}c^{2}+16\,a^{2}bc^{3})x^{6}+a^{2}b^{4}-8\,a^{3}b^{2}c+16\,a^{4}c^{2}+(b^{6}-6\,ab^{4}c+32\,a^{3}c^{3})x^{4}+2\,(ab\cdots\right.}$$
$$-\frac{3\int\frac{4\,Bab-Ab^{2}-4\,Aac-\left(Bb^{2}+4\,(Ba-Ab)c\right)x^{2}}{cx^{4}+bx^{2}+a}\,dx}{8\,(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^4/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(B*b^2*c + 4*(B*a - A*b)*c^2)*x^7 + (5*B*b^3 + 4*A*a*c^2 +
 (16*B*a*b - 19*A*b^2)*c)*x^5 + (19*B*a*b^2 - 5*A*b^3 - 4*(B*a^2
+ 4*A*a*b)*c)*x^3 + 3*(4*B*a^2*b - A*a*b^2 - 4*A*a^2*c)*x)/((b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16
*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a
*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)
*x^2) - 3/8*integrate((4*B*a*b - A*b^2 - 4*A*a*c - (B*b^2 + 4*(B*
```

```
a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^4 - 8*a*b^2*c + 16*a^
2*c^2)
```

Fricas [A] time = 2.09226, size = 7628, normalized size = 20.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^4/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(B*b^2*c + 4*(B*a - A*b)*c^2)*x^7 + 2*(5*B*b^3 + 4*A*a*c^
2 + (16*B*a*b - 19*A*b^2)*c)*x^5 + 2*(19*B*a*b^2 - 5*A*b^3 - 4*(B
*a^2 + 4*A*a*b)*c)*x^3 - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16
*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*
b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x
^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(B^2*a*b^5
 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2
*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)
*c + (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c
^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a
*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 -
640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c -
20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2
*c^5 - 1024*a^6*c^6))*log(-27*(5*B^4*a^2*b^4 - A*B^3*a*b^5 - 16*A
^4*a^2*c^4 + 40*(2*A^3*B*a^2*b - A^4*a*b^2)*c^3 + (16*B^4*a^4 - 8
0*A*B^3*a^3*b + 40*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 + (40*B^4*a^3*b^2
 - 40*A*B^3*a^2*b^3 + A^3*B*b^5)*c)*x + 27/2*sqrt(1/2)*(4*B^3*a^2
*b^7 - A*B^2*a*b^8 - 256*A^3*a^4*c^5 + 128*(2*A*B^2*a^5 + 2*A^2*B
*a^4*b + A^3*a^3*b^2)*c^4 - 64*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 + 3
*A^2*B*a^3*b^3)*c^3 + 8*(24*B^3*a^4*b^3 + 6*A^2*B*a^2*b^5 - A^3*a
*b^6)*c^2 - (48*B^3*a^3*b^5 - 8*A*B^2*a^2*b^6 + 4*A^2*B*a*b^7 - A
^3*b^8)*c - (4096*(2*B*a^8 - 3*A*a^7*b)*c^7 - 2048*(2*B*a^7*b^2 -
 7*A*a^6*b^3)*c^6 - 1280*(2*B*a^6*b^4 + 5*A*a^5*b^5)*c^5 + 1280*(
2*B*a^5*b^6 + A*a^4*b^7)*c^4 - 80*(10*B*a^4*b^8 + A*a^3*b^9)*c^3
+ 8*(14*B*a^3*b^10 - A*a^2*b^11)*c^2 - (6*B*a^2*b^12 - A*a*b^13)*
c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^
3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6
- 1024*a^7*c^7)))*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)
*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2
*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c + (a*b^10*c - 20*a^2*b^8*c^2
 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^
6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 2
0*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*
c^6 - 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^
3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))) + 3*sqrt
(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2
+ (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 1
6*a^3*b*c^2)*x^2)*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)
```

```
*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2
*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c + (a*b^10*c - 20*a^2*b^8*c^2
 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^
6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 2
0*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*
c^6 - 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^
3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))*log(-27*(
5*B^4*a^2*b^4 - A*B^3*a*b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3*B*a^2*b
 - A^4*a*b^2)*c^3 + (16*B^4*a^4 - 80*A*B^3*a^3*b + 40*A^3*B*a*b^3
 - 5*A^4*b^4)*c^2 + (40*B^4*a^3*b^2 - 40*A*B^3*a^2*b^3 + A^3*B*b^5
)*c)*x - 27/2*sqrt(1/2)*(4*B^3*a^2*b^7 - A*B^2*a*b^8 - 256*A^3*a^
4*c^5 + 128*(2*A*B^2*a^5 + 2*A^2*B*a^4*b + A^3*a^3*b^2)*c^4 - 64*
(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 + 3*A^2*B*a^3*b^3)*c^3 + 8*(24*B^3
*a^4*b^3 + 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^2 - (48*B^3*a^3*b^5 - 8
*A*B^2*a^2*b^6 + 4*A^2*B*a*b^7 - A^3*b^8)*c - (4096*(2*B*a^8 - 3*
A*a^7*b)*c^7 - 2048*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2*B*a
^6*b^4 + 5*A*a^5*b^5)*c^5 + 1280*(2*B*a^5*b^6 + A*a^4*b^7)*c^4 -
80*(10*B*a^4*b^8 + A*a^3*b^9)*c^3 + 8*(14*B*a^3*b^10 - A*a^2*b^11
)*c^2 - (6*B*a^2*b^12 - A*a*b^13)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*
c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 6
40*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))*sqrt(-(B^2*a*
b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*
a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b
^5)*c + (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^
4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^
2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4
 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c
 - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*
b^2*c^5 - 1024*a^6*c^6))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 +
 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a
^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3
)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(B^2*a*
b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*
a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b
^5)*c - (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^
4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^
2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4
 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c
 - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*
b^2*c^5 - 1024*a^6*c^6))*log(-27*(5*B^4*a^2*b^4 - A*B^3*a*b^5 - 1
6*A^4*a^2*c^4 + 40*(2*A^3*B*a^2*b - A^4*a*b^2)*c^3 + (16*B^4*a^4
 - 80*A*B^3*a^3*b + 40*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 + (40*B^4*a^3*
b^2 - 40*A*B^3*a^2*b^3 + A^3*B*b^5)*c)*x + 27/2*sqrt(1/2)*(4*B^3*
a^2*b^7 - A*B^2*a*b^8 - 256*A^3*a^4*c^5 + 128*(2*A*B^2*a^5 + 2*A^
2*B*a^4*b + A^3*a^3*b^2)*c^4 - 64*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2
+ 3*A^2*B*a^3*b^3)*c^3 + 8*(24*B^3*a^4*b^3 + 6*A^2*B*a^2*b^5 - A^
3*a*b^6)*c^2 - (48*B^3*a^3*b^5 - 8*A*B^2*a^2*b^6 + 4*A^2*B*a*b^7
 - A^3*b^8)*c + (4096*(2*B*a^8 - 3*A*a^7*b)*c^7 - 2048*(2*B*a^7*b^
2 - 7*A*a^6*b^3)*c^6 - 1280*(2*B*a^6*b^4 + 5*A*a^5*b^5)*c^5 + 128
0*(2*B*a^5*b^6 + A*a^4*b^7)*c^4 - 80*(10*B*a^4*b^8 + A*a^3*b^9)*c
^3 + 8*(14*B*a^3*b^10 - A*a^2*b^11)*c^2 - (6*B*a^2*b^12 - A*a*b^1
3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20
*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c
^6 - 1024*a^7*c^7)))*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2
```

```
*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*
B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c - (a*b^10*c - 20*a^2*b^8*
c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024
*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2
- 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b
^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6
*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))) + 3*s
qrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c
^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c
+ 16*a^3*b*c^2)*x^2)*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2
*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*
B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c - (a*b^10*c - 20*a^2*b^8*
c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024
*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2
- 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b
^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6
*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))*log(-2
7*(5*B^4*a^2*b^4 - A*B^3*a*b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3*B*a^2
*b - A^4*a*b^2)*c^3 + (16*B^4*a^4 - 80*A*B^3*a^3*b + 40*A^3*B*a*b
^3 - 5*A^4*b^4)*c^2 + (40*B^4*a^3*b^2 - 40*A*B^3*a^2*b^3 + A^3*B*
b^5)*c)*x - 27/2*sqrt(1/2)*(4*B^3*a^2*b^7 - A*B^2*a*b^8 - 256*A^3
*a^4*c^5 + 128*(2*A*B^2*a^5 + 2*A^2*B*a^4*b + A^3*a^3*b^2)*c^4 -
64*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 + 3*A^2*B*a^3*b^3)*c^3 + 8*(24*
B^3*a^4*b^3 + 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^2 - (48*B^3*a^3*b^5
- 8*A*B^2*a^2*b^6 + 4*A^2*B*a*b^7 - A^3*b^8)*c + (4096*(2*B*a^8 -
 3*A*a^7*b)*c^7 - 2048*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2*
B*a^6*b^4 + 5*A*a^5*b^5)*c^5 + 1280*(2*B*a^5*b^6 + A*a^4*b^7)*c^4
 - 80*(10*B*a^4*b^8 + A*a^3*b^9)*c^3 + 8*(14*B*a^3*b^10 - A*a^2*b
^11)*c^2 - (6*B*a^2*b^12 - A*a*b^13)*c)*sqrt((B^4*a^2 - 2*A^2*B^2
*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4
- 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))*sqrt(-(B^2
*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A
*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^
2*b^5)*c - (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4
*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2
*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*
c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^1
0*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a
^5*b^2*c^5 - 1024*a^6*c^6))) + 6*(4*B*a^2*b - A*a*b^2 - 4*A*a^2*c
)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 +
 (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16
*a^3*b*c^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 37.0532, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^4/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] Done
```

3.135 $\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=438

$$-\frac{x\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(-A\left(8abc+b^3\right)+cx^2\left(12abB-A\left(20ac+b^2\right)\right)+aB\left(7b^2-4ac\right)\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(A\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)+6aB\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A\left(-b^2\sqrt{b^2-4ac}-20ac\sqrt{b^2-4ac}-52abc+b^3\right)+6aB\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)^2) - (x*(a*B*(7*b^2 - 4*a*c) - A*(b^3 + 8*a*b*c) + c*(
12*a*b*B - A*(b^2 + 20*a*c))*x^2))/(8*a*(b^2 - 4*a*c)^2*(a + b*x^
2 + c*x^4)) + (Sqrt[c]*(6*a*B*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a
*c]) + A*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^
2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]
) - (Sqrt[c]*(6*a*B*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c]) + A*(
b^3 - 52*a*b*c - b^2*Sqrt[b^2 - 4*a*c] - 20*a*c*Sqrt[b^2 - 4*a*c]
))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sq
rt[2]*a*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 2.12677, antiderivative size = 438, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$-\frac{x\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(-A\left(8abc+b^3\right)+cx^2\left(12abB-A\left(20ac+b^2\right)\right)+aB\left(7b^2-4ac\right)\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(A\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)+6aB\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A\left(-b^2\sqrt{b^2-4ac}-20ac\sqrt{b^2-4ac}-52abc+b^3\right)+6aB\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)^2) - (x*(a*B*(7*b^2 - 4*a*c) - A*(b^3 + 8*a*b*c) + c*(
12*a*b*B - A*(b^2 + 20*a*c))*x^2))/(8*a*(b^2 - 4*a*c)^2*(a + b*x^
2 + c*x^4)) + (Sqrt[c]*(6*a*B*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a
*c]) + A*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^
2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]
) - (Sqrt[c]*(6*a*B*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c]) + A*(
b^3 - 52*a*b*c - b^2*Sqrt[b^2 - 4*a*c] - 20*a*c*Sqrt[b^2 - 4*a*c]
))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sq
rt[2]*a*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.89292, size = 436, normalized size = 1.

$$\frac{1}{16}\left(\frac{4x\left(B\left(2a+bx^2\right)-A\left(b+2cx^2\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{2x\left(A\left(8abc+20ac^2x^2+b^3+b^2cx^2\right)+aB\left(4ac-7b^2-12bcx^2\right)\right)}{a(b^2-4ac)^2(a+bx^2+cx^4)}\right.$$
$$+\frac{\sqrt{2}\sqrt{c}\left(A\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)+6aB\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$\left.+\frac{\sqrt{2}\sqrt{c}\left(A\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}+52abc-b^3\right)-6aB\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a(b^2-4ac)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*x*(B*(2*a + b*x^2) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)^2) + (2*x*(a*B*(-7*b^2 + 4*a*c - 12*b*c*x^2) + A*(b^
3 + 8*a*b*c + b^2*c*x^2 + 20*a*c^2*x^2)))/(a*(b^2 - 4*a*c)^2*(a +
 b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(6*a*B*(3*b^2 + 4*a*c - 2*b*S
qrt[b^2 - 4*a*c]) + A*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 2
0*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (Sqrt[2]*Sqrt[c]*(-6*a*B*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4
*a*c]) + A*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt
[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/16
```

Maple [B] time = 0.202, size = 8433, normalized size = 19.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(20\,Aac^3 - (12\,Bab - Ab^2)\,c^2)\,x^7 + (4\,(Ba^2 + 7\,Aab)\,c^2 - (19\,Bab^2 - 2\,Ab^3)\,c)\,x^5 - (5\,Bab^3 - Ab^4 - 36\,Aa^2c^2 + (16\,Ba^2b - \cdots}{8\,((ab^4c^2 - 8\,a^2b^2c^3 + 16\,a^3c^4)x^8 + a^3b^4 - 8\,a^4b^2c + 16\,a^5c^2 + 2\,(ab^5c - 8\,a^2b^3c^2 + 16\,a^3bc^3)x^6 + (ab^6 - 6\,a^2b^4c + \cdots} + \frac{\int \frac{3\,Bab^2 + Ab^3 + (20\,Aac^2 - (12\,Bab - Ab^2)\,c)\,x^2 + 4\,(3\,Ba^2 - 4\,Aab)\,c}{cx^4 + bx^2 + a}\,dx}{8\,(ab^4 - 8\,a^2b^2c + 16\,a^3c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^2/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((20*A*a*c^3 - (12*B*a*b - A*b^2)*c^2)*x^7 + (4*(B*a^2 + 7*A*
a*b)*c^2 - (19*B*a*b^2 - 2*A*b^3)*c)*x^5 - (5*B*a*b^3 - A*b^4 - 3
6*A*a^2*c^2 + (16*B*a^2*b - 5*A*a*b^2)*c)*x^3 - (3*B*a^2*b^2 + A*
a*b^3 + 4*(3*B*a^3 - 4*A*a^2*b)*c)*x)/((a*b^4*c^2 - 8*a^2*b^2*c^3
 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^
5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c +
32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2) +
 1/8*integrate((3*B*a*b^2 + A*b^3 + (20*A*a*c^2 - (12*B*a*b - A*b
^2)*c)*x^2 + 4*(3*B*a^2 - 4*A*a*b)*c)/(c*x^4 + b*x^2 + a), x)/(a*
b^4 - 8*a^2*b^2*c + 16*a^3*c^2)
```

Fricas [A] time = 3.93732, size = 9815, normalized size = 22.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^2/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(20*A*a*c^3 - (12*B*a*b - A*b^2)*c^2)*x^7 + 2*(4*(B*a^2 +
 7*A*a*b)*c^2 - (19*B*a*b^2 - 2*A*b^3)*c)*x^5 - 2*(5*B*a*b^3 - A*
b^4 - 36*A*a^2*c^2 + (16*B*a^2*b - 5*A*a*b^2)*c)*x^3 + sqrt(1/2)*
((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b
^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^
6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b
^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^
2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48
*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^
2*b^4 - 7*A^2*a*b^5)*c + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c
^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*
B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 +
 A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b +
A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^
9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a
^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 -
 1024*a^8*c^5))*log((10000*A^4*a^3*c^5 - 15000*(2*A^3*B*a^3*b - A
^4*a^2*b^2)*c^4 - 3*(432*B^4*a^5 - 3024*A*B^3*a^4*b - 3312*A^2*B^
2*a^3*b^2 + 3864*A^3*B*a^2*b^3 + 497*A^4*a*b^4)*c^3 - 5*(648*B^4*
a^4*b^2 - 216*A*B^3*a^3*b^3 - 648*A^2*B^2*a^2*b^4 - 189*A^3*B*a*b
^5 - 7*A^4*b^6)*c^2 - 15*(27*B^4*a^3*b^4 + 27*A*B^3*a^2*b^5 + 9*A
^2*B^2*a*b^6 + A^3*B*b^7)*c)*x + 1/2*sqrt(1/2)*(27*B^3*a^3*b^8 +
27*A*B^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^3*b^11 + 6400*(3*A^2*B*a^6
- 4*A^3*a^5*b)*c^5 - 64*(108*B^3*a^7 - 72*A*B^2*a^6*b + 66*A^2*B*
a^5*b^2 - 341*A^3*a^4*b^3)*c^4 + 16*(216*B^3*a^6*b^2 - 324*A*B^2*
a^5*b^3 - 288*A^2*B*a^4*b^4 - 427*A^3*a^3*b^5)*c^3 + 20*(108*A*B^
2*a^4*b^5 + 102*A^2*B*a^3*b^6 + 47*A^3*a^2*b^7)*c^2 - (216*B^3*a^
4*b^6 + 396*A*B^2*a^3*b^7 + 267*A^2*B*a^2*b^8 + 53*A^3*a*b^9)*c -
 (3*B*a^4*b^13 + A*a^3*b^14 + 40960*A*a^10*c^7 - 4096*(9*B*a^10*b
 + 8*A*a^9*b^2)*c^6 + 1536*(28*B*a^9*b^3 + A*a^8*b^4)*c^5 - 6400*
(3*B*a^8*b^5 - A*a^7*b^6)*c^4 + 160*(24*B*a^7*b^7 - 17*A*a^6*b^8)
*c^3 - 240*(B*a^6*b^9 - 2*A*a^5*b^10)*c^2 - 2*(12*B*a^5*b^11 + 19
*A*a^4*b^12)*c)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a
^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B
^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c +
 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^1
1*c^5)))*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*
B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*
A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b
^5)*c + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*
c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B
^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^
```

4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^
6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*
a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^
5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)))
 - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b
^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a
^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b
^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(9*B^2*a^2*b^5 + 6*A*
B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^
2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3
 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c + (a^3*b^10 - 20*a^4*b^8*c + 1
60*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^
5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A
^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^
3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*
c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3
*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a
^7*b^2*c^4 - 1024*a^8*c^5))*log((10000*A^4*a^3*c^5 - 15000*(2*A^3
*B*a^3*b - A^4*a^2*b^2)*c^4 - 3*(432*B^4*a^5 - 3024*A*B^3*a^4*b -
 3312*A^2*B^2*a^3*b^2 + 3864*A^3*B*a^2*b^3 + 497*A^4*a*b^4)*c^3 -
 5*(648*B^4*a^4*b^2 - 216*A*B^3*a^3*b^3 - 648*A^2*B^2*a^2*b^4 - 1
89*A^3*B*a*b^5 - 7*A^4*b^6)*c^2 - 15*(27*B^4*a^3*b^4 + 27*A*B^3*a
^2*b^5 + 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)*x - 1/2*sqrt(1/2)*(27*B^
3*a^3*b^8 + 27*A*B^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^3*b^11 + 6400*(
3*A^2*B*a^6 - 4*A^3*a^5*b)*c^5 - 64*(108*B^3*a^7 - 72*A*B^2*a^6*b
 + 66*A^2*B*a^5*b^2 - 341*A^3*a^4*b^3)*c^4 + 16*(216*B^3*a^6*b^2
 - 324*A*B^2*a^5*b^3 - 288*A^2*B*a^4*b^4 - 427*A^3*a^3*b^5)*c^3 +
20*(108*A*B^2*a^4*b^5 + 102*A^2*B*a^3*b^6 + 47*A^3*a^2*b^7)*c^2 -
 (216*B^3*a^4*b^6 + 396*A*B^2*a^3*b^7 + 267*A^2*B*a^2*b^8 + 53*A^
3*a*b^9)*c - (3*B*a^4*b^13 + A*a^3*b^14 + 40960*A*a^10*c^7 - 4096
*(9*B*a^10*b + 8*A*a^9*b^2)*c^6 + 1536*(28*B*a^9*b^3 + A*a^8*b^4)
c^5 - 6400(3*B*a^8*b^5 - A*a^7*b^6)*c^4 + 160*(24*B*a^7*b^7 - 1
7*A*a^6*b^8)*c^3 - 240*(B*a^6*b^9 - 2*A*a^5*b^10)*c^2 - 2*(12*B*a
^5*b^11 + 19*A*a^4*b^12)*c)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b +
54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 -
 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20
*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^
4 - 1024*a^11*c^5)))*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7
 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*
a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4
 - 7*A^2*a*b^5)*c + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 -
640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a
^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*
b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a
*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4
*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^
8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024
*a^8*c^5))) + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)
*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^
3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^
4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(9*B^2*a^
2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3
 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72
*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c - (a^3*b^10 - 20*a

^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 -
 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^
2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^
2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c +
160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11
*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*
c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((10000*A^4*a^3*c^5 -
15000*(2*A^3*B*a^3*b - A^4*a^2*b^2)*c^4 - 3*(432*B^4*a^5 - 3024*A
*B^3*a^4*b - 3312*A^2*B^2*a^3*b^2 + 3864*A^3*B*a^2*b^3 + 497*A^4*
a*b^4)*c^3 - 5*(648*B^4*a^4*b^2 - 216*A*B^3*a^3*b^3 - 648*A^2*B^2
*a^2*b^4 - 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^2 - 15*(27*B^4*a^3*b^4
+ 27*A*B^3*a^2*b^5 + 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)*x + 1/2*sqrt
(1/2)*(27*B^3*a^3*b^8 + 27*A*B^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^3*b
^11 + 6400*(3*A^2*B*a^6 - 4*A^3*a^5*b)*c^5 - 64*(108*B^3*a^7 - 72
*A*B^2*a^6*b + 66*A^2*B*a^5*b^2 - 341*A^3*a^4*b^3)*c^4 + 16*(216*
B^3*a^6*b^2 - 324*A*B^2*a^5*b^3 - 288*A^2*B*a^4*b^4 - 427*A^3*a^3
*b^5)*c^3 + 20*(108*A*B^2*a^4*b^5 + 102*A^2*B*a^3*b^6 + 47*A^3*a^
2*b^7)*c^2 - (216*B^3*a^4*b^6 + 396*A*B^2*a^3*b^7 + 267*A^2*B*a^2
*b^8 + 53*A^3*a*b^9)*c + (3*B*a^4*b^13 + A*a^3*b^14 + 40960*A*a^1
0*c^7 - 4096*(9*B*a^10*b + 8*A*a^9*b^2)*c^6 + 1536*(28*B*a^9*b^3
+ A*a^8*b^4)*c^5 - 6400*(3*B*a^8*b^5 - A*a^7*b^6)*c^4 + 160*(24*B
*a^7*b^7 - 17*A*a^6*b^8)*c^3 - 240*(B*a^6*b^9 - 2*A*a^5*b^10)*c^2
 - 2*(12*B*a^5*b^11 + 19*A*a^4*b^12)*c)*sqrt((81*B^4*a^4 + 108*A*
B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A
^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a
^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280
*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b
^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4
*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12
*A*B*a^2*b^4 - 7*A^2*a*b^5)*c - (a^3*b^10 - 20*a^4*b^8*c + 160*a^
5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sq
rt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*
a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a
^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 -
 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10
 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^
2*c^4 - 1024*a^8*c^5))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 +
 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*
c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32
*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqr
t(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^
2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)
c^2 + 5(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c - (a^3
*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a
^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 5
4*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 -
50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*
a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4
 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 -
640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((10000*A^
4*a^3*c^5 - 15000*(2*A^3*B*a^3*b - A^4*a^2*b^2)*c^4 - 3*(432*B^4*
a^5 - 3024*A*B^3*a^4*b - 3312*A^2*B^2*a^3*b^2 + 3864*A^3*B*a^2*b^
3 + 497*A^4*a*b^4)*c^3 - 5*(648*B^4*a^4*b^2 - 216*A*B^3*a^3*b^3 -
 648*A^2*B^2*a^2*b^4 - 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^2 - 15*(27*

```
B^4*a^3*b^4 + 27*A*B^3*a^2*b^5 + 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)*
x - 1/2*sqrt(1/2)*(27*B^3*a^3*b^8 + 27*A*B^2*a^2*b^9 + 9*A^2*B*a*
b^10 + A^3*b^11 + 6400*(3*A^2*B*a^6 - 4*A^3*a^5*b)*c^5 - 64*(108*
B^3*a^7 - 72*A*B^2*a^6*b + 66*A^2*B*a^5*b^2 - 341*A^3*a^4*b^3)*c^
4 + 16*(216*B^3*a^6*b^2 - 324*A*B^2*a^5*b^3 - 288*A^2*B*a^4*b^4 -
 427*A^3*a^3*b^5)*c^3 + 20*(108*A*B^2*a^4*b^5 + 102*A^2*B*a^3*b^6
 + 47*A^3*a^2*b^7)*c^2 - (216*B^3*a^4*b^6 + 396*A*B^2*a^3*b^7 + 2
67*A^2*B*a^2*b^8 + 53*A^3*a*b^9)*c + (3*B*a^4*b^13 + A*a^3*b^14 +
 40960*A*a^10*c^7 - 4096*(9*B*a^10*b + 8*A*a^9*b^2)*c^6 + 1536*(2
8*B*a^9*b^3 + A*a^8*b^4)*c^5 - 6400*(3*B*a^8*b^5 - A*a^7*b^6)*c^4
 + 160*(24*B*a^7*b^7 - 17*A*a^6*b^8)*c^3 - 240*(B*a^6*b^9 - 2*A*a
^5*b^10)*c^2 - 2*(12*B*a^5*b^11 + 19*A*a^4*b^12)*c)*sqrt((81*B^4*
a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4
*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*
a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^
4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(9*B^2*a^2*b^5
 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40
*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*
a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c - (a^3*b^10 - 20*a^4*b^
8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024
*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2
 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3
 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a
^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)
))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 +
 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) - 2*(3*B*a^2*b^2 + A*a*b^3 +
4*(3*B*a^3 - 4*A*a^2*b)*c)*x)/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^
3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*
a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c
^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.136 $\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=461

$$\frac{x\left(A\left(28a^2c^2-25ab^2c+3b^4\right)+cx^2\left(3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)+abB\left(8ac+b^2\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{3A\left(56a^2c^2-10ab^2c+b^4\right)+abB\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{3A\left(56a^2c^2-10ab^2c+b^4\right)+abB\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x\left(-A\left(b^2-2ac\right)+cx^2(-(Ab-2aB))+abB\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

```
[Out] -(x*(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2))/(4*a*(b^2 -
4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(a*b*B*(b^2 + 8*a*c) + A*(3*b^
4 - 25*a*b^2*c + 28*a^2*c^2) + c*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 -
 8*a*b*c))*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (S
qrt[c]*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c) + (a*b*B*(b^2 -
52*a*c) + 3*A*(b^4 - 10*a*b^2*c + 56*a^2*c^2))/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt
[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(
a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c) - (a*b*B*(b^2 - 52*a*c)
+ 3*A*(b^4 - 10*a*b^2*c + 56*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*
(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 2.85678, antiderivative size = 461, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{x\left(A\left(28a^2c^2-25ab^2c+3b^4\right)+cx^2\left(3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)+abB\left(8ac+b^2\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{3A\left(56a^2c^2-10ab^2c+b^4\right)+abB\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{3A\left(56a^2c^2-10ab^2c+b^4\right)+abB\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(4*a*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(a*b*B*(b^2 + 8*a*c) + A*(3*b^4
 - 25*a*b^2*c + 28*a^2*c^2) + c*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 -
8*a*b*c))*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sq
rt[c]*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c) + (a*b*B*(b^2 - 5
2*a*c) + 3*A*(b^4 - 10*a*b^2*c + 56*a^2*c^2))/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[
2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(a
*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c) - (a*b*B*(b^2 - 52*a*c) +
 3*A*(b^4 - 10*a*b^2*c + 56*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(
b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 4.42074, size = 516, normalized size = 1.12

$$\frac{\frac{2x\left(A\left(28a^2c^2-25ab^2c-24abc^2x^2+3b^4+3b^3cx^2\right)+aB\left(8abc+20ac^2x^2+b^3+b^2cx^2\right)\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(3A\left(56a^2c^2-10ab^2c-8abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)+aB\left(b^2\sqrt{b^2 \cdots}\right.\right.}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2 \cdots}}}}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*a*x*(a*B*(b + 2*c*x^2) - A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 -
 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*x*(a*B*(b^3 + 8*a*b*c + b^2*c
*x^2 + 20*a*c^2*x^2) + A*(3*b^4 - 25*a*b^2*c + 28*a^2*c^2 + 3*b^3
*c*x^2 - 24*a*b*c^2*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4))
+ (Sqrt[2]*Sqrt[c]*(a*B*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] +
 20*a*c*Sqrt[b^2 - 4*a*c]) + 3*A*(b^4 - 10*a*b^2*c + 56*a^2*c^2 +
```

```
 b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)
*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(a*B*(-b^3 + 52*
a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]) + 3*A*(
-b^4 + 10*a*b^2*c - 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*
Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(1
6*a^2)
```

Maple [B] time = 0.286, size = 11936, normalized size = 25.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(4\left(5\,Ba^2-6\,Aab\right)c^3+\left(Bab^2+3\,Ab^3\right)c^2\right)x^7+\left(28\,Aa^2c^3+7\left(4\,Ba^2b-7\,Aab^2\right)c^2+2\left(Bab^3+3\,Ab^4\right)c\right)x^5+\left(Bab^4+3\,Ab^5+\right.}{8\left(\left(a^2b^4c^2-8\,a^3b^2c^3+16\,a^4c^4\right)x^8+a^4b^4-8\,a^5b^2c+16\,a^6c^2+2\left(a^2b^5c-8\,a^3b^3c^2+16\,a^4\right.\right.}-\frac{-\int\frac{Bab^3+3\,Ab^4+84\,Aa^2c^2+\left(4\left(5\,Ba^2-6\,Aab\right)c^2+\left(Bab^2+3\,Ab^3\right)c\right)x^2-\left(16\,Ba^2b+27\,Aab^2\right)c}{cx^4+bx^2+a}\,dx}{8\left(a^2b^4-8\,a^3b^2c+16\,a^4c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((4*(5*B*a^2 - 6*A*a*b)*c^3 + (B*a*b^2 + 3*A*b^3)*c^2)*x^7 +
(28*A*a^2*c^3 + 7*(4*B*a^2*b - 7*A*a*b^2)*c^2 + 2*(B*a*b^3 + 3*A*
b^4)*c)*x^5 + (B*a*b^4 + 3*A*b^5 + 4*(9*B*a^3 - A*a^2*b)*c^2 + 5*
(B*a^2*b^2 - 4*A*a*b^3)*c)*x^3 - (B*a^2*b^3 - 5*A*a*b^4 - 44*A*a^
3*c^2 - (16*B*a^3*b - 37*A*a^2*b^2)*c)*x)/((a^2*b^4*c^2 - 8*a^3*b
^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2
*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^
3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c
^2)*x^2) - 1/8*integrate(-(B*a*b^3 + 3*A*b^4 + 84*A*a^2*c^2 + (4*
(5*B*a^2 - 6*A*a*b)*c^2 + (B*a*b^2 + 3*A*b^3)*c)*x^2 - (16*B*a^2*
b + 27*A*a*b^2)*c)/(c*x^4 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c
 + 16*a^4*c^2)
```

Fricas [A] time = 10.2181, size = 13377, normalized size = 29.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(4*(5*B*a^2 - 6*A*a*b)*c^3 + (B*a*b^2 + 3*A*b^3)*c^2)*x^7
 + 2*(28*A*a^2*c^3 + 7*(4*B*a^2*b - 7*A*a*b^2)*c^2 + 2*(B*a*b^3 +
 3*A*b^4)*c)*x^5 + 2*(B*a*b^4 + 3*A*b^5 + 4*(9*B*a^3 - A*a^2*b)*c
^2 + 5*(B*a^2*b^2 - 4*A*a*b^3)*c)*x^3 - sqrt(1/2)*((a^2*b^4*c^2 -
 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6
*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^
6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16
*a^5*b*c^2)*x^2)*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1
680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*
b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 +
243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2
*a*b^7)*c + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*
b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((B^4*a^4*b^4 + 1
2*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b
^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b +
 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^
2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*
(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*
A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 16
0*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15
*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*
c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log((3111696*A^4*a^4*c^7
 - 1555848*(2*A^3*B*a^4*b + A^4*a^3*b^2)*c^6 - (10000*B^4*a^6 - 9
0000*A*B^3*a^5*b - 863136*A^2*B^2*a^4*b^2 - 1298376*A^3*B*a^3*b^3
 - 339309*A^4*a^2*b^4)*c^5 - 3*(5000*B^4*a^5*b^2 + 32952*A*B^3*a^
4*b^3 + 79488*A^2*B^2*a^3*b^4 + 80919*A^3*B*a^2*b^5 + 12069*A^4*a
*b^6)*c^4 + 21*(71*B^4*a^4*b^4 + 537*A*B^3*a^3*b^5 + 1314*A^2*B^2
*a^2*b^6 + 1053*A^3*B*a*b^7 + 81*A^4*b^8)*c^3 - 35*(B^4*a^3*b^6 +
 9*A*B^3*a^2*b^7 + 27*A^2*B^2*a*b^8 + 27*A^3*B*b^9)*c^2)*x + 1/2*
sqrt(1/2)*(B^3*a^3*b^11 + 9*A*B^2*a^2*b^12 + 27*A^2*B*a*b^13 + 27
*A^3*b^14 - 2370816*A^3*a^7*c^7 + 2688*(50*A*B^2*a^8 + 384*A^2*B*
a^7*b + 1143*A^3*a^6*b^2)*c^6 - 64*(400*B^3*a^8*b + 4062*A*B^2*a^
7*b^2 + 17541*A^2*B*a^6*b^3 + 26865*A^3*a^5*b^4)*c^5 + 8*(2728*B^
3*a^7*b^3 + 20520*A*B^2*a^6*b^4 + 62694*A^2*B*a^5*b^5 + 67797*A^3
*a^4*b^6)*c^4 - 7*(976*B^3*a^6*b^5 + 6744*A*B^2*a^5*b^6 + 16884*A
^2*B*a^4*b^7 + 14985*A^3*a^3*b^8)*c^3 + (940*B^3*a^5*b^7 + 6591*A
*B^2*a^4*b^8 + 15489*A^2*B*a^3*b^9 + 12528*A^3*a^2*b^10)*c^2 - (5
3*B^3*a^4*b^9 + 414*A*B^2*a^3*b^10 + 1053*A^2*B*a^2*b^11 + 864*A^
3*a*b^12)*c - (B*a^6*b^14 + 3*A*a^5*b^15 + 4096*(10*B*a^13 - 33*A
*a^12*b)*c^7 - 2048*(16*B*a^12*b^2 - 99*A*a^11*b^3)*c^6 + 768*(2*
B*a^11*b^4 - 169*A*a^10*b^5)*c^5 + 1280*(5*B*a^10*b^6 + 36*A*a^9*
b^7)*c^4 - 80*(34*B*a^9*b^8 + 123*A*a^8*b^9)*c^3 + 24*(20*B*a^8*b
```

```
^10 + 53*A*a^7*b^11)*c^2 - (38*B*a^7*b^12 + 93*A*a^6*b^13)*c)*sqr
t((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*
B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 +
 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^
3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4
*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B
^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 -
20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b
^2*c^4 - 1024*a^15*c^5)))*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^
2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4
*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a
^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6
 + 27*A^2*a*b^7)*c + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 -
 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((B^4*a^
4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 +
 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*
B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b +
 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)
*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^
4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b
^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 -
1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640
*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) + sqrt(1/2)*((
a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b
^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*
x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a
^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 +
9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b
 - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A
*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2
*b^6 + 27*A^2*a*b^7)*c + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c
^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((B^
4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b
^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*
A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5
*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*
b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^
3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^
11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^
4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 -
 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log((311169
6*A^4*a^4*c^7 - 1555848*(2*A^3*B*a^4*b + A^4*a^3*b^2)*c^6 - (1000
0*B^4*a^6 - 90000*A*B^3*a^5*b - 863136*A^2*B^2*a^4*b^2 - 1298376*
A^3*B*a^3*b^3 - 339309*A^4*a^2*b^4)*c^5 - 3*(5000*B^4*a^5*b^2 + 3
2952*A*B^3*a^4*b^3 + 79488*A^2*B^2*a^3*b^4 + 80919*A^3*B*a^2*b^5
+ 12069*A^4*a*b^6)*c^4 + 21*(71*B^4*a^4*b^4 + 537*A*B^3*a^3*b^5 +
 1314*A^2*B^2*a^2*b^6 + 1053*A^3*B*a*b^7 + 81*A^4*b^8)*c^3 - 35*(
B^4*a^3*b^6 + 9*A*B^3*a^2*b^7 + 27*A^2*B^2*a*b^8 + 27*A^3*B*b^9)*
c^2)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^11 + 9*A*B^2*a^2*b^12 + 27*A^2*
B*a*b^13 + 27*A^3*b^14 - 2370816*A^3*a^7*c^7 + 2688*(50*A*B^2*a^8
 + 384*A^2*B*a^7*b + 1143*A^3*a^6*b^2)*c^6 - 64*(400*B^3*a^8*b +
4062*A*B^2*a^7*b^2 + 17541*A^2*B*a^6*b^3 + 26865*A^3*a^5*b^4)*c^5
 + 8*(2728*B^3*a^7*b^3 + 20520*A*B^2*a^6*b^4 + 62694*A^2*B*a^5*b^
5 + 67797*A^3*a^4*b^6)*c^4 - 7*(976*B^3*a^6*b^5 + 6744*A*B^2*a^5*
```

b^6 + 16884*A^2*B*a^4*b^7 + 14985*A^3*a^3*b^8)*c^3 + (940*B^3*a^5
*b^7 + 6591*A*B^2*a^4*b^8 + 15489*A^2*B*a^3*b^9 + 12528*A^3*a^2*b
^10)*c^2 - (53*B^3*a^4*b^9 + 414*A*B^2*a^3*b^10 + 1053*A^2*B*a^2*
b^11 + 864*A^3*a*b^12)*c - (B*a^6*b^14 + 3*A*a^5*b^15 + 4096*(10*
B*a^13 - 33*A*a^12*b)*c^7 - 2048*(16*B*a^12*b^2 - 99*A*a^11*b^3)*
c^6 + 768*(2*B*a^11*b^4 - 169*A*a^10*b^5)*c^5 + 1280*(5*B*a^10*b^
6 + 36*A*a^9*b^7)*c^4 - 80*(34*B*a^9*b^8 + 123*A*a^8*b^9)*c^3 + 2
4*(20*B*a^8*b^10 + 53*A*a^7*b^11)*c^2 - (38*B*a^7*b^12 + 93*A*a^6
*b^13)*c)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b
^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*
A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^
6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^
3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^
3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/
(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3
+ 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(B^2*a^2*b^7 + 6*A*B
*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*
B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^
3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 2
4*A*B*a^2*b^6 + 27*A^2*a*b^7)*c + (a^5*b^10 - 20*a^6*b^8*c + 160*
a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)
*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*
A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a
^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*
A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739
*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A
^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^1
0 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^
14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*
b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) -
sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*
b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 1
6*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(B^2*a^2*b^7 + 6
*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840
*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^
4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5
+ 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c - (a^5*b^10 - 20*a^6*b^8*c +
160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*
c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 +
108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B
^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5
400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 1
7739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 9
72*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10
*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 128
0*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*
a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)
)*log((3111696*A^4*a^4*c^7 - 1555848*(2*A^3*B*a^4*b + A^4*a^3*b^2
)*c^6 - (10000*B^4*a^6 - 90000*A*B^3*a^5*b - 863136*A^2*B^2*a^4*b
^2 - 1298376*A^3*B*a^3*b^3 - 339309*A^4*a^2*b^4)*c^5 - 3*(5000*B^
4*a^5*b^2 + 32952*A*B^3*a^4*b^3 + 79488*A^2*B^2*a^3*b^4 + 80919*A
^3*B*a^2*b^5 + 12069*A^4*a*b^6)*c^4 + 21*(71*B^4*a^4*b^4 + 537*A*
B^3*a^3*b^5 + 1314*A^2*B^2*a^2*b^6 + 1053*A^3*B*a*b^7 + 81*A^4*b^

8)*c^3 - 35*(B^4*a^3*b^6 + 9*A*B^3*a^2*b^7 + 27*A^2*B^2*a*b^8 + 2
7*A^3*B*b^9)*c^2)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^11 + 9*A*B^2*a^2*b
^12 + 27*A^2*B*a*b^13 + 27*A^3*b^14 - 2370816*A^3*a^7*c^7 + 2688*
(50*A*B^2*a^8 + 384*A^2*B*a^7*b + 1143*A^3*a^6*b^2)*c^6 - 64*(400
*B^3*a^8*b + 4062*A*B^2*a^7*b^2 + 17541*A^2*B*a^6*b^3 + 26865*A^3
*a^5*b^4)*c^5 + 8*(2728*B^3*a^7*b^3 + 20520*A*B^2*a^6*b^4 + 62694
*A^2*B*a^5*b^5 + 67797*A^3*a^4*b^6)*c^4 - 7*(976*B^3*a^6*b^5 + 67
44*A*B^2*a^5*b^6 + 16884*A^2*B*a^4*b^7 + 14985*A^3*a^3*b^8)*c^3 +
 (940*B^3*a^5*b^7 + 6591*A*B^2*a^4*b^8 + 15489*A^2*B*a^3*b^9 + 12
528*A^3*a^2*b^10)*c^2 - (53*B^3*a^4*b^9 + 414*A*B^2*a^3*b^10 + 10
53*A^2*B*a^2*b^11 + 864*A^3*a*b^12)*c + (B*a^6*b^14 + 3*A*a^5*b^1
5 + 4096*(10*B*a^13 - 33*A*a^12*b)*c^7 - 2048*(16*B*a^12*b^2 - 99
*A*a^11*b^3)*c^6 + 768*(2*B*a^11*b^4 - 169*A*a^10*b^5)*c^5 + 1280
*(5*B*a^10*b^6 + 36*A*a^9*b^7)*c^4 - 80*(34*B*a^9*b^8 + 123*A*a^8
*b^9)*c^3 + 24*(20*B*a^8*b^10 + 53*A*a^7*b^11)*c^2 - (38*B*a^7*b^
12 + 93*A*a^6*b^13)*c)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*
A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c
^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3
+ (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676
*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258
*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A
^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*
a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(B^2*a^
2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*
c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(
40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^
2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c - (a^5*b^10 - 20*a^6
*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1
024*a^10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a
^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*
(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^
4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^
3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^
4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)
*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*
c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8
*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*
a^10*c^5))) + sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^
4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^
3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c
^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(B^
2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4
*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 +
 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(
5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c - (a^5*b^10 - 20
*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4
 - 1024*a^10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B
^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 -
882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (62
5*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*
B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^
3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*
b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*
b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6

```
*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1
024*a^10*c^5))*log((3111696*A^4*a^4*c^7 - 1555848*(2*A^3*B*a^4*b
+ A^4*a^3*b^2)*c^6 - (10000*B^4*a^6 - 90000*A*B^3*a^5*b - 863136*
A^2*B^2*a^4*b^2 - 1298376*A^3*B*a^3*b^3 - 339309*A^4*a^2*b^4)*c^5
 - 3*(5000*B^4*a^5*b^2 + 32952*A*B^3*a^4*b^3 + 79488*A^2*B^2*a^3*
b^4 + 80919*A^3*B*a^2*b^5 + 12069*A^4*a*b^6)*c^4 + 21*(71*B^4*a^4
*b^4 + 537*A*B^3*a^3*b^5 + 1314*A^2*B^2*a^2*b^6 + 1053*A^3*B*a*b^
7 + 81*A^4*b^8)*c^3 - 35*(B^4*a^3*b^6 + 9*A*B^3*a^2*b^7 + 27*A^2*
B^2*a*b^8 + 27*A^3*B*b^9)*c^2)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^11 +
9*A*B^2*a^2*b^12 + 27*A^2*B*a*b^13 + 27*A^3*b^14 - 2370816*A^3*a^
7*c^7 + 2688*(50*A*B^2*a^8 + 384*A^2*B*a^7*b + 1143*A^3*a^6*b^2)*
c^6 - 64*(400*B^3*a^8*b + 4062*A*B^2*a^7*b^2 + 17541*A^2*B*a^6*b^
3 + 26865*A^3*a^5*b^4)*c^5 + 8*(2728*B^3*a^7*b^3 + 20520*A*B^2*a^
6*b^4 + 62694*A^2*B*a^5*b^5 + 67797*A^3*a^4*b^6)*c^4 - 7*(976*B^3
*a^6*b^5 + 6744*A*B^2*a^5*b^6 + 16884*A^2*B*a^4*b^7 + 14985*A^3*a
^3*b^8)*c^3 + (940*B^3*a^5*b^7 + 6591*A*B^2*a^4*b^8 + 15489*A^2*B
*a^3*b^9 + 12528*A^3*a^2*b^10)*c^2 - (53*B^3*a^4*b^9 + 414*A*B^2*
a^3*b^10 + 1053*A^2*B*a^2*b^11 + 864*A^3*a*b^12)*c + (B*a^6*b^14
+ 3*A*a^5*b^15 + 4096*(10*B*a^13 - 33*A*a^12*b)*c^7 - 2048*(16*B*
a^12*b^2 - 99*A*a^11*b^3)*c^6 + 768*(2*B*a^11*b^4 - 169*A*a^10*b^
5)*c^5 + 1280*(5*B*a^10*b^6 + 36*A*a^9*b^7)*c^4 - 80*(34*B*a^9*b^
8 + 123*A*a^8*b^9)*c^3 + 24*(20*B*a^8*b^10 + 53*A*a^7*b^11)*c^2 -
 (38*B*a^7*b^12 + 93*A*a^6*b^13)*c)*sqrt((B^4*a^4*b^4 + 12*A*B^3*
a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194
481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*
a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^
4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*
a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^
2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b
^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*
sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 -
9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b
^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*
c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c - (a^5*
b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^
9*b^2*c^4 - 1024*a^10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 +
 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a
^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*
c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 2
6676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 +
 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 8
91*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 -
640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10
 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^
2*c^4 - 1024*a^10*c^5))) - 2*(B*a^2*b^3 - 5*A*a*b^4 - 44*A*a^3*c^
2 - (16*B*a^3*b - 37*A*a^2*b^2)*c)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c
^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^
2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^
4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*
x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(c*x**4+b*x**2+a)**3,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 39.4694, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")

[Out] Done
```

3.137 $\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Rubi [A] time = 0.0506523, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Rubi in Sympy [A] time = 11.855, size = 17, normalized size = 0.68

$$\frac{\log\left(-x^2+1\right)}{2}+\frac{3\log\left(-x^2+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(4*x**2-7)/(x**4-5*x**2+4),x)
```

```
[Out] log(-x**2 + 1)/2 + 3*log(-x**2 + 4)/2
```

Mathematica [A] time = 0.00999147, size = 25, normalized size = 1.

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Maple [A] time = 0.009, size = 18, normalized size = 0.7

$$\frac{\ln\left(x^{2}-1\right)}{2}+\frac{3\ \ln\left(x^{2}-4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(4*x^2-7)/(x^4-5*x^2+4),x)
```

```
[Out] 1/2*ln(x^2-1)+3/2*ln(x^2-4)
```

Maxima [A] time = 0.701921, size = 23, normalized size = 0.92

$$\frac{1}{2}\ \log\left(x^{2}-1\right)+\frac{3}{2}\ \log\left(x^{2}-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 7)*x/(x^4 - 5*x^2 + 4),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

Fricas [A] time = 0.253026, size = 23, normalized size = 0.92

$$\frac{1}{2}\ \log\left(x^{2}-1\right)+\frac{3}{2}\ \log\left(x^{2}-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 7)*x/(x^4 - 5*x^2 + 4),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

Sympy [A] time = 0.231747, size = 17, normalized size = 0.68

$$\frac{3\log\left(x^{2}-4\right)}{2}+\frac{\log\left(x^{2}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(4*x**2-7)/(x**4-5*x**2+4),x)

[Out] 3*log(x**2 - 4)/2 + log(x**2 - 1)/2
```

GIAC/XCAS [A] time = 0.289875, size = 26, normalized size = 1.04

$$\frac{1}{2}\ln\left(\left|x^2-1\right|\right)+\frac{3}{2}\ln\left(\left|x^2-4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 7)*x/(x^4 - 5*x^2 + 4),x, algorithm="giac")

[Out] 1/2*ln(abs(x^2 - 1)) + 3/2*ln(abs(x^2 - 4))
```

3.138 $\int \frac{-7x+4x^3}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Rubi [A] time = 0.0519601, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4),x]

[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Rubi in Sympy [A] time = 13.5832, size = 17, normalized size = 0.68

$$\frac{\log\left(-x^2+1\right)}{2}+\frac{3\log\left(-x^2+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**3-7*x)/(x**4-5*x**2+4),x)

[Out] log(-x**2 + 1)/2 + 3*log(-x**2 + 4)/2
```

Mathematica [A] time = 0.00813973, size = 25, normalized size = 1.

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Maple [A] time = 0.008, size = 18, normalized size = 0.7

$$\frac{\ln\left(x^2-1\right)}{2}+\frac{3\ \ln\left(x^2-4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^3-7*x)/(x^4-5*x^2+4),x)
```

```
[Out] 1/2*ln(x^2-1)+3/2*ln(x^2-4)
```

Maxima [A] time = 0.697192, size = 34, normalized size = 1.36

$$\frac{3}{2}\ \log\left(x+2\right)+\frac{1}{2}\ \log\left(x+1\right)+\frac{1}{2}\ \log\left(x-1\right)+\frac{3}{2}\ \log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^3 - 7*x)/(x^4 - 5*x^2 + 4),x, algorithm="maxima")
```

```
[Out] 3/2*log(x + 2) + 1/2*log(x + 1) + 1/2*log(x - 1) + 3/2*log(x - 2)
```

Fricas [A] time = 0.251158, size = 23, normalized size = 0.92

$$\frac{1}{2}\ \log\left(x^2-1\right)+\frac{3}{2}\ \log\left(x^2-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^3 - 7*x)/(x^4 - 5*x^2 + 4),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

Sympy [A] time = 0.217721, size = 17, normalized size = 0.68

$$\frac{3\log\left(x^2-4\right)}{2}+\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**3-7*x)/(x**4-5*x**2+4),x)
```

```
[Out] 3*log(x**2 - 4)/2 + log(x**2 - 1)/2
```

GIAC/XCAS [A] time = 0.272121, size = 26, normalized size = 1.04

$$\frac{1}{2}\ln\left(\left|x^{2}-1\right|\right)+\frac{3}{2}\ln\left(\left|x^{2}-4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^3 - 7*x)/(x^4 - 5*x^2 + 4),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(x^2 - 1)) + 3/2*ln(abs(x^2 - 4))
```

3.139 $\int \frac{x(2+x^2)}{1+x^2+x^4}\,dx$

Optimal. Leaf size=37

$$\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Rubi [A] time = 0.0727849, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294

$$\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(2 + x^2))/(1 + x^2 + x^4),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Rubi in Sympy [A] time = 11.2756, size = 34, normalized size = 0.92

$$\frac{\log\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}+\frac{1}{3}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(x**2+2)/(x**4+x**2+1),x)
```

```
[Out] log(x**4 + x**2 + 1)/4 + sqrt(3)*atan(sqrt(3)*(2*x**2/3 + 1/3))/2
```

Mathematica [A] time = 0.0157249, size = 37, normalized size = 1.

$$\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(2 + x^2))/(1 + x^2 + x^4),x]

[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Maple [A] time = 0.004, size = 31, normalized size = 0.8

$$\frac{\ln\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}}{2}\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(x^2+2)/(x^4+x^2+1),x)

[Out] 1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 0.784581, size = 41, normalized size = 1.11

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2)*x/(x^4 + x^2 + 1),x, algorithm="maxima")

[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 +
 1)
```

Fricas [A] time = 0.254209, size = 41, normalized size = 1.11

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2)*x/(x^4 + x^2 + 1),x, algorithm="fricas")

[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 +
 1)
```

Sympy [A] time = 0.24943, size = 37, normalized size = 1.

$$\frac{\log\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(x**2+2)/(x**4+x**2+1),x)
```

```
[Out] log(x**4 + x**2 + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/
3)/2
```

GIAC/XCAS [A] time = 0.273147, size = 41, normalized size = 1.11

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{4}\ln\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2)*x/(x^4 + x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*ln(x^4 + x^2 +
1)
```

3.140 $\int \frac{2x+x^3}{1+x^2+x^4}\,dx$

Optimal. Leaf size=37

$$\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Rubi [A] time = 0.0701185, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2*x + x^3)/(1 + x^2 + x^4),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Rubi in Sympy [A] time = 11.9051, size = 34, normalized size = 0.92

$$\frac{\log\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}+\frac{1}{3}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+2*x)/(x**4+x**2+1),x)
```

```
[Out] log(x**4 + x**2 + 1)/4 + sqrt(3)*atan(sqrt(3)*(2*x**2/3 + 1/3))/2
```

Mathematica [A] time = 0.00857202, size = 37, normalized size = 1.

$$\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*x + x^3)/(1 + x^2 + x^4),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Maple [A] time = 0.003, size = 31, normalized size = 0.8

$$\frac{\ln\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}}{2}\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+2*x)/(x^4+x^2+1),x)
```

```
[Out] 1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 0.795853, size = 72, normalized size = 1.95

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)+\frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 2*x)/(x^4 + x^2 + 1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*sqrt(3)*arctan(1
/3*sqrt(3)*(2*x - 1)) + 1/4*log(x^2 + x + 1) + 1/4*log(x^2 - x +
1)
```

Fricas [A] time = 0.249944, size = 41, normalized size = 1.11

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 2*x)/(x^4 + x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 +
 1)
```

Sympy [A] time = 0.22477, size = 37, normalized size = 1.

$$\frac{\log\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+2*x)/(x**4+x**2+1),x)
```

```
[Out] log(x**4 + x**2 + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/
3)/2
```

GIAC/XCAS [A] time = 0.271782, size = 41, normalized size = 1.11

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{4}\ln\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 2*x)/(x^4 + x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*ln(x^4 + x^2 +
1)
```

3.141 $\int \frac{11x+2x^3}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=45

$$\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{9x^2+5}{8\left(x^4+2x^2+3\right)}$$

```
[Out] (5 + 9*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])
/(8*Sqrt[2])
```

Rubi [A] time = 0.0907152, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{9x^2+5}{8\left(x^4+2x^2+3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 + 9*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])
/(8*Sqrt[2])
```

Rubi in Sympy [A] time = 13.9234, size = 41, normalized size = 0.91

$$\frac{18x^2+10}{16\left(x^4+2x^2+3\right)}+\frac{9\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{x^2}{2}+\frac{1}{2}\right)\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**3+11*x)/(x**4+2*x**2+3)**2,x)
```

```
[Out] (18*x**2 + 10)/(16*(x**4 + 2*x**2 + 3)) + 9*sqrt(2)*atan(sqrt(2)*
(x**2/2 + 1/2))/16
```

Mathematica [A] time = 0.0465281, size = 45, normalized size = 1.

$$\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{9x^2+5}{8\left(x^4+2x^2+3\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 + 9*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])
/(8*Sqrt[2])
```

Maple [A] time = 0.011, size = 41, normalized size = 0.9

$$\frac{18\,x^2+10}{16\,x^4+32\,x^2+48}+\frac{9\,\sqrt{2}}{16}\arctan\left(\frac{\left(2\,x^2+2\right)\,\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^3+11*x)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 1/16*(18*x^2+10)/(x^4+2*x^2+3)+9/16*2^(1/2)*arctan(1/4*(2*x^2+2)*
2^(1/2))
```

Maxima [A] time = 0.82534, size = 51, normalized size = 1.13

$$\frac{9}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)+\frac{9\,x^2+5}{8\left(x^4+2\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 11*x)/(x^4 + 2*x^2 + 3)^2,x, algorithm="maxima")
```

```
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/8*(9*x^2 + 5)/(x^4
 + 2*x^2 + 3)
```

Fricas [A] time = 0.281232, size = 70, normalized size = 1.56

$$\frac{\sqrt{2}\left(9\left(x^4+2\,x^2+3\right)\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)+\sqrt{2}\left(9\,x^2+5\right)\right)}{16\left(x^4+2\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 11*x)/(x^4 + 2*x^2 + 3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(2)*(9*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) +
 sqrt(2)*(9*x^2 + 5))/(x^4 + 2*x^2 + 3)
```

Sympy [A] time = 0.376625, size = 44, normalized size = 0.98

$$\frac{9x^2+5}{8x^4+16x^2+24}+\frac{9\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**3+11*x)/(x**4+2*x**2+3)**2,x)
```

```
[Out] (9*x**2 + 5)/(8*x**4 + 16*x**2 + 24) + 9*sqrt(2)*atan(sqrt(2)*x**
2/2 + sqrt(2)/2)/16
```

GIAC/XCAS [A] time = 0.27076, size = 51, normalized size = 1.13

$$\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{9\,x^2+5}{8\,(x^4+2\,x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 + 11*x)/(x^4 + 2*x^2 + 3)^2,x, algorithm="giac")
```

```
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/8*(9*x^2 + 5)/(x^4
 + 2*x^2 + 3)
```

3.142 $\int x^5 \left(2+3x^2\right) \sqrt{3+5x^2+x^4}\,dx$

Optimal. Leaf size=102

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{3/2}x^4+\frac{1}{480}\left(1837-510x^2\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{1633}{256}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{21229}{512}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] (-1633*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/256 + (3*x^4*(3 + 5*x^2
 + x^4)^(3/2))/10 + ((1837 - 510*x^2)*(3 + 5*x^2 + x^4)^(3/2))/48
0 + (21229*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/512
```

Rubi [A] time = 0.205229, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{3/2}x^4+\frac{1}{480}\left(1837-510x^2\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{1633}{256}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{21229}{512}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-1633*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/256 + (3*x^4*(3 + 5*x^2
 + x^4)^(3/2))/10 + ((1837 - 510*x^2)*(3 + 5*x^2 + x^4)^(3/2))/48
0 + (21229*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/512
```

Rubi in Sympy [A] time = 19.8867, size = 95, normalized size = 0.93

$$\frac{3x^4\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{10}+\frac{\left(-\frac{255x^2}{2}+\frac{1837}{4}\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{120}-\frac{1633\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{256}+\frac{21229\,\operatorname{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] 3*x**4*(x**4 + 5*x**2 + 3)**(3/2)/10 + (-255*x**2/2 + 1837/4)*(x*
*4 + 5*x**2 + 3)**(3/2)/120 - 1633*(2*x**2 + 5)*sqrt(x**4 + 5*x**
2 + 3)/256 + 21229*atanh((2*x**2 + 5)/(2*sqrt(x**4 + 5*x**2 + 3))
)/512
```

Mathematica [A] time = 0.0534356, size = 69, normalized size = 0.68

$$\frac{318435\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)+2\sqrt{x^4+5x^2+3}\left(1152x^8+1680x^6-2248x^4+12250x^2-78387\right)}{7680}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(-78387 + 12250*x^2 - 2248*x^4 + 1680*x^
6 + 1152*x^8) + 318435*Log[5 + 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/
7680
```

Maple [A] time = 0.033, size = 91, normalized size = 0.9

$$-\frac{17\,x^2}{16}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}+\frac{1837}{480}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}-\frac{3266\,x^2+8165}{256}\sqrt{x^4+5\,x^2+3}+\frac{21229}{512}\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5\,x^2+3}\right)+\frac{3\,x^4}{10}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -17/16*x^2*(x^4+5*x^2+3)^(3/2)+1837/480*(x^4+5*x^2+3)^(3/2)-1633/
256*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+21229/512*ln(x^2+5/2+(x^4+5*x^2
+3)^(1/2))+3/10*x^4*(x^4+5*x^2+3)^(3/2)
```

Maxima [A] time = 0.767968, size = 140, normalized size = 1.37

$$\frac{3}{10}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^4-\frac{17}{16}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^2-\frac{1633}{128}\sqrt{x^4+5\,x^2+3}x^2+\frac{1837}{480}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}-\frac{8165}{256}\sqrt{x^4+5\,x^2+3}+\frac{21229}{512}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^5,x, algorithm="maxima")
```

```
[Out] 3/10*(x^4 + 5*x^2 + 3)^(3/2)*x^4 - 17/16*(x^4 + 5*x^2 + 3)^(3/2)*
x^2 - 1633/128*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1837/480*(x^4 + 5*x^2
+ 3)^(3/2) - 8165/256*sqrt(x^4 + 5*x^2 + 3) + 21229/512*log(2*x^2
 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 0.259882, size = 360, normalized size = 3.53

$$-\frac{9437184\,x^{20} + 155320320\,x^{18} + 1019084800\,x^{16} + 3448463360\,x^{14} + 6357278720\,x^{12} + 3232834560\,x^{10} - 22104931840\,x^{8} - }{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^5,x, algorithm="fricas")
```

```
[Out] -1/61440*(9437184*x^20 + 155320320*x^18 + 1019084800*x^16 + 34484
63360*x^14 + 6357278720*x^12 + 3232834560*x^10 - 22104931840*x^8
- 76821709600*x^6 - 104453343600*x^4 - 56935072490*x^2 + 2547480*
(512*x^10 + 6400*x^8 + 29920*x^6 + 64400*x^4 + 62690*x^2 - 2*(256
*x^8 + 2560*x^6 + 8976*x^4 + 12880*x^2 + 6269)*sqrt(x^4 + 5*x^2 +
 3) + 21725)*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5) - 2*(47185
92*x^18 + 65863680*x^16 + 352550912*x^14 + 930713600*x^12 + 12113
32608*x^10 - 742387200*x^8 - 8744289280*x^6 - 18704377200*x^4 - 1
4328360240*x^2 - 2499476825)*sqrt(x^4 + 5*x^2 + 3) - 8624708513)/
(512*x^10 + 6400*x^8 + 29920*x^6 + 64400*x^4 + 62690*x^2 - 2*(256
*x^8 + 2560*x^6 + 8976*x^4 + 12880*x^2 + 6269)*sqrt(x^4 + 5*x^2 +
 3) + 21725)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{5}\left(3x^{2}+2\right)\sqrt{x^{4}+5x^{2}+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

GIAC/XCAS [A] time = 0.280093, size = 90, normalized size = 0.88

$$\frac{1}{3840}\sqrt{x^4+5x^2+3}\left(2\left(4\left(6\left(24x^2+35\right)x^2-281\right)x^2+6125\right)x^2-78387\right)-\frac{21229}{512}\ln\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^5,x, algorithm="giac")
```

```
[Out] 1/3840*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(24*x^2 + 35)*x^2 - 281)*x^
2 + 6125)*x^2 - 78387) - 21229/512*ln(2*x^2 - 2*sqrt(x^4 + 5*x^2
+ 3) + 5)
```

3.143 $\int x^3 \left(2+3x^2\right) \sqrt{3+5x^2+x^4} \, dx$

Optimal. Leaf size=81

$$-\frac{1}{48}\left(59-18x^2\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{259}{128}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{3367}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] (259*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/128 - ((59 - 18*x^2)*(3 +
 5*x^2 + x^4)^(3/2))/48 - (3367*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5
*x^2 + x^4])])/256
```

Rubi [A] time = 0.141364, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{1}{48}\left(59-18x^2\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{259}{128}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{3367}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (259*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/128 - ((59 - 18*x^2)*(3 +
 5*x^2 + x^4)^(3/2))/48 - (3367*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5
*x^2 + x^4])])/256
```

Rubi in Sympy [A] time = 14.8191, size = 73, normalized size = 0.9

$$-\frac{\left(-9x^2+\frac{59}{2}\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{24}+\frac{259\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{128}-\frac{3367\,\text{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] -(-9*x**2 + 59/2)*(x**4 + 5*x**2 + 3)**(3/2)/24 + 259*(2*x**2 + 5
)*sqrt(x**4 + 5*x**2 + 3)/128 - 3367*atanh((2*x**2 + 5)/(2*sqrt(x
**4 + 5*x**2 + 3)))/256
```

Mathematica [A] time = 0.0383804, size = 64, normalized size = 0.79

$$\frac{1}{768}\left(2\sqrt{x^4+5x^2+3}\left(144x^6+248x^4-374x^2+2469\right)-10101\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(2469 - 374*x^2 + 248*x^4 + 144*x^6) - 1
0101*Log[5 + 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/768
```

Maple [A] time = 0.021, size = 74, normalized size = 0.9

$$-\frac{59}{48}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{518x^2+1295}{128}\sqrt{x^4+5x^2+3}-\frac{3367}{256}\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)+\frac{3x^2}{8}\left(x^4+5x^2+3\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -59/48*(x^4+5*x^2+3)^(3/2)+259/128*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)-
3367/256*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+3/8*x^2*(x^4+5*x^2+3)^(3
/2)
```

Maxima [A] time = 0.734021, size = 117, normalized size = 1.44

$$\frac{3}{8}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2+\frac{259}{64}\sqrt{x^4+5x^2+3}x^2-\frac{59}{48}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{1295}{128}\sqrt{x^4+5x^2+3}-\frac{3367}{256}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^3,x, algorithm="maxima")
```

```
[Out] 3/8*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 259/64*sqrt(x^4 + 5*x^2 + 3)*x^
2 - 59/48*(x^4 + 5*x^2 + 3)^(3/2) + 1295/128*sqrt(x^4 + 5*x^2 + 3
) - 3367/256*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 0.275768, size = 306, normalized size = 3.78

$$-\frac{294912\,x^{16} + 4194304\,x^{14} + 22577152\,x^{12} + 60047360\,x^{10} + 105251200\,x^{8} + 197316352\,x^{6} + 306023776\,x^{4} + 210683744\,x^{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^3,x, algorithm="fricas")
```

```
[Out] -1/6144*(294912*x^16 + 4194304*x^14 + 22577152*x^12 + 60047360*x^
10 + 105251200*x^8 + 197316352*x^6 + 306023776*x^4 + 210683744*x^
2 - 80808*(128*x^8 + 1280*x^6 + 4384*x^4 + 5920*x^2 - 8*(16*x^6 +
 120*x^4 + 274*x^2 + 185)*sqrt(x^4 + 5*x^2 + 3) + 2569)*log(-2*x^
2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5) - 8*(36864*x^14 + 432128*x^12 +
1801728*x^10 + 3554048*x^8 + 5866640*x^6 + 12112056*x^4 + 1273865
8*x^2 + 2846497)*sqrt(x^4 + 5*x^2 + 3) + 38765495)/(128*x^8 + 128
0*x^6 + 4384*x^4 + 5920*x^2 - 8*(16*x^6 + 120*x^4 + 274*x^2 + 185
)*sqrt(x^4 + 5*x^2 + 3) + 2569)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{3}\left(3x^{2}+2\right)\sqrt{x^{4}+5x^{2}+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

GIAC/XCAS [A] time = 0.275985, size = 81, normalized size = 1.

$$\frac{1}{384}\sqrt{x^{4}+5\,x^{2}+3}\left(2\left(4\left(18\,x^{2}+31\right)x^{2}-187\right)x^{2}+2469\right)+\frac{3367}{256}\ln\left(2\,x^{2}-2\sqrt{x^{4}+5\,x^{2}+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^3,x, algorithm="giac")
```

```
[Out] 1/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(18*x^2 + 31)*x^2 - 187)*x^2 +
2469) + 3367/256*ln(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.144 $\int x\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx$

Optimal. Leaf size=74

$$\frac{1}{2}\left(x^4+5x^2+3\right)^{3/2}-\frac{11}{16}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{143}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] (-11*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/16 + (3 + 5*x^2 + x^4)^(3
/2)/2 + (143*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32
```

Rubi [A] time = 0.10209, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$\frac{1}{2}\left(x^4+5x^2+3\right)^{3/2}-\frac{11}{16}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{143}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-11*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/16 + (3 + 5*x^2 + x^4)^(3
/2)/2 + (143*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32
```

Rubi in Sympy [A] time = 12.0704, size = 65, normalized size = 0.88

$$-\frac{11\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{16}+\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{2}+\frac{143\,\mathrm{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] -11*(2*x**2 + 5)*sqrt(x**4 + 5*x**2 + 3)/16 + (x**4 + 5*x**2 + 3)
**(3/2)/2 + 143*atanh((2*x**2 + 5)/(2*sqrt(x**4 + 5*x**2 + 3)))/3
2
```

Mathematica [A] time = 0.0378313, size = 61, normalized size = 0.82

$$\frac{1}{2}\sqrt{x^4+5x^2+3}\left(x^4+\frac{9x^2}{4}-\frac{31}{8}\right)+\frac{143}{32}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] ((-31/8 + (9*x^2)/4 + x^4)*Sqrt[3 + 5*x^2 + x^4])/2 + (143*Log[5
+ 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/32
```

Maple [A] time = 0.016, size = 57, normalized size = 0.8

$$-\frac{22 x^{2}+55}{16}\sqrt{x^{4}+5 x^{2}+3}+\frac{143}{32}\ln\left(x^{2}+\frac{5}{2}+\sqrt{x^{4}+5 x^{2}+3}\right)+\frac{1}{2}\left(x^{4}+5 x^{2}+3\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -11/16*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+143/32*ln(x^2+5/2+(x^4+5*x^2
+3)^(1/2))+1/2*(x^4+5*x^2+3)^(3/2)
```

Maxima [A] time = 0.730575, size = 95, normalized size = 1.28

$$-\frac{11}{8}\sqrt{x^{4}+5 x^{2}+3}x^{2}+\frac{1}{2}\left(x^{4}+5 x^{2}+3\right)^{\frac{3}{2}}-\frac{55}{16}\sqrt{x^{4}+5 x^{2}+3}+\frac{143}{32}\log\left(2 x^{2}+2\sqrt{x^{4}+5 x^{2}+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x,x, algorithm="maxima")
```

```
[Out] -11/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/2*(x^4 + 5*x^2 + 3)^(3/2) - 5
5/16*sqrt(x^4 + 5*x^2 + 3) + 143/32*log(2*x^2 + 2*sqrt(x^4 + 5*x^
2 + 3) + 5)
```

Fricas [A] time = 0.262023, size = 252, normalized size = 3.41

$$-\frac{2048 x^{12}+25088 x^{10}+106624 x^{8}+172160 x^{6}+45248 x^{4}-79542 x^{2}+572\left(32 x^{6}+240 x^{4}+522 x^{2}-2\left(16 x^{4}+80 x^{2}+8\right.\right.}{128\left(32 x^{6}+240 x^{4}+5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x,x, algorithm="fricas")
```

```
[Out] -1/128*(2048*x^12 + 25088*x^10 + 106624*x^8 + 172160*x^6 + 45248*
x^4 - 79542*x^2 + 572*(32*x^6 + 240*x^4 + 522*x^2 - 2*(16*x^4 + 8
0*x^2 + 87)*sqrt(x^4 + 5*x^2 + 3) + 305)*log(-2*x^2 + 2*sqrt(x^4
+ 5*x^2 + 3) - 5) - 2*(1024*x^10 + 9984*x^8 + 30016*x^6 + 23104*x
^4 - 15168*x^2 - 7805)*sqrt(x^4 + 5*x^2 + 3) - 24231)/(32*x^6 + 2
40*x^4 + 522*x^2 - 2*(16*x^4 + 80*x^2 + 87)*sqrt(x^4 + 5*x^2 + 3)
 + 305)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int x\left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

GIAC/XCAS [A]   time = 0.27468, size = 72, normalized size = 0.97

$$\frac{1}{16}\sqrt{x^4+5\,x^2+3}\left(2\left(4\,x^2+9\right)x^2-31\right)-\frac{143}{32}\ln\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 9)*x^2 - 31) - 143/32*ln(2
*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.145 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x} \, dx$

Optimal. Leaf size=94

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\left(6x^2+23\right)+\frac{1}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] ((23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + ArcTanh[(5 + 2*x^2)/(2*S
qrt[3 + 5*x^2 + x^4])]/16 - Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3
]*Sqrt[3 + 5*x^2 + x^4])]
```

Rubi [A] time = 0.1996, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\left(6x^2+23\right)+\frac{1}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x,x]
```

```
[Out] ((23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + ArcTanh[(5 + 2*x^2)/(2*S
qrt[3 + 5*x^2 + x^4])]/16 - Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3
]*Sqrt[3 + 5*x^2 + x^4])]
```

Rubi in Sympy [A] time = 21.1851, size = 82, normalized size = 0.87

$$\frac{\left(3x^2+\frac{23}{2}\right)\sqrt{x^4+5x^2+3}}{4}+\frac{\operatorname{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{16}-\sqrt{3}\operatorname{atanh}\left(\frac{\sqrt{3}\left(5x^2+6\right)}{6\sqrt{x^4+5x^2+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x,x)
```

```
[Out] (3*x**2 + 23/2)*sqrt(x**4 + 5*x**2 + 3)/4 + atanh((2*x**2 + 5)/(2
*sqrt(x**4 + 5*x**2 + 3)))/16 - sqrt(3)*atanh(sqrt(3)*(5*x**2 + 6
)/(6*sqrt(x**4 + 5*x**2 + 3)))
```

Mathematica [A] time = 0.126806, size = 100, normalized size = 1.06

$$\sqrt{3}\log\left(x^2\right)+\frac{1}{8}\sqrt{x^4+5x^2+3}\left(6x^2+23\right)+\frac{1}{16}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)-\sqrt{3}\log\left(5x^2+2\sqrt{3}\sqrt{x^4+5x^2+3}+6\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x,x]
```

```
[Out] ((23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + Sqrt[3]*Log[x^2] + Log[5
 + 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]]/16 - Sqrt[3]*Log[6 + 5*x^2 +
2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]]
```

Maple [A] time = 0.018, size = 85, normalized size = 0.9

$$\sqrt{x^4+5x^2+3}+\frac{1}{16}\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)-\mathit{Artanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5x^2+3}}\right)\sqrt{3}+\frac{6x^2+15}{8}\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x)
```

```
[Out] (x^4+5*x^2+3)^(1/2)+1/16*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-arctanh(
1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+3/8*(2*x^2+5)*
(x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 0.859078, size = 120, normalized size = 1.28

$$\frac{3}{4}\sqrt{x^4+5x^2+3}x^2-\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{23}{8}\sqrt{x^4+5x^2+3}+\frac{1}{16}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x,x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(x^4 + 5*x^2 + 3)*x^2 - sqrt(3)*log(2*sqrt(3)*sqrt(x^4 +
5*x^2 + 3)/x^2 + 6/x^2 + 5) + 23/8*sqrt(x^4 + 5*x^2 + 3) + 1/16*l
og(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 0.266154, size = 356, normalized size = 3.79

$$-\frac{384\,x^8 + 4352\,x^6 + 15752\,x^4 + 19496\,x^2 + 4\left(8\,x^4 + 40\,x^2 - 4\,\sqrt{x^4 + 5\,x^2 + 3}\left(2\,x^2 + 5\right) + 37\right)\log\left(-2\,x^2 + 2\,\sqrt{x^4 + 5\,x^2 + 3}\right.}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x,x, algorithm="fricas")
```

```
[Out] -1/64*(384*x^8 + 4352*x^6 + 15752*x^4 + 19496*x^2 + 4*(8*x^4 + 40
*x^2 - 4*sqrt(x^4 + 5*x^2 + 3)*(2*x^2 + 5) + 37)*log(-2*x^2 + 2*s
qrt(x^4 + 5*x^2 + 3) - 5) + 64*(4*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)*(
2*x^2 + 5) - sqrt(3)*(8*x^4 + 40*x^2 + 37))*log((2*x^4 + 2*sqrt(3
)*x^2 + 5*x^2 - 2*sqrt(x^4 + 5*x^2 + 3)*(x^2 + sqrt(3)) + 6)/(2*x
^4 - 2*sqrt(x^4 + 5*x^2 + 3)*x^2 + 5*x^2)) - 4*(96*x^6 + 848*x^4
+ 1974*x^2 + 927)*sqrt(x^4 + 5*x^2 + 3) + 5305)/(8*x^4 + 40*x^2 -
 4*sqrt(x^4 + 5*x^2 + 3)*(2*x^2 + 5) + 37)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3x^2 + 2\right)\sqrt{x^4 + 5x^2 + 3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 5\,x^2 + 3}\left(3\,x^2 + 2\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x,x, algorithm="giac")

[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x, x)
```

3.146 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{x^4+5x^2+3}\,(2-3x^2)}{2x^2}+\frac{19}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{7\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

```
[Out] -((2 - 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/(2*x^2) + (19*ArcTanh[(5 + 2
*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/4 - (7*ArcTanh[(6 + 5*x^2)/(2*S
qrt[3]*Sqrt[3 + 5*x^2 + x^4])])/Sqrt[3]
```

Rubi [A] time = 0.208238, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$-\frac{\sqrt{x^4+5x^2+3}\,(2-3x^2)}{2x^2}+\frac{19}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{7\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^3,x]
```

```
[Out] -((2 - 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/(2*x^2) + (19*ArcTanh[(5 + 2
*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/4 - (7*ArcTanh[(6 + 5*x^2)/(2*S
qrt[3]*Sqrt[3 + 5*x^2 + x^4])])/Sqrt[3]
```

Rubi in Sympy [A] time = 21.1162, size = 88, normalized size = 0.91

$$\frac{19\,\mathrm{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{4}-\frac{7\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}(5x^2+6)}{6\sqrt{x^4+5x^2+3}}\right)}{3}-\frac{(-3x^2+2)\sqrt{x^4+5x^2+3}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**3,x)
```

```
[Out] 19*atanh((2*x**2 + 5)/(2*sqrt(x**4 + 5*x**2 + 3)))/4 - 7*sqrt(3)*
atanh(sqrt(3)*(5*x**2 + 6)/(6*sqrt(x**4 + 5*x**2 + 3)))/3 - (-3*x
**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/(2*x**2)
```

Mathematica [A] time = 0.130576, size = 96, normalized size = 0.99

$$\sqrt{x^4+5x^2+3}\left(\frac{3}{2}-\frac{1}{x^2}\right)+\frac{19}{4}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)+\frac{7\left(2\log(x)-\log\left(5x^2+2\sqrt{3}\sqrt{x^4+5x^2+3}+6\right)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^3,x]
```

```
[Out] (3/2 - x^(-2))*Sqrt[3 + 5*x^2 + x^4] + (19*Log[5 + 2*x^2 + 2*Sqrt
[3 + 5*x^2 + x^4]])/4 + (7*(2*Log[x] - Log[6 + 5*x^2 + 2*Sqrt[3]*
Sqrt[3 + 5*x^2 + x^4]]))/Sqrt[3]
```

Maple [A] time = 0.022, size = 104, normalized size = 1.1

$$-\frac{1}{3\,x^2}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}+\frac{7}{3}\sqrt{x^4+5\,x^2+3}+\frac{19}{4}\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5\,x^2+3}\right)-\frac{7\,\sqrt{3}}{3}Artanh\left(\frac{\left(5\,x^2+6\right)\,\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5\,x^2+3}}\right)+\frac{2\,x^2+5}{6}\sqrt{x^4+5\,x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x)
```

```
[Out] -1/3/x^2*(x^4+5*x^2+3)^(3/2)+7/3*(x^4+5*x^2+3)^(1/2)+19/4*ln(x^2+
5/2+(x^4+5*x^2+3)^(1/2))-7/3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5
*x^2+3)^(1/2))*3^(1/2)+1/6*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 0.833406, size = 120, normalized size = 1.24

$$-\frac{7}{3}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{3}{2}\sqrt{x^4+5\,x^2+3}-\frac{\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{19}{4}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^3,x, algorithm="maxima")
```

```
[Out] -7/3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5)
 + 3/2*sqrt(x^4 + 5*x^2 + 3) - sqrt(x^4 + 5*x^2 + 3)/x^2 + 19/4*l
og(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 0.280116, size = 419, normalized size = 4.32

$$-\frac{8\sqrt{3}\left(12\,x^6+45\,x^4-2\,x^2-37\right)\sqrt{x^4+5\,x^2+3}+38\left(4\sqrt{3}\left(2\,x^4+5\,x^2\right)\sqrt{x^4+5\,x^2+3}-\sqrt{3}\left(8\,x^6+40\,x^4+37\,x^2\right)\right)\log\left(-2\right.}{8\left(4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(8*sqrt(3)*(12*x^6 + 45*x^4 - 2*x^2 - 37)*sqrt(x^4 + 5*x^2 +
 3) + 38*(4*sqrt(3)*(2*x^4 + 5*x^2)*sqrt(x^4 + 5*x^2 + 3) - sqrt(
3)*(8*x^6 + 40*x^4 + 37*x^2))*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3
) - 5) + 56*(8*x^6 + 40*x^4 + 37*x^2 - 4*(2*x^4 + 5*x^2)*sqrt(x^4
 + 5*x^2 + 3))*log((6*x^2 + sqrt(3)*(2*x^4 + 5*x^2 + 6) - 2*sqrt(
x^4 + 5*x^2 + 3)*(sqrt(3)*x^2 + 3))/(2*x^4 - 2*sqrt(x^4 + 5*x^2 +
 3)*x^2 + 5*x^2)) - sqrt(3)*(96*x^8 + 600*x^6 + 728*x^4 - 531*x^2
 - 480))/(4*sqrt(3)*(2*x^4 + 5*x^2)*sqrt(x^4 + 5*x^2 + 3) - sqrt(
3)*(8*x^6 + 40*x^4 + 37*x^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**3,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^3,x, algorithm="giac")

[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^3, x)
```

3.147 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx$

Optimal. Leaf size=99

$$-\frac{\sqrt{x^4+5x^2+3}\,(23x^2+6)}{12x^4}+\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{77\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}$$

```
[Out] -((6 + 23*x^2)*Sqrt[3 + 5*x^2 + x^4])/(12*x^4) + (3*ArcTanh[(5 +
2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2 - (77*ArcTanh[(6 + 5*x^2)/(2
*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(24*Sqrt[3])
```

Rubi [A] time = 0.207388, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$-\frac{\sqrt{x^4+5x^2+3}\,(23x^2+6)}{12x^4}+\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{77\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^5,x]
```

```
[Out] -((6 + 23*x^2)*Sqrt[3 + 5*x^2 + x^4])/(12*x^4) + (3*ArcTanh[(5 +
2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2 - (77*ArcTanh[(6 + 5*x^2)/(2
*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(24*Sqrt[3])
```

Rubi in Sympy [A] time = 20.9532, size = 88, normalized size = 0.89

$$\frac{3\,\mathrm{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{2}-\frac{77\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}(5x^2+6)}{6\sqrt{x^4+5x^2+3}}\right)}{72}-\frac{(23x^2+6)\sqrt{x^4+5x^2+3}}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**5,x)
```

```
[Out] 3*atanh((2*x**2 + 5)/(2*sqrt(x**4 + 5*x**2 + 3)))/2 - 77*sqrt(3)*
atanh(sqrt(3)*(5*x**2 + 6)/(6*sqrt(x**4 + 5*x**2 + 3)))/72 - (23*
x**2 + 6)*sqrt(x**4 + 5*x**2 + 3)/(12*x**4)
```

Mathematica [A] time = 0.156872, size = 102, normalized size = 1.03

$$\frac{1}{12}\left(-\frac{\sqrt{x^4+5x^2+3}\,(23x^2+6)}{x^4}+18\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)+\frac{77\left(2\log(x)-\log\left(5x^2+2\sqrt{3}\sqrt{x^4+5x^2+3}+6\right)\right)}{2\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^5,x]
```

```
[Out] (-(((6 + 23*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4) + 18*Log[5 + 2*x^2 +
 2*Sqrt[3 + 5*x^2 + x^4]] + (77*(2*Log[x] - Log[6 + 5*x^2 + 2*Sqr
t[3]*Sqrt[3 + 5*x^2 + x^4]]))/(2*Sqrt[3]))/12
```

Maple [A] time = 0.022, size = 121, normalized size = 1.2

$$-\frac{1}{6x^4}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{13}{36x^2}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{77}{72}\sqrt{x^4+5x^2+3}-\frac{77\sqrt{3}}{72}\mathit{Artanh}\left(\frac{(5x^2+6)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5x^2+3}}\right)+\frac{26x^2+65}{72}\sqrt{x^4+5x^2+3}+\frac{3}{2}\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x)
```

```
[Out] -1/6/x^4*(x^4+5*x^2+3)^(3/2)-13/36/x^2*(x^4+5*x^2+3)^(3/2)+77/72*
(x^4+5*x^2+3)^(1/2)-77/72*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^
2+3)^(1/2))*3^(1/2)+13/72*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+3/2*ln(x^
2+5/2+(x^4+5*x^2+3)^(1/2))
```

Maxima [A] time = 0.818556, size = 143, normalized size = 1.44

$$-\frac{77}{72}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{1}{6}\sqrt{x^4+5x^2+3}-\frac{13\sqrt{x^4+5x^2+3}}{12x^2}-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{6x^4}+\frac{3}{2}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^5,x, algorithm="maxima")
```

```
[Out] -77/72*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 +
5) + 1/6*sqrt(x^4 + 5*x^2 + 3) - 13/12*sqrt(x^4 + 5*x^2 + 3)/x^2
- 1/6*(x^4 + 5*x^2 + 3)^(3/2)/x^4 + 3/2*log(2*x^2 + 2*sqrt(x^4 +
5*x^2 + 3) + 5)
```

Fricas [A] time = 0.26974, size = 405, normalized size = 4.09

$$\frac{2\sqrt{3}\left(508\,x^4+1091\,x^2+222\right)\sqrt{x^4+5\,x^2+3}-36\left(4\sqrt{3}\left(2\,x^6+5\,x^4\right)\sqrt{x^4+5\,x^2+3}-\sqrt{3}\left(8\,x^8+40\,x^6+37\,x^4\right)\right)\log\left(-2\,x^2\right.}{24\left(4\sqrt{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(3)*(508*x^4 + 1091*x^2 + 222)*sqrt(x^4 + 5*x^2 + 3)
- 36*(4*sqrt(3)*(2*x^6 + 5*x^4)*sqrt(x^4 + 5*x^2 + 3) - sqrt(3)*(
8*x^8 + 40*x^6 + 37*x^4))*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) -
5) - 77*(8*x^8 + 40*x^6 + 37*x^4 - 4*(2*x^6 + 5*x^4)*sqrt(x^4 + 5
*x^2 + 3))*log((6*x^2 + sqrt(3)*(2*x^4 + 5*x^2 + 6) - 2*sqrt(x^4
+ 5*x^2 + 3)*(sqrt(3)*x^2 + 3))/(2*x^4 - 2*sqrt(x^4 + 5*x^2 + 3)*
x^2 + 5*x^2)) - 2*sqrt(3)*(508*x^6 + 2361*x^4 + 2124*x^2 + 360))/
(4*sqrt(3)*(2*x^6 + 5*x^4)*sqrt(x^4 + 5*x^2 + 3) - sqrt(3)*(8*x^8
 + 40*x^6 + 37*x^4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**5,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**5, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 5\,x^2 + 3}\left(3\,x^2 + 2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^5, x)
```

3.148 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx$

Optimal. Leaf size=90

$$-\frac{(5x^2+6)\sqrt{x^4+5x^2+3}}{18x^4}+\frac{13\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{36\sqrt{3}}-\frac{(x^4+5x^2+3)^{3/2}}{9x^6}$$

```
[Out] -((6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(18*x^4) - (3 + 5*x^2 + x^4)
^(3/2)/(9*x^6) + (13*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^
2 + x^4])])/(36*Sqrt[3])
```

Rubi [A] time = 0.163155, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{(5x^2+6)\sqrt{x^4+5x^2+3}}{18x^4}+\frac{13\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{36\sqrt{3}}-\frac{(x^4+5x^2+3)^{3/2}}{9x^6}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^7,x]
```

```
[Out] -((6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(18*x^4) - (3 + 5*x^2 + x^4)
^(3/2)/(9*x^6) + (13*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^
2 + x^4])])/(36*Sqrt[3])
```

Rubi in Sympy [A] time = 18.302, size = 80, normalized size = 0.89

$$\frac{13\sqrt{3}\,\operatorname{atanh}\left(\frac{\sqrt{3}(5x^2+6)}{6\sqrt{x^4+5x^2+3}}\right)}{108}-\frac{(5x^2+6)\sqrt{x^4+5x^2+3}}{18x^4}-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**7,x)
```

```
[Out] 13*sqrt(3)*atanh(sqrt(3)*(5*x**2 + 6)/(6*sqrt(x**4 + 5*x**2 + 3))
)/108 - (5*x**2 + 6)*sqrt(x**4 + 5*x**2 + 3)/(18*x**4) - (x**4 +
5*x**2 + 3)**(3/2)/(9*x**6)
```

Mathematica [A] time = 0.100609, size = 79, normalized size = 0.88

$$\frac{1}{108}\left(-13\sqrt{3}\left(\log\left(x^2\right)-\log\left(5x^2+2\sqrt{3}\sqrt{x^4+5x^2+3}+6\right)\right)-\frac{6\sqrt{x^4+5x^2+3}\left(7x^4+16x^2+6\right)}{x^6}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^7,x]
```

```
[Out] ((-6*Sqrt[3 + 5*x^2 + x^4]*(6 + 16*x^2 + 7*x^4))/x^6 - 13*Sqrt[3]
*(Log[x^2] - Log[6 + 5*x^2 + 2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]]))/1
08
```

Maple [A] time = 0.02, size = 118, normalized size = 1.3

$$-\frac{1}{9x^6}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{1}{9x^4}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{5}{54x^2}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{13}{108}\sqrt{x^4+5x^2+3}+\frac{13\sqrt{3}}{108}\mathit{Artanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5x^2+3}}\right)-\frac{10x^2+25}{108}\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x)
```

```
[Out] -1/9*(x^4+5*x^2+3)^(3/2)/x^6-1/9/x^4*(x^4+5*x^2+3)^(3/2)+5/54/x^2
*(x^4+5*x^2+3)^(3/2)-13/108*(x^4+5*x^2+3)^(1/2)+13/108*arctanh(1/
6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-5/108*(2*x^2+5)*
(x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 0.807117, size = 134, normalized size = 1.49

$$\frac{13}{108}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{1}{9}\sqrt{x^4+5x^2+3}+\frac{5\sqrt{x^4+5x^2+3}}{18x^2}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{9x^4}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^7,x, algorithm="maxima")
```

```
[Out] 13/108*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 +
5) + 1/9*sqrt(x^4 + 5*x^2 + 3) + 5/18*sqrt(x^4 + 5*x^2 + 3)/x^2 -
 1/9*(x^4 + 5*x^2 + 3)^(3/2)/x^4 - 1/9*(x^4 + 5*x^2 + 3)^(3/2)/x^
6
```

Fricas [A] time = 0.274619, size = 362, normalized size = 4.02

$$\frac{2\sqrt{3}\left(1072\,x^{8}+6468\,x^{6}+11927\,x^{4}+8012\,x^{2}+1830\right)\sqrt{x^{4}+5\,x^{2}+3}-13\left(32\,x^{12}+240\,x^{10}+522\,x^{8}+305\,x^{6}-2\left(16\,x^{10}+8\right.\right.}{36\left(2\sqrt{3}(16\,x^{10}+80\,x^{8}+87\,x^{6})\sqrt{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/36*(2*sqrt(3)*(1072*x^8 + 6468*x^6 + 11927*x^4 + 8012*x^2 + 183
0)*sqrt(x^4 + 5*x^2 + 3) - 13*(32*x^12 + 240*x^10 + 522*x^8 + 305
*x^6 - 2*(16*x^10 + 80*x^8 + 87*x^6)*sqrt(x^4 + 5*x^2 + 3))*log(-
(6*x^2 - sqrt(3)*(2*x^4 + 5*x^2 + 6) + 2*sqrt(x^4 + 5*x^2 + 3)*(s
qrt(3)*x^2 - 3))/(2*x^4 - 2*sqrt(x^4 + 5*x^2 + 3)*x^2 + 5*x^2)) -
 2*sqrt(3)*(1072*x^10 + 9148*x^8 + 26355*x^6 + 31674*x^4 + 16452*
x^2 + 3132))/(2*sqrt(3)*(16*x^10 + 80*x^8 + 87*x^6)*sqrt(x^4 + 5*
x^2 + 3) - sqrt(3)*(32*x^12 + 240*x^10 + 522*x^8 + 305*x^6))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3x^{2}+2\right)\sqrt{x^{4}+5x^{2}+3}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**7,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**7, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^{4}+5\,x^{2}+3}\left(3\,x^{2}+2\right)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^7, x)
```

3.149 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx$

Optimal. Leaf size=111

$$\frac{67\left(5x^2+6\right)\sqrt{x^4+5x^2+3}}{1728x^4}-\frac{871\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3456\sqrt{3}}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{12x^8}-\frac{11\left(x^4+5x^2+3\right)^{3/2}}{216x^6}$$

```
[Out] (67*(6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(1728*x^4) - (3 + 5*x^2 +
x^4)^(3/2)/(12*x^8) - (11*(3 + 5*x^2 + x^4)^(3/2))/(216*x^6) - (8
71*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(3456*
Sqrt[3])
```

Rubi [A] time = 0.215769, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$\frac{67\left(5x^2+6\right)\sqrt{x^4+5x^2+3}}{1728x^4}-\frac{871\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3456\sqrt{3}}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{12x^8}-\frac{11\left(x^4+5x^2+3\right)^{3/2}}{216x^6}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^9,x]
```

```
[Out] (67*(6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(1728*x^4) - (3 + 5*x^2 +
x^4)^(3/2)/(12*x^8) - (11*(3 + 5*x^2 + x^4)^(3/2))/(216*x^6) - (8
71*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(3456*
Sqrt[3])
```

Rubi in Sympy [A] time = 22.835, size = 102, normalized size = 0.92

$$-\frac{871\sqrt{3}\operatorname{atanh}\left(\frac{\sqrt{3}\left(5x^2+6\right)}{6\sqrt{x^4+5x^2+3}}\right)}{10368}+\frac{67\left(5x^2+6\right)\sqrt{x^4+5x^2+3}}{1728x^4}-\frac{11\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{216x^6}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{12x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**9,x)
```

```
[Out] -871*sqrt(3)*atanh(sqrt(3)*(5*x**2 + 6)/(6*sqrt(x**4 + 5*x**2 + 3
)))/10368 + 67*(5*x**2 + 6)*sqrt(x**4 + 5*x**2 + 3)/(1728*x**4) -
 11*(x**4 + 5*x**2 + 3)**(3/2)/(216*x**6) - (x**4 + 5*x**2 + 3)**
```

```
(3/2)/(12*x**8)
```

Mathematica [A] time = 0.0864444, size = 87, normalized size = 0.78

$$\frac{871\sqrt{3}x^8\left(\log\left(x^2\right)-\log\left(5x^2+2\sqrt{3}\sqrt{x^4+5x^2+3}+6\right)\right)-6\sqrt{x^4+5x^2+3}\left(-247x^6+182x^4+984x^2+432\right)}{10368x^8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^9,x]
```

```
[Out] (-6*Sqrt[3 + 5*x^2 + x^4]*(432 + 984*x^2 + 182*x^4 - 247*x^6) + 8
71*Sqrt[3]*x^8*(Log[x^2] - Log[6 + 5*x^2 + 2*Sqrt[3]*Sqrt[3 + 5*x
^2 + x^4]]))/(10368*x^8)
```

Maple [A] time = 0.023, size = 135, normalized size = 1.2

$$-\frac{1}{12x^8}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{11}{216x^6}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{67}{864x^4}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{335}{5184x^2}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{871}{10368}\sqrt{x^4+5x^2+3}-\frac{871\sqrt{3}}{10368}\mathit{Artanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5x^2+3}}\right)+\frac{670x^2+1675}{10368}\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x)
```

```
[Out] -1/12*(x^4+5*x^2+3)^(3/2)/x^8-11/216*(x^4+5*x^2+3)^(3/2)/x^6+67/8
64/x^4*(x^4+5*x^2+3)^(3/2)-335/5184/x^2*(x^4+5*x^2+3)^(3/2)+871/1
0368*(x^4+5*x^2+3)^(1/2)-871/10368*arctanh(1/6*(5*x^2+6)*3^(1/2)/
(x^4+5*x^2+3)^(1/2))*3^(1/2)+335/10368*(2*x^2+5)*(x^4+5*x^2+3)^(1
/2)
```

Maxima [A] time = 0.834927, size = 157, normalized size = 1.41

$$-\frac{871}{10368}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{67}{864}\sqrt{x^4+5x^2+3}-\frac{335\sqrt{x^4+5x^2+3}}{1728x^2}+\frac{67\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{864x^4}-\frac{11\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{216x^6}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{12x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^9,x, algorithm="maxima")
```

```
[Out] -871/10368*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^
2 + 5) - 67/864*sqrt(x^4 + 5*x^2 + 3) - 335/1728*sqrt(x^4 + 5*x^2
 + 3)/x^2 + 67/864*(x^4 + 5*x^2 + 3)^(3/2)/x^4 - 11/216*(x^4 + 5*
x^2 + 3)^(3/2)/x^6 - 1/12*(x^4 + 5*x^2 + 3)^(3/2)/x^8
```

Fricas [A] time = 0.274378, size = 413, normalized size = 3.72

$$-\frac{2\sqrt{3}\left(55744\,x^{12}+182512\,x^{10}-1016024\,x^{8}-5309713\,x^{6}-8186726\,x^{4}-5085336\,x^{2}-1109808\right)\sqrt{x^4+5\,x^2+3}+871\left(128\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/3456*(2*sqrt(3)*(55744*x^12 + 182512*x^10 - 1016024*x^8 - 5309
713*x^6 - 8186726*x^4 - 5085336*x^2 - 1109808)*sqrt(x^4 + 5*x^2 +
 3) + 871*(128*x^16 + 1280*x^14 + 4384*x^12 + 5920*x^10 + 2569*x^
8 - 8*(16*x^14 + 120*x^12 + 274*x^10 + 185*x^8)*sqrt(x^4 + 5*x^2
+ 3))*log((6*x^2 + sqrt(3)*(2*x^4 + 5*x^2 + 6) - 2*sqrt(x^4 + 5*x
^2 + 3)*(sqrt(3)*x^2 + 3))/(2*x^4 - 2*sqrt(x^4 + 5*x^2 + 3)*x^2 +
 5*x^2)) - 2*sqrt(3)*(55744*x^14 + 321872*x^12 - 650328*x^10 - 79
19895*x^8 - 19708264*x^6 - 21178704*x^4 - 10406592*x^2 - 1918080)
)/(8*sqrt(3)*(16*x^14 + 120*x^12 + 274*x^10 + 185*x^8)*sqrt(x^4 +
 5*x^2 + 3) - sqrt(3)*(128*x^16 + 1280*x^14 + 4384*x^12 + 5920*x^
10 + 2569*x^8))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**9,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**9, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 5x^2 + 3}\,(3x^2 + 2)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^9,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^9, x)
```

3.150 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx$

Optimal. Leaf size=132

$$-\frac{161(5x^2+6)\sqrt{x^4+5x^2+3}}{5184x^4}+\frac{2093\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{10368\sqrt{3}}-\frac{(x^4+5x^2+3)^{3/2}}{15x^{10}}-\frac{(x^4+5x^2+3)^{3/2}}{36x^8}+\frac{173(x^4+5x^2+3)^{3/2}}{3240x^6}$$

```
[Out] (-161*(6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(5184*x^4) - (3 + 5*x^2
+ x^4)^(3/2)/(15*x^10) - (3 + 5*x^2 + x^4)^(3/2)/(36*x^8) + (173*
(3 + 5*x^2 + x^4)^(3/2))/(3240*x^6) + (2093*ArcTanh[(6 + 5*x^2)/(
2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(10368*Sqrt[3])
```

Rubi [A] time = 0.268982, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$-\frac{161(5x^2+6)\sqrt{x^4+5x^2+3}}{5184x^4}+\frac{2093\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{10368\sqrt{3}}-\frac{(x^4+5x^2+3)^{3/2}}{15x^{10}}-\frac{(x^4+5x^2+3)^{3/2}}{36x^8}+\frac{173(x^4+5x^2+3)^{3/2}}{3240x^6}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^11,x]
```

```
[Out] (-161*(6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(5184*x^4) - (3 + 5*x^2
+ x^4)^(3/2)/(15*x^10) - (3 + 5*x^2 + x^4)^(3/2)/(36*x^8) + (173*
(3 + 5*x^2 + x^4)^(3/2))/(3240*x^6) + (2093*ArcTanh[(6 + 5*x^2)/(
2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(10368*Sqrt[3])
```

Rubi in Sympy [A] time = 27.3705, size = 121, normalized size = 0.92

$$\frac{2093\sqrt{3}\operatorname{atanh}\left(\frac{\sqrt{3}(5x^2+6)}{6\sqrt{x^4+5x^2+3}}\right)}{31104}-\frac{161(5x^2+6)\sqrt{x^4+5x^2+3}}{5184x^4}+\frac{173(x^4+5x^2+3)^{\frac{3}{2}}}{3240x^6}-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{36x^8}-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{15x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**11,x)
```

```
[Out] 2093*sqrt(3)*atanh(sqrt(3)*(5*x**2 + 6)/(6*sqrt(x**4 + 5*x**2 + 3
)))/31104 - 161*(5*x**2 + 6)*sqrt(x**4 + 5*x**2 + 3)/(5184*x**4)
+ 173*(x**4 + 5*x**2 + 3)**(3/2)/(3240*x**6) - (x**4 + 5*x**2 + 3
)**(3/2)/(36*x**8) - (x**4 + 5*x**2 + 3)**(3/2)/(15*x**10)
```

Mathematica [A] time = 0.124435, size = 89, normalized size = 0.67

$$\frac{-10465\sqrt{3}\left(\log\left(x^2\right)-\log\left(5x^2+2\sqrt{3}\sqrt{x^4+5x^2+3}+6\right)\right)-\frac{6\sqrt{x^4+5x^2+3}\left(2641x^8-1370x^6+1176x^4+10800x^2+5184\right)}{x^{10}}}{155520}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^11,x]
```

```
[Out] ((-6*Sqrt[3 + 5*x^2 + x^4]*(5184 + 10800*x^2 + 1176*x^4 - 1370*x^
6 + 2641*x^8))/x^10 - 10465*Sqrt[3]*(Log[x^2] - Log[6 + 5*x^2 + 2
*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]]))/155520
```

Maple [A] time = 0.025, size = 152, normalized size = 1.2

$$-\frac{1}{15\,x^{10}}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}-\frac{1}{36\,x^8}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}+\frac{173}{3240\,x^6}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}-\frac{161}{2592\,x^4}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}+\frac{805}{15552\,x^2}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}-\frac{2093}{31104}\sqrt{x^4+5\,x^2+3}+\frac{2093\,\sqrt{3}}{31104}Artanh\left(\frac{\left(5\,x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5\,x^2+3}}\right)-\frac{1610\,x^2+4025}{31104}\sqrt{x^4+5\,x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x)
```

```
[Out] -1/15*(x^4+5*x^2+3)^(3/2)/x^10-1/36*(x^4+5*x^2+3)^(3/2)/x^8+173/3
240*(x^4+5*x^2+3)^(3/2)/x^6-161/2592/x^4*(x^4+5*x^2+3)^(3/2)+805/
15552/x^2*(x^4+5*x^2+3)^(3/2)-2093/31104*(x^4+5*x^2+3)^(1/2)+2093
/31104*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
-805/31104*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 0.815431, size = 180, normalized size = 1.36

$$\frac{2093}{31104}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{161}{2592}\sqrt{x^4+5x^2+3}+\frac{805\sqrt{x^4+5x^2+3}}{5184x^2}-\frac{161\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{2592x^4}+\frac{173\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{3240x^6}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{36x^8}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{15x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^11,x, algorithm="maxima")
```

```
[Out] 2093/31104*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^
2 + 5) + 161/2592*sqrt(x^4 + 5*x^2 + 3) + 805/5184*sqrt(x^4 + 5*x
^2 + 3)/x^2 - 161/2592*(x^4 + 5*x^2 + 3)^(3/2)/x^4 + 173/3240*(x^
4 + 5*x^2 + 3)^(3/2)/x^6 - 1/36*(x^4 + 5*x^2 + 3)^(3/2)/x^8 - 1/1
5*(x^4 + 5*x^2 + 3)^(3/2)/x^10
```

Fricas [A] time = 0.275158, size = 470, normalized size = 3.56

$$\frac{10\sqrt{3}\left(535808x^{16}+4688320x^{14}+14822768x^{12}+30236712x^{10}+80707685x^{8}+178917094x^{6}+207290040x^{4}+11192299\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/51840*(10*sqrt(3)*(535808*x^16 + 4688320*x^14 + 14822768*x^12 +
 30236712*x^10 + 80707685*x^8 + 178917094*x^6 + 207290040*x^4 + 1
11922992*x^2 + 22524480)*sqrt(x^4 + 5*x^2 + 3) - 10465*(512*x^20
+ 6400*x^18 + 29920*x^16 + 64400*x^14 + 62690*x^12 + 21725*x^10 -
 2*(256*x^18 + 2560*x^16 + 8976*x^14 + 12880*x^12 + 6269*x^10)*sq
rt(x^4 + 5*x^2 + 3))*log(-(6*x^2 - sqrt(3)*(2*x^4 + 5*x^2 + 6) +
2*sqrt(x^4 + 5*x^2 + 3)*(sqrt(3)*x^2 - 3))/(2*x^4 - 2*sqrt(x^4 +
5*x^2 + 3)*x^2 + 5*x^2)) - 2*sqrt(3)*(2679040*x^18 + 30139200*x^1
6 + 128364400*x^14 + 309259160*x^12 + 725547665*x^10 + 1784368010
*x^8 + 2899650660*x^6 + 2567795760*x^4 + 1131835680*x^2 + 1949909
76))/(2*sqrt(3)*(256*x^18 + 2560*x^16 + 8976*x^14 + 12880*x^12 +
6269*x^10)*sqrt(x^4 + 5*x^2 + 3) - sqrt(3)*(512*x^20 + 6400*x^18
+ 29920*x^16 + 64400*x^14 + 62690*x^12 + 21725*x^10))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**11,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**11, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5\,x^2+3}\,(3\,x^2+2)}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^11,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^11, x)
```

3.151 $\int x^4 \left(2+3x^2\right) \sqrt{3+5x^2+x^4}\, dx$

Optimal. Leaf size=322

$$\frac{13}{3}\sqrt{x^4+5x^2+3}x-\frac{1924\left(2x^2+\sqrt{13}+5\right)x}{105\sqrt{x^4+5x^2+3}}-\frac{13\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}+\frac{962\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{105\sqrt{x^4+5x^2+3}}+\frac{1}{21}\left(7x^2+11\right)\sqrt{x^4+5x^2+3}x^5-\frac{26}{35}\sqrt{x^4+5x^2+3}x^3$$

```
[Out] (-1924*x*(5 + Sqrt[13] + 2*x^2))/(105*Sqrt[3 + 5*x^2 + x^4]) + (1
3*x*Sqrt[3 + 5*x^2 + x^4])/3 - (26*x^3*Sqrt[3 + 5*x^2 + x^4])/35
+ (x^5*(11 + 7*x^2)*Sqrt[3 + 5*x^2 + x^4])/21 + (962*Sqrt[(2*(5 +
 Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*
x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13
])/6]*x], (-13 + 5*Sqrt[13])/6])/(105*Sqrt[3 + 5*x^2 + x^4]) - (1
3*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5
 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-1
3 + 5*Sqrt[13])/6])/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4]
)
```

Rubi [A] time = 0.602963, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{13}{3}\sqrt{x^4+5x^2+3}x-\frac{1924\left(2x^2+\sqrt{13}+5\right)x}{105\sqrt{x^4+5x^2+3}}-\frac{13\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}+\frac{962\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{105\sqrt{x^4+5x^2+3}}+\frac{1}{21}\left(7x^2+11\right)\sqrt{x^4+5x^2+3}x^5-\frac{26}{35}\sqrt{x^4+5x^2+3}x^3$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-1924*x*(5 + Sqrt[13] + 2*x^2))/(105*Sqrt[3 + 5*x^2 + x^4]) + (1
3*x*Sqrt[3 + 5*x^2 + x^4])/3 - (26*x^3*Sqrt[3 + 5*x^2 + x^4])/35
+ (x^5*(11 + 7*x^2)*Sqrt[3 + 5*x^2 + x^4])/21 + (962*Sqrt[(2*(5 +
 Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*
x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13
])/6]*x], (-13 + 5*Sqrt[13])/6])/(105*Sqrt[3 + 5*x^2 + x^4]) - (1
3*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5
 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-1
3 + 5*Sqrt[13])/6])/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4]
)
```

Rubi in Sympy [A] time = 42.9635, size = 301, normalized size = 0.93

$$\frac{x^5\left(21x^2+33\right)\sqrt{x^4+5x^2+3}}{63}-\frac{26x^3\sqrt{x^4+5x^2+3}}{35}-\frac{1924x\left(2x^2+\sqrt{13}+5\right)}{105\sqrt{x^4+5x^2+3}}+\frac{13x\sqrt{x^4+5x^2+3}}{3}+\frac{962\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{315\sqrt{x^4+5x^2+3}}-\frac{13\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{6\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] x**5*(21*x**2 + 33)*sqrt(x**4 + 5*x**2 + 3)/63 - 26*x**3*sqrt(x**
4 + 5*x**2 + 3)/35 - 1924*x*(2*x**2 + sqrt(13) + 5)/(105*sqrt(x**
4 + 5*x**2 + 3)) + 13*x*sqrt(x**4 + 5*x**2 + 3)/3 + 962*sqrt(6)*s
qrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6))*sqrt(sq
rt(13) + 5)*(x**2*(sqrt(13) + 5) + 6)*elliptic_e(atan(sqrt(6)*x*s
qrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(315*sqrt(x**4 + 5*x*
*2 + 3)) - 13*sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt
(13) + 5) + 6))*(x**2*(sqrt(13) + 5) + 6)*elliptic_f(atan(sqrt(6)
*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(6*sqrt(sqrt(13)
+ 5)*sqrt(x**4 + 5*x**2 + 3))
```

Mathematica [C] time = 0.551457, size = 237, normalized size = 0.74

$$\frac{70x^{11}+460x^9+604x^7+460x^5+4082x^3+13i\sqrt{2}\left(148\sqrt{13}-635\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}\right.}{210\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^4*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2730*x + 4082*x^3 + 460*x^5 + 604*x^7 + 460*x^9 + 70*x^11 - (192
4*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + S
qrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(
5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + (13*I)*Sqrt[2]*(-635
+ 148*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqr
t[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])
]*x], 19/6 + (5*Sqrt[13])/6])/(210*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.3, size = 260, normalized size = 0.8

$$\frac{11x^5}{21}\sqrt{x^4+5x^2+3}-\frac{26x^3}{35}\sqrt{x^4+5x^2+3}+\frac{13x}{3}\sqrt{x^4+5x^2+3}$$
$$-78\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right)x^2}EllipticF\left(1/6x\sqrt{-30+6\sqrt{13}},5/6\sqrt{3}+1/6\sqrt{39}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$$
$$+\frac{46176}{35\sqrt{-30+6\sqrt{13}}\left(5+\sqrt{13}\right)}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-Ell\right.$$
$$+\frac{x^7}{3}\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 11/21*x^5*(x^4+5*x^2+3)^(1/2)-26/35*x^3*(x^4+5*x^2+3)^(1/2)+13/3*
x*(x^4+5*x^2+3)^(1/2)-78/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(
1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^
(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(
1/2))+46176/35/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)
^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+1
3^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*
39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*
39^(1/2)))+1/3*x^7*(x^4+5*x^2+3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(3\,x^6+2\,x^4\right)\sqrt{x^4+5\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^4,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4)*sqrt(x^4 + 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**4*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^4,x, algorithm="giac")

[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^4, x)
```

3.152 $\int x^2 \left(2+3x^2\right) \sqrt{3+5x^2+x^4}\,dx$

Optimal. Leaf size=305

$$-\frac{4}{3}\sqrt{x^4+5x^2+3}x+\frac{1247\left(2x^2+\sqrt{13}+5\right)x}{210\sqrt{x^4+5x^2+3}}+\frac{2\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{1247\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{210\sqrt{x^4+5x^2+3}}+\frac{1}{35}\left(15x^2+29\right)\sqrt{x^4+5x^2+3}x^3$$

```
[Out] (1247*x*(5 + Sqrt[13] + 2*x^2))/(210*Sqrt[3 + 5*x^2 + x^4]) - (4*
x*Sqrt[3 + 5*x^2 + x^4])/3 + (x^3*(29 + 15*x^2)*Sqrt[3 + 5*x^2 +
x^4])/35 - (1247*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*
x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE
[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(210*Sq
rt[3 + 5*x^2 + x^4]) + (2*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5
 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x
^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13]
)/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi [A] time = 0.464907, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{4}{3}\sqrt{x^4+5x^2+3}x+\frac{1247\left(2x^2+\sqrt{13}+5\right)x}{210\sqrt{x^4+5x^2+3}}+\frac{2\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{1247\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{210\sqrt{x^4+5x^2+3}}+\frac{1}{35}\left(15x^2+29\right)\sqrt{x^4+5x^2+3}x^3$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (1247*x*(5 + Sqrt[13] + 2*x^2))/(210*Sqrt[3 + 5*x^2 + x^4]) - (4*
x*Sqrt[3 + 5*x^2 + x^4])/3 + (x^3*(29 + 15*x^2)*Sqrt[3 + 5*x^2 +
x^4])/35 - (1247*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*
x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE
[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(210*Sq
rt[3 + 5*x^2 + x^4]) + (2*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5
 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x
^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13]
)/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi in Sympy [A] time = 37.8938, size = 280, normalized size = 0.92

$$\frac{x^3\left(15x^2+29\right)\sqrt{x^4+5x^2+3}}{35}+\frac{1247x\left(2x^2+\sqrt{13}+5\right)}{210\sqrt{x^4+5x^2+3}}-\frac{4x\sqrt{x^4+5x^2+3}}{3}$$
$$-\frac{1247\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{1260\sqrt{x^4+5x^2+3}}$$
$$+\frac{2\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{3\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] x**3*(15*x**2 + 29)*sqrt(x**4 + 5*x**2 + 3)/35 + 1247*x*(2*x**2 +
 sqrt(13) + 5)/(210*sqrt(x**4 + 5*x**2 + 3)) - 4*x*sqrt(x**4 + 5*
x**2 + 3)/3 - 1247*sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*
(sqrt(13) + 5) + 6))*sqrt(sqrt(13) + 5)*(x**2*(sqrt(13) + 5) + 6)
*elliptic_e(atan(sqrt(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(
13)/6)/(1260*sqrt(x**4 + 5*x**2 + 3)) + 2*sqrt(6)*sqrt((x**2*(-sq
rt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6))*(x**2*(sqrt(13) + 5)
+ 6)*elliptic_f(atan(sqrt(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*s
qrt(13)/6)/(3*sqrt(sqrt(13) + 5)*sqrt(x**4 + 5*x**2 + 3))
```

Mathematica [C] time = 0.523429, size = 234, normalized size = 0.77

$$\frac{-i\sqrt{2}\left(1247\sqrt{13}-5395\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)\middle|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+1247i\sqrt{2}\left(\sqrt{13}-5\right)\sqrt{\frac{-2x^2+\sqrt{13}-}{\sqrt{13}-5}}}{420\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (4*x*(-420 - 439*x^2 + 430*x^4 + 312*x^6 + 45*x^8) + (1247*I)*Sqr
t[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])
]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt
[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-5395 + 1247*Sqrt[
13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[
13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6
+ (5*Sqrt[13])/6])/(420*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.019, size = 243, normalized size = 0.8

$$\frac{29x^3}{35}\sqrt{x^4+5x^2+3}-\frac{4x}{3}\sqrt{x^4+5x^2+3} +24\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right)x^2}\,EllipticF\left(1/6\,x\sqrt{-30+6\sqrt{13}},5/6\sqrt{3}+1/6\sqrt{39}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}} -\frac{14964}{35\sqrt{-30+6\sqrt{13}}\left(5+\sqrt{13}\right)}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-El\right. +\frac{3x^5}{7}\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 29/35*x^3*(x^4+5*x^2+3)^(1/2)-4/3*x*(x^4+5*x^2+3)^(1/2)+24/(-30+6
*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*1
3^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13
^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-14964/35/(-30+6*13^(1/2))
^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x
^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+
6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+
6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))+3/7*x^5*(x^4+5*x^2+3
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^4+5x^2+3}\left(3x^2+2\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(3\,x^{4}+2\,x^{2}\right)\sqrt{x^{4}+5\,x^{2}+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^2,x, algorithm="fricas")
```

```
[Out] integral((3*x^4 + 2*x^2)*sqrt(x^4 + 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{2}\left(3x^{2}+2\right)\sqrt{x^{4}+5x^{2}+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**2*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^{4}+5\,x^{2}+3}\left(3\,x^{2}+2\right)x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^2, x)
```

3.153 $\int \left(2+3x^2\right) \sqrt{3+5x^2+x^4}\, dx$

Optimal. Leaf size=279

$$-\frac{23x\left(2x^2+\sqrt{13}+5\right)}{15\sqrt{x^4+5x^2+3}}+\frac{1}{15}x\left(9x^2+25\right)\sqrt{x^4+5x^2+3}+\frac{\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}+\frac{23\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{15\sqrt{x^4+5x^2+3}}$$

```
[Out] (-23*x*(5 + Sqrt[13] + 2*x^2))/(15*Sqrt[3 + 5*x^2 + x^4]) + (x*(2
5 + 9*x^2)*Sqrt[3 + 5*x^2 + x^4])/15 + (23*Sqrt[(5 + Sqrt[13])/6]
*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5
+ Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
 + 5*Sqrt[13])/6])/(15*Sqrt[3 + 5*x^2 + x^4]) + (Sqrt[(6 + (5 - S
qrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*
EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6]
)/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rubi [A] time = 0.285305, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{23x\left(2x^2+\sqrt{13}+5\right)}{15\sqrt{x^4+5x^2+3}}+\frac{1}{15}x\left(9x^2+25\right)\sqrt{x^4+5x^2+3}+\frac{\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}+\frac{23\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{15\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-23*x*(5 + Sqrt[13] + 2*x^2))/(15*Sqrt[3 + 5*x^2 + x^4]) + (x*(2
5 + 9*x^2)*Sqrt[3 + 5*x^2 + x^4])/15 + (23*Sqrt[(5 + Sqrt[13])/6]
*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5
+ Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
 + 5*Sqrt[13])/6])/(15*Sqrt[3 + 5*x^2 + x^4]) + (Sqrt[(6 + (5 - S
qrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*
EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6]
)/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rubi in Sympy [A] time = 23.4639, size = 258, normalized size = 0.92

$$\frac{x\left(9x^2+25\right)\sqrt{x^4+5x^2+3}}{15}-\frac{23x\left(2x^2+\sqrt{13}+5\right)}{15\sqrt{x^4+5x^2+3}}+\frac{23\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{90\sqrt{x^4+5x^2+3}}+\frac{\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{6\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] x*(9*x**2 + 25)*sqrt(x**4 + 5*x**2 + 3)/15 - 23*x*(2*x**2 + sqrt(
13) + 5)/(15*sqrt(x**4 + 5*x**2 + 3)) + 23*sqrt(6)*sqrt((x**2*(-s
qrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6))*sqrt(sqrt(13) + 5)*(
x**2*(sqrt(13) + 5) + 6)*elliptic_e(atan(sqrt(6)*x*sqrt(sqrt(13)
+ 5)/6), -13/6 + 5*sqrt(13)/6)/(90*sqrt(x**4 + 5*x**2 + 3)) + sqr
t(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6))*(
x**2*(sqrt(13) + 5) + 6)*elliptic_f(atan(sqrt(6)*x*sqrt(sqrt(13)
+ 5)/6), -13/6 + 5*sqrt(13)/6)/(6*sqrt(sqrt(13) + 5)*sqrt(x**4 +
5*x**2 + 3))
```

Mathematica [C] time = 0.516216, size = 229, normalized size = 0.82

$$\frac{i\sqrt{2}\left(23\sqrt{13}-130\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-23i\sqrt{2}\left(\sqrt{13}-5\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2}}{30\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*x*(75 + 152*x^2 + 70*x^4 + 9*x^6) - (23*I)*Sqrt[2]*(-5 + Sqrt[
13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[
13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6
+ (5*Sqrt[13])/6] + I*Sqrt[2]*(-130 + 23*Sqrt[13])*Sqrt[(-5 + Sqr
t[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ellipt
icF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/
(30*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.016, size = 226, normalized size = 0.8

$$\frac{5x}{3}\sqrt{x^4+5x^2+3} +6\,\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right)x^2}\,EllipticF\left(1/6\,x\sqrt{-30+6\sqrt{13}},5/6\sqrt{3}+1/6\sqrt{39}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}} +\frac{552}{5\sqrt{-30+6\sqrt{13}}\left(5+\sqrt{13}\right)}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-Elli\right. +\frac{3x^3}{5}\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 5/3*x*(x^4+5*x^2+3)^(1/2)+6/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*1
3^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+
3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*3
9^(1/2))+552/5/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)
^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+1
3^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*
39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*
39^(1/2)))+3/5*x^3*(x^4+5*x^2+3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+5x^2+3}\left(3x^2+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x^4 + 5\,x^2 + 3}\left(3\,x^2 + 2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(3x^2 + 2\right)\sqrt{x^4 + 5x^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 5\,x^2 + 3}\left(3\,x^2 + 2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2), x)
```

3.154 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx$

Optimal. Leaf size=284

$$-\frac{\sqrt{x^4+5x^2+3}\,(2-x^2)}{x}+\frac{9x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}}+\frac{8\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{3\sqrt{\frac{3}{2}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{2\sqrt{x^4+5x^2+3}}$$

```
[Out] (9*x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - ((2 - x^
2)*Sqrt[3 + 5*x^2 + x^4])/x - (3*Sqrt[(3*(5 + Sqrt[13]))/2]*Sqrt[
(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt
[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*S
qrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4]) + (8*Sqrt[2/(3*(5 + Sqrt[1
3]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6
+ (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x],
 (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi [A] time = 0.297493, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$-\frac{\sqrt{x^4+5x^2+3}\,(2-x^2)}{x}+\frac{9x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}}+\frac{8\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{3\sqrt{\frac{3}{2}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{2\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^2,x]
```

```
[Out] (9*x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - ((2 - x^
2)*Sqrt[3 + 5*x^2 + x^4])/x - (3*Sqrt[(3*(5 + Sqrt[13]))/2]*Sqrt[
```

```
(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt
[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*S
qrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4]) + (8*Sqrt[2/(3*(5 + Sqrt[1
3]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6
+ (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x],
 (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi in Sympy [A] time = 25.2566, size = 260, normalized size = 0.92

$$\frac{9x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}} - \frac{3\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{4\sqrt{x^4+5x^2+3}} + \frac{8\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{3\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}} - \frac{\left(-3x^2+6\right)\sqrt{x^4+5x^2+3}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**2,x)
```

```
[Out] 9*x*(2*x**2 + sqrt(13) + 5)/(2*sqrt(x**4 + 5*x**2 + 3)) - 3*sqrt(
6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6))*sqr
t(sqrt(13) + 5)*(x**2*(sqrt(13) + 5) + 6)*elliptic_e(atan(sqrt(6)
*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(4*sqrt(x**4 + 5*
x**2 + 3)) + 8*sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqr
t(13) + 5) + 6))*(x**2*(sqrt(13) + 5) + 6)*elliptic_f(atan(sqrt(6
)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(3*sqrt(sqrt(13)
 + 5)*sqrt(x**4 + 5*x**2 + 3)) - (-3*x**2 + 6)*sqrt(x**4 + 5*x**2
 + 3)/(3*x)
```

Mathematica [C] time = 0.52857, size = 231, normalized size = 0.81

$$\frac{-i\sqrt{2}\left(9\sqrt{13}-13\right)x\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+9i\sqrt{2}\left(\sqrt{13}-5\right)x\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2}}{4x\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^2,x]
```

```
[Out] (4*(-6 - 7*x^2 + 3*x^4 + x^6) + (9*I)*Sqrt[2]*(-5 + Sqrt[13])*x*S
qrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] +
2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*S
qrt[13])/6] - I*Sqrt[2]*(-13 + 9*Sqrt[13])*x*Sqrt[(-5 + Sqrt[13]
- 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*
ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(4*x*S
qrt[3 + 5*x^2 + x^4])
```

Maple [A]   time = 0.025, size = 225, normalized size = 0.8

$$x\sqrt{x^4+5x^2+3}$$
$$+96\,\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right)x^2}\,\mathit{EllipticF}\left(1/6\,x\sqrt{-30+6\sqrt{13}},5/6\sqrt{3}+1/6\sqrt{39}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$$
$$-324\,\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right)x^2}\left(\mathit{EllipticF}\left(1/6\,x\sqrt{-30+6\sqrt{13}},5/6\sqrt{3}+1/6\sqrt{39}\right)-\mathit{Elliptic}\ldots\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}\left(5+\sqrt{13}\right)}$$
$$-2\,\frac{\sqrt{x^4+5x^2+3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x)
```

```
[Out] x*(x^4+5*x^2+3)^(1/2)+96/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(
1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^
(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(
1/2))-324/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2
)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/
2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1
/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1
/2)))-2*(x^4+5*x^2+3)^(1/2)/x
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5x^2+3}\left(3x^2+2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2,x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**2,x)

[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2,x, algorithm="giac")

[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2, x)
```

3.155 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx$

Optimal. Leaf size=305

$$-\frac{64\sqrt{x^4+5x^2+3}}{9x}+\frac{32x\left(2x^2+\sqrt{13}+5\right)}{9\sqrt{x^4+5x^2+3}}+\frac{49\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{3\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}-\frac{16\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{9\sqrt{x^4+5x^2+3}}-\frac{\sqrt{x^4+5x^2+3}\left(2-9x^2\right)}{3x^3}$$

```
[Out] (32*x*(5 + Sqrt[13] + 2*x^2))/(9*Sqrt[3 + 5*x^2 + x^4]) - (64*Sqr
t[3 + 5*x^2 + x^4])/(9*x) - ((2 - 9*x^2)*Sqrt[3 + 5*x^2 + x^4])/(
3*x^3) - (16*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[13])*
x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE
[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(9*Sqrt
[3 + 5*x^2 + x^4]) + (49*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5
 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(3*Sqrt[6*(5 + Sqrt[13
])]*Sqrt[3 + 5*x^2 + x^4])
```

Rubi [A] time = 0.377835, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{64\sqrt{x^4+5x^2+3}}{9x}+\frac{32x\left(2x^2+\sqrt{13}+5\right)}{9\sqrt{x^4+5x^2+3}}+\frac{49\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{3\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}-\frac{16\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{9\sqrt{x^4+5x^2+3}}-\frac{\sqrt{x^4+5x^2+3}\left(2-9x^2\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4,x]
```

```
[Out] (32*x*(5 + Sqrt[13] + 2*x^2))/(9*Sqrt[3 + 5*x^2 + x^4]) - (64*Sqr
t[3 + 5*x^2 + x^4])/(9*x) - ((2 - 9*x^2)*Sqrt[3 + 5*x^2 + x^4])/(
3*x^3) - (16*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[13])*
x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE
[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(9*Sqrt
[3 + 5*x^2 + x^4]) + (49*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5
 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(3*Sqrt[6*(5 + Sqrt[13
])]*Sqrt[3 + 5*x^2 + x^4])
```

Rubi in Sympy [A] time = 32.6291, size = 280, normalized size = 0.92

$$\frac{32x\left(2x^2+\sqrt{13}+5\right)}{9\sqrt{x^4+5x^2+3}} - \frac{16\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{27\sqrt{x^4+5x^2+3}} + \frac{49\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{18\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}} - \frac{64\sqrt{x^4+5x^2+3}}{9x} - \frac{\left(-9x^2+2\right)\sqrt{x^4+5x^2+3}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**4,x)
```

```
[Out] 32*x*(2*x**2 + sqrt(13) + 5)/(9*sqrt(x**4 + 5*x**2 + 3)) - 16*sqr
t(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6))*s
qrt(sqrt(13) + 5)*(x**2*(sqrt(13) + 5) + 6)*elliptic_e(atan(sqrt(
6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(27*sqrt(x**4 +
 5*x**2 + 3)) + 49*sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*
(sqrt(13) + 5) + 6))*(x**2*(sqrt(13) + 5) + 6)*elliptic_f(atan(sq
rt(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(18*sqrt(sqr
t(13) + 5)*sqrt(x**4 + 5*x**2 + 3)) - 64*sqrt(x**4 + 5*x**2 + 3)/
(9*x) - (-9*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/(3*x**3)
```

Mathematica [C] time = 0.549552, size = 237, normalized size = 0.78

$$\frac{-i\sqrt{2}\left(32\sqrt{13}-13\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}x^3F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+32i\sqrt{2}\left(\sqrt{13}-5\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2}}{18x^3\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4,x]
```

```
[Out] (-2*(18 + 141*x^2 + 191*x^4 + 37*x^6) + (32*I)*Sqrt[2]*(-5 + Sqrt
[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 +
Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x],
19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-13 + 32*Sqrt[13])*x^3*Sqrt[(
-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2
]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[1
3])/6])/(18*x^3*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.025, size = 228, normalized size = 0.8

$$-\frac{2}{3x^3}\sqrt{x^4+5x^2+3}-\frac{37}{9x}\sqrt{x^4+5x^2+3}$$
$$+98\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right)x^2}\,EllipticF\left(1/6\,x\sqrt{-30+6\sqrt{13}},5/6\sqrt{3}+1/6\sqrt{39}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$$
$$-256\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right)x^2}\left(EllipticF\left(1/6\,x\sqrt{-30+6\sqrt{13}},5/6\sqrt{3}+1/6\sqrt{39}\right)-EllipticE\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}\left(5+\sqrt{13}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x)
```

```
[Out] -2/3*(x^4+5*x^2+3)^(1/2)/x^3-37/9*(x^4+5*x^2+3)^(1/2)/x+98/(-30+6
*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*1
3^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13
^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-256/(-30+6*13^(1/2))^(1/2
)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(
1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^
(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^
(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5\,x^2+3}\,(3\,x^2+2)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+5\,x^2+3}\,(3\,x^2+2)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)\,\sqrt{x^4+5x^2+3}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**4,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**4, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5\,x^2+3}\,(3\,x^2+2)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4, x)
```

3.156 $\int x^5 (2+3x^2) (3+5x^2+x^4)^{3/2} \, dx$

Optimal. Leaf size=127

$$\frac{3}{14}\left(x^4+5x^2+3\right)^{5/2}x^4+\frac{\left(3313-1070x^2\right)\left(x^4+5x^2+3\right)^{5/2}}{1680} -\frac{2183}{768}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{28379\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{2048}-\frac{368927\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{4096}$$

```
[Out] (28379*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/2048 - (2183*(5 + 2*x^2
)*(3 + 5*x^2 + x^4)^(3/2))/768 + (3*x^4*(3 + 5*x^2 + x^4)^(5/2))/
14 + ((3313 - 1070*x^2)*(3 + 5*x^2 + x^4)^(5/2))/1680 - (368927*A
rcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/4096
```

Rubi [A] time = 0.229955, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{3}{14}\left(x^4+5x^2+3\right)^{5/2}x^4+\frac{\left(3313-1070x^2\right)\left(x^4+5x^2+3\right)^{5/2}}{1680} -\frac{2183}{768}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{28379\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{2048}-\frac{368927\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{4096}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (28379*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/2048 - (2183*(5 + 2*x^2
)*(3 + 5*x^2 + x^4)^(3/2))/768 + (3*x^4*(3 + 5*x^2 + x^4)^(5/2))/
14 + ((3313 - 1070*x^2)*(3 + 5*x^2 + x^4)^(5/2))/1680 - (368927*A
rcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/4096
```

Rubi in Sympy [A] time = 21.1937, size = 119, normalized size = 0.94

$$\frac{3x^4\left(x^4+5x^2+3\right)^{\frac{5}{2}}}{14}+\frac{\left(-\frac{535x^2}{2}+\frac{3313}{4}\right)\left(x^4+5x^2+3\right)^{\frac{5}{2}}}{420}-\frac{2183\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{768} +\frac{28379\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{2048}-\frac{368927\operatorname{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{4096}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] 3*x**4*(x**4 + 5*x**2 + 3)**(5/2)/14 + (-535*x**2/2 + 3313/4)*(x*
*4 + 5*x**2 + 3)**(5/2)/420 - 2183*(2*x**2 + 5)*(x**4 + 5*x**2 +
3)**(3/2)/768 + 28379*(2*x**2 + 5)*sqrt(x**4 + 5*x**2 + 3)/2048 -
 368927*atanh((2*x**2 + 5)/(2*sqrt(x**4 + 5*x**2 + 3)))/4096
```

Mathematica [A] time = 0.0672841, size = 79, normalized size = 0.62

$$\frac{2\sqrt{x^4+5x^2+3}\left(46080x^{12}+323840x^{10}+482944x^8+154800x^6+283304x^4-1499570x^2+9546951\right)-38737335\log\left(2x^2+\right.}{430080}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(9546951 - 1499570*x^2 + 283304*x^4 + 15
4800*x^6 + 482944*x^8 + 323840*x^10 + 46080*x^12) - 38737335*Log[
5 + 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/430080
```

Maple [A] time = 0.047, size = 138, normalized size = 1.1

$$\frac{253\,x^{10}}{168}\sqrt{x^4+5\,x^2+3}+\frac{539\,x^8}{240}\sqrt{x^4+5\,x^2+3}+\frac{645\,x^6}{896}\sqrt{x^4+5\,x^2+3}+\frac{5059\,x^4}{3840}\sqrt{x^4+5\,x^2+3}-\frac{149957\,x^2}{21504}\sqrt{x^4+5\,x^2+3}+\frac{3182317}{71680}\sqrt{x^4+5\,x^2+3}-\frac{368927}{4096}\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5\,x^2+3}\right)+\frac{3\,x^{12}}{14}\sqrt{x^4+5\,x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 253/168*x^10*(x^4+5*x^2+3)^(1/2)+539/240*x^8*(x^4+5*x^2+3)^(1/2)+
645/896*x^6*(x^4+5*x^2+3)^(1/2)+5059/3840*x^4*(x^4+5*x^2+3)^(1/2)
-149957/21504*x^2*(x^4+5*x^2+3)^(1/2)+3182317/71680*(x^4+5*x^2+3)
^(1/2)-368927/4096*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+3/14*x^12*(x^4
+5*x^2+3)^(1/2)
```

Maxima [A] time = 0.740455, size = 182, normalized size = 1.43

$$\frac{3}{14}\left(x^4+5x^2+3\right)^{\frac{5}{2}}x^4-\frac{107}{168}\left(x^4+5x^2+3\right)^{\frac{5}{2}}x^2-\frac{2183}{384}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2+\frac{3313}{1680}\left(x^4+5x^2+3\right)^{\frac{5}{2}}+\frac{28379}{1024}\sqrt{x^4+5x^2+3}x^2-\frac{10915}{768}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{141895}{2048}\sqrt{x^4+5x^2+3}-\frac{368927}{4096}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^5,x, algorithm="maxima")
```

```
[Out] 3/14*(x^4 + 5*x^2 + 3)^(5/2)*x^4 - 107/168*(x^4 + 5*x^2 + 3)^(5/2
)*x^2 - 2183/384*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 3313/1680*(x^4 + 5
*x^2 + 3)^(5/2) + 28379/1024*sqrt(x^4 + 5*x^2 + 3)*x^2 - 10915/76
8*(x^4 + 5*x^2 + 3)^(3/2) + 141895/2048*sqrt(x^4 + 5*x^2 + 3) - 3
68927/4096*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 0.261605, size = 468, normalized size = 3.69

$$-\frac{6039797760\,x^{28}+163242311680\,x^{26}+1925026152448\,x^{24}+13026354790400\,x^{22}+56020526170112\,x^{20}+1603518758912}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^5,x, algorithm="fricas")
```

```
[Out] -1/3440640*(6039797760*x^28 + 163242311680*x^26 + 1925026152448*x
^24 + 13026354790400*x^22 + 56020526170112*x^20 + 160351875891200
*x^18 + 311486603624448*x^16 + 417859472998400*x^14 + 44654471691
4688*x^12 + 647316389099520*x^10 + 1305695545011200*x^8 + 1940788
986577280*x^6 + 1629316481183552*x^4 + 665049507193190*x^2 - 3098
98680*(8192*x^14 + 143360*x^12 + 1028608*x^10 + 3897600*x^8 + 836
3712*x^6 + 10087840*x^4 + 6288842*x^2 - 2*(4096*x^12 + 61440*x^10
 + 367360*x^8 + 1113600*x^6 + 1792224*x^4 + 1441120*x^2 + 449203)
*sqrt(x^4 + 5*x^2 + 3) + 1556105)*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2
 + 3) - 5) - 2*(3019898880*x^26 + 74071408640*x^24 + 782241890304
*x^22 + 4665670369280*x^20 + 17350973194240*x^18 + 41945865584640
*x^16 + 66830623850496*x^14 + 72809996984320*x^12 + 7910176621363
2*x^10 + 159399745182720*x^8 + 307775900170240*x^6 + 334552245101
760*x^4 + 168959327923136*x^2 + 27616762406365)*sqrt(x^4 + 5*x^2
+ 3) + 95657842018463)/(8192*x^14 + 143360*x^12 + 1028608*x^10 +
3897600*x^8 + 8363712*x^6 + 10087840*x^4 + 6288842*x^2 - 2*(4096*
x^12 + 61440*x^10 + 367360*x^8 + 1113600*x^6 + 1792224*x^4 + 1441
```

```
120*x^2 + 449203)*sqrt(x^4 + 5*x^2 + 3) + 1556105)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \left(3x^2+2\right) \left(x^4+5x^2+3\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

GIAC/XCAS [A] time = 0.282346, size = 109, normalized size = 0.86

$$\frac{1}{215040}\,\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(2\left(8\left(10\left(36\,x^2+253\right)x^2+3773\right)x^2+9675\right)x^2+35413\right)x^2-749785\right)x^2+9546951\right)$$
$$+\frac{368927}{4096}\,\ln\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^5,x, algorithm="giac")
```

```
[Out] 1/215040*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(2*(8*(10*(36*x^2 + 253)*x^2
 + 3773)*x^2 + 9675)*x^2 + 35413)*x^2 - 749785)*x^2 + 9546951) +
368927/4096*ln(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.157 $\int x^3 \left(2+3x^2\right) \left(3+5x^2+x^4\right)^{3/2} \, dx$

Optimal. Leaf size=106

$$-\frac{1}{40}\left(27-10x^2\right)\left(x^4+5x^2+3\right)^{5/2}+\frac{123}{128}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{4797\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{1024}+\frac{62361\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{2048}$$

```
[Out] (-4797*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/1024 + (123*(5 + 2*x^2)
*(3 + 5*x^2 + x^4)^(3/2))/128 - ((27 - 10*x^2)*(3 + 5*x^2 + x^4)^
(5/2))/40 + (62361*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])]
)/2048
```

Rubi [A] time = 0.167, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{1}{40}\left(27-10x^2\right)\left(x^4+5x^2+3\right)^{5/2}+\frac{123}{128}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{4797\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{1024}+\frac{62361\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{2048}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-4797*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/1024 + (123*(5 + 2*x^2)
*(3 + 5*x^2 + x^4)^(3/2))/128 - ((27 - 10*x^2)*(3 + 5*x^2 + x^4)^
(5/2))/40 + (62361*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])]
)/2048
```

Rubi in Sympy [A] time = 15.5935, size = 97, normalized size = 0.92

$$-\frac{\left(-15x^2+\frac{81}{2}\right)\left(x^4+5x^2+3\right)^{\frac{5}{2}}}{60}+\frac{123\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{128}-\frac{4797\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{1024}+\frac{62361\operatorname{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{2048}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] -(-15*x**2 + 81/2)*(x**4 + 5*x**2 + 3)**(5/2)/60 + 123*(2*x**2 +
5)*(x**4 + 5*x**2 + 3)**(3/2)/128 - 4797*(2*x**2 + 5)*sqrt(x**4 +
 5*x**2 + 3)/1024 + 62361*atanh((2*x**2 + 5)/(2*sqrt(x**4 + 5*x**
2 + 3)))/2048
```

Mathematica [A] time = 0.0618153, size = 74, normalized size = 0.7

$$\frac{311805\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)+2\sqrt{x^4+5x^2+3}\left(1280x^{10}+9344x^8+14960x^6+5064x^4+12390x^2-77229\right)}{10240}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(-77229 + 12390*x^2 + 5064*x^4 + 14960*x
^6 + 9344*x^8 + 1280*x^10) + 311805*Log[5 + 2*x^2 + 2*Sqrt[3 + 5*
x^2 + x^4]])/10240
```

Maple [A] time = 0.023, size = 121, normalized size = 1.1

$$\frac{73\,x^8}{40}\sqrt{x^4+5\,x^2+3}+\frac{187\,x^6}{64}\sqrt{x^4+5\,x^2+3}+\frac{633\,x^4}{640}\sqrt{x^4+5\,x^2+3}+\frac{1239\,x^2}{512}\sqrt{x^4+5\,x^2+3}$$
$$-\frac{77229}{5120}\sqrt{x^4+5\,x^2+3}+\frac{62361}{2048}\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5\,x^2+3}\right)+\frac{x^{10}}{4}\sqrt{x^4+5\,x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 73/40*x^8*(x^4+5*x^2+3)^(1/2)+187/64*x^6*(x^4+5*x^2+3)^(1/2)+633/
640*x^4*(x^4+5*x^2+3)^(1/2)+1239/512*x^2*(x^4+5*x^2+3)^(1/2)-7722
9/5120*(x^4+5*x^2+3)^(1/2)+62361/2048*ln(x^2+5/2+(x^4+5*x^2+3)^(1
/2))+1/4*x^10*(x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 0.742762, size = 159, normalized size = 1.5

$$\frac{1}{4}\left(x^4+5\,x^2+3\right)^{\frac{5}{2}}x^2+\frac{123}{64}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^2-\frac{27}{40}\left(x^4+5\,x^2+3\right)^{\frac{5}{2}}-\frac{4797}{512}\sqrt{x^4+5\,x^2+3}x^2$$
$$+\frac{615}{128}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}-\frac{23985}{1024}\sqrt{x^4+5\,x^2+3}+\frac{62361}{2048}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(x^4 + 5*x^2 + 3)^(5/2)*x^2 + 123/64*(x^4 + 5*x^2 + 3)^(3/2)*
x^2 - 27/40*(x^4 + 5*x^2 + 3)^(5/2) - 4797/512*sqrt(x^4 + 5*x^2 +
 3)*x^2 + 615/128*(x^4 + 5*x^2 + 3)^(3/2) - 23985/1024*sqrt(x^4 +
 5*x^2 + 3) + 62361/2048*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 0.254477, size = 414, normalized size = 3.91

$$-\frac{41943040\,x^{24} + 1040187392\,x^{22} + 11080826880\,x^{20} + 66470805504\,x^{18} + 247861248000\,x^{16} + 598956244992\,x^{14} + 9377923}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(41943040*x^24 + 1040187392*x^22 + 11080826880*x^20 + 66
470805504*x^18 + 247861248000*x^16 + 598956244992*x^14 + 93779238
9120*x^12 + 847234711552*x^10 + 61962731520*x^8 - 979548160000*x^
6 - 1260223402440*x^4 - 639355934952*x^2 + 2494440*(2048*x^12 + 3
0720*x^10 + 182016*x^8 + 540160*x^6 + 837768*x^4 + 636840*x^2 - 4
*(512*x^10 + 6400*x^8 + 30336*x^6 + 67520*x^4 + 69814*x^2 + 26535
)*sqrt(x^4 + 5*x^2 + 3) + 183853)*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2
 + 3) - 5) - 4*(10485760*x^22 + 233832448*x^20 + 2202664960*x^18
+ 11448418304*x^16 + 36093992960*x^14 + 71472951296*x^12 + 861429
04320*x^10 + 42426618880*x^8 - 54355909120*x^6 - 118711166720*x^4
 - 79254765310*x^2 - 15608169723)*sqrt(x^4 + 5*x^2 + 3) - 1080881
00605)/(2048*x^12 + 30720*x^10 + 182016*x^8 + 540160*x^6 + 837768
*x^4 + 636840*x^2 - 4*(512*x^10 + 6400*x^8 + 30336*x^6 + 67520*x^
4 + 69814*x^2 + 26535)*sqrt(x^4 + 5*x^2 + 3) + 183853)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{3}\left(3x^{2}+2\right)\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

GIAC/XCAS [A] time = 0.276299, size = 100, normalized size = 0.94

$$\frac{1}{5120}\sqrt{x^4+5x^2+3}\left(2\left(4\left(2\left(8\left(10x^2+73\right)x^2+935\right)x^2+633\right)x^2+6195\right)x^2-77229\right) - \frac{62361}{2048}\ln\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^3,x, algorithm="giac")
```

```
[Out] 1/5120*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(2*(8*(10*x^2 + 73)*x^2 + 935)
*x^2 + 633)*x^2 + 6195)*x^2 - 77229) - 62361/2048*ln(2*x^2 - 2*sq
rt(x^4 + 5*x^2 + 3) + 5)
```

3.158 $\int x\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=99

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{5/2}-\frac{11}{32}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{429}{256}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{5577}{512}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] (429*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/256 - (11*(5 + 2*x^2)*(3
+ 5*x^2 + x^4)^(3/2))/32 + (3*(3 + 5*x^2 + x^4)^(5/2))/10 - (5577
*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/512
```

Rubi [A] time = 0.127127, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{5/2}-\frac{11}{32}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{429}{256}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{5577}{512}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (429*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/256 - (11*(5 + 2*x^2)*(3
+ 5*x^2 + x^4)^(3/2))/32 + (3*(3 + 5*x^2 + x^4)^(5/2))/10 - (5577
*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/512
```

Rubi in Sympy [A] time = 12.8707, size = 90, normalized size = 0.91

$$-\frac{11\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{32}+\frac{429\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{256}+\frac{3\left(x^4+5x^2+3\right)^{\frac{5}{2}}}{10}-\frac{5577\,\mathrm{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] -11*(2*x**2 + 5)*(x**4 + 5*x**2 + 3)**(3/2)/32 + 429*(2*x**2 + 5)
*sqrt(x**4 + 5*x**2 + 3)/256 + 3*(x**4 + 5*x**2 + 3)**(5/2)/10 -
```

```
5577*atanh((2*x**2 + 5)/(2*sqrt(x**4 + 5*x**2 + 3)))/512
```

Mathematica [A] time = 0.0455454, size = 69, normalized size = 0.7

$$\frac{2\sqrt{x^4+5x^2+3}\left(384x^8+2960x^6+5304x^4+2170x^2+7581\right)-27885\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{2560}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(7581 + 2170*x^2 + 5304*x^4 + 2960*x^6 +
 384*x^8) - 27885*Log[5 + 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/2560
```

Maple [A] time = 0.019, size = 104, normalized size = 1.1

$$\frac{37\,x^6}{16}\sqrt{x^4+5\,x^2+3}+\frac{663\,x^4}{160}\sqrt{x^4+5\,x^2+3}+\frac{217\,x^2}{128}\sqrt{x^4+5\,x^2+3}+\frac{7581}{1280}\sqrt{x^4+5\,x^2+3}-\frac{5577}{512}\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5\,x^2+3}\right)+\frac{3\,x^8}{10}\sqrt{x^4+5\,x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 37/16*x^6*(x^4+5*x^2+3)^(1/2)+663/160*x^4*(x^4+5*x^2+3)^(1/2)+217
/128*x^2*(x^4+5*x^2+3)^(1/2)+7581/1280*(x^4+5*x^2+3)^(1/2)-5577/5
12*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+3/10*x^8*(x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 0.722598, size = 136, normalized size = 1.37

$$-\frac{11}{16}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^2+\frac{3}{10}\left(x^4+5\,x^2+3\right)^{\frac{5}{2}}+\frac{429}{128}\sqrt{x^4+5\,x^2+3}x^2-\frac{55}{32}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}+\frac{2145}{256}\sqrt{x^4+5\,x^2+3}-\frac{5577}{512}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x,x, algorithm="maxima")
```

```
[Out] -11/16*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 3/10*(x^4 + 5*x^2 + 3)^(5/2)
 + 429/128*sqrt(x^4 + 5*x^2 + 3)*x^2 - 55/32*(x^4 + 5*x^2 + 3)^(3
/2) + 2145/256*sqrt(x^4 + 5*x^2 + 3) - 5577/512*log(2*x^2 + 2*sqr
t(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 0.282536, size = 360, normalized size = 3.64

$$-\frac{3145728\,x^{20} + 71434240\,x^{18} + 684195840\,x^{16} + 3609026560\,x^{14} + 11552849920\,x^{12} + 23642158080\,x^{10} + 32610490880\,x^{8} + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x,x, algorithm="fricas")
```

```
[Out] -1/20480*(3145728*x^20 + 71434240*x^18 + 684195840*x^16 + 3609026
560*x^14 + 11552849920*x^12 + 23642158080*x^10 + 32610490880*x^8
+ 32442765280*x^6 + 23399554320*x^4 + 10176374950*x^2 - 223080*(5
12*x^10 + 6400*x^8 + 29920*x^6 + 64400*x^4 + 62690*x^2 - 2*(256*x
^8 + 2560*x^6 + 8976*x^4 + 12880*x^2 + 6269)*sqrt(x^4 + 5*x^2 + 3
) + 21725)*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5) - 2*(1572864
*x^18 + 31784960*x^16 + 265191424*x^14 + 1186795520*x^12 + 312929
6896*x^10 + 5158310400*x^8 + 5776010240*x^6 + 4755043600*x^4 + 25
13463120*x^2 + 506839375)*sqrt(x^4 + 5*x^2 + 3) + 1752842119)/(51
2*x^10 + 6400*x^8 + 29920*x^6 + 64400*x^4 + 62690*x^2 - 2*(256*x^
8 + 2560*x^6 + 8976*x^4 + 12880*x^2 + 6269)*sqrt(x^4 + 5*x^2 + 3)
 + 21725)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(3x^{2} + 2\right)\left(x^{4} + 5x^{2} + 3\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

GIAC/XCAS [A] time = 0.276172, size = 90, normalized size = 0.91

$$\frac{1}{1280}\,\sqrt{x^4+5\,x^2+3}\left(2\,\left(4\,\left(2\,\left(24\,x^2+185\right)x^2+663\right)x^2+1085\right)x^2+7581\right)+\frac{5577}{512}\,\ln\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x,x, algorithm="giac")
```

```
[Out] 1/1280*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(2*(24*x^2 + 185)*x^2 + 663)*x
^2 + 1085)*x^2 + 7581) + 5577/512*ln(2*x^2 - 2*sqrt(x^4 + 5*x^2 +
 3) + 5)
```

3.159 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x} \, dx$

Optimal. Leaf size=119

$$\frac{1}{48}\left(18x^2+61\right)\left(x^4+5x^2+3\right)^{3/2} + \frac{1}{128}\left(199-74x^2\right)\sqrt{x^4+5x^2+3} + \frac{2401}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - 3\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] ((199 - 74*x^2)*Sqrt[3 + 5*x^2 + x^4])/128 + ((61 + 18*x^2)*(3 +
5*x^2 + x^4)^(3/2))/48 + (2401*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*
x^2 + x^4])])/256 - 3*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt
[3 + 5*x^2 + x^4])]
```

Rubi [A] time = 0.256481, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{1}{48}\left(18x^2+61\right)\left(x^4+5x^2+3\right)^{3/2} + \frac{1}{128}\left(199-74x^2\right)\sqrt{x^4+5x^2+3} + \frac{2401}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - 3\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x,x]
```

```
[Out] ((199 - 74*x^2)*Sqrt[3 + 5*x^2 + x^4])/128 + ((61 + 18*x^2)*(3 +
5*x^2 + x^4)^(3/2))/48 + (2401*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*
x^2 + x^4])])/256 - 3*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt
[3 + 5*x^2 + x^4])]
```

Rubi in Sympy [A] time = 27.0119, size = 110, normalized size = 0.92

$$\frac{\left(-\frac{37x^2}{2}+\frac{199}{4}\right)\sqrt{x^4+5x^2+3}}{32} + \frac{\left(9x^2+\frac{61}{2}\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{24} + \frac{2401\,\mathrm{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{256} - 3\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\left(5x^2+6\right)}{6\sqrt{x^4+5x^2+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x,x)
```

```
[Out] (-37*x**2/2 + 199/4)*sqrt(x**4 + 5*x**2 + 3)/32 + (9*x**2 + 61/2)
*(x**4 + 5*x**2 + 3)**(3/2)/24 + 2401*atanh((2*x**2 + 5)/(2*sqrt(
x**4 + 5*x**2 + 3)))/256 - 3*sqrt(3)*atanh(sqrt(3)*(5*x**2 + 6)/(
6*sqrt(x**4 + 5*x**2 + 3)))
```

Mathematica [A] time = 0.139443, size = 107, normalized size = 0.9

$$\frac{2401}{256}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)+3\sqrt{3}\left(2\log(x)-\log\left(5x^2+2\sqrt{3}\sqrt{x^4+5x^2+3}+6\right)\right)+\frac{1}{384}\sqrt{x^4+5x^2+3}\left(144x^6+1208x^4+2650x^2+2061\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x,x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(2061 + 2650*x^2 + 1208*x^4 + 144*x^6))/38
4 + (2401*Log[5 + 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/256 + 3*Sqrt[
3]*(2*Log[x] - Log[6 + 5*x^2 + 2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]])
```

Maple [A] time = 0.022, size = 117, normalized size = 1.

$$\frac{151\,x^4}{48}\sqrt{x^4+5\,x^2+3}+\frac{1325\,x^2}{192}\sqrt{x^4+5\,x^2+3}+\frac{687}{128}\sqrt{x^4+5\,x^2+3}+\frac{2401}{256}\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5\,x^2+3}\right)-3\,Artanh\left(1/6\,\frac{\left(5\,x^2+6\right)\sqrt{3}}{\sqrt{x^4+5\,x^2+3}}\right)\sqrt{3}+\frac{3\,x^6}{8}\sqrt{x^4+5\,x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x)
```

```
[Out] 151/48*x^4*(x^4+5*x^2+3)^(1/2)+1325/192*x^2*(x^4+5*x^2+3)^(1/2)+6
87/128*(x^4+5*x^2+3)^(1/2)+2401/256*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2
))-3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+3
/8*x^6*(x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 0.818915, size = 162, normalized size = 1.36

$$\frac{3}{8}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2-\frac{37}{64}\sqrt{x^4+5x^2+3}x^2+\frac{61}{48}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-3\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{199}{128}\sqrt{x^4+5x^2+3}+\frac{2401}{256}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x,x, algorithm="maxima")
```

```
[Out] 3/8*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 37/64*sqrt(x^4 + 5*x^2 + 3)*x^2
 + 61/48*(x^4 + 5*x^2 + 3)^(3/2) - 3*sqrt(3)*log(2*sqrt(3)*sqrt(x
^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 199/128*sqrt(x^4 + 5*x^2 + 3)
+ 2401/256*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 0.265556, size = 491, normalized size = 4.13

$$-\frac{294912x^{16}+6160384x^{14}+53346304x^{12}+249921536x^{10}+691886464x^{8}+1153924864x^{6}+1117721440x^{4}+560180576}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x,x, algorithm="fricas")
```

```
[Out] -1/6144*(294912*x^16 + 6160384*x^14 + 53346304*x^12 + 249921536*x
^10 + 691886464*x^8 + 1153924864*x^6 + 1117721440*x^4 + 560180576
*x^2 + 57624*(128*x^8 + 1280*x^6 + 4384*x^4 + 5920*x^2 - 8*(16*x^
6 + 120*x^4 + 274*x^2 + 185)*sqrt(x^4 + 5*x^2 + 3) + 2569)*log(-2
*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5) + 18432*(8*sqrt(3)*(16*x^6 +
120*x^4 + 274*x^2 + 185)*sqrt(x^4 + 5*x^2 + 3) - sqrt(3)*(128*x^8
 + 1280*x^6 + 4384*x^4 + 5920*x^2 + 2569))*log((2*x^4 + 2*sqrt(3)
*x^2 + 5*x^2 - 2*sqrt(x^4 + 5*x^2 + 3)*(x^2 + sqrt(3)) + 6)/(2*x^
4 - 2*sqrt(x^4 + 5*x^2 + 3)*x^2 + 5*x^2)) - 8*(36864*x^14 + 67788
8*x^12 + 5033472*x^10 + 19608320*x^8 + 43313552*x^6 + 53850552*x^
4 + 33901090*x^2 + 7809793)*sqrt(x^4 + 5*x^2 + 3) + 107814215)/(1
28*x^8 + 1280*x^6 + 4384*x^4 + 5920*x^2 - 8*(16*x^6 + 120*x^4 + 2
74*x^2 + 185)*sqrt(x^4 + 5*x^2 + 3) + 2569)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)\,(x^4+5x^2+3)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x, x)
```

3.160 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=122

$$-\frac{(2-x^2)(x^4+5x^2+3)^{3/2}}{2x^2}+\frac{3}{16}(18x^2+109)\sqrt{x^4+5x^2+3}+\frac{609}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-12\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] (3*(109 + 18*x^2)*Sqrt[3 + 5*x^2 + x^4])/16 - ((2 - x^2)*(3 + 5*x
^2 + x^4)^(3/2))/(2*x^2) + (609*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5
*x^2 + x^4])])/32 - 12*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqr
t[3 + 5*x^2 + x^4])]
```

Rubi [A] time = 0.266192, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$-\frac{(2-x^2)(x^4+5x^2+3)^{3/2}}{2x^2}+\frac{3}{16}(18x^2+109)\sqrt{x^4+5x^2+3}+\frac{609}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-12\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^3,x]
```

```
[Out] (3*(109 + 18*x^2)*Sqrt[3 + 5*x^2 + x^4])/16 - ((2 - x^2)*(3 + 5*x
^2 + x^4)^(3/2))/(2*x^2) + (609*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5
*x^2 + x^4])])/32 - 12*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqr
t[3 + 5*x^2 + x^4])]
```

Rubi in Sympy [A] time = 26.5, size = 110, normalized size = 0.9

$$\frac{\left(27x^2+\frac{327}{2}\right)\sqrt{x^4+5x^2+3}}{8}+\frac{609\,\mathrm{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{32}-12\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}(5x^2+6)}{6\sqrt{x^4+5x^2+3}}\right)-\frac{(-3x^2+6)(x^4+5x^2+3)^{\frac{3}{2}}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**3,x)
```

```
[Out] (27*x**2 + 327/2)*sqrt(x**4 + 5*x**2 + 3)/8 + 609*atanh((2*x**2 +
 5)/(2*sqrt(x**4 + 5*x**2 + 3)))/32 - 12*sqrt(3)*atanh(sqrt(3)*(5
*x**2 + 6)/(6*sqrt(x**4 + 5*x**2 + 3))) - (-3*x**2 + 6)*(x**4 + 5
*x**2 + 3)**(3/2)/(6*x**2)
```

Mathematica [A] time = 0.161482, size = 107, normalized size = 0.88

$$\frac{1}{16}\sqrt{x^4+5x^2+3}\left(8x^4+78x^2-\frac{48}{x^2}+271\right)+\frac{609}{32}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)+12\sqrt{3}\left(2\log(x)-\log\left(5x^2+2\sqrt{3}\sqrt{x^4+5x^2+3}+6\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(271 - 48/x^2 + 78*x^2 + 8*x^4))/16 + (609
*Log[5 + 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/32 + 12*Sqrt[3]*(2*Log
[x] - Log[6 + 5*x^2 + 2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]])
```

Maple [A] time = 0.025, size = 117, normalized size = 1.

$$\frac{39\,x^2}{8}\sqrt{x^4+5\,x^2+3}+\frac{271}{16}\sqrt{x^4+5\,x^2+3}+\frac{609}{32}\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5\,x^2+3}\right)-3\,\frac{\sqrt{x^4+5\,x^2+3}}{x^2}-12\,\mathit{Artanh}\left(1/6\,\frac{\left(5\,x^2+6\right)\sqrt{3}}{\sqrt{x^4+5\,x^2+3}}\right)\sqrt{3}+\frac{x^4}{2}\sqrt{x^4+5\,x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x)
```

```
[Out] 39/8*x^2*(x^4+5*x^2+3)^(1/2)+271/16*(x^4+5*x^2+3)^(1/2)+609/32*ln
(x^2+5/2+(x^4+5*x^2+3)^(1/2))-3*(x^4+5*x^2+3)^(1/2)/x^2-12*arctan
h(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+1/2*x^4*(x^4
+5*x^2+3)^(1/2)
```

Maxima [A] time = 0.819874, size = 162, normalized size = 1.33

$$\frac{27}{8}\sqrt{x^4+5x^2+3}x^2+\frac{1}{2}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-12\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{327}{16}\sqrt{x^4+5x^2+3}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{x^2}+\frac{609}{32}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^3,x, algorithm="maxima")
```

```
[Out] 27/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/2*(x^4 + 5*x^2 + 3)^(3/2) - 12
*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 3
27/16*sqrt(x^4 + 5*x^2 + 3) - (x^4 + 5*x^2 + 3)^(3/2)/x^2 + 609/3
2*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 0.278944, size = 524, normalized size = 4.3

$$-\frac{8192\,x^{16}+182272\,x^{14}+1747968\,x^{12}+8805504\,x^{10}+23858432\,x^{8}+32630816\,x^{6}+18064928\,x^{4}+300307\,x^{2}+2436\left(128\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/128*(8192*x^16 + 182272*x^14 + 1747968*x^12 + 8805504*x^10 + 2
3858432*x^8 + 32630816*x^6 + 18064928*x^4 + 300307*x^2 + 2436*(12
8*x^10 + 1280*x^8 + 4384*x^6 + 5920*x^4 + 2569*x^2 - 8*(16*x^8 +
120*x^6 + 274*x^4 + 185*x^2)*sqrt(x^4 + 5*x^2 + 3))*log(-2*x^2 +
2*sqrt(x^4 + 5*x^2 + 3) - 5) + 1536*(8*sqrt(3)*(16*x^8 + 120*x^6
+ 274*x^4 + 185*x^2)*sqrt(x^4 + 5*x^2 + 3) - sqrt(3)*(128*x^10 +
1280*x^8 + 4384*x^6 + 5920*x^4 + 2569*x^2))*log((2*x^4 + 2*sqrt(3
)*x^2 + 5*x^2 - 2*sqrt(x^4 + 5*x^2 + 3)*(x^2 + sqrt(3)) + 6)/(2*x
^4 - 2*sqrt(x^4 + 5*x^2 + 3)*x^2 + 5*x^2)) - 16*(512*x^14 + 10112
*x^12 + 84800*x^10 + 352696*x^8 + 712008*x^6 + 586018*x^4 + 63477
*x^2 - 61656)*sqrt(x^4 + 5*x^2 + 3) - 1704960)/(128*x^10 + 1280*x
^8 + 4384*x^6 + 5920*x^4 + 2569*x^2 - 8*(16*x^8 + 120*x^6 + 274*x
^4 + 185*x^2)*sqrt(x^4 + 5*x^2 + 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)\,(x^4+5x^2+3)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**3,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4+5\,x^2+3)^{\frac{3}{2}}\,(3\,x^2+2)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^3, x)
```

3.161 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=127

$$-\frac{(2-3x^2)(x^4+5x^2+3)^{3/2}}{4x^4}-\frac{3(28-19x^2)\sqrt{x^4+5x^2+3}}{8x^2}+\frac{453}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{127}{8}\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] (-3*(28 - 19*x^2)*Sqrt[3 + 5*x^2 + x^4])/(8*x^2) - ((2 - 3*x^2)*(
3 + 5*x^2 + x^4)^(3/2))/(4*x^4) + (453*ArcTanh[(5 + 2*x^2)/(2*Sqr
t[3 + 5*x^2 + x^4])])/16 - (127*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sq
rt[3]*Sqrt[3 + 5*x^2 + x^4])])/8
```

Rubi [A] time = 0.266817, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$-\frac{(2-3x^2)(x^4+5x^2+3)^{3/2}}{4x^4}-\frac{3(28-19x^2)\sqrt{x^4+5x^2+3}}{8x^2}+\frac{453}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{127}{8}\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^5,x]
```

```
[Out] (-3*(28 - 19*x^2)*Sqrt[3 + 5*x^2 + x^4])/(8*x^2) - ((2 - 3*x^2)*(
3 + 5*x^2 + x^4)^(3/2))/(4*x^4) + (453*ArcTanh[(5 + 2*x^2)/(2*Sqr
t[3 + 5*x^2 + x^4])])/16 - (127*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sq
rt[3]*Sqrt[3 + 5*x^2 + x^4])])/8
```

Rubi in Sympy [A] time = 26.6163, size = 116, normalized size = 0.91

$$\frac{453\,\mathrm{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{16}-\frac{127\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}(5x^2+6)}{6\sqrt{x^4+5x^2+3}}\right)}{8}-\frac{3(-38x^2+56)\sqrt{x^4+5x^2+3}}{16x^2}-\frac{(-6x^2+4)(x^4+5x^2+3)^{\frac{3}{2}}}{8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**5,x)
```

```
[Out] 453*atanh((2*x**2 + 5)/(2*sqrt(x**4 + 5*x**2 + 3)))/16 - 127*sqrt
(3)*atanh(sqrt(3)*(5*x**2 + 6)/(6*sqrt(x**4 + 5*x**2 + 3)))/8 - 3
*(-38*x**2 + 56)*sqrt(x**4 + 5*x**2 + 3)/(16*x**2) - (-6*x**2 + 4
)*(x**4 + 5*x**2 + 3)**(3/2)/(8*x**4)
```

Mathematica [A] time = 0.230541, size = 110, normalized size = 0.87

$$\frac{1}{16}\left(453\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)+254\sqrt{3}\left(2\log(x)-\log\left(5x^2+2\sqrt{3}\sqrt{x^4+5x^2+3}+6\right)\right)+\frac{2\sqrt{x^4+5x^2+3}\left(6x^6+83x^4-86x^2-12\right)}{x^4}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^5,x]
```

```
[Out] ((2*Sqrt[3 + 5*x^2 + x^4]*(-12 - 86*x^2 + 83*x^4 + 6*x^6))/x^4 +
453*Log[5 + 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]] + 254*Sqrt[3]*(2*Log
[x] - Log[6 + 5*x^2 + 2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]]))/16
```

Maple [A] time = 0.027, size = 117, normalized size = 0.9

$$\frac{83}{8}\sqrt{x^4+5\,x^2+3}+\frac{453}{16}\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5\,x^2+3}\right)-\frac{3}{2\,x^4}\sqrt{x^4+5\,x^2+3}-\frac{43}{4\,x^2}\sqrt{x^4+5\,x^2+3}-\frac{127\,\sqrt{3}}{8}\mathit{Artanh}\left(\frac{\left(5\,x^2+6\right)\,\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5\,x^2+3}}\right)+\frac{3\,x^2}{4}\sqrt{x^4+5\,x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x)
```

```
[Out] 83/8*(x^4+5*x^2+3)^(1/2)+453/16*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-3
/2*(x^4+5*x^2+3)^(1/2)/x^4-43/4*(x^4+5*x^2+3)^(1/2)/x^2-127/8*arc
tanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+3/4*x^2*(
x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 0.819084, size = 185, normalized size = 1.46

$$\frac{7}{2}\sqrt{x^4+5\,x^2+3}x^2+\frac{1}{6}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}-\frac{127}{8}\sqrt{3}\log\left(\frac{2\,\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{197}{8}\sqrt{x^4+5\,x^2+3}-\frac{23\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}}{12\,x^2}-\frac{\left(x^4+5\,x^2+3\right)^{\frac{5}{2}}}{6\,x^4}+\frac{453}{16}\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^5,x, algorithm="maxima")
```

```
[Out] 7/2*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/6*(x^4 + 5*x^2 + 3)^(3/2) - 127
/8*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) +
 197/8*sqrt(x^4 + 5*x^2 + 3) - 23/12*(x^4 + 5*x^2 + 3)^(3/2)/x^2
- 1/6*(x^4 + 5*x^2 + 3)^(5/2)/x^4 + 453/16*log(2*x^2 + 2*sqrt(x^4
 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 0.273077, size = 524, normalized size = 4.13

$$-\frac{6144\,x^{16}+161792\,x^{14}+1319808\,x^{12}+4434176\,x^{10}+5321312\,x^{8}-2151712\,x^{6}-8310171\,x^{4}-4396416\,x^{2}+1812\left(128\,x^{1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/64*(6144*x^16 + 161792*x^14 + 1319808*x^12 + 4434176*x^10 + 53
21312*x^8 - 2151712*x^6 - 8310171*x^4 - 4396416*x^2 + 1812*(128*x
^12 + 1280*x^10 + 4384*x^8 + 5920*x^6 + 2569*x^4 - 8*(16*x^10 + 1
20*x^8 + 274*x^6 + 185*x^4)*sqrt(x^4 + 5*x^2 + 3))*log(-2*x^2 + 2
*sqrt(x^4 + 5*x^2 + 3) - 5) + 1016*(8*sqrt(3)*(16*x^10 + 120*x^8
+ 274*x^6 + 185*x^4)*sqrt(x^4 + 5*x^2 + 3) - sqrt(3)*(128*x^12 +
1280*x^10 + 4384*x^8 + 5920*x^6 + 2569*x^4))*log((2*x^4 + 2*sqrt(
3)*x^2 + 5*x^2 - 2*sqrt(x^4 + 5*x^2 + 3)*(x^2 + sqrt(3)) + 6)/(2*
x^4 - 2*sqrt(x^4 + 5*x^2 + 3)*x^2 + 5*x^2)) - 16*(384*x^14 + 9152
*x^12 + 60232*x^10 + 139868*x^8 + 48016*x^6 - 180448*x^4 - 145987
*x^2 - 15414)*sqrt(x^4 + 5*x^2 + 3) - 426240)/(128*x^12 + 1280*x^
10 + 4384*x^8 + 5920*x^6 + 2569*x^4 - 8*(16*x^10 + 120*x^8 + 274*
x^6 + 185*x^4)*sqrt(x^4 + 5*x^2 + 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)\,(x^4+5x^2+3)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**5,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**5, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4+5\,x^2+3)^{\frac{3}{2}}\,(3\,x^2+2)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^5,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^5, x)
```

3.162 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=127

$$-\frac{(67-32x^2)\sqrt{x^4+5x^2+3}}{12x^2}+\frac{49}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{527\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}-\frac{(7x^2+2)(x^4+5x^2+3)^{3/2}}{6x^6}$$

```
[Out] -((67 - 32*x^2)*Sqrt[3 + 5*x^2 + x^4])/(12*x^2) - ((2 + 7*x^2)*(3
 + 5*x^2 + x^4)^(3/2))/(6*x^6) + (49*ArcTanh[(5 + 2*x^2)/(2*Sqrt[
3 + 5*x^2 + x^4])])/4 - (527*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[
3 + 5*x^2 + x^4])])/(24*Sqrt[3])
```

Rubi [A] time = 0.263227, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$-\frac{(67-32x^2)\sqrt{x^4+5x^2+3}}{12x^2}+\frac{49}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{527\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}-\frac{(7x^2+2)(x^4+5x^2+3)^{3/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^7,x]
```

```
[Out] -((67 - 32*x^2)*Sqrt[3 + 5*x^2 + x^4])/(12*x^2) - ((2 + 7*x^2)*(3
 + 5*x^2 + x^4)^(3/2))/(6*x^6) + (49*ArcTanh[(5 + 2*x^2)/(2*Sqrt[
3 + 5*x^2 + x^4])])/4 - (527*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[
3 + 5*x^2 + x^4])])/(24*Sqrt[3])
```

Rubi in Sympy [A] time = 26.6391, size = 114, normalized size = 0.9

$$\frac{49\,\mathrm{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{4}-\frac{527\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}(5x^2+6)}{6\sqrt{x^4+5x^2+3}}\right)}{72}-\frac{(-64x^2+134)\sqrt{x^4+5x^2+3}}{24x^2}-\frac{(42x^2+12)(x^4+5x^2+3)^{\frac{3}{2}}}{36x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**7,x)
```

```
[Out] 49*atanh((2*x**2 + 5)/(2*sqrt(x**4 + 5*x**2 + 3)))/4 - 527*sqrt(3
)*atanh(sqrt(3)*(5*x**2 + 6)/(6*sqrt(x**4 + 5*x**2 + 3)))/72 - (-
64*x**2 + 134)*sqrt(x**4 + 5*x**2 + 3)/(24*x**2) - (42*x**2 + 12)
*(x**4 + 5*x**2 + 3)**(3/2)/(36*x**6)
```

Mathematica [A] time = 0.192967, size = 112, normalized size = 0.88

$$\frac{49}{4}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)+\frac{527\left(2\log(x)-\log\left(5x^2+2\sqrt{3}\sqrt{x^4+5x^2+3}+6\right)\right)}{24\sqrt{3}}+\sqrt{x^4+5x^2+3}\left(-\frac{1}{x^6}-\frac{31}{6x^4}-\frac{47}{4x^2}+\frac{3}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^7,x]
```

```
[Out] (3/2 - x^(-6) - 31/(6*x^4) - 47/(4*x^2))*Sqrt[3 + 5*x^2 + x^4] +
(49*Log[5 + 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/4 + (527*(2*Log[x]
- Log[6 + 5*x^2 + 2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]]))/(24*Sqrt[3])
```

Maple [A] time = 0.026, size = 117, normalized size = 0.9

$$\frac{49}{4}\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5\,x^2+3}\right)-\frac{1}{x^6}\sqrt{x^4+5\,x^2+3}-\frac{31}{6\,x^4}\sqrt{x^4+5\,x^2+3}-\frac{47}{4\,x^2}\sqrt{x^4+5\,x^2+3}-\frac{527\,\sqrt{3}}{72}\mathit{Artanh}\left(\frac{\left(5\,x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5\,x^2+3}}\right)+\frac{3}{2}\sqrt{x^4+5\,x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x)
```

```
[Out] 49/4*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-(x^4+5*x^2+3)^(1/2)/x^6-31/6
*(x^4+5*x^2+3)^(1/2)/x^4-47/4*(x^4+5*x^2+3)^(1/2)/x^2-527/72*arct
anh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+3/2*(x^4+5
*x^2+3)^(1/2)
```

Maxima [A] time = 0.826634, size = 208, normalized size = 1.64

$$\frac{67}{36}\sqrt{x^4+5x^2+3}x^2+\frac{11}{54}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{527}{72}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{431}{36}\sqrt{x^4+5x^2+3}-\frac{79\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{108x^2}-\frac{11\left(x^4+5x^2+3\right)^{\frac{5}{2}}}{54x^4}-\frac{\left(x^4+5x^2+3\right)^{\frac{5}{2}}}{9x^6}+\frac{49}{4}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^7,x, algorithm="maxima")
```

```
[Out] 67/36*sqrt(x^4 + 5*x^2 + 3)*x^2 + 11/54*(x^4 + 5*x^2 + 3)^(3/2) -
 527/72*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 +
 5) + 431/36*sqrt(x^4 + 5*x^2 + 3) - 79/108*(x^4 + 5*x^2 + 3)^(3/
2)/x^2 - 11/54*(x^4 + 5*x^2 + 3)^(5/2)/x^4 - 1/9*(x^4 + 5*x^2 + 3
)^(5/2)/x^6 + 49/4*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 0.273047, size = 554, normalized size = 4.36

$$-\frac{2\sqrt{3}\left(2304x^{14}+20160x^{12}+4256x^{10}-332728x^8-900266x^6-781877x^4-230318x^2-30828\right)\sqrt{x^4+5x^2+3}+294\left(8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(3)*(2304*x^14 + 20160*x^12 + 4256*x^10 - 332728*x^8
 - 900266*x^6 - 781877*x^4 - 230318*x^2 - 30828)*sqrt(x^4 + 5*x^2
 + 3) + 294*(8*sqrt(3)*(16*x^12 + 120*x^10 + 274*x^8 + 185*x^6)*s
qrt(x^4 + 5*x^2 + 3) - sqrt(3)*(128*x^14 + 1280*x^12 + 4384*x^10
+ 5920*x^8 + 2569*x^6))*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
 + 527*(128*x^14 + 1280*x^12 + 4384*x^10 + 5920*x^8 + 2569*x^6 -
8*(16*x^12 + 120*x^10 + 274*x^8 + 185*x^6)*sqrt(x^4 + 5*x^2 + 3))
*log((6*x^2 + sqrt(3)*(2*x^4 + 5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 +
3)*(sqrt(3)*x^2 + 3))/(2*x^4 - 2*sqrt(x^4 + 5*x^2 + 3)*x^2 + 5*x^
2)) - sqrt(3)*(4608*x^16 + 51840*x^14 + 101824*x^12 - 690976*x^10
 - 3367088*x^8 - 5249243*x^6 - 3352784*x^4 - 885984*x^2 - 106560)
)/(8*sqrt(3)*(16*x^12 + 120*x^10 + 274*x^8 + 185*x^6)*sqrt(x^4 +
5*x^2 + 3) - sqrt(3)*(128*x^14 + 1280*x^12 + 4384*x^10 + 5920*x^8
 + 2569*x^6))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)\,(x^4+5x^2+3)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**7,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**7, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^7,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^7, x)
```

3.163 $\int x^4 \left(2+3x^2\right) \left(3+5x^2+x^4\right)^{3/2} \, dx$

Optimal. Leaf size=356

$$-\frac{4210}{429}\sqrt{x^4+5x^2+3}x+\frac{176723\left(2x^2+\sqrt{13}+5\right)x}{4290\sqrt{x^4+5x^2+3}}$$
$$+\frac{2105\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{143\sqrt{x^4+5x^2+3}}$$
$$-\frac{176723\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{4290\sqrt{x^4+5x^2+3}}$$
$$+\frac{1}{143}\left(33x^2+71\right)\left(x^4+5x^2+3\right)^{3/2}x^5-\frac{1}{429}\left(272x^2+283\right)\sqrt{x^4+5x^2+3}x^5+\frac{1251}{715}\sqrt{x^4+5x^2+3}x^3$$

```
[Out] (176723*x*(5 + Sqrt[13] + 2*x^2))/(4290*Sqrt[3 + 5*x^2 + x^4]) -
(4210*x*Sqrt[3 + 5*x^2 + x^4])/429 + (1251*x^3*Sqrt[3 + 5*x^2 + x
^4])/715 - (x^5*(283 + 272*x^2)*Sqrt[3 + 5*x^2 + x^4])/429 + (x^5
*(71 + 33*x^2)*(3 + 5*x^2 + x^4)^(3/2))/143 - (176723*Sqrt[(5 + S
qrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2
)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/
6]*x], (-13 + 5*Sqrt[13])/6])/(4290*Sqrt[3 + 5*x^2 + x^4]) + (210
5*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (
5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqr
t[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(143*Sqrt[3 + 5*x^
2 + x^4])
```

Rubi [A] time = 0.642794, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{4210}{429}\sqrt{x^4+5x^2+3}x+\frac{176723\left(2x^2+\sqrt{13}+5\right)x}{4290\sqrt{x^4+5x^2+3}}$$
$$+\frac{2105\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{143\sqrt{x^4+5x^2+3}}$$
$$-\frac{176723\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{4290\sqrt{x^4+5x^2+3}}$$
$$+\frac{1}{143}\left(33x^2+71\right)\left(x^4+5x^2+3\right)^{3/2}x^5-\frac{1}{429}\left(272x^2+283\right)\sqrt{x^4+5x^2+3}x^5+\frac{1251}{715}\sqrt{x^4+5x^2+3}x^3$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (176723*x*(5 + Sqrt[13] + 2*x^2))/(4290*Sqrt[3 + 5*x^2 + x^4]) -
(4210*x*Sqrt[3 + 5*x^2 + x^4])/429 + (1251*x^3*Sqrt[3 + 5*x^2 + x
^4])/715 - (x^5*(283 + 272*x^2)*Sqrt[3 + 5*x^2 + x^4])/429 + (x^5
*(71 + 33*x^2)*(3 + 5*x^2 + x^4)^(3/2))/143 - (176723*Sqrt[(5 + S
qrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2
)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/
6]*x], (-13 + 5*Sqrt[13])/6])/(4290*Sqrt[3 + 5*x^2 + x^4]) + (210
5*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (
5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqr
t[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(143*Sqrt[3 + 5*x^
2 + x^4])
```

Rubi in Sympy [A] time = 50.5822, size = 326, normalized size = 0.92

$$\frac{x^5\left(33x^2+71\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{143}-\frac{x^5\left(1904x^2+1981\right)\sqrt{x^4+5x^2+3}}{3003}+\frac{1251x^3\sqrt{x^4+5x^2+3}}{715}+\frac{176723x\left(2x^2+\sqrt{13}+5\right)}{4290\sqrt{x^4+5x^2+3}}-\frac{4210x\sqrt{x^4+5x^2+3}}{429}-\frac{176723\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{25740\sqrt{x^4+5x^2+3}}+\frac{2105\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{429\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] x**5*(33*x**2 + 71)*(x**4 + 5*x**2 + 3)**(3/2)/143 - x**5*(1904*x
**2 + 1981)*sqrt(x**4 + 5*x**2 + 3)/3003 + 1251*x**3*sqrt(x**4 +
5*x**2 + 3)/715 + 176723*x*(2*x**2 + sqrt(13) + 5)/(4290*sqrt(x**
4 + 5*x**2 + 3)) - 4210*x*sqrt(x**4 + 5*x**2 + 3)/429 - 176723*sq
rt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6))*
sqrt(sqrt(13) + 5)*(x**2*(sqrt(13) + 5) + 6)*elliptic_e(atan(sqrt
(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(25740*sqrt(x*
*4 + 5*x**2 + 3)) + 2105*sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/
(x**2*(sqrt(13) + 5) + 6))*(x**2*(sqrt(13) + 5) + 6)*elliptic_f(a
tan(sqrt(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(429*s
qrt(sqrt(13) + 5)*sqrt(x**4 + 5*x**2 + 3))
```

Mathematica [C] time = 0.654941, size = 249, normalized size = 0.7

$$-i\sqrt{2}\left(176723\sqrt{13}-757315\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+176723i\sqrt{2}\left(\sqrt{13}-5\right)\sqrt{-}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^4*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*x*(-63150 - 93991*x^2 + 3055*x^4 + 29003*x^6 + 39650*x^8 + 246
35*x^10 + 6015*x^12 + 495*x^14) + (176723*I)*Sqrt[2]*(-5 + Sqrt[1
3])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[1
3] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 +
 (5*Sqrt[13])/6] - I*Sqrt[2]*(-757315 + 176723*Sqrt[13])*Sqrt[(-5
 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*
EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13]
)/6])/(8580*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.027, size = 294, normalized size = 0.8

$$\frac{236x^9}{143}\sqrt{x^4+5x^2+3}+\frac{1090x^7}{429}\sqrt{x^4+5x^2+3}$$
$$+\frac{356x^5}{429}\sqrt{x^4+5x^2+3}+\frac{1251x^3}{715}\sqrt{x^4+5x^2+3}-\frac{4210x}{429}\sqrt{x^4+5x^2+3}$$
$$+\frac{25260}{143\sqrt{-30+6\sqrt{13}}}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\mathit{EllipticF}\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)\frac{1}{\sqrt{x^4+5x^2+3}}$$
$$-\frac{2120676}{715\sqrt{-30+6\sqrt{13}}\left(5+\sqrt{13}\right)}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\mathit{EllipticF}\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-E\right.$$
$$+\frac{3x^{11}}{13}\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 236/143*x^9*(x^4+5*x^2+3)^(1/2)+1090/429*x^7*(x^4+5*x^2+3)^(1/2)+
356/429*x^5*(x^4+5*x^2+3)^(1/2)+1251/715*x^3*(x^4+5*x^2+3)^(1/2)-
4210/429*x*(x^4+5*x^2+3)^(1/2)+25260/143/(-30+6*13^(1/2))^(1/2)*(
1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2
)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*
```

```
3^(1/2)+1/6*39^(1/2))-2120676/715/(-30+6*13^(1/2))^(1/2)*(1-(-5/6
+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+
5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/
2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/
2),5/6*3^(1/2)+1/6*39^(1/2)))+3/13*x^11*(x^4+5*x^2+3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+5\,x^2+3\right)^{\frac{3}{2}}\left(3\,x^2+2\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^4,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(3\,x^{10}+17\,x^8+19\,x^6+6\,x^4\right)\sqrt{x^4+5\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^4,x, algorithm="fricas")
```

```
[Out] integral((3*x^10 + 17*x^8 + 19*x^6 + 6*x^4)*sqrt(x^4 + 5*x^2 + 3)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**4*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4 + 5x^2 + 3\right)^{\frac{3}{2}} \left(3x^2 + 2\right) x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^4,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^4, x)
```

3.164 $\int x^2 \left(2+3x^2\right) \left(3+5x^2+x^4\right)^{3/2} \, dx$

Optimal. Leaf size=331

$$\frac{353}{99}\sqrt{x^4+5x^2+3}x-\frac{49949\left(2x^2+\sqrt{13}+5\right)x}{3465\sqrt{x^4+5x^2+3}}$$
$$-\frac{353\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{33\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}$$
$$+\frac{49949\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{3465\sqrt{x^4+5x^2+3}}$$
$$+\frac{1}{99}\left(27x^2+67\right)\left(x^4+5x^2+3\right)^{3/2}x^3-\frac{\left(890x^2+911\right)\sqrt{x^4+5x^2+3}x^3}{1155}$$

```
[Out] (-49949*x*(5 + Sqrt[13] + 2*x^2))/(3465*Sqrt[3 + 5*x^2 + x^4]) +
(353*x*Sqrt[3 + 5*x^2 + x^4])/99 - (x^3*(911 + 890*x^2)*Sqrt[3 +
5*x^2 + x^4])/1155 + (x^3*(67 + 27*x^2)*(3 + 5*x^2 + x^4)^(3/2))/
99 + (49949*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/
(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcT
an[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(3465*Sqrt[3
 + 5*x^2 + x^4]) - (353*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + S
qrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5
+ Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(33*Sqrt[6*(5 + Sqrt[13
])]*Sqrt[3 + 5*x^2 + x^4])
```

Rubi [A] time = 0.534762, antiderivative size = 331, normalized size of antiderivative = 1., number

of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{353}{99}\sqrt{x^4+5x^2+3}x-\frac{49949\left(2x^2+\sqrt{13}+5\right)x}{3465\sqrt{x^4+5x^2+3}}-\frac{353\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{33\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}+\frac{49949\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{3465\sqrt{x^4+5x^2+3}}+\frac{1}{99}\left(27x^2+67\right)\left(x^4+5x^2+3\right)^{3/2}x^3-\frac{\left(890x^2+911\right)\sqrt{x^4+5x^2+3}x^3}{1155}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-49949*x*(5 + Sqrt[13] + 2*x^2))/(3465*Sqrt[3 + 5*x^2 + x^4]) +
(353*x*Sqrt[3 + 5*x^2 + x^4])/99 - (x^3*(911 + 890*x^2)*Sqrt[3 +
5*x^2 + x^4])/1155 + (x^3*(67 + 27*x^2)*(3 + 5*x^2 + x^4)^(3/2))/
99 + (49949*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/
(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcT
an[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(3465*Sqrt[3
 + 5*x^2 + x^4]) - (353*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + S
qrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5
+ Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(33*Sqrt[6*(5 + Sqrt[13
])]*Sqrt[3 + 5*x^2 + x^4])
```

Rubi in Sympy [A] time = 47.355, size = 306, normalized size = 0.92

$$\frac{x^3\left(27x^2+67\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{99}-\frac{x^3\left(890x^2+911\right)\sqrt{x^4+5x^2+3}}{1155}-\frac{49949x\left(2x^2+\sqrt{13}+5\right)}{3465\sqrt{x^4+5x^2+3}}+\frac{353x\sqrt{x^4+5x^2+3}}{99}+\frac{49949\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{20790\sqrt{x^4+5x^2+3}}-\frac{353\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{198\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] x**3*(27*x**2 + 67)*(x**4 + 5*x**2 + 3)**(3/2)/99 - x**3*(890*x**
2 + 911)*sqrt(x**4 + 5*x**2 + 3)/1155 - 49949*x*(2*x**2 + sqrt(13
) + 5)/(3465*sqrt(x**4 + 5*x**2 + 3)) + 353*x*sqrt(x**4 + 5*x**2
+ 3)/99 + 49949*sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sq
rt(13) + 5) + 6))*sqrt(sqrt(13) + 5)*(x**2*(sqrt(13) + 5) + 6)*el
liptic_e(atan(sqrt(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)
/6)/(20790*sqrt(x**4 + 5*x**2 + 3)) - 353*sqrt(6)*sqrt((x**2*(-sq
rt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6))*(x**2*(sqrt(13) + 5)
+ 6)*elliptic_f(atan(sqrt(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*s
qrt(13)/6)/(198*sqrt(sqrt(13) + 5)*sqrt(x**4 + 5*x**2 + 3))
```

Mathematica [C] time = 0.557361, size = 244, normalized size = 0.74

$$\frac{i\sqrt{2}\left(49949\sqrt{13}-212680\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-49949i\sqrt{2}\left(\sqrt{13}-5\right)\sqrt{\frac{-2x^2+\cdots}{\sqrt{13}\cdots}}\cdots}{6930\cdots}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*x*(37065 + 74681*x^2 + 69535*x^4 + 84962*x^6 + 50075*x^8 + 117
95*x^10 + 945*x^12) - (49949*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5
+ Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*E
llipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])
/6] + I*Sqrt[2]*(-212680 + 49949*Sqrt[13])*Sqrt[(-5 + Sqrt[13] -
2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*Ar
cSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(6930*Sq
rt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.021, size = 277, normalized size = 0.8

$$\frac{202 x^7}{99}\sqrt{x^4+5x^2+3}+\frac{2378 x^5}{693}\sqrt{x^4+5x^2+3}+\frac{478 x^3}{385}\sqrt{x^4+5x^2+3}+\frac{353 x}{99}\sqrt{x^4+5x^2+3}$$
$$-\frac{706}{11\sqrt{-30+6\sqrt{13}}}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}EllipticF\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)\frac{1}{\sqrt{x^4+5x^2+3}}$$
$$+\frac{399592}{385\sqrt{-30+6\sqrt{13}}\left(5+\sqrt{13}\right)}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-E\right.$$
$$+\frac{3x^9}{11}\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 202/99*x^7*(x^4+5*x^2+3)^(1/2)+2378/693*x^5*(x^4+5*x^2+3)^(1/2)+4
78/385*x^3*(x^4+5*x^2+3)^(1/2)+353/99*x*(x^4+5*x^2+3)^(1/2)-706/1
1/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5
/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(
-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))+399592/385/(-30+6
*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*1
3^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1
/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1
/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))+3/11*x^9*(
x^4+5*x^2+3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^4+5x^2+3\right)^{\frac{3}{2}}\left(3x^2+2\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(3x^8+17x^6+19x^4+6x^2\right)\sqrt{x^4+5x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^2,x, algorithm="fricas")
```

```
[Out] integral((3*x^8 + 17*x^6 + 19*x^4 + 6*x^2)*sqrt(x^4 + 5*x^2 + 3),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(3x^2+2\right) \left(x^4+5x^2+3\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**2*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+5\,x^2+3\right)^{\frac{3}{2}} \left(3\,x^2+2\right) x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^2,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^2, x)
```

3.165 $\int \left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=308

$$\frac{1}{3}x\left(x^2+3\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{1}{15}x\left(12x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{203x\left(2x^2+\sqrt{13}+5\right)}{30\sqrt{x^4+5x^2+3}}+\frac{5\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{203\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{30\sqrt{x^4+5x^2+3}}$$

```
[Out] (203*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (x*(5
 + 12*x^2)*Sqrt[3 + 5*x^2 + x^4])/15 + (x*(3 + x^2)*(3 + 5*x^2 +
x^4)^(3/2))/3 - (203*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[1
3])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellip
ticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(30
*Sqrt[3 + 5*x^2 + x^4]) + (5*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 +
 (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13]
)*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[
13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi [A] time = 0.346488, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{1}{3}x\left(x^2+3\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{1}{15}x\left(12x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{203x\left(2x^2+\sqrt{13}+5\right)}{30\sqrt{x^4+5x^2+3}}+\frac{5\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{203\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{30\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (203*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (x*(5
 + 12*x^2)*Sqrt[3 + 5*x^2 + x^4])/15 + (x*(3 + x^2)*(3 + 5*x^2 +
x^4)^(3/2))/3 - (203*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[1
3])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellip
ticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(30
*Sqrt[3 + 5*x^2 + x^4]) + (5*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 +
 (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13]
)*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[
13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi in Sympy [A] time = 29.4934, size = 284, normalized size = 0.92

$$\frac{x\left(21x^2+63\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{63}-\frac{x\left(252x^2+105\right)\sqrt{x^4+5x^2+3}}{315}+\frac{203x\left(2x^2+\sqrt{13}+5\right)}{30\sqrt{x^4+5x^2+3}}-\frac{203\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{180\sqrt{x^4+5x^2+3}}+\frac{5\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{3\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] x*(21*x**2 + 63)*(x**4 + 5*x**2 + 3)**(3/2)/63 - x*(252*x**2 + 10
5)*sqrt(x**4 + 5*x**2 + 3)/315 + 203*x*(2*x**2 + sqrt(13) + 5)/(3
0*sqrt(x**4 + 5*x**2 + 3)) - 203*sqrt(6)*sqrt((x**2*(-sqrt(13) +
5) + 6)/(x**2*(sqrt(13) + 5) + 6))*sqrt(sqrt(13) + 5)*(x**2*(sqrt
(13) + 5) + 6)*elliptic_e(atan(sqrt(6)*x*sqrt(sqrt(13) + 5)/6), -
13/6 + 5*sqrt(13)/6)/(180*sqrt(x**4 + 5*x**2 + 3)) + 5*sqrt(6)*sq
rt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6))*(x**2*(s
qrt(13) + 5) + 6)*elliptic_f(atan(sqrt(6)*x*sqrt(sqrt(13) + 5)/6)
, -13/6 + 5*sqrt(13)/6)/(3*sqrt(sqrt(13) + 5)*sqrt(x**4 + 5*x**2
+ 3))
```

Mathematica [C] time = 0.500754, size = 239, normalized size = 0.78

$$\frac{-i\sqrt{2}\left(203\sqrt{13}-715\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+203i\sqrt{2}\left(\sqrt{13}-5\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{}}{60\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*x*(120 + 434*x^2 + 550*x^4 + 293*x^6 + 65*x^8 + 5*x^10) + (203
*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sq
rt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5
 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-715 + 203*
Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 +
Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x],
19/6 + (5*Sqrt[13])/6])/(60*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.017, size = 260, normalized size = 0.8

$$\frac{8x^5}{3}\sqrt{x^4+5x^2+3}+\frac{26x^3}{5}\sqrt{x^4+5x^2+3}+\frac{8x}{3}\sqrt{x^4+5x^2+3}$$
$$+60\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right)x^2}\,EllipticF\left(1/6\,x\sqrt{-30+6\sqrt{13}},5/6\sqrt{3}+1/6\sqrt{39}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$$
$$-\frac{2436}{5\sqrt{-30+6\sqrt{13}}\left(5+\sqrt{13}\right)}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-Elli\right.$$
$$+\frac{x^7}{3}\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 8/3*x^5*(x^4+5*x^2+3)^(1/2)+26/5*x^3*(x^4+5*x^2+3)^(1/2)+8/3*x*(x
^4+5*x^2+3)^(1/2)+60/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2)
)*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2
)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)
)-2436/5/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)
*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2
))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/
2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/
2)))+1/3*x^7*(x^4+5*x^2+3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^4+5x^2+3\right)^{\frac{3}{2}}\left(3x^2+2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(3\,x^{6}+17\,x^{4}+19\,x^{2}+6\right)\sqrt{x^{4}+5\,x^{2}+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(3x^{2}+2\right)\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^{4}+5\,x^{2}+3\right)^{\frac{3}{2}}\left(3\,x^{2}+2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2), x)
```

3.166 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=312

$$-\frac{(14-3x^2)(x^4+5x^2+3)^{3/2}}{7x}+\frac{1}{35}x(129x^2+655)\sqrt{x^4+5x^2+3}+\frac{412x\left(2x^2+\sqrt{13}+5\right)}{35\sqrt{x^4+5x^2+3}}+\frac{19\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{206\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{35\sqrt{x^4+5x^2+3}}$$

```
[Out] (412*x*(5 + Sqrt[13] + 2*x^2))/(35*Sqrt[3 + 5*x^2 + x^4]) + (x*(6
55 + 129*x^2)*Sqrt[3 + 5*x^2 + x^4])/35 - ((14 - 3*x^2)*(3 + 5*x^
2 + x^4)^(3/2))/(7*x) - (206*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 +
 (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13]
)*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[
13])/6])/(35*Sqrt[3 + 5*x^2 + x^4]) + (19*Sqrt[3/(2*(5 + Sqrt[13]
))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 +
(5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (
-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi [A] time = 0.357026, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{(14-3x^2)(x^4+5x^2+3)^{3/2}}{7x}+\frac{1}{35}x(129x^2+655)\sqrt{x^4+5x^2+3}+\frac{412x\left(2x^2+\sqrt{13}+5\right)}{35\sqrt{x^4+5x^2+3}}+\frac{19\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{206\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{35\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^2,x]
```

```
[Out] (412*x*(5 + Sqrt[13] + 2*x^2))/(35*Sqrt[3 + 5*x^2 + x^4]) + (x*(6
55 + 129*x^2)*Sqrt[3 + 5*x^2 + x^4])/35 - ((14 - 3*x^2)*(3 + 5*x^
2 + x^4)^(3/2))/(7*x) - (206*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 +
 (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13]
)*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[
13])/6])/(35*Sqrt[3 + 5*x^2 + x^4]) + (19*Sqrt[3/(2*(5 + Sqrt[13]
))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 +
(5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (
-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi in Sympy [A] time = 31.2235, size = 284, normalized size = 0.91

$$\frac{x\left(129x^2+655\right)\sqrt{x^4+5x^2+3}}{35}+\frac{412x\left(2x^2+\sqrt{13}+5\right)}{35\sqrt{x^4+5x^2+3}}-\frac{206\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{105\sqrt{x^4+5x^2+3}}+\frac{19\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{2\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}}-\frac{\left(-3x^2+14\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**2,x)
```

```
[Out] x*(129*x**2 + 655)*sqrt(x**4 + 5*x**2 + 3)/35 + 412*x*(2*x**2 + s
qrt(13) + 5)/(35*sqrt(x**4 + 5*x**2 + 3)) - 206*sqrt(6)*sqrt((x**
2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6))*sqrt(sqrt(13) +
 5)*(x**2*(sqrt(13) + 5) + 6)*elliptic_e(atan(sqrt(6)*x*sqrt(sqrt
(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(105*sqrt(x**4 + 5*x**2 + 3))
 + 19*sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5
) + 6))*(x**2*(sqrt(13) + 5) + 6)*elliptic_f(atan(sqrt(6)*x*sqrt(
sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(2*sqrt(sqrt(13) + 5)*sqr
t(x**4 + 5*x**2 + 3)) - (-3*x**2 + 14)*(x**4 + 5*x**2 + 3)**(3/2)
/(7*x)
```

Mathematica [C] time = 0.558537, size = 235, normalized size = 0.75

$$\frac{30x^{10}+418x^8+2130x^6+3884x^4-i\sqrt{2}\left(412\sqrt{13}-65\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,xF\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+}{70x\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^2,x]
```

```
[Out] (-1260 + 3884*x^4 + 2130*x^6 + 418*x^8 + 30*x^10 + (412*I)*Sqrt[2
]*(-5 + Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]
*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[
13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-65 + 412*Sqrt[13])
*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13
] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 +
(5*Sqrt[13])/6])/(70*x*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.024, size = 260, normalized size = 0.8

$$\frac{3x^5}{7}\sqrt{x^4+5x^2+3}+\frac{134x^3}{35}\sqrt{x^4+5x^2+3}+10x\sqrt{x^4+5x^2+3}$$
$$+342\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right)x^2}\mathit{EllipticF}\left(1/6\,x\sqrt{-30+6\sqrt{13}},5/6\sqrt{3}+1/6\sqrt{39}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$$
$$-\frac{29664}{35\sqrt{-30+6\sqrt{13}}\left(5+\sqrt{13}\right)}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\mathit{EllipticF}\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-\mathit{El}\right.$$
$$-6\frac{\sqrt{x^4+5x^2+3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x)
```

```
[Out] 3/7*x^5*(x^4+5*x^2+3)^(1/2)+134/35*x^3*(x^4+5*x^2+3)^(1/2)+10*x*(
x^4+5*x^2+3)^(1/2)+342/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/
2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1
/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/
2))-29664/35/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(
1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^
(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39
^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39
^(1/2)))-6*(x^4+5*x^2+3)^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4+5x^2+3)^{\frac{3}{2}}(3x^2+2)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(3x^6+17x^4+19x^2+6)\sqrt{x^4+5x^2+3}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3)/x^2,
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)\,(x^4+5x^2+3)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**2,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4+5x^2+3)^{\frac{3}{2}}(3x^2+2)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^2, x)
```

3.167 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4} dx$

Optimal. Leaf size=314

$$-\frac{13(24-5x^2)\sqrt{x^4+5x^2+3}}{15x}+\frac{949x\left(2x^2+\sqrt{13}+5\right)}{30\sqrt{x^4+5x^2+3}}+\frac{65\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{949\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{30\sqrt{x^4+5x^2+3}}-\frac{(10-9x^2)(x^4+5x^2+3)^{3/2}}{15x^3}$$

```
[Out] (949*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (13*(
24 - 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(15*x) - ((10 - 9*x^2)*(3 + 5*
x^2 + x^4)^(3/2))/(15*x^3) - (949*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6
+ (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13
])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt
[13])/6])/(30*Sqrt[3 + 5*x^2 + x^4]) + (65*Sqrt[2/(3*(5 + Sqrt[13
]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 +
 (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x],
(-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi [A] time = 0.400159, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$-\frac{13(24-5x^2)\sqrt{x^4+5x^2+3}}{15x}+\frac{949x\left(2x^2+\sqrt{13}+5\right)}{30\sqrt{x^4+5x^2+3}}+\frac{65\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{949\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{30\sqrt{x^4+5x^2+3}}-\frac{(10-9x^2)(x^4+5x^2+3)^{3/2}}{15x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^4,x]
```

```
[Out] (949*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (13*(
24 - 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(15*x) - ((10 - 9*x^2)*(3 + 5*
x^2 + x^4)^(3/2))/(15*x^3) - (949*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6
+ (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13
])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt
[13])/6])/(30*Sqrt[3 + 5*x^2 + x^4]) + (65*Sqrt[2/(3*(5 + Sqrt[13
]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 +
 (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x],
(-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi in Sympy [A] time = 33.1578, size = 286, normalized size = 0.91

$$\frac{949x\left(2x^2+\sqrt{13}+5\right)}{30\sqrt{x^4+5x^2+3}} - \frac{949\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{180\sqrt{x^4+5x^2+3}} + \frac{65\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{3\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}} - \frac{\left(-65x^2+312\right)\sqrt{x^4+5x^2+3}}{15x} - \frac{\left(-9x^2+10\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{15x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**4,x)
```

```
[Out] 949*x*(2*x**2 + sqrt(13) + 5)/(30*sqrt(x**4 + 5*x**2 + 3)) - 949*
sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6)
)*sqrt(sqrt(13) + 5)*(x**2*(sqrt(13) + 5) + 6)*elliptic_e(atan(sq
rt(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(180*sqrt(x*
*4 + 5*x**2 + 3)) + 65*sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x
**2*(sqrt(13) + 5) + 6))*(x**2*(sqrt(13) + 5) + 6)*elliptic_f(ata
n(sqrt(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(3*sqrt(
sqrt(13) + 5)*sqrt(x**4 + 5*x**2 + 3)) - (-65*x**2 + 312)*sqrt(x*
*4 + 5*x**2 + 3)/(15*x) - (-9*x**2 + 10)*(x**4 + 5*x**2 + 3)**(3/
2)/(15*x**3)
```

Mathematica [C] time = 0.565847, size = 247, normalized size = 0.79

$$\frac{-13i\sqrt{2}\left(73\sqrt{13}-65\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}x^3F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+949i\sqrt{2}\left(\sqrt{13}-5\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}}{60x^3\sqrt{x^4+5x^2+}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^4,x]
```

```
[Out] (4*(-90 - 1155*x^2 - 1405*x^4 + 192*x^6 + 145*x^8 + 9*x^10) + (94
9*I)*Sqrt[2]*(-5 + Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5
 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt
[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - (13*I)*Sqrt[2]*(-
65 + 73*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13]
)]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqr
t[13])]*x], 19/6 + (5*Sqrt[13])/6])/(60*x^3*Sqrt[3 + 5*x^2 + x^4]
)
```

Maple [A] time = 0.025, size = 260, normalized size = 0.8

$$-2\,\frac{\sqrt{x^4+5\,x^2+3}}{x^3}-\frac{67}{3\,x}\sqrt{x^4+5\,x^2+3}+\frac{20\,x}{3}\sqrt{x^4+5\,x^2+3}$$
$$+780\,\frac{\sqrt{1-\left(-5/6+1/6\,\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\,\sqrt{13}\right)x^2}EllipticF\left(1/6\,x\sqrt{-30+6\,\sqrt{13}},5/6\,\sqrt{3}+1/6\,\sqrt{39}\right)}{\sqrt{-30+6\,\sqrt{13}}\sqrt{x^4+5\,x^2+3}}$$
$$-\frac{11388}{5\,\sqrt{-30+6\,\sqrt{13}}\left(5+\sqrt{13}\right)}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-30+6\,\sqrt{13}}}{6},\frac{5\,\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-Elli\right.$$
$$+\frac{3\,x^3}{5}\sqrt{x^4+5\,x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x)
```

```
[Out] -2*(x^4+5*x^2+3)^(1/2)/x^3-67/3*(x^4+5*x^2+3)^(1/2)/x+20/3*x*(x^4
+5*x^2+3)^(1/2)+780/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))
*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)
*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))
-11388/5/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)
*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2
))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/
2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/
```

```
2)))+3/5*x^3*(x^4+5*x^2+3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4 + 5x^2 + 3)^{\frac{3}{2}} (3x^2 + 2)}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(3x^6 + 17x^4 + 19x^2 + 6)\sqrt{x^4 + 5x^2 + 3}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3)/x^4,
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2 + 2)(x^4 + 5x^2 + 3)^{\frac{3}{2}}}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**4,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**4, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4 + 5\,x^2 + 3\right)^{\frac{3}{2}}\left(3\,x^2 + 2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^4, x)
```

3.168 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=331

$$-\frac{722\sqrt{x^4+5x^2+3}}{15x}+\frac{361x\left(2x^2+\sqrt{13}+5\right)}{15\sqrt{x^4+5x^2+3}}+\frac{103\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}-\frac{361\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{15\sqrt{x^4+5x^2+3}}-\frac{(2-5x^2)\left(x^4+5x^2+3\right)^{3/2}}{5x^5}-\frac{(40-87x^2)\sqrt{x^4+5x^2+3}}{5x^3}$$

```
[Out] (361*x*(5 + Sqrt[13] + 2*x^2))/(15*Sqrt[3 + 5*x^2 + x^4]) - (722*
Sqrt[3 + 5*x^2 + x^4])/(15*x) - ((40 - 87*x^2)*Sqrt[3 + 5*x^2 + x
^4])/(5*x^3) - ((2 - 5*x^2)*(3 + 5*x^2 + x^4)^(3/2))/(5*x^5) - (3
61*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
 Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(
5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(15*Sqrt[3 + 5*x^2 +
x^4]) + (103*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^
2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])
/6]*x], (-13 + 5*Sqrt[13])/6])/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5
*x^2 + x^4])
```

Rubi [A] time = 0.493636, antiderivative size = 331, normalized size of antiderivative = 1., number

of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{722\sqrt{x^4+5x^2+3}}{15x}+\frac{361x\left(2x^2+\sqrt{13}+5\right)}{15\sqrt{x^4+5x^2+3}}+\frac{103\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}-\frac{361\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{15\sqrt{x^4+5x^2+3}}-\frac{\left(2-5x^2\right)\left(x^4+5x^2+3\right)^{3/2}}{5x^5}-\frac{\left(40-87x^2\right)\sqrt{x^4+5x^2+3}}{5x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^6,x]
```

```
[Out] (361*x*(5 + Sqrt[13] + 2*x^2))/(15*Sqrt[3 + 5*x^2 + x^4]) - (722*
Sqrt[3 + 5*x^2 + x^4])/(15*x) - ((40 - 87*x^2)*Sqrt[3 + 5*x^2 + x
^4])/(5*x^3) - ((2 - 5*x^2)*(3 + 5*x^2 + x^4)^(3/2))/(5*x^5) - (3
61*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
 Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(
5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(15*Sqrt[3 + 5*x^2 +
x^4]) + (103*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^
2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])
/6]*x], (-13 + 5*Sqrt[13])/6])/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5
*x^2 + x^4])
```

Rubi in Sympy [A] time = 39.8939, size = 306, normalized size = 0.92

$$\frac{361x\left(2x^2+\sqrt{13}+5\right)}{15\sqrt{x^4+5x^2+3}}-\frac{361\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{90\sqrt{x^4+5x^2+3}}+\frac{103\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{6\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}}-\frac{722\sqrt{x^4+5x^2+3}}{15x}-\frac{\left(-261x^2+120\right)\sqrt{x^4+5x^2+3}}{15x^3}-\frac{\left(-15x^2+6\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**6,x)
```

```
[Out] 361*x*(2*x**2 + sqrt(13) + 5)/(15*sqrt(x**4 + 5*x**2 + 3)) - 361*
sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6)
)*sqrt(sqrt(13) + 5)*(x**2*(sqrt(13) + 5) + 6)*elliptic_e(atan(sq
rt(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(90*sqrt(x**
4 + 5*x**2 + 3)) + 103*sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x
**2*(sqrt(13) + 5) + 6))*(x**2*(sqrt(13) + 5) + 6)*elliptic_f(ata
n(sqrt(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(6*sqrt(
sqrt(13) + 5)*sqrt(x**4 + 5*x**2 + 3)) - 722*sqrt(x**4 + 5*x**2 +
 3)/(15*x) - (-261*x**2 + 120)*sqrt(x**4 + 5*x**2 + 3)/(15*x**3)
- (-15*x**2 + 6)*(x**4 + 5*x**2 + 3)**(3/2)/(15*x**5)
```

Mathematica [C] time = 0.604923, size = 244, normalized size = 0.74

$$\frac{30x^{10} - 634x^8 - 4040x^6 - 3438x^4 - 810x^2 - i\sqrt{2}\left(361\sqrt{13} - 260\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2 + \sqrt{13} + 5}\,x^5 F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right) \right|}{30x^5\sqrt{x^4 + 5x^2 + 3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^6,x]
```

```
[Out] (-108 - 810*x^2 - 3438*x^4 - 4040*x^6 - 634*x^8 + 30*x^10 + (361*
I)*Sqrt[2]*(-5 + Sqrt[13])*x^5*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 +
 Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2
/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-260 + 3
61*Sqrt[13])*x^5*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sq
rt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13]
)]*x], 19/6 + (5*Sqrt[13])/6])/(30*x^5*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.028, size = 259, normalized size = 0.8

$$-\frac{6}{5x^5}\sqrt{x^4+5x^2+3}-7\frac{\sqrt{x^4+5x^2+3}}{x^3}-\frac{392}{15x}\sqrt{x^4+5x^2+3}$$
$$+618\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right)x^2}EllipticF\left(1/6x\sqrt{-30+6\sqrt{13}},5/6\sqrt{3}+1/6\sqrt{39}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$$
$$-\frac{8664}{5\sqrt{-30+6\sqrt{13}}\left(5+\sqrt{13}\right)}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-Elli\right.$$
$$+x\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x)
```

```
[Out] -6/5/x^5*(x^4+5*x^2+3)^(1/2)-7*(x^4+5*x^2+3)^(1/2)/x^3-392/15*(x^
4+5*x^2+3)^(1/2)/x+618/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/
2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1
/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/
2))-8664/5/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/
2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1
/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(
1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(
1/2)))+x*(x^4+5*x^2+3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}\left(3x^2+2\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3x^6+17x^4+19x^2+6\right)\sqrt{x^4+5x^2+3}}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^6,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3)/x^6,
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)\,(x^4+5x^2+3)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**6,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**6, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^6, x)
```

3.169 $\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{a+bx^2+cx^4}\left(-16aBc-2cx^2(5bB-6Ac)-18Abc+15b^2B\right)}{48c^3}-\frac{\left(8aAc^2-12abBc-6Ab^2c+5b^3B\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{7/2}}+\frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c}$$

```
[Out] (B*x^4*Sqrt[a + b*x^2 + c*x^4])/(6*c) + ((15*b^2*B - 18*A*b*c - 1
6*a*B*c - 2*c*(5*b*B - 6*A*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(48*c
^3) - ((5*b^3*B - 6*A*b^2*c - 12*a*b*B*c + 8*a*A*c^2)*ArcTanh[(b
+ 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(7/2))
```

Rubi [A] time = 0.507291, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{\sqrt{a+bx^2+cx^4}\left(-16aBc-2cx^2(5bB-6Ac)-18Abc+15b^2B\right)}{48c^3}-\frac{\left(8aAc^2-12abBc-6Ab^2c+5b^3B\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{7/2}}+\frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*x^4*Sqrt[a + b*x^2 + c*x^4])/(6*c) + ((15*b^2*B - 18*A*b*c - 1
6*a*B*c - 2*c*(5*b*B - 6*A*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(48*c
^3) - ((5*b^3*B - 6*A*b^2*c - 12*a*b*B*c + 8*a*A*c^2)*ArcTanh[(b
+ 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(7/2))
```

Rubi in Sympy [A] time = 35.4782, size = 153, normalized size = 1.

$$\frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c}-\frac{\sqrt{a+bx^2+cx^4}\left(4Bac+\frac{3b(6Ac-5Bb)}{4}-\frac{cx^2(6Ac-5Bb)}{2}\right)}{12c^3}-\frac{\left(8Aac^2-6Ab^2c-12Babc+5Bb^3\right)\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] B*x**4*sqrt(a + b*x**2 + c*x**4)/(6*c) - sqrt(a + b*x**2 + c*x**4
)*(4*B*a*c + 3*b*(6*A*c - 5*B*b)/4 - c*x**2*(6*A*c - 5*B*b)/2)/(1
2*c**3) - (8*A*a*c**2 - 6*A*b**2*c - 12*B*a*b*c + 5*B*b**3)*atanh
((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(32*c**(7/
2))
```

Mathematica [A] time = 0.163228, size = 137, normalized size = 0.9

$$\frac{2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(4c\left(-4aB+3Acx^2+2Bcx^4\right)-2bc\left(9A+5Bx^2\right)+15b^2B\right)-3\left(8aAc^2-12abBc-6Ab^2c+5b^3B\right)\log\left(2\right.}{96c^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(15*b^2*B - 2*b*c*(9*A + 5*B*x
^2) + 4*c*(-4*a*B + 3*A*c*x^2 + 2*B*c*x^4)) - 3*(5*b^3*B - 6*A*b^
2*c - 12*a*b*B*c + 8*a*A*c^2)*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a
+ b*x^2 + c*x^4]])/(96*c^(7/2))
```

Maple [B] time = 0.015, size = 286, normalized size = 1.9

$$\frac{Ax^2}{4c}\sqrt{cx^4+bx^2+a}-\frac{3Ab}{8c^2}\sqrt{cx^4+bx^2+a}+\frac{3Ab^2}{16}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{5}{2}}$$
$$-\frac{Aa}{4}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}+\frac{Bx^4}{6c}\sqrt{cx^4+bx^2+a}-\frac{5bBx^2}{24c^2}\sqrt{cx^4+bx^2+a}$$
$$+\frac{5b^2B}{16c^3}\sqrt{cx^4+bx^2+a}-\frac{5b^3B}{32}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{7}{2}}$$
$$+\frac{3abB}{8}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{5}{2}}-\frac{Ba}{3c^2}\sqrt{cx^4+bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*A*x^2/c*(c*x^4+b*x^2+a)^(1/2)-3/8*A*b/c^2*(c*x^4+b*x^2+a)^(1/
2)+3/16*A*b^2/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1
/2))-1/4*A*a/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/
2))+1/6*B*x^4*(c*x^4+b*x^2+a)^(1/2)/c-5/24*B*b/c^2*x^2*(c*x^4+b*x
```

```
^2+a)^(1/2)+5/16*B*b^2/c^3*(c*x^4+b*x^2+a)^(1/2)-5/32*B*b^3/c^(7/
2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/8*B*b/c^(5/2
)*a*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/3*B/c^2*a*(
c*x^4+b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^5/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.35913, size = 1, normalized size = 0.01

$$\left[\frac{4\left(8\,Bc^2x^4+15\,Bb^2-2\left(5\,Bbc-6\,Ac^2\right)x^2-2\left(8\,Ba+9\,Ab\right)c\right)\sqrt{cx^4+bx^2+a}\sqrt{c}+3\left(5\,Bb^3+8\,Aac^2-6\left(2\,Bab+Ab^2\right)c\right)\text{lo}}{192\,c^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^5/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/192*(4*(8*B*c^2*x^4 + 15*B*b^2 - 2*(5*B*b*c - 6*A*c^2)*x^2 - 2
*(8*B*a + 9*A*b)*c)*sqrt(c*x^4 + b*x^2 + a)*sqrt(c) + 3*(5*B*b^3
+ 8*A*a*c^2 - 6*(2*B*a*b + A*b^2)*c)*log(4*sqrt(c*x^4 + b*x^2 + a
)*(2*c^2*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(
c)))/c^(7/2), 1/96*(2*(8*B*c^2*x^4 + 15*B*b^2 - 2*(5*B*b*c - 6*A*
c^2)*x^2 - 2*(8*B*a + 9*A*b)*c)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c)
- 3*(5*B*b^3 + 8*A*a*c^2 - 6*(2*B*a*b + A*b^2)*c)*arctan(1/2*(2*c
*x^2 + b)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a)*c)))/(sqrt(-c)*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [A] time = 0.305565, size = 203, normalized size = 1.33

$$\frac{1}{48}\sqrt{cx^4+bx^2+a}\left(2\left(\frac{4\,Bx^2}{c}-\frac{5\,Bbc^2-6\,Ac^3}{c^4}\right)x^2+\frac{15\,Bb^2c-16\,Bac^2-18\,Abc^2}{c^4}\right)+\frac{\left(5\,Bb^3c-12\,Babc^2-6\,Ab^2c^2+8\,Aac^3\right)\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{32\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^5/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2 + a)*(2*(4*B*x^2/c - (5*B*b*c^2 - 6*A*c^3
)/c^4)*x^2 + (15*B*b^2*c - 16*B*a*c^2 - 18*A*b*c^2)/c^4) + 1/32*(
5*B*b^3*c - 12*B*a*b*c^2 - 6*A*b^2*c^2 + 8*A*a*c^3)*ln(abs(-2*(sq
rt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(9/2)
```

3.170 $\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=100

$$\frac{(-4aBc-4Abc+3b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{\sqrt{a+bx^2+cx^4}\,(-4Ac+3bB-2Bcx^2)}{8c^2}$$

```
[Out] -((3*b*B - 4*A*c - 2*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) +
((3*b^2*B - 4*A*b*c - 4*a*B*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*S
qrt[a + b*x^2 + c*x^4]])]/(16*c^(5/2))
```

Rubi [A] time = 0.252702, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148

$$\frac{(-4aBc-4Abc+3b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{\sqrt{a+bx^2+cx^4}\,(-4Ac+3bB-2Bcx^2)}{8c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] -((3*b*B - 4*A*c - 2*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) +
((3*b^2*B - 4*A*b*c - 4*a*B*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*S
qrt[a + b*x^2 + c*x^4]])]/(16*c^(5/2))
```

Rubi in Sympy [A] time = 21.5702, size = 97, normalized size = 0.97

$$\frac{\sqrt{a+bx^2+cx^4}\left(2Ac-\frac{3Bb}{2}+Bcx^2\right)}{4c^2}+\frac{(-4Abc-4Bac+3Bb^2)\,\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a + b*x**2 + c*x**4)*(2*A*c - 3*B*b/2 + B*c*x**2)/(4*c**2) +
 (-4*A*b*c - 4*B*a*c + 3*B*b**2)*atanh((b + 2*c*x**2)/(2*sqrt(c)*
sqrt(a + b*x**2 + c*x**4)))/(16*c**(5/2))
```

Mathematica [A] time = 0.105575, size = 99, normalized size = 0.99

$$\frac{\left(-4aBc-4Abc+3b^2B\right)\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)+2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(4Ac-3bB+2Bcx^2\right)}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*(-3*b*B + 4*A*c + 2*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4] +
 (3*b^2*B - 4*A*b*c - 4*a*B*c)*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a
 + b*x^2 + c*x^4]])/(16*c^(5/2))
```

Maple [B] time = 0.012, size = 176, normalized size = 1.8

$$\frac{A}{2\,c}\sqrt{cx^4+bx^2+a}-\frac{Ab}{4}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}+\frac{Bx^2}{4\,c}\sqrt{cx^4+bx^2+a}$$
$$-\frac{3\,bB}{8\,c^2}\sqrt{cx^4+bx^2+a}+\frac{3\,b^2B}{16}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{5}{2}}$$
$$-\frac{Ba}{4}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*A/c*(c*x^4+b*x^2+a)^(1/2)-1/4*A*b/c^(3/2)*ln((1/2*b+c*x^2)/c^
(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/4*B*x^2/c*(c*x^4+b*x^2+a)^(1/2)-3/
8*B*b/c^2*(c*x^4+b*x^2+a)^(1/2)+3/16*B*b^2/c^(5/2)*ln((1/2*b+c*x^
2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/4*B*a/c^(3/2)*ln((1/2*b+c*x^2
)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^3/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.342241, size = 1, normalized size = 0.01

$$\left[\frac{4\,\sqrt{cx^4+bx^2+a}\left(2\,Bcx^2-3\,Bb+4\,Ac\right)\sqrt{c}-\left(3\,Bb^2-4\,(Ba+Ab)c\right)\log\left(4\,\sqrt{cx^4+bx^2+a}\left(2\,c^2x^2+bc\right)-\left(8\,c^2x^4+8\,bcx^2\right.\right.}{32\,c^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^3/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(c*x^4 + b*x^2 + a)*(2*B*c*x^2 - 3*B*b + 4*A*c)*sqrt
(c) - (3*B*b^2 - 4*(B*a + A*b)*c)*log(4*sqrt(c*x^4 + b*x^2 + a)*(
2*c^2*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c))
)/c^(5/2), 1/16*(2*sqrt(c*x^4 + b*x^2 + a)*(2*B*c*x^2 - 3*B*b + 4
*A*c)*sqrt(-c) + (3*B*b^2 - 4*(B*a + A*b)*c)*arctan(1/2*(2*c*x^2
+ b)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a)*c)))/(sqrt(-c)*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [A] time = 0.296202, size = 132, normalized size = 1.32

$$\frac{1}{8}\,\sqrt{cx^4+bx^2+a}\left(\frac{2\,Bx^2}{c}-\frac{3\,Bb-4\,Ac}{c^2}\right)-\frac{\left(3\,Bb^2-4\,Bac-4\,Abc\right)\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^3/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*B*x^2/c - (3*B*b - 4*A*c)/c^2) - 1
/16*(3*B*b^2 - 4*B*a*c - 4*A*b*c)*ln(abs(-2*(sqrt(c)*x^2 - sqrt(c
*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(5/2)
```

3.171 $\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=76

$$\frac{B\sqrt{a+bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

```
[Out] (B*Sqrt[a + b*x^2 + c*x^4])/(2*c) - ((b*B - 2*A*c)*ArcTanh[(b + 2
*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2))
```

Rubi [A] time = 0.161301, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$\frac{B\sqrt{a+bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*Sqrt[a + b*x^2 + c*x^4])/(2*c) - ((b*B - 2*A*c)*ArcTanh[(b + 2
*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2))
```

Rubi in Sympy [A] time = 17.4067, size = 66, normalized size = 0.87

$$\frac{B\sqrt{a+bx^2+cx^4}}{2c}+\frac{(2Ac-Bb)\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] B*sqrt(a + b*x**2 + c*x**4)/(2*c) + (2*A*c - B*b)*atanh((b + 2*c*
x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(4*c**(3/2))
```

Mathematica [A] time = 0.0635454, size = 75, normalized size = 0.99

$$\frac{(2Ac - bB)\log\left(2\sqrt{c}\sqrt{a + bx^2 + cx^4} + b + 2cx^2\right)}{4c^{3/2}} + \frac{B\sqrt{a + bx^2 + cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*Sqrt[a + b*x^2 + c*x^4])/(2*c) + ((-(b*B) + 2*A*c)*Log[b + 2*c
*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(4*c^(3/2))
```

Maple [A] time = 0.009, size = 93, normalized size = 1.2

$$\frac{A}{2}\ln\left(1\left(\frac{b}{2} + cx^2\right)\frac{1}{\sqrt{c}} + \sqrt{cx^4 + bx^2 + a}\right)\frac{1}{\sqrt{c}} + \frac{B}{2\,c}\sqrt{cx^4 + bx^2 + a}$$
$$- \frac{bB}{4}\ln\left(1\left(\frac{b}{2} + cx^2\right)\frac{1}{\sqrt{c}} + \sqrt{cx^4 + bx^2 + a}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*A*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+1/2
*B*(c*x^4+b*x^2+a)^(1/2)/c-1/4*B*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/
2)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.313552, size = 1, normalized size = 0.01

$$\left[\frac{4\sqrt{cx^4+bx^2+a}B\sqrt{c}-(Bb-2Ac)\log\left(-4\sqrt{cx^4+bx^2+a}\left(2c^2x^2+bc\right)-\left(8c^2x^4+8bcx^2+b^2+4ac\right)\sqrt{c}\right)}{8c^{\frac{3}{2}}}, \frac{2\sqrt{cx^4+bx^2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(c*x^4 + b*x^2 + a)*B*sqrt(c) - (B*b - 2*A*c)*log(-4*
sqrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^
2 + b^2 + 4*a*c)*sqrt(c)))/c^(3/2), 1/4*(2*sqrt(c*x^4 + b*x^2 + a
)*B*sqrt(-c) - (B*b - 2*A*c)*arctan(1/2*(2*c*x^2 + b)*sqrt(-c)/(s
qrt(c*x^4 + b*x^2 + a)*c)))/(sqrt(-c)*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [A] time = 0.296469, size = 93, normalized size = 1.22

$$\frac{\sqrt{cx^4+bx^2+a}B}{2c}+\frac{(Bb-2Ac)\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{4c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^4 + b*x^2 + a)*B/c + 1/4*(B*b - 2*A*c)*ln(abs(-2*(sq
rt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2)
```

3.172 $\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=90

$$\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}-\frac{A\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

```
[Out] -(A*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(
2*Sqrt[a]) + (B*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 +
 c*x^4])])/(2*Sqrt[c])
```

Rubi [A] time = 0.248373, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185

$$\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}-\frac{A\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(A*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(
2*Sqrt[a]) + (B*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 +
 c*x^4])])/(2*Sqrt[c])
```

Rubi in Sympy [A] time = 23.1527, size = 80, normalized size = 0.89

$$-\frac{A\,\mathrm{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}+\frac{B\,\mathrm{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -A*atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**4)))/(2
*sqrt(a)) + B*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c
*x**4)))/(2*sqrt(c))
```

Mathematica [A] time = 0.300228, size = 96, normalized size = 1.07

$$A\left(\frac{\log(x)}{\sqrt{a}}-\frac{\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)}{2\sqrt{a}}\right)+\frac{B\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] A*(Log[x]/Sqrt[a] - Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2 +
c*x^4]]/(2*Sqrt[a])) + (B*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*
x^2 + c*x^4]])/(2*Sqrt[c])
```

Maple [A] time = 0.009, size = 76, normalized size = 0.8

$$-\frac{A}{2}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{B}{2}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*A/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2
)+1/2*B*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.370738, size = 1, normalized size = 0.01

$$\left[\frac{B\sqrt{a}\log\left(-4\sqrt{cx^4+bx^2+a}\left(2\,c^2x^2+bc\right)-\left(8\,c^2x^4+8\,bcx^2+b^2+4\,ac\right)\sqrt{c}\right)+A\sqrt{c}\log\left(\frac{4\sqrt{cx^4+bx^2+a}\left(abx^2+2\,a^2\right)-\left(\left(b^2+4\,ac\right)x\right.}{x^4}\right.}{4\,\sqrt{a}\sqrt{c}}\right.$$

$$-\frac{2\,A\sqrt{c}\arctan\left(\frac{\left(bx^2+2\,a\right)\sqrt{-a}}{2\,\sqrt{cx^4+bx^2+a}a}\right)-B\sqrt{-a}\log\left(-4\sqrt{cx^4+bx^2+a}\left(2\,c^2x^2+bc\right)-\left(8\,c^2x^4+8\,bcx^2+b^2+4\,ac\right)\sqrt{c}\right)}{4\,\sqrt{-a}\sqrt{c}},$$

$$\left.-\frac{A\sqrt{-c}\arctan\left(\frac{\left(bx^2+2\,a\right)\sqrt{-a}}{2\,\sqrt{cx^4+bx^2+a}a}\right)-B\sqrt{-a}\arctan\left(\frac{\left(2\,cx^2+b\right)\sqrt{-c}}{2\,\sqrt{cx^4+bx^2+a}c}\right)}{2\,\sqrt{-a}\sqrt{-c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] [1/4*(B*sqrt(a)*log(-4*sqrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 + b*c)
- (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)) + A*sqrt(c)*log(
(4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2) - ((b^2 + 4*a*c)*x^4
 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4))/(sqrt(a)*sqrt(c)), 1/4*(2*B*
sqrt(a)*arctan(1/2*(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a
)*c)) + A*sqrt(-c)*log((4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^
2) - ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4))/(sqrt
(a)*sqrt(-c)), -1/4*(2*A*sqrt(c)*arctan(1/2*(b*x^2 + 2*a)*sqrt(-a
)/(sqrt(c*x^4 + b*x^2 + a)*a)) - B*sqrt(-a)*log(-4*sqrt(c*x^4 + b
*x^2 + a)*(2*c^2*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*
c)*sqrt(c)))/(sqrt(-a)*sqrt(c)), -1/2*(A*sqrt(-c)*arctan(1/2*(b*x
^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)*a)) - B*sqrt(-a)*arct
an(1/2*(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a)*c)))/(sqrt
(-a)*sqrt(-c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x), x)
```

3.173 $\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=80

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2}$$

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(2*a*x^2) + ((A*b - 2*a*B)*ArcTanh[(
2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rubi [A] time = 0.232452, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(2*a*x^2) + ((A*b - 2*a*B)*ArcTanh[(
2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rubi in Sympy [A] time = 21.8612, size = 70, normalized size = 0.88

$$-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2}+\frac{(Ab-2Ba)\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(a + b*x**2 + c*x**4)/(2*a*x**2) + (A*b - 2*B*a)*atanh((2*
a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**4)))/(4*a**(3/2))
```

Mathematica [A] time = 0.221566, size = 87, normalized size = 1.09

$$\frac{(2aB-Ab)\left(\log\left(x^2\right)-\log\left(2\sqrt{a}\sqrt{a+x^2\left(b+cx^2\right)}+2a+bx^2\right)\right)}{4a^{3/2}}-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(2*a*x^2) + ((-(A*b) + 2*a*B)*(Log[x
^2] - Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + x^2*(b + c*x^2)]]))/(4
*a^(3/2))
```

Maple [A] time = 0.013, size = 104, normalized size = 1.3

$$-\frac{A}{2\,x^2a}\sqrt{cx^4+bx^2+a}+\frac{Ab}{4}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{3}{2}}-\frac{B}{2}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*A*(c*x^4+b*x^2+a)^(1/2)/a/x^2+1/4*A*b/a^(3/2)*ln((2*a+b*x^2+
2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-1/2*B/a^(1/2)*ln((2*a+b*x^2
+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.32187, size = 1, normalized size = 0.01

$$\left[-\frac{(2\,Ba-Ab)x^2\log\left(-\frac{4\,\sqrt{cx^4+bx^2+a}\left(abx^2+2\,a^2\right)+\left(\left(b^2+4\,ac\right)x^4+8\,abx^2+8\,a^2\right)\sqrt{a}}{x^4}\right)+4\,\sqrt{cx^4+bx^2+a}A\sqrt{a}}{8\,a^{\frac{3}{2}}x^2},\right.$$
$$\left.-\frac{(2\,Ba-Ab)x^2\arctan\left(\frac{\left(bx^2+2\,a\right)\sqrt{-a}}{2\,\sqrt{cx^4+bx^2+a}a}\right)+2\,\sqrt{cx^4+bx^2+a}A\sqrt{-a}}{4\,\sqrt{-a}ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/8*((2*B*a - A*b)*x^2*log(-(4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2
 + 2*a^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4)
 + 4*sqrt(c*x^4 + b*x^2 + a)*A*sqrt(a))/(a^(3/2)*x^2), -1/4*((2*B
*a - A*b)*x^2*arctan(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x
^2 + a)*a)) + 2*sqrt(c*x^4 + b*x^2 + a)*A*sqrt(-a))/(sqrt(-a)*a*x
^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**3*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^3), x)
```

3.174 $\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=124

$$-\frac{\left(-4aAc-4abB+3Ab^2\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4}$$

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(4*a*x^4) + ((3*A*b - 4*a*B)*Sqrt[a
+ b*x^2 + c*x^4])/(8*a^2*x^2) - ((3*A*b^2 - 4*a*b*B - 4*a*A*c)*Ar
cTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(
5/2))
```

Rubi [A] time = 0.382329, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185

$$-\frac{\left(-4aAc-4abB+3Ab^2\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(4*a*x^4) + ((3*A*b - 4*a*B)*Sqrt[a
+ b*x^2 + c*x^4])/(8*a^2*x^2) - ((3*A*b^2 - 4*a*b*B - 4*a*A*c)*Ar
cTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(
5/2))
```

Rubi in Sympy [A] time = 34.6226, size = 114, normalized size = 0.92

$$-\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4}+\frac{(3Ab-4Ba)\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{(-4Aac+b\,(3Ab-4Ba))\,\mathrm{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(a + b*x**2 + c*x**4)/(4*a*x**4) + (3*A*b - 4*B*a)*sqrt(a
+ b*x**2 + c*x**4)/(8*a**2*x**2) - (-4*A*a*c + b*(3*A*b - 4*B*a))
*atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**4)))/(16*
```

```
a**(5/2))
```

Mathematica [A] time = 0.196443, size = 112, normalized size = 0.9

$$\frac{\sqrt{a+bx^2+cx^4}\left(3Abx^2-2a\left(A+2Bx^2\right)\right)}{8a^2x^4}-\frac{\left(4aAc+4abB-3Ab^2\right)\left(\log\left(x^2\right)-\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)\right)}{16a^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(3*A*b*x^2 - 2*a*(A + 2*B*x^2)))/(8*a^2*
x^4) - ((-3*A*b^2 + 4*a*b*B + 4*a*A*c)*(Log[x^2] - Log[2*a + b*x^
2 + 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]]))/(16*a^(5/2))
```

Maple [A] time = 0.014, size = 194, normalized size = 1.6

$$-\frac{A}{4\,ax^4}\sqrt{cx^4+bx^2+a}+\frac{3\,Ab}{8\,a^2x^2}\sqrt{cx^4+bx^2+a}-\frac{3\,Ab^2}{16}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{5}{2}}+\frac{Ac}{4}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{3}{2}}-\frac{B}{2\,x^2a}\sqrt{cx^4+bx^2+a}+\frac{bB}{4}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*A*(c*x^4+b*x^2+a)^(1/2)/a/x^4+3/8*A*b/a^2/x^2*(c*x^4+b*x^2+a
)^(1/2)-3/16*A*b^2/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a
)^(1/2))/x^2)+1/4*A*c/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^
2+a)^(1/2))/x^2)-1/2*B/a/x^2*(c*x^4+b*x^2+a)^(1/2)+1/4*B*b/a^(3/2
)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.346938, size = 1, normalized size = 0.01

$$\left[\frac{\left(4\,Bab-3\,Ab^{2}+4\,Aac\right)x^{4}\log\left(-\frac{4\,\sqrt{cx^{4}+bx^{2}+a}\left(abx^{2}+2\,a^{2}\right)+\left(\left(b^{2}+4\,ac\right)x^{4}+8\,abx^{2}+8\,a^{2}\right)\sqrt{a}}{x^{4}}\right)-4\,\sqrt{cx^{4}+bx^{2}+a}\left((4\,Ba-3\,Ab)x^{2}+2\right.}{32\,a^{\frac{5}{2}}x^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^5),x, algorithm="fricas")
```

```
[Out] [1/32*((4*B*a*b - 3*A*b^2 + 4*A*a*c)*x^4*log(-(4*sqrt(c*x^4 + b*x
^2 + a)*(a*b*x^2 + 2*a^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^
2)*sqrt(a))/x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*((4*B*a - 3*A*b)*x^2
 + 2*A*a)*sqrt(a))/(a^(5/2)*x^4), 1/16*((4*B*a*b - 3*A*b^2 + 4*A*
a*c)*x^4*arctan(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 +
a)*a)) - 2*sqrt(c*x^4 + b*x^2 + a)*((4*B*a - 3*A*b)*x^2 + 2*A*a)*
sqrt(-a))/(sqrt(-a)*a^2*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx^{2}}{x^{5}\sqrt{a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**5*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^{2}+A}{\sqrt{cx^{4}+bx^{2}+a}\,x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^5),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^5), x)
```

3.175 $\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=177

$$-\frac{\sqrt{a+bx^2+cx^4}\left(-16aAc-18abB+15Ab^2\right)}{48a^3x^2}+\frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4}+\frac{\left(8a^2Bc-12aAbc-6ab^2B+5Ab^3\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}-\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6}$$

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(6*a*x^6) + ((5*A*b - 6*a*B)*Sqrt[a
+ b*x^2 + c*x^4])/(24*a^2*x^4) - ((15*A*b^2 - 18*a*b*B - 16*a*A*c
)*Sqrt[a + b*x^2 + c*x^4])/(48*a^3*x^2) + ((5*A*b^3 - 6*a*b^2*B -
 12*a*A*b*c + 8*a^2*B*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a
+ b*x^2 + c*x^4])])/(32*a^(7/2))
```

Rubi [A] time = 0.600473, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$-\frac{\sqrt{a+bx^2+cx^4}\left(-16aAc-18abB+15Ab^2\right)}{48a^3x^2}+\frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4}+\frac{\left(8a^2Bc-12aAbc-6ab^2B+5Ab^3\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}-\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(6*a*x^6) + ((5*A*b - 6*a*B)*Sqrt[a
+ b*x^2 + c*x^4])/(24*a^2*x^4) - ((15*A*b^2 - 18*a*b*B - 16*a*A*c
)*Sqrt[a + b*x^2 + c*x^4])/(48*a^3*x^2) + ((5*A*b^3 - 6*a*b^2*B -
 12*a*A*b*c + 8*a^2*B*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a
+ b*x^2 + c*x^4])])/(32*a^(7/2))
```

Rubi in Sympy [A] time = 54.4566, size = 172, normalized size = 0.97

$$-\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6}+\frac{(5Ab-6Ba)\sqrt{a+bx^2+cx^4}}{24a^2x^4}-\frac{\sqrt{a+bx^2+cx^4}\left(-16Aac+15Ab^2-18Bab\right)}{48a^3x^2}+\frac{\left(-12Aabc+5Ab^3+8Ba^2c-6Bab^2\right)\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(a + b*x**2 + c*x**4)/(6*a*x**6) + (5*A*b - 6*B*a)*sqrt(a
+ b*x**2 + c*x**4)/(24*a**2*x**4) - sqrt(a + b*x**2 + c*x**4)*(-1
6*A*a*c + 15*A*b**2 - 18*B*a*b)/(48*a**3*x**2) + (-12*A*a*b*c + 5
*A*b**3 + 8*B*a**2*c - 6*B*a*b**2)*atanh((2*a + b*x**2)/(2*sqrt(a
)*sqrt(a + b*x**2 + c*x**4)))/(32*a**(7/2))
```

Mathematica [A] time = 0.266458, size = 146, normalized size = 0.82

$$-\frac{(8a^2Bc-12aAbc-6ab^2B+5Ab^3)\left(\log(x^2)-\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)\right)}{32a^{7/2}}-\frac{\sqrt{a+bx^2+cx^4}\left(8a^2A+x^4\left(-16aAc-18abB+15Ab^2\right)+2ax^2(6aB-5Ab)\right)}{48a^3x^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x^2 + c*x^4]*(8*a^2*A + 2*a*(-5*A*b + 6*a*B)*x^2 + (
15*A*b^2 - 18*a*b*B - 16*a*A*c)*x^4))/(48*a^3*x^6) - ((5*A*b^3 -
6*a*b^2*B - 12*a*A*b*c + 8*a^2*B*c)*(Log[x^2] - Log[2*a + b*x^2 +
 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]]))/(32*a^(7/2))
```

Maple [A] time = 0.016, size = 311, normalized size = 1.8

$$-\frac{A}{6\,ax^6}\sqrt{cx^4+bx^2+a}+\frac{5\,Ab}{24\,a^2x^4}\sqrt{cx^4+bx^2+a}-\frac{5\,Ab^2}{16\,a^3x^2}\sqrt{cx^4+bx^2+a}+\frac{5\,Ab^3}{32}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{7}{2}}-\frac{3\,Abc}{8}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{5}{2}}+\frac{Ac}{3\,a^2x^2}\sqrt{cx^4+bx^2+a}-\frac{B}{4\,ax^4}\sqrt{cx^4+bx^2+a}+\frac{3\,bB}{8\,a^2x^2}\sqrt{cx^4+bx^2+a}-\frac{3\,b^2B}{16}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{5}{2}}+\frac{Bc}{4}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/6*A*(c*x^4+b*x^2+a)^(1/2)/a/x^6+5/24*A*b/a^2/x^4*(c*x^4+b*x^2+
a)^(1/2)-5/16*A*b^2/a^3/x^2*(c*x^4+b*x^2+a)^(1/2)+5/32*A*b^3/a^(7
/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-3/8*A*b/a
^(5/2)*c*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/3*
A*c/a^2/x^2*(c*x^4+b*x^2+a)^(1/2)-1/4*B/a/x^4*(c*x^4+b*x^2+a)^(1/
2)+3/8*B*b/a^2/x^2*(c*x^4+b*x^2+a)^(1/2)-3/16*B*b^2/a^(5/2)*ln((2
*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/4*B*c/a^(3/2)*ln
((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^7),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.358216, size = 1, normalized size = 0.01

$$\left[\frac{3\left(6\,Bab^2-5\,Ab^3-4\left(2\,Ba^2-3\,Aab\right)c\right)x^6\log\left(\frac{4\sqrt{cx^4+bx^2+a}\left(abx^2+2\,a^2\right)-\left(\left(b^2+4\,ac\right)x^4+8\,abx^2+8\,a^2\right)\sqrt{a}}{x^4}\right)+4\left(\left(18\,Bab-15\,Ab^2+\right.\right.}{192\,a^{\frac{7}{2}}x^6}\right.$$

$$-\frac{3\left(6\,Bab^2-5\,Ab^3-4\left(2\,Ba^2-3\,Aab\right)c\right)x^6\arctan\left(\frac{\left(bx^2+2\,a\right)\sqrt{-a}}{2\sqrt{cx^4+bx^2+a}a}\right)-2\left(\left(18\,Bab-15\,Ab^2+16\,Aac\right)x^4-8\,Aa^2-2\left(6\,Ba^2\right.\right.}{96\sqrt{-a}a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^7),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(6*B*a*b^2 - 5*A*b^3 - 4*(2*B*a^2 - 3*A*a*b)*c)*x^6*log
((4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2) - ((b^2 + 4*a*c)*x^
4 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4) + 4*((18*B*a*b - 15*A*b^2 +
16*A*a*c)*x^4 - 8*A*a^2 - 2*(6*B*a^2 - 5*A*a*b)*x^2)*sqrt(c*x^4 +
 b*x^2 + a)*sqrt(a))/(a^(7/2)*x^6), -1/96*(3*(6*B*a*b^2 - 5*A*b^3
 - 4*(2*B*a^2 - 3*A*a*b)*c)*x^6*arctan(1/2*(b*x^2 + 2*a)*sqrt(-a)
/(sqrt(c*x^4 + b*x^2 + a)*a)) - 2*((18*B*a*b - 15*A*b^2 + 16*A*a*
```

```
c)*x^4 - 8*A*a^2 - 2*(6*B*a^2 - 5*A*a*b)*x^2)*sqrt(c*x^4 + b*x^2
+ a)*sqrt(-a))/(sqrt(-a)*a^3*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{x^7 \sqrt{a + bx^2 + cx^4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**7*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}x^7} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^7),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^7), x)
```

3.176 $\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=403

$$\frac{x\sqrt{a+bx^2+cx^4}\left(-9aBc-10Abc+8b^2B\right)}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}\sqrt{c}(4bB-5Ac)-9aBc-10Abc+8b^2B\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{11/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-9aBc-10Abc+8b^2B\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{11/4}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}(4bB-5Ac)}{15c^2}+\frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c}$$

```
[Out] -((4*b*B - 5*A*c)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^2) + (B*x^3*Sq
rt[a + b*x^2 + c*x^4])/(5*c) + ((8*b^2*B - 10*A*b*c - 9*a*B*c)*x*
Sqrt[a + b*x^2 + c*x^4])/(15*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (
a^(1/4)*(8*b^2*B - 10*A*b*c - 9*a*B*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(11
/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(8*b^2*B - 10*A*b*c - 9*a
*B*c + Sqrt[a]*Sqrt[c]*(4*b*B - 5*A*c))*(Sqrt[a] + Sqrt[c]*x^2)*S
qrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(1
1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.670803, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{x\sqrt{a+bx^2+cx^4}\left(-9aBc-10Abc+8b^2B\right)}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}\sqrt{c}(4bB-5Ac)-9aBc-10Abc+8b^2B\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{11/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-9aBc-10Abc+8b^2B\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{11/4}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}(4bB-5Ac)}{15c^2}+\frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(x^4*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] -((4*b*B - 5*A*c)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^2) + (B*x^3*Sq
rt[a + b*x^2 + c*x^4])/(5*c) + ((8*b^2*B - 10*A*b*c - 9*a*B*c)*x*
Sqrt[a + b*x^2 + c*x^4])/(15*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (
a^(1/4)*(8*b^2*B - 10*A*b*c - 9*a*B*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(11
/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(8*b^2*B - 10*A*b*c - 9*a
*B*c + Sqrt[a]*Sqrt[c]*(4*b*B - 5*A*c))*(Sqrt[a] + Sqrt[c]*x^2)*S
qrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(1
1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 78.2831, size = 381, normalized size = 0.95

$$\frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c} - \frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-10Abc-9Bac+8Bb^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{15c^{\frac{11}{4}}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-10Abc-9Bac+8Bb^2-\sqrt{a}\sqrt{c}\left(5Ac-4Bb\right)\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{30c^{\frac{11}{4}}\sqrt{a+bx^2+cx^4}} + \frac{x\left(5Ac-4Bb\right)\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{x\sqrt{a+bx^2+cx^4}\left(-10Abc-9Bac+8Bb^2\right)}{15c^{\frac{5}{2}}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] B*x**3*sqrt(a + b*x**2 + c*x**4)/(5*c) - a**(1/4)*sqrt((a + b*x**
2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)
*(-10*A*b*c - 9*B*a*c + 8*B*b**2)*elliptic_e(2*atan(c**(1/4)*x/a*
*(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(15*c**(11/4)*sqrt(a + b*x*
*2 + c*x**4)) + a**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sq
rt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(-10*A*b*c - 9*B*a*c + 8
*B*b**2 - sqrt(a)*sqrt(c)*(5*A*c - 4*B*b))*elliptic_f(2*atan(c**(
1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(30*c**(11/4)*sqrt
(a + b*x**2 + c*x**4)) + x*(5*A*c - 4*B*b)*sqrt(a + b*x**2 + c*x*
*4)/(15*c**2) + x*sqrt(a + b*x**2 + c*x**4)*(-10*A*b*c - 9*B*a*c
+ 8*B*b**2)/(15*c**(5/2)*(sqrt(a) + sqrt(c)*x**2))
```

Mathematica [C] time = 3.77184, size = 532, normalized size = 1.32

$$\frac{i\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\left(-9aBc-10Abc+8b^2B\right)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-4*b*B + 5*A*c + 3*B*c*x^
2)*(a + b*x^2 + c*x^4) + I*(8*b^2*B - 10*A*b*c - 9*a*B*c)*(-b + S
qrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b -
Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[
b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
)] - I*(-8*b^3*B + b*c*(17*a*B - 10*A*Sqrt[b^2 - 4*a*c]) + 2*b^2*
(5*A*c + 4*B*Sqrt[b^2 - 4*a*c]) - a*c*(10*A*c + 9*B*Sqrt[b^2 - 4*
a*c]))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 -
4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c
])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(60*c^3
*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [B] time = 0.012, size = 815, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] A*(1/3/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12/c*a*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2
*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Ellipt
icF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/3*b/c*a*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4
+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(
-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Ellip
ticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*
(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+B*(1/5/c*x^3*(c*x^4+b*x^2+a)
^(1/2)-4/15*b/c^2*x*(c*x^4+b*x^2+a)^(1/2)+1/15*b/c^2*a*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^
2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)
```

```
^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),
1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(-3/5/c*a+8/15
*b^2/c^2)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/
2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*
a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2
))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)\,x^4}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^4/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Bx^6 + Ax^4}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^4/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((B*x^6 + A*x^4)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\,(A + Bx^2)}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^4}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)*x^4/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

3.177 $\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=336

$$-\frac{x\sqrt{a+bx^2+cx^4}(2bB-3Ac)}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}B\sqrt{c}-3Ac+2bB\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(2bB-3Ac)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{Bx\sqrt{a+bx^2+cx^4}}{3c}$$

```
[Out] (B*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) - ((2*b*B - 3*A*c)*x*Sqrt[a +
 b*x^2 + c*x^4])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(
2*b*B - 3*A*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)]
, (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(7/4)*Sqrt[a + b*x^2 + c*x^4
]) - (a^(1/4)*(2*b*B + Sqrt[a]*B*Sqrt[c] - 3*A*c)*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]
)/(6*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.393107, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$-\frac{x\sqrt{a+bx^2+cx^4}(2bB-3Ac)}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}B\sqrt{c}-3Ac+2bB\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(2bB-3Ac)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{Bx\sqrt{a+bx^2+cx^4}}{3c}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(x^2*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) - ((2*b*B - 3*A*c)*x*Sqrt[a +
 b*x^2 + c*x^4])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(
2*b*B - 3*A*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(
```

```
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)]
, (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(7/4)*Sqrt[a + b*x^2 + c*x^4
]) - (a^(1/4)*(2*b*B + Sqrt[a]*B*Sqrt[c] - 3*A*c)*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]
)/(6*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 51.8194, size = 308, normalized size = 0.92

$$\frac{Bx\sqrt{a+bx^2+cx^4}}{3c}-\frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(3Ac-2Bb\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{3c^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(3Ac-B\sqrt{a}\sqrt{c}-2Bb\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{6c^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}}+\frac{x\left(3Ac-2Bb\right)\sqrt{a+bx^2+cx^4}}{3c^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] B*x*sqrt(a + b*x**2 + c*x**4)/(3*c) - a**(1/4)*sqrt((a + b*x**2 +
 c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(3
*A*c - 2*B*b)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*
sqrt(a)*sqrt(c)))/(3*c**(7/4)*sqrt(a + b*x**2 + c*x**4)) + a**(1/
4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(
a) + sqrt(c)*x**2)*(3*A*c - B*sqrt(a)*sqrt(c) - 2*B*b)*elliptic_f
(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(6*c**
(7/4)*sqrt(a + b*x**2 + c*x**4)) + x*(3*A*c - 2*B*b)*sqrt(a + b*x
**2 + c*x**4)/(3*c**(3/2)*(sqrt(a) + sqrt(c)*x**2))
```

Mathematica [C] time = 2.40492, size = 479, normalized size = 1.43

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\left(-3Ac\sqrt{b^2-4ac}+2bB\sqrt{b^2-4ac}+2aBc+3Abc-2b^2B\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*B*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) - I*
(2*b*B - 3*A*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 -
4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sq
rt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])
/(b - Sqrt[b^2 - 4*a*c])] + I*(-2*b^2*B + 3*A*b*c + 2*a*B*c + 2*b
*B*Sqrt[b^2 - 4*a*c] - 3*A*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt
[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*Arc
Sinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(12*c^2*Sqrt[c/(b + Sqrt[b^2 -
4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.01, size = 607, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*A*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1
/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c
+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
+B*(1/3/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12/c*a*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+
2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Ellip
ticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*
(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/3*b/c*a*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(
4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+
(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Elli
pticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)x^2}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^4 + Ax^2}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((B*x^4 + A*x^2)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(A + Bx^2\right)}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)x^2}{\sqrt{cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

3.178 $\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=283

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\frac{A\sqrt{c}}{\sqrt{a}}+B\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{Bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[Out] (B*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) -
 (a^(1/4)*B*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (
2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) +
(a^(1/4)*(B + (A*Sqrt[c])/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[
(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sq
rt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.224684, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\frac{A\sqrt{c}}{\sqrt{a}}+B\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{Bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] (B*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) -
 (a^(1/4)*B*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (
2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) +
(a^(1/4)*(B + (A*Sqrt[c])/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[
(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sq
rt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 33.4146, size = 257, normalized size = 0.91

$$-\frac{B\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{c^{\frac{3}{4}}\sqrt{a+bx^2+cx^4}}+\frac{Bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$+\frac{\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(A\sqrt{c}+B\sqrt{a}\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2\sqrt[4]{a}c^{\frac{3}{4}}\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -B*a**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**
2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)
), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(c**(3/4)*sqrt(a + b*x**2 + c*x**
4)) + B*x*sqrt(a + b*x**2 + c*x**4)/(sqrt(c)*(sqrt(a) + sqrt(c)*x
**2)) + sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(
sqrt(a) + sqrt(c)*x**2)*(A*sqrt(c) + B*sqrt(a))*elliptic_f(2*atan
(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(2*a**(1/4)*c
**(3/4)*sqrt(a + b*x**2 + c*x**4))
```

Mathematica [C] time = 0.394668, size = 302, normalized size = 1.07

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+B\left(\sqrt{b^2-4ac}\right.\right.}{2\sqrt{2}c\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(B*(-b + Sqrt[b
^2 - 4*a*c])*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (b
*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*EllipticF[I*ArcSinh[Sqrt[2]*Sqr
t[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqr
t[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqr
t[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.007, size = 362, normalized size = 1.3

$$\frac{A\sqrt{2}}{4}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^{2}}\right)x^{2}}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^{2}}\right)x^{2}}{a}}EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^{2}}\right)},\frac{1}{2}\sqrt{-4+2\,\frac{b\left(b\right.}{}}\right.$$

$$-\frac{Ba\sqrt{2}}{2}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^{2}}\right)x^{2}}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^{2}}\right)x^{2}}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^{2}}\right)},\frac{1}{2}\sqrt{-4+2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*A*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+
b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/
2*B*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+
b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^
(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^
2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(
1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{Bx^{2}+A}{\sqrt{cx^{4}+bx^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Bx^{2}+A}{\sqrt{cx^{4}+bx^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.179 $\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=312

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B+A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}} - \frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\sqrt{a+bx^2+cx^4}} - \frac{A\sqrt{a+bx^2+cx^4}}{ax} + \frac{A\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[Out] -((A*Sqrt[a + b*x^2 + c*x^4])/(a*x)) + (A*Sqrt[c]*x*Sqrt[a + b*x^
2 + c*x^4])/(a*(Sqrt[a] + Sqrt[c]*x^2)) - (A*c^(1/4)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/(a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + ((Sqrt[a]*B + A*Sqrt[c])
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt
[a]*Sqrt[c]))/4])/(2*a^(3/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.343544, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B+A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}} - \frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\sqrt{a+bx^2+cx^4}} - \frac{A\sqrt{a+bx^2+cx^4}}{ax} + \frac{A\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2 + c*x^4])/(a*x)) + (A*Sqrt[c]*x*Sqrt[a + b*x^
2 + c*x^4])/(a*(Sqrt[a] + Sqrt[c]*x^2)) - (A*c^(1/4)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
```

```
/4])/(a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + ((Sqrt[a]*B + A*Sqrt[c])
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt
[a]*Sqrt[c]))/4])/(2*a^(3/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 50.283, size = 279, normalized size = 0.89

$$\frac{A\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{A\sqrt{a+bx^2+cx^4}}{ax}-\frac{A\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{a^{\frac{3}{4}}\sqrt{a+bx^2+cx^4}}+\frac{\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(A\sqrt{c}+B\sqrt{a}\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2a^{\frac{3}{4}}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(c)*x*sqrt(a + b*x**2 + c*x**4)/(a*(sqrt(a) + sqrt(c)*x**2)
) - A*sqrt(a + b*x**2 + c*x**4)/(a*x) - A*c**(1/4)*sqrt((a + b*x*
*2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2
)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt
(c)))/(a**(3/4)*sqrt(a + b*x**2 + c*x**4)) + sqrt((a + b*x**2 + c
*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(A*s
qrt(c) + B*sqrt(a))*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 -
 b/(4*sqrt(a)*sqrt(c)))/(2*a**(3/4)*c**(1/4)*sqrt(a + b*x**2 + c*
x**4))
```

Mathematica [C] time = 1.83518, size = 448, normalized size = 1.44

$$\frac{-ix\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\left(A\left(\sqrt{b^2-4ac}-b\right)+2aB\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+iAx\left(\sqrt{\right.}{4ax\sqrt{\frac{c}{\sqrt{b^2-4ac}}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-4*A*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4) + I*A*(
-b + Sqrt[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/
```

```
(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^
2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
 + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
- 4*a*c])] - I*(2*a*B + A*(-b + Sqrt[b^2 - 4*a*c]))*x*Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b -
2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[c/(b + Sqrt[b^2
 - 4*a*c])]*x*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.012, size = 386, normalized size = 1.2

$$\frac{B\sqrt{2}}{4}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\,EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\,\frac{b\left(b\right.}{}}\right.$$

$$+A\left(-\frac{1}{ax}\sqrt{cx^4+bx^2+a}\right.$$

$$-\frac{c\sqrt{2}}{2}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\,\frac{b}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*B*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+
b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+A*
(-(c*x^4+b*x^2+a)^(1/2)/a/x-1/2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2
)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c
+b^2)^(1/2))/a/c)^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{x^2\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**2*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^2),x, algorithm="giac")

[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

3.180 $\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=376

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}A\sqrt{c}-3aB+2Ab\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)(2Ab-3aB)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x}-\frac{\sqrt{c}x(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3}$$

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(3*a*x^3) + ((2*A*b - 3*a*B)*Sqrt[a
+ b*x^2 + c*x^4])/(3*a^2*x) - ((2*A*b - 3*a*B)*Sqrt[c]*x*Sqrt[a +
 b*x^2 + c*x^4])/(3*a^2*(Sqrt[a] + Sqrt[c]*x^2)) + ((2*A*b - 3*a*
B)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt
[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2
 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*a^(7/4)*Sqrt[a + b*x^2 + c*x^4]) -
 ((2*A*b - 3*a*B + Sqrt[a]*A*Sqrt[c])*c^(1/4)*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6
*a^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.563975, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}A\sqrt{c}-3aB+2Ab\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)(2Ab-3aB)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x}-\frac{\sqrt{c}x(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(A + B*x^2)/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(3*a*x^3) + ((2*A*b - 3*a*B)*Sqrt[a
+ b*x^2 + c*x^4])/(3*a^2*x) - ((2*A*b - 3*a*B)*Sqrt[c]*x*Sqrt[a +
```

```
 b*x^2 + c*x^4])/(3*a^2*(Sqrt[a] + Sqrt[c]*x^2)) + ((2*A*b - 3*a*
B)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt
[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2
 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*a^(7/4)*Sqrt[a + b*x^2 + c*x^4]) -
 ((2*A*b - 3*a*B + Sqrt[a]*A*Sqrt[c])*c^(1/4)*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6
*a^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 74.2805, size = 345, normalized size = 0.92

$$-\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3}-\frac{\sqrt{c}x\left(2Ab-3Ba\right)\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\left(2Ab-3Ba\right)\sqrt{a+bx^2+cx^4}}{3a^2x}+\frac{\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(2Ab-3Ba\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{3a^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(A\sqrt{a}\sqrt{c}+2Ab-3Ba\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{6a^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(a + b*x**2 + c*x**4)/(3*a*x**3) - sqrt(c)*x*(2*A*b - 3*B*
a)*sqrt(a + b*x**2 + c*x**4)/(3*a**2*(sqrt(a) + sqrt(c)*x**2)) +
(2*A*b - 3*B*a)*sqrt(a + b*x**2 + c*x**4)/(3*a**2*x) + c**(1/4)*s
qrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) +
 sqrt(c)*x**2)*(2*A*b - 3*B*a)*elliptic_e(2*atan(c**(1/4)*x/a**(1
/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(3*a**(7/4)*sqrt(a + b*x**2 +
c*x**4)) - c**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)
*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(A*sqrt(a)*sqrt(c) + 2*A*b -
3*B*a)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)
*sqrt(c)))/(6*a**(7/4)*sqrt(a + b*x**2 + c*x**4))
```

Mathematica [C] time = 1.16413, size = 373, normalized size = 0.99

$$\frac{-\frac{4\left(a+bx^2+cx^4\right)\left(a\left(A+3Bx^2\right)-2Abx^2\right)}{x^3}+\frac{i\sqrt{2}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(\left(2A\left(b\sqrt{b^2-4ac}+ac-b^2\right)+3aB\left(b-\sqrt{b^2-4ac}\right)\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right.\right.\right.}{\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}}}{12a^2\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(A + B*x^2)/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-4*(a + b*x^2 + c*x^4)*(-2*A*b*x^2 + a*(A + 3*B*x^2)))/x^3 + (I
*Sqrt[2]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(-((2*A*b - 3
*a*B)*(-b + Sqrt[b^2 - 4*a*c])*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c
/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b
^2 - 4*a*c])]) + (3*a*B*(b - Sqrt[b^2 - 4*a*c]) + 2*A*(-b^2 + a*c
 + b*Sqrt[b^2 - 4*a*c]))*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b +
Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4
*a*c])]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(12*a^2*Sqrt[a + b*x^2
 + c*x^4])
```

Maple [A] time = 0.013, size = 656, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] A*(-1/3*(c*x^4+b*x^2+a)^(1/2)/a/x^3+2/3*b*(c*x^4+b*x^2+a)^(1/2)/a
^2/x-1/12*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+
(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^
2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(
1/2))+1/3*b*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Elliptic
F(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+
(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^
(1/2))))+B*(-(c*x^4+b*x^2+a)^(1/2)/a/x-1/2*c*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(
4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+
(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Elli
pticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + ax^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^4),x, algorithm="maxima")

[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^4),x, algorithm="fricas")

[Out] integral((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{x^4\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral((A + B*x**2)/(x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^4),x, algorithm="giac")

[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

3.181 $\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=98

$$-\frac{89}{48}\sqrt{x^4+5x^2+3}x^4-\frac{1}{384}\left(24243-3802x^2\right)\sqrt{x^4+5x^2+3}+\frac{32801}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)+\frac{3}{8}\sqrt{x^4+5x^2+3}x^6$$

```
[Out] (-89*x^4*Sqrt[3 + 5*x^2 + x^4])/48 + (3*x^6*Sqrt[3 + 5*x^2 + x^4]
)/8 - ((24243 - 3802*x^2)*Sqrt[3 + 5*x^2 + x^4])/384 + (32801*Arc
Tanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/256
```

Rubi [A] time = 0.247569, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{89}{48}\sqrt{x^4+5x^2+3}x^4-\frac{1}{384}\left(24243-3802x^2\right)\sqrt{x^4+5x^2+3}+\frac{32801}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)+\frac{3}{8}\sqrt{x^4+5x^2+3}x^6$$

Antiderivative was successfully verified.

```
[In]  Int[(x^7*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-89*x^4*Sqrt[3 + 5*x^2 + x^4])/48 + (3*x^6*Sqrt[3 + 5*x^2 + x^4]
)/8 - ((24243 - 3802*x^2)*Sqrt[3 + 5*x^2 + x^4])/384 + (32801*Arc
Tanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/256
```

Rubi in Sympy [A] time = 26.0606, size = 92, normalized size = 0.94

$$\frac{3x^6\sqrt{x^4+5x^2+3}}{8}-\frac{89x^4\sqrt{x^4+5x^2+3}}{48}-\frac{\left(-\frac{1901x^2}{4}+\frac{24243}{8}\right)\sqrt{x^4+5x^2+3}}{48}+\frac{32801\,\mathrm{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] 3*x**6*sqrt(x**4 + 5*x**2 + 3)/8 - 89*x**4*sqrt(x**4 + 5*x**2 + 3
)/48 - (-1901*x**2/4 + 24243/8)*sqrt(x**4 + 5*x**2 + 3)/48 + 3280
```

```
1*atanh((2*x**2 + 5)/(2*sqrt(x**4 + 5*x**2 + 3)))/256
```

Mathematica [A] time = 0.0527585, size = 64, normalized size = 0.65

$$\frac{1}{768}\left(98403\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)+2\sqrt{x^4+5x^2+3}\left(144x^6-712x^4+3802x^2-24243\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^7*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(-24243 + 3802*x^2 - 712*x^4 + 144*x^6)
+ 98403*Log[5 + 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/768
```

Maple [A] time = 0.027, size = 87, normalized size = 0.9

$$-\frac{89\,x^4}{48}\sqrt{x^4+5\,x^2+3}+\frac{1901\,x^2}{192}\sqrt{x^4+5\,x^2+3}-\frac{8081}{128}\sqrt{x^4+5\,x^2+3}+\frac{32801}{256}\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5\,x^2+3}\right)+\frac{3\,x^6}{8}\sqrt{x^4+5\,x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -89/48*x^4*(x^4+5*x^2+3)^(1/2)+1901/192*x^2*(x^4+5*x^2+3)^(1/2)-8
081/128*(x^4+5*x^2+3)^(1/2)+32801/256*ln(x^2+5/2+(x^4+5*x^2+3)^(1
/2))+3/8*x^6*(x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 0.706507, size = 122, normalized size = 1.24

$$\frac{3}{8}\sqrt{x^4+5\,x^2+3}x^6-\frac{89}{48}\sqrt{x^4+5\,x^2+3}x^4+\frac{1901}{192}\sqrt{x^4+5\,x^2+3}x^2-\frac{8081}{128}\sqrt{x^4+5\,x^2+3}+\frac{32801}{256}\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)*x^7/sqrt(x^4 + 5*x^2 + 3),x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(x^4 + 5*x^2 + 3)*x^6 - 89/48*sqrt(x^4 + 5*x^2 + 3)*x^4 +
 1901/192*sqrt(x^4 + 5*x^2 + 3)*x^2 - 8081/128*sqrt(x^4 + 5*x^2 +
 3) + 32801/256*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 0.27254, size = 306, normalized size = 3.12

$$-\frac{294912\,x^{16} + 2228224\,x^{14} + 6553600\,x^{12} - 1048576\,x^{10} - 233473664\,x^{8} - 1270350080\,x^{6} - 2376291488\,x^{4} - 1415183008\,x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)*x^7/sqrt(x^4 + 5*x^2 + 3),x, algorithm="fricas")
```

```
[Out] -1/6144*(294912*x^16 + 2228224*x^14 + 6553600*x^12 - 1048576*x^10
 - 233473664*x^8 - 1270350080*x^6 - 2376291488*x^4 - 1415183008*x
^2 + 787224*(128*x^8 + 1280*x^6 + 4384*x^4 + 5920*x^2 - 8*(16*x^6
 + 120*x^4 + 274*x^2 + 185)*sqrt(x^4 + 5*x^2 + 3) + 2569)*log(-2*
x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5) - 8*(36864*x^14 + 186368*x^12
+ 413184*x^10 - 1010944*x^8 - 26319472*x^6 - 95478600*x^4 - 94593
374*x^2 - 7815359)*sqrt(x^4 + 5*x^2 + 3) - 101042425)/(128*x^8 +
1280*x^6 + 4384*x^4 + 5920*x^2 - 8*(16*x^6 + 120*x^4 + 274*x^2 +
185)*sqrt(x^4 + 5*x^2 + 3) + 2569)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{7}\left(3x^{2}+2\right)}{\sqrt{x^{4}+5x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**7*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

GIAC/XCAS [A] time = 0.282435, size = 81, normalized size = 0.83

$$\frac{1}{384}\,\sqrt{x^{4}+5\,x^{2}+3}\left(2\left(4\left(18\,x^{2}-89\right)x^{2}+1901\right)x^{2}-24243\right)-\frac{32801}{256}\,\ln\left(2\,x^{2}-2\,\sqrt{x^{4}+5\,x^{2}+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^7/sqrt(x^4 + 5*x^2 + 3),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(18*x^2 - 89)*x^2 + 1901)*x^2 -
 24243) - 32801/256*ln(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.182 $\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=77

$$\frac{1}{2}\sqrt{x^4+5x^2+3}x^4+\frac{3}{16}\left(89-14x^2\right)\sqrt{x^4+5x^2+3}-\frac{1083}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] (x^4*Sqrt[3 + 5*x^2 + x^4])/2 + (3*(89 - 14*x^2)*Sqrt[3 + 5*x^2 +
 x^4])/16 - (1083*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])
/32
```

Rubi [A] time = 0.183952, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{1}{2}\sqrt{x^4+5x^2+3}x^4+\frac{3}{16}\left(89-14x^2\right)\sqrt{x^4+5x^2+3}-\frac{1083}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (x^4*Sqrt[3 + 5*x^2 + x^4])/2 + (3*(89 - 14*x^2)*Sqrt[3 + 5*x^2 +
 x^4])/16 - (1083*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])
/32
```

Rubi in Sympy [A] time = 19.4908, size = 70, normalized size = 0.91

$$\frac{x^4\sqrt{x^4+5x^2+3}}{2}+\frac{\left(-\frac{63x^2}{2}+\frac{801}{4}\right)\sqrt{x^4+5x^2+3}}{12}-\frac{1083\,\text{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] x**4*sqrt(x**4 + 5*x**2 + 3)/2 + (-63*x**2/2 + 801/4)*sqrt(x**4 +
 5*x**2 + 3)/12 - 1083*atanh((2*x**2 + 5)/(2*sqrt(x**4 + 5*x**2 +
 3)))/32
```

Mathematica [A] time = 0.0409706, size = 61, normalized size = 0.79

$$\frac{1}{2}\left(x^4-\frac{21x^2}{4}+\frac{267}{8}\right)\sqrt{x^4+5x^2+3}-\frac{1083}{32}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] ((267/8 - (21*x^2)/4 + x^4)*Sqrt[3 + 5*x^2 + x^4])/2 - (1083*Log[
5 + 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/32
```

Maple [A] time = 0.017, size = 70, normalized size = 0.9

$$-\frac{21\,x^2}{8}\sqrt{x^4+5\,x^2+3}+\frac{267}{16}\sqrt{x^4+5\,x^2+3}-\frac{1083}{32}\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5\,x^2+3}\right)+\frac{x^4}{2}\sqrt{x^4+5\,x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -21/8*x^2*(x^4+5*x^2+3)^(1/2)+267/16*(x^4+5*x^2+3)^(1/2)-1083/32*
ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+1/2*x^4*(x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 0.702596, size = 99, normalized size = 1.29

$$\frac{1}{2}\sqrt{x^4+5\,x^2+3}x^4-\frac{21}{8}\sqrt{x^4+5\,x^2+3}x^2+\frac{267}{16}\sqrt{x^4+5\,x^2+3}-\frac{1083}{32}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^5/sqrt(x^4 + 5*x^2 + 3),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^4 + 5*x^2 + 3)*x^4 - 21/8*sqrt(x^4 + 5*x^2 + 3)*x^2 +
267/16*sqrt(x^4 + 5*x^2 + 3) - 1083/32*log(2*x^2 + 2*sqrt(x^4 + 5
*x^2 + 3) + 5)
```

Fricas [A] time = 0.266837, size = 252, normalized size = 3.27

$$-\frac{2048\,x^{12}+9728\,x^{10}+29312\,x^{8}+302080\,x^{6}+1052928\,x^{4}+941238\,x^{2}-4332\left(32\,x^{6}+240\,x^{4}+522\,x^{2}-2\left(16\,x^{4}+80\,x^{2}+\right.\right.}{128\left(32\,x^{6}+240\,x^{4}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)*x^5/sqrt(x^4 + 5*x^2 + 3),x, algorithm="fricas")
```

```
[Out] -1/128*(2048*x^12 + 9728*x^10 + 29312*x^8 + 302080*x^6 + 1052928*
x^4 + 941238*x^2 - 4332*(32*x^6 + 240*x^4 + 522*x^2 - 2*(16*x^4 +
 80*x^2 + 87)*sqrt(x^4 + 5*x^2 + 3) + 305)*log(-2*x^2 + 2*sqrt(x^
4 + 5*x^2 + 3) - 5) - 2*(1024*x^10 + 2304*x^8 + 10560*x^6 + 12422
4*x^4 + 235456*x^2 + 31245)*sqrt(x^4 + 5*x^2 + 3) + 82567)/(32*x^
6 + 240*x^4 + 522*x^2 - 2*(16*x^4 + 80*x^2 + 87)*sqrt(x^4 + 5*x^2
 + 3) + 305)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

GIAC/XCAS [A] time = 0.283432, size = 72, normalized size = 0.94

$$\frac{1}{16}\,\sqrt{x^4+5\,x^2+3}\left(2\left(4\,x^2-21\right)x^2+267\right)+\frac{1083}{32}\,\ln\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)*x^5/sqrt(x^4 + 5*x^2 + 3),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 - 21)*x^2 + 267) + 1083/32*l
n(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.183 $\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=56

$$\frac{149}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{1}{8}\left(37-6x^2\right)\sqrt{x^4+5x^2+3}$$

```
[Out] -((37 - 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + (149*ArcTanh[(5 + 2*x^2
)/(2*Sqrt[3 + 5*x^2 + x^4])])/16
```

Rubi [A] time = 0.12406, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$\frac{149}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{1}{8}\left(37-6x^2\right)\sqrt{x^4+5x^2+3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] -((37 - 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + (149*ArcTanh[(5 + 2*x^2
)/(2*Sqrt[3 + 5*x^2 + x^4])])/16
```

Rubi in Sympy [A] time = 14.4906, size = 49, normalized size = 0.88

$$-\frac{\left(-3x^2+\frac{37}{2}\right)\sqrt{x^4+5x^2+3}}{4}+\frac{149\,\mathrm{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] -(-3*x**2 + 37/2)*sqrt(x**4 + 5*x**2 + 3)/4 + 149*atanh((2*x**2 +
 5)/(2*sqrt(x**4 + 5*x**2 + 3)))/16
```

Mathematica [A] time = 0.025046, size = 54, normalized size = 0.96

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\left(6x^2-37\right)+\frac{149}{16}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] ((-37 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + (149*Log[5 + 2*x^2 + 2*
Sqrt[3 + 5*x^2 + x^4]])/16
```

Maple [A] time = 0.017, size = 53, normalized size = 1.

$$-\frac{37}{8}\sqrt{x^4+5x^2+3}+\frac{149}{16}\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)+\frac{3x^2}{4}\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -37/8*(x^4+5*x^2+3)^(1/2)+149/16*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+
3/4*x^2*(x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 0.703414, size = 76, normalized size = 1.36

$$\frac{3}{4}\sqrt{x^4+5x^2+3}x^2-\frac{37}{8}\sqrt{x^4+5x^2+3}+\frac{149}{16}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)*x^3/sqrt(x^4 + 5*x^2 + 3),x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(x^4 + 5*x^2 + 3)*x^2 - 37/8*sqrt(x^4 + 5*x^2 + 3) + 149/
16*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 0.271298, size = 198, normalized size = 3.54

$$-\frac{384x^8+512x^6-8248x^4-16024x^2+596\left(8x^4+40x^2-4\sqrt{x^4+5x^2+3}\left(2x^2+5\right)+37\right)\log\left(-2x^2+2\sqrt{x^4+5x^2+3}\right.}{64\left(8x^4+40x^2-4\sqrt{x^4+5x^2+3}(2x^2+5)+37\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)*x^3/sqrt(x^4 + 5*x^2 + 3),x, algorithm="fricas")
```

```
[Out] -1/64*(384*x^8 + 512*x^6 - 8248*x^4 - 16024*x^2 + 596*(8*x^4 + 40
*x^2 - 4*sqrt(x^4 + 5*x^2 + 3)*(2*x^2 + 5) + 37)*log(-2*x^2 + 2*s
qrt(x^4 + 5*x^2 + 3) - 5) - 4*(96*x^6 - 112*x^4 - 1626*x^2 - 513)
*sqrt(x^4 + 5*x^2 + 3) - 1295)/(8*x^4 + 40*x^2 - 4*sqrt(x^4 + 5*x
^2 + 3)*(2*x^2 + 5) + 37)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

GIAC/XCAS [A] time = 0.277405, size = 62, normalized size = 1.11

$$\frac{1}{8}\sqrt{x^4+5\,x^2+3}\left(6\,x^2-37\right)-\frac{149}{16}\,\ln\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)*x^3/sqrt(x^4 + 5*x^2 + 3),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 - 37) - 149/16*ln(2*x^2 - 2*sqrt
(x^4 + 5*x^2 + 3) + 5)
```

3.184 $\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=49

$$\frac{3}{2}\sqrt{x^4+5x^2+3}-\frac{11}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] (3*Sqrt[3 + 5*x^2 + x^4])/2 - (11*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 +
 5*x^2 + x^4])])/4
```

Rubi [A] time = 0.0813637, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$\frac{3}{2}\sqrt{x^4+5x^2+3}-\frac{11}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (3*Sqrt[3 + 5*x^2 + x^4])/2 - (11*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 +
 5*x^2 + x^4])])/4
```

Rubi in Sympy [A] time = 12.2295, size = 42, normalized size = 0.86

$$\frac{3\sqrt{x^4+5x^2+3}}{2}-\frac{11\,\mathrm{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] 3*sqrt(x**4 + 5*x**2 + 3)/2 - 11*atanh((2*x**2 + 5)/(2*sqrt(x**4
+ 5*x**2 + 3)))/4
```

Mathematica [A] time = 0.0205119, size = 47, normalized size = 0.96

$$\frac{3}{2}\sqrt{x^4+5x^2+3}-\frac{11}{4}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (3*Sqrt[3 + 5*x^2 + x^4])/2 - (11*Log[5 + 2*x^2 + 2*Sqrt[3 + 5*x^
2 + x^4]])/4
```

Maple [A] time = 0.015, size = 36, normalized size = 0.7

$$-\frac{11}{4}\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)+\frac{3}{2}\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -11/4*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+3/2*(x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 0.698365, size = 53, normalized size = 1.08

$$\frac{3}{2}\sqrt{x^4+5x^2+3}-\frac{11}{4}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x/sqrt(x^4 + 5*x^2 + 3),x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) - 11/4*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 +
 3) + 5)
```

Fricas [A] time = 0.266032, size = 139, normalized size = 2.84

$$-\frac{24x^4+90x^2-22\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)-6\sqrt{x^4+5x^2+3}\left(4x^2+5\right)-3}{8\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x/sqrt(x^4 + 5*x^2 + 3),x, algorithm="fricas")
```

```
[Out] -1/8*(24*x^4 + 90*x^2 - 22*(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)*
log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5) - 6*sqrt(x^4 + 5*x^2 +
3)*(4*x^2 + 5) - 3)/(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

GIAC/XCAS [A] time = 0.277136, size = 53, normalized size = 1.08

$$\frac{3}{2}\sqrt{x^4+5\,x^2+3}+\frac{11}{4}\ln\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x/sqrt(x^4 + 5*x^2 + 3),x, algorithm="giac")
```

```
[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) + 11/4*ln(2*x^2 - 2*sqrt(x^4 + 5*x^2 +
3) + 5)
```

3.185 $\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=69

$$\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

```
[Out] (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2 - ArcTanh[(6
 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/Sqrt[3]
```

Rubi [A] time = 0.145927, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(x*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2 - ArcTanh[(6
 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/Sqrt[3]
```

Rubi in Sympy [A] time = 16.8047, size = 61, normalized size = 0.88

$$\frac{3\,\mathrm{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{2}-\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}(5x^2+6)}{6\sqrt{x^4+5x^2+3}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/x/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] 3*atanh((2*x**2 + 5)/(2*sqrt(x**4 + 5*x**2 + 3)))/2 - sqrt(3)*ata
nh(sqrt(3)*(5*x**2 + 6)/(6*sqrt(x**4 + 5*x**2 + 3)))/3
```

Mathematica [A] time = 0.112107, size = 78, normalized size = 1.13

$$\frac{3}{2}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)+2\left(\frac{\log(x)}{\sqrt{3}}-\frac{\log\left(5x^2+2\sqrt{3}\sqrt{x^4+5x^2+3}+6\right)}{2\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)/(x*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (3*Log[5 + 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/2 + 2*(Log[x]/Sqrt[3
] - Log[6 + 5*x^2 + 2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]]/(2*Sqrt[3]))
```

Maple [A] time = 0.016, size = 52, normalized size = 0.8

$$-\frac{\sqrt{3}}{3}\mathit{Artanh}\left(\frac{(5x^2+6)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5x^2+3}}\right)+\frac{3}{2}\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -1/3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+3
/2*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))
```

Maxima [A] time = 0.780192, size = 78, normalized size = 1.13

$$-\frac{1}{3}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{3}{2}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5)
 + 3/2*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 0.280924, size = 151, normalized size = 2.19

$$-\frac{1}{6}\sqrt{3}\left(3\sqrt{3}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)-2\log\left(\frac{6x^2+\sqrt{3}\left(2x^4+5x^2+6\right)-2\sqrt{x^4+5x^2+3}\left(\sqrt{3}x^2+3\right)}{2x^4-2\sqrt{x^4+5x^2+3}x^2+5x^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*(3*sqrt(3)*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
 - 2*log((6*x^2 + sqrt(3)*(2*x^4 + 5*x^2 + 6) - 2*sqrt(x^4 + 5*x^
2 + 3)*(sqrt(3)*x^2 + 3))/(2*x^4 - 2*sqrt(x^4 + 5*x^2 + 3)*x^2 +
5*x^2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{x\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)/x/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x*sqrt(x**4 + 5*x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{\sqrt{x^4+5x^2+3}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x), x)
```

3.186 $\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{x^4+5x^2+3}}{3x^2}-\frac{2\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(3*x^2) - (2*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3
]*Sqrt[3 + 5*x^2 + x^4])])/(3*Sqrt[3])
```

Rubi [A] time = 0.135296, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$-\frac{\sqrt{x^4+5x^2+3}}{3x^2}-\frac{2\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(x^3*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(3*x^2) - (2*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3
]*Sqrt[3 + 5*x^2 + x^4])])/(3*Sqrt[3])
```

Rubi in Sympy [A] time = 16.2681, size = 56, normalized size = 0.9

$$-\frac{2\sqrt{3}\operatorname{atanh}\left(\frac{\sqrt{3}(5x^2+6)}{6\sqrt{x^4+5x^2+3}}\right)}{9}-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/x**3/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] -2*sqrt(3)*atanh(sqrt(3)*(5*x**2 + 6)/(6*sqrt(x**4 + 5*x**2 + 3))
)/9 - sqrt(x**4 + 5*x**2 + 3)/(3*x**2)
```

Mathematica [A] time = 0.0673446, size = 67, normalized size = 1.08

$$\frac{2\left(\log\left(x^2\right)-\log\left(5x^2+2\sqrt{3}\sqrt{x^4+5x^2+3}+6\right)\right)}{3\sqrt{3}}-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)/(x^3*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(3*x^2) + (2*(Log[x^2] - Log[6 + 5*x^2 + 2
*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]]))/(3*Sqrt[3])
```

Maple [A] time = 0.021, size = 49, normalized size = 0.8

$$-\frac{2\sqrt{3}}{9}Artanh\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5x^2+3}}\right)-\frac{1}{3x^2}\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -2/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1
/3*(x^4+5*x^2+3)^(1/2)/x^2
```

Maxima [A] time = 0.771372, size = 69, normalized size = 1.11

$$-\frac{2}{9}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^3),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5)
 - 1/3*sqrt(x^4 + 5*x^2 + 3)/x^2
```

Fricas [A] time = 0.26806, size = 239, normalized size = 3.85

$$-\frac{2\left(2x^4-2\sqrt{x^4+5x^2+3}x^2+5x^2\right)\log\left(\frac{6x^2+\sqrt{3}\left(2x^4+5x^2+6\right)-2\sqrt{x^4+5x^2+3}\left(\sqrt{3}x^2+3\right)}{2x^4-2\sqrt{x^4+5x^2+3}x^2+5x^2}\right)+\sqrt{3}\left(5x^2+6\right)-5\sqrt{3}\sqrt{x^4+5x^2+3}}{3\left(2\sqrt{3}\sqrt{x^4+5x^2+3}x^2-\sqrt{3}\left(2x^4+5x^2\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^3),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(2*x^4 - 2*sqrt(x^4 + 5*x^2 + 3)*x^2 + 5*x^2)*log((6*x^2
+ sqrt(3)*(2*x^4 + 5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(sqrt(3)*
x^2 + 3))/(2*x^4 - 2*sqrt(x^4 + 5*x^2 + 3)*x^2 + 5*x^2)) + sqrt(3
)*(5*x^2 + 6) - 5*sqrt(3)*sqrt(x^4 + 5*x^2 + 3))/(2*sqrt(3)*sqrt(
x^4 + 5*x^2 + 3)*x^2 - sqrt(3)*(2*x^4 + 5*x^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2 + 2}{x^3\sqrt{x^4 + 5x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**3/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**3*sqrt(x**4 + 5*x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2 + 2}{\sqrt{x^4 + 5\,x^2 + 3}x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^3),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^3), x)
```

3.187 $\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=83

$$-\frac{\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}+\frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(6*x^4) - Sqrt[3 + 5*x^2 + x^4]/(12*x^2) +
 (Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])
/8
```

Rubi [A] time = 0.186838, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}+\frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(x^5*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(6*x^4) - Sqrt[3 + 5*x^2 + x^4]/(12*x^2) +
 (Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])
/8
```

Rubi in Sympy [A] time = 20.3406, size = 71, normalized size = 0.86

$$\frac{\sqrt{3}\operatorname{atanh}\left(\frac{\sqrt{3}(5x^2+6)}{6\sqrt{x^4+5x^2+3}}\right)}{8}-\frac{\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/x**5/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] sqrt(3)*atanh(sqrt(3)*(5*x**2 + 6)/(6*sqrt(x**4 + 5*x**2 + 3)))/8
 - sqrt(x**4 + 5*x**2 + 3)/(12*x**2) - sqrt(x**4 + 5*x**2 + 3)/(6
*x**4)
```

Mathematica [A] time = 0.0896247, size = 72, normalized size = 0.87

$$\frac{1}{8}\sqrt{3}\left(\log\left(5x^2+2\sqrt{3}\sqrt{x^4+5x^2+3}+6\right)-\log\left(x^2\right)\right)-\frac{\left(x^2+2\right)\sqrt{x^4+5x^2+3}}{12x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)/(x^5*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -((2 + x^2)*Sqrt[3 + 5*x^2 + x^4])/(12*x^4) + (Sqrt[3]*(-Log[x^2]
 + Log[6 + 5*x^2 + 2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]]))/8
```

Maple [A] time = 0.019, size = 66, normalized size = 0.8

$$\frac{\sqrt{3}}{8}Artanh\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5x^2+3}}\right)-\frac{1}{6x^4}\sqrt{x^4+5x^2+3}-\frac{1}{12x^2}\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 1/8*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/
6*(x^4+5*x^2+3)^(1/2)/x^4-1/12*(x^4+5*x^2+3)^(1/2)/x^2
```

Maxima [A] time = 0.780186, size = 92, normalized size = 1.11

$$\frac{1}{8}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^5),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5)
- 1/12*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/6*sqrt(x^4 + 5*x^2 + 3)/x^4
```

Fricas [A] time = 0.267834, size = 290, normalized size = 3.49

$$\frac{72\,x^6+414\,x^4+616\,x^2-3\left(4\,\sqrt{3}\left(2\,x^6+5\,x^4\right)\sqrt{x^4+5\,x^2+3}-\sqrt{3}\left(8\,x^8+40\,x^6+37\,x^4\right)\right)\log\left(\frac{2\,x^4-2\,\sqrt{3}x^2+5\,x^2-2\,\sqrt{x^4+5\,x^2+3}\left(x^2\right.}{2\,x^4-2\,\sqrt{x^4+5\,x^2+3}x^2+5\,x^2}\right.}{24\left(8\,x^8+40\,x^6+37\,x^4-4\left(2\,x^6+5\,x^4\right)\sqrt{x^4+5\,x^2+3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^5),x, algorithm="fricas")
```

```
[Out] 1/24*(72*x^6 + 414*x^4 + 616*x^2 - 3*(4*sqrt(3)*(2*x^6 + 5*x^4)*s
qrt(x^4 + 5*x^2 + 3) - sqrt(3)*(8*x^8 + 40*x^6 + 37*x^4))*log((2*
x^4 - 2*sqrt(3)*x^2 + 5*x^2 - 2*sqrt(x^4 + 5*x^2 + 3)*(x^2 - sqrt
(3)) + 6)/(2*x^4 - 2*sqrt(x^4 + 5*x^2 + 3)*x^2 + 5*x^2)) - 2*(36*
x^4 + 117*x^2 + 74)*sqrt(x^4 + 5*x^2 + 3) + 240)/(8*x^8 + 40*x^6
+ 37*x^4 - 4*(2*x^6 + 5*x^4)*sqrt(x^4 + 5*x^2 + 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{x^5\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**5/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**5*sqrt(x**4 + 5*x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^5),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^5), x)
```

3.188 $\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=104

$$\frac{13\sqrt{x^4+5x^2+3}}{108x^2}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}-\frac{61\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{216\sqrt{3}}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}$$

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(9*x^6) - Sqrt[3 + 5*x^2 + x^4]/(54*x^4) +
 (13*Sqrt[3 + 5*x^2 + x^4])/(108*x^2) - (61*ArcTanh[(6 + 5*x^2)/(
2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(216*Sqrt[3])
```

Rubi [A] time = 0.238328, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{13\sqrt{x^4+5x^2+3}}{108x^2}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}-\frac{61\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{216\sqrt{3}}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(x^7*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(9*x^6) - Sqrt[3 + 5*x^2 + x^4]/(54*x^4) +
 (13*Sqrt[3 + 5*x^2 + x^4])/(108*x^2) - (61*ArcTanh[(6 + 5*x^2)/(
2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(216*Sqrt[3])
```

Rubi in Sympy [A] time = 24.956, size = 94, normalized size = 0.9

$$-\frac{61\sqrt{3}\operatorname{atanh}\left(\frac{\sqrt{3}(5x^2+6)}{6\sqrt{x^4+5x^2+3}}\right)}{648}+\frac{13\sqrt{x^4+5x^2+3}}{108x^2}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/x**7/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] -61*sqrt(3)*atanh(sqrt(3)*(5*x**2 + 6)/(6*sqrt(x**4 + 5*x**2 + 3)
))/648 + 13*sqrt(x**4 + 5*x**2 + 3)/(108*x**2) - sqrt(x**4 + 5*x*
*2 + 3)/(54*x**4) - sqrt(x**4 + 5*x**2 + 3)/(9*x**6)
```

Mathematica [A] time = 0.0944119, size = 83, normalized size = 0.8

$$\frac{61\left(\log\left(x^2\right)-\log\left(5x^2+2\sqrt{3}\sqrt{x^4+5x^2+3}+6\right)\right)}{216\sqrt{3}}+\sqrt{x^4+5x^2+3}\left(-\frac{1}{9x^6}-\frac{1}{54x^4}+\frac{13}{108x^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)/(x^7*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (-1/(9*x^6) - 1/(54*x^4) + 13/(108*x^2))*Sqrt[3 + 5*x^2 + x^4] +
(61*(Log[x^2] - Log[6 + 5*x^2 + 2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]])
)/(216*Sqrt[3])
```

Maple [A] time = 0.019, size = 83, normalized size = 0.8

$$-\frac{61\sqrt{3}}{648}\mathit{Artanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5x^2+3}}\right)-\frac{1}{9x^6}\sqrt{x^4+5x^2+3}-\frac{1}{54x^4}\sqrt{x^4+5x^2+3}+\frac{13}{108x^2}\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -61/648*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2
)-1/9*(x^4+5*x^2+3)^(1/2)/x^6-1/54*(x^4+5*x^2+3)^(1/2)/x^4+13/108
*(x^4+5*x^2+3)^(1/2)/x^2
```

Maxima [A] time = 0.782997, size = 115, normalized size = 1.11

$$-\frac{61}{648}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{13\sqrt{x^4+5x^2+3}}{108x^2}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^7),x, algorithm="maxima")
```

```
[Out] -61/648*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 +
 5) + 13/108*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/54*sqrt(x^4 + 5*x^2 +
3)/x^4 - 1/9*sqrt(x^4 + 5*x^2 + 3)/x^6
```

Fricas [A] time = 0.264431, size = 359, normalized size = 3.45

$$-\frac{2\,\sqrt{3}\left(976\,x^{8}+3660\,x^{6}+41\,x^{4}-6874\,x^{2}-3660\right)\sqrt{x^{4}+5\,x^{2}+3}+61\,\left(32\,x^{12}+240\,x^{10}+522\,x^{8}+305\,x^{6}-2\,\left(16\,x^{10}+80\,x\right.\right.}{216\,\left(2\,\sqrt{3}(16\,x^{10}+80\,x^{8}+87\,x^{6})\sqrt{x^{4}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^7),x, algorithm="fricas")
```

```
[Out] -1/216*(2*sqrt(3)*(976*x^8 + 3660*x^6 + 41*x^4 - 6874*x^2 - 3660)
*sqrt(x^4 + 5*x^2 + 3) + 61*(32*x^12 + 240*x^10 + 522*x^8 + 305*x
^6 - 2*(16*x^10 + 80*x^8 + 87*x^6)*sqrt(x^4 + 5*x^2 + 3))*log((6*
x^2 + sqrt(3)*(2*x^4 + 5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(sqrt
(3)*x^2 + 3))/(2*x^4 - 2*sqrt(x^4 + 5*x^2 + 3)*x^2 + 5*x^2)) - 2*
sqrt(3)*(976*x^10 + 6100*x^8 + 7605*x^6 - 8754*x^4 - 17244*x^2 -
6264))/(2*sqrt(3)*(16*x^10 + 80*x^8 + 87*x^6)*sqrt(x^4 + 5*x^2 +
3) - sqrt(3)*(32*x^12 + 240*x^10 + 522*x^8 + 305*x^6))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{x^7\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**7/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**7*sqrt(x**4 + 5*x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^7),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^7), x)
```

3.189 $\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=298

$$-\frac{10}{3}\sqrt{x^4+5x^2+3}x+\frac{419\left(2x^2+\sqrt{13}+5\right)x}{30\sqrt{x^4+5x^2+3}}+\frac{5\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{419\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{30\sqrt{x^4+5x^2+3}}+\frac{3}{5}\sqrt{x^4+5x^2+3}x^3$$

```
[Out] (419*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (10*x
*Sqrt[3 + 5*x^2 + x^4])/3 + (3*x^3*Sqrt[3 + 5*x^2 + x^4])/5 - (41
9*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5
 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(30*Sqrt[3 + 5*x^2 + x
^4]) + (5*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2
)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[Ar
cTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5
*x^2 + x^4]
```

Rubi [A] time = 0.484138, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$-\frac{10}{3}\sqrt{x^4+5x^2+3}x+\frac{419\left(2x^2+\sqrt{13}+5\right)x}{30\sqrt{x^4+5x^2+3}}+\frac{5\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{419\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{30\sqrt{x^4+5x^2+3}}+\frac{3}{5}\sqrt{x^4+5x^2+3}x^3$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (419*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (10*x
*Sqrt[3 + 5*x^2 + x^4])/3 + (3*x^3*Sqrt[3 + 5*x^2 + x^4])/5 - (41
9*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5
 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(30*Sqrt[3 + 5*x^2 + x
^4]) + (5*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2
)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[Ar
cTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5
*x^2 + x^4]
```

Rubi in Sympy [A] time = 34.415, size = 275, normalized size = 0.92

$$\frac{3x^3\sqrt{x^4+5x^2+3}}{5}+\frac{419x\left(2x^2+\sqrt{13}+5\right)}{30\sqrt{x^4+5x^2+3}}-\frac{10x\sqrt{x^4+5x^2+3}}{3}-\frac{419\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{180\sqrt{x^4+5x^2+3}}+\frac{5\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{3\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] 3*x**3*sqrt(x**4 + 5*x**2 + 3)/5 + 419*x*(2*x**2 + sqrt(13) + 5)/
(30*sqrt(x**4 + 5*x**2 + 3)) - 10*x*sqrt(x**4 + 5*x**2 + 3)/3 - 4
19*sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) +
 6))*sqrt(sqrt(13) + 5)*(x**2*(sqrt(13) + 5) + 6)*elliptic_e(atan
(sqrt(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(180*sqrt
(x**4 + 5*x**2 + 3)) + 5*sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/
(x**2*(sqrt(13) + 5) + 6))*(x**2*(sqrt(13) + 5) + 6)*elliptic_f(a
tan(sqrt(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(3*sqr
t(sqrt(13) + 5)*sqrt(x**4 + 5*x**2 + 3))
```

Mathematica [C] time = 0.527985, size = 229, normalized size = 0.77

$$\frac{-i\sqrt{2}\left(419\sqrt{13}-1795\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)\middle|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+419i\sqrt{2}\left(\sqrt{13}-5\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}}{60\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^4*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (4*x*(-150 - 223*x^2 - 5*x^4 + 9*x^6) + (419*I)*Sqrt[2]*(-5 + Sqr
t[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqr
t[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/
6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-1795 + 419*Sqrt[13])*Sqrt[(-5 +
 Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*El
lipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/
6])/(60*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.026, size = 226, normalized size = 0.8

$$-\frac{10\,x}{3}\sqrt{x^4+5\,x^2+3}$$
$$+60\,\frac{\sqrt{1-\left(-5/6+1/6\,\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\,\sqrt{13}\right)x^2}\,EllipticF\left(1/6\,x\sqrt{-30+6\,\sqrt{13}},5/6\,\sqrt{3}+1/6\,\sqrt{39}\right)}{\sqrt{-30+6\,\sqrt{13}}\sqrt{x^4+5\,x^2+3}}$$
$$-\frac{5028}{5\,\sqrt{-30+6\,\sqrt{13}}\left(5+\sqrt{13}\right)}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-30+6\,\sqrt{13}}}{6},\frac{5\,\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-Elli\right.$$
$$+\frac{3\,x^3}{5}\sqrt{x^4+5\,x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -10/3*x*(x^4+5*x^2+3)^(1/2)+60/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/
6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x
^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/
6*39^(1/2))-5028/5/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*
x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/
(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+
1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+
1/6*39^(1/2)))+3/5*x^3*(x^4+5*x^2+3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3\,x^2+2\right)x^4}{\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,x^6+2\,x^4}{\sqrt{x^4+5\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4)/sqrt(x^4 + 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4\left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**4*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(3\,x^2+2\right)x^4}{\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5*x^2 + 3), x)
```

3.190 $\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}} \, dx$

Optimal. Leaf size=270

$$\sqrt{x^4+5x^2+3}\,x-\frac{4\left(2x^2+\sqrt{13}+5\right)x}{\sqrt{x^4+5x^2+3}}-\frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}+\frac{2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}$$

```
[Out] (-4*x*(5 + Sqrt[13] + 2*x^2))/Sqrt[3 + 5*x^2 + x^4] + x*Sqrt[3 +
5*x^2 + x^4] + (2*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[
13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Elli
pticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sq
rt[3 + 5*x^2 + x^4] - (Sqrt[3/(2*(5 + Sqrt[13]))]*Sqrt[(6 + (5 -
Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)
*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi [A] time = 0.333043, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\sqrt{x^4+5x^2+3}\,x-\frac{4\left(2x^2+\sqrt{13}+5\right)x}{\sqrt{x^4+5x^2+3}}-\frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}+\frac{2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-4*x*(5 + Sqrt[13] + 2*x^2))/Sqrt[3 + 5*x^2 + x^4] + x*Sqrt[3 +
5*x^2 + x^4] + (2*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[
```

```
13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Elli
pticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sq
rt[3 + 5*x^2 + x^4] - (Sqrt[3/(2*(5 + Sqrt[13]))]*Sqrt[(6 + (5 -
Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)
*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi in Sympy [A] time = 26.4116, size = 248, normalized size = 0.92

$$-\frac{4x\left(2x^2+\sqrt{13}+5\right)}{\sqrt{x^4+5x^2+3}}+x\sqrt{x^4+5x^2+3}+\frac{2\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{3\sqrt{x^4+5x^2+3}}-\frac{\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{2\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] -4*x*(2*x**2 + sqrt(13) + 5)/sqrt(x**4 + 5*x**2 + 3) + x*sqrt(x**
4 + 5*x**2 + 3) + 2*sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2
*(sqrt(13) + 5) + 6))*sqrt(sqrt(13) + 5)*(x**2*(sqrt(13) + 5) + 6
)*elliptic_e(atan(sqrt(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt
(13)/6)/(3*sqrt(x**4 + 5*x**2 + 3)) - sqrt(6)*sqrt((x**2*(-sqrt(1
3) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6))*(x**2*(sqrt(13) + 5) + 6)
*elliptic_f(atan(sqrt(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(
13)/6)/(2*sqrt(sqrt(13) + 5)*sqrt(x**4 + 5*x**2 + 3))
```

Mathematica [C] time = 0.493743, size = 222, normalized size = 0.82

$$\frac{i\sqrt{2}\left(4\sqrt{13}-17\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-4i\sqrt{2}\left(\sqrt{13}-5\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{}}}{2\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^2*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*x*(3 + 5*x^2 + x^4) - (4*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 +
 Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*El
lipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/
6] + I*Sqrt[2]*(-17 + 4*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-
5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqr
t[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2
 + x^4])
```

Maple [A] time = 0.019, size = 208, normalized size = 0.8

$$288\,\frac{\sqrt{1-\left(-5/6+1/6\,\sqrt{13}\right)x^{2}}\sqrt{1-\left(-5/6-1/6\,\sqrt{13}\right)x^{2}}\left(\mathit{EllipticF}\left(1/6\,x\sqrt{-30+6\,\sqrt{13}},5/6\,\sqrt{3}+1/6\,\sqrt{39}\right)-\mathit{EllipticE}\left(\right.\right.}{\sqrt{-30+6\,\sqrt{13}}\sqrt{x^{4}+5\,x^{2}+3}\left(5+\sqrt{13}\right)}$$
$$+x\sqrt{x^{4}+5\,x^{2}+3}$$
$$-18\,\frac{\sqrt{1-\left(-5/6+1/6\,\sqrt{13}\right)x^{2}}\sqrt{1-\left(-5/6-1/6\,\sqrt{13}\right)x^{2}}\mathit{EllipticF}\left(1/6\,x\sqrt{-30+6\,\sqrt{13}},5/6\,\sqrt{3}+1/6\,\sqrt{39}\right)}{\sqrt{-30+6\,\sqrt{13}}\sqrt{x^{4}+5\,x^{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 288/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(
-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(E
llipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-E
llipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))+
x*(x^4+5*x^2+3)^(1/2)-18/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(
1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^
(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3\,x^{2}+2\right)x^{2}}{\sqrt{x^{4}+5\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,x^4+2\,x^2}{\sqrt{x^4+5\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral((3*x^4 + 2*x^2)/sqrt(x^4 + 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\,(3x^2+2)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**2*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3\,x^2+2)\,x^2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5*x^2 + 3), x)
```

3.191 $\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=257

$$\frac{3x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}}+\frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{\sqrt{\frac{3}{2}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{2\sqrt{x^4+5x^2+3}}$$

```
[Out] (3*x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[(3
*(5 + Sqrt[13]))/2]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[
13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sq
rt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4]) +
 (Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (
5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqr
t[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x
^4]
```

Rubi [A] time = 0.211171, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{3x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}}+\frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{\sqrt{\frac{3}{2}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{2\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (3*x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[(3
*(5 + Sqrt[13]))/2]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[
```

```
13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sq
rt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4]) +
 (Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (
5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqr
t[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x
^4]
```

Rubi in Sympy [A] time = 16.6905, size = 233, normalized size = 0.91

$$\frac{3x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}} - \frac{\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{4\sqrt{x^4+5x^2+3}} + \frac{\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{3\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] 3*x*(2*x**2 + sqrt(13) + 5)/(2*sqrt(x**4 + 5*x**2 + 3)) - sqrt(6)
*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6))*sqrt(
sqrt(13) + 5)*(x**2*(sqrt(13) + 5) + 6)*elliptic_e(atan(sqrt(6)*x
*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(4*sqrt(x**4 + 5*x*
*2 + 3)) + sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13
) + 5) + 6))*(x**2*(sqrt(13) + 5) + 6)*elliptic_f(atan(sqrt(6)*x*
sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(3*sqrt(sqrt(13) + 5
)*sqrt(x**4 + 5*x**2 + 3))
```

Mathematica [C] time = 0.181331, size = 159, normalized size = 0.62

$$\frac{i\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\left(\left(11-3\sqrt{13}\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+3\left(\sqrt{13}-5\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\right.\right.}{2\sqrt{2}\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2 + 3*x^2)/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] ((I/2)*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqr
t[13] + 2*x^2]*(3*(-5 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(5 +
 Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + (11 - 3*Sqrt[13])*Ellipt
icF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6]))
/(Sqrt[2]*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.016, size = 194, normalized size = 0.8

$$12\,\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right)x^2}\,EllipticF\left(1/6\,x\sqrt{-30+6\sqrt{13}},5/6\sqrt{3}+1/6\sqrt{39}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$$
$$-108\,\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right)x^2}\left(EllipticF\left(1/6\,x\sqrt{-30+6\sqrt{13}},5/6\sqrt{3}+1/6\sqrt{39}\right)-EllipticE\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}\left(5+\sqrt{13}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 12/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-
5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*
(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-108/(-30+6*13^(1
/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2
))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(
-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(
-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/sqrt(x^4 + 5*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/sqrt(x^4 + 5*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)/sqrt(x^4 + 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/sqrt(x^4 + 5*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5*x^2 + 3), x)
```

3.192 $\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=278

$$\frac{x\left(2x^2+\sqrt{13}+5\right)}{3\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{3x}+\frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{3\sqrt{x^4+5x^2+3}}$$

```
[Out] (x*(5 + Sqrt[13] + 2*x^2))/(3*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3
+ 5*x^2 + x^4])/(3*x) - (Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sq
rt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*E
llipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])
/(3*Sqrt[3 + 5*x^2 + x^4]) + (Sqrt[3/(2*(5 + Sqrt[13]))]*Sqrt[(6
+ (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13
])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt
[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi [A] time = 0.302223, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$\frac{x\left(2x^2+\sqrt{13}+5\right)}{3\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{3x}+\frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{3\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(x^2*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (x*(5 + Sqrt[13] + 2*x^2))/(3*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3
+ 5*x^2 + x^4])/(3*x) - (Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sq
```

```
rt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*E
llipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])
/(3*Sqrt[3 + 5*x^2 + x^4]) + (Sqrt[3/(2*(5 + Sqrt[13]))]*Sqrt[(6
+ (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13
])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt
[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi in Sympy [A] time = 24.1463, size = 250, normalized size = 0.9

$$\frac{x\left(2x^2+\sqrt{13}+5\right)}{3\sqrt{x^4+5x^2+3}}-\frac{\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{18\sqrt{x^4+5x^2+3}}+\frac{\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{2\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/x**2/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] x*(2*x**2 + sqrt(13) + 5)/(3*sqrt(x**4 + 5*x**2 + 3)) - sqrt(6)*s
qrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6))*sqrt(sq
rt(13) + 5)*(x**2*(sqrt(13) + 5) + 6)*elliptic_e(atan(sqrt(6)*x*s
qrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(18*sqrt(x**4 + 5*x**
2 + 3)) + sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13)
 + 5) + 6))*(x**2*(sqrt(13) + 5) + 6)*elliptic_f(atan(sqrt(6)*x*s
qrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(2*sqrt(sqrt(13) + 5)
*sqrt(x**4 + 5*x**2 + 3)) - 2*sqrt(x**4 + 5*x**2 + 3)/(3*x)
```

Mathematica [C] time = 0.53472, size = 224, normalized size = 0.81

$$\frac{-i\sqrt{2}\left(4+\sqrt{13}\right)x\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+i\sqrt{2}\left(\sqrt{13}-5\right)x\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+}}{6x\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2 + 3*x^2)/(x^2*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (-4*(3 + 5*x^2 + x^4) + I*Sqrt[2]*(-5 + Sqrt[13])*x*Sqrt[(-5 + Sq
rt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ellip
ticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6]
- I*Sqrt[2]*(4 + Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + S
qrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(
5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(6*x*Sqrt[3 + 5*x^2 +
x^4])
```

Maple [A] time = 0.024, size = 211, normalized size = 0.8

$$18\,\frac{\sqrt{1-\left(-5/6+1/6\,\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\,\sqrt{13}\right)x^2}\,EllipticF\left(1/6\,x\sqrt{-30+6\,\sqrt{13}},5/6\,\sqrt{3}+1/6\,\sqrt{39}\right)}{\sqrt{-30+6\,\sqrt{13}}\sqrt{x^4+5\,x^2+3}}$$
$$-\frac{2}{3\,x}\sqrt{x^4+5\,x^2+3}$$
$$-24\,\frac{\sqrt{1-\left(-5/6+1/6\,\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\,\sqrt{13}\right)x^2}\left(EllipticF\left(1/6\,x\sqrt{-30+6\,\sqrt{13}},5/6\,\sqrt{3}+1/6\,\sqrt{39}\right)-EllipticE\right.}{\sqrt{-30+6\,\sqrt{13}}\sqrt{x^4+5\,x^2+3}\left(5+\sqrt{13}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 18/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-
5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*
(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-2/3*(x^4+5*x^2+3
)^(1/2)/x-24/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(
1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^
(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39
^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39
^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^2),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{x^2\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**2/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**2*sqrt(x**4 + 5*x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^2), x)
```

3.193 $\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=302

$$\frac{7x\left(2x^2+\sqrt{13}+5\right)}{54\sqrt{x^4+5x^2+3}}-\frac{7\sqrt{x^4+5x^2+3}}{27x}-\frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{9\sqrt{x^4+5x^2+3}}-\frac{7\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{54\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{9x^3}$$

```
[Out] (7*x*(5 + Sqrt[13] + 2*x^2))/(54*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt
[3 + 5*x^2 + x^4])/(9*x^3) - (7*Sqrt[3 + 5*x^2 + x^4])/(27*x) - (
7*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5
 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(54*Sqrt[3 + 5*x^2 + x
^4]) - (Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/
(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcT
an[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(9*Sqrt[3 +
5*x^2 + x^4])
```

Rubi [A] time = 0.404004, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{7x\left(2x^2+\sqrt{13}+5\right)}{54\sqrt{x^4+5x^2+3}}-\frac{7\sqrt{x^4+5x^2+3}}{27x}-\frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{9\sqrt{x^4+5x^2+3}}-\frac{7\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{54\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{9x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(x^4*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (7*x*(5 + Sqrt[13] + 2*x^2))/(54*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt
[3 + 5*x^2 + x^4])/(9*x^3) - (7*Sqrt[3 + 5*x^2 + x^4])/(27*x) - (
7*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5
 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(54*Sqrt[3 + 5*x^2 + x
^4]) - (Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/
(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcT
an[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(9*Sqrt[3 +
5*x^2 + x^4])
```

Rubi in Sympy [A] time = 32.1287, size = 274, normalized size = 0.91

$$\frac{7x\left(2x^2+\sqrt{13}+5\right)}{54\sqrt{x^4+5x^2+3}} - \frac{7\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{324\sqrt{x^4+5x^2+3}} - \frac{\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{27\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}} - \frac{7\sqrt{x^4+5x^2+3}}{27x} - \frac{2\sqrt{x^4+5x^2+3}}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/x**4/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] 7*x*(2*x**2 + sqrt(13) + 5)/(54*sqrt(x**4 + 5*x**2 + 3)) - 7*sqrt
(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6))*sq
rt(sqrt(13) + 5)*(x**2*(sqrt(13) + 5) + 6)*elliptic_e(atan(sqrt(6
)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(324*sqrt(x**4 +
 5*x**2 + 3)) - sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sq
rt(13) + 5) + 6))*(x**2*(sqrt(13) + 5) + 6)*elliptic_f(atan(sqrt(
6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(27*sqrt(sqrt(1
3) + 5)*sqrt(x**4 + 5*x**2 + 3)) - 7*sqrt(x**4 + 5*x**2 + 3)/(27*
x) - 2*sqrt(x**4 + 5*x**2 + 3)/(9*x**3)
```

Mathematica [C] time = 0.549225, size = 237, normalized size = 0.78

$$\frac{-i\sqrt{2}\left(7\sqrt{13}-47\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}x^3F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+7i\sqrt{2}\left(\sqrt{13}-5\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2}}{108x^3\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2 + 3*x^2)/(x^4*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (-4*(18 + 51*x^2 + 41*x^4 + 7*x^6) + (7*I)*Sqrt[2]*(-5 + Sqrt[13]
)*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt
[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6
 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-47 + 7*Sqrt[13])*x^3*Sqrt[(-5 +
Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ell
ipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6
])/(108*x^3*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.027, size = 228, normalized size = 0.8

$$-\frac{2}{9x^3}\sqrt{x^4+5x^2+3}-\frac{7}{27x}\sqrt{x^4+5x^2+3}$$
$$-\frac{4}{3\sqrt{-30+6\sqrt{13}}}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\mathit{EllipticF}\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)\frac{1}{\sqrt{x^4+5x^2+3}}$$
$$-\frac{28}{3\sqrt{-30+6\sqrt{13}}\left(5+\sqrt{13}\right)}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\mathit{EllipticF}\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-\mathit{Elli}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -2/9*(x^4+5*x^2+3)^(1/2)/x^3-7/27*(x^4+5*x^2+3)^(1/2)/x-4/3/(-30+
6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*
13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*1
3^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-28/3/(-30+6*13^(1/2))^(1
/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)
^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*1
3^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*1
3^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^4),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{x^4\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)/x**4/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**4*sqrt(x**4 + 5*x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^4),x, algorithm="giac")

[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^4), x)
```

3.194 $\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{(47x^2+33)\,x^2}{13\sqrt{x^4+5x^2+3}}+\frac{133}{26}\sqrt{x^4+5x^2+3}-\frac{41}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] -(x^2*(33 + 47*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) + (133*Sqrt[3 + 5
*x^2 + x^4])/26 - (41*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4
])])/4
```

Rubi [A] time = 0.165376, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{(47x^2+33)\,x^2}{13\sqrt{x^4+5x^2+3}}+\frac{133}{26}\sqrt{x^4+5x^2+3}-\frac{41}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] -(x^2*(33 + 47*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) + (133*Sqrt[3 + 5
*x^2 + x^4])/26 - (41*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4
])])/4
```

Rubi in Sympy [A] time = 18.5798, size = 68, normalized size = 0.88

$$-\frac{x^2\,(47x^2+33)}{13\sqrt{x^4+5x^2+3}}+\frac{133\sqrt{x^4+5x^2+3}}{26}-\frac{41\,\operatorname{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] -x**2*(47*x**2 + 33)/(13*sqrt(x**4 + 5*x**2 + 3)) + 133*sqrt(x**4
 + 5*x**2 + 3)/26 - 41*atanh((2*x**2 + 5)/(2*sqrt(x**4 + 5*x**2 +
 3)))/4
```

Mathematica [A] time = 0.0542694, size = 59, normalized size = 0.77

$$\frac{39x^4+599x^2+399}{26\sqrt{x^4+5x^2+3}}-\frac{41}{4}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (399 + 599*x^2 + 39*x^4)/(26*Sqrt[3 + 5*x^2 + x^4]) - (41*Log[5 +
 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/4
```

Maple [A] time = 0.029, size = 91, normalized size = 1.2

$$\frac{41\,x^2}{4}\frac{1}{\sqrt{x^4+5\,x^2+3}}-\frac{133}{8}\frac{1}{\sqrt{x^4+5\,x^2+3}}+\frac{1330\,x^2+3325}{104}\frac{1}{\sqrt{x^4+5\,x^2+3}}-\frac{41}{4}\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5\,x^2+3}\right)+\frac{3\,x^4}{2}\frac{1}{\sqrt{x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 41/4*x^2/(x^4+5*x^2+3)^(1/2)-133/8/(x^4+5*x^2+3)^(1/2)+665/104*(2
*x^2+5)/(x^4+5*x^2+3)^(1/2)-41/4*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+
3/2*x^4/(x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 0.695618, size = 99, normalized size = 1.29

$$\frac{3\,x^4}{2\,\sqrt{x^4+5\,x^2+3}}+\frac{599\,x^2}{26\,\sqrt{x^4+5\,x^2+3}}+\frac{399}{26\,\sqrt{x^4+5\,x^2+3}}-\frac{41}{4}\,\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)*x^5/(x^4 + 5*x^2 + 3)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/2*x^4/sqrt(x^4 + 5*x^2 + 3) + 599/26*x^2/sqrt(x^4 + 5*x^2 + 3)
+ 399/26/sqrt(x^4 + 5*x^2 + 3) - 41/4*log(2*x^2 + 2*sqrt(x^4 + 5*
x^2 + 3) + 5)
```

Fricas [A] time = 0.262763, size = 225, normalized size = 2.92

$$-\frac{96\,x^8+840\,x^6+440\,x^4-5544\,x^2-82\left(8\,x^6+60\,x^4+124\,x^2-\left(8\,x^4+40\,x^2+37\right)\sqrt{x^4+5\,x^2+3}+60\right)\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)-\left(96\,x^6+600\,x^4-904\,x^2-2699\right)\sqrt{x^4+5\,x^2+3}-3816}{8\left(8\,x^6+60\,x^4+124\,x^2-\left(8\,x^4+40\,x^2+37\right)\sqrt{x^4+5\,x^2+3}+60\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)*x^5/(x^4 + 5*x^2 + 3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(96*x^8 + 840*x^6 + 440*x^4 - 5544*x^2 - 82*(8*x^6 + 60*x^4
+ 124*x^2 - (8*x^4 + 40*x^2 + 37)*sqrt(x^4 + 5*x^2 + 3) + 60)*log
(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5) - (96*x^6 + 600*x^4 - 904*
x^2 - 2699)*sqrt(x^4 + 5*x^2 + 3) - 3816)/(8*x^6 + 60*x^4 + 124*x
^2 - (8*x^4 + 40*x^2 + 37)*sqrt(x^4 + 5*x^2 + 3) + 60)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\left(3x^2+2\right)}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

GIAC/XCAS [A] time = 0.287902, size = 70, normalized size = 0.91

$$\frac{\left(39\,x^2+599\right)x^2+399}{26\,\sqrt{x^4+5\,x^2+3}}+\frac{41}{4}\ln\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)*x^5/(x^4 + 5*x^2 + 3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/26*((39*x^2 + 599)*x^2 + 399)/sqrt(x^4 + 5*x^2 + 3) + 41/4*ln(2
*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.195 $\int \frac{x^3\left(2+3x^2\right)}{\left(3+5x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=56

$$\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{47x^2+33}{13\sqrt{x^4+5x^2+3}}$$

```
[Out] -(33 + 47*x^2)/(13*Sqrt[3 + 5*x^2 + x^4]) + (3*ArcTanh[(5 + 2*x^2
)/(2*Sqrt[3 + 5*x^2 + x^4])])/2
```

Rubi [A] time = 0.119032, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{47x^2+33}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] -(33 + 47*x^2)/(13*Sqrt[3 + 5*x^2 + x^4]) + (3*ArcTanh[(5 + 2*x^2
)/(2*Sqrt[3 + 5*x^2 + x^4])])/2
```

Rubi in Sympy [A] time = 14.386, size = 48, normalized size = 0.86

$$-\frac{47x^2+33}{13\sqrt{x^4+5x^2+3}}+\frac{3\operatorname{atanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] -(47*x**2 + 33)/(13*sqrt(x**4 + 5*x**2 + 3)) + 3*atanh((2*x**2 +
5)/(2*sqrt(x**4 + 5*x**2 + 3)))/2
```

Mathematica [A] time = 0.0360589, size = 54, normalized size = 0.96

$$\frac{3}{2}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)-\frac{47x^2+33}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] -(33 + 47*x^2)/(13*Sqrt[3 + 5*x^2 + x^4]) + (3*Log[5 + 2*x^2 + 2*
Sqrt[3 + 5*x^2 + x^4]])/2
```

Maple [B] time = 0.017, size = 95, normalized size = 1.7

$$\frac{10\,x^2+12}{13}\frac{1}{\sqrt{x^4+5\,x^2+3}}-\frac{3\,x^2}{2}\frac{1}{\sqrt{x^4+5\,x^2+3}}+\frac{15}{4}\frac{1}{\sqrt{x^4+5\,x^2+3}}-\frac{150\,x^2+375}{52}\frac{1}{\sqrt{x^4+5\,x^2+3}}+\frac{3}{2}\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5\,x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 2/13/(x^4+5*x^2+3)^(1/2)*(5*x^2+6)-3/2*x^2/(x^4+5*x^2+3)^(1/2)+15
/4/(x^4+5*x^2+3)^(1/2)-75/52*(2*x^2+5)/(x^4+5*x^2+3)^(1/2)+3/2*ln
(x^2+5/2+(x^4+5*x^2+3)^(1/2))
```

Maxima [A] time = 0.706633, size = 76, normalized size = 1.36

$$-\frac{47\,x^2}{13\,\sqrt{x^4+5\,x^2+3}}-\frac{33}{13\,\sqrt{x^4+5\,x^2+3}}+\frac{3}{2}\,\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^3/(x^4 + 5*x^2 + 3)^(3/2),x, algorithm="maxima")
```

```
[Out] -47/13*x^2/sqrt(x^4 + 5*x^2 + 3) - 33/13/sqrt(x^4 + 5*x^2 + 3) +
3/2*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 0.270226, size = 155, normalized size = 2.77

$$-\frac{26\,x^2+3\,\left(2\,x^4+10\,x^2-\sqrt{x^4+5\,x^2+3}(2\,x^2+5)+6\right)\,\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)-26\,\sqrt{x^4+5\,x^2+3}+18}{2\,\left(2\,x^4+10\,x^2-\sqrt{x^4+5\,x^2+3}(2\,x^2+5)+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^3/(x^4 + 5*x^2 + 3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(26*x^2 + 3*(2*x^4 + 10*x^2 - sqrt(x^4 + 5*x^2 + 3)*(2*x^2 +
 5) + 6)*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5) - 26*sqrt(x^4
+ 5*x^2 + 3) + 18)/(2*x^4 + 10*x^2 - sqrt(x^4 + 5*x^2 + 3)*(2*x^2
 + 5) + 6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(3x^2+2\right)}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

GIAC/XCAS [A] time = 0.283003, size = 62, normalized size = 1.11

$$-\frac{47\,x^2+33}{13\,\sqrt{x^4+5\,x^2+3}}-\frac{3}{2}\,\ln\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^3/(x^4 + 5*x^2 + 3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/13*(47*x^2 + 33)/sqrt(x^4 + 5*x^2 + 3) - 3/2*ln(2*x^2 - 2*sqrt
(x^4 + 5*x^2 + 3) + 5)
```

3.196 $\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=25

$$\frac{11x^2 + 8}{13\sqrt{x^4 + 5x^2 + 3}}$$

```
[Out] (8 + 11*x^2)/(13*Sqrt[3 + 5*x^2 + x^4])
```

Rubi [A] time = 0.0639883, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{11x^2 + 8}{13\sqrt{x^4 + 5x^2 + 3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (8 + 11*x^2)/(13*Sqrt[3 + 5*x^2 + x^4])
```

Rubi in Sympy [A] time = 10.8309, size = 20, normalized size = 0.8

$$\frac{22x^2 + 16}{26\sqrt{x^4 + 5x^2 + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] (22*x**2 + 16)/(26*sqrt(x**4 + 5*x**2 + 3))
```

Mathematica [A] time = 0.0216536, size = 25, normalized size = 1.

$$\frac{11x^2 + 8}{13\sqrt{x^4 + 5x^2 + 3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (8 + 11*x^2)/(13*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.007, size = 22, normalized size = 0.9

$$\frac{11\,x^{2}+8}{13}\frac{1}{\sqrt{x^{4}+5\,x^{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 1/13*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 0.706359, size = 43, normalized size = 1.72

$$\frac{11\,x^{2}}{13\,\sqrt{x^{4}+5\,x^{2}+3}}+\frac{8}{13\,\sqrt{x^{4}+5\,x^{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x/(x^4 + 5*x^2 + 3)^(3/2),x, algorithm="maxima")
```

```
[Out] 11/13*x^2/sqrt(x^4 + 5*x^2 + 3) + 8/13/sqrt(x^4 + 5*x^2 + 3)
```

Fricas [A] time = 0.282032, size = 77, normalized size = 3.08

$$\frac{3\,x^{2}-3\,\sqrt{x^{4}+5\,x^{2}+3}+2}{2\,x^{4}+10\,x^{2}-\sqrt{x^{4}+5\,x^{2}+3}(2\,x^{2}+5)+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x/(x^4 + 5*x^2 + 3)^(3/2),x, algorithm="fricas")
```

```
[Out] (3*x^2 - 3*sqrt(x^4 + 5*x^2 + 3) + 2)/(2*x^4 + 10*x^2 - sqrt(x^4
+ 5*x^2 + 3)*(2*x^2 + 5) + 6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(3x^2+2\right)}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

GIAC/XCAS [A] time = 0.281653, size = 28, normalized size = 1.12

$$\frac{11\,x^2+8}{13\,\sqrt{x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x/(x^4 + 5*x^2 + 3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/13*(11*x^2 + 8)/sqrt(x^4 + 5*x^2 + 3)
```

3.197 $\int \frac{2+3x^2}{x(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{8x^2+7}{39\sqrt{x^4+5x^2+3}}-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

```
[Out] -(7 + 8*x^2)/(39*Sqrt[3 + 5*x^2 + x^4]) - ArcTanh[(6 + 5*x^2)/(2*
Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/(3*Sqrt[3])
```

Rubi [A] time = 0.148657, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{8x^2+7}{39\sqrt{x^4+5x^2+3}}-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(x*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] -(7 + 8*x^2)/(39*Sqrt[3 + 5*x^2 + x^4]) - ArcTanh[(6 + 5*x^2)/(2*
Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/(3*Sqrt[3])
```

Rubi in Sympy [A] time = 17.7022, size = 58, normalized size = 0.88

$$-\frac{8x^2+7}{39\sqrt{x^4+5x^2+3}}-\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{\sqrt{3}(5x^2+6)}{6\sqrt{x^4+5x^2+3}}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/x/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] -(8*x**2 + 7)/(39*sqrt(x**4 + 5*x**2 + 3)) - sqrt(3)*atanh(sqrt(3
)*(5*x**2 + 6)/(6*sqrt(x**4 + 5*x**2 + 3)))/9
```

Mathematica [A] time = 0.127906, size = 71, normalized size = 1.08

$$\frac{-8x^2-7}{39\sqrt{x^4+5x^2+3}}+\frac{\log\left(x^2\right)-\log\left(5x^2+2\sqrt{3}\sqrt{x^4+5x^2+3}+6\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)/(x*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-7 - 8*x^2)/(39*Sqrt[3 + 5*x^2 + x^4]) + (Log[x^2] - Log[6 + 5*x
^2 + 2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]])/(3*Sqrt[3])
```

Maple [A] time = 0.027, size = 67, normalized size = 1.

$$\frac{1}{3}\frac{1}{\sqrt{x^4+5x^2+3}}-\frac{8x^2+20}{39}\frac{1}{\sqrt{x^4+5x^2+3}}-\frac{\sqrt{3}}{9}Artanh\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5x^2+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 1/3/(x^4+5*x^2+3)^(1/2)-4/39*(2*x^2+5)/(x^4+5*x^2+3)^(1/2)-1/9*ar
ctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

Maxima [A] time = 0.773884, size = 88, normalized size = 1.33

$$-\frac{8x^2}{39\sqrt{x^4+5x^2+3}}-\frac{1}{9}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{7}{39\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] -8/39*x^2/sqrt(x^4 + 5*x^2 + 3) - 1/9*sqrt(3)*log(2*sqrt(3)*sqrt(
x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 7/39/sqrt(x^4 + 5*x^2 + 3)
```

Fricas [A] time = 0.268539, size = 248, normalized size = 3.76

$$-\frac{\left(2\,x^4+10\,x^2-\sqrt{x^4+5\,x^2+3}\left(2\,x^2+5\right)+6\right)\log\left(\frac{6\,x^2+\sqrt{3}\left(2\,x^4+5\,x^2+6\right)-2\,\sqrt{x^4+5\,x^2+3}\left(\sqrt{3}x^2+3\right)}{2\,x^4-2\,\sqrt{x^4+5\,x^2+3}x^2+5\,x^2}\right)-\sqrt{3}\left(2\,x^2+1\right)+2\,\sqrt{3}\sqrt{x^4+5\,x}}{3\left(\sqrt{3}\sqrt{x^4+5\,x^2+3}(2\,x^2+5)-2\,\sqrt{3}(x^4+5\,x^2+3)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] -1/3*((2*x^4 + 10*x^2 - sqrt(x^4 + 5*x^2 + 3)*(2*x^2 + 5) + 6)*lo
g((6*x^2 + sqrt(3)*(2*x^4 + 5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*
(sqrt(3)*x^2 + 3))/(2*x^4 - 2*sqrt(x^4 + 5*x^2 + 3)*x^2 + 5*x^2))
 - sqrt(3)*(2*x^2 + 1) + 2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3))/(sqrt(3
)*sqrt(x^4 + 5*x^2 + 3)*(2*x^2 + 5) - 2*sqrt(3)*(x^4 + 5*x^2 + 3)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{x\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)/x/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x), x)
```

3.198 $\int \frac{2+3x^2}{x^3(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=90

$$-\frac{8x^2+7}{39x^2\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{39x^2}+\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

```
[Out] -(7 + 8*x^2)/(39*x^2*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3 + 5*x^2 +
 x^4])/(39*x^2) + ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 +
 x^4])]/(3*Sqrt[3])
```

Rubi [A] time = 0.20158, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{8x^2+7}{39x^2\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{39x^2}+\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(x^3*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] -(7 + 8*x^2)/(39*x^2*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3 + 5*x^2 +
 x^4])/(39*x^2) + ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 +
 x^4])]/(3*Sqrt[3])
```

Rubi in Sympy [A] time = 20.9106, size = 80, normalized size = 0.89

$$\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{\sqrt{3}(5x^2+6)}{6\sqrt{x^4+5x^2+3}}\right)}{9}-\frac{8x^2+7}{39x^2\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{39x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/x**3/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] sqrt(3)*atanh(sqrt(3)*(5*x**2 + 6)/(6*sqrt(x**4 + 5*x**2 + 3)))/9
 - (8*x**2 + 7)/(39*x**2*sqrt(x**4 + 5*x**2 + 3)) - 2*sqrt(x**4 +
 5*x**2 + 3)/(39*x**2)
```

Mathematica [A] time = 0.0905552, size = 79, normalized size = 0.88

$$\frac{\log\left(5x^2+2\sqrt{3}\sqrt{x^4+5x^2+3}+6\right)-\log\left(x^2\right)}{3\sqrt{3}}-\frac{2x^4+18x^2+13}{39x^2\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x^2)/(x^3*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] -(13 + 18*x^2 + 2*x^4)/(39*x^2*Sqrt[3 + 5*x^2 + x^4]) + (-Log[x^2
] + Log[6 + 5*x^2 + 2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]])/(3*Sqrt[3])
```

Maple [A] time = 0.023, size = 84, normalized size = 0.9

$$-\frac{1}{3x^2}\frac{1}{\sqrt{x^4+5x^2+3}}-\frac{1}{3}\frac{1}{\sqrt{x^4+5x^2+3}}-\frac{2x^2+5}{39}\frac{1}{\sqrt{x^4+5x^2+3}}+\frac{\sqrt{3}}{9}\mathit{Artanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5x^2+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -1/3/x^2/(x^4+5*x^2+3)^(1/2)-1/3/(x^4+5*x^2+3)^(1/2)-1/39*(2*x^2+
5)/(x^4+5*x^2+3)^(1/2)+1/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x
^2+3)^(1/2))*3^(1/2)
```

Maxima [A] time = 0.780967, size = 111, normalized size = 1.23

$$-\frac{2x^2}{39\sqrt{x^4+5x^2+3}}+\frac{1}{9}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{6}{13\sqrt{x^4+5x^2+3}}-\frac{1}{3\sqrt{x^4+5x^2+3}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^3),x, algorithm="maxima")
```

```
[Out] -2/39*x^2/sqrt(x^4 + 5*x^2 + 3) + 1/9*sqrt(3)*log(2*sqrt(3)*sqrt(
x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 6/13/sqrt(x^4 + 5*x^2 + 3) -
1/3/(sqrt(x^4 + 5*x^2 + 3)*x^2)
```

Fricas [A] time = 0.282852, size = 333, normalized size = 3.7

$$\frac{2\sqrt{3}\left(4x^4+15x^2+10\right)\sqrt{x^4+5x^2+3}-\left(8x^8+60x^6+124x^4+60x^2-\left(8x^6+40x^4+37x^2\right)\sqrt{x^4+5x^2+3}\right)\log\left(-\frac{6x^2-\sqrt{}}{}\right.}{3\left(\sqrt{3}(8x^6+40x^4+37x^2)\sqrt{x^4+5x^2+3}-4\sqrt{3}(2x^8+15x^6+31x^{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^3),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*(4*x^4 + 15*x^2 + 10)*sqrt(x^4 + 5*x^2 + 3) - (8*x
^8 + 60*x^6 + 124*x^4 + 60*x^2 - (8*x^6 + 40*x^4 + 37*x^2)*sqrt(x
^4 + 5*x^2 + 3))*log(-(6*x^2 - sqrt(3)*(2*x^4 + 5*x^2 + 6) + 2*sq
rt(x^4 + 5*x^2 + 3)*(sqrt(3)*x^2 - 3))/(2*x^4 - 2*sqrt(x^4 + 5*x^
2 + 3)*x^2 + 5*x^2)) - sqrt(3)*(8*x^6 + 50*x^4 + 82*x^2 + 37))/(s
qrt(3)*(8*x^6 + 40*x^4 + 37*x^2)*sqrt(x^4 + 5*x^2 + 3) - 4*sqrt(3
)*(2*x^8 + 15*x^6 + 31*x^4 + 15*x^2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**3/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^3),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^3), x)
```

3.199 $\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=307

$$-\frac{11}{13}\sqrt{x^4+5x^2+3}x+\frac{43\left(2x^2+\sqrt{13}+5\right)x}{13\sqrt{x^4+5x^2+3}}$$
$$+\frac{11\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{x^4+5x^2+3}}$$
$$-\frac{43\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{x^4+5x^2+3}}$$
$$+\frac{\left(11x^2+8\right)x^3}{13\sqrt{x^4+5x^2+3}}$$

```
[Out] (43*x*(5 + Sqrt[13] + 2*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) + (x^3*(
8 + 11*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) - (11*x*Sqrt[3 + 5*x^2 +
x^4])/13 - (43*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^
2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[A
rcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(13*Sqrt[
3 + 5*x^2 + x^4]) + (11*Sqrt[3/(2*(5 + Sqrt[13]))]*Sqrt[(6 + (5 -
 Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2
)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/
6])/(13*Sqrt[3 + 5*x^2 + x^4])
```

Rubi [A] time = 0.418515, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{11}{13}\sqrt{x^4+5x^2+3}x+\frac{43\left(2x^2+\sqrt{13}+5\right)x}{13\sqrt{x^4+5x^2+3}}$$
$$+\frac{11\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{x^4+5x^2+3}}$$
$$-\frac{43\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{x^4+5x^2+3}}$$
$$+\frac{\left(11x^2+8\right)x^3}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (43*x*(5 + Sqrt[13] + 2*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) + (x^3*(
8 + 11*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) - (11*x*Sqrt[3 + 5*x^2 +
x^4])/13 - (43*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^
2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[A
rcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(13*Sqrt[
3 + 5*x^2 + x^4]) + (11*Sqrt[3/(2*(5 + Sqrt[13]))]*Sqrt[(6 + (5 -
 Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2
)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/
6])/(13*Sqrt[3 + 5*x^2 + x^4])
```

Rubi in Sympy [A] time = 34.7829, size = 280, normalized size = 0.91

$$\frac{x^3\left(11x^2+8\right)}{13\sqrt{x^4+5x^2+3}}+\frac{43x\left(2x^2+\sqrt{13}+5\right)}{13\sqrt{x^4+5x^2+3}}-\frac{11x\sqrt{x^4+5x^2+3}}{13}-\frac{43\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{78\sqrt{x^4+5x^2+3}}+\frac{11\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{26\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] x**3*(11*x**2 + 8)/(13*sqrt(x**4 + 5*x**2 + 3)) + 43*x*(2*x**2 +
sqrt(13) + 5)/(13*sqrt(x**4 + 5*x**2 + 3)) - 11*x*sqrt(x**4 + 5*x
**2 + 3)/13 - 43*sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(s
qrt(13) + 5) + 6))*sqrt(sqrt(13) + 5)*(x**2*(sqrt(13) + 5) + 6)*e
lliptic_e(atan(sqrt(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13
)/6)/(78*sqrt(x**4 + 5*x**2 + 3)) + 11*sqrt(6)*sqrt((x**2*(-sqrt(
13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6))*(x**2*(sqrt(13) + 5) + 6
)*elliptic_f(atan(sqrt(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt
(13)/6)/(26*sqrt(sqrt(13) + 5)*sqrt(x**4 + 5*x**2 + 3))
```

Mathematica [C] time = 0.494512, size = 219, normalized size = 0.71

$$\frac{-2x\left(47x^2+33\right)-i\sqrt{2}\left(43\sqrt{13}-182\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+43i\sqrt{2}\left(\sqrt{13}-5\right)}{26\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^4*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-2*x*(33 + 47*x^2) + (43*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + S
qrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Elli
pticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6]
 - I*Sqrt[2]*(-182 + 43*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-
5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqr
t[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(26*Sqrt[3 + 5*x^
2 + x^4])
```

Maple [A] time = 0.033, size = 240, normalized size = 0.8

$$-4\frac{1}{\sqrt{x^4+5x^2+3}}\left(-\frac{5x^3}{26}-3/13x\right)$$
$$+\frac{198}{13\sqrt{-30+6\sqrt{13}}}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)\frac{1}{\sqrt{x^4+5x^2+3}}$$
$$-\frac{3096}{13\sqrt{-30+6\sqrt{13}}\left(5+\sqrt{13}\right)}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-El\right.$$
$$-6\frac{1}{\sqrt{x^4+5x^2+3}}\left(\frac{19x^3}{26}+\frac{15x}{26}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -4*(-5/26*x^3-3/13*x)/(x^4+5*x^2+3)^(1/2)+198/13/(-30+6*13^(1/2))
^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x
^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1
/2),5/6*3^(1/2)+1/6*39^(1/2))-3096/13/(-30+6*13^(1/2))^(1/2)*(1-(
-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(
x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))
^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))
^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))-6*(19/26*x^3+15/26*x)/(x^4+5*x^
2+3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3\,x^2+2\right)x^4}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)*x^4/(x^4 + 5*x^2 + 3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,x^6+2\,x^4}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)*x^4/(x^4 + 5*x^2 + 3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4)/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\left(3x^2+2\right)}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**4*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3\,x^2+2\right)x^4}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)*x^4/(x^4 + 5*x^2 + 3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5*x^2 + 3)^(3/2), x)
```

3.200 $\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=286

$$-\frac{11x\left(2x^2+\sqrt{13}+5\right)}{26\sqrt{x^4+5x^2+3}}+\frac{x\left(11x^2+8\right)}{13\sqrt{x^4+5x^2+3}}-\frac{4\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{x^4+5x^2+3}}+\frac{11\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{26\sqrt{x^4+5x^2+3}}$$

```
[Out] (-11*x*(5 + Sqrt[13] + 2*x^2))/(26*Sqrt[3 + 5*x^2 + x^4]) + (x*(8
 + 11*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) + (11*Sqrt[(5 + Sqrt[13])/
6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (
5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-
13 + 5*Sqrt[13])/6])/(26*Sqrt[3 + 5*x^2 + x^4]) - (4*Sqrt[2/(3*(5
 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*
x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13
])/6]*x], (-13 + 5*Sqrt[13])/6])/(13*Sqrt[3 + 5*x^2 + x^4])
```

Rubi [A] time = 0.318153, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$-\frac{11x\left(2x^2+\sqrt{13}+5\right)}{26\sqrt{x^4+5x^2+3}}+\frac{x\left(11x^2+8\right)}{13\sqrt{x^4+5x^2+3}}-\frac{4\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{x^4+5x^2+3}}+\frac{11\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{26\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-11*x*(5 + Sqrt[13] + 2*x^2))/(26*Sqrt[3 + 5*x^2 + x^4]) + (x*(8
 + 11*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) + (11*Sqrt[(5 + Sqrt[13])/
```

```
6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (
5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-
13 + 5*Sqrt[13])/6])/(26*Sqrt[3 + 5*x^2 + x^4]) - (4*Sqrt[2/(3*(5
 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*
x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13
])/6]*x], (-13 + 5*Sqrt[13])/6])/(13*Sqrt[3 + 5*x^2 + x^4])
```

Rubi in Sympy [A] time = 27.73, size = 260, normalized size = 0.91

$$\frac{x\left(11x^2+8\right)}{13\sqrt{x^4+5x^2+3}}-\frac{11x\left(2x^2+\sqrt{13}+5\right)}{26\sqrt{x^4+5x^2+3}}+\frac{11\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{156\sqrt{x^4+5x^2+3}}-\frac{4\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{39\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] x*(11*x**2 + 8)/(13*sqrt(x**4 + 5*x**2 + 3)) - 11*x*(2*x**2 + sqr
t(13) + 5)/(26*sqrt(x**4 + 5*x**2 + 3)) + 11*sqrt(6)*sqrt((x**2*(
-sqrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6))*sqrt(sqrt(13) + 5)
*(x**2*(sqrt(13) + 5) + 6)*elliptic_e(atan(sqrt(6)*x*sqrt(sqrt(13
) + 5)/6), -13/6 + 5*sqrt(13)/6)/(156*sqrt(x**4 + 5*x**2 + 3)) -
4*sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) +
6))*(x**2*(sqrt(13) + 5) + 6)*elliptic_f(atan(sqrt(6)*x*sqrt(sqrt
(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(39*sqrt(sqrt(13) + 5)*sqrt(x
**4 + 5*x**2 + 3))
```

Mathematica [C] time = 0.495049, size = 219, normalized size = 0.77

$$\frac{4x\left(11x^2+8\right)+i\sqrt{2}\left(11\sqrt{13}-39\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-11i\sqrt{2}\left(\sqrt{13}-5\right)\sqrt{-\cdots}}{52\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^2*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*x*(8 + 11*x^2) - (11*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqr
t[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ellipt
icE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] +
 I*Sqrt[2]*(-39 + 11*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 +
 Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2
/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(52*Sqrt[3 + 5*x^2 +
 x^4])
```

Maple [A] time = 0.022, size = 240, normalized size = 0.8

$$-4\,\frac{1}{\sqrt{x^4+5\,x^2+3}}\left(1/13\,x^3+\frac{5\,x}{26}\right)$$
$$-\frac{48}{13\,\sqrt{-30+6\,\sqrt{13}}}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-30+6\,\sqrt{13}}}{6},\frac{5\,\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)\frac{1}{\sqrt{x^4+5\,x^2+3}}$$
$$+\frac{396}{13\,\sqrt{-30+6\,\sqrt{13}}\left(5+\sqrt{13}\right)}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-30+6\,\sqrt{13}}}{6},\frac{5\,\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-Ell\right.$$
$$-6\,\frac{1}{\sqrt{x^4+5\,x^2+3}}\left(-\frac{5\,x^3}{26}-3/13\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -4*(1/13*x^3+5/26*x)/(x^4+5*x^2+3)^(1/2)-48/13/(-30+6*13^(1/2))^(
1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2
)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2
),5/6*3^(1/2)+1/6*39^(1/2))+396/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/
6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4
+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1
/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1
/2),5/6*3^(1/2)+1/6*39^(1/2)))-6*(-5/26*x^3-3/13*x)/(x^4+5*x^2+3)
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(3\,x^2+2\right)x^2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)*x^2/(x^4 + 5*x^2 + 3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,x^{4}+2\,x^{2}}{(x^{4}+5\,x^{2}+3)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)*x^2/(x^4 + 5*x^2 + 3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^4 + 2*x^2)/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}\left(3x^{2}+2\right)}{(x^{4}+5x^{2}+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**2*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3\,x^{2}+2\right)x^{2}}{(x^{4}+5\,x^{2}+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)*x^2/(x^4 + 5*x^2 + 3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5*x^2 + 3)^(3/2), x)
```

3.201 $\int \frac{2+3x^2}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=282

$$\frac{4x\left(2x^2+\sqrt{13}+5\right)}{39\sqrt{x^4+5x^2+3}}-\frac{x\left(8x^2+7\right)}{39\sqrt{x^4+5x^2+3}}+\frac{11\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{39\sqrt{x^4+5x^2+3}}$$

```
[Out] (4*x*(5 + Sqrt[13] + 2*x^2))/(39*Sqrt[3 + 5*x^2 + x^4]) - (x*(7 +
 8*x^2))/(39*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[(2*(5 + Sqrt[13]))/
3]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (
5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-
13 + 5*Sqrt[13])/6])/(39*Sqrt[3 + 5*x^2 + x^4]) + (11*Sqrt[(6 + (
5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*
x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13
])/6])/(13*Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rubi [A] time = 0.26627, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{4x\left(2x^2+\sqrt{13}+5\right)}{39\sqrt{x^4+5x^2+3}}-\frac{x\left(8x^2+7\right)}{39\sqrt{x^4+5x^2+3}}+\frac{11\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{39\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*x*(5 + Sqrt[13] + 2*x^2))/(39*Sqrt[3 + 5*x^2 + x^4]) - (x*(7 +
 8*x^2))/(39*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[(2*(5 + Sqrt[13]))/
3]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (
5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-
13 + 5*Sqrt[13])/6])/(39*Sqrt[3 + 5*x^2 + x^4]) + (11*Sqrt[(6 + (
5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*
x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13
])/6])/(13*Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rubi in Sympy [A] time = 23.3571, size = 260, normalized size = 0.92

$$-\frac{x\left(8x^2+7\right)}{39\sqrt{x^4+5x^2+3}}+\frac{4x\left(2x^2+\sqrt{13}+5\right)}{39\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{117\sqrt{x^4+5x^2+3}}+\frac{11\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{78\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] -x*(8*x**2 + 7)/(39*sqrt(x**4 + 5*x**2 + 3)) + 4*x*(2*x**2 + sqrt
(13) + 5)/(39*sqrt(x**4 + 5*x**2 + 3)) - 2*sqrt(6)*sqrt((x**2*(-s
qrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6))*sqrt(sqrt(13) + 5)*(
x**2*(sqrt(13) + 5) + 6)*elliptic_e(atan(sqrt(6)*x*sqrt(sqrt(13)
+ 5)/6), -13/6 + 5*sqrt(13)/6)/(117*sqrt(x**4 + 5*x**2 + 3)) + 11
*sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6
))*(x**2*(sqrt(13) + 5) + 6)*elliptic_f(atan(sqrt(6)*x*sqrt(sqrt(
13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(78*sqrt(sqrt(13) + 5)*sqrt(x*
*4 + 5*x**2 + 3))
```

Mathematica [C] time = 0.492047, size = 219, normalized size = 0.78

$$\frac{-2x\left(8x^2+7\right)-i\sqrt{2}\left(13+4\sqrt{13}\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)\middle|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+4i\sqrt{2}\left(\sqrt{13}-5\right)\sqrt{\frac{-2x}{}}}{78\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2 + 3*x^2)/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-2*x*(7 + 8*x^2) + (4*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt
[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ellipti
cE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] -
I*Sqrt[2]*(13 + 4*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sq
rt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5
 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(78*Sqrt[3 + 5*x^2 + x^
4])
```

Maple [A] time = 0.021, size = 240, normalized size = 0.9

$$-4\,\frac{1}{\sqrt{x^4+5\,x^2+3}}\left(-\frac{19\,x}{78}-\frac{5\,x^3}{78}\right)$$
$$+\frac{66}{13\,\sqrt{-30+6\,\sqrt{13}}}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-30+6\,\sqrt{13}}}{6},\frac{5\,\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)\frac{1}{\sqrt{x^4+5\,x^2+3}}$$
$$-\frac{96}{13\,\sqrt{-30+6\,\sqrt{13}}\left(5+\sqrt{13}\right)}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-30+6\,\sqrt{13}}}{6},\frac{5\,\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-El\right.$$
$$-\,6\,\frac{1}{\sqrt{x^4+5\,x^2+3}}\left(1/13\,x^3+\frac{5\,x}{26}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -4*(-19/78*x-5/78*x^3)/(x^4+5*x^2+3)^(1/2)+66/13/(-30+6*13^(1/2))
^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x
^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1
/2),5/6*3^(1/2)+1/6*39^(1/2))-96/13/(-30+6*13^(1/2))^(1/2)*(1-(-5
/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^
4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(
1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(
1/2),5/6*3^(1/2)+1/6*39^(1/2)))-6*(1/13*x^3+5/26*x)/(x^4+5*x^2+3)
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(x^4 + 5*x^2 + 3)^(3/2),x, algorithm="maxima")

[Out] integrate((3*x^2 + 2)/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,x^2+2}{(x^4+5\,x^2+3)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(x^4 + 5*x^2 + 3)^(3/2),x, algorithm="fricas")

[Out] integral((3*x^2 + 2)/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)

[Out] Integral((3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{(x^4+5\,x^2+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/(x^4 + 5*x^2 + 3)^(3/2),x, algorithm="giac")

[Out] integrate((3*x^2 + 2)/(x^4 + 5*x^2 + 3)^(3/2), x)
```

3.202 $\int \frac{2+3x^2}{x^2(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=309

$$\frac{19x\left(2x^2+\sqrt{13}+5\right)}{234\sqrt{x^4+5x^2+3}}-\frac{19\sqrt{x^4+5x^2+3}}{117x}-\frac{8x^2+7}{39x\sqrt{x^4+5x^2+3}}-\frac{4\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{39\sqrt{x^4+5x^2+3}}-\frac{19\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{234\sqrt{x^4+5x^2+3}}$$

```
[Out] (19*x*(5 + Sqrt[13] + 2*x^2))/(234*Sqrt[3 + 5*x^2 + x^4]) - (7 +
8*x^2)/(39*x*Sqrt[3 + 5*x^2 + x^4]) - (19*Sqrt[3 + 5*x^2 + x^4])/
(117*x) - (19*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2
)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[Ar
cTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(234*Sqrt[
3 + 5*x^2 + x^4]) - (4*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 -
Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)
*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/(39*Sqrt[3 + 5*x^2 + x^4])
```

Rubi [A] time = 0.401239, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{19x\left(2x^2+\sqrt{13}+5\right)}{234\sqrt{x^4+5x^2+3}}-\frac{19\sqrt{x^4+5x^2+3}}{117x}-\frac{8x^2+7}{39x\sqrt{x^4+5x^2+3}}-\frac{4\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{39\sqrt{x^4+5x^2+3}}-\frac{19\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{234\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(x^2*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (19*x*(5 + Sqrt[13] + 2*x^2))/(234*Sqrt[3 + 5*x^2 + x^4]) - (7 +
8*x^2)/(39*x*Sqrt[3 + 5*x^2 + x^4]) - (19*Sqrt[3 + 5*x^2 + x^4])/
(117*x) - (19*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2
)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[Ar
cTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(234*Sqrt[
3 + 5*x^2 + x^4]) - (4*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 -
Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)
*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/(39*Sqrt[3 + 5*x^2 + x^4])
```

Rubi in Sympy [A] time = 33.1008, size = 279, normalized size = 0.9

$$\frac{19x\left(2x^2+\sqrt{13}+5\right)}{234\sqrt{x^4+5x^2+3}} - \frac{19\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{1404\sqrt{x^4+5x^2+3}} - \frac{4\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{117\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}} - \frac{8x^2+7}{39x\sqrt{x^4+5x^2+3}} - \frac{19\sqrt{x^4+5x^2+3}}{117x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/x**2/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] 19*x*(2*x**2 + sqrt(13) + 5)/(234*sqrt(x**4 + 5*x**2 + 3)) - 19*s
qrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) + 6))
*sqrt(sqrt(13) + 5)*(x**2*(sqrt(13) + 5) + 6)*elliptic_e(atan(sqr
t(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(1404*sqrt(x*
*4 + 5*x**2 + 3)) - 4*sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x*
*2*(sqrt(13) + 5) + 6))*(x**2*(sqrt(13) + 5) + 6)*elliptic_f(atan
(sqrt(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(117*sqrt
(sqrt(13) + 5)*sqrt(x**4 + 5*x**2 + 3)) - (8*x**2 + 7)/(39*x*sqrt
(x**4 + 5*x**2 + 3)) - 19*sqrt(x**4 + 5*x**2 + 3)/(117*x)
```

Mathematica [C] time = 0.537721, size = 228, normalized size = 0.74

$$\frac{-i\sqrt{2}\left(19\sqrt{13}-143\right)x\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)\middle|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+19i\sqrt{2}\left(\sqrt{13}-5\right)x\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{\ }}{468x\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2 + 3*x^2)/(x^2*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-4*(78 + 119*x^2 + 19*x^4) + (19*I)*Sqrt[2]*(-5 + Sqrt[13])*x*Sq
rt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2
*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sq
rt[13])/6] - I*Sqrt[2]*(-143 + 19*Sqrt[13])*x*Sqrt[(-5 + Sqrt[13]
 - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I
*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(468*
x*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.029, size = 257, normalized size = 0.8

$$-6\frac{1}{\sqrt{x^4+5x^2+3}}\left(-\frac{19x}{78}-\frac{5x^3}{78}\right)$$
$$-\frac{16}{13\sqrt{-30+6\sqrt{13}}}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)\frac{1}{\sqrt{x^4+5x^2+3}}$$
$$-\frac{76}{13\sqrt{-30+6\sqrt{13}}\left(5+\sqrt{13}\right)}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-El\right.$$
$$-4\frac{1}{\sqrt{x^4+5x^2+3}}\left(\frac{19x^3}{234}+\frac{40x}{117}\right)-\frac{2}{9x}\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -6*(-19/78*x-5/78*x^3)/(x^4+5*x^2+3)^(1/2)-16/13/(-30+6*13^(1/2))
^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x
^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1
/2),5/6*3^(1/2)+1/6*39^(1/2))-76/13/(-30+6*13^(1/2))^(1/2)*(1-(-5
/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^
4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(
1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(
1/2),5/6*3^(1/2)+1/6*39^(1/2)))-4*(19/234*x^3+40/117*x)/(x^4+5*x^
2+3)^(1/2)-2/9*(x^4+5*x^2+3)^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{3x^2+2}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,x^2+2}{(x^6+5\,x^4+3\,x^2)\sqrt{x^4+5\,x^2+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^2),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)/((x^6 + 5*x^4 + 3*x^2)*sqrt(x^4 + 5*x^2 + 3)
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{x^2\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**2/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**2*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{(x^4+5\,x^2+3)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^2), x)
```

3.203 $\int \frac{2+3x^2}{x^4(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=326

$$-\frac{133x\left(2x^2+\sqrt{13}+5\right)}{1053\sqrt{x^4+5x^2+3}}+\frac{266\sqrt{x^4+5x^2+3}}{1053x}-\frac{5\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{351\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}+\frac{133\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{1053\sqrt{x^4+5x^2+3}}-\frac{5\sqrt{x^4+5x^2+3}}{351x^3}-\frac{8x^2+7}{39x^3\sqrt{x^4+5x^2+3}}$$

```
[Out] (-133*x*(5 + Sqrt[13] + 2*x^2))/(1053*Sqrt[3 + 5*x^2 + x^4]) - (7
 + 8*x^2)/(39*x^3*Sqrt[3 + 5*x^2 + x^4]) - (5*Sqrt[3 + 5*x^2 + x^
4])/(351*x^3) + (266*Sqrt[3 + 5*x^2 + x^4])/(1053*x) + (133*Sqrt[
(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13
])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt
[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(1053*Sqrt[3 + 5*x^2 + x^4])
- (5*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 +
 (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x],
(-13 + 5*Sqrt[13])/6])/(351*Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2
 + x^4])
```

Rubi [A] time = 0.507465, antiderivative size = 326, normalized size of antiderivative = 1., number

of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{133x\left(2x^2+\sqrt{13}+5\right)}{1053\sqrt{x^4+5x^2+3}}+\frac{266\sqrt{x^4+5x^2+3}}{1053x}-\frac{5\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{351\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}+\frac{133\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{1053\sqrt{x^4+5x^2+3}}-\frac{5\sqrt{x^4+5x^2+3}}{351x^3}-\frac{8x^2+7}{39x^3\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x^2)/(x^4*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-133*x*(5 + Sqrt[13] + 2*x^2))/(1053*Sqrt[3 + 5*x^2 + x^4]) - (7
 + 8*x^2)/(39*x^3*Sqrt[3 + 5*x^2 + x^4]) - (5*Sqrt[3 + 5*x^2 + x^
4])/(351*x^3) + (266*Sqrt[3 + 5*x^2 + x^4])/(1053*x) + (133*Sqrt[
(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13
])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt
[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(1053*Sqrt[3 + 5*x^2 + x^4])
- (5*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 +
 (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x],
(-13 + 5*Sqrt[13])/6])/(351*Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2
 + x^4])
```

Rubi in Sympy [A] time = 40.2134, size = 301, normalized size = 0.92

$$-\frac{133x\left(2x^2+\sqrt{13}+5\right)}{1053\sqrt{x^4+5x^2+3}}+\frac{133\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\sqrt{\sqrt{13}+5}\left(x^2\left(\sqrt{13}+5\right)+6\right)E\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{6318\sqrt{x^4+5x^2+3}}-\frac{5\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{13}+5\right)+6}{x^2\left(\sqrt{13}+5\right)+6}}\left(x^2\left(\sqrt{13}+5\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{13}+5}}{6}\right)\middle|-\frac{13}{6}+\frac{5\sqrt{13}}{6}\right)}{2106\sqrt{\sqrt{13}+5}\sqrt{x^4+5x^2+3}}+\frac{266\sqrt{x^4+5x^2+3}}{1053x}-\frac{8x^2+7}{39x^3\sqrt{x^4+5x^2+3}}-\frac{5\sqrt{x^4+5x^2+3}}{351x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2)/x**4/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] -133*x*(2*x**2 + sqrt(13) + 5)/(1053*sqrt(x**4 + 5*x**2 + 3)) + 1
33*sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)/(x**2*(sqrt(13) + 5) +
 6))*sqrt(sqrt(13) + 5)*(x**2*(sqrt(13) + 5) + 6)*elliptic_e(atan
(sqrt(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(6318*sqr
t(x**4 + 5*x**2 + 3)) - 5*sqrt(6)*sqrt((x**2*(-sqrt(13) + 5) + 6)
/(x**2*(sqrt(13) + 5) + 6))*(x**2*(sqrt(13) + 5) + 6)*elliptic_f(
atan(sqrt(6)*x*sqrt(sqrt(13) + 5)/6), -13/6 + 5*sqrt(13)/6)/(2106
*sqrt(sqrt(13) + 5)*sqrt(x**4 + 5*x**2 + 3)) + 266*sqrt(x**4 + 5*
x**2 + 3)/(1053*x) - (8*x**2 + 7)/(39*x**3*sqrt(x**4 + 5*x**2 + 3
)) - 5*sqrt(x**4 + 5*x**2 + 3)/(351*x**3)
```

Mathematica [C] time = 0.551696, size = 234, normalized size = 0.72

$$\frac{532x^6 + 2630x^4 + 1014x^2 + i\sqrt{2}\left(133\sqrt{13} - 650\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2 + \sqrt{13} + 5}x^3F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6} + \frac{5\sqrt{13}}{6}\right) - 133i}{2106x^3\sqrt{x^4 + 5x^2 + 3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(2 + 3*x^2)/(x^4*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-468 + 1014*x^2 + 2630*x^4 + 532*x^6 - (133*I)*Sqrt[2]*(-5 + Sqr
t[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 +
 Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x],
 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*(-650 + 133*Sqrt[13])*x^3*Sqr
t[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*
x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqr
t[13])/6])/(2106*x^3*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.031, size = 274, normalized size = 0.8

$$-\frac{2}{27x^3}\sqrt{x^4+5x^2+3}+\frac{23}{81x}\sqrt{x^4+5x^2+3}-4\frac{1}{\sqrt{x^4+5x^2+3}}\left(-\frac{40x^3}{351}-\frac{343x}{702}\right)$$
$$-\frac{10}{117\sqrt{-30+6\sqrt{13}}}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,EllipticF\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)\frac{1}{\sqrt{x^4+5x^2+3}}$$
$$+\frac{1064}{117\sqrt{-30+6\sqrt{13}}\left(5+\sqrt{13}\right)}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(EllipticF\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-E\right.$$
$$-6\frac{1}{\sqrt{x^4+5x^2+3}}\left(\frac{19x^3}{234}+\frac{40x}{117}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -2/27*(x^4+5*x^2+3)^(1/2)/x^3+23/81*(x^4+5*x^2+3)^(1/2)/x-4*(-40/
351*x^3-343/702*x)/(x^4+5*x^2+3)^(1/2)-10/117/(-30+6*13^(1/2))^(1
/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)
^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2)
,5/6*3^(1/2)+1/6*39^(1/2))+1064/117/(-30+6*13^(1/2))^(1/2)*(1-(-5
/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^
4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(
1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(
1/2),5/6*3^(1/2)+1/6*39^(1/2)))-6*(19/234*x^3+40/117*x)/(x^4+5*x^
2+3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{(x^4+5x^2+3)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^4),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3x^2+2}{(x^8+5x^6+3x^4)\sqrt{x^4+5x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)/((x^8 + 5*x^6 + 3*x^4)*sqrt(x^4 + 5*x^2 + 3)
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2)/x**4/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{(x^4+5\,x^2+3)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^4),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^4), x)
```

3.204 $\int (fx)^{3/2} \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=297

$$\frac{2d(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{9/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{9}{4};-\frac{1}{2},-\frac{1}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*d*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1/2
, 9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])])/(5*f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (2*e*(f*x)^(9/2)*Sq
rt[a + b*x^2 + c*x^4]*AppellF1[9/4, -1/2, -1/2, 13/4, (-2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(9*
f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 1.12305, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$\frac{2d(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{9/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{9}{4};-\frac{1}{2},-\frac{1}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(f*x)^(3/2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] (2*d*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1/2
, 9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])])/(5*f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (2*e*(f*x)^(9/2)*Sq
rt[a + b*x^2 + c*x^4]*AppellF1[9/4, -1/2, -1/2, 13/4, (-2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(9*
f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 80.8413, size = 269, normalized size = 0.91

$$\frac{2d(fx)^{\frac{5}{2}}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{5}{4},-\frac{1}{2},-\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}+\frac{2e(fx)^{\frac{9}{2}}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{9}{4},-\frac{1}{2},-\frac{1}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**(3/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] 2*d*(f*x)**(5/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(5/4, -1/2, -1
/2, 9/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt
(-4*a*c + b**2)))/(5*f*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) +
1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)) + 2*e*(f*x)**(9/
2)*sqrt(a + b*x**2 + c*x**4)*appellf1(9/4, -1/2, -1/2, 13/4, -2*c
*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**
2)))/(9*f**3*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*
c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 7.46563, size = 2835, normalized size = 9.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(f*x)^(3/2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*((4*(13*b*c*d - 7*b^2*e + 18
*a*c*e)*Sqrt[x])/(585*c^2) + (2*(13*c*d + 2*b*e)*x^(5/2))/(117*c)
 + (2*e*x^(9/2))/13))/x^(3/2) + (4*a^3*b*d*(f*x)^(3/2)*(b - Sqrt[
b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF
1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
2)/(-b + Sqrt[b^2 - 4*a*c])])/(9*c*(b - Sqrt[b^2 - 4*a*c])*(b + S
qrt[b^2 - 4*a*c])*x*(a + b*x^2 + c*x^4)^(3/2)*(-5*a*AppellF1[1/4,
 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
 + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/
4, 1/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4,
3/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c])]))) - (28*a^3*b^2*e*(f*x)^(3/2)*(b - Sqrt[b^
2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[
```

1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)
/(-b + Sqrt[b^2 - 4*a*c])])/(117*c^2*(b - Sqrt[b^2 - 4*a*c])*(b +
 Sqrt[b^2 - 4*a*c])*x*(a + b*x^2 + c*x^4)^(3/2)*(-5*a*AppellF1[1/
4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[
5/4, 1/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)
/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4
, 3/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])]))) + (8*a^4*e*(f*x)^(3/2)*(b - Sqrt[b^2 -
 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[1/4
, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])])/(13*c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[
b^2 - 4*a*c])*x*(a + b*x^2 + c*x^4)^(3/2)*(-5*a*AppellF1[1/4, 1/2
, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 1
/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 3/2,
 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sq
rt[b^2 - 4*a*c])]))) - (8*a^3*d*x*(f*x)^(3/2)*(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1/
2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
Sqrt[b^2 - 4*a*c])])/(5*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4
*a*c])*(a + b*x^2 + c*x^4)^(3/2)*(-9*a*AppellF1[5/4, 1/2, 1/2, 9/
4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 1/2, 3/2,
13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^
2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 3/2, 1/2, 13
/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
- 4*a*c])]))) + (12*a^2*b^2*d*x*(f*x)^(3/2)*(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1/2,
 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sq
rt[b^2 - 4*a*c])])/(25*c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 -
4*a*c])*(a + b*x^2 + c*x^4)^(3/2)*(-9*a*AppellF1[5/4, 1/2, 1/2, 9
/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
- 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 1/2, 3/2,
 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b
^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 3/2, 1/2, 1
3/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
 - 4*a*c])]))) - (84*a^2*b^3*e*x*(f*x)^(3/2)*(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1/2
, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])])/(325*c^2*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^
2 - 4*a*c])*(a + b*x^2 + c*x^4)^(3/2)*(-9*a*AppellF1[5/4, 1/2, 1/
2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[
b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 1/2,
3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sq
rt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 3/2, 1/
2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])]))) + (316*a^3*b*e*x*(f*x)^(3/2)*(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4,
1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c])])/(325*c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b
^2 - 4*a*c])*(a + b*x^2 + c*x^4)^(3/2)*(-9*a*AppellF1[5/4, 1/2, 1
/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt

```
[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 1/2,
 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 3/2, 1
/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqr
t[b^2 - 4*a*c])])))
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int (fx)^{\frac{3}{2}} \left(e x^2 + d\right) \sqrt{c x^4 + b x^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c x^4 + b x^2 + a}\left(e x^2 + d\right) (fx)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(efx^3 + dfx\right)\sqrt{c x^4 + b x^2 + a}\sqrt{fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e*f*x^3 + d*f*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (fx)^{\frac{3}{2}} \left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**(3/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((f*x)**(3/2)*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+bx^2+a}\left(ex^2+d\right)(fx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^(3/2), x)
```

3.205 $\int \sqrt{fx}\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=297

$$\frac{2d(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{7/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{7}{4};-\frac{1}{2},-\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*d*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2, -1/2
, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])])/(3*f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (2*e*(f*x)^(7/2)*Sq
rt[a + b*x^2 + c*x^4]*AppellF1[7/4, -1/2, -1/2, 11/4, (-2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*
f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.999333, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$\frac{2d(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{7/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{7}{4};-\frac{1}{2},-\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[f*x]*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] (2*d*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2, -1/2
, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])])/(3*f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (2*e*(f*x)^(7/2)*Sq
rt[a + b*x^2 + c*x^4]*AppellF1[7/4, -1/2, -1/2, 11/4, (-2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*
f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 79.7549, size = 269, normalized size = 0.91

$$\frac{2d(fx)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{3}{4},-\frac{1}{2},-\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}+\frac{2e(fx)^{\frac{7}{2}}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{7}{4},-\frac{1}{2},-\frac{1}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**(1/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] 2*d*(f*x)**(3/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(3/4, -1/2, -1
/2, 7/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt
(-4*a*c + b**2)))/(3*f*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) +
1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)) + 2*e*(f*x)**(7/
2)*sqrt(a + b*x**2 + c*x**4)*appellf1(7/4, -1/2, -1/2, 11/4, -2*c
*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**
2)))/(7*f**3*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*
c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 4.83457, size = 1717, normalized size = 5.78

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[f*x]*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[f*x]*(42*c*(11*c*d + 2*b*e + 7*c*e*x^2)*(a + b*x^2 + c*x^
4)^2 + (1078*a^2*c*d*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2)*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(7*a
*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c
])*AppellF1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])
*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])) - (147*a^2*b*e*(b - Sqrt[
b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF
1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
2)/(-b + Sqrt[b^2 - 4*a*c])])/(7*a*AppellF1[3/4, 1/2, 1/2, 7/4, (
-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a
```

```
*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 1/2, 3/2, 11/4
, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/4,
(-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*
a*c])])) + (363*a*b*c*d*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]
)])/(11*a*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b
^2 - 4*a*c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^
2 - 4*a*c])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])) + (462*a^2*c*
e*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(11*a*AppellF1[7/4
, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[
11/4, 1/2, 3/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[1
1/4, 3/2, 1/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2
)/(-b + Sqrt[b^2 - 4*a*c])])) + (165*a*b^2*e*x^2*(b - Sqrt[b^2 -
4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[7/4,
 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])])/(-11*a*AppellF1[7/4, 1/2, 1/2, 11/4, (-2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c
])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 1/2, 3/2, 15/4,
 (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4
*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 3/2, 1/2, 15/4,
(-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*
a*c])]))))/(1617*c^2*(a + b*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \sqrt{fx}\,(e x^2 + d)\,\sqrt{c x^4 + b x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)

[Out] int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c x^4 + b x^2 + a}\,(e x^2 + d)\,\sqrt{fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + bx^2 + a}(ex^2 + d)\sqrt{fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{fx}\left(d + ex^2\right)\sqrt{a + bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(1/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f*x)*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}(ex^2 + d)\sqrt{fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x), x)
```

3.206 $\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx$

Optimal. Leaf size=295

$$\frac{2d\sqrt{fx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*d*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -1/2, -1/2,
5/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])])/(f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (2*e*(f*x)^(5/2)*Sqrt[a
 + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1/2, 9/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f^3*S
qrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.994622, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$\frac{2d\sqrt{fx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/Sqrt[f*x],x]
```


```
[Out] (2*d*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -1/2, -1/2,
5/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])])/(f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (2*e*(f*x)^(5/2)*Sqrt[a
 + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1/2, 9/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f^3*S
qrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 79.8827, size = 267, normalized size = 0.91

$$\frac{2d\sqrt{fx}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{f\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}+\frac{2e(fx)^{\frac{5}{2}}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{5}{4},-\frac{1}{2},-\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(1/2)/(f*x)**(1/2),x)
```

```
[Out] 2*d*sqrt(f*x)*sqrt(a + b*x**2 + c*x**4)*appellf1(1/4, -1/2, -1/2,
 5/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4
*a*c + b**2)))/(f*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sq
rt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)) + 2*e*(f*x)**(5/2)*sq
rt(a + b*x**2 + c*x**4)*appellf1(5/4, -1/2, -1/2, 9/4, -2*c*x**2/
(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**2)))/(
5*f**3*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x**2
/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 4.93912, size = 1717, normalized size = 5.82

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/Sqrt[f*x],x]
```

```
[Out] (x*(10*c*(9*c*d + 2*b*e + 5*c*e*x^2)*(a + b*x^2 + c*x^4)^2 + (450
*a^2*c*d*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(5*a*AppellF1[1/
4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[
5/4, 1/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)
/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4
, 3/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])])) - (25*a^2*b*e*(b - Sqrt[b^2 - 4*a*c] +
2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[1/4, 1/2, 1/2
, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b
^2 - 4*a*c])])/(5*a*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b
```

```
 + Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 1/2, 3/2, 9/4, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b -
Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 3/2, 1/2, 9/4, (-2*c*x^2)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])) + (81*a*
b*c*d*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^2)*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(9*a*AppellF1[5
/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/
(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1
[9/4, 1/2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9
/4, 3/2, 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)
/(-b + Sqrt[b^2 - 4*a*c])])) + (90*a^2*c*e*x^2*(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1
/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])])/(9*a*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] -
x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 1/2, 3/2, 13/4, (-2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]
 + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 3/2, 1/2, 13/4, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))
 + (27*a*b^2*e*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(-9*a*
AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c]
)*AppellF1[9/4, 1/2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*
AppellF1[9/4, 3/2, 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))))/(225*c^2*Sqrt[f*x]*(a +
b*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int (ex^2 + d)\sqrt{cx^4 + bx^2 + a}\frac{1}{\sqrt{fx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x)
```

```
[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)}{\sqrt{fx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/sqrt(f*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/sqrt(f*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)}{\sqrt{fx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/sqrt(f*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/sqrt(f*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex^2\right)\sqrt{a + bx^2 + cx^4}}{\sqrt{fx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(1/2)/(f*x)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)/sqrt(f*x), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)}{\sqrt{fx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/sqrt(f*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/sqrt(f*x), x)
```

3.207 $\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx$

Optimal. Leaf size=295

$$\frac{2e(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{2d\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (-2*d*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -1/2, -1/2, 3/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])])/(f*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (2*e*(f*x)^(3/2)*Sqrt
[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2, -1/2, 7/4, (-2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f^3
*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.999009, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$\frac{2e(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{2d\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(f*x)^(3/2),x]
```

```
[Out] (-2*d*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -1/2, -1/2, 3/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])])/(f*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (2*e*(f*x)^(3/2)*Sqrt
[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2, -1/2, 7/4, (-2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f^3
*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 80.8162, size = 269, normalized size = 0.91

$$-\frac{2d\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(-\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}+\frac{2e\,(fx)^{\frac{3}{2}}\,\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{3}{4},-\frac{1}{2},-\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(1/2)/(f*x)**(3/2),x)
```

```
[Out] -2*d*sqrt(a + b*x**2 + c*x**4)*appellf1(-1/4, -1/2, -1/2, 3/4, -2
*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b
**2)))/(f*sqrt(f*x)*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*
sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)) + 2*e*(f*x)**(3/2)*
sqrt(a + b*x**2 + c*x**4)*appellf1(3/4, -1/2, -1/2, 7/4, -2*c*x**
2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**2)))
/(3*f**3*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x*
*2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 2.73966, size = 1383, normalized size = 4.69

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(f*x)^(3/2),x]
```

```
[Out] (x*(42*(-7*d + e*x^2)*(a + b*x^2 + c*x^4)^2 + (343*a*b*d*x^2*(b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*A
ppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(c*(7*a*AppellF1[3/4, 1/2, 1/
2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[
b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 1/2,
3/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sq
rt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 3/2, 1/
2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])]))) + (98*a^2*e*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x
^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[3/4, 1/2, 1/2, 7/4
, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
4*a*c])])/(c*(7*a*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b +
```

```
 Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - S
qrt[b^2 - 4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) + (462*
a*d*x^4*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(11*a*AppellF1[7
/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)
/(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF
1[11/4, 1/2, 3/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1
[11/4, 3/2, 1/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^2)/(-b + Sqrt[b^2 - 4*a*c])])) + (33*a*b*e*x^4*(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[7/4,
1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
 + Sqrt[b^2 - 4*a*c])])/(c*(11*a*AppellF1[7/4, 1/2, 1/2, 11/4, (-
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*
c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 1/2, 3/2, 15/4
, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 3/2, 1/2, 15/4,
 (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4
*a*c])])))))/(147*(f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int (ex^2 + d)\sqrt{cx^4 + bx^2 + a}\,(fx)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x)

[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}\,(ex^2 + d)}{(fx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/(f*x)^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/(f*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}(ex^2+d)}{\sqrt{fx}fx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/(f*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/(sqrt(f*x)*f*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(1/2)/(f*x)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)/(f*x)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{cx^4+bx^2+a}(ex^2+d)}{(fx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/(f*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/(f*x)^(3/2), x)
```

3.208 $\int (fx)^{3/2} \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=299

$$\frac{2ad(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{9/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{9}{4};-\frac{3}{2},-\frac{3}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*a*d*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2, -3
/2, 9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])])/(5*f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (2*a*e*(f*x)^(9/2
)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[9/4, -3/2, -3/2, 13/4, (-2*c*x
^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
/(9*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 1.00454, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$\frac{2ad(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{9/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{9}{4};-\frac{3}{2},-\frac{3}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```



```
[Out] (2*a*d*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2, -3
/2, 9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])])/(5*f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (2*a*e*(f*x)^(9/2
)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[9/4, -3/2, -3/2, 13/4, (-2*c*x
^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
/(9*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 91.9624, size = 272, normalized size = 0.91

$$\frac{2ad\,(fx)^{\frac{5}{2}}\,\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(\frac{5}{4},-\frac{3}{2},-\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}+\frac{2ae\,(fx)^{\frac{9}{2}}\,\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(\frac{9}{4},-\frac{3}{2},-\frac{3}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**(3/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] 2*a*d*(f*x)**(5/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(5/4, -3/2,
-3/2, 9/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sq
rt(-4*a*c + b**2)))/(5*f*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2))
+ 1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)) + 2*a*e*(f*x)*
*(9/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(9/4, -3/2, -3/2, 13/4,
-2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c +
 b**2)))/(9*f**3*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sqr
t(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 6.16801, size = 4499, normalized size = 15.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*((8*(-147*b^3*c*d + 924*a*b*
c^2*d + 77*b^4*e - 501*a*b^2*c*e + 612*a^2*c^2*e)*Sqrt[x])/(69615
*c^3) + (2*(84*b^2*c*d + 1911*a*c^2*d - 44*b^3*e + 240*a*b*c*e)*x
^(5/2))/(13923*c^2) + (2*(399*b*c*d + 12*b^2*e + 425*a*c*e)*x^(9/
2))/(4641*c) + (2*(21*c*d + 23*b*e)*x^(13/2))/357 + (2*c*e*x^(17/
2))/21))/x^(3/2) - (56*a^3*b^3*d*(f*x)^(3/2)*(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[1/4, 1/2
, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])])/(663*c^2*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^
2 - 4*a*c])*x*(a + b*x^2 + c*x^4)^(3/2)*(-5*a*AppellF1[1/4, 1/2,
1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqr
t[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 1/2
, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 3/2, 1
```

/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])]))) + (352*a^4*b*d*(f*x)^(3/2)*(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[1/4, 1/
2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
Sqrt[b^2 - 4*a*c])])/(663*c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2
 - 4*a*c])*x*(a + b*x^2 + c*x^4)^(3/2)*(-5*a*AppellF1[1/4, 1/2, 1
/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 1/2,
 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sq
rt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 3/2, 1/
2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[
b^2 - 4*a*c])]))) + (88*a^3*b^4*e*(f*x)^(3/2)*(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[1/4, 1/
2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
Sqrt[b^2 - 4*a*c])])/(1989*c^3*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[
b^2 - 4*a*c])*x*(a + b*x^2 + c*x^4)^(3/2)*(-5*a*AppellF1[1/4, 1/2
, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 1
/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 3/2,
 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sq
rt[b^2 - 4*a*c])]))) - (1336*a^4*b^2*e*(f*x)^(3/2)*(b - Sqrt[b^2
- 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[1/
4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])])/(4641*c^2*(b - Sqrt[b^2 - 4*a*c])*(b +
Sqrt[b^2 - 4*a*c])*x*(a + b*x^2 + c*x^4)^(3/2)*(-5*a*AppellF1[1/4
, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5
/4, 1/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/
(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4,
 3/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
 + Sqrt[b^2 - 4*a*c])]))) + (32*a^5*e*(f*x)^(3/2)*(b - Sqrt[b^2 -
 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[1/4
, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])])/(91*c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[
b^2 - 4*a*c])*x*(a + b*x^2 + c*x^4)^(3/2)*(-5*a*AppellF1[1/4, 1/2
, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 1
/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 3/2,
 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sq
rt[b^2 - 4*a*c])]))) - (96*a^4*d*x*(f*x)^(3/2)*(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1
/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])])/(85*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 -
 4*a*c])*(a + b*x^2 + c*x^4)^(3/2)*(-9*a*AppellF1[5/4, 1/2, 1/2,
9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 1/2, 3/2
, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[
b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 3/2, 1/2,
13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^
2 - 4*a*c])]))) - (504*a^2*b^4*d*x*(f*x)^(3/2)*(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1
/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +

```
 Sqrt[b^2 - 4*a*c])])/(5525*c^2*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt
[b^2 - 4*a*c])*(a + b*x^2 + c*x^4)^(3/2)*(-9*a*AppellF1[5/4, 1/2,
 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sq
rt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 1/
2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 3/2,
 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])]))) + (3768*a^3*b^2*d*x*(f*x)^(3/2)*(b - Sqrt[b
^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1
[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2
)/(-b + Sqrt[b^2 - 4*a*c])])/(5525*c*(b - Sqrt[b^2 - 4*a*c])*(b +
 Sqrt[b^2 - 4*a*c])*(a + b*x^2 + c*x^4)^(3/2)*(-9*a*AppellF1[5/4,
 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
 + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[9/
4, 1/2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/
(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/4,
 3/2, 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])]))) + (264*a^2*b^5*e*x*(f*x)^(3/2)*(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*Appe
llF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(5525*c^3*(b - Sqrt[b^2 - 4*a*c]
)*(b + Sqrt[b^2 - 4*a*c])*(a + b*x^2 + c*x^4)^(3/2)*(-9*a*AppellF
1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*Appel
lF1[9/4, 1/2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF
1[9/4, 3/2, 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^2)/(-b + Sqrt[b^2 - 4*a*c])]))) - (2032*a^3*b^3*e*x*(f*x)^(3/2)*
(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2)*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(5525*c^2*(b - Sqrt[b^2 -
 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + b*x^2 + c*x^4)^(3/2)*(-9*a*
AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c]
)*AppellF1[9/4, 1/2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*
AppellF1[9/4, 3/2, 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) + (26688*a^4*b*e*x*(f*x)^
(3/2)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(38675*c*(b - Sqrt[
b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + b*x^2 + c*x^4)^(3/2)*(
-9*a*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4
*a*c])*AppellF1[9/4, 1/2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a
*c])*AppellF1[9/4, 3/2, 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int (fx)^{\frac{3}{2}} \left(e x^{2}+d\right) \left(c x^{4}+b x^{2}+a\right)^{\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int {\left(c x^{4} + b x^{2} + a\right)}^{\frac{3}{2}} {\left(e x^{2} + d\right)} \left(f x\right)^{\frac{3}{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^(3/2),x, algorithm="maxima
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left({\left(c e f x^{7} + {\left(c d + b e\right)} f x^{5} + {\left(b d + a e\right)} f x^{3} + a d f x\right)} \sqrt{c x^{4} + b x^{2} + a} \sqrt{f x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^(3/2),x, algorithm="fricas
```

```
[Out] integral((c*e*f*x^7 + (c*d + b*e)*f*x^5 + (b*d + a*e)*f*x^3 + a*d
*f*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**(3/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.796536, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^(3/2),x, algorithm="giac")
```

```
[Out] Done
```

3.209 $\int \sqrt{fx}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=299

$$\frac{2ad(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{7/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{7}{4};-\frac{3}{2},-\frac{3}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*a*d*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3/2, -3
/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])])/(3*f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (2*a*e*(f*x)^(7/2
)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[7/4, -3/2, -3/2, 11/4, (-2*c*x
^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
/(7*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 1.00885, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$\frac{2ad(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{7/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{7}{4};-\frac{3}{2},-\frac{3}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (2*a*d*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3/2, -3
/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])])/(3*f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (2*a*e*(f*x)^(7/2
)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[7/4, -3/2, -3/2, 11/4, (-2*c*x
^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
/(7*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 91.8881, size = 272, normalized size = 0.91

$$\frac{2ad(fx)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{3}{4},-\frac{3}{2},-\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}+\frac{2ae(fx)^{\frac{7}{2}}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{7}{4},-\frac{3}{2},-\frac{3}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**(1/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] 2*a*d*(f*x)**(3/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(3/4, -3/2,
-3/2, 7/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sq
rt(-4*a*c + b**2)))/(3*f*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2))
+ 1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)) + 2*a*e*(f*x)*
*(7/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(7/4, -3/2, -3/2, 11/4,
-2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c +
 b**2)))/(7*f**3*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sqr
t(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 6.15074, size = 3656, normalized size = 12.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]*((2*(228*b^2*c*d + 3971*a*c^2*
d - 108*b^3*e + 624*a*b*c*e)*x^(3/2))/(21945*c^2) + (2*(323*b*c*d
 + 12*b^2*e + 345*a*c*e)*x^(7/2))/(3135*c) + (2*(19*c*d + 21*b*e)
*x^(11/2))/285 + (2*c*e*x^(15/2))/19))/Sqrt[x] - (32*a^4*d*x*Sqrt
[f*x]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(15*(b - Sqrt[b^2 -
 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + b*x^2 + c*x^4)^(3/2)*(-7*a*
AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c]
)*AppellF1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*
AppellF1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) + (8*a^3*b^2*d*x*Sqrt[f*x
```

```
]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(55*c*(b - Sqrt[b^2 - 4
*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + b*x^2 + c*x^4)^(3/2)*(-7*a*Ap
pellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*
AppellF1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Ap
pellF1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) - (72*a^3*b^3*e*x*Sqrt[f*x]
*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2)*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(1045*c^2*(b - Sqrt[b^2
- 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + b*x^2 + c*x^4)^(3/2)*(-7*a
*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c
])*AppellF1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])
*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) + (416*a^4*b*e*x*Sqrt[f*
x]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2)*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(1045*c*(b - Sqrt[b^2
- 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + b*x^2 + c*x^4)^(3/2)*(-7*a
*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c
])*AppellF1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])
*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) - (288*a^3*b*d*x^3*Sqrt[
f*x]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(245*(b - Sqrt[b^2
- 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + b*x^2 + c*x^4)^(3/2)*(-11*
a*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a
*c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*
c])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) + (8*a^2*b^3*d*x^3*S
qrt[f*x]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(49*c*(b - Sqrt
[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + b*x^2 + c*x^4)^(3/2)*
(-11*a*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2
- 4*a*c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 -
 4*a*c])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) - (96*a^4*e*x^3
*Sqrt[f*x]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(133*(b - Sqr
t[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + b*x^2 + c*x^4)^(3/2)
```

```
*(-11*a*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2
 - 4*a*c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2
- 4*a*c])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) - (72*a^2*b^4*
e*x^3*Sqrt[f*x]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(931*c^2
*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + b*x^2 + c*x
^4)^(3/2)*(-11*a*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b +
 Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b -
Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) + (24
72*a^3*b^2*e*x^3*Sqrt[f*x]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])
])/(4655*c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + b
*x^2 + c*x^4)^(3/2)*(-11*a*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] +
 x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]
)] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])
])))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \sqrt{fx}\,\left(e x^2+d\right)\,\left(c x^4+b x^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)

[Out] int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c x^4+b x^2+a\right)^{\frac{3}{2}}\left(e x^2+d\right)\sqrt{fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*sqrt(f*x),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*sqrt(f*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cex^6 + (cd + be)x^4 + (bd + ae)x^2 + ad\right)\sqrt{cx^4 + bx^2 + a}\sqrt{fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*sqrt(f*x),x, algorithm="fricas")
```

```
[Out] integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt
(c*x^4 + b*x^2 + a)*sqrt(f*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{fx}\left(d + ex^2\right)\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**(1/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(sqrt(f*x)*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [A] time = 0.754784, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*sqrt(f*x),x, algorithm="giac")
```

```
[Out] Done
```

3.210 $\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx$

Optimal. Leaf size=297

$$\frac{2ad\sqrt{fx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*a*d*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -3/2, -3/2
, 5/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])])/(f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (2*a*e*(f*x)^(5/2)*Sq
rt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2, -3/2, 9/4, (-2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f
^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 1.01361, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$\frac{2ad\sqrt{fx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/Sqrt[f*x],x]
```

```
[Out] (2*a*d*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -3/2, -3/2
, 5/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])])/(f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (2*a*e*(f*x)^(5/2)*Sq
rt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2, -3/2, 9/4, (-2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f
^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 90.8512, size = 270, normalized size = 0.91

$$\frac{2ad\sqrt{fx}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{f\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}+\frac{2ae\,(fx)^{\frac{5}{2}}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{5}{4},-\frac{3}{2},-\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(3/2)/(f*x)**(1/2),x)
```

```
[Out] 2*a*d*sqrt(f*x)*sqrt(a + b*x**2 + c*x**4)*appellf1(1/4, -3/2, -3/
2, 5/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(
-4*a*c + b**2)))/(f*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*
sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)) + 2*a*e*(f*x)**(5/2
)*sqrt(a + b*x**2 + c*x**4)*appellf1(5/4, -3/2, -3/2, 9/4, -2*c*x
**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**2)
))/(5*f**3*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*
x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 6.14324, size = 3656, normalized size = 12.31

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/Sqrt[f*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*((2*(68*b^2*c*d + 867*a*c^2*d -
28*b^3*e + 176*a*b*c*e)*Sqrt[x])/(3315*c^2) + (2*(85*b*c*d + 4*b^
2*e + 91*a*c*e)*x^(5/2))/(663*c) + (2*(17*c*d + 19*b*e)*x^(9/2))/
221 + (2*c*e*x^(13/2))/17))/Sqrt[f*x] - (96*a^4*d*x*(b - Sqrt[b^2
 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[1
/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/
(-b + Sqrt[b^2 - 4*a*c])])/(13*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[
b^2 - 4*a*c])*Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)*(-5*a*AppellF1[
1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)
/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF
1[5/4, 1/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5
/4, 3/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/
(-b + Sqrt[b^2 - 4*a*c])]))) + (8*a^3*b^2*d*x*(b - Sqrt[b^2 - 4*a
```

*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[1/4, 1/
2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
Sqrt[b^2 - 4*a*c])])/(39*c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2
- 4*a*c])*Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)*(-5*a*AppellF1[1/4,
 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
 + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/
4, 1/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4,
3/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c])]))) - (56*a^3*b^3*e*x*(b - Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[1/4, 1/2,
1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqr
t[b^2 - 4*a*c])])/(663*c^2*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2
- 4*a*c])*Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)*(-5*a*AppellF1[1/4,
 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
 + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/
4, 1/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4,
3/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c])]))) + (352*a^4*b*e*x*(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[1/4, 1/2, 1
/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])])/(663*c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4
*a*c])*Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)*(-5*a*AppellF1[1/4, 1/
2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/4,
1/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 3/2
, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])]))) - (672*a^3*b*d*x^3*(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1/2, 1/
2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[
b^2 - 4*a*c])])/(325*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*
c])*Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)*(-9*a*AppellF1[5/4, 1/2,
1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqr
t[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 1/2
, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 3/2,
1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sq
rt[b^2 - 4*a*c])]))) + (72*a^2*b^3*d*x^3*(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1/2, 1/
2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[
b^2 - 4*a*c])])/(325*c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*
a*c])*Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)*(-9*a*AppellF1[5/4, 1/2
, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 1
/2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 3/2
, 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
Sqrt[b^2 - 4*a*c])]))) - (96*a^4*e*x^3*(b - Sqrt[b^2 - 4*a*c] + 2
*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1/2, 1/2,
 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^
2 - 4*a*c])])/(85*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])
*Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)*(-9*a*AppellF1[5/4, 1/2, 1/2

```
, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b
^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 1/2, 3
/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqr
t[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 3/2, 1/2
, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[
b^2 - 4*a*c])]))) - (504*a^2*b^4*e*x^3*(b - Sqrt[b^2 - 4*a*c] + 2
*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1/2, 1/2,
 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^
2 - 4*a*c])])/(5525*c^2*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4
*a*c])*Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)*(-9*a*AppellF1[5/4, 1/
2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[9/4,
1/2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
 + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 3/
2, 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])]))) + (3768*a^3*b^2*e*x^3*(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1/
2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
Sqrt[b^2 - 4*a*c])])/(5525*c*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^
2 - 4*a*c])*Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)*(-9*a*AppellF1[5/
4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[
9/4, 1/2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2
)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/
4, 3/2, 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/
(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \left(e x^{2}+d\right) \left(c x^{4}+b x^{2}+a\right)^{\frac{3}{2}} \frac{1}{\sqrt{f x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x)

[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c x^{4}+b x^{2}+a\right)^{\frac{3}{2}} \left(e x^{2}+d\right)}{\sqrt{f x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/sqrt(f*x),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/sqrt(f*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cex^6 + (cd + be)x^4 + (bd + ae)x^2 + ad\right)\sqrt{cx^4 + bx^2 + a}}{\sqrt{fx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/sqrt(f*x),x, algorithm="fricas")
```

```
[Out] integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt
(c*x^4 + b*x^2 + a)/sqrt(f*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex^2\right)\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{\sqrt{fx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(3/2)/(f*x)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)/sqrt(f*x), x)
```

GIAC/XCAS [A] time = 0.691602, size = 1, normalized size = 0.

$$\mathit{Done}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/sqrt(f*x),x, algorithm="giac")
```

```
[Out] Done
```

3.211 $\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx$

Optimal. Leaf size=297

$$\frac{2ae(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{2ad\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (-2*a*d*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -3/2, -3/2, 3/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])])/(f*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (2*a*e*(f*x)^(3/2)*
Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3/2, -3/2, 7/4, (-2*c*x^2)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3
*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 1.02826, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$\frac{2ae(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{2ad\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2),x]
```

```
[Out] (-2*a*d*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -3/2, -3/2, 3/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])])/(f*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (2*a*e*(f*x)^(3/2)*
Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3/2, -3/2, 7/4, (-2*c*x^2)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3
*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 91.8341, size = 272, normalized size = 0.92

$$-\frac{2ad\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(-\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}+\frac{2ae\,(fx)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(\frac{3}{4},-\frac{3}{2},-\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(3/2)/(f*x)**(3/2),x)
```

```
[Out] -2*a*d*sqrt(a + b*x**2 + c*x**4)*appellf1(-1/4, -3/2, -3/2, 3/4,
-2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c +
 b**2)))/(f*sqrt(f*x)*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1
)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)) + 2*a*e*(f*x)**(3
/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(3/4, -3/2, -3/2, 7/4, -2*c
*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**
2)))/(3*f**3*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*
c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 6.14399, size = 2839, normalized size = 9.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2),x]
```

```
[Out] (x^(3/2)*Sqrt[a + b*x^2 + c*x^4]*((-2*a*d)/Sqrt[x] + (2*(195*b*c*
d + 12*b^2*e + 209*a*c*e)*x^(3/2))/(1155*c) + (2*(15*c*d + 17*b*e
)*x^(7/2))/165 + (2*c*e*x^(11/2))/15))/(f*x)^(3/2) - (128*a^3*b*d
*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2)*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(11*(b - Sqrt[b^2 -
4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(f*x)^(3/2)*(a + b*x^2 + c*x^4)^(
3/2)*(-7*a*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b
^2 - 4*a*c])*AppellF1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2
 - 4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) - (32*a^4*e*x^
3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*
```

x^2)*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(15*(b - Sqrt[b^2 - 4*a
c])(b + Sqrt[b^2 - 4*a*c])*(f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2
)*(-7*a*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2
- 4*a*c])*AppellF1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 -
4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) + (8*a^3*b^2*e*x^
3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(55*c*(b - Sqrt[b^2 - 4
*a*c])*(b + Sqrt[b^2 - 4*a*c])*(f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3
/2)*(-7*a*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^
2 - 4*a*c])*AppellF1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2
- 4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) - (24*a^2*b^2*d
x^5(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(49*(b - Sqrt[b^2 -
4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(f*x)^(3/2)*(a + b*x^2 + c*x^4)^
(3/2)*(-11*a*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqr
t[b^2 - 4*a*c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt
[b^2 - 4*a*c])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) - (96*a^3
*c*d*x^5*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(7*(b - Sqrt[b^
2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(f*x)^(3/2)*(a + b*x^2 + c*x^
4)^(3/2)*(-11*a*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b +
Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - S
qrt[b^2 - 4*a*c])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) - (288
*a^3*b*e*x^5*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(245*(b - S
qrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(f*x)^(3/2)*(a + b*x^2
+ c*x^4)^(3/2)*(-11*a*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*
((b + Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] +
(b - Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))
+ (8*a^2*b^3*e*x^5*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(49*c
*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(f*x)^(3/2)*(a +
b*x^2 + c*x^4)^(3/2)*(-11*a*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*

```
x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]
 + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*
c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c
])])))
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int \left(e x^{2}+d\right) \left(c x^{4}+b x^{2}+a\right)^{\frac{3}{2}} \left(f x\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x)

[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(c x^{4} + b x^{2} + a\right)}^{\frac{3}{2}} {\left(e x^{2} + d\right)}}{\left(f x\right)^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/(f*x)^(3/2),x, algorithm="maxima

[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/(f*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{{\left(c e x^{6} + {\left(c d + b e\right)} x^{4} + {\left(b d + a e\right)} x^{2} + a d\right)} \sqrt{c x^{4} + b x^{2} + a}}{\sqrt{f x} f x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/(f*x)^(3/2),x, algorithm="fricas
```

```
[Out] integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt
(c*x^4 + b*x^2 + a)/(sqrt(f*x)*f*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex^2)\,(a + bx^2 + cx^4)^{\frac{3}{2}}}{(fx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(3/2)/(f*x)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)/(f*x)**(3/2),
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^2 + a)^{\frac{3}{2}}(ex^2 + d)}{(fx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/(f*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/(f*x)^(3/2), x)
```

3.212 $\int \frac{(fx)^{3/2}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=297

$$\frac{2d(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{9/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{9}{4};\frac{1}{2},\frac{1}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*d*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9
/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])/(5*f*Sqrt[a + b*x^2 + c*x^4]) + (2*e*(f*x)^(9/2)*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[9/4, 1/2, 1/2, 13/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(9*f^3*S
qrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 1.00123, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$\frac{2d(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{9/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{9}{4};\frac{1}{2},\frac{1}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[((f*x)^(3/2)*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] (2*d*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9
/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])/(5*f*Sqrt[a + b*x^2 + c*x^4]) + (2*e*(f*x)^(9/2)*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[9/4, 1/2, 1/2, 13/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(9*f^3*S
qrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 81.5301, size = 265, normalized size = 0.89

$$\frac{2d(fx)^{\frac{5}{2}}\sqrt{a+bx^2+cx^4}\,\mathrm{appellf_1}\left(\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{5af\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}+\frac{2e(fx)^{\frac{9}{2}}\sqrt{a+bx^2+cx^4}\,\mathrm{appellf_1}\left(\frac{9}{4},\frac{1}{2},\frac{1}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{9af^3\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**(3/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] 2*d*(f*x)**(5/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(5/4, 1/2, 1/2
, 9/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-
4*a*c + b**2)))/(5*a*f*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) +
1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)) + 2*e*(f*x)**(9/
2)*sqrt(a + b*x**2 + c*x**4)*appellf1(9/4, 1/2, 1/2, 13/4, -2*c*x
**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**2)
))/(9*a*f**3*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*
c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 2.28991, size = 1037, normalized size = 3.49

$$f\sqrt{fx}\left(\frac{25e\left(-2cx^2-b+\sqrt{b^2-4ac}\right)\left(2cx^2+b+\sqrt{b^2-4ac}\right)F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)a^2}{5aF_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-x^2\left(\left(b+\sqrt{b^2-4ac}\right)F_1\left(\frac{5}{4};\frac{1}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{5}{4};\frac{3}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}\ldots}\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((f*x)^(3/2)*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (f*Sqrt[f*x]*(20*c*e*(a + b*x^2 + c*x^4)^2 + (25*a^2*e*(-b + Sqrt
[b^2 - 4*a*c] - 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*Appell
F1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^2)/(-b + Sqrt[b^2 - 4*a*c])])/(5*a*AppellF1[1/4, 1/2, 1/2, 5/4,
(-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*
a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 1/2, 3/2, 9/4
, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 3/2, 1/2, 9/4, (
-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a
*c])])) + (45*a*c*d*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(
```

```
9*a*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*
a*c])*AppellF1[9/4, 1/2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*
c])*AppellF1[9/4, 3/2, 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])) + (27*a*b*e*x^2*(b - S
qrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*App
ellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(-9*a*AppellF1[5/4, 1/2, 1/2, 9
/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
- 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 1/2, 3/2,
 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b
^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 3/2, 1/2, 1
3/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
 - 4*a*c])]))))/(50*c^2*(a + b*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \left(e x^{2}+d\right) \left(f x\right)^{\frac{3}{2}} \frac{1}{\sqrt{c x^{4}+b x^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e x^{2}+d\right) \left(f x\right)^{\frac{3}{2}}}{\sqrt{c x^{4}+b x^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*(f*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(e f x^{3}+d f x\right) \sqrt{f x}}{\sqrt{c x^{4}+b x^{2}+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*(f*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((e*f*x^3 + d*f*x)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^{\frac{3}{2}}\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**(3/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((f*x)**(3/2)*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)(fx)^{\frac{3}{2}}}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*(f*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.213 $\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=297

$$\frac{2d(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{7/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*d*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7
/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])/(3*f*Sqrt[a + b*x^2 + c*x^4]) + (2*e*(f*x)^(7/2)*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*f^3*S
qrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 1.01356, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$\frac{2d(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{7/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[f*x]*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] (2*d*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7
/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])/(3*f*Sqrt[a + b*x^2 + c*x^4]) + (2*e*(f*x)^(7/2)*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*f^3*S
qrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 82.8591, size = 265, normalized size = 0.89

$$\frac{2d\,(fx)^{\frac{3}{2}}\,\sqrt{a+bx^2+cx^4}\operatorname{appellf}_1\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{3af\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}+\frac{2e\,(fx)^{\frac{7}{2}}\,\sqrt{a+bx^2+cx^4}\operatorname{appellf}_1\left(\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{7af^3\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**(1/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] 2*d*(f*x)**(3/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(3/4, 1/2, 1/2
, 7/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-
4*a*c + b**2)))/(3*a*f*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) +
1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)) + 2*e*(f*x)**(7/
2)*sqrt(a + b*x**2 + c*x**4)*appellf1(7/4, 1/2, 1/2, 11/4, -2*c*x
**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**2)
))/(7*a*f**3*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*
c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 0.797241, size = 642, normalized size = 2.16

$$ax\sqrt{fx}\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(-\frac{49dF_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2c}{b+\sqrt{b}}\right.}{x^2\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{7}{4};\frac{1}{2},\frac{3}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{7}{4};\frac{3}{2}\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(Sqrt[f*x]*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (a*x*Sqrt[f*x]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 -
4*a*c] + 2*c*x^2)*((-49*d*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(-
7*a*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*
a*c])*AppellF1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*
c])*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])) - (33*e*x^2*AppellF1[7
/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)
/(-b + Sqrt[b^2 - 4*a*c])])/(-11*a*AppellF1[7/4, 1/2, 1/2, 11/4,
(-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*
```

```
a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 1/2, 3/2, 15
/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
- 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 3/2, 1/2, 15/
4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
 4*a*c])]))))/(42*c*(a + b*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int (ex^2 + d)\sqrt{fx}\frac{1}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)

[Out] int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)\sqrt{fx}}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^2 + d)\sqrt{fx}}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")

[Out] integral((e*x^2 + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{fx}\,(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**(1/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f*x)*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)\sqrt{fx}}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.214 $\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=295

$$\frac{2d\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*d*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4
, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(f*Sqrt[a + b*x^2 + c*x^4]) + (2*e*(f*x)^(5/2)*Sqrt[1 +
 (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f^3*Sqrt[a
 + b*x^2 + c*x^4])
```

Rubi [A] time = 1.00926, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$\frac{2d\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*d*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4
, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(f*Sqrt[a + b*x^2 + c*x^4]) + (2*e*(f*x)^(5/2)*Sqrt[1 +
 (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f^3*Sqrt[a
 + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 83.2078, size = 264, normalized size = 0.89

$$\frac{2d\sqrt{fx}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{af\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}+\frac{2e(fx)^{\frac{5}{2}}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{5af^3\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(f*x)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] 2*d*sqrt(f*x)*sqrt(a + b*x**2 + c*x**4)*appellf1(1/4, 1/2, 1/2, 5
/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a
*c + b**2)))/(a*f*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sq
rt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)) + 2*e*(f*x)**(5/2)*sq
rt(a + b*x**2 + c*x**4)*appellf1(5/4, 1/2, 1/2, 9/4, -2*c*x**2/(b
 - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**2)))/(5*
a*f**3*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x**2
/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 1.00167, size = 642, normalized size = 2.18

$$ax\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(-\frac{25dF_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\right.}{x^2\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{5}{4};\frac{1}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{5}{4};\frac{3}{2},\frac{1}{2};\frac{9}{4};-\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x^2)/(Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (a*x*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2)*((-25*d*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(-5*a*Appell
F1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*Appe
llF1[5/4, 1/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF
1[5/4, 3/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
2)/(-b + Sqrt[b^2 - 4*a*c])])) - (9*e*x^2*AppellF1[5/4, 1/2, 1/2,
 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^
2 - 4*a*c])])/(-9*a*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b
```

```
 + Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 1/2, 3/2, 13/4, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b -
 Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 3/2, 1/2, 13/4, (-2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))/(10*
c*Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int (ex^2+d)\frac{1}{\sqrt{fx}}\frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2+d}{\sqrt{cx^4+bx^2+a}\sqrt{fx}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(f*x)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/(sqrt(f*x)*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)), x)
```

3.215 $\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=295

$$\frac{2e(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{a+bx^2+cx^4}}-\frac{2d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{a+bx^2+cx^4}}$$

```
[Out] (-2*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-1/4, 1/2, 1/2, 3/4, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
)/(f*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]) + (2*e*(f*x)^(3/2)*Sqrt[1
 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sqr
t[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f^3*Sqrt
[a + b*x^2 + c*x^4])
```

Rubi [A] time = 1.01709, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$\frac{2e(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{a+bx^2+cx^4}}-\frac{2d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/((f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```


```
[Out] (-2*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-1/4, 1/2, 1/2, 3/4, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
)/(f*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]) + (2*e*(f*x)^(3/2)*Sqrt[1
 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sqr
t[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f^3*Sqrt
[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 82.7094, size = 265, normalized size = 0.9

$$-\frac{2d\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(-\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{af\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}+\frac{2e\,(fx)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{3af^3\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(f*x)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -2*d*sqrt(a + b*x**2 + c*x**4)*appellf1(-1/4, 1/2, 1/2, 3/4, -2*c
*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**
2)))/(a*f*sqrt(f*x)*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*
sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)) + 2*e*(f*x)**(3/2)*
sqrt(a + b*x**2 + c*x**4)*appellf1(3/4, 1/2, 1/2, 7/4, -2*c*x**2/
(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**2)))/(
3*a*f**3*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x*
*2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 1.65255, size = 1049, normalized size = 3.56

$$2x\left(\frac{99ad\left(2cx^2+b-\sqrt{b^2-4ac}\right)\left(2cx^2+b+\sqrt{b^2-4ac}\right)F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)x^4}{44aF_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-4x^2\left(\left(b+\sqrt{b^2-4ac}\right)F_1\left(\frac{11}{4};\frac{1}{2},\frac{3}{2};\frac{15}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{11}{4};\frac{3}{2},\frac{1}{2};\frac{15}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx}{\sqrt{b^2-4}}\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x^2)/((f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*x*(-21*d*(a + b*x^2 + c*x^4)^2 + (49*a*b*d*x^2*(b - Sqrt[b^2 -
 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[3/4
, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])])/(4*c*(7*a*AppellF1[3/4, 1/2, 1/2, 7/4, (
-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a
*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 1/2, 3/2, 11/4
, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/4,
(-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*
a*c])]))) + (49*a^2*e*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])
```

```
/(4*c*(7*a*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b
^2 - 4*a*c])*AppellF1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2
 - 4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) + (99*a*d*x^4*
(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(44*a*AppellF1[7/4, 1/2,
 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])] - 4*x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[11/4
, 1/2, 3/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[11/4,
 3/2, 1/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])]))))/(21*a*(f*x)^(3/2)*(a + b*x^2 + c*x^4)
^(3/2))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \left(e x^2+d\right) (f x)^{-\frac{3}{2}} \frac{1}{\sqrt{c x^4+b x^2+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)

[Out] int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e x^2+d}{\sqrt{c x^4+b x^2+a}\,(f x)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*(f*x)^(3/2)),x, algorithm="maxima

[Out] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*(f*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + d}{\sqrt{cx^4 + bx^2 + a}\sqrt{fx}fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*(f*x)^(3/2)),x, algorithm="fricas
```

```
[Out] integral((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)*f*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^2}{(fx)^{\frac{3}{2}}\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(f*x)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/((f*x)**(3/2)*sqrt(a + b*x**2 + c*x**4)), x
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{cx^4 + bx^2 + a}\,(fx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*(f*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*(f*x)^(3/2)), x)
```

3.216 $\int \frac{(fx)^{3/2}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=303

$$\frac{2d(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{9/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{9}{4};\frac{3}{2},\frac{3}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9af^3\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*d*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9
/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])/(5*a*f*Sqrt[a + b*x^2 + c*x^4]) + (2*e*(f*x)^(9/2)*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*AppellF1[9/4, 3/2, 3/2, 13/4, (-2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(9*a*f
^3*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 1.02874, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$\frac{2d(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{9/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{9}{4};\frac{3}{2},\frac{3}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9af^3\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (2*d*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9
/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])/(5*a*f*Sqrt[a + b*x^2 + c*x^4]) + (2*e*(f*x)^(9/2)*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*AppellF1[9/4, 3/2, 3/2, 13/4, (-2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(9*a*f
^3*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 102.03, size = 269, normalized size = 0.89

$$\frac{2d(fx)^{\frac{5}{2}}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{5}{4},\frac{3}{2},\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{5a^2f\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}+\frac{2e(fx)^{\frac{9}{2}}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{9}{4},\frac{3}{2},\frac{3}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{9a^2f^3\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**(3/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] 2*d*(f*x)**(5/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(5/4, 3/2, 3/2
, 9/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-
4*a*c + b**2)))/(5*a**2*f*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2))
 + 1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)) + 2*e*(f*x)**
(9/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(9/4, 3/2, 3/2, 13/4, -2*
c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b*
*2)))/(9*a**2*f**3*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*s
qrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 3.76852, size = 1404, normalized size = 4.63

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (f*Sqrt[f*x]*(5*(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)*(a + b*x^2
 + c*x^4) + (25*a^2*e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)*(b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(2
*c*(5*a*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2
- 4*a*c])*AppellF1[5/4, 1/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4
*a*c])*AppellF1[5/4, 3/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) + (25*a*b*d*(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*Appe
llF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(4*c*(5*a*AppellF1[1/4, 1/2, 1/2
, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b
^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 1/2, 3
/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
```

```
[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 3/2, 1/2,
 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^
2 - 4*a*c])]))) + (9*a*d*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]
)])/(18*a*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 2*x^2*((b + Sqrt[
b^2 - 4*a*c])*AppellF1[9/4, 1/2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^
2 - 4*a*c])*AppellF1[9/4, 3/2, 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])) - (9*a*b*e*x^2
*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2)*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(4*c*(9*a*AppellF1[5/4,
1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[9/4
, 1/2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/4,
3/2, 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
 + Sqrt[b^2 - 4*a*c])])))))/(5*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^
(3/2))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \left(e x^{2}+d\right) \left(f x\right)^{\frac{3}{2}} \left(c x^{4}+b x^{2}+a\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)

[Out] int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e x^{2}+d\right) \left(f x\right)^{\frac{3}{2}}}{\left(c x^{4}+b x^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*(f*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="maxima

[Out] integrate((e*x^2 + d)*(f*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(efx^3+dfx)\sqrt{fx}}{(cx^4+bx^2+a)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*(f*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="fricas
```

```
[Out] integral((e*f*x^3 + d*f*x)*sqrt(f*x)/(c*x^4 + b*x^2 + a)^(3/2), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(3/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ex^2+d)(fx)^{\frac{3}{2}}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*(f*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.217 $\int \frac{\sqrt{fx}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=303

$$\frac{2d(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{7/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{7}{4};\frac{3}{2},\frac{3}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7af^3\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*d*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7
/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])/(3*a*f*Sqrt[a + b*x^2 + c*x^4]) + (2*e*(f*x)^(7/2)*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 3/2, 3/2, 11/4, (-2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*a*f
^3*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 1.00402, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$\frac{2d(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{7/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{7}{4};\frac{3}{2},\frac{3}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7af^3\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[f*x]*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*d*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7
/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])/(3*a*f*Sqrt[a + b*x^2 + c*x^4]) + (2*e*(f*x)^(7/2)*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 3/2, 3/2, 11/4, (-2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*a*f
^3*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 101.061, size = 269, normalized size = 0.89

$$\frac{2d\,(fx)^{\frac{3}{2}}\,\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(\frac{3}{4},\frac{3}{2},\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{3a^2f\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}+\frac{2e\,(fx)^{\frac{7}{2}}\,\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(\frac{7}{4},\frac{3}{2},\frac{3}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{7a^2f^3\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**(1/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] 2*d*(f*x)**(3/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(3/4, 3/2, 3/2
, 7/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-
4*a*c + b**2)))/(3*a**2*f*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2))
 + 1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)) + 2*e*(f*x)**
(7/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(7/4, 3/2, 3/2, 11/4, -2*
c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b*
*2)))/(7*a**2*f**3*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*s
qrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 4.52473, size = 1740, normalized size = 5.74

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(Sqrt[f*x]*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[f*x]*(84*(a + b*x^2 + c*x^4)*(-(b^2*d) + b*(a*e - c*d*x^2
) + 2*a*c*(d + e*x^2)) + (196*a^2*d*(b - Sqrt[b^2 - 4*a*c] + 2*c*
x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[3/4, 1/2, 1/2, 7/
4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
 4*a*c])])/(14*a*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 2*x^2*((b
+ Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b -
Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])) + (49*a
*b^2*d*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2)*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(c*(7*a*AppellF1[3
/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/
(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1
[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
```

```
2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[7
/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)
/(-b + Sqrt[b^2 - 4*a*c])]))) - (147*a^2*b*e*(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[3/4, 1/2
, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])])/(c*(7*a*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] -
 x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 1/2, 3/2, 11/4, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])
] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])
)) - (99*a*b*d*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(-11*
a*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a
*c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*
c])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])) + (198*a^2*e*x^2*(b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*A
ppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(-11*a*AppellF1[7/4, 1/2, 1/
2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 1/2
, 3/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 3/2,
 1/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])]))))/(84*a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^(
3/2))
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int (ex^2+d)\sqrt{fx}\left(cx^4+bx^2+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)\sqrt{fx}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*sqrt(f*x)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*sqrt(f*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^2+d)\sqrt{fx}}{(cx^4+bx^2+a)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*sqrt(f*x)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*sqrt(f*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**(1/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)\sqrt{fx}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*sqrt(f*x)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*sqrt(f*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.218 $\int \frac{d+ex^2}{\sqrt{fx}(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=301

$$\frac{2d\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af^3\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*d*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 3/2, 3/2, 5/4
, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(a*f*Sqrt[a + b*x^2 + c*x^4]) + (2*e*(f*x)^(5/2)*Sqrt[1
 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4, (-2*c*x^2)/(b - Sqr
t[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*a*f^3*Sq
rt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 1.00633, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$\frac{2d\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af^3\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/(Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```


```
[Out] (2*d*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 3/2, 3/2, 5/4
, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(a*f*Sqrt[a + b*x^2 + c*x^4]) + (2*e*(f*x)^(5/2)*Sqrt[1
 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4, (-2*c*x^2)/(b - Sqr
t[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*a*f^3*Sq
rt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 99.151, size = 267, normalized size = 0.89

$$\frac{2d\sqrt{fx}\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{a^2f\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}} + \frac{2e(fx)^{\frac{5}{2}}\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(\frac{5}{4},\frac{3}{2},\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{5a^2f^3\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(f*x)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] 2*d*sqrt(f*x)*sqrt(a + b*x**2 + c*x**4)*appellf1(1/4, 3/2, 3/2, 5
/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a
*c + b**2)))/(a**2*f*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)
*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)) + 2*e*(f*x)**(5/2)
*sqrt(a + b*x**2 + c*x**4)*appellf1(5/4, 3/2, 3/2, 9/4, -2*c*x**2
/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**2)))/
(5*a**2*f**3*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*
c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 3.96555, size = 1740, normalized size = 5.78

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x^2)/(Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (x*(20*(a + b*x^2 + c*x^4)*(-(b^2*d) + b*(a*e - c*d*x^2) + 2*a*c*
(d + e*x^2)) + (300*a^2*d*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])
/(10*a*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 2*x^2*((b + Sqrt[b^2
 - 4*a*c])*AppellF1[5/4, 1/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 -
4*a*c])*AppellF1[5/4, 3/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])) - (25*a*b^2*d*(b -
Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*Ap
pellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(c*(5*a*AppellF1[1/4, 1/2, 1/2
, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b
^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 1/2, 3
/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
```

```
[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 3/2, 1/2,
 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^
2 - 4*a*c])]))) - (25*a^2*b*e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[1/4, 1/2, 1/2, 5/4, (-2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c
])])/(c*(5*a*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt
[b^2 - 4*a*c])*AppellF1[5/4, 1/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^
2 - 4*a*c])*AppellF1[5/4, 3/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) - (9*a*b*d*x^2
*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2)*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(-9*a*AppellF1[5/4, 1/2,
 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sq
rt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 1/
2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 3/2,
 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])])) + (18*a^2*e*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c
*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1/2, 1/2, 9
/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
- 4*a*c])])/(-9*a*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b +
 Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 1/2, 3/2, 13/4, (-2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - S
qrt[b^2 - 4*a*c])*AppellF1[9/4, 3/2, 1/2, 13/4, (-2*c*x^2)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))))/(20*a*
(-b^2 + 4*a*c)*Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int (ex^2+d)\frac{1}{\sqrt{fx}}\left(cx^4+bx^2+a\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)

[Out] int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{(cx^4+bx^2+a)^{\frac{3}{2}}\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(f*x)),x, algorithm="maxima

[Out] integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(f*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + d}{(cx^4 + bx^2 + a)^{\frac{3}{2}}\sqrt{fx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(f*x)),x, algorithm="fricas

[Out] integral((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(f*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(f*x)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{(cx^4 + bx^2 + a)^{\frac{3}{2}}\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(f*x)),x, algorithm="giac")

[Out] integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(f*x)), x)
```

3.219 $\int \frac{d+ex^2}{(fx)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=301

$$\frac{2e(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af^3\sqrt{a+bx^2+cx^4}}-\frac{2d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{fx}\sqrt{a+bx^2+cx^4}}$$

```
[Out] (-2*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-1/4, 3/2, 3/2, 3/4, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
)/(a*f*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]) + (2*e*(f*x)^(3/2)*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4, (-2*c*x^2)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*f^3*
Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 1.01925, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$\frac{2e(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af^3\sqrt{a+bx^2+cx^4}}-\frac{2d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{fx}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```


```
[Out] (-2*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-1/4, 3/2, 3/2, 3/4, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
)/(a*f*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]) + (2*e*(f*x)^(3/2)*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4, (-2*c*x^2)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*f^3*
Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 99.4317, size = 269, normalized size = 0.89

$$-\frac{2d\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(-\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{a^2f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}+\frac{2e(fx)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(\frac{3}{4},\frac{3}{2},\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{3a^2f^3\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)/(f*x)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -2*d*sqrt(a + b*x**2 + c*x**4)*appellf1(-1/4, 3/2, 3/2, 3/4, -2*c
*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**
2)))/(a**2*f*sqrt(f*x)*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) +
1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)) + 2*e*(f*x)**(3/
2)*sqrt(a + b*x**2 + c*x**4)*appellf1(3/4, 3/2, 3/2, 7/4, -2*c*x*
*2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**2))
)/(3*a**2*f**3*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(
2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 6.18786, size = 2959, normalized size = 9.83

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (x^(3/2)*Sqrt[a + b*x^2 + c*x^4]*((-2*d)/(a^2*Sqrt[x]) + (b^3*d*x
^(3/2) - 3*a*b*c*d*x^(3/2) - a*b^2*e*x^(3/2) + 2*a^2*c*e*x^(3/2)
+ b^2*c*d*x^(7/2) - 2*a*c^2*d*x^(7/2) - a*b*c*e*x^(7/2))/(a^2*(-b
^2 + 4*a*c)*(a + b*x^2 + c*x^4))))/(f*x)^(3/2) + (7*b^3*d*x^3*(b
- Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*
AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(b - Sqrt[b^
2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(f*x)^(3/2)*(a + b*x^2 + c*x^
4)^(3/2)*(-7*a*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sq
rt[b^2 - 4*a*c])*AppellF1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt
[b^2 - 4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) - (21*a*b*
c*d*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2
```

```
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((-b^2 + 4*a*c)*(
b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(f*x)^(3/2)*(a + b
*x^2 + c*x^4)^(3/2)*(-7*a*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x
^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 1/2, 3/2, 11/4, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]
+ (b - Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))
 - (7*a*b^2*e*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(3*(-b^
2 + 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(f*x)^
(3/2)*(a + b*x^2 + c*x^4)^(3/2)*(-7*a*AppellF1[3/4, 1/2, 1/2, 7/4
, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 1/2, 3/2, 1
1/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/
4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
 4*a*c])]))) - (14*a^2*c*e*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[3/4, 1/2, 1/2, 7/4, (-
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*
c])])/(3*(-b^2 + 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4
*a*c])*(f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)*(-7*a*AppellF1[3/4,
1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[7/4
, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[7/4,
3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
 + Sqrt[b^2 - 4*a*c])]))) + (99*b^2*c*d*x^5*(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[7/4, 1/2,
 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])])/(7*(-b^2 + 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*(b
+ Sqrt[b^2 - 4*a*c])*(f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)*(-11*a
*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*
c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c
])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) - (330*a*c^2*d*x^5*(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(7*(-b^2 + 4*a*c)*(b - Sqr
t[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(f*x)^(3/2)*(a + b*x^2 +
c*x^4)^(3/2)*(-11*a*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((
b + Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b
 - Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) -
(33*a*b*c*e*x^5*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(7*(-b^2
 + 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(f*x)^(
3/2)*(a + b*x^2 + c*x^4)^(3/2)*(-11*a*AppellF1[7/4, 1/2, 1/2, 11/
```

```
4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 1/2, 3/2,
 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b
^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 3/2, 1/2,
15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^
2 - 4*a*c])])))
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int \left(e x^2+d\right) (f x)^{-\frac{3}{2}} \left(c x^4+b x^2+a\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)

[Out] int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e x^2+d}{\left(c x^4+b x^2+a\right)^{\frac{3}{2}} (f x)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*(f*x)^(3/2)),x, algorithm="maxi

[Out] integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*(f*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e x^2+d}{\left(c f x^5+b f x^3+a f x\right)\sqrt{c x^4+b x^2+a}\sqrt{f x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*(f*x)^(3/2)),x, algorithm="frica
```

```
[Out] integral((e*x^2 + d)/((c*f*x^5 + b*f*x^3 + a*f*x)*sqrt(c*x^4 + b*
x^2 + a)*sqrt(f*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)/(f*x)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} (fx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*(f*x)^(3/2)),x, algorithm="giac
```

```
[Out] integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*(f*x)^(3/2)), x)
```

3.220 $\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^3 \, dx$

Optimal. Leaf size=243

$$\frac{a^3d(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+7}\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{f^7(m+7)}+\frac{a^2(fx)^{m+3}(ae+3bd)}{f^3(m+3)}+\frac{3c(fx)^{m+11}\left(ace+b^2e+bcd\right)}{f^{11}(m+11)}+\frac{3a(fx)^{m+5}\left(abe+acd+b^2d\right)}{f^5(m+5)}+\frac{(fx)^{m+9}\left(6abce+3ac^2d+b^3e+3b^2cd\right)}{f^9(m+9)}+\frac{c^2(fx)^{m+13}(3be+cd)}{f^{13}(m+13)}+\frac{c^3e(fx)^{m+15}}{f^{15}(m+15)}$$

```
[Out] (a^3*d*(f*x)^(1 + m))/(f*(1 + m)) + (a^2*(3*b*d + a*e)*(f*x)^(3 +
 m))/(f^3*(3 + m)) + (3*a*(b^2*d + a*c*d + a*b*e)*(f*x)^(5 + m))/
(f^5*(5 + m)) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*(f*x
)^(7 + m))/(f^7*(7 + m)) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*
b*c*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (3*c*(b*c*d + b^2*e + a*c*e
)*(f*x)^(11 + m))/(f^11*(11 + m)) + (c^2*(c*d + 3*b*e)*(f*x)^(13
+ m))/(f^13*(13 + m)) + (c^3*e*(f*x)^(15 + m))/(f^15*(15 + m))
```

Rubi [A] time = 0.399543, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$

$$\frac{a^3d(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+7}\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{f^7(m+7)}+\frac{a^2(fx)^{m+3}(ae+3bd)}{f^3(m+3)}+\frac{3c(fx)^{m+11}\left(ace+b^2e+bcd\right)}{f^{11}(m+11)}+\frac{3a(fx)^{m+5}\left(abe+acd+b^2d\right)}{f^5(m+5)}+\frac{(fx)^{m+9}\left(6abce+3ac^2d+b^3e+3b^2cd\right)}{f^9(m+9)}+\frac{c^2(fx)^{m+13}(3be+cd)}{f^{13}(m+13)}+\frac{c^3e(fx)^{m+15}}{f^{15}(m+15)}$$

Antiderivative was successfully verified.

```
[In]  Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x]
```


```
[Out] (a^3*d*(f*x)^(1 + m))/(f*(1 + m)) + (a^2*(3*b*d + a*e)*(f*x)^(3 +
 m))/(f^3*(3 + m)) + (3*a*(b^2*d + a*c*d + a*b*e)*(f*x)^(5 + m))/
(f^5*(5 + m)) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*(f*x
)^(7 + m))/(f^7*(7 + m)) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*
b*c*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (3*c*(b*c*d + b^2*e + a*c*e
)*(f*x)^(11 + m))/(f^11*(11 + m)) + (c^2*(c*d + 3*b*e)*(f*x)^(13
+ m))/(f^13*(13 + m)) + (c^3*e*(f*x)^(15 + m))/(f^15*(15 + m))
```

Rubi in Sympy [A] time = 72.1371, size = 238, normalized size = 0.98

$$\frac{a^3d(fx)^{m+1}}{f(m+1)}+\frac{a^2(fx)^{m+3}(ae+3bd)}{f^3(m+3)}+\frac{3a(fx)^{m+5}\left(abe+acd+b^2d\right)}{f^5(m+5)}+\frac{c^3e(fx)^{m+15}}{f^{15}(m+15)}+\frac{c^2(fx)^{m+13}(3be+cd)}{f^{13}(m+13)}+\frac{3c(fx)^{m+11}\left(ace+b^2e+bcd\right)}{f^{11}(m+11)}+\frac{(fx)^{m+7}\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{f^7(m+7)}+\frac{(fx)^{m+9}\left(6abce+3ac^2d+b^3e+3b^2cd\right)}{f^9(m+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] a**3*d*(f*x)**(m + 1)/(f*(m + 1)) + a**2*(f*x)**(m + 3)*(a*e + 3*
b*d)/(f**3*(m + 3)) + 3*a*(f*x)**(m + 5)*(a*b*e + a*c*d + b**2*d)
/(f**5*(m + 5)) + c**3*e*(f*x)**(m + 15)/(f**15*(m + 15)) + c**2*
(f*x)**(m + 13)*(3*b*e + c*d)/(f**13*(m + 13)) + 3*c*(f*x)**(m +
11)*(a*c*e + b**2*e + b*c*d)/(f**11*(m + 11)) + (f*x)**(m + 7)*(3
*a**2*c*e + 3*a*b**2*e + 6*a*b*c*d + b**3*d)/(f**7*(m + 7)) + (f*
x)**(m + 9)*(6*a*b*c*e + 3*a*c**2*d + b**3*e + 3*b**2*c*d)/(f**9*
(m + 9))
```

Mathematica [A] time = 0.528569, size = 191, normalized size = 0.79

$$(fx)^m\left(\frac{a^3dx}{m+1}+\frac{x^7\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{m+7}+\frac{a^2x^3(ae+3bd)}{m+3}+\frac{3cx^{11}\left(ace+b^2e+bcd\right)}{m+11}+\frac{3ax^5\left(abe+acd+b^2d\right)}{m+5}+\frac{x^9\left(6abce+3ac^2d+b^3e+3b^2cd\right)}{m+9}+\frac{c^2x^{13}(3be+cd)}{m+13}+\frac{c^3ex^{15}}{m+15}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (f*x)^m*((a^3*d*x)/(1 + m) + (a^2*(3*b*d + a*e)*x^3)/(3 + m) + (3
*a*(b^2*d + a*c*d + a*b*e)*x^5)/(5 + m) + ((b^3*d + 6*a*b*c*d + 3
*a*b^2*e + 3*a^2*c*e)*x^7)/(7 + m) + ((3*b^2*c*d + 3*a*c^2*d + b^
3*e + 6*a*b*c*e)*x^9)/(9 + m) + (3*c*(b*c*d + b^2*e + a*c*e)*x^11
)/(11 + m) + (c^2*(c*d + 3*b*e)*x^13)/(13 + m) + (c^3*e*x^15)/(15
 + m))
```

Maple [B] time = 0.013, size = 1935, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] x*(c^3*e*m^7*x^14+49*c^3*e*m^6*x^14+3*b*c^2*e*m^7*x^12+c^3*d*m^7*
x^12+973*c^3*e*m^5*x^14+153*b*c^2*e*m^6*x^12+51*c^3*d*m^6*x^12+10
045*c^3*e*m^4*x^14+3*a*c^2*e*m^7*x^10+3*b^2*c*e*m^7*x^10+3*b*c^2*
d*m^7*x^10+3135*b*c^2*e*m^5*x^12+1045*c^3*d*m^5*x^12+57379*c^3*e*
m^3*x^14+159*a*c^2*e*m^6*x^10+159*b^2*c*e*m^6*x^10+159*b*c^2*d*m^
6*x^10+33165*b*c^2*e*m^4*x^12+11055*c^3*d*m^4*x^12+177331*c^3*e*m
^2*x^14+6*a*b*c*e*m^7*x^8+3*a*c^2*d*m^7*x^8+3375*a*c^2*e*m^5*x^10
+b^3*e*m^7*x^8+3*b^2*c*d*m^7*x^8+3375*b^2*c*e*m^5*x^10+3375*b*c^2
*d*m^5*x^10+193017*b*c^2*e*m^3*x^12+64339*c^3*d*m^3*x^12+264207*c
^3*e*m*x^14+330*a*b*c*e*m^6*x^8+165*a*c^2*d*m^6*x^8+36795*a*c^2*e
*m^4*x^10+55*b^3*e*m^6*x^8+165*b^2*c*d*m^6*x^8+36795*b^2*c*e*m^4*
x^10+36795*b*c^2*d*m^4*x^10+604827*b*c^2*e*m^2*x^12+201609*c^3*d*
m^2*x^12+135135*c^3*e*x^14+3*a^2*c*e*m^7*x^6+3*a*b^2*e*m^7*x^6+6*
a*b*c*d*m^7*x^6+7278*a*b*c*e*m^5*x^8+3639*a*c^2*d*m^5*x^8+219417*
a*c^2*e*m^3*x^10+b^3*d*m^7*x^6+1213*b^3*e*m^5*x^8+3639*b^2*c*d*m^
5*x^8+219417*b^2*c*e*m^3*x^10+219417*b*c^2*d*m^3*x^10+909765*b*c^
2*e*m*x^12+303255*c^3*d*m*x^12+171*a^2*c*e*m^6*x^6+171*a*b^2*e*m^
6*x^6+342*a*b*c*d*m^6*x^6+82338*a*b*c*e*m^4*x^8+41169*a*c^2*d*m^4
*x^8+700461*a*c^2*e*m^2*x^10+57*b^3*d*m^6*x^6+13723*b^3*e*m^4*x^8
+41169*b^2*c*d*m^4*x^8+700461*b^2*c*e*m^2*x^10+700461*b*c^2*d*m^2
*x^10+467775*b*c^2*e*x^12+155925*c^3*d*x^12+3*a^2*b*e*m^7*x^4+3*a
^2*c*d*m^7*x^4+3927*a^2*c*e*m^5*x^6+3*a*b^2*d*m^7*x^4+3927*a*b^2*
e*m^5*x^6+7854*a*b*c*d*m^5*x^6+507282*a*b*c*e*m^3*x^8+253641*a*c^
2*d*m^3*x^8+1067445*a*c^2*e*m*x^10+1309*b^3*d*m^5*x^6+84547*b^3*e
*m^3*x^8+253641*b^2*c*d*m^3*x^8+1067445*b^2*c*e*m*x^10+1067445*b*
c^2*d*m*x^10+177*a^2*b*e*m^6*x^4+177*a^2*c*d*m^6*x^4+46431*a^2*c*
e*m^4*x^6+177*a*b^2*d*m^6*x^4+46431*a*b^2*e*m^4*x^6+92862*a*b*c*d
*m^4*x^6+1662558*a*b*c*e*m^2*x^8+831279*a*c^2*d*m^2*x^8+552825*a*
c^2*e*x^10+15477*b^3*d*m^4*x^6+277093*b^3*e*m^2*x^8+831279*b^2*c*
d*m^2*x^8+552825*b^2*c*e*x^10+552825*b*c^2*d*x^10+a^3*e*m^7*x^2+3
*a^2*b*d*m^7*x^2+4239*a^2*b*e*m^5*x^4+4239*a^2*c*d*m^5*x^4+299145
*a^2*c*e*m^3*x^6+4239*a*b^2*d*m^5*x^4+299145*a*b^2*e*m^3*x^6+5982
90*a*b*c*d*m^3*x^6+2582010*a*b*c*e*m*x^8+1291005*a*c^2*d*m*x^8+99
715*b^3*d*m^3*x^6+430335*b^3*e*m*x^8+1291005*b^2*c*d*m*x^8+61*a^3
*e*m^6*x^2+183*a^2*b*d*m^6*x^2+52725*a^2*b*e*m^4*x^4+52725*a^2*c*
d*m^4*x^4+1020033*a^2*c*e*m^2*x^6+52725*a*b^2*d*m^4*x^4+1020033*a
*b^2*e*m^2*x^6+2040066*a*b*c*d*m^2*x^6+1351350*a*b*c*e*x^8+675675
*a*c^2*d*x^8+340011*b^3*d*m^2*x^6+225225*b^3*e*x^8+675675*b^2*c*d
*x^8+a^3*d*m^7+1525*a^3*e*m^5*x^2+4575*a^2*b*d*m^5*x^2+360537*a^2
*b*e*m^3*x^4+360537*a^2*c*d*m^3*x^4+1632285*a^2*c*e*m*x^6+360537*
a*b^2*d*m^3*x^4+1632285*a*b^2*e*m*x^6+3264570*a*b*c*d*m*x^6+54409
5*b^3*d*m*x^6+63*a^3*d*m^6+20065*a^3*e*m^4*x^2+60195*a^2*b*d*m^4*
x^2+1311363*a^2*b*e*m^2*x^4+1311363*a^2*c*d*m^2*x^4+868725*a^2*c*
e*x^6+1311363*a*b^2*d*m^2*x^4+868725*a*b^2*e*x^6+1737450*a*b*c*d*
x^6+289575*b^3*d*x^6+1645*a^3*d*m^5+147859*a^3*e*m^3*x^2+443577*a
^2*b*d*m^3*x^2+2215701*a^2*b*e*m*x^4+2215701*a^2*c*d*m*x^4+221570
1*a*b^2*d*m*x^4+22995*a^3*d*m^4+594439*a^3*e*m^2*x^2+1783317*a^2*
b*d*m^2*x^2+1216215*a^2*b*e*x^4+1216215*a^2*c*d*x^4+1216215*a*b^2
*d*x^4+185059*a^3*d*m^3+1140855*a^3*e*m*x^2+3422565*a^2*b*d*m*x^2
```

```
+852957*a^3*d*m^2+675675*a^3*e*x^2+2027025*a^2*b*d*x^2+2071215*a^
3*d*m+2027025*a^3*d)*(f*x)^m/(1+m)/(3+m)/(5+m)/(7+m)/(9+m)/(11+m)
/(13+m)/(15+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^3*(e*x^2 + d)*(f*x)^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.315144, size = 1832, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^3*(e*x^2 + d)*(f*x)^m,x, algorithm="fricas")
```

```
[Out] ((c^3*e*m^7 + 49*c^3*e*m^6 + 973*c^3*e*m^5 + 10045*c^3*e*m^4 + 57
379*c^3*e*m^3 + 177331*c^3*e*m^2 + 264207*c^3*e*m + 135135*c^3*e)
*x^15 + ((c^3*d + 3*b*c^2*e)*m^7 + 51*(c^3*d + 3*b*c^2*e)*m^6 + 1
045*(c^3*d + 3*b*c^2*e)*m^5 + 11055*(c^3*d + 3*b*c^2*e)*m^4 + 155
925*c^3*d + 467775*b*c^2*e + 64339*(c^3*d + 3*b*c^2*e)*m^3 + 2016
09*(c^3*d + 3*b*c^2*e)*m^2 + 303255*(c^3*d + 3*b*c^2*e)*m)*x^13 +
 3*((b*c^2*d + (b^2*c + a*c^2)*e)*m^7 + 53*(b*c^2*d + (b^2*c + a*
c^2)*e)*m^6 + 1125*(b*c^2*d + (b^2*c + a*c^2)*e)*m^5 + 12265*(b*c
^2*d + (b^2*c + a*c^2)*e)*m^4 + 184275*b*c^2*d + 73139*(b*c^2*d +
 (b^2*c + a*c^2)*e)*m^3 + 233487*(b*c^2*d + (b^2*c + a*c^2)*e)*m^
2 + 184275*(b^2*c + a*c^2)*e + 355815*(b*c^2*d + (b^2*c + a*c^2)*
e)*m)*x^11 + ((3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^7 + 55*
(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^6 + 1213*(3*(b^2*c +
a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^5 + 13723*(3*(b^2*c + a*c^2)*d +
(b^3 + 6*a*b*c)*e)*m^4 + 84547*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*
b*c)*e)*m^3 + 277093*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^
2 + 675675*(b^2*c + a*c^2)*d + 225225*(b^3 + 6*a*b*c)*e + 430335*
(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m)*x^9 + (((b^3 + 6*a*b
*c)*d + 3*(a*b^2 + a^2*c)*e)*m^7 + 57*((b^3 + 6*a*b*c)*d + 3*(a*b
^2 + a^2*c)*e)*m^6 + 1309*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*
e)*m^5 + 15477*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^4 + 99
715*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^3 + 340011*((b^3
+ 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^2 + 289575*(b^3 + 6*a*b*c)*
```

```
d + 868725*(a*b^2 + a^2*c)*e + 544095*((b^3 + 6*a*b*c)*d + 3*(a*b
^2 + a^2*c)*e)*m)*x^7 + 3*((a^2*b*e + (a*b^2 + a^2*c)*d)*m^7 + 59
*(a^2*b*e + (a*b^2 + a^2*c)*d)*m^6 + 1413*(a^2*b*e + (a*b^2 + a^2
*c)*d)*m^5 + 17575*(a^2*b*e + (a*b^2 + a^2*c)*d)*m^4 + 405405*a^2
*b*e + 120179*(a^2*b*e + (a*b^2 + a^2*c)*d)*m^3 + 437121*(a^2*b*e
 + (a*b^2 + a^2*c)*d)*m^2 + 405405*(a*b^2 + a^2*c)*d + 738567*(a^
2*b*e + (a*b^2 + a^2*c)*d)*m)*x^5 + ((3*a^2*b*d + a^3*e)*m^7 + 61
*(3*a^2*b*d + a^3*e)*m^6 + 1525*(3*a^2*b*d + a^3*e)*m^5 + 20065*(
3*a^2*b*d + a^3*e)*m^4 + 2027025*a^2*b*d + 675675*a^3*e + 147859*
(3*a^2*b*d + a^3*e)*m^3 + 594439*(3*a^2*b*d + a^3*e)*m^2 + 114085
5*(3*a^2*b*d + a^3*e)*m)*x^3 + (a^3*d*m^7 + 63*a^3*d*m^6 + 1645*a
^3*d*m^5 + 22995*a^3*d*m^4 + 185059*a^3*d*m^3 + 852957*a^3*d*m^2
+ 2071215*a^3*d*m + 2027025*a^3*d)*x)*(f*x)^m/(m^8 + 64*m^7 + 170
8*m^6 + 24640*m^5 + 208054*m^4 + 1038016*m^3 + 2924172*m^2 + 4098
240*m + 2027025)
```

Sympy [A] time = 39.9844, size = 11538, normalized size = 47.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Piecewise(((-a**3*d/(14*x**14) - a**3*e/(12*x**12) - a**2*b*d/(4*
x**12) - 3*a**2*b*e/(10*x**10) - 3*a**2*c*d/(10*x**10) - 3*a**2*c
*e/(8*x**8) - 3*a*b**2*d/(10*x**10) - 3*a*b**2*e/(8*x**8) - 3*a*b
*c*d/(4*x**8) - a*b*c*e/x**6 - a*c**2*d/(2*x**6) - 3*a*c**2*e/(4*
x**4) - b**3*d/(8*x**8) - b**3*e/(6*x**6) - b**2*c*d/(2*x**6) - 3
*b**2*c*e/(4*x**4) - 3*b*c**2*d/(4*x**4) - 3*b*c**2*e/(2*x**2) -
c**3*d/(2*x**2) + c**3*e*log(x))/f**15, Eq(m, -15)), ((-a**3*d/(1
2*x**12) - a**3*e/(10*x**10) - 3*a**2*b*d/(10*x**10) - 3*a**2*b*e
/(8*x**8) - 3*a**2*c*d/(8*x**8) - a**2*c*e/(2*x**6) - 3*a*b**2*d/
(8*x**8) - a*b**2*e/(2*x**6) - a*b*c*d/x**6 - 3*a*b*c*e/(2*x**4)
- 3*a*c**2*d/(4*x**4) - 3*a*c**2*e/(2*x**2) - b**3*d/(6*x**6) - b
**3*e/(4*x**4) - 3*b**2*c*d/(4*x**4) - 3*b**2*c*e/(2*x**2) - 3*b*
c**2*d/(2*x**2) + 3*b*c**2*e*log(x) + c**3*d*log(x) + c**3*e*x**2
/2)/f**13, Eq(m, -13)), ((-a**3*d/(10*x**10) - a**3*e/(8*x**8) -
3*a**2*b*d/(8*x**8) - a**2*b*e/(2*x**6) - a**2*c*d/(2*x**6) - 3*a
**2*c*e/(4*x**4) - a*b**2*d/(2*x**6) - 3*a*b**2*e/(4*x**4) - 3*a*
b*c*d/(2*x**4) - 3*a*b*c*e/x**2 - 3*a*c**2*d/(2*x**2) + 3*a*c**2*
e*log(x) - b**3*d/(4*x**4) - b**3*e/(2*x**2) - 3*b**2*c*d/(2*x**2
) + 3*b**2*c*e*log(x) + 3*b*c**2*d*log(x) + 3*b*c**2*e*x**2/2 + c
**3*d*x**2/2 + c**3*e*x**4/4)/f**11, Eq(m, -11)), ((-a**3*d/(8*x*
*8) - a**3*e/(6*x**6) - a**2*b*d/(2*x**6) - 3*a**2*b*e/(4*x**4) -
 3*a**2*c*d/(4*x**4) - 3*a**2*c*e/(2*x**2) - 3*a*b**2*d/(4*x**4)
- 3*a*b**2*e/(2*x**2) - 3*a*b*c*d/x**2 + 6*a*b*c*e*log(x) + 3*a*c
**2*d*log(x) + 3*a*c**2*e*x**2/2 - b**3*d/(2*x**2) + b**3*e*log(x
) + 3*b**2*c*d*log(x) + 3*b**2*c*e*x**2/2 + 3*b*c**2*d*x**2/2 + 3
```

```
*b*c**2*e*x**4/4 + c**3*d*x**4/4 + c**3*e*x**6/6)/f**9, Eq(m, -9)
), ((-a**3*d/(6*x**6) - a**3*e/(4*x**4) - 3*a**2*b*d/(4*x**4) - 3
*a**2*b*e/(2*x**2) - 3*a**2*c*d/(2*x**2) + 3*a**2*c*e*log(x) - 3*
a*b**2*d/(2*x**2) + 3*a*b**2*e*log(x) + 6*a*b*c*d*log(x) + 3*a*b*
c*e*x**2 + 3*a*c**2*d*x**2/2 + 3*a*c**2*e*x**4/4 + b**3*d*log(x)
+ b**3*e*x**2/2 + 3*b**2*c*d*x**2/2 + 3*b**2*c*e*x**4/4 + 3*b*c**
2*d*x**4/4 + b*c**2*e*x**6/2 + c**3*d*x**6/6 + c**3*e*x**8/8)/f**
7, Eq(m, -7)), ((-a**3*d/(4*x**4) - a**3*e/(2*x**2) - 3*a**2*b*d/
(2*x**2) + 3*a**2*b*e*log(x) + 3*a**2*c*d*log(x) + 3*a**2*c*e*x**
2/2 + 3*a*b**2*d*log(x) + 3*a*b**2*e*x**2/2 + 3*a*b*c*d*x**2 + 3*
a*b*c*e*x**4/2 + 3*a*c**2*d*x**4/4 + a*c**2*e*x**6/2 + b**3*d*x**
2/2 + b**3*e*x**4/4 + 3*b**2*c*d*x**4/4 + b**2*c*e*x**6/2 + b*c**
2*d*x**6/2 + 3*b*c**2*e*x**8/8 + c**3*d*x**8/8 + c**3*e*x**10/10)
/f**5, Eq(m, -5)), ((-a**3*d/(2*x**2) + a**3*e*log(x) + 3*a**2*b*
d*log(x) + 3*a**2*b*e*x**2/2 + 3*a**2*c*d*x**2/2 + 3*a**2*c*e*x**
4/4 + 3*a*b**2*d*x**2/2 + 3*a*b**2*e*x**4/4 + 3*a*b*c*d*x**4/2 +
a*b*c*e*x**6 + a*c**2*d*x**6/2 + 3*a*c**2*e*x**8/8 + b**3*d*x**4/
4 + b**3*e*x**6/6 + b**2*c*d*x**6/2 + 3*b**2*c*e*x**8/8 + 3*b*c**
2*d*x**8/8 + 3*b*c**2*e*x**10/10 + c**3*d*x**10/10 + c**3*e*x**12
/12)/f**3, Eq(m, -3)), ((a**3*d*log(x) + a**3*e*x**2/2 + 3*a**2*b
*d*x**2/2 + 3*a**2*b*e*x**4/4 + 3*a**2*c*d*x**4/4 + a**2*c*e*x**6
/2 + 3*a*b**2*d*x**4/4 + a*b**2*e*x**6/2 + a*b*c*d*x**6 + 3*a*b*c
*e*x**8/4 + 3*a*c**2*d*x**8/8 + 3*a*c**2*e*x**10/10 + b**3*d*x**6
/6 + b**3*e*x**8/8 + 3*b**2*c*d*x**8/8 + 3*b**2*c*e*x**10/10 + 3*
b*c**2*d*x**10/10 + b*c**2*e*x**12/4 + c**3*d*x**12/12 + c**3*e*x
**14/14)/f, Eq(m, -1)), (a**3*d*f**m*m**7*x*x**m/(m**8 + 64*m**7
+ 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m
**2 + 4098240*m + 2027025) + 63*a**3*d*f**m*m**6*x*x**m/(m**8 + 6
4*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 29
24172*m**2 + 4098240*m + 2027025) + 1645*a**3*d*f**m*m**5*x*x**m/
(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*
m**3 + 2924172*m**2 + 4098240*m + 2027025) + 22995*a**3*d*f**m*m*
*4*x*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4
+ 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 185059*a**
3*d*f**m*m**3*x*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2
08054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) +
 852957*a**3*d*f**m*m**2*x*x**m/(m**8 + 64*m**7 + 1708*m**6 + 246
40*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
 2027025) + 2071215*a**3*d*f**m*m*x*x**m/(m**8 + 64*m**7 + 1708*m
**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40
98240*m + 2027025) + 2027025*a**3*d*f**m*x*x**m/(m**8 + 64*m**7 +
 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m*
*2 + 4098240*m + 2027025) + a**3*e*f**m*m**7*x**3*x**m/(m**8 + 64
*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292
4172*m**2 + 4098240*m + 2027025) + 61*a**3*e*f**m*m**6*x**3*x**m/
(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*
m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1525*a**3*e*f**m*m**
5*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**
4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 20065*a*
*3*e*f**m*m**4*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202702
5) + 147859*a**3*e*f**m*m**3*x**3*x**m/(m**8 + 64*m**7 + 1708*m**
6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098
240*m + 2027025) + 594439*a**3*e*f**m*m**2*x**3*x**m/(m**8 + 64*m
```

7 + 1708*m6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 29241
72*m**2 + 4098240*m + 2027025) + 1140855*a**3*e*f**m*m*x**3*x**m/
(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*
m**3 + 2924172*m**2 + 4098240*m + 2027025) + 675675*a**3*e*f**m*x
3*xm/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 +
1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*a**2*b*d*
f**m*m**7*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20
8054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) +
183*a**2*b*d*f**m*m**6*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24
640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
+ 2027025) + 4575*a**2*b*d*f**m*m**5*x**3*x**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**
2 + 4098240*m + 2027025) + 60195*a**2*b*d*f**m*m**4*x**3*x**m/(m*
*8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**
3 + 2924172*m**2 + 4098240*m + 2027025) + 443577*a**2*b*d*f**m*m*
*3*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m*
*4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1783317
*a**2*b*d*f**m*m**2*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640
*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2
027025) + 3422565*a**2*b*d*f**m*m*x**3*x**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 2027025*a**2*b*d*f**m*x**3*x**m/(m**8 + 6
4*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 29
24172*m**2 + 4098240*m + 2027025) + 3*a**2*b*e*f**m*m**7*x**5*x**
m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103801
6*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 177*a**2*b*e*f**m*
m**6*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*
m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 4239*
a**2*b*e*f**m*m**5*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*
m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20
27025) + 52725*a**2*b*e*f**m*m**4*x**5*x**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 360537*a**2*b*e*f**m*m**3*x**5*x**m/(m**8
+ 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3
+ 2924172*m**2 + 4098240*m + 2027025) + 1311363*a**2*b*e*f**m*m**
2*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**
4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2215701*
a**2*b*e*f**m*m*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20270
25) + 1216215*a**2*b*e*f**m*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6
+ 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40982
40*m + 2027025) + 3*a**2*c*d*f**m*m**7*x**5*x**m/(m**8 + 64*m**7
+ 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m
2 + 4098240*m + 2027025) + 177*a2*c*d*f**m*m**6*x**5*x**m/(m*
*8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**
3 + 2924172*m**2 + 4098240*m + 2027025) + 4239*a**2*c*d*f**m*m**5
*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4
+ 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 52725*a**
2*c*d*f**m*m**4*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20270
25) + 360537*a**2*c*d*f**m*m**3*x**5*x**m/(m**8 + 64*m**7 + 1708*
m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4
098240*m + 2027025) + 1311363*a**2*c*d*f**m*m**2*x**5*x**m/(m**8
+ 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +

2924172*m**2 + 4098240*m + 2027025) + 2215701*a**2*c*d*f**m*m*x*
*5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 +
1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1216215*a**2
*c*d*f**m*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20
8054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) +
3*a**2*c*e*f**m*m**7*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 2464
0*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 171*a**2*c*e*f**m*m**6*x**7*x**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 3927*a**2*c*e*f**m*m**5*x**7*x**m/(m**8 +
64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
2924172*m**2 + 4098240*m + 2027025) + 46431*a**2*c*e*f**m*m**4*x*
*7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 +
1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 299145*a**2*
c*e*f**m*m**3*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
+ 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025
) + 1020033*a**2*c*e*f**m*m**2*x**7*x**m/(m**8 + 64*m**7 + 1708*m
6 + 24640*m5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40
98240*m + 2027025) + 1632285*a**2*c*e*f**m*m*x**7*x**m/(m**8 + 64
*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292
4172*m**2 + 4098240*m + 2027025) + 868725*a**2*c*e*f**m*x**7*x**m
/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016
*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*a*b**2*d*f**m*m**
7*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**
4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 177*a*b*
*2*d*f**m*m**6*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
+ 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202702
5) + 4239*a*b**2*d*f**m*m**5*x**5*x**m/(m**8 + 64*m**7 + 1708*m**
6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098
240*m + 2027025) + 52725*a*b**2*d*f**m*m**4*x**5*x**m/(m**8 + 64*
m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924
172*m**2 + 4098240*m + 2027025) + 360537*a*b**2*d*f**m*m**3*x**5*
x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103
8016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1311363*a*b**2*
d*f**m*m**2*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 +
208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
+ 2215701*a*b**2*d*f**m*m*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240
*m + 2027025) + 1216215*a*b**2*d*f**m*x**5*x**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m*
*2 + 4098240*m + 2027025) + 3*a*b**2*e*f**m*m**7*x**7*x**m/(m**8
+ 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
2924172*m**2 + 4098240*m + 2027025) + 171*a*b**2*e*f**m*m**6*x**
7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1
038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3927*a*b**2*e
*f**m*m**5*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2
08054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) +
46431*a*b**2*e*f**m*m**4*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240
*m + 2027025) + 299145*a*b**2*e*f**m*m**3*x**7*x**m/(m**8 + 64*m*
*7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 1020033*a*b**2*e*f**m*m**2*x**7*x
m/(m8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038
016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1632285*a*b**2*e

*f**m*m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2080
54*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 86
8725*a*b**2*e*f**m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*
m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20
27025) + 6*a*b*c*d*f**m*m**7*x**7*x**m/(m**8 + 64*m**7 + 1708*m**
6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098
240*m + 2027025) + 342*a*b*c*d*f**m*m**6*x**7*x**m/(m**8 + 64*m**
7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172
*m**2 + 4098240*m + 2027025) + 7854*a*b*c*d*f**m*m**5*x**7*x**m/(
m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m
3 + 2924172*m2 + 4098240*m + 2027025) + 92862*a*b*c*d*f**m*m*
*4*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m*
*4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 598290*
a*b*c*d*f**m*m**3*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m
5 + 208054*m4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202
7025) + 2040066*a*b*c*d*f**m*m**2*x**7*x**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
 4098240*m + 2027025) + 3264570*a*b*c*d*f**m*m*x**7*x**m/(m**8 +
64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2
924172*m**2 + 4098240*m + 2027025) + 1737450*a*b*c*d*f**m*x**7*x*
*m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10380
16*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 6*a*b*c*e*f**m*m*
*7*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m*
*4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 330*a*b
*c*e*f**m*m**6*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202702
5) + 7278*a*b*c*e*f**m*m**5*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6
 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40982
40*m + 2027025) + 82338*a*b*c*e*f**m*m**4*x**9*x**m/(m**8 + 64*m*
*7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 507282*a*b*c*e*f**m*m**3*x**9*x**
m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103801
6*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1662558*a*b*c*e*f*
*m*m**2*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2080
54*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 25
82010*a*b*c*e*f**m*m*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 2464
0*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 1351350*a*b*c*e*f**m*x**9*x**m/(m**8 + 64*m**7 + 1708*
m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4
098240*m + 2027025) + 3*a*c**2*d*f**m*m**7*x**9*x**m/(m**8 + 64*m
7 + 1708*m6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 29241
72*m**2 + 4098240*m + 2027025) + 165*a*c**2*d*f**m*m**6*x**9*x**m
/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016
*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3639*a*c**2*d*f**m*
m**5*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*
m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 41169
*a*c**2*d*f**m*m**4*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640
*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2
027025) + 253641*a*c**2*d*f**m*m**3*x**9*x**m/(m**8 + 64*m**7 + 1
708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2
 + 4098240*m + 2027025) + 831279*a*c**2*d*f**m*m**2*x**9*x**m/(m*
*8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**
3 + 2924172*m**2 + 4098240*m + 2027025) + 1291005*a*c**2*d*f**m*m
*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4

```
 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 675675*a*
c**2*d*f**m*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 +
208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
+ 3*a*c**2*e*f**m*m**7*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 2
4640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
 + 2027025) + 159*a*c**2*e*f**m*m**6*x**11*x**m/(m**8 + 64*m**7 +
 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m*
*2 + 4098240*m + 2027025) + 3375*a*c**2*e*f**m*m**5*x**11*x**m/(m
**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m*
*3 + 2924172*m**2 + 4098240*m + 2027025) + 36795*a*c**2*e*f**m*m*
*4*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m
**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 219417
*a*c**2*e*f**m*m**3*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 2464
0*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 700461*a*c**2*e*f**m*m**2*x**11*x**m/(m**8 + 64*m**7 +
 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m*
*2 + 4098240*m + 2027025) + 1067445*a*c**2*e*f**m*m*x**11*x**m/(m
**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m*
*3 + 2924172*m**2 + 4098240*m + 2027025) + 552825*a*c**2*e*f**m*x
**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4
+ 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + b**3*d*f**
m*m**7*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20805
4*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 57*
b**3*d*f**m*m**6*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m*
*5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027
025) + 1309*b**3*d*f**m*m**5*x**7*x**m/(m**8 + 64*m**7 + 1708*m**
6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098
240*m + 2027025) + 15477*b**3*d*f**m*m**4*x**7*x**m/(m**8 + 64*m*
*7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 99715*b**3*d*f**m*m**3*x**7*x**m/
(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*
m**3 + 2924172*m**2 + 4098240*m + 2027025) + 340011*b**3*d*f**m*m
**2*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m
**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 544095
*b**3*d*f**m*m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202702
5) + 289575*b**3*d*f**m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 2
4640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
 + 2027025) + b**3*e*f**m*m**7*x**9*x**m/(m**8 + 64*m**7 + 1708*m
**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40
98240*m + 2027025) + 55*b**3*e*f**m*m**6*x**9*x**m/(m**8 + 64*m**
7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172
*m**2 + 4098240*m + 2027025) + 1213*b**3*e*f**m*m**5*x**9*x**m/(m
**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m*
*3 + 2924172*m**2 + 4098240*m + 2027025) + 13723*b**3*e*f**m*m**4
*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4
 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 84547*b**
3*e*f**m*m**3*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
+ 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025
) + 277093*b**3*e*f**m*m**2*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6
 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40982
40*m + 2027025) + 430335*b**3*e*f**m*m*x**9*x**m/(m**8 + 64*m**7
+ 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m
**2 + 4098240*m + 2027025) + 225225*b**3*e*f**m*x**9*x**m/(m**8 +
```

64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
2924172*m**2 + 4098240*m + 2027025) + 3*b**2*c*d*f**m*m**7*x**9*x
m/(m8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038
016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 165*b**2*c*d*f**
m*m**6*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20805
4*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 363
9*b**2*c*d*f**m*m**5*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 2464
0*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 41169*b**2*c*d*f**m*m**4*x**9*x**m/(m**8 + 64*m**7 + 1
708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2
 + 4098240*m + 2027025) + 253641*b**2*c*d*f**m*m**3*x**9*x**m/(m*
*8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**
3 + 2924172*m**2 + 4098240*m + 2027025) + 831279*b**2*c*d*f**m*m*
*2*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m*
*4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1291005
*b**2*c*d*f**m*m*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m*
*5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027
025) + 675675*b**2*c*d*f**m*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6
 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40982
40*m + 2027025) + 3*b**2*c*e*f**m*m**7*x**11*x**m/(m**8 + 64*m**7
 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*
m**2 + 4098240*m + 2027025) + 159*b**2*c*e*f**m*m**6*x**11*x**m/(
m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m
3 + 2924172*m2 + 4098240*m + 2027025) + 3375*b**2*c*e*f**m*m*
*5*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m
4 + 1038016*m3 + 2924172*m**2 + 4098240*m + 2027025) + 36795*
b**2*c*e*f**m*m**4*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640
*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2
027025) + 219417*b**2*c*e*f**m*m**3*x**11*x**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**
2 + 4098240*m + 2027025) + 700461*b**2*c*e*f**m*m**2*x**11*x**m/(
m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m
3 + 2924172*m2 + 4098240*m + 2027025) + 1067445*b**2*c*e*f**m
*m*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m
4 + 1038016*m3 + 2924172*m**2 + 4098240*m + 2027025) + 552825
*b**2*c*e*f**m*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20270
25) + 3*b*c**2*d*f**m*m**7*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6
 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40982
40*m + 2027025) + 159*b*c**2*d*f**m*m**6*x**11*x**m/(m**8 + 64*m*
*7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 3375*b*c**2*d*f**m*m**5*x**11*x**
m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103801
6*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 36795*b*c**2*d*f**
m*m**4*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2080
54*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 21
9417*b*c**2*d*f**m*m**3*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*
m + 2027025) + 700461*b*c**2*d*f**m*m**2*x**11*x**m/(m**8 + 64*m*
*7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 1067445*b*c**2*d*f**m*m*x**11*x**
m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103801
6*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 552825*b*c**2*d*f*
*m*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m

```
**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*b*c*
*2*e*f**m*m**7*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20270
25) + 153*b*c**2*e*f**m*m**6*x**13*x**m/(m**8 + 64*m**7 + 1708*m*
*6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409
8240*m + 2027025) + 3135*b*c**2*e*f**m*m**5*x**13*x**m/(m**8 + 64
*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292
4172*m**2 + 4098240*m + 2027025) + 33165*b*c**2*e*f**m*m**4*x**13
*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10
38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 193017*b*c**2*
e*f**m*m**3*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 +
 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
 + 604827*b*c**2*e*f**m*m**2*x**13*x**m/(m**8 + 64*m**7 + 1708*m*
*6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409
8240*m + 2027025) + 909765*b*c**2*e*f**m*m*x**13*x**m/(m**8 + 64*
m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924
172*m**2 + 4098240*m + 2027025) + 467775*b*c**2*e*f**m*x**13*x**m
/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016
*m**3 + 2924172*m**2 + 4098240*m + 2027025) + c**3*d*f**m*m**7*x*
*13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 +
 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 51*c**3*d*f
**m*m**6*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20
8054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) +
1045*c**3*d*f**m*m**5*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24
640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
+ 2027025) + 11055*c**3*d*f**m*m**4*x**13*x**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**
2 + 4098240*m + 2027025) + 64339*c**3*d*f**m*m**3*x**13*x**m/(m**
8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3
 + 2924172*m**2 + 4098240*m + 2027025) + 201609*c**3*d*f**m*m**2*
x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4
 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 303255*c*
*3*d*f**m*m*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 +
 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
 + 155925*c**3*d*f**m*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24
640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
+ 2027025) + c**3*e*f**m*m**7*x**15*x**m/(m**8 + 64*m**7 + 1708*m
**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40
98240*m + 2027025) + 49*c**3*e*f**m*m**6*x**15*x**m/(m**8 + 64*m*
*7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 973*c**3*e*f**m*m**5*x**15*x**m/(
m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m
**3 + 2924172*m**2 + 4098240*m + 2027025) + 10045*c**3*e*f**m*m**
4*x**15*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m*
*4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 57379*c
**3*e*f**m*m**3*x**15*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m*
*5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027
025) + 177331*c**3*e*f**m*m**2*x**15*x**m/(m**8 + 64*m**7 + 1708*
m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4
098240*m + 2027025) + 264207*c**3*e*f**m*m*x**15*x**m/(m**8 + 64*
m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924
172*m**2 + 4098240*m + 2027025) + 135135*c**3*e*f**m*x**15*x**m/(
m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m
**3 + 2924172*m**2 + 4098240*m + 2027025), True))
```

GIAC/XCAS [A] time = 0.292848, size = 1, normalized size = 0.

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^3*(e*x^2 + d)*(f*x)^m,x, algorithm="giac")

[Out] Done
```

3.221 $\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=155

$$\frac{a^2d(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+7}\left(2ace+b^2e+2bcd\right)}{f^7(m+7)}+\frac{(fx)^{m+5}\left(2abe+2acd+b^2d\right)}{f^5(m+5)}+\frac{a(fx)^{m+3}(ae+2bd)}{f^3(m+3)}+\frac{c(fx)^{m+9}(2be+cd)}{f^9(m+9)}+\frac{c^2e(fx)^{m+11}}{f^{11}(m+11)}$$

```
[Out] (a^2*d*(f*x)^(1 + m))/(f*(1 + m)) + (a*(2*b*d + a*e)*(f*x)^(3 + m
))/(f^3*(3 + m)) + ((b^2*d + 2*a*c*d + 2*a*b*e)*(f*x)^(5 + m))/(f
^5*(5 + m)) + ((2*b*c*d + b^2*e + 2*a*c*e)*(f*x)^(7 + m))/(f^7*(7
 + m)) + (c*(c*d + 2*b*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (c^2*e*(
f*x)^(11 + m))/(f^11*(11 + m))
```

Rubi [A] time = 0.236653, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$

$$\frac{a^2d(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+7}\left(2ace+b^2e+2bcd\right)}{f^7(m+7)}+\frac{(fx)^{m+5}\left(2abe+2acd+b^2d\right)}{f^5(m+5)}+\frac{a(fx)^{m+3}(ae+2bd)}{f^3(m+3)}+\frac{c(fx)^{m+9}(2be+cd)}{f^9(m+9)}+\frac{c^2e(fx)^{m+11}}{f^{11}(m+11)}$$

Antiderivative was successfully verified.

```
[In]  Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*d*(f*x)^(1 + m))/(f*(1 + m)) + (a*(2*b*d + a*e)*(f*x)^(3 + m
))/(f^3*(3 + m)) + ((b^2*d + 2*a*c*d + 2*a*b*e)*(f*x)^(5 + m))/(f
^5*(5 + m)) + ((2*b*c*d + b^2*e + 2*a*c*e)*(f*x)^(7 + m))/(f^7*(7
 + m)) + (c*(c*d + 2*b*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (c^2*e*(
f*x)^(11 + m))/(f^11*(11 + m))
```

Rubi in Sympy [A] time = 43.5034, size = 146, normalized size = 0.94

$$\frac{a^2d\,(fx)^{m+1}}{f\,(m+1)}+\frac{a\,(fx)^{m+3}\,(ae+2bd)}{f^3\,(m+3)}+\frac{c^2e\,(fx)^{m+11}}{f^{11}\,(m+11)}+\frac{c\,(fx)^{m+9}\,(2be+cd)}{f^9\,(m+9)}+\frac{(fx)^{m+5}\left(2abe+2acd+b^2d\right)}{f^5\,(m+5)}+\frac{(fx)^{m+7}\left(2ace+b^2e+2bcd\right)}{f^7\,(m+7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d*(f*x)**(m + 1)/(f*(m + 1)) + a*(f*x)**(m + 3)*(a*e + 2*b*d
)/(f**3*(m + 3)) + c**2*e*(f*x)**(m + 11)/(f**11*(m + 11)) + c*(f
*x)**(m + 9)*(2*b*e + c*d)/(f**9*(m + 9)) + (f*x)**(m + 5)*(2*a*b
*e + 2*a*c*d + b**2*d)/(f**5*(m + 5)) + (f*x)**(m + 7)*(2*a*c*e +
 b**2*e + 2*b*c*d)/(f**7*(m + 7))
```

Mathematica [A] time = 0.24354, size = 117, normalized size = 0.75

$$(fx)^m\left(\frac{a^2dx}{m+1}+\frac{x^7\left(2ace+b^2e+2bcd\right)}{m+7}+\frac{x^5\left(2abe+2acd+b^2d\right)}{m+5}+\frac{ax^3(ae+2bd)}{m+3}+\frac{cx^9(2be+cd)}{m+9}+\frac{c^2ex^{11}}{m+11}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (f*x)^m*((a^2*d*x)/(1 + m) + (a*(2*b*d + a*e)*x^3)/(3 + m) + ((b^
2*d + 2*a*c*d + 2*a*b*e)*x^5)/(5 + m) + ((2*b*c*d + b^2*e + 2*a*c
*e)*x^7)/(7 + m) + (c*(c*d + 2*b*e)*x^9)/(9 + m) + (c^2*e*x^11)/(
11 + m))
```

Maple [B] time = 0.011, size = 783, normalized size = 5.1

$$\frac{\left(c^2em^5x^{10}+25\,c^2em^4x^{10}+2\,bcem^5x^8+c^2dm^5x^8+230\,c^2em^3x^{10}+54\,bcem^4x^8+27\,c^2dm^4x^8+950\,c^2em^2x^{10}+2\,acem^5x^6+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] x*(c^2*e*m^5*x^10+25*c^2*e*m^4*x^10+2*b*c*e*m^5*x^8+c^2*d*m^5*x^8
+230*c^2*e*m^3*x^10+54*b*c*e*m^4*x^8+27*c^2*d*m^4*x^8+950*c^2*e*m
^2*x^10+2*a*c*e*m^5*x^6+b^2*e*m^5*x^6+2*b*c*d*m^5*x^6+524*b*c*e*m
^3*x^8+262*c^2*d*m^3*x^8+1689*c^2*e*m*x^10+58*a*c*e*m^4*x^6+29*b^
2*e*m^4*x^6+58*b*c*d*m^4*x^6+2244*b*c*e*m^2*x^8+1122*c^2*d*m^2*x^
8+945*c^2*e*x^10+2*a*b*e*m^5*x^4+2*a*c*d*m^5*x^4+604*a*c*e*m^3*x^
6+b^2*d*m^5*x^4+302*b^2*e*m^3*x^6+604*b*c*d*m^3*x^6+4082*b*c*e*m*
x^8+2041*c^2*d*m*x^8+62*a*b*e*m^4*x^4+62*a*c*d*m^4*x^4+2732*a*c*e
*m^2*x^6+31*b^2*d*m^4*x^4+1366*b^2*e*m^2*x^6+2732*b*c*d*m^2*x^6+2
310*b*c*e*x^8+1155*c^2*d*x^8+a^2*e*m^5*x^2+2*a*b*d*m^5*x^2+700*a*
b*e*m^3*x^4+700*a*c*d*m^3*x^4+5154*a*c*e*m*x^6+350*b^2*d*m^3*x^4+
2577*b^2*e*m*x^6+5154*b*c*d*m*x^6+33*a^2*e*m^4*x^2+66*a*b*d*m^4*x
```

```
^2+3460*a*b*e*m^2*x^4+3460*a*c*d*m^2*x^4+2970*a*c*e*x^6+1730*b^2*
d*m^2*x^4+1485*b^2*e*x^6+2970*b*c*d*x^6+a^2*d*m^5+406*a^2*e*m^3*x
^2+812*a*b*d*m^3*x^2+6978*a*b*e*m*x^4+6978*a*c*d*m*x^4+3489*b^2*d
*m*x^4+35*a^2*d*m^4+2262*a^2*e*m^2*x^2+4524*a*b*d*m^2*x^2+4158*a*
b*e*x^4+4158*a*c*d*x^4+2079*b^2*d*x^4+470*a^2*d*m^3+5353*a^2*e*m*
x^2+10706*a*b*d*m*x^2+3010*a^2*d*m^2+3465*a^2*e*x^2+6930*a*b*d*x^
2+9129*a^2*d*m+10395*a^2*d)*(f*x)^m/(11+m)/(9+m)/(7+m)/(5+m)/(3+m
)/(1+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(e*x^2 + d)*(f*x)^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.312559, size = 774, normalized size = 4.99

$$\frac{\left(\left(c^2em^5 + 25\,c^2em^4 + 230\,c^2em^3 + 950\,c^2em^2 + 1689\,c^2em + 945\,c^2e\right)x^{11} + \left(\left(c^2d + 2\,bce\right)m^5 + 27\,\left(c^2d + 2\,bce\right)m^4 + 262\,\left(c^2\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(e*x^2 + d)*(f*x)^m,x, algorithm="fricas")
```

```
[Out] ((c^2*e*m^5 + 25*c^2*e*m^4 + 230*c^2*e*m^3 + 950*c^2*e*m^2 + 1689
*c^2*e*m + 945*c^2*e)*x^11 + ((c^2*d + 2*b*c*e)*m^5 + 27*(c^2*d +
 2*b*c*e)*m^4 + 262*(c^2*d + 2*b*c*e)*m^3 + 1155*c^2*d + 2310*b*c
*e + 1122*(c^2*d + 2*b*c*e)*m^2 + 2041*(c^2*d + 2*b*c*e)*m)*x^9 +
 ((2*b*c*d + (b^2 + 2*a*c)*e)*m^5 + 29*(2*b*c*d + (b^2 + 2*a*c)*e
)*m^4 + 302*(2*b*c*d + (b^2 + 2*a*c)*e)*m^3 + 2970*b*c*d + 1366*(
2*b*c*d + (b^2 + 2*a*c)*e)*m^2 + 1485*(b^2 + 2*a*c)*e + 2577*(2*b
*c*d + (b^2 + 2*a*c)*e)*m)*x^7 + ((2*a*b*e + (b^2 + 2*a*c)*d)*m^5
 + 31*(2*a*b*e + (b^2 + 2*a*c)*d)*m^4 + 350*(2*a*b*e + (b^2 + 2*a
*c)*d)*m^3 + 4158*a*b*e + 1730*(2*a*b*e + (b^2 + 2*a*c)*d)*m^2 +
2079*(b^2 + 2*a*c)*d + 3489*(2*a*b*e + (b^2 + 2*a*c)*d)*m)*x^5 +
((2*a*b*d + a^2*e)*m^5 + 33*(2*a*b*d + a^2*e)*m^4 + 406*(2*a*b*d
+ a^2*e)*m^3 + 6930*a*b*d + 3465*a^2*e + 2262*(2*a*b*d + a^2*e)*m
^2 + 5353*(2*a*b*d + a^2*e)*m)*x^3 + (a^2*d*m^5 + 35*a^2*d*m^4 +
470*a^2*d*m^3 + 3010*a^2*d*m^2 + 9129*a^2*d*m + 10395*a^2*d)*x)*(
f*x)^m/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m +
```

```
10395)
```

Sympy [A] time = 16.8543, size = 4190, normalized size = 27.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Piecewise(((-a**2*d/(10*x**10) - a**2*e/(8*x**8) - a*b*d/(4*x**8)
 - a*b*e/(3*x**6) - a*c*d/(3*x**6) - a*c*e/(2*x**4) - b**2*d/(6*x
**6) - b**2*e/(4*x**4) - b*c*d/(2*x**4) - b*c*e/x**2 - c**2*d/(2*
x**2) + c**2*e*log(x))/f**11, Eq(m, -11)), ((-a**2*d/(8*x**8) - a
**2*e/(6*x**6) - a*b*d/(3*x**6) - a*b*e/(2*x**4) - a*c*d/(2*x**4)
 - a*c*e/x**2 - b**2*d/(4*x**4) - b**2*e/(2*x**2) - b*c*d/x**2 +
2*b*c*e*log(x) + c**2*d*log(x) + c**2*e*x**2/2)/f**9, Eq(m, -9)),
 ((-a**2*d/(6*x**6) - a**2*e/(4*x**4) - a*b*d/(2*x**4) - a*b*e/x*
*2 - a*c*d/x**2 + 2*a*c*e*log(x) - b**2*d/(2*x**2) + b**2*e*log(x
) + 2*b*c*d*log(x) + b*c*e*x**2 + c**2*d*x**2/2 + c**2*e*x**4/4)/
f**7, Eq(m, -7)), ((-a**2*d/(4*x**4) - a**2*e/(2*x**2) - a*b*d/x*
*2 + 2*a*b*e*log(x) + 2*a*c*d*log(x) + a*c*e*x**2 + b**2*d*log(x)
 + b**2*e*x**2/2 + b*c*d*x**2 + b*c*e*x**4/2 + c**2*d*x**4/4 + c*
*2*e*x**6/6)/f**5, Eq(m, -5)), ((-a**2*d/(2*x**2) + a**2*e*log(x)
 + 2*a*b*d*log(x) + a*b*e*x**2 + a*c*d*x**2 + a*c*e*x**4/2 + b**2
*d*x**2/2 + b**2*e*x**4/4 + b*c*d*x**4/2 + b*c*e*x**6/3 + c**2*d*
x**6/6 + c**2*e*x**8/8)/f**3, Eq(m, -3)), ((a**2*d*log(x) + a**2*
e*x**2/2 + a*b*d*x**2 + a*b*e*x**4/2 + a*c*d*x**4/2 + a*c*e*x**6/
3 + b**2*d*x**4/4 + b**2*e*x**6/6 + b*c*d*x**6/3 + b*c*e*x**8/4 +
 c**2*d*x**8/8 + c**2*e*x**10/10)/f, Eq(m, -1)), (a**2*d*f**m*m**
5*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19
524*m + 10395) + 35*a**2*d*f**m*m**4*x*x**m/(m**6 + 36*m**5 + 505
*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 470*a**2*d*f*
*m*m**3*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**
2 + 19524*m + 10395) + 3010*a**2*d*f**m*m**2*x*x**m/(m**6 + 36*m*
*5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 9129*
a**2*d*f**m*m*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + 10395*a**2*d*f**m*x*x**m/(m**6 + 36*
m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + a**
2*e*f**m*m**5*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 +
12139*m**2 + 19524*m + 10395) + 33*a**2*e*f**m*m**4*x**3*x**m/(m*
*6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1039
5) + 406*a**2*e*f**m*m**3*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 +
3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2262*a**2*e*f**m*m**2
*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
19524*m + 10395) + 5353*a**2*e*f**m*m*x**3*x**m/(m**6 + 36*m**5 +
 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3465*a**2
*e*f**m*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*
m**2 + 19524*m + 10395) + 2*a*b*d*f**m*m**5*x**3*x**m/(m**6 + 36*
```

```
m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 66*
a*b*d*f**m*m**4*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
+ 12139*m**2 + 19524*m + 10395) + 812*a*b*d*f**m*m**3*x**3*x**m/(
m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10
395) + 4524*a*b*d*f**m*m**2*x**3*x**m/(m**6 + 36*m**5 + 505*m**4
+ 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10706*a*b*d*f**m*m*
x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1
9524*m + 10395) + 6930*a*b*d*f**m*x**3*x**m/(m**6 + 36*m**5 + 505
*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2*a*b*e*f**m*
m**5*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**
2 + 19524*m + 10395) + 62*a*b*e*f**m*m**4*x**5*x**m/(m**6 + 36*m*
*5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 700*a
*b*e*f**m*m**3*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 +
 12139*m**2 + 19524*m + 10395) + 3460*a*b*e*f**m*m**2*x**5*x**m/(
m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10
395) + 6978*a*b*e*f**m*m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3
480*m**3 + 12139*m**2 + 19524*m + 10395) + 4158*a*b*e*f**m*x**5*x
**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
 + 10395) + 2*a*c*d*f**m*m**5*x**5*x**m/(m**6 + 36*m**5 + 505*m**
4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 62*a*c*d*f**m*m**
4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
 19524*m + 10395) + 700*a*c*d*f**m*m**3*x**5*x**m/(m**6 + 36*m**5
 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3460*a*
c*d*f**m*m**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 +
12139*m**2 + 19524*m + 10395) + 6978*a*c*d*f**m*m*x**5*x**m/(m**6
 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
 + 4158*a*c*d*f**m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m*
*3 + 12139*m**2 + 19524*m + 10395) + 2*a*c*e*f**m*m**5*x**7*x**m/
(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1
0395) + 58*a*c*e*f**m*m**4*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 +
 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 604*a*c*e*f**m*m**3*
x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1
9524*m + 10395) + 2732*a*c*e*f**m*m**2*x**7*x**m/(m**6 + 36*m**5
+ 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 5154*a*c
*e*f**m*m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1213
9*m**2 + 19524*m + 10395) + 2970*a*c*e*f**m*x**7*x**m/(m**6 + 36*
m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + b**
2*d*f**m*m**5*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 +
12139*m**2 + 19524*m + 10395) + 31*b**2*d*f**m*m**4*x**5*x**m/(m*
*6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1039
5) + 350*b**2*d*f**m*m**3*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 +
3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1730*b**2*d*f**m*m**2
*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
19524*m + 10395) + 3489*b**2*d*f**m*m*x**5*x**m/(m**6 + 36*m**5 +
 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2079*b**2
*d*f**m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*
m**2 + 19524*m + 10395) + b**2*e*f**m*m**5*x**7*x**m/(m**6 + 36*m
**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 29*b
**2*e*f**m*m**4*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
+ 12139*m**2 + 19524*m + 10395) + 302*b**2*e*f**m*m**3*x**7*x**m/
(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1
0395) + 1366*b**2*e*f**m*m**2*x**7*x**m/(m**6 + 36*m**5 + 505*m**
4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2577*b**2*e*f**m*
m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
```

```
 19524*m + 10395) + 1485*b**2*e*f**m*x**7*x**m/(m**6 + 36*m**5 +
505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2*b*c*d*f*
*m*m**5*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*
m**2 + 19524*m + 10395) + 58*b*c*d*f**m*m**4*x**7*x**m/(m**6 + 36
*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 60
4*b*c*d*f**m*m**3*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**
3 + 12139*m**2 + 19524*m + 10395) + 2732*b*c*d*f**m*m**2*x**7*x**
m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m +
 10395) + 5154*b*c*d*f**m*m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4
+ 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2970*b*c*d*f**m*x**
7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1952
4*m + 10395) + 2*b*c*e*f**m*m**5*x**9*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 54*b*c*e*f**m*
m**4*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**
2 + 19524*m + 10395) + 524*b*c*e*f**m*m**3*x**9*x**m/(m**6 + 36*m
**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2244
*b*c*e*f**m*m**2*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
 + 12139*m**2 + 19524*m + 10395) + 4082*b*c*e*f**m*m*x**9*x**m/(m
**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 103
95) + 2310*b*c*e*f**m*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480
*m**3 + 12139*m**2 + 19524*m + 10395) + c**2*d*f**m*m**5*x**9*x**
m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m +
 10395) + 27*c**2*d*f**m*m**4*x**9*x**m/(m**6 + 36*m**5 + 505*m**
4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 262*c**2*d*f**m*m
**3*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2
 + 19524*m + 10395) + 1122*c**2*d*f**m*m**2*x**9*x**m/(m**6 + 36*
m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 204
1*c**2*d*f**m*m*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
+ 12139*m**2 + 19524*m + 10395) + 1155*c**2*d*f**m*x**9*x**m/(m**
6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395
) + c**2*e*f**m*m**5*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480
*m**3 + 12139*m**2 + 19524*m + 10395) + 25*c**2*e*f**m*m**4*x**11
*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524
*m + 10395) + 230*c**2*e*f**m*m**3*x**11*x**m/(m**6 + 36*m**5 + 5
05*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 950*c**2*e*
f**m*m**2*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + 1689*c**2*e*f**m*m*x**11*x**m/(m**6
+ 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
+ 945*c**2*e*f**m*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m*
*3 + 12139*m**2 + 19524*m + 10395), True))
```

GIAC/XCAS [A] time = 0.27698, size = 1, normalized size = 0.01

$$\text{Done}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^2*(e*x^2 + d)*(f*x)^m,x, algorithm="giac")
```

```
[Out] Done
```

3.222 $\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right) \, dx$

Optimal. Leaf size=83

$$\frac{(fx)^{m+3}(ae+bd)}{f^3(m+3)}+\frac{ad(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+5}(be+cd)}{f^5(m+5)}+\frac{ce(fx)^{m+7}}{f^7(m+7)}$$

```
[Out] (a*d*(f*x)^(1 + m))/(f*(1 + m)) + ((b*d + a*e)*(f*x)^(3 + m))/(f^
3*(3 + m)) + ((c*d + b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (c*e*(f*
x)^(7 + m))/(f^7*(7 + m))
```

Rubi [A] time = 0.11259, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{(fx)^{m+3}(ae+bd)}{f^3(m+3)}+\frac{ad(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+5}(be+cd)}{f^5(m+5)}+\frac{ce(fx)^{m+7}}{f^7(m+7)}$$

Antiderivative was successfully verified.

```
[In]  Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d*(f*x)^(1 + m))/(f*(1 + m)) + ((b*d + a*e)*(f*x)^(3 + m))/(f^
3*(3 + m)) + ((c*d + b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (c*e*(f*
x)^(7 + m))/(f^7*(7 + m))
```

Rubi in Sympy [A] time = 21.1202, size = 71, normalized size = 0.86

$$\frac{ad\,(fx)^{m+1}}{f\,(m+1)}+\frac{ce\,(fx)^{m+7}}{f^7\,(m+7)}+\frac{(fx)^{m+3}\,(ae+bd)}{f^3\,(m+3)}+\frac{(fx)^{m+5}\,(be+cd)}{f^5\,(m+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d*(f*x)**(m + 1)/(f*(m + 1)) + c*e*(f*x)**(m + 7)/(f**7*(m + 7)
) + (f*x)**(m + 3)*(a*e + b*d)/(f**3*(m + 3)) + (f*x)**(m + 5)*(b
*e + c*d)/(f**5*(m + 5))
```

Mathematica [A] time = 0.0806152, size = 59, normalized size = 0.71

$$(fx)^m\left(\frac{x^3(ae+bd)}{m+3}+\frac{adx}{m+1}+\frac{x^5(be+cd)}{m+5}+\frac{cex^7}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (f*x)^m*((a*d*x)/(1 + m) + ((b*d + a*e)*x^3)/(3 + m) + ((c*d + b*
e)*x^5)/(5 + m) + (c*e*x^7)/(7 + m))
```

Maple [B] time = 0.006, size = 221, normalized size = 2.7

$$\frac{\left(cem^3x^6 + 9\,cem^2x^6 + bem^3x^4 + cdm^3x^4 + 23\,cemx^6 + 11\,bem^2x^4 + 11\,cdm^2x^4 + 15\,cex^6 + aem^3x^2 + bdm^3x^2 + 31\,bemx^4 + \right.}{(7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x)
```

```
[Out] x*(c*e*m^3*x^6+9*c*e*m^2*x^6+b*e*m^3*x^4+c*d*m^3*x^4+23*c*e*m*x^6
+11*b*e*m^2*x^4+11*c*d*m^2*x^4+15*c*e*x^6+a*e*m^3*x^2+b*d*m^3*x^2
+31*b*e*m*x^4+31*c*d*m*x^4+13*a*e*m^2*x^2+13*b*d*m^2*x^2+21*b*e*x
^4+21*c*d*x^4+a*d*m^3+47*a*e*m*x^2+47*b*d*m*x^2+15*a*d*m^2+35*a*e
*x^2+35*b*d*x^2+71*a*d*m+105*a*d)*(f*x)^m/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.268219, size = 231, normalized size = 2.78

$$\frac{\left(\left(cem^3 + 9\,cem^2 + 23\,cem + 15\,ce\right)x^7 + \left((cd + be)m^3 + 11\,(cd + be)m^2 + 21\,cd + 21\,be + 31\,(cd + be)m\right)x^5 + \left((bd + ae)m^3 + 1\right.\right.}{m^4 + 16\,m^3 + 86\,m^2 + 176\,m -}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m,x, algorithm="fricas")
```

```
[Out] ((c*e*m^3 + 9*c*e*m^2 + 23*c*e*m + 15*c*e)*x^7 + ((c*d + b*e)*m^3
 + 11*(c*d + b*e)*m^2 + 21*c*d + 21*b*e + 31*(c*d + b*e)*m)*x^5 +
 ((b*d + a*e)*m^3 + 13*(b*d + a*e)*m^2 + 35*b*d + 35*a*e + 47*(b*
d + a*e)*m)*x^3 + (a*d*m^3 + 15*a*d*m^2 + 71*a*d*m + 105*a*d)*x)*
(f*x)^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Sympy [A] time = 5.42355, size = 1056, normalized size = 12.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] Piecewise(((-a*d/(6*x**6) - a*e/(4*x**4) - b*d/(4*x**4) - b*e/(2*
x**2) - c*d/(2*x**2) + c*e*log(x))/f**7, Eq(m, -7)), ((-a*d/(4*x*
*4) - a*e/(2*x**2) - b*d/(2*x**2) + b*e*log(x) + c*d*log(x) + c*e
*x**2/2)/f**5, Eq(m, -5)), ((-a*d/(2*x**2) + a*e*log(x) + b*d*log
(x) + b*e*x**2/2 + c*d*x**2/2 + c*e*x**4/4)/f**3, Eq(m, -3)), ((a
*d*log(x) + a*e*x**2/2 + b*d*x**2/2 + b*e*x**4/4 + c*d*x**4/4 + c
*e*x**6/6)/f, Eq(m, -1)), (a*d*f**m*m**3*x*x**m/(m**4 + 16*m**3 +
 86*m**2 + 176*m + 105) + 15*a*d*f**m*m**2*x*x**m/(m**4 + 16*m**3
 + 86*m**2 + 176*m + 105) + 71*a*d*f**m*m*x*x**m/(m**4 + 16*m**3
+ 86*m**2 + 176*m + 105) + 105*a*d*f**m*x*x**m/(m**4 + 16*m**3 +
86*m**2 + 176*m + 105) + a*e*f**m*m**3*x**3*x**m/(m**4 + 16*m**3
+ 86*m**2 + 176*m + 105) + 13*a*e*f**m*m**2*x**3*x**m/(m**4 + 16*
m**3 + 86*m**2 + 176*m + 105) + 47*a*e*f**m*m*x**3*x**m/(m**4 + 1
6*m**3 + 86*m**2 + 176*m + 105) + 35*a*e*f**m*x**3*x**m/(m**4 + 1
6*m**3 + 86*m**2 + 176*m + 105) + b*d*f**m*m**3*x**3*x**m/(m**4 +
 16*m**3 + 86*m**2 + 176*m + 105) + 13*b*d*f**m*m**2*x**3*x**m/(m
**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 47*b*d*f**m*m*x**3*x**m/
(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 35*b*d*f**m*x**3*x**m/
(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + b*e*f**m*m**3*x**5*x**
m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*b*e*f**m*m**2*x**
5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 31*b*e*f**m*m*x
**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 21*b*e*f**m*x
**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + c*d*f**m*m**3
*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*c*d*f**m
*m**2*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 31*c*d
*f**m*m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 21*c
*d*f**m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + c*e*
f**m*m**3*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 9*
c*e*f**m*m**2*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105)
+ 23*c*e*f**m*m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105
```

```
) + 15*c*e*f**m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105
), True))
```

GIAC/XCAS [A] time = 0.276233, size = 537, normalized size = 6.47

$$\frac{cm^3x^7e^{(m\ln(fx)+1)} + 9\,cm^2x^7e^{(m\ln(fx)+1)} + cdm^3x^5e^{(m\ln(fx))} + bm^3x^5e^{(m\ln(fx)+1)} + 23\,cmx^7e^{(m\ln(fx)+1)} + 11\,cdm^2x^5e^{(m\ln(fx))} + \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m,x, algorithm="giac")
```

```
[Out] (c*m^3*x^7*e^(m*ln(f*x) + 1) + 9*c*m^2*x^7*e^(m*ln(f*x) + 1) + c*
d*m^3*x^5*e^(m*ln(f*x)) + b*m^3*x^5*e^(m*ln(f*x) + 1) + 23*c*m*x^
7*e^(m*ln(f*x) + 1) + 11*c*d*m^2*x^5*e^(m*ln(f*x)) + 11*b*m^2*x^5
*e^(m*ln(f*x) + 1) + 15*c*x^7*e^(m*ln(f*x) + 1) + b*d*m^3*x^3*e^(
m*ln(f*x)) + 31*c*d*m*x^5*e^(m*ln(f*x)) + a*m^3*x^3*e^(m*ln(f*x)
+ 1) + 31*b*m*x^5*e^(m*ln(f*x) + 1) + 13*b*d*m^2*x^3*e^(m*ln(f*x)
) + 21*c*d*x^5*e^(m*ln(f*x)) + 13*a*m^2*x^3*e^(m*ln(f*x) + 1) + 2
1*b*x^5*e^(m*ln(f*x) + 1) + a*d*m^3*x*e^(m*ln(f*x)) + 47*b*d*m*x^
3*e^(m*ln(f*x)) + 47*a*m*x^3*e^(m*ln(f*x) + 1) + 15*a*d*m^2*x*e^(
m*ln(f*x)) + 35*b*d*x^3*e^(m*ln(f*x)) + 35*a*x^3*e^(m*ln(f*x) + 1
) + 71*a*d*m*x*e^(m*ln(f*x)) + 105*a*d*x*e^(m*ln(f*x)))/(m^4 + 16
*m^3 + 86*m^2 + 176*m + 105)
```

3.223 $\int \frac{(fx)^m\left(d+ex^2\right)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=194

$$\frac{(fx)^{m+1}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{f(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(fx)^{m+1}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometr
ic2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
])/((b - Sqrt[b^2 - 4*a*c])*f*(1 + m)) + ((e - (2*c*d - b*e)/Sqrt
[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 +
 m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/((b + Sqrt[b^2 - 4*a*
c])*f*(1 + m))
```

Rubi [A] time = 0.627567, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074

$$\frac{(fx)^{m+1}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{f(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(fx)^{m+1}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4),x]
```


```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometr
ic2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
])/((b - Sqrt[b^2 - 4*a*c])*f*(1 + m)) + ((e - (2*c*d - b*e)/Sqrt
[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 +
 m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/((b + Sqrt[b^2 - 4*a*
c])*f*(1 + m))
```

Rubi in Sympy [A] time = 48.5086, size = 185, normalized size = 0.95

$$\frac{(fx)^{m+1}\left(be-2cd+e\sqrt{-4ac+b^2}\right){}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{f\left(b+\sqrt{-4ac+b^2}\right)(m+1)\sqrt{-4ac+b^2}}-\frac{(fx)^{m+1}\left(be-2cd-e\sqrt{-4ac+b^2}\right){}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{2cx^2}{b-\sqrt{-4ac+b^2}}\right)}{f\left(b-\sqrt{-4ac+b^2}\right)(m+1)\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] (f*x)**(m + 1)*(b*e - 2*c*d + e*sqrt(-4*a*c + b**2))*hyper((1, m/
2 + 1/2), (m/2 + 3/2,), -2*c*x**2/(b + sqrt(-4*a*c + b**2)))/(f*(
b + sqrt(-4*a*c + b**2))*(m + 1)*sqrt(-4*a*c + b**2)) - (f*x)**(m
 + 1)*(b*e - 2*c*d - e*sqrt(-4*a*c + b**2))*hyper((1, m/2 + 1/2),
 (m/2 + 3/2,), -2*c*x**2/(b - sqrt(-4*a*c + b**2)))/(f*(b - sqrt(
-4*a*c + b**2))*(m + 1)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.47532, size = 316, normalized size = 1.63

$$\frac{d(fx)^m\text{RootSum}\left[\#1^4c+\#1^2b+a\&,\frac{\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)}{2\#1^3c+\#1b}\&\right]}{2m}+\frac{e(fx)^m\text{RootSum}\left[\#1^4c+\#1^2b+a\&,\frac{\#1^2m^2\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)+3\#1^2m\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)+2\#1^2\left(\frac{x}{x-\#1}\cdots\right.}{2\#1^3c+\#1b}\cdots\right]}{2m(m+1)(m+2)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (d*(f*x)^m*RootSum[a + b*#1^2 + c*#1^4 & , Hypergeometric2F1[-m,
-m, 1 - m, -(#1/(x - #1))]/((x/(x - #1))^m*(b*#1 + 2*c*#1^3)) & ]
)/(2*m) + (e*(f*x)^m*RootSum[a + b*#1^2 + c*#1^4 & , (m*x^2 + m^2
*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*Hypergeometric2F1[-m, -m, 1 - m,
-(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (3*m*Hypergeometric2F1[-m,
 -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m^2*Hypergeom
etric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m
*#1^2)/(x/#1)^m)/(b*#1 + 2*c*#1^3) & ])/(2*m*(1 + m)*(2 + m))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\,(e x^2+d)}{c x^4+b x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x)

[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e x^2+d)\,(fx)^m}{c x^4+b x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(e x^2+d)\,(fx)^m}{c x^4+b x^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a),x, algorithm="fricas")

[Out] integral((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m\,(d+e x^2)}{a+b x^2+c x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)(fx)^m}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

3.224 $\int \frac{(fx)^m(d+ex^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=392

$$\frac{c(fx)^{m+1}\left(b\left(d(1-m)\sqrt{b^2-4ac}+4ae\right)-2a\left(e(1-m)\sqrt{b^2-4ac}+2cd(3-m)\right)+b^2(d-dm)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2af(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(fx)^{m+1}\left(b\left(4ae-d(1-m)\sqrt{b^2-4ac}\right)+2a\left(e(1-m)\sqrt{b^2-4ac}-2cd(3-m)\right)+b^2d(1-m)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{2af(m+1)\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}+\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^2))/(
2*a*(b^2 - 4*a*c)*f*(a + b*x^2 + c*x^4)) + (c*(b*(4*a*e + Sqrt[b^
2 - 4*a*c]*d*(1 - m)) - 2*a*(Sqrt[b^2 - 4*a*c]*e*(1 - m) + 2*c*d*
(3 - m)) + b^2*(d - d*m))*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 +
 m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/(2*a*(b^2
- 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*f*(1 + m)) - (c*(b*(4*a*e
- Sqrt[b^2 - 4*a*c]*d*(1 - m)) + 2*a*(Sqrt[b^2 - 4*a*c]*e*(1 - m)
 - 2*c*d*(3 - m)) + b^2*d*(1 - m))*(f*x)^(1 + m)*Hypergeometric2F
1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(
2*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*f*(1 + m))
```

Rubi [A] time = 5.09435, antiderivative size = 358, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{c(fx)^{m+1}\left((1-m)\sqrt{b^2-4ac}(bd-2ae)+4abe-4acd(3-m)+b^2(d-dm)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2af(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(fx)^{m+1}\left(-(1-m)\sqrt{b^2-4ac}(bd-2ae)+4abe-4acd(3-m)+b^2(d-dm)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{2af(m+1)\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}+\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^2))/(
2*a*(b^2 - 4*a*c)*f*(a + b*x^2 + c*x^4)) + (c*(4*a*b*e + Sqrt[b^2
```

```
- 4*a*c]*(b*d - 2*a*e)*(1 - m) - 4*a*c*d*(3 - m) + b^2*(d - d*m)
)*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt
[b^2 - 4*a*c])*f*(1 + m)) - (c*(4*a*b*e - Sqrt[b^2 - 4*a*c]*(b*d
- 2*a*e)*(1 - m) - 4*a*c*d*(3 - m) + b^2*(d - d*m))*(f*x)^(1 + m)
*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*
f*(1 + m))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 1.93904, size = 692, normalized size = 1.77

$$ax(fx)^m\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(\frac{d(m+3)^2F_1\left(\frac{m}{2}\ldots\right.}{(m+1)\left(a(m+3)F_1\left(\frac{m+1}{2};2,2;\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-2x^2\left(\left(\sqrt{b^2-4ac}+b\right)F_1\ldots\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a*x*(f*x)^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)*((d*(3 + m)^2*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2
, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
4*a*c])])/((1 + m)*(a*(3 + m)*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2
, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
4*a*c])] - 2*x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(3 + m)/2, 2,
3, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(3 + m)/2
, 3, 2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/
(-b + Sqrt[b^2 - 4*a*c])]))) + (e*(5 + m)*x^2*AppellF1[(3 + m)/2,
 2, 2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])])/(a*(5 + m)*AppellF1[(3 + m)/2, 2, 2, (5
 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])] - 2*x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(5 + m)
```

```
/2, 2, 3, (7 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2
)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(5
 + m)/2, 3, 2, (7 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))/(4*c*(3 + m)*(a + b*x^2 + c*
x^4)^3)
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\,(ex^2+d)}{(cx^4+bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)\,(fx)^m}{(cx^4+bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^2+d)\,(fx)^m}{c^2x^8+2\,bcx^6+(b^2+2\,ac)x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*(f*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)
*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.225 $\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^{3/2} dx$

Optimal. Leaf size=319

$$\frac{ad(fx)^{m+1}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+1}{2};-\frac{3}{2},-\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{ae(fx)^{m+3}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+3}{2};-\frac{3}{2},-\frac{3}{2};\frac{m+5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (a*d*(f*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -3
/2, -3/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]])/(f*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - S
qrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) +
 (a*e*(f*x)^(3 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(3 + m)/2, -
3/2, -3/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]])/(f^3*(3 + m)*Sqrt[1 + (2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
)
```

Rubi [A] time = 1.1055, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{ad(fx)^{m+1}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+1}{2};-\frac{3}{2},-\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{ae(fx)^{m+3}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+3}{2};-\frac{3}{2},-\frac{3}{2};\frac{m+5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (a*d*(f*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -3
/2, -3/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]])/(f*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - S
qrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) +
 (a*e*(f*x)^(3 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(3 + m)/2, -
3/2, -3/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]])/(f^3*(3 + m)*Sqrt[1 + (2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
```

)

Rubi in Sympy [A] time = 94.8986, size = 286, normalized size = 0.9

$$\frac{ad\,(fx)^{m+1}\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(\frac{m}{2}+\frac{1}{2},-\frac{3}{2},-\frac{3}{2},\frac{m}{2}+\frac{3}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{f\,(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}+\frac{ae\,(fx)^{m+3}\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(\frac{m}{2}+\frac{3}{2},-\frac{3}{2},-\frac{3}{2},\frac{m}{2}+\frac{5}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{f^3\,(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] a*d*(f*x)**(m + 1)*sqrt(a + b*x**2 + c*x**4)*appellf1(m/2 + 1/2,
-3/2, -3/2, m/2 + 3/2, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*
x**2/(b + sqrt(-4*a*c + b**2)))/(f*(m + 1)*sqrt(2*c*x**2/(b - sqr
t(-4*a*c + b**2)) + 1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) +
1)) + a*e*(f*x)**(m + 3)*sqrt(a + b*x**2 + c*x**4)*appellf1(m/2 +
 3/2, -3/2, -3/2, m/2 + 5/2, -2*c*x**2/(b - sqrt(-4*a*c + b**2)),
 -2*c*x**2/(b + sqrt(-4*a*c + b**2)))/(f**3*(m + 3)*sqrt(2*c*x**2
/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c +
b**2)) + 1))
```

Mathematica [B] time = 5.37283, size = 2559, normalized size = 8.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (a*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*d*(3 + m)*x*(f
*x)^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)*AppellF1[(1 + m)/2, -1/2, -1/2, (3 + m)/2, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(8*c^
2*(1 + m)*Sqrt[a + b*x^2 + c*x^4]*(2*a*(3 + m)*AppellF1[(1 + m)/2
, -1/2, -1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*Ap
pellF1[(3 + m)/2, -1/2, 1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 -
4*a*c])*AppellF1[(3 + m)/2, 1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b +
```

```
 Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) + (b*
(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*d*(5 + m)*x^3*(f*
x)^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2)*AppellF1[(3 + m)/2, -1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(8*c^2
*(3 + m)*Sqrt[a + b*x^2 + c*x^4]*(2*a*(5 + m)*AppellF1[(3 + m)/2,
 -1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*App
ellF1[(5 + m)/2, -1/2, 1/2, (7 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4
*a*c])*AppellF1[(5 + m)/2, 1/2, -1/2, (7 + m)/2, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) + (a*(
b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*e*(5 + m)*x^3*(f*x
)^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)*AppellF1[(3 + m)/2, -1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(8*c^2*
(3 + m)*Sqrt[a + b*x^2 + c*x^4]*(2*a*(5 + m)*AppellF1[(3 + m)/2,
-1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*Appe
llF1[(5 + m)/2, -1/2, 1/2, (7 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*
a*c])*AppellF1[(5 + m)/2, 1/2, -1/2, (7 + m)/2, (-2*c*x^2)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) + ((b -
 Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*d*(7 + m)*x^5*(f*x)^m
*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2)*AppellF1[(5 + m)/2, -1/2, -1/2, (7 + m)/2, (-2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(8*c*(5 +
m)*Sqrt[a + b*x^2 + c*x^4]*(2*a*(7 + m)*AppellF1[(5 + m)/2, -1/2,
 -1/2, (7 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[
(7 + m)/2, -1/2, 1/2, (9 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])
*AppellF1[(7 + m)/2, 1/2, -1/2, (9 + m)/2, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) + (b*(b - Sq
rt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*e*(7 + m)*x^5*(f*x)^m*(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
*AppellF1[(5 + m)/2, -1/2, -1/2, (7 + m)/2, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(8*c^2*(5 + m
)*Sqrt[a + b*x^2 + c*x^4]*(2*a*(7 + m)*AppellF1[(5 + m)/2, -1/2,
-1/2, (7 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(
7 + m)/2, -1/2, 1/2, (9 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*
AppellF1[(7 + m)/2, 1/2, -1/2, (9 + m)/2, (-2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) + ((b - Sqrt[
b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*e*(9 + m)*x^7*(f*x)^m*(b -
Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*Ap
pellF1[(7 + m)/2, -1/2, -1/2, (9 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(8*c*(7 + m)*Sqr
t[a + b*x^2 + c*x^4]*(2*a*(9 + m)*AppellF1[(7 + m)/2, -1/2, -1/2,
 (9 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(9 + m
)/2, -1/2, 1/2, (11 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Appe
```

```
llF1[(9 + m)/2, 1/2, -1/2, (11 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int (fx)^m (ex^2+d) (cx^4+bx^2+a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^4+bx^2+a)^{\frac{3}{2}} (ex^2+d) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cex^6+(cd+be)x^4+(bd+ae)x^2+ad\right)\sqrt{cx^4+bx^2+a}\,(fx)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^m,x, algorithm="fricas")
```

```
[Out] integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt
(c*x^4 + b*x^2 + a)*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)(fx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^m,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^m, x)
```

3.226 $\int (fx)^m (d+ex^2) \sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=317

$$\frac{d(fx)^{m+1}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{e(fx)^{m+3}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+3}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (d*(f*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -1/2
, -1/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])])/(f*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (
e*(f*x)^(3 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(3 + m)/2, -1/2,
 -1/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[1 + (2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 1.06432, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{d(fx)^{m+1}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{e(fx)^{m+3}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+3}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(f*x)^m*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (d*(f*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -1/2
, -1/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])])/(f*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (
e*(f*x)^(3 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(3 + m)/2, -1/2,
 -1/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[1 + (2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 82.5343, size = 282, normalized size = 0.89

$$\frac{d (fx)^{m+1}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{m}{2}+\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{m}{2}+\frac{3}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}+\frac{e (fx)^{m+3}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{m}{2}+\frac{3}{2},-\frac{1}{2},-\frac{1}{2},\frac{m}{2}+\frac{5}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] d*(f*x)**(m + 1)*sqrt(a + b*x**2 + c*x**4)*appellf1(m/2 + 1/2, -1
/2, -1/2, m/2 + 3/2, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x*
*2/(b + sqrt(-4*a*c + b**2)))/(f*(m + 1)*sqrt(2*c*x**2/(b - sqrt(
-4*a*c + b**2)) + 1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)
) + e*(f*x)**(m + 3)*sqrt(a + b*x**2 + c*x**4)*appellf1(m/2 + 3/2
, -1/2, -1/2, m/2 + 5/2, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*
c*x**2/(b + sqrt(-4*a*c + b**2)))/(f**3*(m + 3)*sqrt(2*c*x**2/(b
- sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2
)) + 1))
```

Mathematica [B] time = 0.562517, size = 755, normalized size = 2.38

$$\frac{x\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)(fx)^m\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(\frac{}{(m+1)\left(x^2\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{m+3}{2};-\frac{1}{2}\right.\right.\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(f*x)^m*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*x*(f*x)^m*(b - S
qrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*((d
*(3 + m)^2*AppellF1[(1 + m)/2, -1/2, -1/2, (3 + m)/2, (-2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((1
 + m)*(2*a*(3 + m)*AppellF1[(1 + m)/2, -1/2, -1/2, (3 + m)/2, (-2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c
])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(3 + m)/2, -1/2, 1/2,
 (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(3 + m)/2,
```

```
1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
2)/(-b + Sqrt[b^2 - 4*a*c])]))) + (e*(5 + m)*x^2*AppellF1[(3 + m)
/2, -1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(2*a*(5 + m)*AppellF1[(3 + m)/
2, -1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*A
ppellF1[(5 + m)/2, -1/2, 1/2, (7 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 -
 4*a*c])*AppellF1[(5 + m)/2, 1/2, -1/2, (7 + m)/2, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))))/(8*
c^2*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.015, size = 0, normalized size = 0.

$$\int (fx)^m \left(ex^2+d\right) \sqrt{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+bx^2+a}\left(ex^2+d\right)(fx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)(fx)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (fx)^m \left(d + ex^2\right) \sqrt{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral((f*x)**m*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)(fx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m,x, algorithm="giac")

[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m, x)
```

3.227 $\int \frac{(fx)^m(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=317

$$\frac{d(fx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{1}{2},\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{m+3}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+3}{2};\frac{1}{2},\frac{1}{2};\frac{m+5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(m+3)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (d*(f*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 1/2,
1/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])])/(f*(1 + m)*Sqrt[a + b*x^2 + c*x^4]) + (e*
(f*x)^(3 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
+ (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3 + m)/2, 1/2, 1/2
, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 1.06078, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{d(fx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{1}{2},\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{m+3}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+3}{2};\frac{1}{2},\frac{1}{2};\frac{m+5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(m+3)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[((f*x)^m*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (d*(f*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 1/2,
1/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])])/(f*(1 + m)*Sqrt[a + b*x^2 + c*x^4]) + (e*
(f*x)^(3 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
+ (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3 + m)/2, 1/2, 1/2
, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 86.8298, size = 279, normalized size = 0.88

$$\frac{d\left(fx\right)^{m+1}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{m}{2}+\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{m}{2}+\frac{3}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{af\left(m+1\right)\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}+\frac{e\left(fx\right)^{m+3}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{m}{2}+\frac{3}{2},\frac{1}{2},\frac{1}{2},\frac{m}{2}+\frac{5}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{af^3\left(m+3\right)\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] d*(f*x)**(m + 1)*sqrt(a + b*x**2 + c*x**4)*appellf1(m/2 + 1/2, 1/
2, 1/2, m/2 + 3/2, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2
/(b + sqrt(-4*a*c + b**2)))/(a*f*(m + 1)*sqrt(2*c*x**2/(b - sqrt(
-4*a*c + b**2)) + 1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)
) + e*(f*x)**(m + 3)*sqrt(a + b*x**2 + c*x**4)*appellf1(m/2 + 3/2
, 1/2, 1/2, m/2 + 5/2, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*
x**2/(b + sqrt(-4*a*c + b**2)))/(a*f**3*(m + 3)*sqrt(2*c*x**2/(b
- sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2
)) + 1))
```

Mathematica [B] time = 2.67015, size = 728, normalized size = 2.3

$$ax(fx)^m\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(\frac{d(m+3)^2F_1\left(\ldots\right.}{(m+1)\left(2a(m+3)F_1\left(\frac{m+1}{2};\frac{1}{2},\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-x^2\left(\left(\sqrt{b^2-4ac}+b\right)\ldots\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((f*x)^m*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (a*x*(f*x)^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)*((d*(3 + m)^2*AppellF1[(1 + m)/2, 1/2, 1/2, (3 +
m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^
2 - 4*a*c])])/((1 + m)*(2*a*(3 + m)*AppellF1[(1 + m)/2, 1/2, 1/2,
 (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(3 + m
)/2, 1/2, 3/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Appell
F1[(3 + m)/2, 3/2, 1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) - (e*(5 + m)*x^2*App
ellF1[(3 + m)/2, 1/2, 1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
```

```
4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(-2*a*(5 + m)*Appel
lF1[(3 + m)/2, 1/2, 1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 -
 4*a*c])*AppellF1[(5 + m)/2, 1/2, 3/2, (7 + m)/2, (-2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - S
qrt[b^2 - 4*a*c])*AppellF1[(5 + m)/2, 3/2, 1/2, (7 + m)/2, (-2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]
))))/(2*c*(3 + m)*(a + b*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int (fx)^m \left(ex^2+d\right) \frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)(fx)^m}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex^2+d\right)(fx)^m}{\sqrt{cx^4+bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)(fx)^m}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

3.228 $\int \frac{(fx)^m(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=323

$$\frac{d(fx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af(m+1)\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{m+3}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+3}{2};\frac{3}{2},\frac{3}{2};\frac{m+5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af^3(m+3)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (d*(f*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 3/2,
3/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])])/(a*f*(1 + m)*Sqrt[a + b*x^2 + c*x^4]) + (
e*(f*x)^(3 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3 + m)/2, 3/2, 3
/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])])/(a*f^3*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 1.08536, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$\frac{d(fx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af(m+1)\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{m+3}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+3}{2};\frac{3}{2},\frac{3}{2};\frac{m+5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af^3(m+3)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (d*(f*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 3/2,
3/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])])/(a*f*(1 + m)*Sqrt[a + b*x^2 + c*x^4]) + (
e*(f*x)^(3 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3 + m)/2, 3/2, 3
/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])])/(a*f^3*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 111.897, size = 282, normalized size = 0.87

$$\frac{d(fx)^{m+1}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{m}{2}+\frac{1}{2},\frac{3}{2},\frac{3}{2},\frac{m}{2}+\frac{3}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{a^2f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}+\frac{e(fx)^{m+3}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{m}{2}+\frac{3}{2},\frac{3}{2},\frac{3}{2},\frac{m}{2}+\frac{5}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{a^2f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] d*(f*x)**(m + 1)*sqrt(a + b*x**2 + c*x**4)*appellf1(m/2 + 1/2, 3/
2, 3/2, m/2 + 3/2, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2
/(b + sqrt(-4*a*c + b**2)))/(a**2*f*(m + 1)*sqrt(2*c*x**2/(b - sq
rt(-4*a*c + b**2)) + 1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) +
 1)) + e*(f*x)**(m + 3)*sqrt(a + b*x**2 + c*x**4)*appellf1(m/2 +
3/2, 3/2, 3/2, m/2 + 5/2, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2
*c*x**2/(b + sqrt(-4*a*c + b**2)))/(a**2*f**3*(m + 3)*sqrt(2*c*x*
*2/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c
+ b**2)) + 1))
```

Mathematica [B] time = 3.02777, size = 728, normalized size = 2.25

$$ax(fx)^m\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(\frac{d(m+3)^2F_1\left(\ldots\right.}{(m+1)\left(2a(m+3)F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-3x^2\left(\left(\sqrt{b^2-4ac}+b\right)\ldots\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (a*x*(f*x)^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)*((d*(3 + m)^2*AppellF1[(1 + m)/2, 3/2, 3/2, (3 +
m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^
2 - 4*a*c])])/((1 + m)*(2*a*(3 + m)*AppellF1[(1 + m)/2, 3/2, 3/2,
 (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])] - 3*x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(3 +
 m)/2, 3/2, 5/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Appe
llF1[(3 + m)/2, 5/2, 3/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) + (e*(5 + m)*x^2*A
ppellF1[(3 + m)/2, 3/2, 3/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2
```

```
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(2*a*(5 + m)*Appe
llF1[(3 + m)/2, 3/2, 3/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 3*x^2*((b + Sqrt[b^
2 - 4*a*c])*AppellF1[(5 + m)/2, 3/2, 5/2, (7 + m)/2, (-2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b
- Sqrt[b^2 - 4*a*c])*AppellF1[(5 + m)/2, 5/2, 3/2, (7 + m)/2, (-2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c
])])))))/(2*c*(3 + m)*(a + b*x^2 + c*x^4)^(5/2))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int (fx)^m \left(ex^2+d\right) \left(cx^4+bx^2+a\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)(fx)^m}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex^2+d\right)(fx)^m}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)\left(fx\right)^m}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.229 $\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=134

$$\frac{a^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}-\frac{a^2e\log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(ae^2+cd^2\right)}-\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}$$

```
[Out] -(d*x^2)/(2*c*e^2) + x^4/(4*c*e) + (a^(3/2)*d*ArcTan[(Sqrt[c]*x^2
)/Sqrt[a]])/(2*c^(3/2)*(c*d^2 + a*e^2)) + (d^4*Log[d + e*x^2])/(2
*e^3*(c*d^2 + a*e^2)) - (a^2*e*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*
e^2))
```

Rubi [A] time = 0.322171, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{a^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}-\frac{a^2e\log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(ae^2+cd^2\right)}-\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(d*x^2)/(2*c*e^2) + x^4/(4*c*e) + (a^(3/2)*d*ArcTan[(Sqrt[c]*x^2
)/Sqrt[a]])/(2*c^(3/2)*(c*d^2 + a*e^2)) + (d^4*Log[d + e*x^2])/(2
*e^3*(c*d^2 + a*e^2)) - (a^2*e*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*
e^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^{\frac{3}{2}}d\,\mathrm{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}\left(ae^2+cd^2\right)}-\frac{a^2e\log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(ae^2+cd^2\right)}+\frac{\int^{x^2}x\,dx}{2ce}-\frac{\int^{x^2}d\,dx}{2ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] a**(3/2)*d*atan(sqrt(c)*x**2/sqrt(a))/(2*c**(3/2)*(a*e**2 + c*d**
2)) - a**2*e*log(a + c*x**4)/(4*c**2*(a*e**2 + c*d**2)) + d**4*lo
g(d + e*x**2)/(2*e**3*(a*e**2 + c*d**2)) + Integral(x, (x, x**2))
```

```
/(2*c*e) - Integral(d, (x, x**2))/(2*c*e**2)
```

Mathematica [A] time = 0.107434, size = 134, normalized size = 1.

$$\frac{a^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}-\frac{a^2e\log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(ae^2+cd^2\right)}-\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(d*x^2)/(2*c*e^2) + x^4/(4*c*e) + (a^(3/2)*d*ArcTan[(Sqrt[c]*x^2
)/Sqrt[a]])/(2*c^(3/2)*(c*d^2 + a*e^2)) + (d^4*Log[d + e*x^2])/(2
*e^3*(c*d^2 + a*e^2)) - (a^2*e*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*
e^2))
```

Maple [A] time = 0.016, size = 122, normalized size = 0.9

$$\frac{x^4}{4\,ce}-\frac{dx^2}{2\,e^2c}-\frac{a^2e\ln\left(cx^4+a\right)}{4\,c^2\left(ae^2+cd^2\right)}+\frac{a^2d}{\left(2\,ae^2+2\,cd^2\right)c}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{d^4\ln\left(ex^2+d\right)}{2\,e^3\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/4*x^4/c/e-1/2*d*x^2/e^2/c-1/4*a^2*e*ln(c*x^4+a)/c^2/(a*e^2+c*d^
2)+1/2*a^2/(a*e^2+c*d^2)/c*d/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)
)+1/2*d^4*ln(e*x^2+d)/e^3/(a*e^2+c*d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.19803, size = 1, normalized size = 0.01

$$\left[\frac{acde^3\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4+2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)-a^2e^4\log\left(cx^4+a\right)+2\,c^2d^4\log\left(ex^2+d\right)+\left(c^2d^2e^2+ace^4\right)x^4-2\left(c^2d^3e+acde^3\right)x^2}{4\left(c^3d^2e^3+ac^2e^5\right)},\ 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/4*(a*c*d*e^3*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sqrt(-a/c) - a)/(
c*x^4 + a)) - a^2*e^4*log(c*x^4 + a) + 2*c^2*d^4*log(e*x^2 + d) +
 (c^2*d^2*e^2 + a*c*e^4)*x^4 - 2*(c^2*d^3*e + a*c*d*e^3)*x^2)/(c^
3*d^2*e^3 + a*c^2*e^5), 1/4*(2*a*c*d*e^3*sqrt(a/c)*arctan(x^2/sqr
t(a/c)) - a^2*e^4*log(c*x^4 + a) + 2*c^2*d^4*log(e*x^2 + d) + (c^
2*d^2*e^2 + a*c*e^4)*x^4 - 2*(c^2*d^3*e + a*c*d*e^3)*x^2)/(c^3*d^
2*e^3 + a*c^2*e^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275585, size = 163, normalized size = 1.22

$$\frac{d^4\ln\left(\left|x^2e+d\right|\right)}{2\left(cd^2e^3+ae^5\right)}-\frac{a^2e\ln\left(cx^4+a\right)}{4\left(c^3d^2+ac^2e^2\right)}+\frac{a^2d\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(c^2d^2+ace^2\right)\sqrt{ac}}+\frac{\left(cx^4e-2\,cdx^2\right)e^{(-2)}}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] 1/2*d^4*ln(abs(x^2*e + d))/(c*d^2*e^3 + a*e^5) - 1/4*a^2*e*ln(c*x
^4 + a)/(c^3*d^2 + a*c^2*e^2) + 1/2*a^2*d*arctan(c*x^2/sqrt(a*c))
```

```
/((c^2*d^2 + a*c*e^2)*sqrt(a*c)) + 1/4*(c*x^4*e - 2*c*d*x^2)*e^(-
2)/c^2
```

3.230 $\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=118

$$-\frac{a^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}-\frac{ad\log\left(a+cx^4\right)}{4c\left(ae^2+cd^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(ae^2+cd^2\right)}+\frac{x^2}{2ce}$$

```
[Out] x^2/(2*c*e) - (a^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*c^(3/2
)*(c*d^2 + a*e^2)) - (d^3*Log[d + e*x^2])/(2*e^2*(c*d^2 + a*e^2))
 - (a*d*Log[a + c*x^4])/(4*c*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.277518, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$-\frac{a^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}-\frac{ad\log\left(a+cx^4\right)}{4c\left(ae^2+cd^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(ae^2+cd^2\right)}+\frac{x^2}{2ce}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] x^2/(2*c*e) - (a^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*c^(3/2
)*(c*d^2 + a*e^2)) - (d^3*Log[d + e*x^2])/(2*e^2*(c*d^2 + a*e^2))
 - (a*d*Log[a + c*x^4])/(4*c*(c*d^2 + a*e^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{\frac{3}{2}}e\operatorname{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}\left(ae^2+cd^2\right)}-\frac{ad\log\left(a+cx^4\right)}{4c\left(ae^2+cd^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(ae^2+cd^2\right)}+\frac{\int^{x^2}\frac{1}{c}\,dx}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] -a**(3/2)*e*atan(sqrt(c)*x**2/sqrt(a))/(2*c**(3/2)*(a*e**2 + c*d*
*2)) - a*d*log(a + c*x**4)/(4*c*(a*e**2 + c*d**2)) - d**3*log(d +
 e*x**2)/(2*e**2*(a*e**2 + c*d**2)) + Integral(1/c, (x, x**2))/(2
*e)
```

Mathematica [A] time = 0.159741, size = 99, normalized size = 0.84

$$\frac{-\frac{2a^{3/2}e^3\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{c^{3/2}}+\frac{e\left(2x^2\left(ae^2+cd^2\right)-ade\log\left(a+cx^4\right)\right)}{c}-2d^3\log\left(d+ex^2\right)}{4e^2\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] ((-2*a^(3/2)*e^3*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/c^(3/2) - 2*d^3*L
og[d + e*x^2] + (e*(2*(c*d^2 + a*e^2)*x^2 - a*d*e*Log[a + c*x^4])
)/c)/(4*e^2*(c*d^2 + a*e^2))
```

Maple [A] time = 0.012, size = 108, normalized size = 0.9

$$\frac{x^2}{2\,ce}-\frac{ad\ln\left(cx^4+a\right)}{4\,\left(ae^2+cd^2\right)c}-\frac{a^2e}{\left(2\,ae^2+2\,cd^2\right)c}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}-\frac{d^3\ln\left(ex^2+d\right)}{2\,e^2\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/2*x^2/c/e-1/4*a*d*ln(c*x^4+a)/c/(a*e^2+c*d^2)-1/2*a^2/(a*e^2+c*
d^2)/c*e/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))-1/2*d^3*ln(e*x^2+d
)/e^2/(a*e^2+c*d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83586, size = 1, normalized size = 0.01

$$\left[\frac{ae^3\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4-2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)-ade^2\log\left(cx^4+a\right)-2\,cd^3\log\left(ex^2+d\right)+2\left(cd^2e+ae^3\right)x^2}{4\left(c^2d^2e^2+ace^4\right)},\right.$$
$$\left.-\frac{2\,ae^3\sqrt{\frac{a}{c}}\arctan\left(\frac{x^2}{\sqrt{\frac{a}{c}}}\right)+ade^2\log\left(cx^4+a\right)+2\,cd^3\log\left(ex^2+d\right)-2\left(cd^2e+ae^3\right)x^2}{4\left(c^2d^2e^2+ace^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/4*(a*e^3*sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^
4 + a)) - a*d*e^2*log(c*x^4 + a) - 2*c*d^3*log(e*x^2 + d) + 2*(c*
d^2*e + a*e^3)*x^2)/(c^2*d^2*e^2 + a*c*e^4), -1/4*(2*a*e^3*sqrt(a
/c)*arctan(x^2/sqrt(a/c)) + a*d*e^2*log(c*x^4 + a) + 2*c*d^3*log(
e*x^2 + d) - 2*(c*d^2*e + a*e^3)*x^2)/(c^2*d^2*e^2 + a*c*e^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.27395, size = 142, normalized size = 1.2

$$-\frac{d^3\ln\left(\left|x^2e+d\right|\right)}{2\left(cd^2e^2+ae^4\right)}-\frac{a^2\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2\left(c^2d^2+ace^2\right)\sqrt{ac}}+\frac{x^2e^{(-1)}}{2\,c}-\frac{ad\ln\left(cx^4+a\right)}{4\left(c^2d^2+ace^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] -1/2*d^3*ln(abs(x^2*e + d))/(c*d^2*e^2 + a*e^4) - 1/2*a^2*arctan(
c*x^2/sqrt(a*c))*e/((c^2*d^2 + a*c*e^2)*sqrt(a*c)) + 1/2*x^2*e^(-
1)/c - 1/4*a*d*ln(c*x^4 + a)/(c^2*d^2 + a*c*e^2)
```

3.231 $\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=105

$$\frac{ae\log\left(a+cx^4\right)}{4c\left(ae^2+cd^2\right)}+\frac{d^2\log\left(d+ex^2\right)}{2e\left(ae^2+cd^2\right)}-\frac{\sqrt{a}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}\left(ae^2+cd^2\right)}$$

```
[Out] -(Sqrt[a]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[c]*(c*d^2 + a*
e^2)) + (d^2*Log[d + e*x^2])/(2*e*(c*d^2 + a*e^2)) + (a*e*Log[a +
 c*x^4])/(4*c*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.250944, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{ae\log\left(a+cx^4\right)}{4c\left(ae^2+cd^2\right)}+\frac{d^2\log\left(d+ex^2\right)}{2e\left(ae^2+cd^2\right)}-\frac{\sqrt{a}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(Sqrt[a]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[c]*(c*d^2 + a*
e^2)) + (d^2*Log[d + e*x^2])/(2*e*(c*d^2 + a*e^2)) + (a*e*Log[a +
 c*x^4])/(4*c*(c*d^2 + a*e^2))
```

Rubi in Sympy [A] time = 35.6023, size = 88, normalized size = 0.84

$$-\frac{\sqrt{a}d\,\mathrm{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}\left(ae^2+cd^2\right)}+\frac{ae\log\left(a+cx^4\right)}{4c\left(ae^2+cd^2\right)}+\frac{d^2\log\left(d+ex^2\right)}{2e\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] -sqrt(a)*d*atan(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c)*(a*e**2 + c*d**2
)) + a*e*log(a + c*x**4)/(4*c*(a*e**2 + c*d**2)) + d**2*log(d + e
*x**2)/(2*e*(a*e**2 + c*d**2))
```

Mathematica [A] time = 0.0580545, size = 77, normalized size = 0.73

$$\frac{-2\sqrt{a}\sqrt{c}de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)+ae^2\log\left(a+cx^4\right)+2cd^2\log\left(d+ex^2\right)}{4ace^3+4c^2d^2e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[c]*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]] + 2*c*d^2*L
og[d + e*x^2] + a*e^2*Log[a + c*x^4])/(4*c^2*d^2*e + 4*a*c*e^3)
```

Maple [A] time = 0.01, size = 92, normalized size = 0.9

$$\frac{ae\ln\left(cx^4+a\right)}{4c\left(ae^2+cd^2\right)}-\frac{ad}{2ae^2+2cd^2}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{d^2\ln\left(ex^2+d\right)}{2e\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/4*a*e*ln(c*x^4+a)/c/(a*e^2+c*d^2)-1/2*a/(a*e^2+c*d^2)*d/(a*c)^(
1/2)*arctan(c*x^2/(a*c)^(1/2))+1/2*d^2*ln(e*x^2+d)/e/(a*e^2+c*d^2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.08229, size = 1, normalized size = 0.01

$$\left[\frac{cde\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4-2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)+ae^2\log\left(cx^4+a\right)+2\,cd^2\log\left(ex^2+d\right)}{4\,(c^2d^2e+ace^3)},\right.$$
$$\left.-\frac{2\,cde\sqrt{\frac{a}{c}}\arctan\left(\frac{x^2}{\sqrt{\frac{a}{c}}}\right)-ae^2\log\left(cx^4+a\right)-2\,cd^2\log\left(ex^2+d\right)}{4\,(c^2d^2e+ace^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/4*(c*d*e*sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^
4 + a)) + a*e^2*log(c*x^4 + a) + 2*c*d^2*log(e*x^2 + d))/(c^2*d^2
*e + a*c*e^3), -1/4*(2*c*d*e*sqrt(a/c)*arctan(x^2/sqrt(a/c)) - a*
e^2*log(c*x^4 + a) - 2*c*d^2*log(e*x^2 + d))/(c^2*d^2*e + a*c*e^3
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275688, size = 122, normalized size = 1.16

$$\frac{ae\ln\left(cx^4+a\right)}{4\,(c^2d^2+ace^2)}+\frac{d^2\ln\left(\left|x^2e+d\right|\right)}{2\,(cd^2e+ae^3)}-\frac{ad\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\,(cd^2+ae^2)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] 1/4*a*e*ln(c*x^4 + a)/(c^2*d^2 + a*c*e^2) + 1/2*d^2*ln(abs(x^2*e
+ d))/(c*d^2*e + a*e^3) - 1/2*a*d*arctan(c*x^2/sqrt(a*c))/((c*d^2
 + a*e^2)*sqrt(a*c))
```

3.232 $\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=96

$$\frac{d\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)}-\frac{d\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)}+\frac{\sqrt{a}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}\left(ae^2+cd^2\right)}$$

```
[Out] (Sqrt[a]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[c]*(c*d^2 + a*e
^2)) - (d*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)) + (d*Log[a + c*x^4]
)/(4*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.198127, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{d\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)}-\frac{d\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)}+\frac{\sqrt{a}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (Sqrt[a]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[c]*(c*d^2 + a*e
^2)) - (d*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)) + (d*Log[a + c*x^4]
)/(4*(c*d^2 + a*e^2))
```

Rubi in Sympy [A] time = 27.1887, size = 85, normalized size = 0.89

$$\frac{\sqrt{a}e\,\mathrm{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}\left(ae^2+cd^2\right)}+\frac{d\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)}-\frac{d\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] sqrt(a)*e*atan(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c)*(a*e**2 + c*d**2)
) + d*log(a + c*x**4)/(4*(a*e**2 + c*d**2)) - d*log(d + e*x**2)/(
2*(a*e**2 + c*d**2))
```

Mathematica [A] time = 0.0661565, size = 66, normalized size = 0.69

$$\frac{d\log\left(a+cx^4\right)+\frac{2\sqrt{a}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{c}}-2d\log\left(d+ex^2\right)}{4ae^2+4cd^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] ((2*Sqrt[a]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/Sqrt[c] - 2*d*Log[d
+ e*x^2] + d*Log[a + c*x^4])/(4*c*d^2 + 4*a*e^2)
```

Maple [A] time = 0.009, size = 83, normalized size = 0.9

$$\frac{d\ln\left(cx^4+a\right)}{4\,ae^2+4\,cd^2}+\frac{ae}{2\,ae^2+2\,cd^2}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}-\frac{d\ln\left(ex^2+d\right)}{2\,ae^2+2\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/4*d*ln(c*x^4+a)/(a*e^2+c*d^2)+1/2/(a*e^2+c*d^2)*a*e/(a*c)^(1/2)
*arctan(c*x^2/(a*c)^(1/2))-1/2*d*ln(e*x^2+d)/(a*e^2+c*d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.444012, size = 1, normalized size = 0.01

$$\left[\frac{e\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4+2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)+d\log\left(cx^4+a\right)-2\,d\log\left(ex^2+d\right)}{4\left(cd^2+ae^2\right)},\frac{2\,e\sqrt{\frac{a}{c}}\arctan\left(\frac{x^2}{\sqrt{\frac{a}{c}}}\right)+d\log\left(cx^4+a\right)-2\,d\log\left(ex^2+d\right.}{4\left(cd^2+ae^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/4*(e*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 +
a)) + d*log(c*x^4 + a) - 2*d*log(e*x^2 + d))/(c*d^2 + a*e^2), 1/4
*(2*e*sqrt(a/c)*arctan(x^2/sqrt(a/c)) + d*log(c*x^4 + a) - 2*d*lo
g(e*x^2 + d))/(c*d^2 + a*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275192, size = 116, normalized size = 1.21

$$-\frac{de\ln\left(\left|x^2e+d\right|\right)}{2\left(cd^2e+ae^3\right)}+\frac{a\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2\left(cd^2+ae^2\right)\sqrt{ac}}+\frac{d\ln\left(cx^4+a\right)}{4\left(cd^2+ae^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] -1/2*d*e*ln(abs(x^2*e + d))/(c*d^2*e + a*e^3) + 1/2*a*arctan(c*x^
2/sqrt(a*c))*e/((c*d^2 + a*e^2)*sqrt(a*c)) + 1/4*d*ln(c*x^4 + a)/
(c*d^2 + a*e^2)
```

3.233 $\int \frac{x}{(d+ex^2)(a+cx^4)} dx$

Optimal. Leaf size=96

$$-\frac{e\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)}+\frac{e\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)}+\frac{\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)}$$

```
[Out] (Sqrt[c]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e
^2)) + (e*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)) - (e*Log[a + c*x^4]
)/(4*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.138898, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{e\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)}+\frac{e\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)}+\frac{\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (Sqrt[c]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e
^2)) + (e*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)) - (e*Log[a + c*x^4]
)/(4*(c*d^2 + a*e^2))
```

Rubi in Sympy [A] time = 25.0017, size = 85, normalized size = 0.89

$$-\frac{e\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)}+\frac{e\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)}+\frac{\sqrt{c}d\,\mathrm{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] -e*log(a + c*x**4)/(4*(a*e**2 + c*d**2)) + e*log(d + e*x**2)/(2*(
a*e**2 + c*d**2)) + sqrt(c)*d*atan(sqrt(c)*x**2/sqrt(a))/(2*sqrt(
a)*(a*e**2 + c*d**2))
```

Mathematica [A] time = 0.0604976, size = 67, normalized size = 0.7

$$\frac{\frac{2\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}} - e\log\left(a+cx^4\right) + 2e\log\left(d+ex^2\right)}{4ae^2+4cd^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] ((2*Sqrt[c]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/Sqrt[a] + 2*e*Log[d
+ e*x^2] - e*Log[a + c*x^4])/(4*c*d^2 + 4*a*e^2)
```

Maple [A] time = 0.009, size = 83, normalized size = 0.9

$$-\frac{e\ln\left(cx^4+a\right)}{4\,ae^2+4\,cd^2}+\frac{cd}{2\,ae^2+2\,cd^2}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{e\ln\left(ex^2+d\right)}{2\,ae^2+2\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/4*e*ln(c*x^4+a)/(a*e^2+c*d^2)+1/2*c/(a*e^2+c*d^2)*d/(a*c)^(1/2
)*arctan(c*x^2/(a*c)^(1/2))+1/2*e*ln(e*x^2+d)/(a*e^2+c*d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.555051, size = 1, normalized size = 0.01

$$\left[\frac{d\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4+2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)-e\log\left(cx^4+a\right)+2\,e\log\left(ex^2+d\right)}{4\left(cd^2+ae^2\right)},\ -\frac{2\,d\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right)+e\log\left(cx^4+a\right)-2\,e\log\left(ex^2+d\right)}{4\left(cd^2+ae^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/4*(d*sqrt(-c/a)*log((c*x^4 + 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 +
a)) - e*log(c*x^4 + a) + 2*e*log(e*x^2 + d))/(c*d^2 + a*e^2), -1/
4*(2*d*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) + e*log(c*x^4 + a) -
 2*e*log(e*x^2 + d))/(c*d^2 + a*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.274714, size = 115, normalized size = 1.2

$$\frac{cd\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(cd^2+ae^2\right)\sqrt{ac}}-\frac{e\ln\left(cx^4+a\right)}{4\left(cd^2+ae^2\right)}+\frac{e^2\ln\left(\left|x^2e+d\right|\right)}{2\left(cd^2e+ae^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] 1/2*c*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/4
*e*ln(c*x^4 + a)/(c*d^2 + a*e^2) + 1/2*e^2*ln(abs(x^2*e + d))/(c*
```

```
d^2*e + a*e^3)
```

3.234 $\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=114

$$-\frac{cd\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)}-\frac{\sqrt{c}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)}+\frac{\log(x)}{ad}$$

```
[Out] -(Sqrt[c]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*
e^2)) + Log[x]/(a*d) - (e^2*Log[d + e*x^2])/(2*d*(c*d^2 + a*e^2))
 - (c*d*Log[a + c*x^4])/(4*a*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.263241, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$-\frac{cd\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)}-\frac{\sqrt{c}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)}+\frac{\log(x)}{ad}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(Sqrt[c]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*
e^2)) + Log[x]/(a*d) - (e^2*Log[d + e*x^2])/(2*d*(c*d^2 + a*e^2))
 - (c*d*Log[a + c*x^4])/(4*a*(c*d^2 + a*e^2))
```

Rubi in Sympy [A] time = 43.9538, size = 99, normalized size = 0.87

$$-\frac{e^2\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)}-\frac{cd\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}+\frac{\log\left(x^2\right)}{2ad}-\frac{\sqrt{c}e\,\mathrm{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] -e**2*log(d + e*x**2)/(2*d*(a*e**2 + c*d**2)) - c*d*log(a + c*x**
4)/(4*a*(a*e**2 + c*d**2)) + log(x**2)/(2*a*d) - sqrt(c)*e*atan(s
qrt(c)*x**2/sqrt(a))/(2*sqrt(a)*(a*e**2 + c*d**2))
```

Mathematica [A] time = 0.118285, size = 134, normalized size = 1.18

$$\frac{-cd^2\log\left(a+cx^4\right)+2\sqrt{a}\sqrt{c}de\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\sqrt{a}\sqrt{c}de\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)-2ae^2\log\left(d+ex^2\right)+4ae^2\log(x)+4cd}{4a^2de^2+4acd^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[c]*d*e*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] +
2*Sqrt[a]*Sqrt[c]*d*e*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4
*c*d^2*Log[x] + 4*a*e^2*Log[x] - 2*a*e^2*Log[d + e*x^2] - c*d^2*L
og[a + c*x^4])/(4*a*c*d^3 + 4*a^2*d*e^2)
```

Maple [A] time = 0.014, size = 101, normalized size = 0.9

$$\frac{\ln(x)}{ad}-\frac{cd\ln\left(cx^4+a\right)}{4\left(ae^2+cd^2\right)a}-\frac{ce}{2\,ae^2+2\,cd^2}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}-\frac{e^2\ln\left(ex^2+d\right)}{2\,d\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] ln(x)/a/d-1/4*c*d*ln(c*x^4+a)/a/(a*e^2+c*d^2)-1/2*c/(a*e^2+c*d^2)
*e/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))-1/2*e^2*ln(e*x^2+d)/d/(a
*e^2+c*d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.64565, size = 1, normalized size = 0.01

$$\left[\frac{ade\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4-2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)-cd^2\log\left(cx^4+a\right)-2\,ae^2\log\left(ex^2+d\right)+4\left(cd^2+ae^2\right)\log\left(x\right)}{4\left(acd^3+a^2de^2\right)},\frac{2\,ade\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right)-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)*x),x, algorithm="fricas")
```

```
[Out] [1/4*(a*d*e*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^
4 + a)) - c*d^2*log(c*x^4 + a) - 2*a*e^2*log(e*x^2 + d) + 4*(c*d^
2 + a*e^2)*log(x))/(a*c*d^3 + a^2*d*e^2), 1/4*(2*a*d*e*sqrt(c/a)*
arctan(a*sqrt(c/a)/(c*x^2)) - c*d^2*log(c*x^4 + a) - 2*a*e^2*log(
e*x^2 + d) + 4*(c*d^2 + a*e^2)*log(x))/(a*c*d^3 + a^2*d*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.274363, size = 138, normalized size = 1.21

$$-\frac{cd\ln\left(cx^4+a\right)}{4\left(acd^2+a^2e^2\right)}-\frac{c\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2\left(cd^2+ae^2\right)\sqrt{ac}}-\frac{e^3\ln\left(\left|x^2e+d\right|\right)}{2\left(cd^3e+ade^3\right)}+\frac{\ln\left(x^2\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)*x),x, algorithm="giac")
```

```
[Out] -1/4*c*d*ln(c*x^4 + a)/(a*c*d^2 + a^2*e^2) - 1/2*c*arctan(c*x^2/s
qrt(a*c))*e/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/2*e^3*ln(abs(x^2*e +
d))/(c*d^3*e + a*d*e^3) + 1/2*ln(x^2)/(a*d)
```

3.235 $\int \frac{1}{x^3(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=129

$$-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}+\frac{ce\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(ae^2+cd^2\right)}-\frac{e\log(x)}{ad^2}-\frac{1}{2adx^2}$$

```
[Out] -1/(2*a*d*x^2) - (c^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(
3/2)*(c*d^2 + a*e^2)) - (e*Log[x])/(a*d^2) + (e^3*Log[d + e*x^2])
/(2*d^2*(c*d^2 + a*e^2)) + (c*e*Log[a + c*x^4])/(4*a*(c*d^2 + a*e
^2))
```

Rubi [A] time = 0.307174, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}+\frac{ce\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(ae^2+cd^2\right)}-\frac{e\log(x)}{ad^2}-\frac{1}{2adx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/(2*a*d*x^2) - (c^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(
3/2)*(c*d^2 + a*e^2)) - (e*Log[x])/(a*d^2) + (e^3*Log[d + e*x^2])
/(2*d^2*(c*d^2 + a*e^2)) + (c*e*Log[a + c*x^4])/(4*a*(c*d^2 + a*e
^2))
```

Rubi in Sympy [A] time = 45.0328, size = 114, normalized size = 0.88

$$\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(ae^2+cd^2\right)}+\frac{ce\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}-\frac{1}{2adx^2}-\frac{e\log\left(x^2\right)}{2ad^2}-\frac{c^{\frac{3}{2}}d\,\mathrm{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] e**3*log(d + e*x**2)/(2*d**2*(a*e**2 + c*d**2)) + c*e*log(a + c*x
**4)/(4*a*(a*e**2 + c*d**2)) - 1/(2*a*d*x**2) - e*log(x**2)/(2*a*
d**2) - c**(3/2)*d*atan(sqrt(c)*x**2/sqrt(a))/(2*a**(3/2)*(a*e**2
```

```
+ c*d**2))
```

Mathematica [A] time = 0.174805, size = 169, normalized size = 1.31

$$\frac{2c^{3/2}d^3x^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2c^{3/2}d^3x^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)+\sqrt{a}\left(-4ex^2\log(x)\left(ae^2+cd^2\right)+cd^2ex^2\log\left(a+cx^4\right)+2ae^3\right.}{4a^{3/2}d^2x^2\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (2*c^(3/2)*d^3*x^2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*c^
(3/2)*d^3*x^2*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[a]*(
-2*c*d^3 - 2*a*d*e^2 - 4*e*(c*d^2 + a*e^2)*x^2*Log[x] + 2*a*e^3*x
^2*Log[d + e*x^2] + c*d^2*e*x^2*Log[a + c*x^4]))/(4*a^(3/2)*d^2*(
c*d^2 + a*e^2)*x^2)
```

Maple [A] time = 0.016, size = 119, normalized size = 0.9

$$-\frac{1}{2\,adx^2}-\frac{\ln(x)\,e}{ad^2}+\frac{ce\ln\left(cx^4+a\right)}{4\,\left(ae^2+cd^2\right)a}-\frac{c^2d}{\left(2\,ae^2+2\,cd^2\right)a}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{e^3\ln\left(ex^2+d\right)}{2\,d^2\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/2/a/d/x^2-e*ln(x)/a/d^2+1/4*c*e*ln(c*x^4+a)/a/(a*e^2+c*d^2)-1/
2/(a*e^2+c*d^2)*c^2/a*d/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))+1/2
*e^3*ln(e*x^2+d)/d^2/(a*e^2+c*d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)*x^3),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 39.8634, size = 1, normalized size = 0.01

$$\left[\frac{cd^3x^2\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4-2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)+cd^2ex^2\log\left(cx^4+a\right)+2\,ae^3x^2\log\left(ex^2+d\right)-2\,cd^3-2\,ade^2-4\left(cd^2e+ae^3\right)x^2\log\left(x\right.}{4\left(acd^4+a^2d^2e^2\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)*x^3),x, algorithm="fricas")
```

```
[Out] [1/4*(c*d^3*x^2*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(
c*x^4 + a)) + c*d^2*e*x^2*log(c*x^4 + a) + 2*a*e^3*x^2*log(e*x^2
+ d) - 2*c*d^3 - 2*a*d*e^2 - 4*(c*d^2*e + a*e^3)*x^2*log(x))/((a*
c*d^4 + a^2*d^2*e^2)*x^2), 1/4*(2*c*d^3*x^2*sqrt(c/a)*arctan(a*sq
rt(c/a)/(c*x^2)) + c*d^2*e*x^2*log(c*x^4 + a) + 2*a*e^3*x^2*log(e
*x^2 + d) - 2*c*d^3 - 2*a*d*e^2 - 4*(c*d^2*e + a*e^3)*x^2*log(x))
/((a*c*d^4 + a^2*d^2*e^2)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275578, size = 178, normalized size = 1.38

$$-\frac{c^2d\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(acd^2+a^2e^2\right)\sqrt{ac}}+\frac{ce\ln\left(cx^4+a\right)}{4\left(acd^2+a^2e^2\right)}+\frac{e^4\ln\left(\left|x^2e+d\right|\right)}{2\left(cd^4e+ad^2e^3\right)}-\frac{e\ln\left(x^2\right)}{2\,ad^2}+\frac{x^2e-d}{2\,ad^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)*x^3),x, algorithm="giac")
```

```
[Out] -1/2*c^2*d*arctan(c*x^2/sqrt(a*c))/((a*c*d^2 + a^2*e^2)*sqrt(a*c)
) + 1/4*c*e*ln(c*x^4 + a)/(a*c*d^2 + a^2*e^2) + 1/2*e^4*ln(abs(x^
2*e + d))/(c*d^4*e + a*d^2*e^3) - 1/2*e*ln(x^2)/(a*d^2) + 1/2*(x^
2*e - d)/(a*d^2*x^2)
```

3.236 $\int \frac{1}{x^5(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=156

$$\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}+\frac{c^2d\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)}-\frac{\log(x)\left(cd^2-ae^2\right)}{a^2d^3}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(ae^2+cd^2\right)}+\frac{e}{2ad^2x^2}-\frac{1}{4adx^4}$$

```
[Out] -1/(4*a*d*x^4) + e/(2*a*d^2*x^2) + (c^(3/2)*e*ArcTan[(Sqrt[c]*x^2
)/Sqrt[a]])/(2*a^(3/2)*(c*d^2 + a*e^2)) - ((c*d^2 - a*e^2)*Log[x]
)/(a^2*d^3) - (e^4*Log[d + e*x^2])/(2*d^3*(c*d^2 + a*e^2)) + (c^2
*d*Log[a + c*x^4])/(4*a^2*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.374493, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}+\frac{c^2d\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)}-\frac{\log(x)\left(cd^2-ae^2\right)}{a^2d^3}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(ae^2+cd^2\right)}+\frac{e}{2ad^2x^2}-\frac{1}{4adx^4}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/(4*a*d*x^4) + e/(2*a*d^2*x^2) + (c^(3/2)*e*ArcTan[(Sqrt[c]*x^2
)/Sqrt[a]])/(2*a^(3/2)*(c*d^2 + a*e^2)) - ((c*d^2 - a*e^2)*Log[x]
)/(a^2*d^3) - (e^4*Log[d + e*x^2])/(2*d^3*(c*d^2 + a*e^2)) + (c^2
*d*Log[a + c*x^4])/(4*a^2*(c*d^2 + a*e^2))
```

Rubi in Sympy [A] time = 54.4211, size = 139, normalized size = 0.89

$$-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(ae^2+cd^2\right)}-\frac{1}{4adx^4}+\frac{e}{2ad^2x^2}+\frac{c^2d\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)}+\frac{\left(ae^2-cd^2\right)\log\left(x^2\right)}{2a^2d^3}+\frac{c^{\frac{3}{2}}e\,\mathrm{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] -e**4*log(d + e*x**2)/(2*d**3*(a*e**2 + c*d**2)) - 1/(4*a*d*x**4)
 + e/(2*a*d**2*x**2) + c**2*d*log(a + c*x**4)/(4*a**2*(a*e**2 + c
*d**2)) + (a*e**2 - c*d**2)*log(x**2)/(2*a**2*d**3) + c**(3/2)*e*
```

```
atan(sqrt(c)*x**2/sqrt(a))/(2*a**(3/2)*(a*e**2 + c*d**2))
```

Mathematica [A] time = 0.161162, size = 209, normalized size = 1.34

$$-\frac{a^2d^2e^2+2a^2e^4x^4\log\left(d+ex^2\right)-2a^2de^3x^2-4a^2e^4x^4\log(x)+2\sqrt{a}c^{3/2}d^3ex^4\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\sqrt{a}c^{3/2}d^3ex^4\tan^{-1}\left(\right.}{4a^2d^3x^4\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(a*c*d^4 + a^2*d^2*e^2 - 2*a*c*d^3*e*x^2 - 2*a^2*d*e^3*x^2 + 2*S
qrt[a]*c^(3/2)*d^3*e*x^4*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
+ 2*Sqrt[a]*c^(3/2)*d^3*e*x^4*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1
/4)] + 4*c^2*d^4*x^4*Log[x] - 4*a^2*e^4*x^4*Log[x] + 2*a^2*e^4*x^
4*Log[d + e*x^2] - c^2*d^4*x^4*Log[a + c*x^4])/(4*a^2*d^3*(c*d^2
+ a*e^2)*x^4)
```

Maple [A] time = 0.017, size = 145, normalized size = 0.9

$$-\frac{1}{4\,ad{x}^{4}}+\frac{\ln(x)\,{e}^{2}}{{d}^{3}a}-\frac{\ln(x)\,c}{{a}^{2}d}+\frac{e}{2\,a{d}^{2}{x}^{2}}+\frac{{c}^{2}d\ln\left(c{x}^{4}+a\right)}{4\,\left(a{e}^{2}+c{d}^{2}\right){a}^{2}}+\frac{{c}^{2}e}{\left(2\,a{e}^{2}+2\,c{d}^{2}\right)a}\arctan\left(c{x}^{2}\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}-\frac{{e}^{4}\ln\left(e{x}^{2}+d\right)}{2\,{d}^{3}\left(a{e}^{2}+c{d}^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/4/a/d/x^4+1/d^3/a*ln(x)*e^2-1/d/a^2*ln(x)*c+1/2*e/a/d^2/x^2+1/
4*c^2*d*ln(c*x^4+a)/a^2/(a*e^2+c*d^2)+1/2*c^2/(a*e^2+c*d^2)/a*e/(
a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))-1/2*e^4*ln(e*x^2+d)/d^3/(a*e
^2+c*d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 87.5516, size = 1, normalized size = 0.01

$$\left[\frac{acd^3ex^4\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4+2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)+c^2d^4x^4\log\left(cx^4+a\right)-2\,a^2e^4x^4\log\left(ex^2+d\right)-acd^4-a^2d^2e^2-4\left(c^2d^4-a^2e^4\right)x^4\log\ldots}{4\,(a^2cd^5+a^3d^3e^2)x^4},\right.$$
$$\left.-\frac{2\,acd^3ex^4\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right)-c^2d^4x^4\log\left(cx^4+a\right)+2\,a^2e^4x^4\log\left(ex^2+d\right)+acd^4+a^2d^2e^2+4\left(c^2d^4-a^2e^4\right)x^4\log\left(x\right)-\ldots}{4\,(a^2cd^5+a^3d^3e^2)x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)*x^5),x, algorithm="fricas")
```

```
[Out] [1/4*(a*c*d^3*e*x^4*sqrt(-c/a)*log((c*x^4 + 2*a*x^2*sqrt(-c/a) -
a)/(c*x^4 + a)) + c^2*d^4*x^4*log(c*x^4 + a) - 2*a^2*e^4*x^4*log(
e*x^2 + d) - a*c*d^4 - a^2*d^2*e^2 - 4*(c^2*d^4 - a^2*e^4)*x^4*lo
g(x) + 2*(a*c*d^3*e + a^2*d*e^3)*x^2)/((a^2*c*d^5 + a^3*d^3*e^2)*
x^4), -1/4*(2*a*c*d^3*e*x^4*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2))
 - c^2*d^4*x^4*log(c*x^4 + a) + 2*a^2*e^4*x^4*log(e*x^2 + d) + a*
c*d^4 + a^2*d^2*e^2 + 4*(c^2*d^4 - a^2*e^4)*x^4*log(x) - 2*(a*c*d
^3*e + a^2*d*e^3)*x^2)/((a^2*c*d^5 + a^3*d^3*e^2)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.276662, size = 227, normalized size = 1.46

$$\frac{c^2 d\ln\left(cx^4+a\right)}{4\left(a^2cd^2+a^3e^2\right)}+\frac{c^2\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2\left(acd^2+a^2e^2\right)\sqrt{ac}}-\frac{e^5\ln\left(\left|x^2e+d\right|\right)}{2\left(cd^5e+ad^3e^3\right)}-\frac{\left(cd^2-ae^2\right)\ln\left(x^2\right)}{2\,a^2d^3}+\frac{3\,cd^2x^4-3\,ax^4e^2+2\,adx^2e-ad^2}{4\,a^2d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)*x^5),x, algorithm="giac")
```

```
[Out] 1/4*c^2*d*ln(c*x^4 + a)/(a^2*c*d^2 + a^3*e^2) + 1/2*c^2*arctan(c*
x^2/sqrt(a*c))*e/((a*c*d^2 + a^2*e^2)*sqrt(a*c)) - 1/2*e^5*ln(abs
(x^2*e + d))/(c*d^5*e + a*d^3*e^3) - 1/2*(c*d^2 - a*e^2)*ln(x^2)/
(a^2*d^3) + 1/4*(3*c*d^2*x^4 - 3*a*x^4*e^2 + 2*a*d*x^2*e - a*d^2)
/(a^2*d^3*x^4)
```

3.237 $\int \frac{x^8}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=359

$$-\frac{a^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}+\frac{a^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}-\frac{a^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}+\frac{a^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\left(ae^2+cd^2\right)}-\frac{dx}{ce^2}+\frac{x^3}{3ce}$$

```
[Out] -((d*x)/(c*e^2)) + x^3/(3*c*e) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(e^(5/2)*(c*d^2 + a*e^2)) - (a^(5/4)*(Sqrt[c]*d - Sqrt[a]*e
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(7/4)*(c*
d^2 + a*e^2)) + (a^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - (a
^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) + (a^(5
/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.653974, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$-\frac{a^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}+\frac{a^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}-\frac{a^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}+\frac{a^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\left(ae^2+cd^2\right)}-\frac{dx}{ce^2}+\frac{x^3}{3ce}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -((d*x)/(c*e^2)) + x^3/(3*c*e) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(e^(5/2)*(c*d^2 + a*e^2)) - (a^(5/4)*(Sqrt[c]*d - Sqrt[a]*e
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(7/4)*(c*
```

```
d^2 + a*e^2)) + (a^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - (a
^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) + (a^(5
/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}a^{\frac{5}{4}}\left(\sqrt{a}e-\sqrt{c}d\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4c^{\frac{7}{4}}\left(ae^{2}+cd^{2}\right)}-\frac{\sqrt{2}a^{\frac{5}{4}}\left(\sqrt{a}e-\sqrt{c}d\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4c^{\frac{7}{4}}\left(ae^{2}+cd^{2}\right)}-\frac{\sqrt{2}a^{\frac{5}{4}}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^{2}\right)}{8c^{\frac{7}{4}}\left(ae^{2}+cd^{2}\right)}+\frac{\sqrt{2}a^{\frac{5}{4}}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^{2}\right)}{8c^{\frac{7}{4}}\left(ae^{2}+cd^{2}\right)}+\frac{d^{\frac{7}{2}}\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{\frac{5}{2}}\left(ae^{2}+cd^{2}\right)}+\frac{x^{3}}{3ce}-\frac{\int d\,dx}{ce^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] sqrt(2)*a**(5/4)*(sqrt(a)*e - sqrt(c)*d)*atan(1 - sqrt(2)*c**(1/4
)*x/a**(1/4))/(4*c**(7/4)*(a*e**2 + c*d**2)) - sqrt(2)*a**(5/4)*(
sqrt(a)*e - sqrt(c)*d)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/(4*c
**(7/4)*(a*e**2 + c*d**2)) - sqrt(2)*a**(5/4)*(sqrt(a)*e + sqrt(c
)*d)*log(-sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)
/(8*c**(7/4)*(a*e**2 + c*d**2)) + sqrt(2)*a**(5/4)*(sqrt(a)*e + s
qrt(c)*d)*log(sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x
**2)/(8*c**(7/4)*(a*e**2 + c*d**2)) + d**(7/2)*atan(sqrt(e)*x/sqr
t(d))/(e**(5/2)*(a*e**2 + c*d**2)) + x**3/(3*c*e) - Integral(d, x
)/(c*e**2)
```

Mathematica [A] time = 0.864583, size = 344, normalized size = 0.96

$$-3\sqrt{2}ae^{5/2}\left(a^{3/4}e+\sqrt[4]{a}\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^{2}\right)+3\sqrt{2}ae^{5/2}\left(a^{3/4}e+\sqrt[4]{a}\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^{2}\right)+$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-24*c^(3/4)*d*Sqrt[e]*(c*d^2 + a*e^2)*x + 8*c^(3/4)*e^(3/2)*(c*d
^2 + a*e^2)*x^3 + 24*c^(7/4)*d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
+ 6*Sqrt[2]*a^(5/4)*e^(5/2)*(-(Sqrt[c]*d) + Sqrt[a]*e)*ArcTan[1 -
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - 6*Sqrt[2]*a^(5/4)*e^(5/2)*(-(Sqrt
[c]*d) + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - 3*S
qrt[2]*a*e^(5/2)*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2]*a*e^(5/2)*(a^(
1/4)*Sqrt[c]*d + a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(24*c^(7/4)*e^(5/2)*(c*d^2 + a*e^2))
```

Maple [A] time = 0.014, size = 405, normalized size = 1.1

$$\frac{x^3}{3\,ce}-\frac{dx}{e^2c}+\frac{ad\sqrt{2}}{(8\,ae^2+8\,cd^2)\,c}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)$$
$$+\frac{ad\sqrt{2}}{(4\,ae^2+4\,cd^2)\,c}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{ad\sqrt{2}}{(4\,ae^2+4\,cd^2)\,c}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)$$
$$-\frac{a^2e\sqrt{2}}{(8\,ae^2+8\,cd^2)\,c^2}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$-\frac{a^2e\sqrt{2}}{(4\,ae^2+4\,cd^2)\,c^2}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$-\frac{a^2e\sqrt{2}}{(4\,ae^2+4\,cd^2)\,c^2}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{d^4}{e^2\,(ae^2+cd^2)}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/3*x^3/c/e-d*x/e^2/c+1/8*a/(a*e^2+c*d^2)/c*d*(1/c*a)^(1/4)*2^(1/
2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1
/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/4*a/(a*e^2+c*d^2)/c*d*(1/c*a)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/4*a/(a*e^2+c*d^2)/
c*d*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)-1/8*a
^2/(a*e^2+c*d^2)/c^2*e/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4
)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(
1/2)))-1/4*a^2/(a*e^2+c*d^2)/c^2*e/(1/c*a)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(1/c*a)^(1/4)*x+1)-1/4*a^2/(a*e^2+c*d^2)/c^2*e/(1/c*a)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)+1/e^2*d^4/(a*e^2+c*d
```

```
^2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 8.75617, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/12*(6*c*d^3*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x
^2 + d)) + 4*(c*d^2*e + a*e^3)*x^3 - 3*(c^2*d^2*e^2 + a*c*e^4)*sq
rt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(
a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6
*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^
5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*c*d^2 - a^4*e^2
)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^4*e + 2*a*c^6*d^2*e^3
 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(
c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e
^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2
+ a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c
^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^
6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) +
3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^
2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e
^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*
d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4
))*log(-(a^3*c*d^2 - a^4*e^2)*x - (a^2*c^3*d^3 - a^3*c^2*d*e^2 +
(c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 -
2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c
^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e +
 (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2
*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^
9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4
*d^2*e^2 + a^2*c^3*e^4))) - 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3
```

```
*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d
^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*
a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2
*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*c*d^2 - a^4*e^2)*x + (a^
2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^
5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8
+ 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*
c^7*e^8)))*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3
*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 +
 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c
^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) + 3*(c^2*d^
2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a
^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11
*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 +
 a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(
a^3*c*d^2 - a^4*e^2)*x - (a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*
e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d
^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^
4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e - (c^5*d^4
 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^
2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4
 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2
+ a^2*c^3*e^4))) - 12*(c*d^3 + a*d*e^2)*x)/(c^2*d^2*e^2 + a*c*e^4
), 1/12*(12*c*d^3*sqrt(d/e)*arctan(x/sqrt(d/e)) + 4*(c*d^2*e + a*
e^3)*x^3 - 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e + (c^5*d^4 +
 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*
e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 +
 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 +
a^2*c^3*e^4))*log(-(a^3*c*d^2 - a^4*e^2)*x + (a^2*c^3*d^3 - a^3*c
^2*d*e^2 + (c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5
*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^
2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*sqrt((
2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*
c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2
 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^
4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) + 3*(c^2*d^2*e^2 + a*c*e^4)*
sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(
-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d
^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(
c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*c*d^2 - a^4*e
^2)*x - (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^4*e + 2*a*c^6*d^2*e
^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)
/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2
*e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^
2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/
(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*
e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)))
- 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*
d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7
*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^
8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e
^4))*log(-(a^3*c*d^2 - a^4*e^2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2
- (c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4
- 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2
```

```
*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e
 - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 -
 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*
c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c
^4*d^2*e^2 + a^2*c^3*e^4))) + 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a
^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2
*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 +
6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 +
 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*c*d^2 - a^4*e^2)*x - (
a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*
c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^
8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^
4*c^7*e^8)))*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c
^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8
 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4
*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) - 12*(c*d
^3 + a*d*e^2)*x)/(c^2*d^2*e^2 + a*c*e^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.282771, size = 490, normalized size = 1.36

$$\frac{d^{\frac{7}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{cd^2e^2+ae^4}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d-\left(ac^3\right)^{\frac{3}{4}}ae\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^5d^2+\sqrt{2}ac^4e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d-\left(ac^3\right)^{\frac{3}{4}}ae\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^5d^2+\sqrt{2}ac^4e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d+\left(ac^3\right)^{\frac{3}{4}}ae\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^5d^2+\sqrt{2}ac^4e^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d+\left(ac^3\right)^{\frac{3}{4}}ae\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^5d^2+\sqrt{2}ac^4e^2\right)}+\frac{\left(c^2x^3e^2-3\,c^2dxe\right)e^{(-3)}}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] d^(7/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e^2 + a*e^4) +
1/2*((a*c^3)^(1/4)*a*c^2*d - (a*c^3)^(3/4)*a*e)*arctan(1/2*sqrt(2
)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^5*d^2 + sqr
t(2)*a*c^4*e^2) + 1/2*((a*c^3)^(1/4)*a*c^2*d - (a*c^3)^(3/4)*a*e)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqr
t(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) + 1/4*((a*c^3)^(1/4)*a*c^2*d +
(a*c^3)^(3/4)*a*e)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(s
qrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) - 1/4*((a*c^3)^(1/4)*a*c^2*d
+ (a*c^3)^(3/4)*a*e)*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/
(sqrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) + 1/3*(c^2*x^3*e^2 - 3*c^2*
d*x*e)*e^(-3)/c^3
```

3.238 $\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=345

$$-\frac{a^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{5/4}\left(ae^2+cd^2\right)}+\frac{a^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{5/4}\left(ae^2+cd^2\right)}+\frac{a^{3/4}\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{5/4}\left(ae^2+cd^2\right)}-\frac{a^{3/4}\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}c^{5/4}\left(ae^2+cd^2\right)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(ae^2+cd^2\right)}+\frac{x}{ce}$$

```
[Out] x/(c*e) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 +
 a*e^2)) + (a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) - (a^(3/4
)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
)/(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) - (a^(3/4)*(Sqrt[c]*d - Sqr
t[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
4*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) + (a^(3/4)*(Sqrt[c]*d - Sqrt[a
]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*S
qrt[2]*c^(5/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.576568, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$-\frac{a^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{5/4}\left(ae^2+cd^2\right)}+\frac{a^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{5/4}\left(ae^2+cd^2\right)}+\frac{a^{3/4}\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{5/4}\left(ae^2+cd^2\right)}-\frac{a^{3/4}\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}c^{5/4}\left(ae^2+cd^2\right)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(ae^2+cd^2\right)}+\frac{x}{ce}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] x/(c*e) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 +
 a*e^2)) + (a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) - (a^(3/4
```

```
)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
)/(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) - (a^(3/4)*(Sqrt[c]*d - Sqr
t[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
4*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) + (a^(3/4)*(Sqrt[c]*d - Sqrt[a
]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*S
qrt[2]*c^(5/4)*(c*d^2 + a*e^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}a^{\frac{3}{4}}\left(\sqrt{a}e-\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8c^{\frac{5}{4}}\left(ae^2+cd^2\right)} - \frac{\sqrt{2}a^{\frac{3}{4}}\left(\sqrt{a}e-\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8c^{\frac{5}{4}}\left(ae^2+cd^2\right)} + \frac{\sqrt{2}a^{\frac{3}{4}}\left(\sqrt{a}e+\sqrt{c}d\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4c^{\frac{5}{4}}\left(ae^2+cd^2\right)} - \frac{\sqrt{2}a^{\frac{3}{4}}\left(\sqrt{a}e+\sqrt{c}d\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4c^{\frac{5}{4}}\left(ae^2+cd^2\right)} - \frac{d^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{\frac{3}{2}}\left(ae^2+cd^2\right)} + \frac{\int\frac{1}{c}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] sqrt(2)*a**(3/4)*(sqrt(a)*e - sqrt(c)*d)*log(-sqrt(2)*a**(1/4)*c*
*(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(8*c**(5/4)*(a*e**2 + c*d**2
)) - sqrt(2)*a**(3/4)*(sqrt(a)*e - sqrt(c)*d)*log(sqrt(2)*a**(1/4
)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(8*c**(5/4)*(a*e**2 + c*
d**2)) + sqrt(2)*a**(3/4)*(sqrt(a)*e + sqrt(c)*d)*atan(1 - sqrt(2
)*c**(1/4)*x/a**(1/4))/(4*c**(5/4)*(a*e**2 + c*d**2)) - sqrt(2)*a
**(3/4)*(sqrt(a)*e + sqrt(c)*d)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1
/4))/(4*c**(5/4)*(a*e**2 + c*d**2)) - d**(5/2)*atan(sqrt(e)*x/sqr
t(d))/(e**(3/2)*(a*e**2 + c*d**2)) + Integral(1/c, x)/e
```

Mathematica [A] time = 0.474167, size = 373, normalized size = 1.08

$$-\frac{\left(a^{3/4}cd-a^{5/4}\sqrt{c}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)} + \frac{\left(a^{3/4}cd-a^{5/4}\sqrt{c}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)} - \frac{\left(a^{3/4}cd+a^{5/4}\sqrt{c}e\right)\tan^{-1}\left(\frac{2\sqrt[4]{c}x-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)} - \frac{\left(a^{3/4}cd+a^{5/4}\sqrt{c}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{c}x}{\sqrt{2}\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(ae^2+cd^2\right)} + \frac{x}{ce}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] x/(c*e) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 +
 a*e^2)) - ((a^(3/4)*c*d + a^(5/4)*Sqrt[c]*e)*ArcTan[(-(Sqrt[2]*a
^(1/4)) + 2*c^(1/4)*x)/(Sqrt[2]*a^(1/4))])/(2*Sqrt[2]*c^(7/4)*(c*
d^2 + a*e^2)) - ((a^(3/4)*c*d + a^(5/4)*Sqrt[c]*e)*ArcTan[(Sqrt[2
]*a^(1/4) + 2*c^(1/4)*x)/(Sqrt[2]*a^(1/4))])/(2*Sqrt[2]*c^(7/4)*(
c*d^2 + a*e^2)) - ((a^(3/4)*c*d - a^(5/4)*Sqrt[c]*e)*Log[Sqrt[a]
- Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c
*d^2 + a*e^2)) + ((a^(3/4)*c*d - a^(5/4)*Sqrt[c]*e)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*
d^2 + a*e^2))
```

Maple [A] time = 0.011, size = 387, normalized size = 1.1

$$\frac{x}{ce}-\frac{ae\sqrt{2}}{(4\,ae^2+4\,cd^2)\,c}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)$$
$$-\frac{ae\sqrt{2}}{(8\,ae^2+8\,cd^2)\,c}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)$$
$$-\frac{ae\sqrt{2}}{(4\,ae^2+4\,cd^2)\,c}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)$$
$$-\frac{ad\sqrt{2}}{(8\,ae^2+8\,cd^2)\,c}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$-\frac{ad\sqrt{2}}{(4\,ae^2+4\,cd^2)\,c}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$-\frac{ad\sqrt{2}}{(4\,ae^2+4\,cd^2)\,c}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}-\frac{d^3}{e\,(ae^2+cd^2)}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] x/c/e-1/4*a/(a*e^2+c*d^2)/c*e*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(1/c*a)^(1/4)*x-1)-1/8*a/(a*e^2+c*d^2)/c*e*(1/c*a)^(1/4)*2^(1/2
)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/
4)*x*2^(1/2)+(1/c*a)^(1/2)))-1/4*a/(a*e^2+c*d^2)/c*e*(1/c*a)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)-1/8*a/(a*e^2+c*d^2)/c
*d/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^
(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))-1/4*a/(a*e^2+
c*d^2)/c*d/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1
)-1/4*a/(a*e^2+c*d^2)/c*d/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1
/c*a)^(1/4)*x-1)-1/e*d^3/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(x*e/(d*
e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45829, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/4*(2*c*d^2*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^
2 + d)) + (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a
*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2
+ a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3
*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^
2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3
 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4
 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*
c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e
 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 -
 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^
7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3
*d^2*e^2 + a^2*c^2*e^4))) - (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*
e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4
- 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c
```

```
^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^
3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x - (a^2*c^2
*d^2*e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*
sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^
8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8))
)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sq
rt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*
d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/
(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) + (c^2*d^2*e + a*c*e^
3)*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*s
qrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8
*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))
/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3
*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 + (c^6*d^5 + 2*a*c^5*d^3*e^2
 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)
/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e
^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2
 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(
c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6
 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) - (
c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^
2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/
(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^
6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log
(-(a^2*c*d^2 - a^3*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 + (c^6*d^5
 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*
d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4
 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e - (c^4*d^4
 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^
2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 +
 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 +
a^2*c^2*e^4))) + 4*(c*d^2 + a*e^2)*x)/(c^2*d^2*e + a*c*e^3), -1/4
*(4*c*d^2*sqrt(d/e)*arctan(x/sqrt(d/e)) - (c^2*d^2*e + a*c*e^3)*s
qrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(
-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6
*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^
4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2
)*x + (a^2*c^2*d^2*e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a
^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^
9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 +
 a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a
^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*
d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a
^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) + (c^2*
d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 +
a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9
*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 +
a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a
^2*c*d^2 - a^3*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 - (c^6*d^5 + 2
*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*
e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4
*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2
*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^
2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a
```

```
^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*
c^2*e^4))) - (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e - (c^4*d^4 +
2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e
^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*
a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2
*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x + (a^2*c^2*d^2*e - a^3*c*
e^3 + (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*
d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a
^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*
d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^
4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2
*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*
c^3*d^2*e^2 + a^2*c^2*e^4))) + (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2
*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d
^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^
2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a
*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x - (a^2*
c^2*d^2*e - a^3*c*e^3 + (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^
4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a
*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^
8)))*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)
*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c
^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)
))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) - 4*(c*d^2 + a*e^2
)*x)/(c^2*d^2*e + a*c*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.282327, size = 450, normalized size = 1.3

$$-\frac{d^{\frac{5}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{cd^2e+ae^3}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}+\frac{xe^{(-1)}}{c}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace-\left(ac^3\right)^{\frac{3}{4}}d\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace-\left(ac^3\right)^{\frac{3}{4}}d\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] -d^(5/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e + a*e^3) - 1
/2*((a*c^3)^(1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*
a*c^3*e^2) - 1/2*((a*c^3)^(1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1
/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^4*
d^2 + sqrt(2)*a*c^3*e^2) + x*e^(-1)/c - 1/4*((a*c^3)^(1/4)*a*c*e
- (a*c^3)^(3/4)*d)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(s
qrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) + 1/4*((a*c^3)^(1/4)*a*c*e -
(a*c^3)^(3/4)*d)*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqr
t(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2)
```

3.239 $\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=336

$$\frac{\sqrt[4]{a}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(ae^2+cd^2\right)}$$

```
[Out] (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 + a*e^2)) +
 (a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*c^(3/4)*(c*d^2 + a*e^2)) + (a^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*
c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(
3/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.53499, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$\frac{\sqrt[4]{a}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 + a*e^2)) +
 (a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*c^(3/4)*(c*d^2 + a*e^2)) + (a^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*
c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(
3/4)*(c*d^2 + a*e^2))
```

Rubi in Sympy [A] time = 100.285, size = 304, normalized size = 0.9

$$-\frac{\sqrt{2}\sqrt[4]{a}\left(\sqrt{a}e-\sqrt{c}d\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4c^{\frac{3}{4}}\left(ae^{2}+cd^{2}\right)}+\frac{\sqrt{2}\sqrt[4]{a}\left(\sqrt{a}e-\sqrt{c}d\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4c^{\frac{3}{4}}\left(ae^{2}+cd^{2}\right)}+\frac{\sqrt{2}\sqrt[4]{a}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^{2}\right)}{8c^{\frac{3}{4}}\left(ae^{2}+cd^{2}\right)}-\frac{\sqrt{2}\sqrt[4]{a}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^{2}\right)}{8c^{\frac{3}{4}}\left(ae^{2}+cd^{2}\right)}+\frac{d^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(ae^{2}+cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] -sqrt(2)*a**(1/4)*(sqrt(a)*e - sqrt(c)*d)*atan(1 - sqrt(2)*c**(1/
4)*x/a**(1/4))/(4*c**(3/4)*(a*e**2 + c*d**2)) + sqrt(2)*a**(1/4)*
(sqrt(a)*e - sqrt(c)*d)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/(4*
c**(3/4)*(a*e**2 + c*d**2)) + sqrt(2)*a**(1/4)*(sqrt(a)*e + sqrt(
c)*d)*log(-sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2
)/(8*c**(3/4)*(a*e**2 + c*d**2)) - sqrt(2)*a**(1/4)*(sqrt(a)*e +
sqrt(c)*d)*log(sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*
x**2)/(8*c**(3/4)*(a*e**2 + c*d**2)) + d**(3/2)*atan(sqrt(e)*x/sq
rt(d))/(sqrt(e)*(a*e**2 + c*d**2))
```

Mathematica [A] time = 0.364328, size = 233, normalized size = 0.69

$$\frac{\sqrt{2}\sqrt[4]{a}\sqrt{e}\left(\left(\sqrt{a}e+\sqrt{c}d\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^{2}\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^{2}\right)\right)+2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\cdots}{\sqrt[4]{\cdots}}\right.\cdots}{8c^{3/4}\sqrt{e}\left(ae^{2}+cd^{2}\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (8*c^(3/4)*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*a^(1/4)*
Sqrt[e]*(2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)
/a^(1/4)] + (-2*Sqrt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*x)/a^(1/4)] + (Sqrt[c]*d + Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
c^(1/4)*x + Sqrt[c]*x^2])))/(8*c^(3/4)*Sqrt[e]*(c*d^2 + a*e^2))
```

Maple [A] time = 0.01, size = 363, normalized size = 1.1

$$-\frac{d\sqrt{2}}{8\,ae^{2}+8\,cd^{2}}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^{2}+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^{2}-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)$$
$$-\frac{d\sqrt{2}}{4\,ae^{2}+4\,cd^{2}}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)-\frac{d\sqrt{2}}{4\,ae^{2}+4\,cd^{2}}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)$$
$$+\frac{ae\sqrt{2}}{(8\,ae^{2}+8\,cd^{2})\,c}\ln\left(1\left(x^{2}-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^{2}+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$+\frac{ae\sqrt{2}}{(4\,ae^{2}+4\,cd^{2})\,c}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$+\frac{ae\sqrt{2}}{(4\,ae^{2}+4\,cd^{2})\,c}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{d^{2}}{ae^{2}+cd^{2}}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/8/(a*e^2+c*d^2)*d*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*
x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/
2)))-1/4/(a*e^2+c*d^2)*d*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
c*a)^(1/4)*x+1)-1/4/(a*e^2+c*d^2)*d*(1/c*a)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(1/c*a)^(1/4)*x-1)+1/8*a/(a*e^2+c*d^2)*e/c/(1/c*a)^(1/4)*
2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*
a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/4*a/(a*e^2+c*d^2)*e/c/(1/c*a
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/4*a/(a*e^2+c*
d^2)*e/c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)+
d^2/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.514177, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/4*((c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2
+ a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d
^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^
4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2
 - a*e^2)*x + (c^2*d^3 - a*c*d*e^2 + (c^4*d^4*e + 2*a*c^3*d^2*e^3
 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^
7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 +
 a^4*c^3*e^8)))*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*
c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4
*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*
e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) - (c*d^2 + a*e^2
)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(
a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2
 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^
4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x - (c^2*d
^3 - a*c*d*e^2 + (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt
(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*
e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt
((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*
d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a
^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*
a*c^2*d^2*e^2 + a^2*c*e^4))) + (c*d^2 + a*e^2)*sqrt((2*a*d*e - (c
^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*
d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4
 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2
+ a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x + (c^2*d^3 - a*c*d*e^2 - (c^
4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2
*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*
e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e - (c^3*d^4
 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^
2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a
^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*
c*e^4))) - (c*d^2 + a*e^2)*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^2*d^2
*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(
c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6
 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(
c*d^2 - a*e^2)*x - (c^2*d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c^3*d^
```

```
2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4
)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*
e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^2*d^2*e^2 +
 a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^
8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4
*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) + 2*d*sqrt(
-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)))/(c*d^2 + a
*e^2), 1/4*(4*d*sqrt(d/e)*arctan(x/sqrt(d/e)) + (c*d^2 + a*e^2)*s
qrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c
^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 +
6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 +
 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x + (c^2*d^3
- a*c*d*e^2 + (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(
a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2
 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2
*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4
 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*
c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c
^2*d^2*e^2 + a^2*c*e^4))) - (c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*
d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2
*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 +
4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a
^2*c*e^4))*log(-(c*d^2 - a*e^2)*x - (c^2*d^3 - a*c*d*e^2 + (c^4*d
^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*
d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4
 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e + (c^3*d^4 +
2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 +
 a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*
c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e
^4))) + (c*d^2 + a*e^2)*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^2*d^2*e^
2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7
*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 +
a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d
^2 - a*e^2)*x + (c^2*d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c^3*d^2*e
^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(
c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6
 + a^4*c^3*e^8)))*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^
2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 +
 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^
3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) - (c*d^2 + a*e
^2)*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(
-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e
^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*
d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x - (c^2
*d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sq
rt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^
6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sq
rt((2*a*d*e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^
2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6
*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 +
2*a*c^2*d^2*e^2 + a^2*c*e^4))))/(c*d^2 + a*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.281036, size = 441, normalized size = 1.31

$$\frac{d^{\frac{3}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{cd^2+ae^2}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(ac^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(ac^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] d^(3/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2 + a*e^2) - 1/2*
((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x +
 sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c
^3*e^2) - 1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^4*d^2
 + sqrt(2)*a*c^3*e^2) - 1/4*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*
e)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4*d^2 +
 sqrt(2)*a*c^3*e^2) + 1/4*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)
*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4*d^2 + s
qrt(2)*a*c^3*e^2)
```

3.240 $\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=337

$$\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}-\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{ae^2+cd^2}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}$$

```
[Out] -((Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2))
- ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]*d + Sq
rt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1
/4)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)
*c^(1/4)*(c*d^2 + a*e^2)) - ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^
(1/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.519013, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}-\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{ae^2+cd^2}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((d + e*x^2)*(a + c*x^4)),x]
```


```
[Out] -((Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2))
- ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]*d + Sq
rt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1
/4)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)
*c^(1/4)*(c*d^2 + a*e^2)) - ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^
(1/4)*(c*d^2 + a*e^2))
```

Rubi in Sympy [A] time = 91.7333, size = 304, normalized size = 0.9

$$-\frac{\sqrt{d}\sqrt{e}\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{ae^2+cd^2}-\frac{\sqrt{2}\left(\sqrt{a}e-\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}+\frac{\sqrt{2}\left(\sqrt{a}e-\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}-\frac{\sqrt{2}\left(\sqrt{a}e+\sqrt{c}d\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}+\frac{\sqrt{2}\left(\sqrt{a}e+\sqrt{c}d\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] -sqrt(d)*sqrt(e)*atan(sqrt(e)*x/sqrt(d))/(a*e**2 + c*d**2) - sqrt
(2)*(sqrt(a)*e - sqrt(c)*d)*log(-sqrt(2)*a**(1/4)*c**(3/4)*x + sq
rt(a)*sqrt(c) + c*x**2)/(8*a**(1/4)*c**(1/4)*(a*e**2 + c*d**2)) +
 sqrt(2)*(sqrt(a)*e - sqrt(c)*d)*log(sqrt(2)*a**(1/4)*c**(3/4)*x
+ sqrt(a)*sqrt(c) + c*x**2)/(8*a**(1/4)*c**(1/4)*(a*e**2 + c*d**2
)) - sqrt(2)*(sqrt(a)*e + sqrt(c)*d)*atan(1 - sqrt(2)*c**(1/4)*x/
a**(1/4))/(4*a**(1/4)*c**(1/4)*(a*e**2 + c*d**2)) + sqrt(2)*(sqrt
(a)*e + sqrt(c)*d)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/(4*a**(1
/4)*c**(1/4)*(a*e**2 + c*d**2))
```

Mathematica [A] time = 0.266186, size = 232, normalized size = 0.69

$$\frac{\sqrt{2}\left(\left(\sqrt{c}d-\sqrt{a}e\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)\right)-2\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-8*a^(1/4)*c^(1/4)*Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] +
 Sqrt[2]*(-2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*
x)/a^(1/4)] + 2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/
4)*x)/a^(1/4)] + (Sqrt[c]*d - Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2))
```

Maple [A] time = 0.01, size = 351, normalized size = 1.

$$\frac{e\sqrt{2}}{4\,ae^2+4\,cd^2}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{e\sqrt{2}}{4\,ae^2+4\,cd^2}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)$$
$$+\frac{e\sqrt{2}}{8\,ae^2+8\,cd^2}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)$$
$$+\frac{d\sqrt{2}}{8\,ae^2+8\,cd^2}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$+\frac{d\sqrt{2}}{4\,ae^2+4\,cd^2}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$+\frac{d\sqrt{2}}{4\,ae^2+4\,cd^2}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}-\frac{de}{ae^2+cd^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/4/(a*e^2+c*d^2)*e*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^
(1/4)*x+1)+1/4/(a*e^2+c*d^2)*e*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(1/c*a)^(1/4)*x-1)+1/8/(a*e^2+c*d^2)*e*(1/c*a)^(1/4)*2^(1/2)*l
n((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*
x*2^(1/2)+(1/c*a)^(1/2)))+1/8/(a*e^2+c*d^2)*d/(1/c*a)^(1/4)*2^(1/
2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1
/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/4/(a*e^2+c*d^2)*d/(1/c*a)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/4/(a*e^2+c*d^2)*d/(1/c
*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)-e*d/(a*e^2+c*
d^2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.32268, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [-1/4*((c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4
*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*
e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))*log(-(c*d^2 - a*e^2)*
x + (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c
*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*
a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e
^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c
^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2
+ 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 +
 2*a*c*d^2*e^2 + a^2*e^4))) - (c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2
*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 +
a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a
^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)
)*log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 + 2*
a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a
^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^
4*c^2*d^2*e^6 + a^5*c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*
e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d
^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 +
a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) + (c*d^2 + a*e
^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2
*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 +
6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2
*a*c*d^2*e^2 + a^2*e^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^
2*e^3 + (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*
d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6
*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))*sqrt(-(2*d*e
- (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*
e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^
2*e^4))) - (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^
2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8
 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^
5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))*log(-(c*d^2 - a*e
^2)*x - (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a
^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8
+ 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5
*c*e^8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt
(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*
```

```
e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d
^4 + 2*a*c*d^2*e^2 + a^2*e^4))) - 2*sqrt(-d*e)*log((e*x^2 - 2*sqr
t(-d*e)*x - d)/(e*x^2 + d)))/(c*d^2 + a*e^2), -1/4*((c*d^2 + a*e^
2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*
d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6
*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*
a*c*d^2*e^2 + a^2*e^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2
*e^3 - (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d
^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*
a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))*sqrt(-(2*d*e +
 (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e
^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
+ 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2
*e^4))) - (c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2
 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8
+ 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5
*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))*log(-(c*d^2 - a*e^
2)*x - (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^
3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 +
 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*
c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(
-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e
^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^
4 + 2*a*c*d^2*e^2 + a^2*e^4))) + (c*d^2 + a*e^2)*sqrt(-(2*d*e - (
c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2
 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 +
4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e
^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 +
 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2
+ a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4
*a^4*c^2*d^2*e^6 + a^5*c*e^8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d
^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^
5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6
 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) - (c*d^2 +
a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(
c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2
 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4
+ 2*a*c*d^2*e^2 + a^2*e^4))*log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e -
 a^2*e^3 + (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c
^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2
+ 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))*sqrt(-(2*d
*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d
^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*
e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 +
 a^2*e^4))) + 4*sqrt(d*e)*arctan(e*x/sqrt(d*e)))/(c*d^2 + a*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.282283, size = 454, normalized size = 1.35

$$-\frac{\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{cd^2+ae^2}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}$$
$$+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}$$
$$+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace-\left(ac^3\right)^{\frac{3}{4}}d\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}$$
$$-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace-\left(ac^3\right)^{\frac{3}{4}}d\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/(c*d^2 + a*e^2) + 1/2*
((a*c^3)^(1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x +
 sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a
^2*c^2*e^2) + 1/2*((a*c^3)^(1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c
^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*((a*c^3)^(1/4)*a*c*e - (a*c^3
)^(3/4)*d)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a
*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*a*c*e - (a*c
^3)^(3/4)*d)*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)
*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2)
```

3.241 $\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=336

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2+cd^2\right)}$$

```
[Out] (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) -
 (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*
a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(
3/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.494912, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)*(a + c*x^4)),x]
```



```
[Out] (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) -
 (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*
a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(
3/4)*(c*d^2 + a*e^2))
```

Rubi in Sympy [A] time = 95.3475, size = 304, normalized size = 0.9

$$\frac{e^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2+cd^2)}+\frac{\sqrt{2}\sqrt[4]{c}\left(\sqrt{a}e-\sqrt{c}d\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\,(ae^2+cd^2)}-\frac{\sqrt{2}\sqrt[4]{c}\left(\sqrt{a}e-\sqrt{c}d\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\,(ae^2+cd^2)}-\frac{\sqrt{2}\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}\,(ae^2+cd^2)}+\frac{\sqrt{2}\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}\,(ae^2+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] e**(3/2)*atan(sqrt(e)*x/sqrt(d))/(sqrt(d)*(a*e**2 + c*d**2)) + sq
rt(2)*c**(1/4)*(sqrt(a)*e - sqrt(c)*d)*atan(1 - sqrt(2)*c**(1/4)*
x/a**(1/4))/(4*a**(3/4)*(a*e**2 + c*d**2)) - sqrt(2)*c**(1/4)*(sq
rt(a)*e - sqrt(c)*d)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/(4*a**
(3/4)*(a*e**2 + c*d**2)) - sqrt(2)*c**(1/4)*(sqrt(a)*e + sqrt(c)*
d)*log(-sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(
8*a**(3/4)*(a*e**2 + c*d**2)) + sqrt(2)*c**(1/4)*(sqrt(a)*e + sqr
t(c)*d)*log(sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**
2)/(8*a**(3/4)*(a*e**2 + c*d**2))
```

Mathematica [A] time = 0.315887, size = 234, normalized size = 0.7

$$\frac{8a^{3/4}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\sqrt{2}\sqrt[4]{c}\sqrt{d}\left(-\left(\sqrt{a}e+\sqrt{c}d\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)\right)+(2\cdots}{8a^{3/4}\sqrt{d}\,(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (8*a^(3/4)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*c^(1/4)*
Sqrt[d]*((-2*Sqrt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)
*x)/a^(1/4)] + 2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*x)/a^(1/4)] - (Sqrt[c]*d + Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
c^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(3/4)*Sqrt[d]*(c*d^2 + a*e^2))
```

Maple [A] time = 0.003, size = 363, normalized size = 1.1

$$\frac{cd\sqrt{2}}{(8\,ae^2+8\,cd^2)\,a}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)$$
$$+\frac{cd\sqrt{2}}{(4\,ae^2+4\,cd^2)\,a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{cd\sqrt{2}}{(4\,ae^2+4\,cd^2)\,a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)$$
$$-\frac{e\sqrt{2}}{8\,ae^2+8\,cd^2}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$-\frac{e\sqrt{2}}{4\,ae^2+4\,cd^2}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$-\frac{e\sqrt{2}}{4\,ae^2+4\,cd^2}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{e^2}{ae^2+cd^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/8*c/(a*e^2+c*d^2)*d*(1/c*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/c*a)^(1/
4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^
(1/2)))+1/4*c/(a*e^2+c*d^2)*d*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1
/2)/(1/c*a)^(1/4)*x+1)+1/4*c/(a*e^2+c*d^2)*d*(1/c*a)^(1/4)/a*2^(1
/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)-1/8/(a*e^2+c*d^2)*e/(1/c*a)
^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^
2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))-1/4/(a*e^2+c*d^2)*e/(1/
c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)-1/4/(a*e^2+c
*d^2)*e/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)+e
^2/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.853446, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [-1/4*((c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e
^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*
c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6
 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*
d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*
a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c
*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^
6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2
*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^
3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e
^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*d^
2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4
)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*
a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))
)/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^
2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^
3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c
^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6
+ a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e
^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 +
4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8
)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*s
qrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3
*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*
e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4
 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^
2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)
*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a
^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))
*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c
^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^
6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d
^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e
 - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c
^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*
c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d
^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x - (a*c^2*d^3 - a^2*
```

```
c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*
d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e
^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d
*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a
*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^
5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + a^3*e^4))) - 2*e*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e
/d) - d)/(e*x^2 + d)))/(c*d^2 + a*e^2), 1/4*(4*e*sqrt(e/d)*arctan
(e*x/(d*sqrt(e/d))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4
+ 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a
^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 +
4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e
^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 + (a^3
*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*
d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2
*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^
4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 +
 a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4
+ 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3
*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^
2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a
^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*
e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c
^2*d^2 - a*c*e^2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e +
 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^
2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4
*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*
d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/
(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^
2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c
*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*
e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 +
 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^
8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c
*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2
*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^
3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e
^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^
3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8
 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*
e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2
)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(
c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d
^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x - (a
*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e
^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 +
4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8
)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(
-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3
*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^
2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))))/(c*d^2 + a*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.282068, size = 458, normalized size = 1.36

$$\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(ac^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(ac^3\right)^{\frac{3}{4}}e\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}+\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{3}{2}}}{(cd^2+ae^2)\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] 1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2
*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(
2)*a^2*c^2*e^2) + 1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)
*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*((a*c^3)^(1/4)*c^2*d + (a
*c^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(
2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*c^2*d +
(a*c^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqr
t(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + arctan(x*e^(1/2)/sqrt(d))
*e^(3/2)/((c*d^2 + a*e^2)*sqrt(d))
```

3.242 $\int \frac{1}{x^2(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=348

$$-\frac{c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}\left(ae^2+cd^2\right)}-\frac{c^{3/4}\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{5/4}\left(ae^2+cd^2\right)}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(ae^2+cd^2\right)}-\frac{1}{adx}$$

```
[Out] -(1/(a*d*x)) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*
d^2 + a*e^2)) + (c^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) - (c
^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(2*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) - (c^(3/4)*(Sqrt[c]*d
- Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) + (c^(3/4)*(Sqrt[c]*d - S
qrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(4*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.580027, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$-\frac{c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}\left(ae^2+cd^2\right)}-\frac{c^{3/4}\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{5/4}\left(ae^2+cd^2\right)}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(ae^2+cd^2\right)}-\frac{1}{adx}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(d + e*x^2)*(a + c*x^4)),x]
```


```
[Out] -(1/(a*d*x)) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*
d^2 + a*e^2)) + (c^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) - (c
```

```
^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(2*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) - (c^(3/4)*(Sqrt[c]*d
- Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) + (c^(3/4)*(Sqrt[c]*d - S
qrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(4*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2))
```

Rubi in Sympy [A] time = 109.165, size = 311, normalized size = 0.89

$$-\frac{e^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{\frac{3}{2}}\left(ae^2+cd^2\right)}-\frac{1}{adx}+\frac{\sqrt{2}c^{\frac{3}{4}}\left(\sqrt{a}e-\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{5}{4}}\left(ae^2+cd^2\right)}-\frac{\sqrt{2}c^{\frac{3}{4}}\left(\sqrt{a}e-\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{5}{4}}\left(ae^2+cd^2\right)}+\frac{\sqrt{2}c^{\frac{3}{4}}\left(\sqrt{a}e+\sqrt{c}d\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{5}{4}}\left(ae^2+cd^2\right)}-\frac{\sqrt{2}c^{\frac{3}{4}}\left(\sqrt{a}e+\sqrt{c}d\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{5}{4}}\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] -e**(5/2)*atan(sqrt(e)*x/sqrt(d))/(d**(3/2)*(a*e**2 + c*d**2)) -
1/(a*d*x) + sqrt(2)*c**(3/4)*(sqrt(a)*e - sqrt(c)*d)*log(-sqrt(2)
*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(8*a**(5/4)*(a*e
**2 + c*d**2)) - sqrt(2)*c**(3/4)*(sqrt(a)*e - sqrt(c)*d)*log(sqr
t(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(8*a**(5/4)*
(a*e**2 + c*d**2)) + sqrt(2)*c**(3/4)*(sqrt(a)*e + sqrt(c)*d)*ata
n(1 - sqrt(2)*c**(1/4)*x/a**(1/4))/(4*a**(5/4)*(a*e**2 + c*d**2))
 - sqrt(2)*c**(3/4)*(sqrt(a)*e + sqrt(c)*d)*atan(1 + sqrt(2)*c**(
1/4)*x/a**(1/4))/(4*a**(5/4)*(a*e**2 + c*d**2))
```

Mathematica [A] time = 0.458417, size = 389, normalized size = 1.12

$$\frac{-\sqrt{d}\left(8a^{5/4}e^2+\sqrt{2}c^{5/4}d^2x\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\sqrt{2}c^{5/4}d^2x\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\sqrt{2}\sqrt{a}c^{3/4}dex\log\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-8*a^(5/4)*e^(5/2)*x*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - Sqrt[d]*(8*a^
(1/4)*c*d^2 + 8*a^(5/4)*e^2 - 2*Sqrt[2]*c^(3/4)*d*(Sqrt[c]*d + Sq
rt[a]*e)*x*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*c^
(3/4)*d*(Sqrt[c]*d + Sqrt[a]*e)*x*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/
a^(1/4)] + Sqrt[2]*c^(5/4)*d^2*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^
(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*Sqrt[a]*c^(3/4)*d*e*x*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*c^(5/4)*d^
2*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt
[2]*Sqrt[a]*c^(3/4)*d*e*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2]))/(8*a^(5/4)*d^(3/2)*(c*d^2 + a*e^2)*x)
```

Maple [A] time = 0.013, size = 390, normalized size = 1.1

$$
\begin{aligned}
&-\frac{ce\sqrt{2}}{(4\,ae^2+4\,cd^2)\,a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\\
&-\frac{ce\sqrt{2}}{(8\,ae^2+8\,cd^2)\,a}\sqrt[4]{\frac{a}{c}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\\
&-\frac{ce\sqrt{2}}{(4\,ae^2+4\,cd^2)\,a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\\
&-\frac{cd\sqrt{2}}{(8\,ae^2+8\,cd^2)\,a}\ln\left(1\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}\\
&-\frac{cd\sqrt{2}}{(4\,ae^2+4\,cd^2)\,a}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}\\
&-\frac{cd\sqrt{2}}{(4\,ae^2+4\,cd^2)\,a}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}-\frac{e^3}{d\,(ae^2+cd^2)}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{1}{adx}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/4*c/(a*e^2+c*d^2)/a*e*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
c*a)^(1/4)*x-1)-1/8*c/(a*e^2+c*d^2)/a*e*(1/c*a)^(1/4)*2^(1/2)*ln(
(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*
2^(1/2)+(1/c*a)^(1/2)))-1/4*c/(a*e^2+c*d^2)/a*e*(1/c*a)^(1/4)*2^(
```

```
1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)-1/8*c/(a*e^2+c*d^2)/a*d/(1
/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)
)/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))-1/4*c/(a*e^2+c*d^2
)/a*d/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)-1/4
*c/(a*e^2+c*d^2)/a*d/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)
^(1/4)*x-1)-1/d*e^3/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1
/2))-1/a/d/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.95273, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)*x^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*e^2*x*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-e/d) - d)/(e*
x^2 + d)) + (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e + (a^2*c^2*d
^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2
+ a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e
^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2
+ a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x + (a^2*c^2*d^2*e - a^3*c
*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4
 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^
2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2
*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 -
 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2
+ 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 +
 2*a^3*c*d^2*e^2 + a^4*e^4))) - (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*
c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^
4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e
^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^
4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x - (a
^2*c^2*d^2*e - a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d
*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^
```

```
8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9
*e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e
^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8
+ 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e
^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) + (a*c*d^3 + a^
2*d*e^2)*x*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^
4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d
^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^
9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2
 - a*c^2*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 + (a^4*c^2*d^5 + 2*a
^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*
c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4
*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a
^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^
3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a
^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^
4))) - (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 +
2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2
*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 +
4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4
*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3
+ (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*
a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6
*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e
- (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*
c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a
^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^
3*c*d^2*e^2 + a^4*e^4))) - 4*c*d^2 - 4*a*e^2)/((a*c*d^3 + a^2*d*e
^2)*x), -1/4*(4*a*e^2*x*sqrt(e/d)*arctan(e*x/(d*sqrt(e/d))) - (a*
c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^
2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/
(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^
2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log
(-(c^3*d^2 - a*c^2*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 - (a^4*c^2
*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*
e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d
^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2
*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^
2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4
*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^
2 + a^4*e^4))) + (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e + (a^2*
c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2
*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*
d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2
*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x - (a^2*c^2*d^2*e -
a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^
5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d
^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(
2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*
d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6
*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*
d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) - (a*c*d^3 + a^2*d*e^2)*x*sqrt
(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c
^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*
```

```
d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c
^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x
 + (a^2*c^2*d^2*e - a^3*c*e^3 + (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 +
a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c
^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6
+ a^9*e^8)))*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 +
a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4
*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 +
a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) + (a*c*d^3
 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2
 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*
c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6
 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^
3*d^2 - a*c^2*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 + (a^4*c^2*d^5
+ 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 +
 a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^
4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4
+ 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a
^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4
+ 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a
^4*e^4))) + 4*c*d^2 + 4*a*e^2)/((a*c*d^3 + a^2*d*e^2)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280687, size = 470, normalized size = 1.35

$$-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace-\left(ac^3\right)^{\frac{3}{4}}d\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace-\left(ac^3\right)^{\frac{3}{4}}d\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)}-\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{5}{2}}}{\left(cd^3+ade^2\right)\sqrt{d}}-\frac{1}{adx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)*x^2),x, algorithm="giac")
```

```
[Out] -1/2*((a*c^3)^(1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(
2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sq
rt(2)*a^3*c*e^2) - 1/2*((a*c^3)^(1/4)*a*c*e + (a*c^3)^(3/4)*d)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2
)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1/4*((a*c^3)^(1/4)*a*c*e - (
a*c^3)^(3/4)*d)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt
(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) + 1/4*((a*c^3)^(1/4)*a*c*e -
 (a*c^3)^(3/4)*d)*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sq
rt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - arctan(x*e^(1/2)/sqrt(d)
)*e^(5/2)/((c*d^3 + a*d*e^2)*sqrt(d)) - 1/(a*d*x)
```

3.243 $\int \frac{1}{x^4(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=360

$$\frac{c^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)} - \frac{c^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)} + \frac{c^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)} - \frac{c^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(ae^2+cd^2\right)} + \frac{e}{ad^2x} - \frac{1}{3adx^3}$$

```
[Out] -1/(3*a*d*x^3) + e/(a*d^2*x) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d
]])/(d^(5/2)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*
ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(7/4)*(c*d^
2 + a*e^2)) - (c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(
5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(5/4
)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.602339, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$\frac{c^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)} - \frac{c^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)} + \frac{c^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)} - \frac{c^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(ae^2+cd^2\right)} + \frac{e}{ad^2x} - \frac{1}{3adx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/(3*a*d*x^3) + e/(a*d^2*x) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d
]])/(d^(5/2)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*
ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(7/4)*(c*d^
```

```
2 + a*e^2)) - (c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(
5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(5/4
)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Rubi in Sympy [A] time = 119.6, size = 323, normalized size = 0.9

$$\frac{e^{\frac{7}{2}}\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{\frac{5}{2}}\left(ae^2+cd^2\right)}-\frac{1}{3adx^3}+\frac{e}{ad^2x}-\frac{\sqrt{2}c^{\frac{5}{4}}\left(\sqrt{a}e-\sqrt{c}d\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{7}{4}}\left(ae^2+cd^2\right)}+\frac{\sqrt{2}c^{\frac{5}{4}}\left(\sqrt{a}e-\sqrt{c}d\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{7}{4}}\left(ae^2+cd^2\right)}+\frac{\sqrt{2}c^{\frac{5}{4}}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{7}{4}}\left(ae^2+cd^2\right)}-\frac{\sqrt{2}c^{\frac{5}{4}}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{7}{4}}\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] e**(7/2)*atan(sqrt(e)*x/sqrt(d))/(d**(5/2)*(a*e**2 + c*d**2)) - 1
/(3*a*d*x**3) + e/(a*d**2*x) - sqrt(2)*c**(5/4)*(sqrt(a)*e - sqrt
(c)*d)*atan(1 - sqrt(2)*c**(1/4)*x/a**(1/4))/(4*a**(7/4)*(a*e**2
+ c*d**2)) + sqrt(2)*c**(5/4)*(sqrt(a)*e - sqrt(c)*d)*atan(1 + sq
rt(2)*c**(1/4)*x/a**(1/4))/(4*a**(7/4)*(a*e**2 + c*d**2)) + sqrt(
2)*c**(5/4)*(sqrt(a)*e + sqrt(c)*d)*log(-sqrt(2)*a**(1/4)*c**(3/4
)*x + sqrt(a)*sqrt(c) + c*x**2)/(8*a**(7/4)*(a*e**2 + c*d**2)) -
sqrt(2)*c**(5/4)*(sqrt(a)*e + sqrt(c)*d)*log(sqrt(2)*a**(1/4)*c**
(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(8*a**(7/4)*(a*e**2 + c*d**2)
)
```

Mathematica [A] time = 0.99711, size = 367, normalized size = 1.02

$$3\sqrt{2}c^{5/4}d^{5/2}x^3\left(a^{3/4}e+\sqrt[4]{a}\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-3\sqrt{2}c^{5/4}d^{5/2}x^3\left(a^{3/4}e+\sqrt[4]{a}\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\cdots\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-8*a*d^(3/2)*(c*d^2 + a*e^2) + 24*a*Sqrt[d]*e*(c*d^2 + a*e^2)*x^
2 + 24*a^2*e^(7/2)*x^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + 6*Sqrt[2]*a^
(1/4)*c^(5/4)*d^(5/2)*(Sqrt[c]*d - Sqrt[a]*e)*x^3*ArcTan[1 - (Sqr
t[2]*c^(1/4)*x)/a^(1/4)] + 6*Sqrt[2]*a^(1/4)*c^(5/4)*d^(5/2)*(-(S
qrt[c]*d) + Sqrt[a]*e)*x^3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
] + 3*Sqrt[2]*c^(5/4)*d^(5/2)*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*x^3
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - 3*Sqrt[
2]*c^(5/4)*d^(5/2)*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*x^3*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(24*a^2*d^(5/2)*(c*
d^2 + a*e^2)*x^3)
```

Maple [A] time = 0.017, size = 406, normalized size = 1.1

$$-\frac{1}{3\,ad{x}^{3}}+\frac{e}{a{d}^{2}x}-\frac{{c}^{2}d\sqrt{2}}{(8\,a{e}^{2}+8\,c{d}^{2}){a}^{2}}\sqrt[4]{\frac{a}{c}}\ln\left(1\left({x}^{2}+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left({x}^{2}-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)$$
$$-\frac{{c}^{2}d\sqrt{2}}{(4\,a{e}^{2}+4\,c{d}^{2}){a}^{2}}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)-\frac{{c}^{2}d\sqrt{2}}{(4\,a{e}^{2}+4\,c{d}^{2}){a}^{2}}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)$$
$$+\frac{ce\sqrt{2}}{(8\,a{e}^{2}+8\,c{d}^{2})a}\ln\left(1\left({x}^{2}-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left({x}^{2}+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$+\frac{ce\sqrt{2}}{(4\,a{e}^{2}+4\,c{d}^{2})a}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}$$
$$+\frac{ce\sqrt{2}}{(4\,a{e}^{2}+4\,c{d}^{2})a}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{c}}}+\frac{{e}^{4}}{{d}^{2}(a{e}^{2}+c{d}^{2})}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/3/a/d/x^3+e/a/d^2/x-1/8*c^2/(a*e^2+c*d^2)/a^2*d*(1/c*a)^(1/4)*
2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*
a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))-1/4*c^2/(a*e^2+c*d^2)/a^2*d*(1
/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)-1/4*c^2/(a*
e^2+c*d^2)/a^2*d*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/
4)*x-1)+1/8*c/(a*e^2+c*d^2)/a*e/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/
c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+
(1/c*a)^(1/2)))+1/4*c/(a*e^2+c*d^2)/a*e/(1/c*a)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/4*c/(a*e^2+c*d^2)/a*e/(1/c*a)^(1
```

```
/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)+1/d^2*e^4/(a*e^2+c*
d^2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 10.0751, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)*x^4),x, algorithm="fricas")
```

```
[Out] [1/12*(6*a*e^3*x^3*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/
(e*x^2 + d)) + 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e + (a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*
d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c
^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*
c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x + (a^2*c^4*d^3
 - a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sq
rt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^
8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8))
)*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqr
t(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8
*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))
/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a*c*d^4 + a^2*d
^2*e^2)*x^3*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^
5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d
^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a
^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d
^2 - a*c^4*e^2)*x - (a^2*c^4*d^3 - a^3*c^3*d*e^2 + (a^6*c^2*d^4*e
 + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 +
a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4
 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e + (a^3*c^2*d^4
+ 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a
^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4
+ 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 +
```

```
 a^5*e^4))) + 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e - (a^
3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d
^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^
2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c
*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x + (a^2*c^4*d^3
- a^3*c^3*d*e^2 - (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqr
t(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8
*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))
*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt
(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*
c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/
(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a*c*d^4 + a^2*d^
2*e^2)*x^3*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5
*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^
8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^
11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^
2 - a*c^4*e^2)*x - (a^2*c^4*d^3 - a^3*c^3*d*e^2 - (a^6*c^2*d^4*e
+ 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a
^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4
+ 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (a^3*c^2*d^4 +
 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^
2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 +
 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 +
a^5*e^4))) - 4*c*d^3 - 4*a*d*e^2 + 12*(c*d^2*e + a*e^3)*x^2)/((a*
c*d^4 + a^2*d^2*e^2)*x^3), 1/12*(12*a*e^3*x^3*sqrt(e/d)*arctan(e*
x/(d*sqrt(e/d))) + 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e
+ (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*
c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a
^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*
a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x + (a^2*c^4
*d^3 - a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5
)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 +
4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e
^8)))*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)
*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4
*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^
8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a*c*d^4 + a
^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2
+ a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c
^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6
 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c
^5*d^2 - a*c^4*e^2)*x - (a^2*c^4*d^3 - a^3*c^3*d*e^2 + (a^6*c^2*d
^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^
2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4
*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e + (a^3*c^2*
d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2
 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*
e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e
^2 + a^5*e^4))) + 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e -
 (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c
^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^
9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a
^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x + (a^2*c^4*
d^3 - a^3*c^3*d*e^2 - (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)
```

```
*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4
*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^
8)))*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*
sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*
a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8
)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a*c*d^4 + a^
2*d^2*e^2)*x^3*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 +
 a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^
4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6
+ a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^
5*d^2 - a*c^4*e^2)*x - (a^2*c^4*d^3 - a^3*c^3*d*e^2 - (a^6*c^2*d^
4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2
 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*
e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (a^3*c^2*d
^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2
+ a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e
^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^
2 + a^5*e^4))) - 4*c*d^3 - 4*a*d*e^2 + 12*(c*d^2*e + a*e^3)*x^2)/
((a*c*d^4 + a^2*d^2*e^2)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.281618, size = 491, normalized size = 1.36

$$-\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d-\left(ac^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^{2}c^{2}d^{2}+\sqrt{2}a^{3}ce^{2}\right)}-\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d-\left(ac^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^{2}c^{2}d^{2}+\sqrt{2}a^{3}ce^{2}\right)}-\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d+\left(ac^{3}\right)^{\frac{3}{4}}e\right)\ln\left(x^{2}+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}a^{2}c^{2}d^{2}+\sqrt{2}a^{3}ce^{2}\right)}+\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d+\left(ac^{3}\right)^{\frac{3}{4}}e\right)\ln\left(x^{2}-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}a^{2}c^{2}d^{2}+\sqrt{2}a^{3}ce^{2}\right)}+\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{7}{2}}}{\left(cd^{4}+ad^{2}e^{2}\right)\sqrt{d}}+\frac{3x^{2}e-d}{3ad^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)*(e*x^2 + d)*x^4),x, algorithm="giac")
```

```
[Out] -1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(
2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sq
rt(2)*a^3*c*e^2) - 1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2
)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1/4*((a*c^3)^(1/4)*c^2*d + (
a*c^3)^(3/4)*e)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt
(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) + 1/4*((a*c^3)^(1/4)*c^2*d +
 (a*c^3)^(3/4)*e)*ln(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sq
rt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) + arctan(x*e^(1/2)/sqrt(d)
)*e^(7/2)/((c*d^4 + a*d^2*e^2)*sqrt(d)) + 1/3*(3*x^2*e - d)/(a*d^
2*x^3)
```

3.244 $\int \frac{x^9}{(d+ex^2)(a+cx^4)^2} \, dx$

Optimal. Leaf size=169

$$-\frac{\sqrt{a}d\left(ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2}\left(ae^2+cd^2\right)^2}+\frac{ae\left(ae^2+2cd^2\right)\log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)^2}+\frac{a\left(ae+cdx^2\right)}{4c^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e\left(ae^2+cd^2\right)^2}$$

```
[Out] (a*(a*e + c*d*x^2))/(4*c^2*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[a
]*d*(3*c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*c^(3/2)*(
c*d^2 + a*e^2)^2) + (d^4*Log[d + e*x^2])/(2*e*(c*d^2 + a*e^2)^2)
+ (a*e*(2*c*d^2 + a*e^2)*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*e^2)^2
)
```

Rubi [A] time = 0.630753, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$-\frac{\sqrt{a}d\left(ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2}\left(ae^2+cd^2\right)^2}+\frac{ae\left(ae^2+2cd^2\right)\log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)^2}+\frac{a\left(ae+cdx^2\right)}{4c^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*(a*e + c*d*x^2))/(4*c^2*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[a
]*d*(3*c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*c^(3/2)*(
c*d^2 + a*e^2)^2) + (d^4*Log[d + e*x^2])/(2*e*(c*d^2 + a*e^2)^2)
+ (a*e*(2*c*d^2 + a*e^2)*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*e^2)^2
)
```

Rubi in Sympy [A] time = 67.401, size = 192, normalized size = 1.14

$$\frac{\sqrt{a}d\,\mathrm{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{\frac{3}{2}}\left(ae^2+cd^2\right)}-\frac{\sqrt{a}d\left(ae^2+2cd^2\right)\mathrm{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}\left(ae^2+cd^2\right)^2}+\frac{ae\left(ae^2+2cd^2\right)\log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)^2}+\frac{a\left(ae+cdx^2\right)}{4c^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e\left(ae^2+cd^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] sqrt(a)*d*atan(sqrt(c)*x**2/sqrt(a))/(4*c**(3/2)*(a*e**2 + c*d**2
)) - sqrt(a)*d*(a*e**2 + 2*c*d**2)*atan(sqrt(c)*x**2/sqrt(a))/(2*
c**(3/2)*(a*e**2 + c*d**2)**2) + a*e*(a*e**2 + 2*c*d**2)*log(a +
c*x**4)/(4*c**2*(a*e**2 + c*d**2)**2) + a*(a*e + c*d*x**2)/(4*c**
2*(a + c*x**4)*(a*e**2 + c*d**2)) + d**4*log(d + e*x**2)/(2*e*(a*
e**2 + c*d**2)**2)
```

Mathematica [A] time = 0.365544, size = 135, normalized size = 0.8

$$\frac{-\frac{\sqrt{a}d\left(ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{c^{3/2}}+\frac{ae\left(ae^2+2cd^2\right)\log\left(a+cx^4\right)}{c^2}+\frac{a\left(ae^2+cd^2\right)\left(ae+cdx^2\right)}{c^2\left(a+cx^4\right)}+\frac{2d^4\log\left(d+ex^2\right)}{e}}{4\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((a*(c*d^2 + a*e^2)*(a*e + c*d*x^2))/(c^2*(a + c*x^4)) - (Sqrt[a]
*d*(3*c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/c^(3/2) + (2*
d^4*Log[d + e*x^2])/e + (a*e*(2*c*d^2 + a*e^2)*Log[a + c*x^4])/c^
2)/(4*(c*d^2 + a*e^2)^2)
```

Maple [B] time = 0.033, size = 309, normalized size = 1.8

$$\frac{a^2dx^2e^2}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)c}+\frac{ax^2d^3}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)}+\frac{a^3e^3}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)c^2}+\frac{a^2ed^2}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)c}+\frac{a^2\ln\left(\left(cx^4+a\right)c\right)e^3}{4\left(ae^2+cd^2\right)^2c^2}+\frac{a\ln\left(\left(cx^4+a\right)c\right)d^2e}{2\left(ae^2+cd^2\right)^2c}-\frac{da^2e^2}{4\left(ae^2+cd^2\right)^2c}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}-\frac{3\,ad^3}{4\left(ae^2+cd^2\right)^2}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{d^4\ln\left(ex^2+d\right)}{2\,e\left(ae^2+cd^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*a^2/(a*e^2+c*d^2)^2/(c*x^4+a)*d/c*x^2*e^2+1/4*a/(a*e^2+c*d^2)
^2/(c*x^4+a)*x^2*d^3+1/4*a^3/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3/c^2+1/
4*a^2/(a*e^2+c*d^2)^2/(c*x^4+a)*e/c*d^2+1/4*a^2/(a*e^2+c*d^2)^2/c
^2*ln((c*x^4+a)*c)*e^3+1/2*a/(a*e^2+c*d^2)^2/c*ln((c*x^4+a)*c)*d^
2*e-1/4*a^2/(a*e^2+c*d^2)^2/c/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2
))*d*e^2-3/4*a/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/
2))*d^3+1/2*d^4*ln(e*x^2+d)/e/(a*e^2+c*d^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 15.3437, size = 1, normalized size = 0.01

$$\left[\frac{2\,a^2cd^2e^2 + 2\,a^3e^4 + 2\left(ac^2d^3e + a^2cde^3\right)x^2 + \left(3\,ac^2d^3e + a^2cde^3 + \left(3\,c^3d^3e + ac^2de^3\right)x^4\right)\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4-2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right) + 2\,(2}{8\left(ac^4d^4e + 2\,a^2c^3d^2e^3 + a^3c^2e^5 + (c^5d^4e + 2\,ac^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/8*(2*a^2*c*d^2*e^2 + 2*a^3*e^4 + 2*(a*c^2*d^3*e + a^2*c*d*e^3)
*x^2 + (3*a*c^2*d^3*e + a^2*c*d*e^3 + (3*c^3*d^3*e + a*c^2*d*e^3)
*x^4)*sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)
) + 2*(2*a^2*c*d^2*e^2 + a^3*e^4 + (2*a*c^2*d^2*e^2 + a^2*c*e^4)*
x^4)*log(c*x^4 + a) + 4*(c^3*d^4*x^4 + a*c^2*d^4)*log(e*x^2 + d))
/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*
a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^4), 1/4*(a^2*c*d^2*e^2 + a^3*e^4 +
 (a*c^2*d^3*e + a^2*c*d*e^3)*x^2 - (3*a*c^2*d^3*e + a^2*c*d*e^3 +
 (3*c^3*d^3*e + a*c^2*d*e^3)*x^4)*sqrt(a/c)*arctan(x^2/sqrt(a/c))
 + (2*a^2*c*d^2*e^2 + a^3*e^4 + (2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4
)*log(c*x^4 + a) + 2*(c^3*d^4*x^4 + a*c^2*d^4)*log(e*x^2 + d))/(a
*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*a*c
^4*d^2*e^3 + a^2*c^3*e^5)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.276606, size = 339, normalized size = 2.01

$$\frac{d^4\ln\left(\left|x^2e+d\right|\right)}{2\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)}+\frac{\left(2\,acd^2e+a^2e^3\right)\ln\left(cx^4+a\right)}{4\left(c^4d^4+2\,ac^3d^2e^2+a^2c^2e^4\right)}-\frac{\left(3\,acd^3+a^2de^2\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)\sqrt{ac}}-\frac{2\,acd^2x^4e-acd^3x^2+a^2x^4e^3-a^2dx^2e^2+a^2d^2e}{4\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)\left(cx^4+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] 1/2*d^4*ln(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)
+ 1/4*(2*a*c*d^2*e + a^2*e^3)*ln(c*x^4 + a)/(c^4*d^4 + 2*a*c^3*d^
2*e^2 + a^2*c^2*e^4) - 1/4*(3*a*c*d^3 + a^2*d*e^2)*arctan(c*x^2/s
qrt(a*c))/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(a*c)) - 1
/4*(2*a*c*d^2*x^4*e - a*c*d^3*x^2 + a^2*x^4*e^3 - a^2*d*x^2*e^2 +
 a^2*d^2*e)/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*(c*x^4 + a))
```

3.245 $\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{a}e\left(ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2}\left(ae^2+cd^2\right)^2}+\frac{a\left(d-ex^2\right)}{4c\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{d^3\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}-\frac{d^3\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}$$

```
[Out] (a*(d - e*x^2))/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) + (Sqrt[a]*e*(3
*c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*c^(3/2)*(c*d^2
+ a*e^2)^2) - (d^3*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) + (d^3*L
og[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 0.485226, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{\sqrt{a}e\left(ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2}\left(ae^2+cd^2\right)^2}+\frac{a\left(d-ex^2\right)}{4c\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{d^3\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}-\frac{d^3\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*(d - e*x^2))/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) + (Sqrt[a]*e*(3
*c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*c^(3/2)*(c*d^2
+ a*e^2)^2) - (d^3*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) + (d^3*L
og[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

Rubi in Sympy [A] time = 70.3934, size = 170, normalized size = 1.13

$$-\frac{\sqrt{a}e\,\text{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{\frac{3}{2}}\left(ae^2+cd^2\right)}+\frac{\sqrt{a}e\left(ae^2+2cd^2\right)\text{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}\left(ae^2+cd^2\right)^2}+\frac{a\left(d-ex^2\right)}{4c\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{d^3\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}-\frac{d^3\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] -sqrt(a)*e*atan(sqrt(c)*x**2/sqrt(a))/(4*c**(3/2)*(a*e**2 + c*d**
2)) + sqrt(a)*e*(a*e**2 + 2*c*d**2)*atan(sqrt(c)*x**2/sqrt(a))/(2
*c**(3/2)*(a*e**2 + c*d**2)**2) + a*(d - e*x**2)/(4*c*(a + c*x**4
)*(a*e**2 + c*d**2)) + d**3*log(a + c*x**4)/(4*(a*e**2 + c*d**2)*
*2) - d**3*log(d + e*x**2)/(2*(a*e**2 + c*d**2)**2)
```

Mathematica [A] time = 0.190637, size = 142, normalized size = 0.95

$$\frac{\sqrt{a}e\left(a+cx^4\right)\left(ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)+\sqrt{c}\left(-2cd^3\left(a+cx^4\right)\log\left(d+ex^2\right)+cd^3\left(a+cx^4\right)\log\left(a+cx^4\right)+a\left(d-ex^2\right)\left(a\right.\right.}{4c^{3/2}\left(a+cx^4\right)\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (Sqrt[a]*e*(3*c*d^2 + a*e^2)*(a + c*x^4)*ArcTan[(Sqrt[c]*x^2)/Sqr
t[a]] + Sqrt[c]*(a*(c*d^2 + a*e^2)*(d - e*x^2) - 2*c*d^3*(a + c*x
^4)*Log[d + e*x^2] + c*d^3*(a + c*x^4)*Log[a + c*x^4]))/(4*c^(3/2
)*(c*d^2 + a*e^2)^2*(a + c*x^4))
```

Maple [A] time = 0.023, size = 262, normalized size = 1.8

$$-\frac{a^2e^3x^2}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)c}-\frac{aex^2d^2}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)}+\frac{da^2e^2}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)c}+\frac{ad^3}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)}+\frac{d^3\ln\left(\left(cx^4+a\right)c\right)}{4\left(ae^2+cd^2\right)^2}+\frac{a^2e^3}{4\left(ae^2+cd^2\right)^2c}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{3\,d^2ea}{4\left(ae^2+cd^2\right)^2}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}-\frac{d^3\ln\left(ex^2+d\right)}{2\left(ae^2+cd^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a^2*e^3/c*x^2-1/4/(a*e^2+c*d^2)^2/
(c*x^4+a)*a*e*x^2*d^2+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a^2*d/c*e^2+1
/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a*d^3+1/4/(a*e^2+c*d^2)^2*d^3*ln((c*
x^4+a)*c)+1/4/(a*e^2+c*d^2)^2/c/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1
/2))*a^2*e^3+3/4/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(
1/2))*a*d^2*e-1/2*d^3*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.11036, size = 1, normalized size = 0.01

$$\left[\frac{2\,acd^3 + 2\,a^2de^2 - 2\,\left(acd^2e + a^2e^3\right)x^2 + \left(3\,acd^2e + a^2e^3 + \left(3\,c^2d^2e + ace^3\right)x^4\right)\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4+2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right) + 2\,\left(c^2d^3x^4 + ac\right.}{8\,(ac^3d^4 + 2\,a^2c^2d^2e^2 + a^3ce^4 + (c^4d^4 + 2\,ac^3d^2e^2 + a^2c^2e^4)x^4)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*c*d^3 + 2*a^2*d*e^2 - 2*(a*c*d^2*e + a^2*e^3)*x^2 + (3*
a*c*d^2*e + a^2*e^3 + (3*c^2*d^2*e + a*c*e^3)*x^4)*sqrt(-a/c)*log
((c*x^4 + 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) + 2*(c^2*d^3*x^4 +
 a*c*d^3)*log(c*x^4 + a) - 4*(c^2*d^3*x^4 + a*c*d^3)*log(e*x^2 +
d))/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c
^3*d^2*e^2 + a^2*c^2*e^4)*x^4), 1/4*(a*c*d^3 + a^2*d*e^2 - (a*c*d
^2*e + a^2*e^3)*x^2 + (3*a*c*d^2*e + a^2*e^3 + (3*c^2*d^2*e + a*c
*e^3)*x^4)*sqrt(a/c)*arctan(x^2/sqrt(a/c)) + (c^2*d^3*x^4 + a*c*d
^3)*log(c*x^4 + a) - 2*(c^2*d^3*x^4 + a*c*d^3)*log(e*x^2 + d))/(a
*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2
*e^2 + a^2*c^2*e^4)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.274248, size = 301, normalized size = 2.01

$$-\frac{d^3 e \ln\left(\left|x^2 e + d\right|\right)}{2\left(c^2 d^4 e + 2\,acd^2 e^3 + a^2 e^5\right)} + \frac{d^3 \ln\left(cx^4 + a\right)}{4\left(c^2 d^4 + 2\,acd^2 e^2 + a^2 e^4\right)} + \frac{\left(3\,acd^2 e + a^2 e^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^3 d^4 + 2\,ac^2 d^2 e^2 + a^2 ce^4\right)\sqrt{ac}} - \frac{c^2 d^3 x^4 + acd^2 x^2 e + a^2 x^2 e^3 - a^2 de^2}{4\left(c^3 d^4 + 2\,ac^2 d^2 e^2 + a^2 ce^4\right)\left(cx^4 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] -1/2*d^3*e*ln(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^
5) + 1/4*d^3*ln(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) +
1/4*(3*a*c*d^2*e + a^2*e^3)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2
*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(a*c)) - 1/4*(c^2*d^3*x^4 + a*c*d
^2*x^2*e + a^2*x^2*e^3 - a^2*d*e^2)/((c^3*d^4 + 2*a*c^2*d^2*e^2 +
 a^2*c*e^4)*(c*x^4 + a))
```

3.246 $\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=153

$$-\frac{d^2e\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}+\frac{d^2e\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}+\frac{d\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)^2}-\frac{ae+cdx^2}{4c\left(a+cx^4\right)\left(ae^2+cd^2\right)}$$

```
[Out] -(a*e + c*d*x^2)/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) + (d*(c*d^2 -
a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*Sqrt[a]*Sqrt[c]*(c*d^2 +
 a*e^2)^2) + (d^2*e*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) - (d^2*
e*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 0.482651, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{d^2e\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}+\frac{d^2e\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}+\frac{d\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)^2}-\frac{ae+cdx^2}{4c\left(a+cx^4\right)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(a*e + c*d*x^2)/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) + (d*(c*d^2 -
a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*Sqrt[a]*Sqrt[c]*(c*d^2 +
 a*e^2)^2) + (d^2*e*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) - (d^2*
e*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

Rubi in Sympy [A] time = 55.5867, size = 165, normalized size = 1.08

$$-\frac{d^2e\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}+\frac{d^2e\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}-\frac{ae+cdx^2}{4c\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{\sqrt{c}d^3\,\mathrm{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)^2}-\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] -d**2*e*log(a + c*x**4)/(4*(a*e**2 + c*d**2)**2) + d**2*e*log(d +
 e*x**2)/(2*(a*e**2 + c*d**2)**2) - (a*e + c*d*x**2)/(4*c*(a + c*
x**4)*(a*e**2 + c*d**2)) + sqrt(c)*d**3*atan(sqrt(c)*x**2/sqrt(a)
```

```
)/(2*sqrt(a)*(a*e**2 + c*d**2)**2) - d*atan(sqrt(c)*x**2/sqrt(a))
/(4*sqrt(a)*sqrt(c)*(a*e**2 + c*d**2))
```

Mathematica [A] time = 0.274019, size = 120, normalized size = 0.78

$$\frac{\frac{d(cd^2-ae^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}-\frac{(ae^2+cd^2)(ae+cdx^2)}{c(a+cx^4)}-d^2e\log\left(a+cx^4\right)+2d^2e\log\left(d+ex^2\right)}{4\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-(((c*d^2 + a*e^2)*(a*e + c*d*x^2))/(c*(a + c*x^4))) + (d*(c*d^2
 - a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) + 2*d^
2*e*Log[d + e*x^2] - d^2*e*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

Maple [A] time = 0.023, size = 252, normalized size = 1.7

$$-\frac{e^2x^2da}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)}-\frac{cx^2d^3}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)}-\frac{a^2e^3}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)c}-\frac{aed^2}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)}-\frac{d^2e\ln\left(cx^4+a\right)}{4\left(ae^2+cd^2\right)^2}-\frac{e^2da}{4\left(ae^2+cd^2\right)^2}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{cd^3}{4\left(ae^2+cd^2\right)^2}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{d^2e\ln\left(ex^2+d\right)}{2\left(ae^2+cd^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^2*e^2*d*a-1/4/(a*e^2+c*d^2)^2/(c
*x^4+a)*x^2*c*d^3-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a^2*e^3/c-1/4/(a*
e^2+c*d^2)^2/(c*x^4+a)*a*e*d^2-1/4*d^2*e*ln(c*x^4+a)/(a*e^2+c*d^2
)^2-1/4/(a*e^2+c*d^2)^2*d/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))*a
*e^2+1/4/(a*e^2+c*d^2)^2*d^3/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)
)*c+1/2*d^2*e*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.84151, size = 1, normalized size = 0.01

$$\left[-\frac{\left(ac^2d^3 - a^2cde^2 + \left(c^3d^3 - ac^2de^2\right)x^4\right)\log\left(-\frac{2\,acx^2-\left(cx^4-a\right)\sqrt{-ac}}{cx^4+a}\right) + 2\left(acd^2e + a^2e^3 + \left(c^2d^3 + acde^2\right)x^2 + \left(c^2d^2ex^4 + acd\right.\right.}{8\left(ac^3d^4 + 2\,a^2c^2d^2e^2 + a^3ce^4 + (c^4d^4 + 2\,ac^3d^2e^2 + a^2c^2e^4)x^4\right)\sqrt{-}}\right.$$

$$-\frac{\left(ac^2d^3 - a^2cde^2 + \left(c^3d^3 - ac^2de^2\right)x^4\right)\arctan\left(\frac{a}{\sqrt{ac}x^2}\right) + \left(acd^2e + a^2e^3 + \left(c^2d^3 + acde^2\right)x^2 + \left(c^2d^2ex^4 + acd^2e\right)\log\left(cx^4 + \right.\right.}{4\left(ac^3d^4 + 2\,a^2c^2d^2e^2 + a^3ce^4 + (c^4d^4 + 2\,ac^3d^2e^2 + a^2c^2e^4)x^4\right)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [-1/8*((a*c^2*d^3 - a^2*c*d*e^2 + (c^3*d^3 - a*c^2*d*e^2)*x^4)*lo
g(-(2*a*c*x^2 - (c*x^4 - a)*sqrt(-a*c))/(c*x^4 + a)) + 2*(a*c*d^2
*e + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x^2 + (c^2*d^2*e*x^4 + a*c*d
^2*e)*log(c*x^4 + a) - 2*(c^2*d^2*e*x^4 + a*c*d^2*e)*log(e*x^2 +
d))*sqrt(-a*c))/((a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^
4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-a*c)), -1/4*((a
*c^2*d^3 - a^2*c*d*e^2 + (c^3*d^3 - a*c^2*d*e^2)*x^4)*arctan(a/(s
qrt(a*c)*x^2)) + (a*c*d^2*e + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x^2
 + (c^2*d^2*e*x^4 + a*c*d^2*e)*log(c*x^4 + a) - 2*(c^2*d^2*e*x^4
+ a*c*d^2*e)*log(e*x^2 + d))*sqrt(a*c))/((a*c^3*d^4 + 2*a^2*c^2*d
^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^
4)*sqrt(a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275551, size = 297, normalized size = 1.94

$$-\frac{d^{2}e\ln\left(cx^{4}+a\right)}{4\left(c^{2}d^{4}+2\,acd^{2}e^{2}+a^{2}e^{4}\right)}+\frac{d^{2}e^{2}\ln\left(\left|x^{2}e+d\right|\right)}{2\left(c^{2}d^{4}e+2\,acd^{2}e^{3}+a^{2}e^{5}\right)}+\frac{\left(cd^{3}-ade^{2}\right)\arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)}{4\left(c^{2}d^{4}+2\,acd^{2}e^{2}+a^{2}e^{4}\right)\sqrt{ac}}+\frac{c^{2}d^{2}x^{4}e-c^{2}d^{3}x^{2}-acdx^{2}e^{2}-a^{2}e^{3}}{4\left(c^{3}d^{4}+2\,ac^{2}d^{2}e^{2}+a^{2}ce^{4}\right)\left(cx^{4}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] -1/4*d^2*e*ln(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/
2*d^2*e^2*ln(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5
) + 1/4*(c*d^3 - a*d*e^2)*arctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a
*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)) + 1/4*(c^2*d^2*x^4*e - c^2*d^3*x
^2 - a*c*d*x^2*e^2 - a^2*e^3)/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c
*e^4)*(c*x^4 + a))
```

3.247 $\int \frac{x^3}{(d+ex^2)(a+cx^4)^2} dx$

Optimal. Leaf size=148

$$\frac{de^2 \log \left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}-\frac{de^2 \log \left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}-\frac{e\left(cd^2-ae^2\right) \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)^2}-\frac{d-ex^2}{4\left(a+cx^4\right)\left(ae^2+cd^2\right)}$$

```
[Out] -(d - e*x^2)/(4*(c*d^2 + a*e^2)*(a + c*x^4)) - (e*(c*d^2 - a*e^2)
*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2
)^2) - (d*e^2*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) + (d*e^2*Log[
a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 0.387509, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{de^2 \log \left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}-\frac{de^2 \log \left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}-\frac{e\left(cd^2-ae^2\right) \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)^2}-\frac{d-ex^2}{4\left(a+cx^4\right)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(d - e*x^2)/(4*(c*d^2 + a*e^2)*(a + c*x^4)) - (e*(c*d^2 - a*e^2)
*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2
)^2) - (d*e^2*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) + (d*e^2*Log[
a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

Rubi in Sympy [A] time = 67.4309, size = 129, normalized size = 0.87

$$\frac{de^2 \log \left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}-\frac{de^2 \log \left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}-\frac{d-ex^2}{4\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{e\left(ae^2-cd^2\right) \operatorname{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] d*e**2*log(a + c*x**4)/(4*(a*e**2 + c*d**2)**2) - d*e**2*log(d +
e*x**2)/(2*(a*e**2 + c*d**2)**2) - (d - e*x**2)/(4*(a + c*x**4)*(
a*e**2 + c*d**2)) + e*(a*e**2 - c*d**2)*atan(sqrt(c)*x**2/sqrt(a)
```

```
)/(4*sqrt(a)*sqrt(c)*(a*e**2 + c*d**2)**2)
```

Mathematica [A] time = 0.258924, size = 114, normalized size = 0.77

$$\frac{\frac{e(ae^2-cd^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}+\frac{(ex^2-d)(ae^2+cd^2)}{a+cx^4}+de^2\log\left(a+cx^4\right)-2de^2\log\left(d+ex^2\right)}{4\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (((c*d^2 + a*e^2)*(-d + e*x^2))/(a + c*x^4) + (e*(-(c*d^2) + a*e^
2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - 2*d*e^2*Log
[d + e*x^2] + d*e^2*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

Maple [A] time = 0.021, size = 247, normalized size = 1.7

$$\frac{x^2e^3a}{4\left(ae^2+cd^2\right)^2(cx^4+a)}+\frac{x^2d^2ec}{4\left(ae^2+cd^2\right)^2(cx^4+a)}-\frac{e^2da}{4\left(ae^2+cd^2\right)^2(cx^4+a)}-\frac{cd^3}{4\left(ae^2+cd^2\right)^2(cx^4+a)}+\frac{e^2d\ln\left(cx^4+a\right)}{4\left(ae^2+cd^2\right)^2}+\frac{e^3a}{4\left(ae^2+cd^2\right)^2}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}-\frac{d^2ec}{4\left(ae^2+cd^2\right)^2}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}-\frac{e^2d\ln\left(ex^2+d\right)}{2\left(ae^2+cd^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^2*e^3*a+1/4/(a*e^2+c*d^2)^2/(c*x^
4+a)*x^2*d^2*e*c-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*e^2*d*a-1/4/(a*e^2
+c*d^2)^2/(c*x^4+a)*c*d^3+1/4*d*e^2*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1
/4/(a*e^2+c*d^2)^2*e^3/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))*a-1/
4/(a*e^2+c*d^2)^2*e/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))*c*d^2-1
/2*d*e^2*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A]   time = 2.82016, size = 1, normalized size = 0.01

$$\left[-\frac{\left(acd^2e - a^2e^3 + \left(c^2d^2e - ace^3\right)x^4\right)\log\left(\frac{2\,acx^2+\left(cx^4-a\right)\sqrt{-ac}}{cx^4+a}\right) + 2\left(cd^3 + ade^2 - \left(cd^2e + ae^3\right)x^2 - \left(cde^2x^4 + ade^2\right)\log\left(cx\right.\right.}{8\left(ac^2d^4 + 2\,a^2cd^2e^2 + a^3e^4 + \left(c^3d^4 + 2\,ac^2d^2e^2 + a^2ce^4\right)x^4\right)\sqrt{-ac}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [-1/8*((a*c*d^2*e - a^2*e^3 + (c^2*d^2*e - a*c*e^3)*x^4)*log((2*a
*c*x^2 + (c*x^4 - a)*sqrt(-a*c))/(c*x^4 + a)) + 2*(c*d^3 + a*d*e^
2 - (c*d^2*e + a*e^3)*x^2 - (c*d*e^2*x^4 + a*d*e^2)*log(c*x^4 + a
) + 2*(c*d*e^2*x^4 + a*d*e^2)*log(e*x^2 + d))*sqrt(-a*c))/((a*c^2
*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a
^2*c*e^4)*x^4)*sqrt(-a*c)), 1/4*((a*c*d^2*e - a^2*e^3 + (c^2*d^2*
e - a*c*e^3)*x^4)*arctan(a/(sqrt(a*c)*x^2)) - (c*d^3 + a*d*e^2 -
(c*d^2*e + a*e^3)*x^2 - (c*d*e^2*x^4 + a*d*e^2)*log(c*x^4 + a) +
2*(c*d*e^2*x^4 + a*d*e^2)*log(e*x^2 + d))*sqrt(a*c))/((a*c^2*d^4
+ 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*
e^4)*x^4)*sqrt(a*c))]
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275832, size = 254, normalized size = 1.72

$$\frac{de^2\ln\left(cx^4+a\right)}{4\,(c^2d^4+2\,acd^2e^2+a^2e^4)}-\frac{de^3\ln\left(\left|x^2e+d\right|\right)}{2\,(c^2d^4e+2\,acd^2e^3+a^2e^5)}-\frac{\left(cd^2e-ae^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{ac}}-\frac{cd^3-\left(cd^2e+ae^3\right)x^2+ade^2}{4\,(cx^4+a)(cd^2+ae^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] 1/4*d*e^2*ln(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/2
*d*e^3*ln(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) -
 1/4*(c*d^2*e - a*e^3)*arctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*
d^2*e^2 + a^2*e^4)*sqrt(a*c)) - 1/4*(c*d^3 - (c*d^2*e + a*e^3)*x^
2 + a*d*e^2)/((c*x^4 + a)*(c*d^2 + a*e^2)^2)
```

3.248 $\int \frac{x}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{c}d\left(3ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(ae^2+cd^2\right)^2}+\frac{ae+cdx^2}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{e^3\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}+\frac{e^3\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}$$

```
[Out] (a*e + c*d*x^2)/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (Sqrt[c]*d*(c
*d^2 + 3*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^2
+ a*e^2)^2) + (e^3*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) - (e^3*L
og[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 0.363493, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{\sqrt{c}d\left(3ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(ae^2+cd^2\right)^2}+\frac{ae+cdx^2}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{e^3\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}+\frac{e^3\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*e + c*d*x^2)/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (Sqrt[c]*d*(c
*d^2 + 3*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^2
+ a*e^2)^2) + (e^3*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) - (e^3*L
og[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

Rubi in Sympy [A] time = 58.7762, size = 133, normalized size = 0.88

$$-\frac{e^3\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}+\frac{e^3\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}+\frac{ae+cdx^2}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{\sqrt{c}d\left(3ae^2+cd^2\right)\operatorname{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\left(ae^2+cd^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] -e**3*log(a + c*x**4)/(4*(a*e**2 + c*d**2)**2) + e**3*log(d + e*x
**2)/(2*(a*e**2 + c*d**2)**2) + (a*e + c*d*x**2)/(4*a*(a + c*x**4
)*(a*e**2 + c*d**2)) + sqrt(c)*d*(3*a*e**2 + c*d**2)*atan(sqrt(c)
```

```
*x**2/sqrt(a))/(4*a**(3/2)*(a*e**2 + c*d**2)**2)
```

Mathematica [A] time = 0.249135, size = 117, normalized size = 0.77

$$\frac{\frac{\sqrt{c}d(3ae^2+cd^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{a^{3/2}}+\frac{(ae^2+cd^2)(ae+cdx^2)}{a(a+cx^4)}-e^3\log\left(a+cx^4\right)+2e^3\log\left(d+ex^2\right)}{4\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (((c*d^2 + a*e^2)*(a*e + c*d*x^2))/(a*(a + c*x^4)) + (Sqrt[c]*d*(
c*d^2 + 3*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/a^(3/2) + 2*e^3*L
og[d + e*x^2] - e^3*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

Maple [A] time = 0.023, size = 257, normalized size = 1.7

$$\frac{cx^2e^2d}{4\left(ae^2+cd^2\right)^2(cx^4+a)}+\frac{c^2d^3x^2}{4\left(ae^2+cd^2\right)^2(cx^4+a)a}+\frac{e^3a}{4\left(ae^2+cd^2\right)^2(cx^4+a)}+\frac{d^2ec}{4\left(ae^2+cd^2\right)^2(cx^4+a)}-\frac{e^3\ln\left(a\left(cx^4+a\right)\right)}{4\left(ae^2+cd^2\right)^2}+\frac{3\,cde^2}{4\left(ae^2+cd^2\right)^2}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{c^2d^3}{4\left(ae^2+cd^2\right)^2a}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{e^3\ln\left(ex^2+d\right)}{2\left(ae^2+cd^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*x^2*e^2*d+1/4*c^2/(a*e^2+c*d^2)^2
/(c*x^4+a)*d^3/a*x^2+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3*a+1/4*c/(a
*e^2+c*d^2)^2/(c*x^4+a)*d^2*e-1/4/(a*e^2+c*d^2)^2*e^3*ln(a*(c*x^4
+a))+3/4*c/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))*
e^2*d+1/4*c^2/(a*e^2+c*d^2)^2/a/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1
/2))*d^3+1/2*e^3*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.24755, size = 1, normalized size = 0.01

$$\left[\frac{2\,acd^2e+2\,a^2e^3+2\left(c^2d^3+acde^2\right)x^2+\left(acd^3+3\,a^2de^2+\left(c^2d^3+3\,acde^2\right)x^4\right)\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4+2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)-2\left(ace^3x^4+a^2\right.}{8\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4+(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4)x^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*c*d^2*e + 2*a^2*e^3 + 2*(c^2*d^3 + a*c*d*e^2)*x^2 + (a*
c*d^3 + 3*a^2*d*e^2 + (c^2*d^3 + 3*a*c*d*e^2)*x^4)*sqrt(-c/a)*log
((c*x^4 + 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) - 2*(a*c*e^3*x^4 +
 a^2*e^3)*log(c*x^4 + a) + 4*(a*c*e^3*x^4 + a^2*e^3)*log(e*x^2 +
d))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2
*c^2*d^2*e^2 + a^3*c*e^4)*x^4), 1/4*(a*c*d^2*e + a^2*e^3 + (c^2*d
^3 + a*c*d*e^2)*x^2 - (a*c*d^3 + 3*a^2*d*e^2 + (c^2*d^3 + 3*a*c*d
*e^2)*x^4)*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) - (a*c*e^3*x^4 +
 a^2*e^3)*log(c*x^4 + a) + 2*(a*c*e^3*x^4 + a^2*e^3)*log(e*x^2 +
d))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2
*c^2*d^2*e^2 + a^3*c*e^4)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275941, size = 269, normalized size = 1.78

$$-\frac{e^3\ln\left(cx^4+a\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{e^4\ln\left(\left|x^2e+d\right|\right)}{2\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)}+\frac{\left(c^2d^3+3\,acde^2\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)\sqrt{ac}}+\frac{acd^2e+\left(c^2d^3+acde^2\right)x^2+a^2e^3}{4\left(cx^4+a\right)\left(cd^2+ae^2\right)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] -1/4*e^3*ln(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*
e^4*ln(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/
4*(c^2*d^3 + 3*a*c*d*e^2)*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2
*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(a*c)) + 1/4*(a*c*d^2*e + (c^2*d^3
+ a*c*d*e^2)*x^2 + a^2*e^3)/((c*x^4 + a)*(c*d^2 + a*e^2)^2*a)
```

3.249 $\int \frac{1}{x(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=209

$$-\frac{\sqrt{c}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(ae^2+cd^2\right)}-\frac{cd\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)^2}+\frac{\log(x)}{a^2d}+\frac{c\left(d-ex^2\right)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{e^4\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)^2}-\frac{\sqrt{c}e^3\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)^2}$$

```
[Out] (c*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[c]*e^3*
ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e^2)^2) - (S
qrt[c]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^2 + a*e^2
)) + Log[x]/(a^2*d) - (e^4*Log[d + e*x^2])/(2*d*(c*d^2 + a*e^2)^2
) - (c*d*(c*d^2 + 2*a*e^2)*Log[a + c*x^4])/(4*a^2*(c*d^2 + a*e^2)
^2)
```

Rubi [A] time = 0.485347, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{\sqrt{c}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(ae^2+cd^2\right)}-\frac{cd\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)^2}+\frac{\log(x)}{a^2d}+\frac{c\left(d-ex^2\right)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{e^4\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)^2}-\frac{\sqrt{c}e^3\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (c*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[c]*e^3*
ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e^2)^2) - (S
qrt[c]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^2 + a*e^2
)) + Log[x]/(a^2*d) - (e^4*Log[d + e*x^2])/(2*d*(c*d^2 + a*e^2)^2
) - (c*d*(c*d^2 + 2*a*e^2)*Log[a + c*x^4])/(4*a^2*(c*d^2 + a*e^2)
^2)
```

Rubi in Sympy [A] time = 74.1124, size = 189, normalized size = 0.9

$$-\frac{e^4\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)^2}+\frac{c\left(d-ex^2\right)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{cd\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)^2}+\frac{\log\left(x^2\right)}{2a^2d}-\frac{\sqrt{c}e^3\,\mathrm{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)^2}-\frac{\sqrt{c}e\,\mathrm{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] -e**4*log(d + e*x**2)/(2*d*(a*e**2 + c*d**2)**2) + c*(d - e*x**2)
/(4*a*(a + c*x**4)*(a*e**2 + c*d**2)) - c*d*(2*a*e**2 + c*d**2)*l
og(a + c*x**4)/(4*a**2*(a*e**2 + c*d**2)**2) + log(x**2)/(2*a**2*
d) - sqrt(c)*e**3*atan(sqrt(c)*x**2/sqrt(a))/(2*sqrt(a)*(a*e**2 +
 c*d**2)**2) - sqrt(c)*e*atan(sqrt(c)*x**2/sqrt(a))/(4*a**(3/2)*(
a*e**2 + c*d**2))
```

Mathematica [A] time = 0.320713, size = 241, normalized size = 1.15

$$\frac{-2a^2e^4\left(a+cx^4\right)\log\left(d+ex^2\right)+4\log(x)\left(a+cx^4\right)\left(ae^2+cd^2\right)^2-cd^2\left(a+cx^4\right)\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)+\sqrt{a}\sqrt{c}de\left(a+\cdots\right.}{4a^2d\left(a+cx^4\right)\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*c*d*(c*d^2 + a*e^2)*(d - e*x^2) + Sqrt[a]*Sqrt[c]*d*e*(c*d^2 +
 3*a*e^2)*(a + c*x^4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + S
qrt[a]*Sqrt[c]*d*e*(c*d^2 + 3*a*e^2)*(a + c*x^4)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)] + 4*(c*d^2 + a*e^2)^2*(a + c*x^4)*Log[x]
- 2*a^2*e^4*(a + c*x^4)*Log[d + e*x^2] - c*d^2*(c*d^2 + 2*a*e^2)*
(a + c*x^4)*Log[a + c*x^4])/(4*a^2*d*(c*d^2 + a*e^2)^2*(a + c*x^4
))
```

Maple [A] time = 0.029, size = 309, normalized size = 1.5

$$\frac{\ln(x)}{a^2 d} - \frac{c x^2 e^3}{4(ae^2+cd^2)^2(cx^4+a)} - \frac{c^2 x^2 d^2 e}{4(ae^2+cd^2)^2 a(cx^4+a)} + \frac{cde^2}{4(ae^2+cd^2)^2(cx^4+a)} + \frac{c^2 d^3}{4(ae^2+cd^2)^2 a(cx^4+a)} - \frac{c\ln(cx^4+a)de^2}{2(ae^2+cd^2)^2 a} - \frac{c^2\ln(cx^4+a)d^3}{4(ae^2+cd^2)^2 a^2} - \frac{3ce^3}{4(ae^2+cd^2)^2}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}} - \frac{d^2ec^2}{4(ae^2+cd^2)^2 a}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}} - \frac{e^4\ln(ex^2+d)}{2d(ae^2+cd^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] ln(x)/a^2/d-1/4/(a*e^2+c*d^2)^2*c/(c*x^4+a)*x^2*e^3-1/4/(a*e^2+c*
d^2)^2*c^2/a/(c*x^4+a)*x^2*d^2*e+1/4/(a*e^2+c*d^2)^2*c/(c*x^4+a)*
d*e^2+1/4/(a*e^2+c*d^2)^2*c^2/a/(c*x^4+a)*d^3-1/2/(a*e^2+c*d^2)^2
*c/a*ln(c*x^4+a)*d*e^2-1/4/(a*e^2+c*d^2)^2*c^2/a^2*ln(c*x^4+a)*d^
3-3/4/(a*e^2+c*d^2)^2*c/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))*e^3
-1/4/(a*e^2+c*d^2)^2*c^2/a/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))*
d^2*e-1/2*e^4*ln(e*x^2+d)/d/(a*e^2+c*d^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 130.493, size = 1, normalized size = 0.

$$\left[\frac{2ac^2d^4 + 2a^2cd^2e^2 - 2\left(ac^2d^3e + a^2cde^3\right)x^2 + \left(a^2cd^3e + 3a^3de^3 + \left(ac^2d^3e + 3a^2cde^3\right)x^4\right)\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4 - 2ax^2\sqrt{-\frac{c}{a}} - a}{cx^4+a}\right) - 2}{8\left(a^3c^2d^5 + 2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)*x),x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*c^2*d^4 + 2*a^2*c*d^2*e^2 - 2*(a*c^2*d^3*e + a^2*c*d*e^
3)*x^2 + (a^2*c*d^3*e + 3*a^3*d*e^3 + (a*c^2*d^3*e + 3*a^2*c*d*e^
3)*x^4)*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 +
a)) - 2*(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + (c^3*d^4 + 2*a*c^2*d^2*e^2
)*x^4)*log(c*x^4 + a) - 4*(a^2*c*e^4*x^4 + a^3*e^4)*log(e*x^2 + d
) + 8*(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2
*d^2*e^2 + a^2*c*e^4)*x^4)*log(x))/(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2
 + a^5*d*e^4 + (a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^
4), 1/4*(a*c^2*d^4 + a^2*c*d^2*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*
x^2 + (a^2*c*d^3*e + 3*a^3*d*e^3 + (a*c^2*d^3*e + 3*a^2*c*d*e^3)*
x^4)*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) - (a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + (c^3*d^4 + 2*a*c^2*d^2*e^2)*x^4)*log(c*x^4 + a) - 2*(a
^2*c*e^4*x^4 + a^3*e^4)*log(e*x^2 + d) + 4*(a*c^2*d^4 + 2*a^2*c*d
^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*l
og(x))/(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5
+ 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.276621, size = 377, normalized size = 1.8

$$-\frac{\left(c^2d^3+2\,acde^2\right)\ln\left(cx^4+a\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)}-\frac{e^5\ln\left(\left|x^2e+d\right|\right)}{2\left(c^2d^5e+2\,acd^3e^3+a^2de^5\right)}-\frac{\left(c^2d^2e+3\,ace^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)\sqrt{ac}}+\frac{c^3d^3x^4+2\,ac^2dx^4e^2-ac^2d^2x^2e+2\,ac^2d^3-a^2cx^2e^3+3\,a^2cde^2}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)\left(cx^4+a\right)}+\frac{\ln\left(x^2\right)}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)*x),x, algorithm="giac")
```

```
[Out] -1/4*(c^2*d^3 + 2*a*c*d*e^2)*ln(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c
*d^2*e^2 + a^4*e^4) - 1/2*e^5*ln(abs(x^2*e + d))/(c^2*d^5*e + 2*a
*c*d^3*e^3 + a^2*d*e^5) - 1/4*(c^2*d^2*e + 3*a*c*e^3)*arctan(c*x^
2/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(a*c))
+ 1/4*(c^3*d^3*x^4 + 2*a*c^2*d*x^4*e^2 - a*c^2*d^2*x^2*e + 2*a*c^
```

```
2*d^3 - a^2*c*x^2*e^3 + 3*a^2*c*d*e^2)/((a^2*c^2*d^4 + 2*a^3*c*d^
2*e^2 + a^4*e^4)*(c*x^4 + a)) + 1/2*ln(x^2)/(a^2*d)
```

3.250 $\int \frac{1}{x^3(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=236

$$-\frac{c^{3/2}d\left(2ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}\left(ae^2+cd^2\right)^2}-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(ae^2+cd^2\right)}+\frac{ce\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)^2}-\frac{c\left(ae+cdx^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{e\log(x)}{a^2d^2}-\frac{1}{2a^2dx^2}+\frac{e^5\log\left(d+ex^2\right)}{2d^2\left(ae^2+cd^2\right)^2}$$

```
[Out] -1/(2*a^2*d*x^2) - (c*(a*e + c*d*x^2))/(4*a^2*(c*d^2 + a*e^2)*(a
+ c*x^4)) - (c^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(5/2)*
(c*d^2 + a*e^2)) - (c^(3/2)*d*(c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*x
^2)/Sqrt[a]])/(2*a^(5/2)*(c*d^2 + a*e^2)^2) - (e*Log[x])/(a^2*d^2
) + (e^5*Log[d + e*x^2])/(2*d^2*(c*d^2 + a*e^2)^2) + (c*e*(c*d^2
+ 2*a*e^2)*Log[a + c*x^4])/(4*a^2*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 0.53423, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{c^{3/2}d\left(2ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}\left(ae^2+cd^2\right)^2}-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(ae^2+cd^2\right)}+\frac{ce\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)^2}-\frac{c\left(ae+cdx^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{e\log(x)}{a^2d^2}-\frac{1}{2a^2dx^2}+\frac{e^5\log\left(d+ex^2\right)}{2d^2\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(d + e*x^2)*(a + c*x^4)^2),x]
```


```
[Out] -1/(2*a^2*d*x^2) - (c*(a*e + c*d*x^2))/(4*a^2*(c*d^2 + a*e^2)*(a
+ c*x^4)) - (c^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(5/2)*
(c*d^2 + a*e^2)) - (c^(3/2)*d*(c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*x
^2)/Sqrt[a]])/(2*a^(5/2)*(c*d^2 + a*e^2)^2) - (e*Log[x])/(a^2*d^2
) + (e^5*Log[d + e*x^2])/(2*d^2*(c*d^2 + a*e^2)^2) + (c*e*(c*d^2
+ 2*a*e^2)*Log[a + c*x^4])/(4*a^2*(c*d^2 + a*e^2)^2)
```

Rubi in Sympy [A] time = 76.2225, size = 221, normalized size = 0.94

$$\frac{e^5\log\left(d+ex^2\right)}{2d^2\left(ae^2+cd^2\right)^2}+\frac{ce\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)^2}-\frac{c\left(ae+cdx^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{1}{2a^2dx^2}-\frac{e\log\left(x^2\right)}{2a^2d^2}-\frac{c^{\frac{3}{2}}d\,\mathrm{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}\left(ae^2+cd^2\right)}-\frac{c^{\frac{3}{2}}d\left(2ae^2+cd^2\right)\mathrm{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{\frac{5}{2}}\left(ae^2+cd^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] e**5*log(d + e*x**2)/(2*d**2*(a*e**2 + c*d**2)**2) + c*e*(2*a*e**
2 + c*d**2)*log(a + c*x**4)/(4*a**2*(a*e**2 + c*d**2)**2) - c*(a*
e + c*d*x**2)/(4*a**2*(a + c*x**4)*(a*e**2 + c*d**2)) - 1/(2*a**2
*d*x**2) - e*log(x**2)/(2*a**2*d**2) - c**(3/2)*d*atan(sqrt(c)*x*
*2/sqrt(a))/(4*a**(5/2)*(a*e**2 + c*d**2)) - c**(3/2)*d*(2*a*e**2
 + c*d**2)*atan(sqrt(c)*x**2/sqrt(a))/(2*a**(5/2)*(a*e**2 + c*d**
2)**2)
```

Mathematica [A] time = 1.30543, size = 248, normalized size = 1.05

$$\frac{1}{4}\left(\frac{c^{3/2}d\left(5ae^2+3cd^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{5/2}\left(ae^2+cd^2\right)^2}+\frac{c^{3/2}d\left(5ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{a^{5/2}\left(ae^2+cd^2\right)^2}-\frac{c\left(ae+cdx^2\right)}{a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{c\left(2ae^3+cd^2e\right)\log\left(a+cx^4\right)}{a^2\left(ae^2+cd^2\right)^2}-\frac{4e\log(x)}{a^2d^2}-\frac{2}{a^2dx^2}+\frac{2e^5\log\left(d+ex^2\right)}{\left(ade^2+cd^3\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-2/(a^2*d*x^2) - (c*(a*e + c*d*x^2))/(a^2*(c*d^2 + a*e^2)*(a + c
*x^4)) + (c^(3/2)*d*(3*c*d^2 + 5*a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2) + (c^(3/2)*d*(3*c*d^2
 + 5*a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*
d^2 + a*e^2)^2) - (4*e*Log[x])/(a^2*d^2) + (2*e^5*Log[d + e*x^2])
/(c*d^3 + a*d*e^2)^2 + (c*(c*d^2*e + 2*a*e^3)*Log[a + c*x^4])/(a^
2*(c*d^2 + a*e^2)^2))/4
```

Maple [A] time = 0.032, size = 332, normalized size = 1.4

$$
\begin{aligned}
&-\frac{1}{2\,a^2dx^2}-\frac{\ln(x)\,e}{a^2d^2}-\frac{c^2x^2e^2d}{4\,(ae^2+cd^2)^2\,a\,(cx^4+a)}-\frac{c^3x^2d^3}{4\,(ae^2+cd^2)^2\,a^2\,(cx^4+a)}\\
&-\frac{ce^3}{4\,(ae^2+cd^2)^2\,(cx^4+a)}-\frac{d^2ec^2}{4\,(ae^2+cd^2)^2\,a\,(cx^4+a)}+\frac{c\ln\left(cx^4+a\right)\,e^3}{2\,(ae^2+cd^2)^2\,a}\\
&+\frac{c^2\ln\left(cx^4+a\right)\,d^2e}{4\,(ae^2+cd^2)^2\,a^2}-\frac{5\,c^2e^2d}{4\,(ae^2+cd^2)^2\,a}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}\\
&-\frac{3\,c^3d^3}{4\,(ae^2+cd^2)^2\,a^2}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{e^5\ln\left(ex^2+d\right)}{2\,d^2\,(ae^2+cd^2)^2}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/2/a^2/d/x^2-e*ln(x)/a^2/d^2-1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a
)*x^2*e^2*d-1/4*c^3/(a*e^2+c*d^2)^2/a^2/(c*x^4+a)*x^2*d^3-1/4*c/(
a*e^2+c*d^2)^2/(c*x^4+a)*e^3-1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*
e*d^2+1/2*c/(a*e^2+c*d^2)^2/a*ln(c*x^4+a)*e^3+1/4*c^2/(a*e^2+c*d^
2)^2/a^2*ln(c*x^4+a)*d^2*e-5/4*c^2/(a*e^2+c*d^2)^2/a/(a*c)^(1/2)*
arctan(c*x^2/(a*c)^(1/2))*e^2*d-3/4*c^3/(a*e^2+c*d^2)^2/a^2/(a*c)
^(1/2)*arctan(c*x^2/(a*c)^(1/2))*d^3+1/2*e^5*ln(e*x^2+d)/d^2/(a*e
^2+c*d^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.276167, size = 464, normalized size = 1.97

$$\frac{\left(c^{2} d^{2} e + 2\,ace^{3}\right) \ln\left(cx^{4} + a\right)}{4\left(a^{2} c^{2} d^{4} + 2\,a^{3} c d^{2} e^{2} + a^{4} e^{4}\right)} + \frac{e^{6} \ln\left(\left|x^{2} e + d\right|\right)}{2\left(c^{2} d^{6} e + 2\,acd^{4} e^{3} + a^{2} d^{2} e^{5}\right)} - \frac{\left(3\,c^{3} d^{3} + 5\,ac^{2} de^{2}\right) \arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)}{4\left(a^{2} c^{2} d^{4} + 2\,a^{3} c d^{2} e^{2} + a^{4} e^{4}\right)\sqrt{ac}} - \frac{9\,c^{3} d^{5} x^{4} + 15\,ac^{2} d^{3} x^{4} e^{2} - 2\,a^{2} c x^{6} e^{5} + 3\,ac^{2} d^{4} x^{2} e + 6\,a^{2} c d x^{4} e^{4} + 6\,ac^{2} d^{5} + 3\,a^{2} c d^{2} x^{2} e^{3} + 12\,a^{2} c d^{3} e^{2} - 2\,a^{3} x^{2} e^{5} + 6\,a^{3} d e^{4}}{12\left(a^{2} c^{2} d^{6} + 2\,a^{3} c d^{4} e^{2} + a^{4} d^{2} e^{4}\right)\left(cx^{6} + ax^{2}\right)} - \frac{e \ln\left(x^{2}\right)}{2\,a^{2} d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)*x^3),x, algorithm="giac")
```

```
[Out] 1/4*(c^2*d^2*e + 2*a*c*e^3)*ln(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*
d^2*e^2 + a^4*e^4) + 1/2*e^6*ln(abs(x^2*e + d))/(c^2*d^6*e + 2*a*
c*d^4*e^3 + a^2*d^2*e^5) - 1/4*(3*c^3*d^3 + 5*a*c^2*d*e^2)*arctan
(c*x^2/sqrt(a*c))/((a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt
(a*c)) - 1/12*(9*c^3*d^5*x^4 + 15*a*c^2*d^3*x^4*e^2 - 2*a^2*c*x^6
*e^5 + 3*a*c^2*d^4*x^2*e + 6*a^2*c*d*x^4*e^4 + 6*a*c^2*d^5 + 3*a^
2*c*d^2*x^2*e^3 + 12*a^2*c*d^3*e^2 - 2*a^3*x^2*e^5 + 6*a^3*d*e^4)
/((a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + a^4*d^2*e^4)*(c*x^6 + a*x^2))
- 1/2*e*ln(x^2)/(a^2*d^2)
```

3.251 $\int \frac{1}{x^5(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=265

$$\frac{c^{3/2}e\left(2ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}\left(ae^2+cd^2\right)^2}+\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(ae^2+cd^2\right)}+\frac{c^2d\left(3ae^2+2cd^2\right)\log\left(a+cx^4\right)}{4a^3\left(ae^2+cd^2\right)^2}-\frac{\log(x)\left(2cd^2-ae^2\right)}{a^3d^3}-\frac{c^2\left(d-ex^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{e}{2a^2d^2x^2}-\frac{1}{4a^2dx^4}-\frac{e^6\log\left(d+ex^2\right)}{2d^3\left(ae^2+cd^2\right)^2}$$

```
[Out] -1/(4*a^2*d*x^4) + e/(2*a^2*d^2*x^2) - (c^2*(d - e*x^2))/(4*a^2*(
c*d^2 + a*e^2)*(a + c*x^4)) + (c^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqr
t[a]])/(4*a^(5/2)*(c*d^2 + a*e^2)) + (c^(3/2)*e*(c*d^2 + 2*a*e^2)
*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(5/2)*(c*d^2 + a*e^2)^2) - (
(2*c*d^2 - a*e^2)*Log[x])/(a^3*d^3) - (e^6*Log[d + e*x^2])/(2*d^3
*(c*d^2 + a*e^2)^2) + (c^2*d*(2*c*d^2 + 3*a*e^2)*Log[a + c*x^4])/
(4*a^3*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 0.664066, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{c^{3/2}e\left(2ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}\left(ae^2+cd^2\right)^2}+\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(ae^2+cd^2\right)}+\frac{c^2d\left(3ae^2+2cd^2\right)\log\left(a+cx^4\right)}{4a^3\left(ae^2+cd^2\right)^2}-\frac{\log(x)\left(2cd^2-ae^2\right)}{a^3d^3}-\frac{c^2\left(d-ex^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{e}{2a^2d^2x^2}-\frac{1}{4a^2dx^4}-\frac{e^6\log\left(d+ex^2\right)}{2d^3\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(d + e*x^2)*(a + c*x^4)^2),x]
```


```
[Out] -1/(4*a^2*d*x^4) + e/(2*a^2*d^2*x^2) - (c^2*(d - e*x^2))/(4*a^2*(
c*d^2 + a*e^2)*(a + c*x^4)) + (c^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqr
t[a]])/(4*a^(5/2)*(c*d^2 + a*e^2)) + (c^(3/2)*e*(c*d^2 + 2*a*e^2)
*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(5/2)*(c*d^2 + a*e^2)^2) - (
(2*c*d^2 - a*e^2)*Log[x])/(a^3*d^3) - (e^6*Log[d + e*x^2])/(2*d^3
*(c*d^2 + a*e^2)^2) + (c^2*d*(2*c*d^2 + 3*a*e^2)*Log[a + c*x^4])/
(4*a^3*(c*d^2 + a*e^2)^2)
```

Rubi in Sympy [A] time = 96.4187, size = 246, normalized size = 0.93

$$-\frac{e^6\log\left(d+ex^2\right)}{2d^3\left(ae^2+cd^2\right)^2}-\frac{c^2\left(d-ex^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{1}{4a^2dx^4}+\frac{e}{2a^2d^2x^2}+\frac{c^2d\left(3ae^2+2cd^2\right)\log\left(a+cx^4\right)}{4a^3\left(ae^2+cd^2\right)^2}+\frac{\left(ae^2-2cd^2\right)\log\left(x^2\right)}{2a^3d^3}+\frac{c^{\frac{3}{2}}e\operatorname{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}\left(ae^2+cd^2\right)}+\frac{c^{\frac{3}{2}}e\left(2ae^2+cd^2\right)\operatorname{atan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{\frac{5}{2}}\left(ae^2+cd^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] -e**6*log(d + e*x**2)/(2*d**3*(a*e**2 + c*d**2)**2) - c**2*(d - e
*x**2)/(4*a**2*(a + c*x**4)*(a*e**2 + c*d**2)) - 1/(4*a**2*d*x**4
) + e/(2*a**2*d**2*x**2) + c**2*d*(3*a*e**2 + 2*c*d**2)*log(a + c
*x**4)/(4*a**3*(a*e**2 + c*d**2)**2) + (a*e**2 - 2*c*d**2)*log(x*
*2)/(2*a**3*d**3) + c**(3/2)*e*atan(sqrt(c)*x**2/sqrt(a))/(4*a**(
5/2)*(a*e**2 + c*d**2)) + c**(3/2)*e*(2*a*e**2 + c*d**2)*atan(sqr
t(c)*x**2/sqrt(a))/(2*a**(5/2)*(a*e**2 + c*d**2)**2)
```

Mathematica [A] time = 0.80328, size = 278, normalized size = 1.05

$$\frac{1}{4}\left(-\frac{c^{3/2}e\left(5ae^2+3cd^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{5/2}\left(ae^2+cd^2\right)^2}-\frac{c^{3/2}e\left(5ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{a^{5/2}\left(ae^2+cd^2\right)^2}+\frac{c^2\left(3ade^2+2cd^3\right)\log\left(a+cx^4\right)}{a^3\left(ae^2+cd^2\right)^2}+\frac{4\log(x)\left(ae^2-2cd^2\right)}{a^3d^3}+\frac{c^2\left(ex^2-d\right)}{a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{2e}{a^2d^2x^2}-\frac{1}{a^2dx^4}-\frac{2e^6\log\left(d+ex^2\right)}{d^3\left(ae^2+cd^2\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-(1/(a^2*d*x^4)) + (2*e)/(a^2*d^2*x^2) + (c^2*(-d + e*x^2))/(a^2
*(c*d^2 + a*e^2)*(a + c*x^4)) - (c^(3/2)*e*(3*c*d^2 + 5*a*e^2)*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2
) - (c^(3/2)*e*(3*c*d^2 + 5*a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)
/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2) + (4*(-2*c*d^2 + a*e^2)*Lo
g[x])/(a^3*d^3) - (2*e^6*Log[d + e*x^2])/(d^3*(c*d^2 + a*e^2)^2)
+ (c^2*(2*c*d^3 + 3*a*d*e^2)*Log[a + c*x^4])/(a^3*(c*d^2 + a*e^2)
```

```
^2))/4
```

Maple [A] time = 0.035, size = 363, normalized size = 1.4

$$-\frac{1}{4a^2dx^4}+\frac{\ln(x)e^2}{d^3a^2}-2\frac{\ln(x)c}{a^3d}+\frac{e}{2a^2d^2x^2}+\frac{c^2x^2e^3}{4(ae^2+cd^2)^2a(cx^4+a)}+\frac{c^3x^2d^2e}{4(ae^2+cd^2)^2a^2(cx^4+a)}-\frac{c^2de^2}{4(ae^2+cd^2)^2a(cx^4+a)}-\frac{c^3d^3}{4(ae^2+cd^2)^2a^2(cx^4+a)}+\frac{3c^2\ln(cx^4+a)de^2}{4(ae^2+cd^2)^2a^2}+\frac{c^3\ln(cx^4+a)d^3}{2(ae^2+cd^2)^2a^3}+\frac{5c^2e^3}{4(ae^2+cd^2)^2a}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{3c^3d^2e}{4(ae^2+cd^2)^2a^2}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}-\frac{e^6\ln(ex^2+d)}{2d^3(ae^2+cd^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4/a^2/d/x^4+1/a^2/d^3*ln(x)*e^2-2/a^3/d*ln(x)*c+1/2*e/a^2/d^2/
x^2+1/4/(a*e^2+c*d^2)^2*c^2/a/(c*x^4+a)*x^2*e^3+1/4/(a*e^2+c*d^2)
^2*c^3/a^2/(c*x^4+a)*x^2*d^2*e-1/4/(a*e^2+c*d^2)^2*c^2/a/(c*x^4+a
)*d*e^2-1/4/(a*e^2+c*d^2)^2*c^3/a^2/(c*x^4+a)*d^3+3/4/(a*e^2+c*d^
2)^2*c^2/a^2*ln(c*x^4+a)*d*e^2+1/2/(a*e^2+c*d^2)^2*c^3/a^3*ln(c*x
^4+a)*d^3+5/4/(a*e^2+c*d^2)^2*c^2/a/(a*c)^(1/2)*arctan(c*x^2/(a*c
)^(1/2))*e^3+3/4/(a*e^2+c*d^2)^2*c^3/a^2/(a*c)^(1/2)*arctan(c*x^2
/(a*c)^(1/2))*d^2*e-1/2*e^6*ln(e*x^2+d)/d^3/(a*e^2+c*d^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)*x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.278368, size = 473, normalized size = 1.78

$$\frac{\left(2\,c^{3}d^{3}+3\,ac^{2}de^{2}\right)\ln\left(cx^{4}+a\right)}{4\left(a^{3}c^{2}d^{4}+2\,a^{4}cd^{2}e^{2}+a^{5}e^{4}\right)}-\frac{e^{7}\ln\left(\left|x^{2}e+d\right|\right)}{2\left(c^{2}d^{7}e+2\,acd^{5}e^{3}+a^{2}d^{3}e^{5}\right)}+\frac{\left(3\,c^{3}d^{2}e+5\,ac^{2}e^{3}\right)\arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)}{4\left(a^{2}c^{2}d^{4}+2\,a^{3}cd^{2}e^{2}+a^{4}e^{4}\right)\sqrt{ac}}-\frac{2\,c^{4}d^{3}x^{4}+3\,ac^{3}dx^{4}e^{2}-ac^{3}d^{2}x^{2}e+3\,ac^{3}d^{3}-a^{2}c^{2}x^{2}e^{3}+4\,a^{2}c^{2}de^{2}}{4\left(a^{3}c^{2}d^{4}+2\,a^{4}cd^{2}e^{2}+a^{5}e^{4}\right)\left(cx^{4}+a\right)}-\frac{\left(2\,cd^{2}-ae^{2}\right)\ln\left(x^{2}\right)}{2\,a^{3}d^{3}}+\frac{6\,cd^{2}x^{4}-3\,ax^{4}e^{2}+2\,adx^{2}e-ad^{2}}{4\,a^{3}d^{3}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)*x^5),x, algorithm="giac")
```

```
[Out] 1/4*(2*c^3*d^3 + 3*a*c^2*d*e^2)*ln(c*x^4 + a)/(a^3*c^2*d^4 + 2*a^
4*c*d^2*e^2 + a^5*e^4) - 1/2*e^7*ln(abs(x^2*e + d))/(c^2*d^7*e +
2*a*c*d^5*e^3 + a^2*d^3*e^5) + 1/4*(3*c^3*d^2*e + 5*a*c^2*e^3)*ar
ctan(c*x^2/sqrt(a*c))/((a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*
sqrt(a*c)) - 1/4*(2*c^4*d^3*x^4 + 3*a*c^3*d*x^4*e^2 - a*c^3*d^2*x
^2*e + 3*a*c^3*d^3 - a^2*c^2*x^2*e^3 + 4*a^2*c^2*d*e^2)/((a^3*c^2
*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*(c*x^4 + a)) - 1/2*(2*c*d^2 - a
*e^2)*ln(x^2)/(a^3*d^3) + 1/4*(6*c*d^2*x^4 - 3*a*x^4*e^2 + 2*a*d*
x^2*e - a*d^2)/(a^3*d^3*x^4)
```

3.252 $\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=712

$$\frac{\sqrt[4]{a}d^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{a}d^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{a}\left(3\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{a}\left(3\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{a}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{a}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{a}\left(\sqrt{c}d-3\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{c}d-3\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}-\frac{x^3\left(ae+cdx^2\right)}{4c\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{dx}{4c\left(ae^2+cd^2\right)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(ae^2+cd^2\right)^2}$$

```
[Out] (d*x)/(4*c*(c*d^2 + a*e^2)) - (x^3*(a*e + c*d*x^2))/(4*c*(c*d^2 +
 a*e^2)*(a + c*x^4)) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqr
t[e]*(c*d^2 + a*e^2)^2) + (a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2
+ a*e^2)^2) + (a^(1/4)*(Sqrt[c]*d - 3*Sqrt[a]*e)*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - (a
^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)
/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) - (a^(1/4)*(Sqrt
[c]*d - 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*
Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) + (a^(1/4)*d^2*(Sqrt[c]*d + Sqrt
[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4
*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) + (a^(1/4)*(Sqrt[c]*d + 3*Sqr
t[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
16*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - (a^(1/4)*d^2*(Sqrt[c]*d + S
qrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) - (a^(1/4)*(Sqrt[c]*d + 3*
Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
)/(16*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 1.38015, antiderivative size = 712, normalized size of antiderivative = 1., number of

steps used = 24, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{\sqrt[4]{a}d^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)^2} - \frac{\sqrt[4]{a}d^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)^2} + \frac{\sqrt[4]{a}\left(3\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)} - \frac{\sqrt[4]{a}\left(3\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)} + \frac{\sqrt[4]{a}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)^2} - \frac{\sqrt[4]{a}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)^2} + \frac{\sqrt[4]{a}\left(\sqrt{c}d-3\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)} - \frac{\sqrt[4]{a}\left(\sqrt{c}d-3\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)} - \frac{x^3\left(ae+cdx^2\right)}{4c\left(a+cx^4\right)\left(ae^2+cd^2\right)} + \frac{dx}{4c\left(ae^2+cd^2\right)} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (d*x)/(4*c*(c*d^2 + a*e^2)) - (x^3*(a*e + c*d*x^2))/(4*c*(c*d^2 +
 a*e^2)*(a + c*x^4)) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqr
t[e]*(c*d^2 + a*e^2)^2) + (a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2
+ a*e^2)^2) + (a^(1/4)*(Sqrt[c]*d - 3*Sqrt[a]*e)*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - (a
^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)
/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) - (a^(1/4)*(Sqrt
[c]*d - 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*
Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) + (a^(1/4)*d^2*(Sqrt[c]*d + Sqrt
[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4
*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) + (a^(1/4)*(Sqrt[c]*d + 3*Sqr
t[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
16*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - (a^(1/4)*d^2*(Sqrt[c]*d + S
qrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) - (a^(1/4)*(Sqrt[c]*d + 3*
Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
)/(16*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.535359, size = 431, normalized size = 0.61

$$\frac{\frac{\sqrt{2}\sqrt[4]{a}\left(3a^{3/2}e^3+7\sqrt{a}cd^2e+a\sqrt{c}de^2+5c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{7/4}}-\frac{\sqrt{2}\sqrt[4]{a}\left(3a^{3/2}e^3+7\sqrt{a}cd^2e+a\sqrt{c}de^2+5c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{7/4}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*a*(c*d^2 + a*e^2)*x*(d - e*x^2))/(c*(a + c*x^4)) + (32*d^(7/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*Sqrt[2]*a^(1/4)*(-5*c
^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*e - a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(7/4) + (2*Sqrt[2]*a^
(1/4)*(-5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*e - a*Sqrt[c]*d*e^2 + 3*a
^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(7/4) + (S
qrt[2]*a^(1/4)*(5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e
^2 + 3*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/c^(7/4) - (Sqrt[2]*a^(1/4)*(5*c^(3/2)*d^3 + 7*Sqrt[a]*
c*d^2*e + a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A] time = 0.021, size = 873, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4*a^2/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3/c*x^3-1/4*a/(a*e^2+c*d^2)^
2/(c*x^4+a)*x^3*d^2*e+1/4*a^2/(a*e^2+c*d^2)^2/(c*x^4+a)*d/c*x*e^2
```

```
+1/4*a/(a*e^2+c*d^2)^2/(c*x^4+a)*d^3*x-1/16*a/(a*e^2+c*d^2)^2*(1/
c*a)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d*e^2-5/16
/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/
4)*x-1)*d^3-1/32*a/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)/c*2^(1/2)*ln((x^
2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(
1/2)+(1/c*a)^(1/2)))*d*e^2-5/32/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)*2^(
1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^
(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^3-1/16*a/(a*e^2+c*d^2)^2*(1/c*a
)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d*e^2-5/16/(a
*e^2+c*d^2)^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*
x+1)*d^3+3/32*a^2/(a*e^2+c*d^2)^2/c^2/(1/c*a)^(1/4)*2^(1/2)*ln((x
^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^
(1/2)+(1/c*a)^(1/2)))*e^3+7/32*a/(a*e^2+c*d^2)^2/c/(1/c*a)^(1/4)*
2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*
a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^2*e+3/16*a^2/(a*e^2+c*d^2)^2
/c^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*e^3+
7/16*a/(a*e^2+c*d^2)^2/c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
c*a)^(1/4)*x-1)*d^2*e+3/16*a^2/(a*e^2+c*d^2)^2/c^2/(1/c*a)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*e^3+7/16*a/(a*e^2+c*d^2
)^2/c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^2
*e+d^4/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 15.9624, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(a*c*d^2*e + a^2*e^3)*x^3 - (a*c^3*d^4 + 2*a^2*c^2*d^2*
e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*
sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8
+ 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c
^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c
```

^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^
6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a
^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 5
6*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 +
a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 +
4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log(-(625*c^4*d^8 - 750*a*c^3*d
^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x
 + (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^
3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 + (7*c^10*d^10*e + 31*a*c^9*d^8*e
^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9
 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2
- 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e
^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14
*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*
d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*
d^2*e^14 + a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^
3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4
+ 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a
^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 23
83*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16
 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^
6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e
^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6
*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) + (
a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^
2*e^2 + a^2*c^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3
 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 +
 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^
2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 238
3*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16
+ 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6
 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^
12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*
e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log(-
(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3
*c*d^2*e^6 - 81*a^4*e^8)*x - (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 2
44*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 + (7*c^
10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^
4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6*d^1
2 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^
6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/
(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^
12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6
*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((70*a*c
^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^
6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt
(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 +
 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10
 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12
*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d
^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16
)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^
2*e^6 + a^4*c^3*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e
^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt((70*a*c^

```
2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6
*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(
-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 +
2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10
+ 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*
e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^
6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)
))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2
*e^6 + a^4*c^3*e^8))*log(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376
*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (125*c^6*d
^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6
 - 9*a^4*c^2*d*e^8 - (7*c^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c
^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*
e^11)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4
*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*
c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2
*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*
a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^
8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e
^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d
^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e
^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^
4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d
^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^
11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c
^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*
c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) + (a*c^3*d^4 + 2
*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c
^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^
5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^
2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^
2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4
*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^
14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^1
1*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^
8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c
^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log(-(625*c^4*d^8
- 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 -
81*a^4*e^8)*x - (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^
5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 - (7*c^10*d^10*e + 3
1*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^
4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*
c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*
a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 +
8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 +
 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12
 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e + 44
*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2
*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*
d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3
*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^1
2)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3
*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*
a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8
```

```
+ 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c
^3*e^8))) - 8*(c^2*d^3*x^4 + a*c*d^3)*sqrt(-d/e)*log((e*x^2 + 2*e
*x*sqrt(-d/e) - d)/(e*x^2 + d)) - 4*(a*c*d^3 + a^2*d*e^2)*x)/(a*c
^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e
^2 + a^2*c^2*e^4)*x^4), -1/16*(4*(a*c*d^2*e + a^2*e^3)*x^3 - 16*(
c^2*d^3*x^4 + a*c*d^3)*sqrt(d/e)*arctan(x/sqrt(d/e)) - (a*c^3*d^4
 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a
^2*c^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*
d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^
4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^1
0*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2
*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^1
4*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4
*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^
7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a
^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log(-(625*c^4*
d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^
6 - 81*a^4*e^8)*x + (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^
4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 + (7*c^10*d^10*e
 + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 1
9*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6*d^12 - 1950*
a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2
383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^1
6 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e
^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*
e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e
+ 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6
*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*
c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4
*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7
*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56
*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 +
 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*
d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a
^4*c^3*e^8))) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4
*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e +
 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*
a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c
^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*
c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*
e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*
a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 +
28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d
^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^
4*c^3*e^8))*log(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*
d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (125*c^6*d^9 - 170*
a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*
c^2*d*e^8 + (7*c^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^
5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqr
t(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4
+ 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^1
0 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^1
2*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*
d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^1
```

6)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7
*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 +
a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*
a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 7
38*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 +
28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^
8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^
14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e
^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*
d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x
^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*
d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a
^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a
^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 73
8*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 +
28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8
 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^1
4 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^
4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log(-(625*c^4*d^8 - 750*a*c
^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^
8)*x + (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 8
6*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 - (7*c^10*d^10*e + 31*a*c^9*d
^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2
*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*
e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d
^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*
d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c
^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*
c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^
3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*
e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 19
50*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6
+ 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*
d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^1
0*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d
^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6
*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))
 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^
3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3
*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e
^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 195
0*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 +
 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d
^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10
*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^
4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*
d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*l
og(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594
*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (125*c^6*d^9 - 170*a*c^5*d^7*e^2
 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 - (
7*c^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^
7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6
*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^
3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^

```
12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^
3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28
*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((70
*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^
6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*
sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e
^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*
e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*
d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^
10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*
e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^
4*d^2*e^6 + a^4*c^3*e^8))) - 4*(a*c*d^3 + a^2*d*e^2)*x)/(a*c^3*d^
4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 +
a^2*c^2*e^4)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.287558, size = 784, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] d^(7/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c^2*d^4 + 2*a*c*d^2*e
^2 + a^2*e^4) - 1/8*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^
2*d*e^2 - 7*(a*c^3)^(3/4)*c*d^2*e - 3*(a*c^3)^(3/4)*a*e^3)*arctan
(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^
6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2 + sqrt(2)*a^2*c^4*e^4) - 1/8*(5*(
a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 - 7*(a*c^3)^(3/4
)*c*d^2*e - 3*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*
d^2*e^2 + sqrt(2)*a^2*c^4*e^4) - 1/16*(5*(a*c^3)^(1/4)*c^3*d^3 +
(a*c^3)^(1/4)*a*c^2*d*e^2 + 7*(a*c^3)^(3/4)*c*d^2*e + 3*(a*c^3)^(
3/4)*a*e^3)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*
c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2 + sqrt(2)*a^2*c^4*e^4) + 1/16*(
```

```
5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 + 7*(a*c^3)^(
3/4)*c*d^2*e + 3*(a*c^3)^(3/4)*a*e^3)*ln(x^2 - sqrt(2)*x*(a/c)^(1
/4) + sqrt(a/c))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2 + sqr
t(2)*a^2*c^4*e^4) - 1/4*(a*x^3*e - a*d*x)/((c*x^4 + a)*(c^2*d^2 +
 a*c*e^2))
```

3.253 $\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=687

$$-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}\left(ae^2+cd^2\right)}$$
$$+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}c^{5/4}\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\sqrt[4]{a}c^{5/4}\left(ae^2+cd^2\right)}$$
$$+\frac{d^2\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)^2}$$
$$-\frac{d^2\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)^2}$$
$$-\frac{x\left(ae+cdx^2\right)}{4c\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{d^2\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)^2}$$
$$+\frac{d^2\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)^2}-\frac{d^{5/2}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(ae^2+cd^2\right)^2}$$

```
[Out] -(x*(a*e + c*d*x^2))/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) - (d^(5/2)
*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 - (d^2*(S
qrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2
*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)^2) + ((Sqrt[c]*d - Sqrt[
a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(1/4)
*c^(5/4)*(c*d^2 + a*e^2)) + (d^2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^
2 + a*e^2)^2) - ((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2)) + (d
^2*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*
x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)^2) -
 ((Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2)) - (
d^2*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)^2)
+ ((Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*
x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 1.19639, antiderivative size = 687, normalized size of antiderivative = 1., number of

steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$

$$-\frac{(\sqrt{a}e+\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}(ae^2+cd^2)}+\frac{(\sqrt{a}e+\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}(ae^2+cd^2)}+\frac{(\sqrt{c}d-\sqrt{a}e)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}c^{5/4}(ae^2+cd^2)}-\frac{(\sqrt{c}d-\sqrt{a}e)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\sqrt[4]{a}c^{5/4}(ae^2+cd^2)}+\frac{d^2(\sqrt{c}d-\sqrt{a}e)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(ae^2+cd^2)^2}-\frac{d^2(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(ae^2+cd^2)^2}-\frac{x(ae+cdx^2)}{4c(a+cx^4)(ae^2+cd^2)}-\frac{d^2(\sqrt{a}e+\sqrt{c}d)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(ae^2+cd^2)^2}+\frac{d^2(\sqrt{a}e+\sqrt{c}d)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(ae^2+cd^2)^2}-\frac{d^{5/2}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(x*(a*e + c*d*x^2))/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) - (d^(5/2)
*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 - (d^2*(S
qrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2
*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)^2) + ((Sqrt[c]*d - Sqrt[
a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(1/4)
*c^(5/4)*(c*d^2 + a*e^2)) + (d^2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^
2 + a*e^2)^2) - ((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2)) + (d
^2*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*
x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)^2) -
 ((Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2)) - (
d^2*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)^2)
+ ((Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*
x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2))
```

Rubi in Sympy [A] time = 161.664, size = 631, normalized size = 0.92

$$-\frac{d^{\frac{5}{2}}\sqrt{e}\,\mathrm{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(ae^2+cd^2)^2}-\frac{x\left(ae+cdx^2\right)}{4c\left(a+cx^4\right)(ae^2+cd^2)} - \frac{\sqrt{2}d^2\left(\sqrt{a}e-\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8\sqrt[4]{a}\sqrt[4]{c}\,(ae^2+cd^2)^2} + \frac{\sqrt{2}d^2\left(\sqrt{a}e-\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8\sqrt[4]{a}\sqrt[4]{c}\,(ae^2+cd^2)^2}-\frac{\sqrt{2}d^2\left(\sqrt{a}e+\sqrt{c}d\right)\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\,(ae^2+cd^2)^2} + \frac{\sqrt{2}d^2\left(\sqrt{a}e+\sqrt{c}d\right)\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\,(ae^2+cd^2)^2}-\frac{\sqrt{2}\left(\sqrt{a}e-\sqrt{c}d\right)\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16\sqrt[4]{a}c^{\frac{5}{4}}\,(ae^2+cd^2)} + \frac{\sqrt{2}\left(\sqrt{a}e-\sqrt{c}d\right)\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16\sqrt[4]{a}c^{\frac{5}{4}}\,(ae^2+cd^2)}-\frac{\sqrt{2}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{32\sqrt[4]{a}c^{\frac{5}{4}}\,(ae^2+cd^2)} + \frac{\sqrt{2}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{32\sqrt[4]{a}c^{\frac{5}{4}}\,(ae^2+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] -d**(5/2)*sqrt(e)*atan(sqrt(e)*x/sqrt(d))/(a*e**2 + c*d**2)**2 -
x*(a*e + c*d*x**2)/(4*c*(a + c*x**4)*(a*e**2 + c*d**2)) - sqrt(2)
*d**2*(sqrt(a)*e - sqrt(c)*d)*log(-sqrt(2)*a**(1/4)*c**(3/4)*x +
sqrt(a)*sqrt(c) + c*x**2)/(8*a**(1/4)*c**(1/4)*(a*e**2 + c*d**2)*
*2) + sqrt(2)*d**2*(sqrt(a)*e - sqrt(c)*d)*log(sqrt(2)*a**(1/4)*c
**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(8*a**(1/4)*c**(1/4)*(a*e**
2 + c*d**2)**2) - sqrt(2)*d**2*(sqrt(a)*e + sqrt(c)*d)*atan(1 - s
qrt(2)*c**(1/4)*x/a**(1/4))/(4*a**(1/4)*c**(1/4)*(a*e**2 + c*d**2
)**2) + sqrt(2)*d**2*(sqrt(a)*e + sqrt(c)*d)*atan(1 + sqrt(2)*c**
(1/4)*x/a**(1/4))/(4*a**(1/4)*c**(1/4)*(a*e**2 + c*d**2)**2) - sq
rt(2)*(sqrt(a)*e - sqrt(c)*d)*atan(1 - sqrt(2)*c**(1/4)*x/a**(1/4
))/(16*a**(1/4)*c**(5/4)*(a*e**2 + c*d**2)) + sqrt(2)*(sqrt(a)*e
- sqrt(c)*d)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/(16*a**(1/4)*c
**(5/4)*(a*e**2 + c*d**2)) - sqrt(2)*(sqrt(a)*e + sqrt(c)*d)*log(
-sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(32*a**(
1/4)*c**(5/4)*(a*e**2 + c*d**2)) + sqrt(2)*(sqrt(a)*e + sqrt(c)*d
)*log(sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(32
*a**(1/4)*c**(5/4)*(a*e**2 + c*d**2))
```

Mathematica [A] time = 0.628598, size = 428, normalized size = 0.62

$$-\frac{\frac{\sqrt{2}\left(a^{3/2}e^3+5\sqrt{a}cd^2e+a\sqrt{c}de^2-3c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{a}c^{5/4}}-\frac{\sqrt{2}\left(a^{3/2}e^3+5\sqrt{a}cd^2e+a\sqrt{c}de^2-3c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{a}c^{5/4}}+\frac{2\sqrt{2}(\cdots}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -((8*(c*d^2 + a*e^2)*(a*e*x + c*d*x^3))/(c*(a + c*x^4)) + 32*d^(5
/2)*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (2*Sqrt[2]*(3*c^(3/2)*d
^3 + 5*Sqrt[a]*c*d^2*e - a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1
- (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(1/4)*c^(5/4)) - (2*Sqrt[2]*(3
*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e - a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(1/4)*c^(5/4)) + (Sq
rt[2]*(-3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 + a^(
3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(a^(1/4)*c^(5/4)) - (Sqrt[2]*(-3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e
 + a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(a^(1/4)*c^(5/4)))/(32*(c*d^2 + a*e^2)^2
)
```

Maple [A] time = 0.02, size = 852, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^3*e^2*d*a-1/4/(a*e^2+c*d^2)^2/(c
*x^4+a)*x^3*c*d^3-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a^2*e^3/c*x-1/4/(
a*e^2+c*d^2)^2/(c*x^4+a)*e*a*d^2*x+1/16/(a*e^2+c*d^2)^2*(1/c*a)^(
1/4)*a/c*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*e^3+5/16/(a*e^
2+c*d^2)^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1
)*d^2*e+1/16/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)*a/c*2^(1/2)*arctan(2^(
1/2)/(1/c*a)^(1/4)*x-1)*e^3+5/16/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^2*e+1/32/(a*e^2+c*d^2)^
2*(1/c*a)^(1/4)*a/c*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*
a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*e^3+5/32/(
a*e^2+c*d^2)^2*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1
/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d
^2*e-1/32/(a*e^2+c*d^2)^2/c/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)
^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c
*a)^(1/2)))*a*d*e^2+3/32/(a*e^2+c*d^2)^2/(1/c*a)^(1/4)*2^(1/2)*ln
```

```
((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x
*2^(1/2)+(1/c*a)^(1/2)))*d^3-1/16/(a*e^2+c*d^2)^2/c/(1/c*a)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*a*d*e^2+3/16/(a*e^2+c*
d^2)^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^
3-1/16/(a*e^2+c*d^2)^2/c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
c*a)^(1/4)*x-1)*a*d*e^2+3/16/(a*e^2+c*d^2)^2/(1/c*a)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^3-e/(a*e^2+c*d^2)^2*d^3/(d*
e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 11.3628, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(c^2*d^3 + a*c*d*e^2)*x^3 + (a*c^3*d^4 + 2*a^2*c^2*d^2*
e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*
sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*
a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e
^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4
 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10
+ a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12
*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6
*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16))
)/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*
e^6 + a^4*c^2*e^8))*log(-(81*c^4*d^8 - 270*a*c^3*d^6*e^2 - 112*a^
2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*x + (45*a*c^5*d^8*e -
 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^2*e^7 -
a^5*c*e^9 - (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e
^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-
(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3
*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12
)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*
```

a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28
*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))*sqrt(-(3
0*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^
6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt
(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a
^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^
12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 5
6*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 +
28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d
^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^
4*c^2*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*
d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4
*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c
^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12
- 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6
+ 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^1
6 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10
*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*
e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^
6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log
(-(81*c^4*d^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*
c*d^2*e^6 - a^4*e^8)*x - (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 -
76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^2*e^7 - a^5*c*e^9 - (3*a*c^9*d^
11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4 + 6*a^4*c^6*d^5*e^6
- a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d^12 - 558*a*c^
5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*
c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*
c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*
a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a
^8*c^6*d^2*e^14 + a^9*c^5*e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3
*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e
^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*
c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^
4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^
2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 7
0*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8
*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 +
6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))) + (a*c^3*d
^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 +
a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*
e^5 - (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*
d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 +
799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 +
18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2
+ 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e
^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^
14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e
^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^4*d^8 - 270*a*c
^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*x
+ (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14
*a^4*c^2*d^2*e^7 - a^5*c*e^9 + (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2
+ 14*a^3*c^7*d^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6
*c^4*d*e^10)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^
4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*

```
d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*
c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^
6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c
^5*e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c
^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6
+ a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*
c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*
c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^
3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*
a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9
*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a
^3*c^3*d^2*e^6 + a^4*c^2*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2
+ a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt
(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^
5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*
sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 5
40*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^
6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4
 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^1
0 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c
^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6
+ a^4*c^2*e^8))*log(-(81*c^4*d^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^
2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*x - (45*a*c^5*d^8*e - 146
*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^2*e^7 - a^5*
c*e^9 + (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4 +
 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*
c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3
*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a
*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*
c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7
*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))*sqrt(-(30*c^
2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e^
2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(8
1*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c
^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/
(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^
4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a
^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 +
 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^
2*e^8))) - 8*(c^2*d^2*x^4 + a*c*d^2)*sqrt(-d*e)*log((e*x^2 - 2*sq
rt(-d*e)*x - d)/(e*x^2 + d)) + 4*(a*c*d^2*e + a^2*e^3)*x)/(a*c^3*
d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2
+ a^2*c^2*e^4)*x^4), -1/16*(4*(c^2*d^3 + a*c*d*e^2)*x^3 + 16*(c^2
*d^2*x^4 + a*c*d^2)*sqrt(d*e)*arctan(e*x/sqrt(d*e)) + (a*c^3*d^4
+ 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^
2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5
 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2
*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799
*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18
*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 +
28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8
+ 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14
+ a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4
+ 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^4*d^8 - 270*a*c^3*
```

d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*x + (
45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^
4*c^2*d^2*e^7 - a^5*c*e^9 - (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 +
14*a^3*c^7*d^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^
4*d*e^10)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d
^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2
*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^1
1*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c
^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*
e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*
d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a
^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4
*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d
^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c
^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6
*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^
5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*
c^3*d^2*e^6 + a^4*c^2*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a
^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(
30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d
^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqr
t(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*
a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e
^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 +
56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 +
 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*
d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a
^4*c^2*e^8))*log(-(81*c^4*d^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d
^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*x - (45*a*c^5*d^8*e - 146*a^
2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^2*e^7 - a^5*c*e
^9 - (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4 + 6*
a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6
*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^
6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^
13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^1
0*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^
7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))*sqrt(-(30*c^2*d
^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 +
 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c
^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*
d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*
c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c
^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*
c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*
a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e
^8))) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2
*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*c*d^
3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*
e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a
*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a
^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a
^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 +
70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 +
8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 +

```
 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c
^4*d^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e
^6 - a^4*e^8)*x + (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*
c^3*d^4*e^5 - 14*a^4*c^2*d^2*e^7 - a^5*c*e^9 + (3*a*c^9*d^11 + 11
*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c
^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*
e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4
*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^
14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9
*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*
d^2*e^14 + a^9*c^5*e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 -
2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*
a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^1
0*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d
^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*
d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c
^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^
6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c
^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))) - (a*c^3*d^4 + 2*
a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^
2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (
c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6
 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2
*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5
*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a
^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56
*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^
9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*
a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^4*d^8 - 270*a*c^3*d^6*
e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*x - (45*a
*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^
2*d^2*e^7 - a^5*c*e^9 + (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a
^3*c^7*d^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*
e^10)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e
^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^1
0 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^
12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d
^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16
)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8
+ 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c
^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8
*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e
^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*
d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8
*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^
16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*
d^2*e^6 + a^4*c^2*e^8))) + 4*(a*c*d^2*e + a^2*e^3)*x)/(a*c^3*d^4
+ 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^
2*c^2*e^4)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.287294, size = 803, normalized size = 1.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] -d^(5/2)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/(c^2*d^4 + 2*a*c*d^2*e
^2 + a^2*e^4) + 1/8*(5*(a*c^3)^(1/4)*a*c^2*d^2*e + 3*(a*c^3)^(3/4
)*c*d^3 + (a*c^3)^(1/4)*a^2*c*e^3 - (a*c^3)^(3/4)*a*d*e^2)*arctan
(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*
c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) + 1/8*
(5*(a*c^3)^(1/4)*a*c^2*d^2*e + 3*(a*c^3)^(3/4)*c*d^3 + (a*c^3)^(1
/4)*a^2*c*e^3 - (a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*
a^2*c^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) + 1/16*(5*(a*c^3)^(1/4)*a*
c^2*d^2*e - 3*(a*c^3)^(3/4)*c*d^3 + (a*c^3)^(1/4)*a^2*c*e^3 + (a*
c^3)^(3/4)*a*d*e^2)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*e^2 + sqrt(2)*a^3*c^3*e
^4) - 1/16*(5*(a*c^3)^(1/4)*a*c^2*d^2*e - 3*(a*c^3)^(3/4)*c*d^3 +
 (a*c^3)^(1/4)*a^2*c*e^3 + (a*c^3)^(3/4)*a*d*e^2)*ln(x^2 - sqrt(2
)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c
^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) - 1/4*(c*d*x^3 + a*x*e)/((c*x^4
 + a)*(c^2*d^2 + a*c*e^2))
```

3.254 $\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=685

$$\frac{\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}+\frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}d^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}d^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{x\left(d-ex^2\right)}{4\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(ae^2+cd^2\right)^2}$$

```
[Out] -(x*(d - e*x^2))/(4*(c*d^2 + a*e^2)*(a + c*x^4)) + (d^(3/2)*e^(3/
2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 - (c^(1/4)*d^2*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + ((3*Sqrt[c]*d - Sqrt[a]*e
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(3/4)*c^(
3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 +
a*e^2)^2) - ((3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(8*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2)) - (c^(
1/4)*d^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^
(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (
(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2)) + (
c^(1/4)*d^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2)
- ((3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 1.14968, antiderivative size = 685, normalized size of antiderivative = 1., number of

steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$

$$\frac{\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}+\frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}d^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}d^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{x\left(d-ex^2\right)}{4\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(x*(d - e*x^2))/(4*(c*d^2 + a*e^2)*(a + c*x^4)) + (d^(3/2)*e^(3/
2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 - (c^(1/4)*d^2*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + ((3*Sqrt[c]*d - Sqrt[a]*e
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(3/4)*c^(
3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 +
a*e^2)^2) - ((3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(8*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2)) - (c^(
1/4)*d^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^
(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (
(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2)) + (
c^(1/4)*d^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2)
- ((3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2))
```

Rubi in Sympy [A] time = 163.398, size = 632, normalized size = 0.92

$$\frac{d^{\frac{3}{2}}e^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(ae^2+cd^2)^2}-\frac{x(d-ex^2)}{4(a+cx^4)(ae^2+cd^2)}+\frac{\sqrt{2}\sqrt[4]{c}d^2\left(\sqrt{a}e-\sqrt{c}d\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}(ae^2+cd^2)^2}-\frac{\sqrt{2}\sqrt[4]{c}d^2\left(\sqrt{a}e-\sqrt{c}d\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}(ae^2+cd^2)^2}-\frac{\sqrt{2}\sqrt[4]{c}d^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}(ae^2+cd^2)^2}+\frac{\sqrt{2}\sqrt[4]{c}d^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}(ae^2+cd^2)^2}-\frac{\sqrt{2}\left(\sqrt{a}e-3\sqrt{c}d\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{3}{4}}c^{\frac{3}{4}}(ae^2+cd^2)}+\frac{\sqrt{2}\left(\sqrt{a}e-3\sqrt{c}d\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{3}{4}}c^{\frac{3}{4}}(ae^2+cd^2)}+\frac{\sqrt{2}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{32a^{\frac{3}{4}}c^{\frac{3}{4}}(ae^2+cd^2)}-\frac{\sqrt{2}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{32a^{\frac{3}{4}}c^{\frac{3}{4}}(ae^2+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] d**(3/2)*e**(3/2)*atan(sqrt(e)*x/sqrt(d))/(a*e**2 + c*d**2)**2 -
x*(d - e*x**2)/(4*(a + c*x**4)*(a*e**2 + c*d**2)) + sqrt(2)*c**(1
/4)*d**2*(sqrt(a)*e - sqrt(c)*d)*atan(1 - sqrt(2)*c**(1/4)*x/a**(
1/4))/(4*a**(3/4)*(a*e**2 + c*d**2)**2) - sqrt(2)*c**(1/4)*d**2*(
sqrt(a)*e - sqrt(c)*d)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/(4*a
**(3/4)*(a*e**2 + c*d**2)**2) - sqrt(2)*c**(1/4)*d**2*(sqrt(a)*e
+ sqrt(c)*d)*log(-sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) +
 c*x**2)/(8*a**(3/4)*(a*e**2 + c*d**2)**2) + sqrt(2)*c**(1/4)*d**
2*(sqrt(a)*e + sqrt(c)*d)*log(sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(
a)*sqrt(c) + c*x**2)/(8*a**(3/4)*(a*e**2 + c*d**2)**2) - sqrt(2)*
(sqrt(a)*e - 3*sqrt(c)*d)*atan(1 - sqrt(2)*c**(1/4)*x/a**(1/4))/(
16*a**(3/4)*c**(3/4)*(a*e**2 + c*d**2)) + sqrt(2)*(sqrt(a)*e - 3*
sqrt(c)*d)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/(16*a**(3/4)*c**
(3/4)*(a*e**2 + c*d**2)) + sqrt(2)*(sqrt(a)*e + 3*sqrt(c)*d)*log(
-sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(32*a**(
3/4)*c**(3/4)*(a*e**2 + c*d**2)) - sqrt(2)*(sqrt(a)*e + 3*sqrt(c)
*d)*log(sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*x**2)/(
32*a**(3/4)*c**(3/4)*(a*e**2 + c*d**2))
```

Mathematica [A] time = 0.487336, size = 423, normalized size = 0.62

$$\frac{\frac{\sqrt{2}\left(a^{3/2}e^3-3\sqrt{a}cd^2e+3a\sqrt{c}de^2-c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{3/4}c^{3/4}}+\frac{\sqrt{2}\left(-a^{3/2}e^3+3\sqrt{a}cd^2e-3a\sqrt{c}de^2+c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{3/4}c^{3/4}}-\frac{2\sqrt{2}(\ldots}{}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*(c*d^2 + a*e^2)*(-(d*x) + e*x^3))/(a + c*x^4) + 32*d^(3/2)*e^
(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - (2*Sqrt[2]*(c^(3/2)*d^3 - 3*S
qrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*x)/a^(1/4)])/(a^(3/4)*c^(3/4)) + (2*Sqrt[2]*(c^(3/2)
*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTa
n[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(3/4)*c^(3/4)) + (Sqrt[2]*
(-(c^(3/2)*d^3) - 3*Sqrt[a]*c*d^2*e + 3*a*Sqrt[c]*d*e^2 + a^(3/2)
*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^
(3/4)*c^(3/4)) + (Sqrt[2]*(c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e - 3*a*
Sqrt[c]*d*e^2 - a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(a^(3/4)*c^(3/4)))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A] time = 0.019, size = 848, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^3*a*e^3+1/4/(a*e^2+c*d^2)^2/(c*x^
4+a)*x^3*c*d^2*e-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*e^2*d*a*x-1/4/(a*e
^2+c*d^2)^2/(c*x^4+a)*d^3*c*x-3/16/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d*e^2+1/16/(a*e^2+c*d^2
)^2*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*c*d
^3-3/32/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/
4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^
(1/2)))*d*e^2+1/32/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)/a*2^(1/2)*ln((x^
2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(
1/2)+(1/c*a)^(1/2)))*c*d^3-3/16/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d*e^2+1/16/(a*e^2+c*d^2)^2
*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*c*d^3+
1/32/(a*e^2+c*d^2)^2/c/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4
)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(
1/2)))*a*e^3-3/32/(a*e^2+c*d^2)^2/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(
1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2
```

```
)+(1/c*a)^(1/2)))*d^2*e+1/16/(a*e^2+c*d^2)^2/c/(1/c*a)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*a*e^3-3/16/(a*e^2+c*d^2)^2/
(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^2*e+1/1
6/(a*e^2+c*d^2)^2/c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^
(1/4)*x+1)*a*e^3-3/16/(a*e^2+c*d^2)^2/(1/c*a)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^2*e+d^2*e^2/(a*e^2+c*d^2)^2/(d*e)^
(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 9.19886, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(c*d^2*e + a*e^3)*x^3 - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a
^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*sqrt((6*c^2
*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^
6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-
(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3
*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a
^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6
*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9
*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8
 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^
5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d^2*e^6 - a
^4*e^8)*x + (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4
- 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 + (3*a^3*c^7*d^10*e + 11*a^4
*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d
^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*
a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*
a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 +
 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 +
 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14
```

```
+ a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e
^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c
^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255
*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30
*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2
+ 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8
+ 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14
 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^
4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) + (a*c^2*d^4 + 2*a^2*c*d
^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*s
qrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*
a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e
^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 4
52*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^
6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^
4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^1
0 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/
(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^
2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d
^2*e^6 - a^4*e^8)*x - (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a^3*c^
3*d^5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 + (3*a^3*c^7*d^10*
e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 -
 a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*
e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4
*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*
d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7
*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4
*d^2*e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 +
 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10
*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^
4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10
*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^
7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^
4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*
a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) - (a*c^2*d^4 +
 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e
^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - (a*c^
5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6
 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d
^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2
*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c
^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c
^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^
3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*
a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*d^6*e^2 +
14*a^3*c*d^2*e^6 - a^4*e^8)*x + (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 +
 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 - (3*a^3
*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4
*d^4*e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 - 30*a
*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a
^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8
*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 +
70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 +
```

```
8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c
*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c
^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*
a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*
a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 +
8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 +
 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 +
 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^
6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) + (a
*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2
 + a^2*c*e^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e
^5 - (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c
^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255
*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30
*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2
+ 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8
+ 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14
 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^
4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*
d^6*e^2 + 14*a^3*c*d^2*e^6 - a^4*e^8)*x - (a*c^5*d^9 - 18*a^2*c^4
*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^
8 - (3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 +
 6*a^6*c^4*d^4*e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d
^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e
^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^1
1*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d
^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d
^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*
e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2
 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*
d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*
e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^
11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*
d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*
d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*
a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e
^8))) + 8*(c*d*e*x^4 + a*d*e)*sqrt(-d*e)*log((e*x^2 + 2*sqrt(-d*e
)*x - d)/(e*x^2 + d)) - 4*(c*d^3 + a*d*e^2)*x)/(a*c^2*d^4 + 2*a^2
*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^
4), 1/16*(4*(c*d^2*e + a*e^3)*x^3 + 16*(c*d*e*x^4 + a*d*e)*sqrt(d
*e)*arctan(e*x/sqrt(d*e)) - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^
4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*sqrt((6*c^2*d^5*
e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2
 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*
d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*
e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^
11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*
d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*
d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*
a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e
^8))*log(-(c^4*d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d^2*e^6 - a^4*e^
8)*x + (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*
a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 + (3*a^3*c^7*d^10*e + 11*a^4*c^6*
d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d^2*e^
```

9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c
^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c
*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a
^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a
^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^1
1*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 +
(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^
2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*
c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*
c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*
a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*
a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^
11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) + (a*c^2*d^4 + 2*a^2*c*d^2*e^
2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*sqrt((
6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c
^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*s
qrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^
3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^1
2)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 5
6*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 2
8*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^
5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6
 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d^2*e^
6 - a^4*e^8)*x - (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a^3*c^3*d^5
*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 + (3*a^3*c^7*d^10*e + 1
1*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*
c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 +
 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8
- 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*
e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*
e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*
e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^
2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*
a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2
+ 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8
 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14
*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8
*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2
*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c
^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) - (a*c^2*d^4 + 2*a^
2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x
^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8
 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^
5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^
4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10
 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^
12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^
6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^1
6)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c
^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*d^6*e^2 + 14*a^
3*c*d^2*e^6 - a^4*e^8)*x + (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a
^3*c^3*d^5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 - (3*a^3*c^7*
d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*

```
e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 - 30*a*c^5*
d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^
2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*
c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^
7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^1
0*c^4*d^2*e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*
e^3 + 6*a^2*d*e^5 - (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^
4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5
*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c
^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4
*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a
^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^
10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2
 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) + (a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^
2*c*e^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 -
(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^
2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*
c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*
c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*
a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*
a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^
11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*d^6*e
^2 + 14*a^3*c*d^2*e^6 - a^4*e^8)*x - (a*c^5*d^9 - 18*a^2*c^4*d^7*
e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 - (
3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^
6*c^4*d^4*e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 -
 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 +
255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^1
6 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e
^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^
12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 2
0*a*c*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*
a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12
- 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 +
 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^
16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*
e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e
^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c
^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))
 - 4*(c*d^3 + a*d*e^2)*x)/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4
+ (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(e*x**2+d)/(c*x**4+a)**2,x)
```

[Out] Timed out

GIAC/XCAS [A] time = 0.286276, size = 791, normalized size = 1.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] d^(3/2)*arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/(c^2*d^4 + 2*a*c*d^2*e^
2 + a^2*e^4) + 1/8*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2
*d*e^2 - 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5
*d^4 + 2*sqrt(2)*a^2*c^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) + 1/8*((a
*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(3/
4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(
2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c
^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) + 1/16*((a*c^3)^(1/4)*c^3*d^3 -
 3*(a*c^3)^(1/4)*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^
(3/4)*a*e^3)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)
*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) - 1
/16*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 + 3*(a*c
^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*ln(x^2 - sqrt(2)*x*(a/c)
^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*e^
2 + sqrt(2)*a^3*c^3*e^4) + 1/4*(x^3*e - d*x)/((c*x^4 + a)*(c*d^2
+ a*e^2))
```

3.255 $\int \frac{x^2}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=685

$$\frac{\sqrt[4]{c}de\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}de\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\left(\sqrt{c}d-3\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}\sqrt[4]{c}\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{c}d-3\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}\sqrt[4]{c}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}de\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}de\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\left(3\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}\sqrt[4]{c}\left(ae^2+cd^2\right)}+\frac{\left(3\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{5/4}\sqrt[4]{c}\left(ae^2+cd^2\right)}+\frac{x\left(ae+cdx^2\right)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{\sqrt{d}e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(ae^2+cd^2\right)^2}$$

```
[Out] (x*(a*e + c*d*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[d]*
e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (c^(1/4)
*d*e*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - ((Sqrt[c]*d + 3*Sqrt
[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(5/4
)*c^(1/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d*e*(Sqrt[c]*d - Sqrt[a]*e)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d
^2 + a*e^2)^2) + ((Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)) +
 (c^(1/4)*d*e*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2
) + ((Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)
) - (c^(1/4)*d*e*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) - ((Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e
^2))
```

Rubi [A] time = 1.09613, antiderivative size = 685, normalized size of antiderivative = 1., number of

steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$

$$\frac{\sqrt[4]{c}de\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2} - \frac{\sqrt[4]{c}de\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2} + \frac{\left(\sqrt{c}d-3\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}\sqrt[4]{c}\left(ae^2+cd^2\right)} - \frac{\left(\sqrt{c}d-3\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}\sqrt[4]{c}\left(ae^2+cd^2\right)} + \frac{\sqrt[4]{c}de\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2} - \frac{\sqrt[4]{c}de\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2} - \frac{\left(3\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}\sqrt[4]{c}\left(ae^2+cd^2\right)} + \frac{\left(3\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{5/4}\sqrt[4]{c}\left(ae^2+cd^2\right)} + \frac{x\left(ae+cdx^2\right)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)} - \frac{\sqrt{d}e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (x*(a*e + c*d*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[d]*
e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (c^(1/4)
*d*e*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - ((Sqrt[c]*d + 3*Sqrt
[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(5/4
)*c^(1/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d*e*(Sqrt[c]*d - Sqrt[a]*e)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d
^2 + a*e^2)^2) + ((Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)) +
 (c^(1/4)*d*e*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2
) + ((Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)
) - (c^(1/4)*d*e*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) - ((Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e
^2))
```

Rubi in Sympy [A] time = 160.217, size = 638, normalized size = 0.93

$$-\frac{\sqrt{d}e^{\frac{5}{2}}\,\mathrm{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(ae^2+cd^2)^2}+\frac{x\,(ae+cdx^2)}{4a\,(a+cx^4)\,(ae^2+cd^2)} - \frac{\sqrt{2}\sqrt[4]{c}de\,(\sqrt{a}e-\sqrt{c}d)\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\,(ae^2+cd^2)^2}+\frac{\sqrt{2}\sqrt[4]{c}de\,(\sqrt{a}e-\sqrt{c}d)\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\,(ae^2+cd^2)^2} + \frac{\sqrt{2}\sqrt[4]{c}de\,(\sqrt{a}e+\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}\,(ae^2+cd^2)^2} - \frac{\sqrt{2}\sqrt[4]{c}de\,(\sqrt{a}e+\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{8a^{\frac{3}{4}}\,(ae^2+cd^2)^2} - \frac{\sqrt{2}\,(3\sqrt{a}e-\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{32a^{\frac{5}{4}}\sqrt[4]{c}\,(ae^2+cd^2)} + \frac{\sqrt{2}\,(3\sqrt{a}e-\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^2\right)}{32a^{\frac{5}{4}}\sqrt[4]{c}\,(ae^2+cd^2)} - \frac{\sqrt{2}\,(3\sqrt{a}e+\sqrt{c}d)\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{5}{4}}\sqrt[4]{c}\,(ae^2+cd^2)}+\frac{\sqrt{2}\,(3\sqrt{a}e+\sqrt{c}d)\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{5}{4}}\sqrt[4]{c}\,(ae^2+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] -sqrt(d)*e**(5/2)*atan(sqrt(e)*x/sqrt(d))/(a*e**2 + c*d**2)**2 +
x*(a*e + c*d*x**2)/(4*a*(a + c*x**4)*(a*e**2 + c*d**2)) - sqrt(2)
*c**(1/4)*d*e*(sqrt(a)*e - sqrt(c)*d)*atan(1 - sqrt(2)*c**(1/4)*x
/a**(1/4))/(4*a**(3/4)*(a*e**2 + c*d**2)**2) + sqrt(2)*c**(1/4)*d
*e*(sqrt(a)*e - sqrt(c)*d)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/
(4*a**(3/4)*(a*e**2 + c*d**2)**2) + sqrt(2)*c**(1/4)*d*e*(sqrt(a)
*e + sqrt(c)*d)*log(-sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c
) + c*x**2)/(8*a**(3/4)*(a*e**2 + c*d**2)**2) - sqrt(2)*c**(1/4)*
d*e*(sqrt(a)*e + sqrt(c)*d)*log(sqrt(2)*a**(1/4)*c**(3/4)*x + sqr
t(a)*sqrt(c) + c*x**2)/(8*a**(3/4)*(a*e**2 + c*d**2)**2) - sqrt(2
)*(3*sqrt(a)*e - sqrt(c)*d)*log(-sqrt(2)*a**(1/4)*c**(3/4)*x + sq
rt(a)*sqrt(c) + c*x**2)/(32*a**(5/4)*c**(1/4)*(a*e**2 + c*d**2))
+ sqrt(2)*(3*sqrt(a)*e - sqrt(c)*d)*log(sqrt(2)*a**(1/4)*c**(3/4)
*x + sqrt(a)*sqrt(c) + c*x**2)/(32*a**(5/4)*c**(1/4)*(a*e**2 + c*
d**2)) - sqrt(2)*(3*sqrt(a)*e + sqrt(c)*d)*atan(1 - sqrt(2)*c**(1
/4)*x/a**(1/4))/(16*a**(5/4)*c**(1/4)*(a*e**2 + c*d**2)) + sqrt(2
)*(3*sqrt(a)*e + sqrt(c)*d)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))
/(16*a**(5/4)*c**(1/4)*(a*e**2 + c*d**2))
```

Mathematica [A] time = 0.493501, size = 428, normalized size = 0.62

$$\frac{\sqrt{2}\left(-3a^{3/2}e^3+\sqrt{a}cd^2e+5a\sqrt{c}de^2+c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{5/4}\sqrt[4]{c}} - \frac{\sqrt{2}\left(-3a^{3/2}e^3+\sqrt{a}cd^2e+5a\sqrt{c}de^2+c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{5/4}\sqrt[4]{c}} - 2\sqrt{2}(\ldots$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*(c*d^2 + a*e^2)*(a*e*x + c*d*x^3))/(a*(a + c*x^4)) - 32*Sqrt[
d]*e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - (2*Sqrt[2]*(c^(3/2)*d^3
- Sqrt[a]*c*d^2*e + 5*a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 -
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(5/4)*c^(1/4)) + (2*Sqrt[2]*(c^
(3/2)*d^3 - Sqrt[a]*c*d^2*e + 5*a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*
ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(5/4)*c^(1/4)) + (Sqr
t[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 5*a*Sqrt[c]*d*e^2 - 3*a^(3/
2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
a^(5/4)*c^(1/4)) - (Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 5*a*
Sqrt[c]*d*e^2 - 3*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(a^(5/4)*c^(1/4)))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A] time = 0.02, size = 852, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*c*d*x^3*e^2+1/4/(a*e^2+c*d^2)^2/(c*
x^4+a)*c^2*d^3/a*x^3+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x*e^3*a+1/4/(a
*e^2+c*d^2)^2/(c*x^4+a)*x*d^2*e*c+3/16/(a*e^2+c*d^2)^2*(1/c*a)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*e^3-1/16/(a*e^2+c*d
^2)^2/a*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*c
*d^2*e+3/32/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)
^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c
*a)^(1/2)))*e^3-1/32/(a*e^2+c*d^2)^2/a*(1/c*a)^(1/4)*2^(1/2)*ln((
x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2
^(1/2)+(1/c*a)^(1/2)))*c*d^2*e+3/16/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*e^3-1/16/(a*e^2+c*d^2)
^2/a*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*c*d^
2*e+5/32/(a*e^2+c*d^2)^2/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1
/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)
^(1/2)))*d*e^2+1/32/(a*e^2+c*d^2)^2/a*c/(1/c*a)^(1/4)*2^(1/2)*ln(
(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*
```

```
2^(1/2)+(1/c*a)^(1/2)))*d^3+5/16/(a*e^2+c*d^2)^2/(1/c*a)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d*e^2+1/16/(a*e^2+c*d^2)^
2/a*c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^3
+5/16/(a*e^2+c*d^2)^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a
)^(1/4)*x+1)*d*e^2+1/16/(a*e^2+c*d^2)^2/a*c/(1/c*a)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^3-d*e^3/(a*e^2+c*d^2)^2/(d*e
)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 10.8122, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="fricas")

[Out] [1/16*(4*(c^2*d^3 + a*c*d*e^2)*x^3 + (a^2*c^2*d^4 + 2*a^3*c*d^2*e
^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*s
qrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 +
4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8
)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540
*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*
a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^
4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^
10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/
(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^
2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^2*c^2*
d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (a^2*c^4*d^8*e + 6*
a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*
e^9 - (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34
*a^7*c^3*d^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^
6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^
6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(
a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*
c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^1
```

```
1*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*c^2
*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*
d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c
^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d
^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/
(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8
*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^
11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d
^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^
6*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4
+ 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d
^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*
c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c
^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4
*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 +
8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 +
70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12
+ 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d
^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^
4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^
6 - 81*a^4*e^8)*x - (a^2*c^4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^
2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^9 - (a^4*c^6*d^11 + 9*a^
5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5*e^6 + 21*a^8*
c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2
 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^
8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d
^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*
d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^
2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 3
0*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^
4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^
2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e
^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*
d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5
*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c
^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6
*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))) + (a^2*c^2*d^4 +
2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*
c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a
^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*
e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*
d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d
^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^
7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^
10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^
13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4
 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*e^2 +
 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (a^2*c
^4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*
e^7 + 27*a^6*e^9 + (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4
*d^7*e^4 + 34*a^7*c^3*d^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^
10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 5
40*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 8
1*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*
```

```
e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*
e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16))
)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8
 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*
e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 +
540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 +
81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12
*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6
*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)
))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c
*d^2*e^6 + a^6*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4
+ (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^
5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6
*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*
d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*
e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^
5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^
6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*
c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8
+ 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e
^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^2*c^2*d^4*e^4 + 270
*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (a^2*c^4*d^8*e + 6*a^3*c^3*d^6*e
^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^9 + (a^4*c^
6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5*
e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a
*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a
^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16
+ 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6
+ 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^1
2 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*c^2*d^5*e + 4*a*
c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a
^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*
a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*
a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16
 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6
 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^
12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^
3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))) + 8*
(a*c*e^2*x^4 + a^2*e^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x -
d)/(e*x^2 + d)) + 4*(a*c*d^2*e + a^2*e^3)*x)/(a^2*c^2*d^4 + 2*a^3
*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4
)*x^4), 1/16*(4*(c^2*d^3 + a*c*d*e^2)*x^3 - 16*(a*c*e^2*x^4 + a^2
*e^2)*sqrt(d*e)*arctan(e*x/sqrt(d*e)) + (a^2*c^2*d^4 + 2*a^3*c*d^
2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4
)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8
 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*
e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 +
540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 +
81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12
*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6
*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)
))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c
*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^2*c
^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (a^2*c^4*d^8*e +
```

```
6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a
^6*e^9 - (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 +
 34*a^7*c^3*d^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-
(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3
*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12
)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a
^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*
a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*
c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c
^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(
-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^
3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^1
2)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*
a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28
*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^
4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 +
 a^6*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d
^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*
c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a
^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*
a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*
a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16
 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6
 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^
12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^
3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-
(c^4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2
*e^6 - 81*a^4*e^8)*x - (a^2*c^4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4
*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^9 - (a^4*c^6*d^11 + 9
*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5*e^6 + 21*a
^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*
e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4
*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^
8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c
^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12
*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3
- 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4
*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10
*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^
4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c
^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*
c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^1
2*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2
+ 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))) + (a^2*c^2*d^4
 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a
^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 -
 (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d
^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c
^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*
c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28
*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56
*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 +
 a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*
e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*e^
```

```
2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (a^
2*c^4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d
^2*e^7 + 27*a^6*e^9 + (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*
c^4*d^7*e^4 + 34*a^7*c^3*d^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d
*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4
+ 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10
+ 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^
12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d
^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^1
6)))*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*
d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a
^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4
 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10
 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d
^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*
d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^
16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^
5*c*d^2*e^6 + a^6*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e
^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2
*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*
d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c
^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d
^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/
(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8
*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^
11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d
^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^
6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^2*c^2*d^4*e^4 +
270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (a^2*c^4*d^8*e + 6*a^3*c^3*d^
6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^9 + (a^4
*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d
^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 1
8*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 79
9*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^
16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e
^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*
e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*c^2*d^5*e + 4
*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 +
6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 +
18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 7
99*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d
^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*
e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4
*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3
*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))) +
 4*(a*c*d^2*e + a^2*e^3)*x)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*
e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.287142, size = 814, normalized size = 1.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(5/2)/(c^2*d^4 + 2*a*c*d^2*e
^2 + a^2*e^4) - 1/8*((a*c^3)^(1/4)*a*c^2*d^2*e - (a*c^3)^(3/4)*c*
d^3 - 3*(a*c^3)^(1/4)*a^2*c*e^3 - 5*(a*c^3)^(3/4)*a*d*e^2)*arctan
(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^
2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/
8*((a*c^3)^(1/4)*a*c^2*d^2*e - (a*c^3)^(3/4)*c*d^3 - 3*(a*c^3)^(1
/4)*a^2*c*e^3 - 5*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x
- sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt
(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16*((a*c^3)^(1/4)*
a*c^2*d^2*e + (a*c^3)^(3/4)*c*d^3 - 3*(a*c^3)^(1/4)*a^2*c*e^3 + 5
*(a*c^3)^(3/4)*a*d*e^2)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c
))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4
*c^2*e^4) + 1/16*((a*c^3)^(1/4)*a*c^2*d^2*e + (a*c^3)^(3/4)*c*d^3
 - 3*(a*c^3)^(1/4)*a^2*c*e^3 + 5*(a*c^3)^(3/4)*a*d*e^2)*ln(x^2 -
sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(
2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/4*(c*d*x^3 + a*x*e)
/((c*x^4 + a)*(a*c*d^2 + a^2*e^2))
```

3.256 $\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=689

$$-\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}+\frac{cx\left(d-ex^2\right)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2+cd^2\right)^2}$$

```
[Out] (c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)
*e^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*
d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d - Sqrt[a]*e)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d
^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (S
qrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) -
 (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2
) - (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)
) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e
^2))
```

Rubi [A] time = 1.13855, antiderivative size = 689, normalized size of antiderivative = 1., number of

steps used = 22, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$

$$-\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}+\frac{cx\left(d-ex^2\right)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)
*e^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*
d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d - Sqrt[a]*e)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d
^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (S
qrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) -
 (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2
) - (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)
) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e
^2))
```

Rubi in Sympy [A] time = 171.294, size = 636, normalized size = 0.92

$$\frac{e^{\frac{7}{2}}\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^{2}+cd^{2}\right)^{2}}+\frac{cx\left(d-ex^{2}\right)}{4a\left(a+cx^{4}\right)\left(ae^{2}+cd^{2}\right)}+\frac{\sqrt{2}\sqrt[4]{c}e^{2}\left(\sqrt{a}e-\sqrt{c}d\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\left(ae^{2}+cd^{2}\right)^{2}}-\frac{\sqrt{2}\sqrt[4]{c}e^{2}\left(\sqrt{a}e-\sqrt{c}d\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\left(ae^{2}+cd^{2}\right)^{2}}-\frac{\sqrt{2}\sqrt[4]{c}e^{2}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^{2}\right)}{8a^{\frac{3}{4}}\left(ae^{2}+cd^{2}\right)^{2}}+\frac{\sqrt{2}\sqrt[4]{c}e^{2}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^{2}\right)}{8a^{\frac{3}{4}}\left(ae^{2}+cd^{2}\right)^{2}}+\frac{\sqrt{2}\sqrt[4]{c}\left(\sqrt{a}e-3\sqrt{c}d\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}\left(ae^{2}+cd^{2}\right)}-\frac{\sqrt{2}\sqrt[4]{c}\left(\sqrt{a}e-3\sqrt{c}d\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}\left(ae^{2}+cd^{2}\right)}-\frac{\sqrt{2}\sqrt[4]{c}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^{2}\right)}{32a^{\frac{7}{4}}\left(ae^{2}+cd^{2}\right)}+\frac{\sqrt{2}\sqrt[4]{c}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}c^{\frac{3}{4}}x+\sqrt{a}\sqrt{c}+cx^{2}\right)}{32a^{\frac{7}{4}}\left(ae^{2}+cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] e**(7/2)*atan(sqrt(e)*x/sqrt(d))/(sqrt(d)*(a*e**2 + c*d**2)**2) +
 c*x*(d - e*x**2)/(4*a*(a + c*x**4)*(a*e**2 + c*d**2)) + sqrt(2)*
c**(1/4)*e**2*(sqrt(a)*e - sqrt(c)*d)*atan(1 - sqrt(2)*c**(1/4)*x
/a**(1/4))/(4*a**(3/4)*(a*e**2 + c*d**2)**2) - sqrt(2)*c**(1/4)*e
**2*(sqrt(a)*e - sqrt(c)*d)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))
/(4*a**(3/4)*(a*e**2 + c*d**2)**2) - sqrt(2)*c**(1/4)*e**2*(sqrt(
a)*e + sqrt(c)*d)*log(-sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt
(c) + c*x**2)/(8*a**(3/4)*(a*e**2 + c*d**2)**2) + sqrt(2)*c**(1/4
)*e**2*(sqrt(a)*e + sqrt(c)*d)*log(sqrt(2)*a**(1/4)*c**(3/4)*x +
sqrt(a)*sqrt(c) + c*x**2)/(8*a**(3/4)*(a*e**2 + c*d**2)**2) + sqr
t(2)*c**(1/4)*(sqrt(a)*e - 3*sqrt(c)*d)*atan(1 - sqrt(2)*c**(1/4)
*x/a**(1/4))/(16*a**(7/4)*(a*e**2 + c*d**2)) - sqrt(2)*c**(1/4)*(
sqrt(a)*e - 3*sqrt(c)*d)*atan(1 + sqrt(2)*c**(1/4)*x/a**(1/4))/(1
6*a**(7/4)*(a*e**2 + c*d**2)) - sqrt(2)*c**(1/4)*(sqrt(a)*e + 3*s
qrt(c)*d)*log(-sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(c) + c*
x**2)/(32*a**(7/4)*(a*e**2 + c*d**2)) + sqrt(2)*c**(1/4)*(sqrt(a)
*e + 3*sqrt(c)*d)*log(sqrt(2)*a**(1/4)*c**(3/4)*x + sqrt(a)*sqrt(
c) + c*x**2)/(32*a**(7/4)*(a*e**2 + c*d**2))
```

Mathematica [A] time = 0.523047, size = 429, normalized size = 0.62

$$\frac{-\frac{\sqrt{2}\sqrt[4]{c}\left(5a^{3/2}e^3+\sqrt{a}cd^2e+7a\sqrt{c}de^2+3c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{7/4}}+\frac{\sqrt{2}\sqrt[4]{c}\left(5a^{3/2}e^3+\sqrt{a}cd^2e+7a\sqrt{c}de^2+3c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{7/4}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*c*(c*d^2 + a*e^2)*x*(d - e*x^2))/(a*(a + c*x^4)) + (32*e^(7/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[2]*c^(1/4)*(-3*c
^(3/2)*d^3 + Sqrt[a]*c*d^2*e - 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^
(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e - 7*a*Sqrt[c]*d*e^2 + 5*a
^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) - (S
qrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqrt[c]*d*e
^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*
d^2*e + 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A] time = 0.002, size = 873, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*x^3*e^3-1/4*c^2/(a*e^2+c*d^2)^2/
(c*x^4+a)*e/a*x^3*d^2+1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*x*d*e^2+1/4
*c^2/(a*e^2+c*d^2)^2/(c*x^4+a)*d^3/a*x+7/16*c/(a*e^2+c*d^2)^2/a*(
1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d*e^2+3/16
*c^2/(a*e^2+c*d^2)^2/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
c*a)^(1/4)*x+1)*d^3+7/16*c/(a*e^2+c*d^2)^2/a*(1/c*a)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d*e^2+3/16*c^2/(a*e^2+c*d^2)^
2/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^3
+7/32*c/(a*e^2+c*d^2)^2/a*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(
1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a
)^(1/2)))*d*e^2+3/32*c^2/(a*e^2+c*d^2)^2/a^2*(1/c*a)^(1/4)*2^(1/2
)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/
4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^3-5/32/(a*e^2+c*d^2)^2/(1/c*a)^(1/
4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1
```

```
/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*e^3-1/32*c/(a*e^2+c*d^2)^2/
a/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(
1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^2*e-5/16/(a*
e^2+c*d^2)^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x
+1)*e^3-1/16*c/(a*e^2+c*d^2)^2/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(1/c*a)^(1/4)*x+1)*d^2*e-5/16/(a*e^2+c*d^2)^2/(1/c*a)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*e^3-1/16*c/(a*e^2+c*d^2
)^2/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^2
*e+e^4/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 21.0875, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(c^2*d^2*e + a*c*e^3)*x^3 + (a^2*c^2*d^4 + 2*a^3*c*d^2*
e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*
sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*
d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a
^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^
8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2
*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 +
28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8
+ 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14
 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*
e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^
6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^
8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^
4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a
^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^1
0*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^
2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4
```

```
*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*
a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 7
0*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12
+ 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d
^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^
5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 7
38*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 -
 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a
^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*
c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^
13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 +
 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^
8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*
a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^
3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5
*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 73
8*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 -
529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^
7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c
^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^1
3*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 +
4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8
))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 +
750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^
3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a
^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6
*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt
(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748
*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 +
625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d
^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^
3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16
)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*
c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6
 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^
5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5
*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^
2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*
e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*
e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*
d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) + (a^2*c^2*d^4 + 2*a^3*c*d
^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^
4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c
^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6
+ a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5
*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*
c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2
 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e
^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e
^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d
^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4
*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c
*e^8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5
*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 - (a^6*c^5*d^10*e +
```

```
9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*
a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10
*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*
d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 +
 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6
+ 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^
12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^
2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6
*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12
+ 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^
6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)
/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^
10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28
*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^
8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7
*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 +
 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2
*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*
a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 +
 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6
 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/
(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^1
0*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*
a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8
 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*
e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4
 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186
*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 17
5*a^6*c*d*e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*
d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*s
qrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2
748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10
 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^
6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12
*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e
^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a
^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*
e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2
*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*
a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14
*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d
^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d
^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c
^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - 8*(a*c*e^3*x^4 + a^2*
e^3)*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)) -
 4*(c^2*d^3 + a*c*d*e^2)*x)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*
e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4), -1/16*(4*
(c^2*d^2*e + a*c*e^3)*x^3 - 16*(a*c*e^3*x^4 + a^2*e^3)*sqrt(e/d)*
arctan(e*x/(d*sqrt(e/d))) + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*
e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^
3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^
4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sq
rt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 27
48*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10
```

+ 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4

```
*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)
*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 +
 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^
10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*
c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^
12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15
*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4
*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 +
 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x +
(27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198
*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*
d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2
*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 238
3*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 -
1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7
*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*
c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^1
4*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3
+ 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d
^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^
6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^
4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*
d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^1
0*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*
d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*
c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) -
(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2
*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 +
 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^
4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6
*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4
*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d
^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10
*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d
^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c
^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(
-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3
*c^2*d^2*e^6 - 625*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d
^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*
e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 +
34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c
^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^
4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6
*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4
 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e
^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqr
t((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8
 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*
e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e
^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^
2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*
a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 5
6*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 +
a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4
```

```
 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - 4*(c^2*d^3 + a*c*d*e^2)*x)/(a^2
*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2
*e^2 + a^3*c*e^4)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.286476, size = 814, normalized size = 1.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c
^3)^(3/4)*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sq
rt(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/8*(3*(a*c^3)^(1/
4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/4)*c*d^2*e
- 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^
(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*
e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a
*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)
*a*e^3)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*
c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16
*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)
^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*ln(x^2 - sqrt(2)*x*(a/c)^
(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e
^2 + sqrt(2)*a^4*c^2*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(7/2)/((c
^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d)) - 1/4*(c*x^3*e - c*d*x
)/((c*x^4 + a)*(a*c*d^2 + a^2*e^2))
```

3.257 $\int \frac{1}{x^2(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=745

$$-\frac{c^{3/4}\left(\sqrt{c}d-3\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\sqrt{c}d-3\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(3\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)}-\frac{c^{3/4}\left(3\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(a^{3/2}e^3-\sqrt{c}d\left(2ae^2+cd^2\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)^2}-\frac{c^{3/4}\left(a^{3/2}e^3-\sqrt{c}d\left(2ae^2+cd^2\right)\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)^2}+\frac{c^{3/4}\left(a^{3/2}e^3+\sqrt{c}d\left(2ae^2+cd^2\right)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)^2}-\frac{c^{3/4}\left(a^{3/2}e^3+\sqrt{c}d\left(2ae^2+cd^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)^2}-\frac{cx\left(ae+cdx^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{1}{a^2dx}-\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(ae^2+cd^2\right)^2}$$

```
[Out] -(1/(a^2*d*x)) - (c*x*(a*e + c*d*x^2))/(4*a^2*(c*d^2 + a*e^2)*(a
+ c*x^4)) - (e^(9/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2
 + a*e^2)^2) + (c^(3/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) + (
c^(3/4)*(a^(3/2)*e^3 + Sqrt[c]*d*(c*d^2 + 2*a*e^2))*ArcTan[1 - (S
qrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)^2)
 - (c^(3/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*x)/a^(1/4)])/(8*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) - (c^(3/4)*(a^(
3/2)*e^3 + Sqrt[c]*d*(c*d^2 + 2*a*e^2))*ArcTan[1 + (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*
(Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) + (c^(3/4)*
(a^(3/2)*e^3 - Sqrt[c]*d*(c*d^2 + 2*a*e^2))*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(9/4)*(c*d^2 + a*
e^2)^2) + (c^(3/4)*(Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(9/4)*(c*d^2 +
a*e^2)) - (c^(3/4)*(a^(3/2)*e^3 - Sqrt[c]*d*(c*d^2 + 2*a*e^2))*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*
```

```
a^(9/4)*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 1.46005, antiderivative size = 745, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$

$$-\frac{c^{3/4}\left(\sqrt{c}d-3\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\sqrt{c}d-3\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(3\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)}-\frac{c^{3/4}\left(3\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(a^{3/2}e^3-\sqrt{c}d\left(2ae^2+cd^2\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)^2}-\frac{c^{3/4}\left(a^{3/2}e^3-\sqrt{c}d\left(2ae^2+cd^2\right)\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)^2}+\frac{c^{3/4}\left(a^{3/2}e^3+\sqrt{c}d\left(2ae^2+cd^2\right)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)^2}-\frac{c^{3/4}\left(a^{3/2}e^3+\sqrt{c}d\left(2ae^2+cd^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)^2}-\frac{cx\left(ae+cdx^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{1}{a^2dx}-\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(1/(a^2*d*x)) - (c*x*(a*e + c*d*x^2))/(4*a^2*(c*d^2 + a*e^2)*(a
+ c*x^4)) - (e^(9/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2
 + a*e^2)^2) + (c^(3/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) + (
c^(3/4)*(a^(3/2)*e^3 + Sqrt[c]*d*(c*d^2 + 2*a*e^2))*ArcTan[1 - (S
qrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)^2)
 - (c^(3/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*x)/a^(1/4)])/(8*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) - (c^(3/4)*(a^(
3/2)*e^3 + Sqrt[c]*d*(c*d^2 + 2*a*e^2))*ArcTan[1 + (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*
(Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) + (c^(3/4)*
```

```
(a^(3/2)*e^3 - Sqrt[c]*d*(c*d^2 + 2*a*e^2))*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(9/4)*(c*d^2 + a*
e^2)^2) + (c^(3/4)*(Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(9/4)*(c*d^2 +
a*e^2)) - (c^(3/4)*(a^(3/2)*e^3 - Sqrt[c]*d*(c*d^2 + 2*a*e^2))*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*
a^(9/4)*(c*d^2 + a*e^2)^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.69227, size = 499, normalized size = 0.67

$$\frac{1}{32}\left(\frac{\sqrt{2}c^{3/4}\left(7a^{3/2}e^3+3\sqrt{a}cd^2e-9a\sqrt{c}de^2-5c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{9/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt{2}c^{3/4}\left(-7a^{3/2}e^3-3\sqrt{a}cd^2e+9a\sqrt{c}de^2+5c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{9/4}\left(ae^2+cd^2\right)^2}+\frac{2\sqrt{2}c^{3/4}\left(7a^{3/2}e^3+3\sqrt{a}cd^2e+9a\sqrt{c}de^2+5c^{3/2}d^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{9/4}\left(ae^2+cd^2\right)^2}-\frac{2\sqrt{2}c^{3/4}\left(7a^{3/2}e^3+3\sqrt{a}cd^2e+9a\sqrt{c}de^2+5c^{3/2}d^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{a^{9/4}\left(ae^2+cd^2\right)^2}-\frac{8cx\left(ae+cdx^2\right)}{a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{32}{a^2dx}-\frac{32e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(ae^2+cd^2\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-32/(a^2*d*x) - (8*c*x*(a*e + c*d*x^2))/(a^2*(c*d^2 + a*e^2)*(a
+ c*x^4)) - (32*e^(9/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*
d^2 + a*e^2)^2) + (2*Sqrt[2]*c^(3/4)*(5*c^(3/2)*d^3 + 3*Sqrt[a]*c
*d^2*e + 9*a*Sqrt[c]*d*e^2 + 7*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(a^(9/4)*(c*d^2 + a*e^2)^2) - (2*Sqrt[2]*c^(3
/4)*(5*c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[c]*d*e^2 + 7*a^
(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(9/4)*(c*d
^2 + a*e^2)^2) + (Sqrt[2]*c^(3/4)*(-5*c^(3/2)*d^3 + 3*Sqrt[a]*c*d
^2*e - 9*a*Sqrt[c]*d*e^2 + 7*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(9/4)*(c*d^2 + a*e^2)^2) + (S
qrt[2]*c^(3/4)*(5*c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[c]*d
*e^2 - 7*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + S
qrt[c]*x^2])/(a^(9/4)*(c*d^2 + a*e^2)^2))/32
```

Maple [A] time = 0.025, size = 911, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/a^2/d/x-1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x^3*d*e^2-1/4*c^3/
(a*e^2+c*d^2)^2/a^2/(c*x^4+a)*d^3*x^3-1/4*c/(a*e^2+c*d^2)^2/(c*x^
4+a)*x*e^3-1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x*d^2*e-7/16*c/(a*
e^2+c*d^2)^2/a*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)
*x-1)*e^3-3/16*c^2/(a*e^2+c*d^2)^2/a^2*(1/c*a)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^2*e-7/32*c/(a*e^2+c*d^2)^2/a*(1/c
*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/
(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*e^3-3/32*c^2/(a*e^2+
c*d^2)^2/a^2*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2
)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^2
*e-7/16*c/(a*e^2+c*d^2)^2/a*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(1/c*a)^(1/4)*x+1)*e^3-3/16*c^2/(a*e^2+c*d^2)^2/a^2*(1/c*a)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^2*e-9/32*c/(a*e^2+c*
d^2)^2/a/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1
/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d*e^2-5
/32*c^2/(a*e^2+c*d^2)^2/a^2/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)
^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c
*a)^(1/2)))*d^3-9/16*c/(a*e^2+c*d^2)^2/a/(1/c*a)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d*e^2-5/16*c^2/(a*e^2+c*d^2)^2/a^
2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^3-9/1
6*c/(a*e^2+c*d^2)^2/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a
)^(1/4)*x+1)*d*e^2-5/16*c^2/(a*e^2+c*d^2)^2/a^2/(1/c*a)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^3-1/d*e^5/(a*e^2+c*d^2)^
2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 46.7797, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a*c^2*d^4 + 32*a^2*c*d^2*e^2 + 16*a^3*e^4 + 4*(5*c^3*d
^4 + 9*a*c^2*d^2*e^2 + 4*a^2*c*e^4)*x^4 + 4*(a*c^2*d^3*e + a^2*c*
d*e^3)*x^2 - ((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5
 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x)*sqrt(-(30*c^4*d
^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 + (a^4*c^4*d^8 + 4*a
^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*s
qrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 +
 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e
^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28
*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 +
 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14
+ a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e
^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*log(-(625*c^6*d^8 + 3250*a*c^5*d
^6*e^2 + 4944*a^2*c^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2401*a^4*c^
2*e^8)*x + (75*a^3*c^5*d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*
d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c*e^9 - (5*a^7*c^5*d^11 +
 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5*e^6 +
41*a^11*c*d^3*e^8 + 9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 4050*a*c
^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*
a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9
*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*
c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^
15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))*sqrt(-(30*c^4*
d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 + (a^4*c^4*d^8 + 4*
a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*
sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4
+ 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*
e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 2
8*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8
```

```
+ 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14
 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*
e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))) + ((a^2*c^3*d^5 + 2*a^3*c^2*d^
3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d
*e^4)*x)*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*
e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^
7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2
+ 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*
e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 +
8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6
 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e
^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d
^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*log(-(62
5*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c^4*d^4*e^4 + 686*a^3*c
^3*d^2*e^6 - 2401*a^4*c^2*e^8)*x - (75*a^3*c^5*d^8*e + 418*a^4*c^
4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c
*e^9 - (5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4
+ 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12*d*e^10)*sqrt(-
(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868
*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 +
 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11
*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a
^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^1
7*e^16)))*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d
*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a
^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2
 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4
*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 +
 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^
6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*
e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*
d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))) - ((a^
2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 +
 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x)*sqrt(-(30*c^4*d^5*e + 124*a*c^3*
d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 +
6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^1
2 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6
*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^
3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^
4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*
e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a
^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*
e^6 + a^8*e^8))*log(-(625*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2
*c^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2401*a^4*c^2*e^8)*x + (75*a^
3*c^5*d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126*a^6
*c^2*d^2*e^7 - 343*a^7*c*e^9 + (5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e
^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8
 + 9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 851
1*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 -
 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^1
0*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70
*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 +
 8*a^16*c*d^2*e^14 + a^17*e^16)))*sqrt(-(30*c^4*d^5*e + 124*a*c^3
*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 +
```

6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^
12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^
6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c
^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e
^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6
*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(
a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2
*e^6 + a^8*e^8))) + ((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e
^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x)*sqrt(-(3
0*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^
8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8
*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^
8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^
4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e
^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^
8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^
2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^
2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*log(-(625*c^6*d^8 + 3250*
a*c^5*d^6*e^2 + 4944*a^2*c^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2401
*a^4*c^2*e^8)*x - (75*a^3*c^5*d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a
^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c*e^9 + (5*a^7*c^5
*d^11 + 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5
*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 4
050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6
- 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^1
2)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 5
6*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10
+ 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))*sqrt(-(
30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d
^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^
8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d
^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c
^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*
e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d
^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d
^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c
^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))) - 8*(a^2*c*e^4*x^5 + a^
3*e^4*x)*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d
)))/((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c
^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x), -1/16*(16*a*c^2*d^4 + 3
2*a^2*c*d^2*e^2 + 16*a^3*e^4 + 4*(5*c^3*d^4 + 9*a*c^2*d^2*e^2 + 4
*a^2*c*e^4)*x^4 + 4*(a*c^2*d^3*e + a^2*c*d*e^3)*x^2 + 16*(a^2*c*e
^4*x^5 + a^3*e^4*x)*sqrt(e/d)*arctan(e*x/(d*sqrt(e/d))) - ((a^2*c
^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*
a^4*c*d^3*e^2 + a^5*d*e^4)*x)*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3
*e^3 + 126*a^2*c^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a
^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 +
4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^
6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e
^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 +
56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^1
0 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*
c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6
+ a^8*e^8))*log(-(625*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c^

4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2401*a^4*c^2*e^8)*x + (75*a^3*c
^5*d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126*a^6*c^
2*d^2*e^7 - 343*a^7*c*e^9 - (5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e^2
+ 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 +
9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a
^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 38
22*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c
^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^
13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*
a^16*c*d^2*e^14 + a^17*e^16)))*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^
3*e^3 + 126*a^2*c^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*
a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12
+ 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e
^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*
e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4
+ 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^
10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4
*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^
6 + a^8*e^8))) + ((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)
*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x)*sqrt(-(30*c
^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 + (a^4*c^4*d^8 +
4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^
8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e
^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d
^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2
+ 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e
^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e
^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d
^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*log(-(625*c^6*d^8 + 3250*a*c
^5*d^6*e^2 + 4944*a^2*c^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2401*a^
4*c^2*e^8)*x - (75*a^3*c^5*d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*
c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c*e^9 - (5*a^7*c^5*d^
11 + 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5*e^
6 + 41*a^11*c*d^3*e^8 + 9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 4050
*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6
417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/
(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a
^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 2
8*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))*sqrt(-(30*
c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 + (a^4*c^4*d^8
+ 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e
^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*
e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*
d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2
+ 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*
e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*
e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*
d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))) - ((a^2*c^3*d^5 + 2*a^3*c^
2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a
^5*d*e^4)*x)*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^
2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 +
4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*
e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*
d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^1

6 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10
*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d
^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c
^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*log(
-(625*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944*a^2*c^4*d^4*e^4 + 686*a
^3*c^3*d^2*e^6 - 2401*a^4*c^2*e^8)*x + (75*a^3*c^5*d^8*e + 418*a^
4*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a
^7*c*e^9 + (5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*
e^4 + 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12*d*e^10)*sq
rt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 +
3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^
10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*
a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 +
56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 +
a^17*e^16)))*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c
^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 +
4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10
*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5
*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^
16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^1
0*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*
d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*
c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))) +
((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^5 + (a^3*c^2*d
^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x)*sqrt(-(30*c^4*d^5*e + 124*a*
c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^
2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9
*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6
*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^
6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^1
2*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*
d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16))
)/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*
d^2*e^6 + a^8*e^8))*log(-(625*c^6*d^8 + 3250*a*c^5*d^6*e^2 + 4944
*a^2*c^4*d^4*e^4 + 686*a^3*c^3*d^2*e^6 - 2401*a^4*c^2*e^8)*x - (7
5*a^3*c^5*d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126
*a^6*c^2*d^2*e^7 - 343*a^7*c*e^9 + (5*a^7*c^5*d^11 + 29*a^8*c^4*d
^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3
*e^8 + 9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 +
8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e
^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8
*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6
+ 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^
12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))*sqrt(-(30*c^4*d^5*e + 124*a
*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e
^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^
9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^
6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a
^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^
12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3
*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)
))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c
*d^2*e^6 + a^8*e^8))))/((a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*
d*e^4)*x^5 + (a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4)*x)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(e*x**2+d)/(c*x**4+a)**2,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.288575, size = 863, normalized size = 1.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)*x^2),x, algorithm="giac")

[Out] -1/8*(3*(a*c^3)^(1/4)*a*c^2*d^2*e + 5*(a*c^3)^(3/4)*c*d^3 + 7*(a*
c^3)^(1/4)*a^2*c*e^3 + 9*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2
)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 +
 2*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/8*(3*(a*c^3)^
(1/4)*a*c^2*d^2*e + 5*(a*c^3)^(3/4)*c*d^3 + 7*(a*c^3)^(1/4)*a^2*c
*e^3 + 9*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)
*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c
^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/16*(3*(a*c^3)^(1/4)*a*c^2*d^2
*e - 5*(a*c^3)^(3/4)*c*d^3 + 7*(a*c^3)^(1/4)*a^2*c*e^3 - 9*(a*c^3
)^(3/4)*a*d*e^2)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqr
t(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4)
 + 1/16*(3*(a*c^3)^(1/4)*a*c^2*d^2*e - 5*(a*c^3)^(3/4)*c*d^3 + 7*
(a*c^3)^(1/4)*a^2*c*e^3 - 9*(a*c^3)^(3/4)*a*d*e^2)*ln(x^2 - sqrt(
2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^
4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - arctan(x*e^(1/2)/sqrt(d))*e^
(9/2)/((c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4)*sqrt(d)) - 1/4*(5*c^
2*d^2*x^4 + 4*a*c*x^4*e^2 + a*c*d*x^2*e + 4*a*c*d^2 + 4*a^2*e^2)/
((a^2*c*d^3 + a^3*d*e^2)*(c*x^5 + a*x))
```

3.258 $\int \frac{1}{x^4(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=751

$$\frac{c^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\left(2ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)^2}+\frac{c^{5/4}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)}-\frac{c^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\left(2ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)^2}-\frac{c^{5/4}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)}+\frac{c^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\left(2ae^2+cd^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)^2}+\frac{c^{5/4}\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)}-\frac{c^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\left(2ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)^2}-\frac{c^{5/4}\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)}-\frac{c^2x\left(d-ex^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{e}{a^2d^2x}-\frac{1}{3a^2dx^3}+\frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(ae^2+cd^2\right)^2}$$

```
[Out] -1/(3*a^2*d*x^3) + e/(a^2*d^2*x) - (c^2*x*(d - e*x^2))/(4*a^2*(c*
d^2 + a*e^2)*(a + c*x^4)) + (e^(11/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(d^(5/2)*(c*d^2 + a*e^2)^2) + (c^(5/4)*(3*Sqrt[c]*d - Sqrt[a]*e
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)*(c
*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + 2*a*e^
2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(11/4)*(
c*d^2 + a*e^2)^2) - (c^(5/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 +
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2
)) - (c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*ArcTan[1
+ (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^
2)^2) + (c^(5/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(11/4)*(c*d^2 + a
*e^2)) + (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(
11/4)*(c*d^2 + a*e^2)^2) - (c^(5/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*
a^(11/4)*(c*d^2 + a*e^2)) - (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*(c*d
^2 + 2*a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(4*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 1.32395, antiderivative size = 751, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409

$$\frac{c^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\left(2ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)^2}+\frac{c^{5/4}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)}-\frac{c^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\left(2ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)^2}-\frac{c^{5/4}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)}+\frac{c^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\left(2ae^2+cd^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)^2}+\frac{c^{5/4}\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)}-\frac{c^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\left(2ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)^2}-\frac{c^{5/4}\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)}-\frac{c^2x\left(d-ex^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{e}{a^2d^2x}-\frac{1}{3a^2dx^3}+\frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/(3*a^2*d*x^3) + e/(a^2*d^2*x) - (c^2*x*(d - e*x^2))/(4*a^2*(c*
d^2 + a*e^2)*(a + c*x^4)) + (e^(11/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(d^(5/2)*(c*d^2 + a*e^2)^2) + (c^(5/4)*(3*Sqrt[c]*d - Sqrt[a]*e
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)*(c
*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + 2*a*e^
2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(11/4)*(
c*d^2 + a*e^2)^2) - (c^(5/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 +
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2
)) - (c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*ArcTan[1
+ (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^
2)^2) + (c^(5/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(11/4)*(c*d^2 + a
*e^2)) + (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(
11/4)*(c*d^2 + a*e^2)^2) - (c^(5/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*
a^(11/4)*(c*d^2 + a*e^2)) - (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*(c*d
^2 + 2*a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(4*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.714258, size = 513, normalized size = 0.68

$$\frac{1}{96}\left(\frac{3\sqrt{2}c^{5/4}\left(9a^{3/2}e^3+5\sqrt{a}cd^2e+11a\sqrt{c}de^2+7c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{11/4}\left(ae^2+cd^2\right)^2}-\frac{3\sqrt{2}c^{5/4}\left(9a^{3/2}e^3+5\sqrt{a}cd^2e+11a\sqrt{c}de^2+7c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{11/4}\left(ae^2+cd^2\right)^2}+\frac{6\sqrt{2}c^{5/4}\left(-9a^{3/2}e^3-5\sqrt{a}cd^2e+11a\sqrt{c}de^2+7c^{3/2}d^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{11/4}\left(ae^2+cd^2\right)^2}+\frac{6\sqrt{2}c^{5/4}\left(9a^{3/2}e^3+5\sqrt{a}cd^2e-11a\sqrt{c}de^2-7c^{3/2}d^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{a^{11/4}\left(ae^2+cd^2\right)^2}-\frac{24c^2x\left(d-ex^2\right)}{a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{96e}{a^2d^2x}-\frac{32}{a^2dx^3}+\frac{96e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(ae^2+cd^2\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-32/(a^2*d*x^3) + (96*e)/(a^2*d^2*x) - (24*c^2*x*(d - e*x^2))/(a
^2*(c*d^2 + a*e^2)*(a + c*x^4)) + (96*e^(11/2)*ArcTan[(Sqrt[e]*x)
/Sqrt[d]])/(d^(5/2)*(c*d^2 + a*e^2)^2) + (6*Sqrt[2]*c^(5/4)*(7*c^
(3/2)*d^3 - 5*Sqrt[a]*c*d^2*e + 11*a*Sqrt[c]*d*e^2 - 9*a^(3/2)*e^
3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(11/4)*(c*d^2 + a*
e^2)^2) + (6*Sqrt[2]*c^(5/4)*(-7*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e
- 11*a*Sqrt[c]*d*e^2 + 9*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*x)/a^(1/4)])/(a^(11/4)*(c*d^2 + a*e^2)^2) + (3*Sqrt[2]*c^(5/4)*(
```

```
7*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + 11*a*Sqrt[c]*d*e^2 + 9*a^(3/2
)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a
^(11/4)*(c*d^2 + a*e^2)^2) - (3*Sqrt[2]*c^(5/4)*(7*c^(3/2)*d^3 +
5*Sqrt[a]*c*d^2*e + 11*a*Sqrt[c]*d*e^2 + 9*a^(3/2)*e^3)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(11/4)*(c*d^2 +
 a*e^2)^2))/96
```

Maple [A] time = 0.03, size = 932, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/3/a^2/d/x^3+e/a^2/d^2/x+1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x^
3*e^3+1/4*c^3/(a*e^2+c*d^2)^2/a^2/(c*x^4+a)*x^3*d^2*e-1/4*c^2/(a*
e^2+c*d^2)^2/a/(c*x^4+a)*e^2*d*x-1/4*c^3/(a*e^2+c*d^2)^2/a^2/(c*x
^4+a)*d^3*x-11/16*c^2/(a*e^2+c*d^2)^2/a^2*(1/c*a)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d*e^2-7/16*c^3/(a*e^2+c*d^2)^2/a
^3*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^3-11
/16*c^2/(a*e^2+c*d^2)^2/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(1/c*a)^(1/4)*x-1)*d*e^2-7/16*c^3/(a*e^2+c*d^2)^2/a^3*(1/c*a)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^3-11/32*c^2/(a*e^2
+c*d^2)^2/a^2*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/
2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d*
e^2-7/32*c^3/(a*e^2+c*d^2)^2/a^3*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1
/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)
+(1/c*a)^(1/2)))*d^3+9/32*c/(a*e^2+c*d^2)^2/a/(1/c*a)^(1/4)*2^(1/
2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1
/4)*x*2^(1/2)+(1/c*a)^(1/2)))*e^3+5/32*c^2/(a*e^2+c*d^2)^2/a^2/(1
/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)
)/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^2*e+9/16*c/(a*e^
2+c*d^2)^2/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x
+1)*e^3+5/16*c^2/(a*e^2+c*d^2)^2/a^2/(1/c*a)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^2*e+9/16*c/(a*e^2+c*d^2)^2/a/(1/c*a
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*e^3+5/16*c^2/(a
*e^2+c*d^2)^2/a^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1
/4)*x-1)*d^2*e+1/d^2*e^6/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(x*e/(
d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 106.225, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)*x^4),x, algorithm="fricas")
```

```
[Out] [-1/48*(16*a*c^2*d^5 + 32*a^2*c*d^3*e^2 + 16*a^3*d*e^4 - 12*(5*c^
3*d^4*e + 9*a*c^2*d^2*e^3 + 4*a^2*c*e^5)*x^6 + 4*(7*c^3*d^5 + 11*
a*c^2*d^3*e^2 + 4*a^2*c*d*e^4)*x^4 - 48*(a*c^2*d^4*e + 2*a^2*c*d^
2*e^3 + a^3*e^5)*x^2 - 3*((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*
c*d^2*e^4)*x^7 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x^
3)*sqrt((70*c^5*d^5*e + 236*a*c^4*d^3*e^3 + 198*a^2*c^3*d*e^5 + (
a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2
*e^6 + a^9*e^8)*sqrt(-(2401*c^11*d^12 + 12642*a*c^10*d^10*e^2 + 1
9679*a^2*c^9*d^8*e^4 + 60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8
 - 5022*a^5*c^6*d^2*e^10 + 6561*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*
a^12*c^7*d^14*e^2 + 28*a^13*c^6*d^12*e^4 + 56*a^14*c^5*d^10*e^6 +
 70*a^15*c^4*d^8*e^8 + 56*a^16*c^3*d^6*e^10 + 28*a^17*c^2*d^4*e^1
2 + 8*a^18*c*d^2*e^14 + a^19*e^16)))/(a^5*c^4*d^8 + 4*a^6*c^3*d^6
*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8))*log(-(2401
*c^8*d^8 + 10290*a*c^7*d^6*e^2 + 11968*a^2*c^6*d^4*e^4 - 1458*a^3
*c^5*d^2*e^6 - 6561*a^4*c^4*e^8)*x + (343*a^3*c^7*d^9 + 1442*a^4*
c^6*d^7*e^2 + 1636*a^5*c^5*d^5*e^4 - 226*a^6*c^4*d^3*e^6 - 891*a^
7*c^3*d*e^8 + (5*a^9*c^5*d^10*e + 29*a^10*c^4*d^8*e^3 + 66*a^11*c
^3*d^6*e^5 + 74*a^12*c^2*d^4*e^7 + 41*a^13*c*d^2*e^9 + 9*a^14*e^1
1)*sqrt(-(2401*c^11*d^12 + 12642*a*c^10*d^10*e^2 + 19679*a^2*c^9*
d^8*e^4 + 60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8 - 5022*a^5*c
^6*d^2*e^10 + 6561*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c^7*d^14
*e^2 + 28*a^13*c^6*d^12*e^4 + 56*a^14*c^5*d^10*e^6 + 70*a^15*c^4*
d^8*e^8 + 56*a^16*c^3*d^6*e^10 + 28*a^17*c^2*d^4*e^12 + 8*a^18*c*
d^2*e^14 + a^19*e^16)))*sqrt((70*c^5*d^5*e + 236*a*c^4*d^3*e^3 +
198*a^2*c^3*d*e^5 + (a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*
d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)*sqrt(-(2401*c^11*d^12 + 1264
2*a*c^10*d^10*e^2 + 19679*a^2*c^9*d^8*e^4 + 60*a^3*c^8*d^6*e^6 -
19937*a^4*c^7*d^4*e^8 - 5022*a^5*c^6*d^2*e^10 + 6561*a^6*c^5*e^12
)/(a^11*c^8*d^16 + 8*a^12*c^7*d^14*e^2 + 28*a^13*c^6*d^12*e^4 + 5
6*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d^8*e^8 + 56*a^16*c^3*d^6*e^10
+ 28*a^17*c^2*d^4*e^12 + 8*a^18*c*d^2*e^14 + a^19*e^16)))/(a^5*c^
4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 +
 a^9*e^8))) + 3*((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e^4
)*x^7 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x^3)*sqrt((
70*c^5*d^5*e + 236*a*c^4*d^3*e^3 + 198*a^2*c^3*d*e^5 + (a^5*c^4*d
^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^
```

9*e^8)*sqrt(-(2401*c^11*d^12 + 12642*a*c^10*d^10*e^2 + 19679*a^2*
c^9*d^8*e^4 + 60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8 - 5022*a
^5*c^6*d^2*e^10 + 6561*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c^7*
d^14*e^2 + 28*a^13*c^6*d^12*e^4 + 56*a^14*c^5*d^10*e^6 + 70*a^15*
c^4*d^8*e^8 + 56*a^16*c^3*d^6*e^10 + 28*a^17*c^2*d^4*e^12 + 8*a^1
8*c*d^2*e^14 + a^19*e^16)))/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*
a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8))*log(-(2401*c^8*d^8
+ 10290*a*c^7*d^6*e^2 + 11968*a^2*c^6*d^4*e^4 - 1458*a^3*c^5*d^2*
e^6 - 6561*a^4*c^4*e^8)*x - (343*a^3*c^7*d^9 + 1442*a^4*c^6*d^7*e
^2 + 1636*a^5*c^5*d^5*e^4 - 226*a^6*c^4*d^3*e^6 - 891*a^7*c^3*d*e
^8 + (5*a^9*c^5*d^10*e + 29*a^10*c^4*d^8*e^3 + 66*a^11*c^3*d^6*e^
5 + 74*a^12*c^2*d^4*e^7 + 41*a^13*c*d^2*e^9 + 9*a^14*e^11)*sqrt(-
(2401*c^11*d^12 + 12642*a*c^10*d^10*e^2 + 19679*a^2*c^9*d^8*e^4 +
60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8 - 5022*a^5*c^6*d^2*e^
10 + 6561*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c^7*d^14*e^2 + 28
*a^13*c^6*d^12*e^4 + 56*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d^8*e^8 +
56*a^16*c^3*d^6*e^10 + 28*a^17*c^2*d^4*e^12 + 8*a^18*c*d^2*e^14
+ a^19*e^16)))*sqrt((70*c^5*d^5*e + 236*a*c^4*d^3*e^3 + 198*a^2*c
^3*d*e^5 + (a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 +
4*a^8*c*d^2*e^6 + a^9*e^8)*sqrt(-(2401*c^11*d^12 + 12642*a*c^10*
d^10*e^2 + 19679*a^2*c^9*d^8*e^4 + 60*a^3*c^8*d^6*e^6 - 19937*a^4
*c^7*d^4*e^8 - 5022*a^5*c^6*d^2*e^10 + 6561*a^6*c^5*e^12)/(a^11*c
^8*d^16 + 8*a^12*c^7*d^14*e^2 + 28*a^13*c^6*d^12*e^4 + 56*a^14*c^
5*d^10*e^6 + 70*a^15*c^4*d^8*e^8 + 56*a^16*c^3*d^6*e^10 + 28*a^17
*c^2*d^4*e^12 + 8*a^18*c*d^2*e^14 + a^19*e^16)))/(a^5*c^4*d^8 + 4
*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)
)) - 3*((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e^4)*x^7 + (
a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x^3)*sqrt((70*c^5*d^
5*e + 236*a*c^4*d^3*e^3 + 198*a^2*c^3*d*e^5 - (a^5*c^4*d^8 + 4*a^
6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)*sq
rt(-(2401*c^11*d^12 + 12642*a*c^10*d^10*e^2 + 19679*a^2*c^9*d^8*e
^4 + 60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8 - 5022*a^5*c^6*d^
2*e^10 + 6561*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c^7*d^14*e^2
+ 28*a^13*c^6*d^12*e^4 + 56*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d^8*e
^8 + 56*a^16*c^3*d^6*e^10 + 28*a^17*c^2*d^4*e^12 + 8*a^18*c*d^2*e
^14 + a^19*e^16)))/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d
^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8))*log(-(2401*c^8*d^8 + 10290*a
*c^7*d^6*e^2 + 11968*a^2*c^6*d^4*e^4 - 1458*a^3*c^5*d^2*e^6 - 656
1*a^4*c^4*e^8)*x + (343*a^3*c^7*d^9 + 1442*a^4*c^6*d^7*e^2 + 1636
*a^5*c^5*d^5*e^4 - 226*a^6*c^4*d^3*e^6 - 891*a^7*c^3*d*e^8 - (5*a
^9*c^5*d^10*e + 29*a^10*c^4*d^8*e^3 + 66*a^11*c^3*d^6*e^5 + 74*a^
12*c^2*d^4*e^7 + 41*a^13*c*d^2*e^9 + 9*a^14*e^11)*sqrt(-(2401*c^1
1*d^12 + 12642*a*c^10*d^10*e^2 + 19679*a^2*c^9*d^8*e^4 + 60*a^3*c
^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8 - 5022*a^5*c^6*d^2*e^10 + 6561
*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c^7*d^14*e^2 + 28*a^13*c^6
*d^12*e^4 + 56*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d^8*e^8 + 56*a^16*
c^3*d^6*e^10 + 28*a^17*c^2*d^4*e^12 + 8*a^18*c*d^2*e^14 + a^19*e^
16)))*sqrt((70*c^5*d^5*e + 236*a*c^4*d^3*e^3 + 198*a^2*c^3*d*e^5
- (a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*
d^2*e^6 + a^9*e^8)*sqrt(-(2401*c^11*d^12 + 12642*a*c^10*d^10*e^2
+ 19679*a^2*c^9*d^8*e^4 + 60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*
e^8 - 5022*a^5*c^6*d^2*e^10 + 6561*a^6*c^5*e^12)/(a^11*c^8*d^16 +
8*a^12*c^7*d^14*e^2 + 28*a^13*c^6*d^12*e^4 + 56*a^14*c^5*d^10*e^
6 + 70*a^15*c^4*d^8*e^8 + 56*a^16*c^3*d^6*e^10 + 28*a^17*c^2*d^4*

```
e^12 + 8*a^18*c*d^2*e^14 + a^19*e^16)))/(a^5*c^4*d^8 + 4*a^6*c^3*
d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8))) + 3*((
a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e^4)*x^7 + (a^3*c^2*d
^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x^3)*sqrt((70*c^5*d^5*e + 236
*a*c^4*d^3*e^3 + 198*a^2*c^3*d*e^5 - (a^5*c^4*d^8 + 4*a^6*c^3*d^6
*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)*sqrt(-(2401
*c^11*d^12 + 12642*a*c^10*d^10*e^2 + 19679*a^2*c^9*d^8*e^4 + 60*a
^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8 - 5022*a^5*c^6*d^2*e^10 +
6561*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c^7*d^14*e^2 + 28*a^13
*c^6*d^12*e^4 + 56*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d^8*e^8 + 56*a
^16*c^3*d^6*e^10 + 28*a^17*c^2*d^4*e^12 + 8*a^18*c*d^2*e^14 + a^1
9*e^16)))/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 +
4*a^8*c*d^2*e^6 + a^9*e^8))*log(-(2401*c^8*d^8 + 10290*a*c^7*d^6*
e^2 + 11968*a^2*c^6*d^4*e^4 - 1458*a^3*c^5*d^2*e^6 - 6561*a^4*c^4
*e^8)*x - (343*a^3*c^7*d^9 + 1442*a^4*c^6*d^7*e^2 + 1636*a^5*c^5*
d^5*e^4 - 226*a^6*c^4*d^3*e^6 - 891*a^7*c^3*d*e^8 - (5*a^9*c^5*d^
10*e + 29*a^10*c^4*d^8*e^3 + 66*a^11*c^3*d^6*e^5 + 74*a^12*c^2*d^
4*e^7 + 41*a^13*c*d^2*e^9 + 9*a^14*e^11)*sqrt(-(2401*c^11*d^12 +
12642*a*c^10*d^10*e^2 + 19679*a^2*c^9*d^8*e^4 + 60*a^3*c^8*d^6*e^
6 - 19937*a^4*c^7*d^4*e^8 - 5022*a^5*c^6*d^2*e^10 + 6561*a^6*c^5*
e^12)/(a^11*c^8*d^16 + 8*a^12*c^7*d^14*e^2 + 28*a^13*c^6*d^12*e^4
 + 56*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d^8*e^8 + 56*a^16*c^3*d^6*e
^10 + 28*a^17*c^2*d^4*e^12 + 8*a^18*c*d^2*e^14 + a^19*e^16)))*sqr
t((70*c^5*d^5*e + 236*a*c^4*d^3*e^3 + 198*a^2*c^3*d*e^5 - (a^5*c^
4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 +
 a^9*e^8)*sqrt(-(2401*c^11*d^12 + 12642*a*c^10*d^10*e^2 + 19679*a
^2*c^9*d^8*e^4 + 60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8 - 502
2*a^5*c^6*d^2*e^10 + 6561*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c
^7*d^14*e^2 + 28*a^13*c^6*d^12*e^4 + 56*a^14*c^5*d^10*e^6 + 70*a^
15*c^4*d^8*e^8 + 56*a^16*c^3*d^6*e^10 + 28*a^17*c^2*d^4*e^12 + 8*
a^18*c*d^2*e^14 + a^19*e^16)))/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 +
 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8))) - 24*(a^2*c*e^5
*x^7 + a^3*e^5*x^3)*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)
/(e*x^2 + d)))/((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e^4)
*x^7 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x^3), -1/48*
(16*a*c^2*d^5 + 32*a^2*c*d^3*e^2 + 16*a^3*d*e^4 - 12*(5*c^3*d^4*e
 + 9*a*c^2*d^2*e^3 + 4*a^2*c*e^5)*x^6 + 4*(7*c^3*d^5 + 11*a*c^2*d
^3*e^2 + 4*a^2*c*d*e^4)*x^4 - 48*(a*c^2*d^4*e + 2*a^2*c*d^2*e^3 +
 a^3*e^5)*x^2 - 48*(a^2*c*e^5*x^7 + a^3*e^5*x^3)*sqrt(e/d)*arctan
(e*x/(d*sqrt(e/d))) - 3*((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c
*d^2*e^4)*x^7 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x^3
)*sqrt((70*c^5*d^5*e + 236*a*c^4*d^3*e^3 + 198*a^2*c^3*d*e^5 + (a
^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*
e^6 + a^9*e^8)*sqrt(-(2401*c^11*d^12 + 12642*a*c^10*d^10*e^2 + 19
679*a^2*c^9*d^8*e^4 + 60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8
- 5022*a^5*c^6*d^2*e^10 + 6561*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a
^12*c^7*d^14*e^2 + 28*a^13*c^6*d^12*e^4 + 56*a^14*c^5*d^10*e^6 +
70*a^15*c^4*d^8*e^8 + 56*a^16*c^3*d^6*e^10 + 28*a^17*c^2*d^4*e^12
 + 8*a^18*c*d^2*e^14 + a^19*e^16)))/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*
e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8))*log(-(2401*
c^8*d^8 + 10290*a*c^7*d^6*e^2 + 11968*a^2*c^6*d^4*e^4 - 1458*a^3*
c^5*d^2*e^6 - 6561*a^4*c^4*e^8)*x + (343*a^3*c^7*d^9 + 1442*a^4*c
^6*d^7*e^2 + 1636*a^5*c^5*d^5*e^4 - 226*a^6*c^4*d^3*e^6 - 891*a^7
*c^3*d*e^8 + (5*a^9*c^5*d^10*e + 29*a^10*c^4*d^8*e^3 + 66*a^11*c^
```

3*d^6*e^5 + 74*a^12*c^2*d^4*e^7 + 41*a^13*c*d^2*e^9 + 9*a^14*e^11
)*sqrt(-(2401*c^11*d^12 + 12642*a*c^10*d^10*e^2 + 19679*a^2*c^9*d
^8*e^4 + 60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8 - 5022*a^5*c^
6*d^2*e^10 + 6561*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c^7*d^14*
e^2 + 28*a^13*c^6*d^12*e^4 + 56*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d
^8*e^8 + 56*a^16*c^3*d^6*e^10 + 28*a^17*c^2*d^4*e^12 + 8*a^18*c*d
^2*e^14 + a^19*e^16)))*sqrt((70*c^5*d^5*e + 236*a*c^4*d^3*e^3 + 1
98*a^2*c^3*d*e^5 + (a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d
^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)*sqrt(-(2401*c^11*d^12 + 12642
*a*c^10*d^10*e^2 + 19679*a^2*c^9*d^8*e^4 + 60*a^3*c^8*d^6*e^6 - 1
9937*a^4*c^7*d^4*e^8 - 5022*a^5*c^6*d^2*e^10 + 6561*a^6*c^5*e^12)
/(a^11*c^8*d^16 + 8*a^12*c^7*d^14*e^2 + 28*a^13*c^6*d^12*e^4 + 56
*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d^8*e^8 + 56*a^16*c^3*d^6*e^10 +
 28*a^17*c^2*d^4*e^12 + 8*a^18*c*d^2*e^14 + a^19*e^16)))/(a^5*c^4
*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 +
a^9*e^8))) + 3*((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e^4)
*x^7 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x^3)*sqrt((7
0*c^5*d^5*e + 236*a*c^4*d^3*e^3 + 198*a^2*c^3*d*e^5 + (a^5*c^4*d^
8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9
*e^8)*sqrt(-(2401*c^11*d^12 + 12642*a*c^10*d^10*e^2 + 19679*a^2*c
^9*d^8*e^4 + 60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8 - 5022*a^
5*c^6*d^2*e^10 + 6561*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c^7*d
^14*e^2 + 28*a^13*c^6*d^12*e^4 + 56*a^14*c^5*d^10*e^6 + 70*a^15*c
^4*d^8*e^8 + 56*a^16*c^3*d^6*e^10 + 28*a^17*c^2*d^4*e^12 + 8*a^18
*c*d^2*e^14 + a^19*e^16)))/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a
^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8))*log(-(2401*c^8*d^8 +
 10290*a*c^7*d^6*e^2 + 11968*a^2*c^6*d^4*e^4 - 1458*a^3*c^5*d^2*e
^6 - 6561*a^4*c^4*e^8)*x - (343*a^3*c^7*d^9 + 1442*a^4*c^6*d^7*e^
2 + 1636*a^5*c^5*d^5*e^4 - 226*a^6*c^4*d^3*e^6 - 891*a^7*c^3*d*e^
8 + (5*a^9*c^5*d^10*e + 29*a^10*c^4*d^8*e^3 + 66*a^11*c^3*d^6*e^5
 + 74*a^12*c^2*d^4*e^7 + 41*a^13*c*d^2*e^9 + 9*a^14*e^11)*sqrt(-(
2401*c^11*d^12 + 12642*a*c^10*d^10*e^2 + 19679*a^2*c^9*d^8*e^4 +
60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8 - 5022*a^5*c^6*d^2*e^1
0 + 6561*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c^7*d^14*e^2 + 28*
a^13*c^6*d^12*e^4 + 56*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d^8*e^8 +
56*a^16*c^3*d^6*e^10 + 28*a^17*c^2*d^4*e^12 + 8*a^18*c*d^2*e^14 +
 a^19*e^16)))*sqrt((70*c^5*d^5*e + 236*a*c^4*d^3*e^3 + 198*a^2*c^
3*d*e^5 + (a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 +
4*a^8*c*d^2*e^6 + a^9*e^8)*sqrt(-(2401*c^11*d^12 + 12642*a*c^10*d
^10*e^2 + 19679*a^2*c^9*d^8*e^4 + 60*a^3*c^8*d^6*e^6 - 19937*a^4*
c^7*d^4*e^8 - 5022*a^5*c^6*d^2*e^10 + 6561*a^6*c^5*e^12)/(a^11*c^
8*d^16 + 8*a^12*c^7*d^14*e^2 + 28*a^13*c^6*d^12*e^4 + 56*a^14*c^5
*d^10*e^6 + 70*a^15*c^4*d^8*e^8 + 56*a^16*c^3*d^6*e^10 + 28*a^17*
c^2*d^4*e^12 + 8*a^18*c*d^2*e^14 + a^19*e^16)))/(a^5*c^4*d^8 + 4*
a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8))
) - 3*((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e^4)*x^7 + (a
^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x^3)*sqrt((70*c^5*d^5
*e + 236*a*c^4*d^3*e^3 + 198*a^2*c^3*d*e^5 - (a^5*c^4*d^8 + 4*a^6
*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)*sqr
t(-(2401*c^11*d^12 + 12642*a*c^10*d^10*e^2 + 19679*a^2*c^9*d^8*e^
4 + 60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8 - 5022*a^5*c^6*d^2
*e^10 + 6561*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c^7*d^14*e^2 +
 28*a^13*c^6*d^12*e^4 + 56*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d^8*e^
8 + 56*a^16*c^3*d^6*e^10 + 28*a^17*c^2*d^4*e^12 + 8*a^18*c*d^2*e^

```
14 + a^19*e^16)))/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^
4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8))*log(-(2401*c^8*d^8 + 10290*a*
c^7*d^6*e^2 + 11968*a^2*c^6*d^4*e^4 - 1458*a^3*c^5*d^2*e^6 - 6561
*a^4*c^4*e^8)*x + (343*a^3*c^7*d^9 + 1442*a^4*c^6*d^7*e^2 + 1636*
a^5*c^5*d^5*e^4 - 226*a^6*c^4*d^3*e^6 - 891*a^7*c^3*d*e^8 - (5*a^
9*c^5*d^10*e + 29*a^10*c^4*d^8*e^3 + 66*a^11*c^3*d^6*e^5 + 74*a^1
2*c^2*d^4*e^7 + 41*a^13*c*d^2*e^9 + 9*a^14*e^11)*sqrt(-(2401*c^11
*d^12 + 12642*a*c^10*d^10*e^2 + 19679*a^2*c^9*d^8*e^4 + 60*a^3*c^
8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8 - 5022*a^5*c^6*d^2*e^10 + 6561*
a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c^7*d^14*e^2 + 28*a^13*c^6*
d^12*e^4 + 56*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d^8*e^8 + 56*a^16*c
^3*d^6*e^10 + 28*a^17*c^2*d^4*e^12 + 8*a^18*c*d^2*e^14 + a^19*e^1
6)))*sqrt((70*c^5*d^5*e + 236*a*c^4*d^3*e^3 + 198*a^2*c^3*d*e^5 -
 (a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d
^2*e^6 + a^9*e^8)*sqrt(-(2401*c^11*d^12 + 12642*a*c^10*d^10*e^2 +
 19679*a^2*c^9*d^8*e^4 + 60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e
^8 - 5022*a^5*c^6*d^2*e^10 + 6561*a^6*c^5*e^12)/(a^11*c^8*d^16 +
8*a^12*c^7*d^14*e^2 + 28*a^13*c^6*d^12*e^4 + 56*a^14*c^5*d^10*e^6
 + 70*a^15*c^4*d^8*e^8 + 56*a^16*c^3*d^6*e^10 + 28*a^17*c^2*d^4*e
^12 + 8*a^18*c*d^2*e^14 + a^19*e^16)))/(a^5*c^4*d^8 + 4*a^6*c^3*d
^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8))) + 3*((a
^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e^4)*x^7 + (a^3*c^2*d^
6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x^3)*sqrt((70*c^5*d^5*e + 236*
a*c^4*d^3*e^3 + 198*a^2*c^3*d*e^5 - (a^5*c^4*d^8 + 4*a^6*c^3*d^6*
e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)*sqrt(-(2401*
c^11*d^12 + 12642*a*c^10*d^10*e^2 + 19679*a^2*c^9*d^8*e^4 + 60*a^
3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8 - 5022*a^5*c^6*d^2*e^10 + 6
561*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c^7*d^14*e^2 + 28*a^13*
c^6*d^12*e^4 + 56*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d^8*e^8 + 56*a^
16*c^3*d^6*e^10 + 28*a^17*c^2*d^4*e^12 + 8*a^18*c*d^2*e^14 + a^19
*e^16)))/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4
*a^8*c*d^2*e^6 + a^9*e^8))*log(-(2401*c^8*d^8 + 10290*a*c^7*d^6*e
^2 + 11968*a^2*c^6*d^4*e^4 - 1458*a^3*c^5*d^2*e^6 - 6561*a^4*c^4*
e^8)*x - (343*a^3*c^7*d^9 + 1442*a^4*c^6*d^7*e^2 + 1636*a^5*c^5*d
^5*e^4 - 226*a^6*c^4*d^3*e^6 - 891*a^7*c^3*d*e^8 - (5*a^9*c^5*d^1
0*e + 29*a^10*c^4*d^8*e^3 + 66*a^11*c^3*d^6*e^5 + 74*a^12*c^2*d^4
*e^7 + 41*a^13*c*d^2*e^9 + 9*a^14*e^11)*sqrt(-(2401*c^11*d^12 + 1
2642*a*c^10*d^10*e^2 + 19679*a^2*c^9*d^8*e^4 + 60*a^3*c^8*d^6*e^6
 - 19937*a^4*c^7*d^4*e^8 - 5022*a^5*c^6*d^2*e^10 + 6561*a^6*c^5*e
^12)/(a^11*c^8*d^16 + 8*a^12*c^7*d^14*e^2 + 28*a^13*c^6*d^12*e^4
+ 56*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d^8*e^8 + 56*a^16*c^3*d^6*e^
10 + 28*a^17*c^2*d^4*e^12 + 8*a^18*c*d^2*e^14 + a^19*e^16)))*sqrt
((70*c^5*d^5*e + 236*a*c^4*d^3*e^3 + 198*a^2*c^3*d*e^5 - (a^5*c^4
*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 +
a^9*e^8)*sqrt(-(2401*c^11*d^12 + 12642*a*c^10*d^10*e^2 + 19679*a^
2*c^9*d^8*e^4 + 60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8 - 5022
*a^5*c^6*d^2*e^10 + 6561*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c^
7*d^14*e^2 + 28*a^13*c^6*d^12*e^4 + 56*a^14*c^5*d^10*e^6 + 70*a^1
5*c^4*d^8*e^8 + 56*a^16*c^3*d^6*e^10 + 28*a^17*c^2*d^4*e^12 + 8*a
^18*c*d^2*e^14 + a^19*e^16)))/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 +
6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8))))/((a^2*c^3*d^6 +
 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e^4)*x^7 + (a^3*c^2*d^6 + 2*a^4*c*
d^4*e^2 + a^5*d^2*e^4)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.28641, size = 848, normalized size = 1.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + a)^2*(e*x^2 + d)*x^4),x, algorithm="giac")
```

```
[Out] -1/8*(7*(a*c^3)^(1/4)*c^3*d^3 + 11*(a*c^3)^(1/4)*a*c^2*d*e^2 - 5*
(a*c^3)^(3/4)*c*d^2*e - 9*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)
*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 +
2*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/8*(7*(a*c^3)^(
1/4)*c^3*d^3 + 11*(a*c^3)^(1/4)*a*c^2*d*e^2 - 5*(a*c^3)^(3/4)*c*d
^2*e - 9*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(
a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2
*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/16*(7*(a*c^3)^(1/4)*c^3*d^3 + 1
1*(a*c^3)^(1/4)*a*c^2*d*e^2 + 5*(a*c^3)^(3/4)*c*d^2*e + 9*(a*c^3)
^(3/4)*a*e^3)*ln(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2
)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) +
1/16*(7*(a*c^3)^(1/4)*c^3*d^3 + 11*(a*c^3)^(1/4)*a*c^2*d*e^2 + 5*
(a*c^3)^(3/4)*c*d^2*e + 9*(a*c^3)^(3/4)*a*e^3)*ln(x^2 - sqrt(2)*x
*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^
2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(11/
2)/((c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4)*sqrt(d)) + 1/4*(c^2*x
^3*e - c^2*d*x)/((a^2*c*d^2 + a^3*e^2)*(c*x^4 + a)) + 1/3*(3*x^2*
e - d)/(a^2*d^2*x^3)
```

3.259 $\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=243

$$\frac{c^2\sqrt{a^2+2abx^2+b^2x^4}(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}(a+bx^2)}-\frac{cx\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(bc-2ad)}{16d^2(a+bx^2)}+\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{6d(a+bx^2)}-\frac{x^3\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(bc-2ad)}{8d(a+bx^2)}$$

```
[Out] -(c*(b*c - 2*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(16*d^2*(a + b*x^2)) - ((b*c - 2*a*d)*x^3*Sqrt[c + d*x^2]*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*d*(a + b*x^2)) + (b*x^3*(c + d*x
^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*d*(a + b*x^2)) + (c
^2*(b*c - 2*a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(Sqrt[d]
*x)/Sqrt[c + d*x^2]])/(16*d^(5/2)*(a + b*x^2))
```

Rubi [A] time = 0.398316, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162

$$\frac{c^2\sqrt{a^2+2abx^2+b^2x^4}(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}(a+bx^2)}-\frac{cx\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(bc-2ad)}{16d^2(a+bx^2)}+\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{6d(a+bx^2)}-\frac{x^3\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(bc-2ad)}{8d(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```


```
[Out] -(c*(b*c - 2*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(16*d^2*(a + b*x^2)) - ((b*c - 2*a*d)*x^3*Sqrt[c + d*x^2]*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*d*(a + b*x^2)) + (b*x^3*(c + d*x
^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*d*(a + b*x^2)) + (c
^2*(b*c - 2*a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(Sqrt[d]
*x)/Sqrt[c + d*x^2]])/(16*d^(5/2)*(a + b*x^2))
```

Rubi in Sympy [A] time = 27.6088, size = 182, normalized size = 0.75

$$\frac{bx^3\left(c+dx^2\right)^{\frac{3}{2}}\sqrt{(a+bx^2)^2}}{6d\left(a+bx^2\right)}-\frac{c^2\left(2ad-bc\right)\sqrt{(a+bx^2)^2}\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{\frac{5}{2}}\left(a+bx^2\right)}+\frac{cx\sqrt{c+dx^2}\left(2ad-bc\right)\sqrt{(a+bx^2)^2}}{16d^2\left(a+bx^2\right)}+\frac{x^3\sqrt{c+dx^2}\left(2ad-bc\right)\sqrt{(a+bx^2)^2}}{8d\left(a+bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] b*x**3*(c + d*x**2)**(3/2)*sqrt((a + b*x**2)**2)/(6*d*(a + b*x**2
)) - c**2*(2*a*d - b*c)*sqrt((a + b*x**2)**2)*atanh(sqrt(d)*x/sqr
t(c + d*x**2))/(16*d**(5/2)*(a + b*x**2)) + c*x*sqrt(c + d*x**2)*
(2*a*d - b*c)*sqrt((a + b*x**2)**2)/(16*d**2*(a + b*x**2)) + x**3
*sqrt(c + d*x**2)*(2*a*d - b*c)*sqrt((a + b*x**2)**2)/(8*d*(a + b
*x**2))
```

Mathematica [A] time = 0.138333, size = 121, normalized size = 0.5

$$\frac{\sqrt{(a+bx^2)^2}\left(\sqrt{d}x\sqrt{c+dx^2}\left(6ad\left(c+2dx^2\right)+b\left(-3c^2+2cdx^2+8d^2x^4\right)\right)+3c^2(bc-2ad)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)\right)}{48d^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(Sqrt[d]*x*Sqrt[c + d*x^2]*(6*a*d*(c + 2*d*x
^2) + b*(-3*c^2 + 2*c*d*x^2 + 8*d^2*x^4)) + 3*c^2*(b*c - 2*a*d)*L
og[d*x + Sqrt[d]*Sqrt[c + d*x^2]]))/(48*d^(5/2)*(a + b*x^2))
```

Maple [A] time = 0.016, size = 164, normalized size = 0.7

$$\frac{1}{48\,bx^2+48\,a}\sqrt{(bx^2+a)^2}\left(8\,bx^3\left(dx^2+c\right)^{3/2}d^{7/2}+12\,ax\left(dx^2+c\right)^{3/2}d^{7/2}-6\,bcx\left(dx^2+c\right)^{3/2}d^{5/2}-6\,acx\sqrt{dx^2+c}d^{7/2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/48*((b*x^2+a)^2)^(1/2)*(8*b*x^3*(d*x^2+c)^(3/2)*d^(7/2)+12*a*x*
(d*x^2+c)^(3/2)*d^(7/2)-6*b*c*x*(d*x^2+c)^(3/2)*d^(5/2)-6*a*c*x*(
d*x^2+c)^(1/2)*d^(7/2)+3*b*c^2*x*(d*x^2+c)^(1/2)*d^(5/2)-6*a*c^2*
ln(x*d^(1/2)+(d*x^2+c)^(1/2))*d^3+3*b*c^3*ln(x*d^(1/2)+(d*x^2+c)^
(1/2))*d^2)/(b*x^2+a)/d^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.295258, size = 1, normalized size = 0.

$$\left[\frac{2\left(8\,bd^2x^5+2\left(bcd+6\,ad^2\right)x^3-3\left(bc^2-2\,acd\right)x\right)\sqrt{dx^2+c}\sqrt{d}-3\left(bc^3-2\,ac^2d\right)\log\left(2\sqrt{dx^2+c}dx-\left(2\,dx^2+c\right)\sqrt{d}\right)}{96\,d^{\frac{5}{2}}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/96*(2*(8*b*d^2*x^5 + 2*(b*c*d + 6*a*d^2)*x^3 - 3*(b*c^2 - 2*a*
c*d)*x)*sqrt(d*x^2 + c)*sqrt(d) - 3*(b*c^3 - 2*a*c^2*d)*log(2*sqr
t(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)))/d^(5/2), 1/48*((8*b*d^
2*x^5 + 2*(b*c*d + 6*a*d^2)*x^3 - 3*(b*c^2 - 2*a*c*d)*x)*sqrt(d*x
^2 + c)*sqrt(-d) + 3*(b*c^3 - 2*a*c^2*d)*arctan(sqrt(-d)*x/sqrt(d
*x^2 + c)))/(sqrt(-d)*d^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

[Out] Timed out

GIAC/XCAS [A] time = 0.277472, size = 211, normalized size = 0.87

$$\frac{1}{48}\Bigg(2\bigg(4\,bx^2\mathrm{sign}\left(bx^2+a\right)+\frac{bcd^3\mathrm{sign}\left(bx^2+a\right)+6\,ad^4\mathrm{sign}\left(bx^2+a\right)}{d^4}\bigg)x^2-\frac{3\,\big(bc^2d^2\mathrm{sign}\left(bx^2+a\right)-2\,acd^3\mathrm{sign}\,(bx^2+a}{d^4}$$
$$-\frac{\left(bc^3\mathrm{sign}\left(bx^2+a\right)-2\,ac^2d\mathrm{sign}\left(bx^2+a\right)\right)\ln\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{16\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2)*x^2,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b*x^2*sign(b*x^2 + a) + (b*c*d^3*sign(b*x^2 + a) + 6*a
*d^4*sign(b*x^2 + a))/d^4)*x^2 - 3*(b*c^2*d^2*sign(b*x^2 + a) - 2
*a*c*d^3*sign(b*x^2 + a))/d^4)*sqrt(d*x^2 + c)*x - 1/16*(b*c^3*si
gn(b*x^2 + a) - 2*a*c^2*d*sign(b*x^2 + a))*ln(abs(-sqrt(d)*x + sq
rt(d*x^2 + c)))/d^(5/2)
```

3.260 $\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=108

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{5/2}}{5d^2\left(a+bx^2\right)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}(bc-ad)}{3d^2\left(a+bx^2\right)}$$

```
[Out] -((b*c - a*d)*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/
(3*d^2*(a + b*x^2)) + (b*(c + d*x^2)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(5*d^2*(a + b*x^2))
```

Rubi [A] time = 0.280404, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{5/2}}{5d^2\left(a+bx^2\right)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}(bc-ad)}{3d^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((b*c - a*d)*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/
(3*d^2*(a + b*x^2)) + (b*(c + d*x^2)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(5*d^2*(a + b*x^2))
```

Rubi in Sympy [A] time = 18.1331, size = 75, normalized size = 0.69

$$\frac{b\left(c+dx^2\right)^{\frac{5}{2}}\sqrt{\left(a+bx^2\right)^2}}{5d^2\left(a+bx^2\right)}+\frac{\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)\sqrt{\left(a+bx^2\right)^2}}{3d^2\left(a+bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] b*(c + d*x**2)**(5/2)*sqrt((a + b*x**2)**2)/(5*d**2*(a + b*x**2))
 + (c + d*x**2)**(3/2)*(a*d - b*c)*sqrt((a + b*x**2)**2)/(3*d**2*
(a + b*x**2))
```

Mathematica [A] time = 0.0578292, size = 56, normalized size = 0.52

$$\frac{\sqrt{(a+bx^2)^2}\,(c+dx^2)^{3/2}\,(5ad-2bc+3bdx^2)}{15d^2\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(c + d*x^2)^(3/2)*(-2*b*c + 5*a*d + 3*b*d*x^
2))/(15*d^2*(a + b*x^2))
```

Maple [A] time = 0.006, size = 51, normalized size = 0.5

$$\frac{3\,bx^2d+5\,ad-2\,bc}{15\,d^2\,(bx^2+a)}\,(dx^2+c)^{\frac{3}{2}}\,\sqrt{(bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/15*(d*x^2+c)^(3/2)*(3*b*d*x^2+5*a*d-2*b*c)*((b*x^2+a)^2)^(1/2)/
d^2/(b*x^2+a)
```

Maxima [A] time = 0.708693, size = 68, normalized size = 0.63

$$\frac{\left(3\,bd^2x^4-2\,bc^2+5\,acd+\left(bcd+5\,ad^2\right)x^2\right)\sqrt{dx^2+c}}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2)*x,x, algorithm="maxima")
```

```
[Out] 1/15*(3*b*d^2*x^4 - 2*b*c^2 + 5*a*c*d + (b*c*d + 5*a*d^2)*x^2)*sq
rt(d*x^2 + c)/d^2
```

Fricas [A] time = 0.269568, size = 68, normalized size = 0.63

$$\frac{\left(3\,bd^2x^4-2\,bc^2+5\,acd+\left(bcd+5\,ad^2\right)x^2\right)\sqrt{dx^2+c}}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2)*x,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*d^2*x^4 - 2*b*c^2 + 5*a*c*d + (b*c*d + 5*a*d^2)*x^2)*sq
rt(d*x^2 + c)/d^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.271715, size = 85, normalized size = 0.79

$$\frac{5\left(dx^2+c\right)^{\frac{3}{2}}a\mathrm{sign}\left(bx^2+a\right)+\frac{\left(3\left(dx^2+c\right)^{\frac{5}{2}}-5\left(dx^2+c\right)^{\frac{3}{2}}c\right)b\mathrm{sign}\left(bx^2+a\right)}{d}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2)*x,x, algorithm="giac")
```

```
[Out] 1/15*(5*(d*x^2 + c)^(3/2)*a*sign(b*x^2 + a) + (3*(d*x^2 + c)^(5/2
) - 5*(d*x^2 + c)^(3/2)*c)*b*sign(b*x^2 + a)/d)/d
```

3.261 $\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=178

$$-\frac{c\sqrt{a^2+2abx^2+b^2x^4}(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx^2)}+\frac{bx\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{4d(a+bx^2)}-\frac{x\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(bc-4ad)}{8d(a+bx^2)}$$

```
[Out] -((b*c - 4*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(8*d*(a + b*x^2)) + (b*x*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(4*d*(a + b*x^2)) - (c*(b*c - 4*a*d)*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(3/2)
*(a + b*x^2))
```

Rubi [A] time = 0.190048, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$

$$-\frac{c\sqrt{a^2+2abx^2+b^2x^4}(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx^2)}+\frac{bx\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{4d(a+bx^2)}-\frac{x\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(bc-4ad)}{8d(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((b*c - 4*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(8*d*(a + b*x^2)) + (b*x*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(4*d*(a + b*x^2)) - (c*(b*c - 4*a*d)*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(3/2)
*(a + b*x^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c+dx^2}\sqrt{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2), x)
```

Mathematica [A] time = 0.084479, size = 98, normalized size = 0.55

$$\frac{\sqrt{(a+bx^2)^2}\left(\sqrt{d}x\sqrt{c+dx^2}\left(4ad+b\left(c+2dx^2\right)\right)+c(4ad-bc)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)\right)}{8d^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(Sqrt[d]*x*Sqrt[c + d*x^2]*(4*a*d + b*(c + 2
*d*x^2)) + c*(-(b*c) + 4*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])
)/(8*d^(3/2)*(a + b*x^2))
```

Maple [A] time = 0.01, size = 122, normalized size = 0.7

$$\frac{1}{8\,bx^2+8\,a}\sqrt{(bx^2+a)^2}\left(2\,bx\left(dx^2+c\right)^{3/2}d^{3/2}+4\,ax\sqrt{dx^2+c}d^{5/2}-bcx\sqrt{dx^2+c}d^{\frac{3}{2}}+4\,ac\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)d^2-bc^2\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/8*((b*x^2+a)^2)^(1/2)*(2*b*x*(d*x^2+c)^(3/2)*d^(3/2)+4*a*x*(d*x
^2+c)^(1/2)*d^(5/2)-b*c*x*(d*x^2+c)^(1/2)*d^(3/2)+4*a*c*ln(x*d^(1
/2)+(d*x^2+c)^(1/2))*d^2-b*c^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2))*d)/(
b*x^2+a)/d^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 0.2857, size = 1, normalized size = 0.01

$$\left[\frac{2\left(2\,bdx^3+(bc+4\,ad)x\right)\sqrt{dx^2+c}\sqrt{d}-\left(bc^2-4\,acd\right)\log\left(-2\sqrt{dx^2+c}dx-\left(2\,dx^2+c\right)\sqrt{d}\right)}{16\,d^{\frac{3}{2}}},\frac{\left(2\,bdx^3+(bc+4\,ad)x\right)\sqrt{dx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(2*b*d*x^3 + (b*c + 4*a*d)*x)*sqrt(d*x^2 + c)*sqrt(d) -
(b*c^2 - 4*a*c*d)*log(-2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt
(d)))/d^(3/2), 1/8*((2*b*d*x^3 + (b*c + 4*a*d)*x)*sqrt(d*x^2 + c)
*sqrt(-d) - (b*c^2 - 4*a*c*d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))
/(sqrt(-d)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.276751, size = 147, normalized size = 0.83

$$\frac{1}{8}\left(2\,bx^2\mathrm{sign}\left(bx^2+a\right)+\frac{bcd\mathrm{sign}\left(bx^2+a\right)+4\,ad^2\mathrm{sign}\left(bx^2+a\right)}{d^2}\right)\sqrt{dx^2+c}x+\frac{\left(bc^2\mathrm{sign}\left(bx^2+a\right)-4\,acd\mathrm{sign}\left(bx^2+a\right)\right)\ln\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{8\,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x^2*sign(b*x^2 + a) + (b*c*d*sign(b*x^2 + a) + 4*a*d^2*s
ign(b*x^2 + a))/d^2)*sqrt(d*x^2 + c)*x + 1/8*(b*c^2*sign(b*x^2 +
a) - 4*a*c*d*sign(b*x^2 + a))*ln(abs(-sqrt(d)*x + sqrt(d*x^2 + c)
))/d^(3/2)
```

3.262 $\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx$

Optimal. Leaf size=152

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{3d\left(a+bx^2\right)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}}{a+bx^2}-\frac{a\sqrt{c}\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx^2}$$

```
[Out] (a*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) +
 (b*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a +
b*x^2)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[Sqrt
[c + d*x^2]/Sqrt[c]])/(a + b*x^2)
```

Rubi [A] time = 0.305133, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{3d\left(a+bx^2\right)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}}{a+bx^2}-\frac{a\sqrt{c}\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
```

```
[Out] (a*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) +
 (b*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a +
b*x^2)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[Sqrt
[c + d*x^2]/Sqrt[c]])/(a + b*x^2)
```

Rubi in Sympy [A] time = 20.0972, size = 105, normalized size = 0.69

$$-\frac{a\sqrt{c}\sqrt{\left(a+bx^2\right)^2}\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx^2}+\frac{a\sqrt{c+dx^2}\sqrt{\left(a+bx^2\right)^2}}{a+bx^2}+\frac{b\left(c+dx^2\right)^{\frac{3}{2}}\sqrt{\left(a+bx^2\right)^2}}{3d\left(a+bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x,x)
```

```
[Out] -a*sqrt(c)*sqrt((a + b*x**2)**2)*atanh(sqrt(c + d*x**2)/sqrt(c))/
(a + b*x**2) + a*sqrt(c + d*x**2)*sqrt((a + b*x**2)**2)/(a + b*x*
*2) + b*(c + d*x**2)**(3/2)*sqrt((a + b*x**2)**2)/(3*d*(a + b*x**
2))
```

Mathematica [A] time = 0.113774, size = 96, normalized size = 0.63

$$\frac{\sqrt{(a+bx^2)^2}\left(\sqrt{c+dx^2}\left(3ad+bc+bdx^2\right)-3a\sqrt{c}d\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)+3a\sqrt{c}d\log(x)\right)}{3d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(Sqrt[c + d*x^2]*(b*c + 3*a*d + b*d*x^2) + 3
*a*Sqrt[c]*d*Log[x] - 3*a*Sqrt[c]*d*Log[c + Sqrt[c]*Sqrt[c + d*x^
2]]))/(3*d*(a + b*x^2))
```

Maple [A] time = 0.01, size = 79, normalized size = 0.5

$$\frac{1}{(3\,bx^2+3\,a)\,d}\sqrt{(bx^2+a)^2}\left(b\left(dx^2+c\right)^{\frac{3}{2}}-3\sqrt{c}\ln\left(2\,\frac{\sqrt{c}\sqrt{dx^2+c}+c}{x}\right)ad+3\sqrt{dx^2+c}ad\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x)
```

```
[Out] 1/3*((b*x^2+a)^2)^(1/2)*(b*(d*x^2+c)^(3/2)-3*c^(1/2)*ln(2*(c^(1/2
)*(d*x^2+c)^(1/2)+c)/x)*a*d+3*(d*x^2+c)^(1/2)*a*d)/(b*x^2+a)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.279204, size = 1, normalized size = 0.01

$$\left[\frac{3\,a\sqrt{c}d\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,\left(bdx^2+bc+3\,ad\right)\sqrt{dx^2+c}}{6\,d},\ -\frac{3\,a\sqrt{-c}d\arctan\left(\frac{c}{\sqrt{dx^2+c}\sqrt{-c}}\right)-\left(bdx^2+bc+3\,ad\right)\sqrt{dx^2+c}}{3\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2)/x,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*sqrt(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c
)/x^2) + 2*(b*d*x^2 + b*c + 3*a*d)*sqrt(d*x^2 + c))/d, -1/3*(3*a*
sqrt(-c)*d*arctan(c/(sqrt(d*x^2 + c)*sqrt(-c))) - (b*d*x^2 + b*c
+ 3*a*d)*sqrt(d*x^2 + c))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275687, size = 113, normalized size = 0.74

$$\frac{ac\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)\mathrm{sign}\left(bx^2+a\right)}{\sqrt{-c}}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}bd^2\mathrm{sign}\left(bx^2+a\right)+3\,\sqrt{dx^2+c}\,cad^3\mathrm{sign}\left(bx^2+a\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2)/x,x, algorithm="giac")
```

```
[Out] a*c*arctan(sqrt(d*x^2 + c)/sqrt(-c))*sign(b*x^2 + a)/sqrt(-c) + 1
/3*((d*x^2 + c)^(3/2)*b*d^2*sign(b*x^2 + a) + 3*sqrt(d*x^2 + c)*a
*d^3*sign(b*x^2 + a))/d^3
```

3.263 $\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx$

Optimal. Leaf size=177

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{cx\left(a+bx^2\right)}+\frac{x\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(2ad+bc)}{2c\left(a+bx^2\right)}+\frac{\sqrt{a^2+2abx^2+b^2x^4}(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\left(a+bx^2\right)}$$

```
[Out] ((b*c + 2*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
/(2*c*(a + b*x^2)) - (a*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(c*x*(a + b*x^2)) + ((b*c + 2*a*d)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*(a +
 b*x^2))
```

Rubi [A] time = 0.287868, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{cx\left(a+bx^2\right)}+\frac{x\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(2ad+bc)}{2c\left(a+bx^2\right)}+\frac{\sqrt{a^2+2abx^2+b^2x^4}(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

```
[Out] ((b*c + 2*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
/(2*c*(a + b*x^2)) - (a*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(c*x*(a + b*x^2)) + ((b*c + 2*a*d)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*(a +
 b*x^2))
```

Rubi in Sympy [A] time = 17.751, size = 128, normalized size = 0.72

$$-\frac{a\left(c+dx^2\right)^{\frac{3}{2}}\sqrt{\left(a+bx^2\right)^2}}{cx\left(a+bx^2\right)}+\frac{(2ad+bc)\sqrt{\left(a+bx^2\right)^2}\operatorname{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\left(a+bx^2\right)}+\frac{x\sqrt{c+dx^2}\,(2ad+bc)\sqrt{\left(a+bx^2\right)^2}}{2c\left(a+bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**2,x)
```

```
[Out] -a*(c + d*x**2)**(3/2)*sqrt((a + b*x**2)**2)/(c*x*(a + b*x**2)) +
 (2*a*d + b*c)*sqrt((a + b*x**2)**2)*atanh(sqrt(d)*x/sqrt(c + d*x
**2))/(2*sqrt(d)*(a + b*x**2)) + x*sqrt(c + d*x**2)*(2*a*d + b*c)
*sqrt((a + b*x**2)**2)/(2*c*(a + b*x**2))
```

Mathematica [A] time = 0.080046, size = 93, normalized size = 0.53

$$\frac{\sqrt{(a+bx^2)^2}\left(\sqrt{d}\left(bx^2-2a\right)\sqrt{c+dx^2}+x(2ad+bc)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)\right)}{2\sqrt{d}x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(Sqrt[d]*(-2*a + b*x^2)*Sqrt[c + d*x^2] + (b
*c + 2*a*d)*x*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]]))/(2*Sqrt[d]*x*(
a + b*x^2))
```

Maple [A] time = 0.013, size = 128, normalized size = 0.7

$$\frac{1}{(2\,bx^2+2\,a)\,cx}\sqrt{(bx^2+a)^2}\left(2\,ad\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)cx+2\,ad^{3/2}x^2\sqrt{dx^2+c}+bx^2\sqrt{dx^2+c}\sqrt{d}c-2\,a\left(dx^2+c\right)^{3/2}\sqrt{d}+b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x)
```

```
[Out] 1/2*((b*x^2+a)^2)^(1/2)*(2*a*d*ln(x*d^(1/2)+(d*x^2+c)^(1/2))*c*x+
2*a*d^(3/2)*x^2*(d*x^2+c)^(1/2)+b*x^2*(d*x^2+c)^(1/2)*d^(1/2)*c-2
*a*(d*x^2+c)^(3/2)*d^(1/2)+b*c^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2))*x)
/(b*x^2+a)/d^(1/2)/c/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.289159, size = 1, normalized size = 0.01

$$\left[\frac{(bc+2\,ad)x\log\left(-2\,\sqrt{dx^2+c}dx-\left(2\,dx^2+c\right)\sqrt{d}\right)+2\left(bx^2-2\,a\right)\sqrt{dx^2+c}\sqrt{d}}{4\,\sqrt{d}x}, \frac{(bc+2\,ad)x\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)+\left(bx^2-2\,a\right.}{2\,\sqrt{-d}x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*((b*c + 2*a*d)*x*log(-2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*
sqrt(d)) + 2*(b*x^2 - 2*a)*sqrt(d*x^2 + c)*sqrt(d))/(sqrt(d)*x),
1/2*((b*c + 2*a*d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b*x^2
- 2*a)*sqrt(d*x^2 + c)*sqrt(-d))/(sqrt(-d)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.282439, size = 157, normalized size = 0.89

$$\frac{1}{2}\,\sqrt{dx^2+c}bx\mathrm{sign}\left(bx^2+a\right)+\frac{2\,ac\sqrt{d}\mathrm{sign}\left(bx^2+a\right)}{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c} -\frac{\left(bc\sqrt{d}\mathrm{sign}\left(bx^2+a\right)+2\,ad^{\frac{3}{2}}\mathrm{sign}\left(bx^2+a\right)\right)\ln\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b*x*sign(b*x^2 + a) + 2*a*c*sqrt(d)*sign(b*x^
2 + a)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c) - 1/4*(b*c*sqrt(d)*s
ign(b*x^2 + a) + 2*a*d^(3/2)*sign(b*x^2 + a))*ln((sqrt(d)*x - sqr
t(d*x^2 + c))^2)/d
```

3.264 $\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$

Optimal. Leaf size=177

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,(c+dx^2)^{3/2}}{2cx^2(a+bx^2)}+\frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(ad+2bc)}{2c(a+bx^2)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}\,(a+bx^2)}$$

```
[Out] ((2*b*c + a*d)*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(
2*c*(a + b*x^2)) - (a*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(2*c*x^2*(a + b*x^2)) - ((2*b*c + a*d)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*Sqrt[c]*(a + b
*x^2))
```

Rubi [A] time = 0.362906, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,(c+dx^2)^{3/2}}{2cx^2(a+bx^2)}+\frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(ad+2bc)}{2c(a+bx^2)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

```
[Out] ((2*b*c + a*d)*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(
2*c*(a + b*x^2)) - (a*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(2*c*x^2*(a + b*x^2)) - ((2*b*c + a*d)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*Sqrt[c]*(a + b
*x^2))
```

Rubi in Sympy [A] time = 22.7992, size = 124, normalized size = 0.7

$$-\frac{a\left(c+dx^2\right)^{\frac{3}{2}}\sqrt{\left(a+bx^2\right)^2}}{2cx^2\left(a+bx^2\right)}+\frac{\sqrt{c+dx^2}\left(\frac{ad}{2}+bc\right)\sqrt{\left(a+bx^2\right)^2}}{c\left(a+bx^2\right)}-\frac{\left(\frac{ad}{2}+bc\right)\sqrt{\left(a+bx^2\right)^2}\operatorname{atanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}\left(a+bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**3,x)
```

```
[Out] -a*(c + d*x**2)**(3/2)*sqrt((a + b*x**2)**2)/(2*c*x**2*(a + b*x**
2)) + sqrt(c + d*x**2)*(a*d/2 + b*c)*sqrt((a + b*x**2)**2)/(c*(a
+ b*x**2)) - (a*d/2 + b*c)*sqrt((a + b*x**2)**2)*atanh(sqrt(c + d
*x**2)/sqrt(c))/(sqrt(c)*(a + b*x**2))
```

Mathematica [A] time = 0.111068, size = 109, normalized size = 0.62

$$\frac{\sqrt{(a+bx^2)^2}\left(\sqrt{c}\left(2bx^2-a\right)\sqrt{c+dx^2}+x^2\log(x)(ad+2bc)-x^2(ad+2bc)\log\left(\sqrt{c}\sqrt{c+dx^2}+c\right)\right)}{2\sqrt{c}x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(Sqrt[c]*(-a + 2*b*x^2)*Sqrt[c + d*x^2] + (2
*b*c + a*d)*x^2*Log[x] - (2*b*c + a*d)*x^2*Log[c + Sqrt[c]*Sqrt[c
 + d*x^2]]))/(2*Sqrt[c]*x^2*(a + b*x^2))
```

Maple [A] time = 0.015, size = 139, normalized size = 0.8

$$-\frac{1}{(2\,bx^2+2\,a)\,x^2}\sqrt{(bx^2+a)^2}\left(2\,c^2\ln\left(2\,\frac{\sqrt{c}\sqrt{dx^2+c}+c}{x}\right)bx^2+ad\ln\left(2\,\frac{\sqrt{c}\sqrt{dx^2+c}+c}{x}\right)x^2c-ad\sqrt{dx^2+c}x^2\sqrt{c}-2\,\sqrt{dx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x)
```

```
[Out] -1/2*((b*x^2+a)^2)^(1/2)*(2*c^2*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/
x)*b*x^2+a*d*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*x^2*c-a*d*(d*x^2
+c)^(1/2)*x^2*c^(1/2)-2*(d*x^2+c)^(1/2)*b*x^2*c^(3/2)+a*(d*x^2+c)
^(3/2)*c^(1/2))/(b*x^2+a)/x^2/c^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.287476, size = 1, normalized size = 0.01

$$\left[\frac{(2\,bc+ad)x^2\log\left(-\frac{(dx^2+2\,c)\sqrt{c}-2\,\sqrt{dx^2+c}c}{x^2}\right)+2\,\left(2\,bx^2-a\right)\sqrt{dx^2+c}\sqrt{c}}{4\,\sqrt{c}x^2},\ -\frac{(2\,bc+ad)x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-\left(2\,bx^2-a\right)\sqrt{dx^2+c}\sqrt{-c}}{2\,\sqrt{-c}x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*((2*b*c + a*d)*x^2*log(-((d*x^2 + 2*c)*sqrt(c) - 2*sqrt(d*x^
2 + c)*c)/x^2) + 2*(2*b*x^2 - a)*sqrt(d*x^2 + c)*sqrt(c))/(sqrt(c
)*x^2), -1/2*((2*b*c + a*d)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c))
- (2*b*x^2 - a)*sqrt(d*x^2 + c)*sqrt(-c))/(sqrt(-c)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.27651, size = 135, normalized size = 0.76

$$\frac{2\,\sqrt{dx^2+c}bd\mathrm{sign}\left(bx^2+a\right)+\frac{\left(2\,bcd\mathrm{sign}\left(bx^2+a\right)+ad^2\mathrm{sign}\left(bx^2+a\right)\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{dx^2+c}ad\mathrm{sign}\left(bx^2+a\right)}{x^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(d*x^2 + c)*b*d*sign(b*x^2 + a) + (2*b*c*d*sign(b*x^2
+ a) + a*d^2*sign(b*x^2 + a))*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sq
rt(-c) - sqrt(d*x^2 + c)*a*d*sign(b*x^2 + a)/x^2)/d
```

3.265 $\int x^3 \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)\, dx$

Optimal. Leaf size=78

$$\frac{1}{8}x^8\left(e(ae+2bd)+cd^2\right)+\frac{1}{6}dx^6(2ae+bd)+\frac{1}{4}ad^2x^4+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{12}ce^2x^{12}$$

```
[Out] (a*d^2*x^4)/4 + (d*(b*d + 2*a*e)*x^6)/6 + ((c*d^2 + e*(2*b*d + a*
e))*x^8)/8 + (e*(2*c*d + b*e)*x^10)/10 + (c*e^2*x^12)/12
```

Rubi [A] time = 0.322713, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{1}{8}x^8\left(e(ae+2bd)+cd^2\right)+\frac{1}{6}dx^6(2ae+bd)+\frac{1}{4}ad^2x^4+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{12}ce^2x^{12}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d^2*x^4)/4 + (d*(b*d + 2*a*e)*x^6)/6 + ((c*d^2 + e*(2*b*d + a*
e))*x^8)/8 + (e*(2*c*d + b*e)*x^10)/10 + (c*e^2*x^12)/12
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ad^2\int^{x^2} x\,dx}{2}+\frac{ce^2x^{12}}{12}+\frac{dx^6\,(2ae+bd)}{6}+\frac{ex^{10}\,(be+2cd)}{10}+x^8\left(\frac{ae^2}{8}+\frac{bde}{4}+\frac{cd^2}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*Integral(x, (x, x**2))/2 + c*e**2*x**12/12 + d*x**6*(2*a*e
 + b*d)/6 + e*x**10*(b*e + 2*c*d)/10 + x**8*(a*e**2/8 + b*d*e/4 +
 c*d**2/8)
```

Mathematica [A] time = 0.0401243, size = 72, normalized size = 0.92

$$\frac{1}{120}x^4\left(15x^4\left(e(ae+2bd)+cd^2\right)+20dx^2(2ae+bd)+30ad^2+12ex^6(be+2cd)+10ce^2x^8\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (x^4*(30*a*d^2 + 20*d*(b*d + 2*a*e)*x^2 + 15*(c*d^2 + e*(2*b*d +
a*e))*x^4 + 12*e*(2*c*d + b*e)*x^6 + 10*c*e^2*x^8))/120
```

Maple [A] time = 0.002, size = 73, normalized size = 0.9

$$\frac{ce^2x^{12}}{12}+\frac{\left(be^2+2\,cde\right)x^{10}}{10}+\frac{\left(ae^2+2\,bde+cd^2\right)x^8}{8}+\frac{\left(2\,ade+bd^2\right)x^6}{6}+\frac{ad^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/12*c*e^2*x^12+1/10*(b*e^2+2*c*d*e)*x^10+1/8*(a*e^2+2*b*d*e+c*d^
2)*x^8+1/6*(2*a*d*e+b*d^2)*x^6+1/4*a*d^2*x^4
```

Maxima [A] time = 0.702282, size = 97, normalized size = 1.24

$$\frac{1}{12}\,ce^2x^{12}+\frac{1}{10}\left(2\,cde+be^2\right)x^{10}+\frac{1}{8}\left(cd^2+2\,bde+ae^2\right)x^8+\frac{1}{4}\,ad^2x^4+\frac{1}{6}\left(bd^2+2\,ade\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2*x^3,x, algorithm="maxima")
```

```
[Out] 1/12*c*e^2*x^12 + 1/10*(2*c*d*e + b*e^2)*x^10 + 1/8*(c*d^2 + 2*b*
d*e + a*e^2)*x^8 + 1/4*a*d^2*x^4 + 1/6*(b*d^2 + 2*a*d*e)*x^6
```

Fricas [A] time = 0.232526, size = 1, normalized size = 0.01

$$\frac{1}{12}x^{12}e^2c+\frac{1}{5}x^{10}edc+\frac{1}{10}x^{10}e^2b+\frac{1}{8}x^8d^2c+\frac{1}{4}x^8edb+\frac{1}{8}x^8e^2a+\frac{1}{6}x^6d^2b+\frac{1}{3}x^6eda+\frac{1}{4}x^4d^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2*x^3,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*e^2*c + 1/5*x^10*e*d*c + 1/10*x^10*e^2*b + 1/8*x^8*d^2*
c + 1/4*x^8*e*d*b + 1/8*x^8*e^2*a + 1/6*x^6*d^2*b + 1/3*x^6*e*d*a
 + 1/4*x^4*d^2*a
```

Sympy [A] time = 0.127716, size = 76, normalized size = 0.97

$$\frac{ad^2x^4}{4} + \frac{ce^2x^{12}}{12} + x^{10}\left(\frac{be^2}{10} + \frac{cde}{5}\right) + x^8\left(\frac{ae^2}{8} + \frac{bde}{4} + \frac{cd^2}{8}\right) + x^6\left(\frac{ade}{3} + \frac{bd^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x**4/4 + c*e**2*x**12/12 + x**10*(b*e**2/10 + c*d*e/5) + x
**8*(a*e**2/8 + b*d*e/4 + c*d**2/8) + x**6*(a*d*e/3 + b*d**2/6)
```

GIAC/XCAS [A] time = 0.26878, size = 107, normalized size = 1.37

$$\frac{1}{12}\,cx^{12}e^2 + \frac{1}{5}\,cdx^{10}e + \frac{1}{10}\,bx^{10}e^2 + \frac{1}{8}\,cd^2x^8 + \frac{1}{4}\,bdx^8e + \frac{1}{8}\,ax^8e^2 + \frac{1}{6}\,bd^2x^6 + \frac{1}{3}\,adx^6e + \frac{1}{4}\,ad^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2*x^3,x, algorithm="giac")
```

```
[Out] 1/12*c*x^12*e^2 + 1/5*c*d*x^10*e + 1/10*b*x^10*e^2 + 1/8*c*d^2*x^
8 + 1/4*b*d*x^8*e + 1/8*a*x^8*e^2 + 1/6*b*d^2*x^6 + 1/3*a*d*x^6*e
 + 1/4*a*d^2*x^4
```

3.266 $\int x^2 \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=78

$$\frac{1}{7}x^7\left(e(ae+2bd)+cd^2\right)+\frac{1}{5}dx^5(2ae+bd)+\frac{1}{3}ad^2x^3+\frac{1}{9}ex^9(be+2cd)+\frac{1}{11}ce^2x^{11}$$

```
[Out] (a*d^2*x^3)/3 + (d*(b*d + 2*a*e)*x^5)/5 + ((c*d^2 + e*(2*b*d + a*
e))*x^7)/7 + (e*(2*c*d + b*e)*x^9)/9 + (c*e^2*x^11)/11
```

Rubi [A] time = 0.166276, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04

$$\frac{1}{7}x^7\left(e(ae+2bd)+cd^2\right)+\frac{1}{5}dx^5(2ae+bd)+\frac{1}{3}ad^2x^3+\frac{1}{9}ex^9(be+2cd)+\frac{1}{11}ce^2x^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d^2*x^3)/3 + (d*(b*d + 2*a*e)*x^5)/5 + ((c*d^2 + e*(2*b*d + a*
e))*x^7)/7 + (e*(2*c*d + b*e)*x^9)/9 + (c*e^2*x^11)/11
```

Rubi in Sympy [A] time = 23.8642, size = 75, normalized size = 0.96

$$\frac{ad^2x^3}{3}+\frac{ce^2x^{11}}{11}+\frac{dx^5\left(2ae+bd\right)}{5}+\frac{ex^9\left(be+2cd\right)}{9}+x^7\left(\frac{ae^2}{7}+\frac{2bde}{7}+\frac{cd^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x**3/3 + c*e**2*x**11/11 + d*x**5*(2*a*e + b*d)/5 + e*x**9
*(b*e + 2*c*d)/9 + x**7*(a*e**2/7 + 2*b*d*e/7 + c*d**2/7)
```

Mathematica [A] time = 0.0271464, size = 78, normalized size = 1.

$$\frac{1}{7}x^7\left(ae^2+2bde+cd^2\right)+\frac{1}{5}dx^5(2ae+bd)+\frac{1}{3}ad^2x^3+\frac{1}{9}ex^9(be+2cd)+\frac{1}{11}ce^2x^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d^2*x^3)/3 + (d*(b*d + 2*a*e)*x^5)/5 + ((c*d^2 + 2*b*d*e + a*e
^2)*x^7)/7 + (e*(2*c*d + b*e)*x^9)/9 + (c*e^2*x^11)/11
```

Maple [A] time = 0.001, size = 73, normalized size = 0.9

$$\frac{ce^2x^{11}}{11}+\frac{\left(be^2+2\,cde\right)x^9}{9}+\frac{\left(ae^2+2\,bde+cd^2\right)x^7}{7}+\frac{\left(2\,ade+bd^2\right)x^5}{5}+\frac{ad^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/11*c*e^2*x^11+1/9*(b*e^2+2*c*d*e)*x^9+1/7*(a*e^2+2*b*d*e+c*d^2)
*x^7+1/5*(2*a*d*e+b*d^2)*x^5+1/3*a*d^2*x^3
```

Maxima [A] time = 0.70196, size = 97, normalized size = 1.24

$$\frac{1}{11}\,ce^2x^{11}+\frac{1}{9}\left(2\,cde+be^2\right)x^9+\frac{1}{7}\left(cd^2+2\,bde+ae^2\right)x^7+\frac{1}{3}\,ad^2x^3+\frac{1}{5}\left(bd^2+2\,ade\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2*x^2,x, algorithm="maxima")
```

```
[Out] 1/11*c*e^2*x^11 + 1/9*(2*c*d*e + b*e^2)*x^9 + 1/7*(c*d^2 + 2*b*d*
e + a*e^2)*x^7 + 1/3*a*d^2*x^3 + 1/5*(b*d^2 + 2*a*d*e)*x^5
```

Fricas [A] time = 0.228654, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}e^2c+\frac{2}{9}x^9edc+\frac{1}{9}x^9e^2b+\frac{1}{7}x^7d^2c+\frac{2}{7}x^7edb+\frac{1}{7}x^7e^2a+\frac{1}{5}x^5d^2b+\frac{2}{5}x^5eda+\frac{1}{3}x^3d^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2*x^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^2*c + 2/9*x^9*e*d*c + 1/9*x^9*e^2*b + 1/7*x^7*d^2*c +
 2/7*x^7*e*d*b + 1/7*x^7*e^2*a + 1/5*x^5*d^2*b + 2/5*x^5*e*d*a +
1/3*x^3*d^2*a
```

Sympy [A] time = 0.130898, size = 82, normalized size = 1.05

$$\frac{ad^2x^3}{3}+\frac{ce^2x^{11}}{11}+x^9\left(\frac{be^2}{9}+\frac{2cde}{9}\right)+x^7\left(\frac{ae^2}{7}+\frac{2bde}{7}+\frac{cd^2}{7}\right)+x^5\left(\frac{2ade}{5}+\frac{bd^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x**3/3 + c*e**2*x**11/11 + x**9*(b*e**2/9 + 2*c*d*e/9) + x
**7*(a*e**2/7 + 2*b*d*e/7 + c*d**2/7) + x**5*(2*a*d*e/5 + b*d**2/
5)
```

GIAC/XCAS [A] time = 0.266618, size = 107, normalized size = 1.37

$$\frac{1}{11}\,cx^{11}e^2+\frac{2}{9}\,cdx^9e+\frac{1}{9}\,bx^9e^2+\frac{1}{7}\,cd^2x^7+\frac{2}{7}\,bdx^7e+\frac{1}{7}\,ax^7e^2+\frac{1}{5}\,bd^2x^5+\frac{2}{5}\,adx^5e+\frac{1}{3}\,ad^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2*x^2,x, algorithm="giac")
```

```
[Out] 1/11*c*x^11*e^2 + 2/9*c*d*x^9*e + 1/9*b*x^9*e^2 + 1/7*c*d^2*x^7 +
 2/7*b*d*x^7*e + 1/7*a*x^7*e^2 + 1/5*b*d^2*x^5 + 2/5*a*d*x^5*e +
1/3*a*d^2*x^3
```

3.267 $\int x \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) \, dx$

Optimal. Leaf size=75

$$\frac{\left(d+ex^2\right)^3 \left(ae^2-bde+cd^2\right)}{6e^3}-\frac{\left(d+ex^2\right)^4 (2cd-be)}{8e^3}+\frac{c \left(d+ex^2\right)^5}{10e^3}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(d + e*x^2)^3)/(6*e^3) - ((2*c*d - b*e)*
(d + e*x^2)^4)/(8*e^3) + (c*(d + e*x^2)^5)/(10*e^3)
```

Rubi [A] time = 0.286615, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{\left(d+ex^2\right)^3 \left(ae^2-bde+cd^2\right)}{6e^3}-\frac{\left(d+ex^2\right)^4 (2cd-be)}{8e^3}+\frac{c \left(d+ex^2\right)^5}{10e^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(d + e*x^2)^3)/(6*e^3) - ((2*c*d - b*e)*
(d + e*x^2)^4)/(8*e^3) + (c*(d + e*x^2)^5)/(10*e^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ce^2x^{10}}{10}+\frac{d^2 \int^{x^2} a\,dx}{2}+\frac{d\,(2ae+bd) \int^{x^2} x\,dx}{2}+\frac{ex^8\,(be+2cd)}{8}+x^6 \left(\frac{ae^2}{6}+\frac{bde}{3}+\frac{cd^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] c*e**2*x**10/10 + d**2*Integral(a, (x, x**2))/2 + d*(2*a*e + b*d)
*Integral(x, (x, x**2))/2 + e*x**8*(b*e + 2*c*d)/8 + x**6*(a*e**2
/6 + b*d*e/3 + c*d**2/6)
```

Mathematica [A] time = 0.0397876, size = 72, normalized size = 0.96

$$\frac{1}{120}x^2 \left(20x^4 \left(e(ae+2bd)+cd^2\right)+30dx^2(2ae+bd)+60ad^2+15ex^6(be+2cd)+12ce^2x^8\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (x^2*(60*a*d^2 + 30*d*(b*d + 2*a*e)*x^2 + 20*(c*d^2 + e*(2*b*d +
a*e))*x^4 + 15*e*(2*c*d + b*e)*x^6 + 12*c*e^2*x^8))/120
```

Maple [A] time = 0.001, size = 73, normalized size = 1.

$$\frac{ce^2x^{10}}{10}+\frac{\left(be^2+2\,cde\right)x^8}{8}+\frac{\left(ae^2+2\,bde+cd^2\right)x^6}{6}+\frac{\left(2\,ade+bd^2\right)x^4}{4}+\frac{ad^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/10*c*e^2*x^10+1/8*(b*e^2+2*c*d*e)*x^8+1/6*(a*e^2+2*b*d*e+c*d^2)
*x^6+1/4*(2*a*d*e+b*d^2)*x^4+1/2*a*d^2*x^2
```

Maxima [A] time = 0.694096, size = 97, normalized size = 1.29

$$\frac{1}{10}\,ce^2x^{10}+\frac{1}{8}\left(2\,cde+be^2\right)x^8+\frac{1}{6}\left(cd^2+2\,bde+ae^2\right)x^6+\frac{1}{2}\,ad^2x^2+\frac{1}{4}\left(bd^2+2\,ade\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2*x,x, algorithm="maxima")
```

```
[Out] 1/10*c*e^2*x^10 + 1/8*(2*c*d*e + b*e^2)*x^8 + 1/6*(c*d^2 + 2*b*d*
e + a*e^2)*x^6 + 1/2*a*d^2*x^2 + 1/4*(b*d^2 + 2*a*d*e)*x^4
```

Fricas [A] time = 0.229945, size = 1, normalized size = 0.01

$$\frac{1}{10}x^{10}e^2c+\frac{1}{4}x^8edc+\frac{1}{8}x^8e^2b+\frac{1}{6}x^6d^2c+\frac{1}{3}x^6edb+\frac{1}{6}x^6e^2a+\frac{1}{4}x^4d^2b+\frac{1}{2}x^4eda+\frac{1}{2}x^2d^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2*x,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e^2*c + 1/4*x^8*e*d*c + 1/8*x^8*e^2*b + 1/6*x^6*d^2*c +
 1/3*x^6*e*d*b + 1/6*x^6*e^2*a + 1/4*x^4*d^2*b + 1/2*x^4*e*d*a +
1/2*x^2*d^2*a
```

Sympy [A] time = 0.130887, size = 76, normalized size = 1.01

$$\frac{ad^2x^2}{2}+\frac{ce^2x^{10}}{10}+x^8\left(\frac{be^2}{8}+\frac{cde}{4}\right)+x^6\left(\frac{ae^2}{6}+\frac{bde}{3}+\frac{cd^2}{6}\right)+x^4\left(\frac{ade}{2}+\frac{bd^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x**2/2 + c*e**2*x**10/10 + x**8*(b*e**2/8 + c*d*e/4) + x**
6*(a*e**2/6 + b*d*e/3 + c*d**2/6) + x**4*(a*d*e/2 + b*d**2/4)
```

GIAC/XCAS [A] time = 0.267565, size = 107, normalized size = 1.43

$$\frac{1}{10}\,cx^{10}e^2+\frac{1}{4}\,cdx^8e+\frac{1}{8}\,bx^8e^2+\frac{1}{6}\,cd^2x^6+\frac{1}{3}\,bdx^6e+\frac{1}{6}\,ax^6e^2+\frac{1}{4}\,bd^2x^4+\frac{1}{2}\,adx^4e+\frac{1}{2}\,ad^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2*x,x, algorithm="giac")
```

```
[Out] 1/10*c*x^10*e^2 + 1/4*c*d*x^8*e + 1/8*b*x^8*e^2 + 1/6*c*d^2*x^6 +
 1/3*b*d*x^6*e + 1/6*a*x^6*e^2 + 1/4*b*d^2*x^4 + 1/2*a*d*x^4*e +
1/2*a*d^2*x^2
```

3.268 $\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) \, dx$

Optimal. Leaf size=73

$$\frac{1}{5}x^5\left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^
5)/5 + (e*(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Rubi [A] time = 0.116278, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{1}{5}x^5\left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^
5)/5 + (e*(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ce^2x^9}{9}+d^2\int a\,dx+\frac{dx^3\,(2ae+bd)}{3}+\frac{ex^7\,(be+2cd)}{7}+x^5\left(\frac{ae^2}{5}+\frac{2bde}{5}+\frac{cd^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] c*e**2*x**9/9 + d**2*Integral(a, x) + d*x**3*(2*a*e + b*d)/3 + e*
x**7*(b*e + 2*c*d)/7 + x**5*(a*e**2/5 + 2*b*d*e/5 + c*d**2/5)
```

Mathematica [A] time = 0.0277905, size = 73, normalized size = 1.

$$\frac{1}{5}x^5\left(ae^2+2bde+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + 2*b*d*e + a*e^2)*x^
5)/5 + (e*(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Maple [A] time = 0., size = 70, normalized size = 1.

$$\frac{ce^2x^9}{9}+\frac{\left(be^2+2\,cde\right)x^7}{7}+\frac{\left(ae^2+2\,bde+cd^2\right)x^5}{5}+\frac{\left(2\,ade+bd^2\right)x^3}{3}+ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/9*c*e^2*x^9+1/7*(b*e^2+2*c*d*e)*x^7+1/5*(a*e^2+2*b*d*e+c*d^2)*x
^5+1/3*(2*a*d*e+b*d^2)*x^3+a*d^2*x
```

Maxima [A] time = 0.700414, size = 93, normalized size = 1.27

$$\frac{1}{9}\,ce^2x^9+\frac{1}{7}\left(2\,cde+be^2\right)x^7+\frac{1}{5}\left(cd^2+2\,bde+ae^2\right)x^5+ad^2x+\frac{1}{3}\left(bd^2+2\,ade\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c*e^2*x^9 + 1/7*(2*c*d*e + b*e^2)*x^7 + 1/5*(c*d^2 + 2*b*d*e
+ a*e^2)*x^5 + a*d^2*x + 1/3*(b*d^2 + 2*a*d*e)*x^3
```

Fricas [A] time = 0.246286, size = 1, normalized size = 0.01

$$\frac{1}{9}x^9e^2c+\frac{2}{7}x^7edc+\frac{1}{7}x^7e^2b+\frac{1}{5}x^5d^2c+\frac{2}{5}x^5edb+\frac{1}{5}x^5e^2a+\frac{1}{3}x^3d^2b+\frac{2}{3}x^3eda+xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^2*c + 2/7*x^7*e*d*c + 1/7*x^7*e^2*b + 1/5*x^5*d^2*c + 2
/5*x^5*e*d*b + 1/5*x^5*e^2*a + 1/3*x^3*d^2*b + 2/3*x^3*e*d*a + x*
d^2*a
```

Sympy [A] time = 0.126845, size = 78, normalized size = 1.07

$$ad^2x + \frac{ce^2x^9}{9} + x^7\left(\frac{be^2}{7} + \frac{2cde}{7}\right) + x^5\left(\frac{ae^2}{5} + \frac{2bde}{5} + \frac{cd^2}{5}\right) + x^3\left(\frac{2ade}{3} + \frac{bd^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x + c*e**2*x**9/9 + x**7*(b*e**2/7 + 2*c*d*e/7) + x**5*(a*
e**2/5 + 2*b*d*e/5 + c*d**2/5) + x**3*(2*a*d*e/3 + b*d**2/3)
```

GIAC/XCAS [A] time = 0.268713, size = 103, normalized size = 1.41

$$\frac{1}{9}\,cx^9e^2 + \frac{2}{7}\,cdx^7e + \frac{1}{7}\,bx^7e^2 + \frac{1}{5}\,cd^2x^5 + \frac{2}{5}\,bdx^5e + \frac{1}{5}\,ax^5e^2 + \frac{1}{3}\,bd^2x^3 + \frac{2}{3}\,adx^3e + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2,x, algorithm="giac")
```

```
[Out] 1/9*c*x^9*e^2 + 2/7*c*d*x^7*e + 1/7*b*x^7*e^2 + 1/5*c*d^2*x^5 + 2
/5*b*d*x^5*e + 1/5*a*x^5*e^2 + 1/3*b*d^2*x^3 + 2/3*a*d*x^3*e + a*
d^2*x
```

3.269 $\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x}\,dx$

Optimal. Leaf size=74

$$\frac{1}{4}x^4\left(e(ae+2bd)+cd^2\right)+\frac{1}{2}dx^2(2ae+bd)+ad^2\log(x)+\frac{1}{6}ex^6(be+2cd)+\frac{1}{8}ce^2x^8$$

```
[Out] (d*(b*d + 2*a*e)*x^2)/2 + ((c*d^2 + e*(2*b*d + a*e))*x^4)/4 + (e*
(2*c*d + b*e)*x^6)/6 + (c*e^2*x^8)/8 + a*d^2*Log[x]
```

Rubi [A] time = 0.194187, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{1}{4}x^4\left(e(ae+2bd)+cd^2\right)+\frac{1}{2}dx^2(2ae+bd)+ad^2\log(x)+\frac{1}{6}ex^6(be+2cd)+\frac{1}{8}ce^2x^8$$

Antiderivative was successfully verified.

```
[In]  Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] (d*(b*d + 2*a*e)*x^2)/2 + ((c*d^2 + e*(2*b*d + a*e))*x^4)/4 + (e*
(2*c*d + b*e)*x^6)/6 + (c*e^2*x^8)/8 + a*d^2*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ad^2\log\left(x^2\right)}{2}+\frac{ce^2x^8}{8}+\frac{ex^6\left(be+2cd\right)}{6}+\left(ae+\frac{bd}{2}\right)\int^{x^2}d\,dx+\left(\frac{ae^2}{2}+bde+\frac{cd^2}{2}\right)\int^{x^2}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x,x)
```

```
[Out] a*d**2*log(x**2)/2 + c*e**2*x**8/8 + e*x**6*(b*e + 2*c*d)/6 + (a*
e + b*d/2)*Integral(d, (x, x**2)) + (a*e**2/2 + b*d*e + c*d**2/2)
*Integral(x, (x, x**2))
```

Mathematica [A] time = 0.0387653, size = 74, normalized size = 1.

$$\frac{1}{4}x^4\left(ae^2+2bde+cd^2\right)+\frac{1}{2}dx^2(2ae+bd)+ad^2\log(x)+\frac{1}{6}ex^6(be+2cd)+\frac{1}{8}ce^2x^8$$

Antiderivative was successfully verified.

```
[In]  Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] (d*(b*d + 2*a*e)*x^2)/2 + ((c*d^2 + 2*b*d*e + a*e^2)*x^4)/4 + (e*
(2*c*d + b*e)*x^6)/6 + (c*e^2*x^8)/8 + a*d^2*Log[x]
```

Maple [A] time = 0.004, size = 77, normalized size = 1.

$$\frac{ce^2x^8}{8}+\frac{x^6be^2}{6}+\frac{x^6cde}{3}+\frac{x^4ae^2}{4}+\frac{x^4bde}{2}+\frac{x^4cd^2}{4}+x^2ade+\frac{x^2bd^2}{2}+ad^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x)
```

```
[Out] 1/8*c*e^2*x^8+1/6*x^6*b*e^2+1/3*x^6*c*d*e+1/4*x^4*a*e^2+1/2*x^4*b
*d*e+1/4*x^4*c*d^2+x^2*a*d*e+1/2*x^2*b*d^2+a*d^2*ln(x)
```

Maxima [A] time = 0.700413, size = 99, normalized size = 1.34

$$\frac{1}{8}ce^2x^8+\frac{1}{6}\left(2\,cde+be^2\right)x^6+\frac{1}{4}\left(cd^2+2\,bde+ae^2\right)x^4+\frac{1}{2}ad^2\log\left(x^2\right)+\frac{1}{2}\left(bd^2+2\,ade\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*c*e^2*x^8 + 1/6*(2*c*d*e + b*e^2)*x^6 + 1/4*(c*d^2 + 2*b*d*e
+ a*e^2)*x^4 + 1/2*a*d^2*log(x^2) + 1/2*(b*d^2 + 2*a*d*e)*x^2
```

Fricas [A] time = 0.252087, size = 95, normalized size = 1.28

$$\frac{1}{8}ce^2x^8+\frac{1}{6}\left(2\,cde+be^2\right)x^6+\frac{1}{4}\left(cd^2+2\,bde+ae^2\right)x^4+ad^2\log\left(x\right)+\frac{1}{2}\left(bd^2+2\,ade\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*c*e^2*x^8 + 1/6*(2*c*d*e + b*e^2)*x^6 + 1/4*(c*d^2 + 2*b*d*e
+ a*e^2)*x^4 + a*d^2*log(x) + 1/2*(b*d^2 + 2*a*d*e)*x^2
```

Sympy [A] time = 1.26214, size = 73, normalized size = 0.99

$$ad^2 \log(x) + \frac{ce^2x^8}{8} + x^6\left(\frac{be^2}{6} + \frac{cde}{3}\right) + x^4\left(\frac{ae^2}{4} + \frac{bde}{2} + \frac{cd^2}{4}\right) + x^2\left(ade + \frac{bd^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x,x)
```

```
[Out] a*d**2*log(x) + c*e**2*x**8/8 + x**6*(b*e**2/6 + c*d*e/3) + x**4*
(a*e**2/4 + b*d*e/2 + c*d**2/4) + x**2*(a*d*e + b*d**2/2)
```

GIAC/XCAS [A] time = 0.269057, size = 107, normalized size = 1.45

$$\frac{1}{8}cx^8e^2 + \frac{1}{3}cdx^6e + \frac{1}{6}bx^6e^2 + \frac{1}{4}cd^2x^4 + \frac{1}{2}bdx^4e + \frac{1}{4}ax^4e^2 + \frac{1}{2}bd^2x^2 + adx^2e + \frac{1}{2}ad^2\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*c*x^8*e^2 + 1/3*c*d*x^6*e + 1/6*b*x^6*e^2 + 1/4*c*d^2*x^4 + 1
/2*b*d*x^4*e + 1/4*a*x^4*e^2 + 1/2*b*d^2*x^2 + a*d*x^2*e + 1/2*a*
d^2*ln(x^2)
```

3.270 $\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2}\,dx$

Optimal. Leaf size=71

$$\frac{1}{3}x^3\left(e(ae+2bd)+cd^2\right)+dx(2ae+bd)-\frac{ad^2}{x}+\frac{1}{5}ex^5(be+2cd)+\frac{1}{7}ce^2x^7$$

```
[Out] -((a*d^2)/x) + d*(b*d + 2*a*e)*x + ((c*d^2 + e*(2*b*d + a*e))*x^3
)/3 + (e*(2*c*d + b*e)*x^5)/5 + (c*e^2*x^7)/7
```

Rubi [A] time = 0.12503, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{1}{3}x^3\left(e(ae+2bd)+cd^2\right)+dx(2ae+bd)-\frac{ad^2}{x}+\frac{1}{5}ex^5(be+2cd)+\frac{1}{7}ce^2x^7$$

Antiderivative was successfully verified.

```
[In]  Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*d^2)/x) + d*(b*d + 2*a*e)*x + ((c*d^2 + e*(2*b*d + a*e))*x^3
)/3 + (e*(2*c*d + b*e)*x^5)/5 + (c*e^2*x^7)/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ad^2}{x}+\frac{ce^2x^7}{7}+\frac{ex^5\,(be+2cd)}{5}+x^3\left(\frac{ae^2}{3}+\frac{2bde}{3}+\frac{cd^2}{3}\right)+\frac{d\,(2ae+bd)\int b\,dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x**2,x)
```

```
[Out] -a*d**2/x + c*e**2*x**7/7 + e*x**5*(b*e + 2*c*d)/5 + x**3*(a*e**2
/3 + 2*b*d*e/3 + c*d**2/3) + d*(2*a*e + b*d)*Integral(b, x)/b
```

Mathematica [A] time = 0.0559375, size = 71, normalized size = 1.

$$\frac{1}{3}x^3\left(ae^2+2bde+cd^2\right)+dx(2ae+bd)-\frac{ad^2}{x}+\frac{1}{5}ex^5(be+2cd)+\frac{1}{7}ce^2x^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*d^2)/x) + d*(b*d + 2*a*e)*x + ((c*d^2 + 2*b*d*e + a*e^2)*x^3
)/3 + (e*(2*c*d + b*e)*x^5)/5 + (c*e^2*x^7)/7
```

Maple [A] time = 0.006, size = 75, normalized size = 1.1

$$\frac{ce^2x^7}{7}+\frac{x^5be^2}{5}+\frac{2\,x^5cde}{5}+\frac{x^3ae^2}{3}+\frac{2\,x^3bde}{3}+\frac{x^3cd^2}{3}+2\,xade+xbd^2-\frac{ad^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x)
```

```
[Out] 1/7*c*e^2*x^7+1/5*x^5*b*e^2+2/5*x^5*c*d*e+1/3*x^3*a*e^2+2/3*x^3*b
*d*e+1/3*x^3*c*d^2+2*x*a*d*e+x*b*d^2-a*d^2/x
```

Maxima [A] time = 0.700418, size = 93, normalized size = 1.31

$$\frac{1}{7}\,ce^2x^7+\frac{1}{5}\left(2\,cde+be^2\right)x^5+\frac{1}{3}\left(cd^2+2\,bde+ae^2\right)x^3-\frac{ad^2}{x}+\left(bd^2+2\,ade\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c*e^2*x^7 + 1/5*(2*c*d*e + b*e^2)*x^5 + 1/3*(c*d^2 + 2*b*d*e
+ a*e^2)*x^3 - a*d^2/x + (b*d^2 + 2*a*d*e)*x
```

Fricas [A] time = 0.245399, size = 100, normalized size = 1.41

$$\frac{15\,ce^2x^8+21\left(2\,cde+be^2\right)x^6+35\left(cd^2+2\,bde+ae^2\right)x^4-105\,ad^2+105\left(bd^2+2\,ade\right)x^2}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*c*e^2*x^8 + 21*(2*c*d*e + b*e^2)*x^6 + 35*(c*d^2 + 2*b*
d*e + a*e^2)*x^4 - 105*a*d^2 + 105*(b*d^2 + 2*a*d*e)*x^2)/x
```

Sympy [A] time = 1.25525, size = 73, normalized size = 1.03

$$-\frac{ad^2}{x} + \frac{ce^2x^7}{7} + x^5\left(\frac{be^2}{5} + \frac{2cde}{5}\right) + x^3\left(\frac{ae^2}{3} + \frac{2bde}{3} + \frac{cd^2}{3}\right) + x\left(2ade + bd^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x**2,x)
```

```
[Out] -a*d**2/x + c*e**2*x**7/7 + x**5*(b*e**2/5 + 2*c*d*e/5) + x**3*(a
*e**2/3 + 2*b*d*e/3 + c*d**2/3) + x*(2*a*d*e + b*d**2)
```

GIAC/XCAS [A] time = 0.266909, size = 100, normalized size = 1.41

$$\frac{1}{7}\,cx^7e^2 + \frac{2}{5}\,cdx^5e + \frac{1}{5}\,bx^5e^2 + \frac{1}{3}\,cd^2x^3 + \frac{2}{3}\,bdx^3e + \frac{1}{3}\,ax^3e^2 + bd^2x + 2\,adxe - \frac{ad^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*c*x^7*e^2 + 2/5*c*d*x^5*e + 1/5*b*x^5*e^2 + 1/3*c*d^2*x^3 + 2
/3*b*d*x^3*e + 1/3*a*x^3*e^2 + b*d^2*x + 2*a*d*x*e - a*d^2/x
```

3.271 $\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^3}\,dx$

Optimal. Leaf size=74

$$\frac{1}{2}x^2\left(e(ae+2bd)+cd^2\right)+d\log(x)(2ae+bd)-\frac{ad^2}{2x^2}+\frac{1}{4}ex^4(be+2cd)+\frac{1}{6}ce^2x^6$$

```
[Out] -(a*d^2)/(2*x^2) + ((c*d^2 + e*(2*b*d + a*e))*x^2)/2 + (e*(2*c*d
+ b*e)*x^4)/4 + (c*e^2*x^6)/6 + d*(b*d + 2*a*e)*Log[x]
```

Rubi [A] time = 0.230301, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{1}{2}x^2\left(e(ae+2bd)+cd^2\right)+d\log(x)(2ae+bd)-\frac{ad^2}{2x^2}+\frac{1}{4}ex^4(be+2cd)+\frac{1}{6}ce^2x^6$$

Antiderivative was successfully verified.

```
[In]  Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] -(a*d^2)/(2*x^2) + ((c*d^2 + e*(2*b*d + a*e))*x^2)/2 + (e*(2*c*d
+ b*e)*x^4)/4 + (c*e^2*x^6)/6 + d*(b*d + 2*a*e)*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ad^2}{2x^2}+\frac{ce^2x^6}{6}+\frac{d\,(2ae+bd)\log\left(x^2\right)}{2}+\frac{e\,(be+2cd)\int^{x^2}x\,dx}{2}+\frac{\left(ae^2+d\,(2be+cd)\right)\int^{x^2}a\,dx}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x**3,x)
```

```
[Out] -a*d**2/(2*x**2) + c*e**2*x**6/6 + d*(2*a*e + b*d)*log(x**2)/2 +
e*(b*e + 2*c*d)*Integral(x, (x, x**2))/2 + (a*e**2 + d*(2*b*e + c
*d))*Integral(a, (x, x**2))/(2*a)
```

Mathematica [A] time = 0.0828625, size = 71, normalized size = 0.96

$$\frac{1}{12}\left(6x^2\left(e(ae+2bd)+cd^2\right)+12d\log(x)(2ae+bd)-\frac{6ad^2}{x^2}+3ex^4(be+2cd)+2ce^2x^6\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] ((-6*a*d^2)/x^2 + 6*(c*d^2 + e*(2*b*d + a*e))*x^2 + 3*e*(2*c*d +
b*e)*x^4 + 2*c*e^2*x^6 + 12*d*(b*d + 2*a*e)*Log[x])/12
```

Maple [A] time = 0.011, size = 76, normalized size = 1.

$$\frac{ce^2x^6}{6}+\frac{x^4be^2}{4}+\frac{x^4cde}{2}+\frac{x^2ae^2}{2}+x^2bde+\frac{x^2cd^2}{2}+2\ln(x)\,ade+\ln(x)\,bd^2-\frac{ad^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x)
```

```
[Out] 1/6*c*e^2*x^6+1/4*x^4*b*e^2+1/2*x^4*c*d*e+1/2*x^2*a*e^2+x^2*b*d*e
+1/2*x^2*c*d^2+2*ln(x)*a*d*e+ln(x)*b*d^2-1/2*a*d^2/x^2
```

Maxima [A] time = 0.691421, size = 99, normalized size = 1.34

$$\frac{1}{6}\,ce^2x^6+\frac{1}{4}\left(2\,cde+be^2\right)x^4+\frac{1}{2}\left(cd^2+2\,bde+ae^2\right)x^2+\frac{1}{2}\left(bd^2+2\,ade\right)\log\left(x^2\right)-\frac{ad^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*c*e^2*x^6 + 1/4*(2*c*d*e + b*e^2)*x^4 + 1/2*(c*d^2 + 2*b*d*e
+ a*e^2)*x^2 + 1/2*(b*d^2 + 2*a*d*e)*log(x^2) - 1/2*a*d^2/x^2
```

Fricas [A] time = 0.259345, size = 103, normalized size = 1.39

$$\frac{2\,ce^2x^8+3\left(2\,cde+be^2\right)x^6+6\left(cd^2+2\,bde+ae^2\right)x^4+12\left(bd^2+2\,ade\right)x^2\log(x)-6\,ad^2}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(2*c*e^2*x^8 + 3*(2*c*d*e + b*e^2)*x^6 + 6*(c*d^2 + 2*b*d*e
+ a*e^2)*x^4 + 12*(b*d^2 + 2*a*d*e)*x^2*log(x) - 6*a*d^2)/x^2
```

Sympy [A] time = 1.59322, size = 71, normalized size = 0.96

$$-\frac{ad^2}{2x^2}+\frac{ce^2x^6}{6}+d\left(2ae+bd\right)\log\left(x\right)+x^4\left(\frac{be^2}{4}+\frac{cde}{2}\right)+x^2\left(\frac{ae^2}{2}+bde+\frac{cd^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x**3,x)
```

```
[Out] -a*d**2/(2*x**2) + c*e**2*x**6/6 + d*(2*a*e + b*d)*log(x) + x**4*
(b*e**2/4 + c*d*e/2) + x**2*(a*e**2/2 + b*d*e + c*d**2/2)
```

GIAC/XCAS [A] time = 0.270727, size = 131, normalized size = 1.77

$$\frac{1}{6}\,cx^6e^2+\frac{1}{2}\,cdx^4e+\frac{1}{4}\,bx^4e^2+\frac{1}{2}\,cd^2x^2+bdx^2e+\frac{1}{2}\,ax^2e^2+\frac{1}{2}\,\left(bd^2+2\,ade\right)\ln\left(x^2\right)-\frac{bd^2x^2+2\,adx^2e+ad^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(e*x^2 + d)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*c*x^6*e^2 + 1/2*c*d*x^4*e + 1/4*b*x^4*e^2 + 1/2*c*d^2*x^2 + b
*d*x^2*e + 1/2*a*x^2*e^2 + 1/2*(b*d^2 + 2*a*d*e)*ln(x^2) - 1/2*(b
*d^2*x^2 + 2*a*d*x^2*e + a*d^2)/x^2
```

3.272 $\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=181

$$-\frac{dx\left(9cd^2-e(7bd-5ae)\right)}{2e^5}+\frac{x^3\left(9cd^2-e(7bd-5ae)\right)}{6e^4}-\frac{x^5\left(9cd^2-e(7bd-5ae)\right)}{10de^3}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(9cd^2-e(7bd-5ae)\right)}{2e^{11/2}}+\frac{x^7\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx^7}{7e^2}$$

```
[Out] -(d*(9*c*d^2 - e*(7*b*d - 5*a*e))*x)/(2*e^5) + ((9*c*d^2 - e*(7*b
*d - 5*a*e))*x^3)/(6*e^4) - ((9*c*d^2 - e*(7*b*d - 5*a*e))*x^5)/(
10*d*e^3) + (c*x^7)/(7*e^2) + ((a + (d*(c*d - b*e))/e^2)*x^7)/(2*
d*(d + e*x^2)) + (d^(3/2)*(9*c*d^2 - e*(7*b*d - 5*a*e))*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(2*e^(11/2))
```

Rubi [A] time = 0.505352, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{dx\left(9cd^2-e(7bd-5ae)\right)}{2e^5}+\frac{x^3\left(9cd^2-e(7bd-5ae)\right)}{6e^4}-\frac{x^5\left(9cd^2-e(7bd-5ae)\right)}{10de^3}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(9cd^2-e(7bd-5ae)\right)}{2e^{11/2}}+\frac{x^7\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx^7}{7e^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] -(d*(9*c*d^2 - e*(7*b*d - 5*a*e))*x)/(2*e^5) + ((9*c*d^2 - e*(7*b
*d - 5*a*e))*x^3)/(6*e^4) - ((9*c*d^2 - e*(7*b*d - 5*a*e))*x^5)/(
10*d*e^3) + (c*x^7)/(7*e^2) + ((a + (d*(c*d - b*e))/e^2)*x^7)/(2*
d*(d + e*x^2)) + (d^(3/2)*(9*c*d^2 - e*(7*b*d - 5*a*e))*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(2*e^(11/2))
```

Rubi in Sympy [A] time = 86.3138, size = 158, normalized size = 0.87

$$\frac{cx^7}{7e^2}+\frac{d^{\frac{3}{2}}\left(5ae^2-7bde+9cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{\frac{11}{2}}}-\frac{d^2x\left(ae^2-bde+cd^2\right)}{2e^5\left(d+ex^2\right)}-\frac{dx\left(2ae^2-3bde+4cd^2\right)}{e^5}+\frac{x^5\left(be-2cd\right)}{5e^3}+\frac{x^3\left(ae^2-2bde+3cd^2\right)}{3e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**7/(7*e**2) + d**(3/2)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)*atan(s
qrt(e)*x/sqrt(d))/(2*e**(11/2)) - d**2*x*(a*e**2 - b*d*e + c*d**2
)/(2*e**5*(d + e*x**2)) - d*x*(2*a*e**2 - 3*b*d*e + 4*c*d**2)/e**
5 + x**5*(b*e - 2*c*d)/(5*e**3) + x**3*(a*e**2 - 2*b*d*e + 3*c*d*
*2)/(3*e**4)
```

Mathematica [A] time = 0.264586, size = 165, normalized size = 0.91

$$-\frac{dx\left(2ae^2-3bde+4cd^2\right)}{e^5}+\frac{x^3\left(ae^2-2bde+3cd^2\right)}{3e^4}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5ae^2-7bde+9cd^2\right)}{2e^{11/2}}-\frac{x\left(ad^2e^2-bd^3e+cd^4\right)}{2e^5\left(d+ex^2\right)}+\frac{x^5(be-2cd)}{5e^3}+\frac{cx^7}{7e^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] -((d*(4*c*d^2 - 3*b*d*e + 2*a*e^2)*x)/e^5) + ((3*c*d^2 - 2*b*d*e
+ a*e^2)*x^3)/(3*e^4) + ((-2*c*d + b*e)*x^5)/(5*e^3) + (c*x^7)/(7
*e^2) - ((c*d^4 - b*d^3*e + a*d^2*e^2)*x)/(2*e^5*(d + e*x^2)) + (
d^(3/2)*(9*c*d^2 - 7*b*d*e + 5*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(2*e^(11/2))
```

Maple [A] time = 0.015, size = 214, normalized size = 1.2

$$\frac{cx^7}{7\,e^2}+\frac{x^5b}{5\,e^2}-\frac{2\,cdx^5}{5\,e^3}+\frac{x^3a}{3\,e^2}-\frac{2\,bx^3d}{3\,e^3}+\frac{x^3cd^2}{e^4}-2\,\frac{adx}{e^3}+3\,\frac{xbd^2}{e^4}-4\,\frac{cd^3x}{e^5}-\frac{ad^2x}{2\,e^3\left(ex^2+d\right)}+\frac{d^3xb}{2\,e^4\left(ex^2+d\right)}-\frac{d^4xc}{2\,e^5\left(ex^2+d\right)}+\frac{5\,ad^2}{2\,e^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{7\,bd^3}{2\,e^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{9\,cd^4}{2\,e^5}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] 1/7*c*x^7/e^2+1/5/e^2*x^5*b-2/5/e^3*c*d*x^5+1/3/e^2*x^3*a-2/3/e^3
*x^3*b*d+1/e^4*x^3*c*d^2-2/e^3*d*a*x+3/e^4*b*d^2*x-4/e^5*d^3*c*x-
```

```
1/2*d^2/e^3*x/(e*x^2+d)*a+1/2*d^3/e^4*x/(e*x^2+d)*b-1/2*d^4/e^5*x
/(e*x^2+d)*c+5/2*d^2/e^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a-7/
2*d^3/e^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b+9/2*d^4/e^5/(d*e)
^(1/2)*arctan(x*e/(d*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*x^6/(e*x^2 + d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.265578, size = 1, normalized size = 0.01

$$\left[\frac{60\,ce^4x^9 - 12\left(9\,cde^3 - 7\,be^4\right)x^7 + 28\left(9\,cd^2e^2 - 7\,bde^3 + 5\,ae^4\right)x^5 - 140\left(9\,cd^3e - 7\,bd^2e^2 + 5\,ade^3\right)x^3 + 105\left(9\,cd^4 - 7\,bd\right.}{420\left(e^6x^2 + de^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*x^6/(e*x^2 + d)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*c*e^4*x^9 - 12*(9*c*d*e^3 - 7*b*e^4)*x^7 + 28*(9*c*d^2
*e^2 - 7*b*d*e^3 + 5*a*e^4)*x^5 - 140*(9*c*d^3*e - 7*b*d^2*e^2 +
5*a*d*e^3)*x^3 + 105*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2 + (9*c*d^
3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^2)*sqrt(-d/e)*log((e*x^2 + 2*e*x
*sqrt(-d/e) - d)/(e*x^2 + d)) - 210*(9*c*d^4 - 7*b*d^3*e + 5*a*d^
2*e^2)*x)/(e^6*x^2 + d*e^5), 1/210*(30*c*e^4*x^9 - 6*(9*c*d*e^3 -
 7*b*e^4)*x^7 + 14*(9*c*d^2*e^2 - 7*b*d*e^3 + 5*a*e^4)*x^5 - 70*(
9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^3 + 105*(9*c*d^4 - 7*b*d^3
*e + 5*a*d^2*e^2 + (9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^2)*sqr
t(d/e)*arctan(x/sqrt(d/e)) - 105*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e
^2)*x)/(e^6*x^2 + d*e^5)]
```

Sympy [A] time = 5.5182, size = 313, normalized size = 1.73

$$\frac{cx^7}{7e^2} - \frac{x\left(ad^2e^2 - bd^3e + cd^4\right)}{2de^5 + 2e^6x^2} - \frac{\sqrt{-\frac{d^3}{e^{11}}}\left(5ae^2 - 7bde + 9cd^2\right)\log\left(-\frac{e^5\sqrt{-\frac{d^3}{e^{11}}}(5ae^2-7bde+9cd^2)}{5ade^2-7bd^2e+9cd^3} + x\right)}{4} + \frac{\sqrt{-\frac{d^3}{e^{11}}}\left(5ae^2 - 7bde + 9cd^2\right)\log\left(\frac{e^5\sqrt{-\frac{d^3}{e^{11}}}(5ae^2-7bde+9cd^2)}{5ade^2-7bd^2e+9cd^3} + x\right)}{4} + \frac{x^5(be - 2cd)}{5e^3} + \frac{x^3\left(ae^2 - 2bde + 3cd^2\right)}{3e^4} - \frac{x\left(2ade^2 - 3bd^2e + 4cd^3\right)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**7/(7*e**2) - x*(a*d**2*e**2 - b*d**3*e + c*d**4)/(2*d*e**5 +
 2*e**6*x**2) - sqrt(-d**3/e**11)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)
*log(-e**5*sqrt(-d**3/e**11)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)/(5*a
*d*e**2 - 7*b*d**2*e + 9*c*d**3) + x)/4 + sqrt(-d**3/e**11)*(5*a*
e**2 - 7*b*d*e + 9*c*d**2)*log(e**5*sqrt(-d**3/e**11)*(5*a*e**2 -
 7*b*d*e + 9*c*d**2)/(5*a*d*e**2 - 7*b*d**2*e + 9*c*d**3) + x)/4
+ x**5*(b*e - 2*c*d)/(5*e**3) + x**3*(a*e**2 - 2*b*d*e + 3*c*d**2
)/(3*e**4) - x*(2*a*d*e**2 - 3*b*d**2*e + 4*c*d**3)/e**5
```

GIAC/XCAS [A] time = 0.270438, size = 216, normalized size = 1.19

$$\frac{\left(9\,cd^4 - 7\,bd^3e + 5\,ad^2e^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{11}{2}\right)}}{2\sqrt{d}} + \frac{1}{105}\left(15\,cx^7e^{12} - 42\,cdx^5e^{11} + 21\,bx^5e^{12} + 105\,cd^2x^3e^{10} - 70\,bdx^3e^{11} - 420\,cd^3xe^9 + 35\,ax^3e^{12} + 315\,bd^2xe^{10} - 210\,adxe^{11}\right. - \frac{\left(cd^4x - bd^3xe + ad^2xe^2\right)e^{(-5)}}{2\left(x^2e + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^6/(e*x^2 + d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*arctan(x*e^(1/2)/sqrt(d))
*e^(-11/2)/sqrt(d) + 1/105*(15*c*x^7*e^12 - 42*c*d*x^5*e^11 + 21*
b*x^5*e^12 + 105*c*d^2*x^3*e^10 - 70*b*d*x^3*e^11 - 420*c*d^3*x*e
^9 + 35*a*x^3*e^12 + 315*b*d^2*x*e^10 - 210*a*d*x*e^11)*e^(-14) -
 1/2*(c*d^4*x - b*d^3*x*e + a*d^2*x*e^2)*e^(-5)/(x^2*e + d)
```

3.273 $\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=151

$$-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(7cd^2-e(5bd-3ae)\right)}{2e^{9/2}}+\frac{x\left(7cd^2-e(5bd-3ae)\right)}{2e^4}-\frac{x^3\left(7cd^2-e(5bd-3ae)\right)}{6de^3}+\frac{x^5\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx^5}{5e^2}$$

```
[Out] ((7*c*d^2 - e*(5*b*d - 3*a*e))*x)/(2*e^4) - ((7*c*d^2 - e*(5*b*d
- 3*a*e))*x^3)/(6*d*e^3) + (c*x^5)/(5*e^2) + ((a + (d*(c*d - b*e)
)/e^2)*x^5)/(2*d*(d + e*x^2)) - (Sqrt[d]*(7*c*d^2 - e*(5*b*d - 3*
a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(9/2))
```

Rubi [A] time = 0.430298, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(7cd^2-e(5bd-3ae)\right)}{2e^{9/2}}+\frac{x\left(7cd^2-e(5bd-3ae)\right)}{2e^4}-\frac{x^3\left(7cd^2-e(5bd-3ae)\right)}{6de^3}+\frac{x^5\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] ((7*c*d^2 - e*(5*b*d - 3*a*e))*x)/(2*e^4) - ((7*c*d^2 - e*(5*b*d
- 3*a*e))*x^3)/(6*d*e^3) + (c*x^5)/(5*e^2) + ((a + (d*(c*d - b*e)
)/e^2)*x^5)/(2*d*(d + e*x^2)) - (Sqrt[d]*(7*c*d^2 - e*(5*b*d - 3*
a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(9/2))
```

Rubi in Sympy [A] time = 64.1523, size = 131, normalized size = 0.87

$$\frac{ax}{e^2}-\frac{2bdx}{e^3}+\frac{3cd^2x}{e^4}+\frac{cx^5}{5e^2}-\frac{\sqrt{d}\left(3ae^2-5bde+7cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{\frac{9}{2}}}+\frac{dx\left(ae^2-bde+cd^2\right)}{2e^4\left(d+ex^2\right)}+\frac{x^3\left(be-2cd\right)}{3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] a*x/e**2 - 2*b*d*x/e**3 + 3*c*d**2*x/e**4 + c*x**5/(5*e**2) - sqr
t(d)*(3*a*e**2 - 5*b*d*e + 7*c*d**2)*atan(sqrt(e)*x/sqrt(d))/(2*e
**(9/2)) + d*x*(a*e**2 - b*d*e + c*d**2)/(2*e**4*(d + e*x**2)) +
x**3*(b*e - 2*c*d)/(3*e**3)
```

Mathematica [A] time = 0.131685, size = 133, normalized size = 0.88

$$-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3ae^2-5bde+7cd^2\right)}{2e^{9/2}}+\frac{x\left(ae^2-2bde+3cd^2\right)}{e^4}+\frac{x\left(ade^2-bd^2e+cd^3\right)}{2e^4\left(d+ex^2\right)}+\frac{x^3(be-2cd)}{3e^3}+\frac{cx^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] ((3*c*d^2 - 2*b*d*e + a*e^2)*x)/e^4 + ((-2*c*d + b*e)*x^3)/(3*e^3
) + (c*x^5)/(5*e^2) + ((c*d^3 - b*d^2*e + a*d*e^2)*x)/(2*e^4*(d +
 e*x^2)) - (Sqrt[d]*(7*c*d^2 - 5*b*d*e + 3*a*e^2)*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(2*e^(9/2))
```

Maple [A] time = 0.014, size = 176, normalized size = 1.2

$$\frac{cx^5}{5\,e^2}+\frac{bx^3}{3\,e^2}-\frac{2\,cdx^3}{3\,e^3}+\frac{ax}{e^2}-2\,\frac{bxd}{e^3}+3\,\frac{cd^2x}{e^4}+\frac{adx}{2\,e^2\,(ex^2+d)}-\frac{xbd^2}{2\,e^3\,(ex^2+d)}+\frac{d^3xc}{2\,e^4\,(ex^2+d)}-\frac{3\,ad}{2\,e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{5\,bd^2}{2\,e^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{7\,cd^3}{2\,e^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] 1/5*c*x^5/e^2+1/3/e^2*x^3*b-2/3/e^3*c*d*x^3+1/e^2*a*x-2/e^3*x*b*d
+3/e^4*c*d^2*x+1/2*d/e^2*x/(e*x^2+d)*a-1/2*d^2/e^3*x/(e*x^2+d)*b+
1/2*d^3/e^4*x/(e*x^2+d)*c-3/2*d/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^
(1/2))*a+5/2*d^2/e^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b-7/2*d^
3/e^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*x^4/(e*x^2 + d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276914, size = 1, normalized size = 0.01

$$\left[\frac{12\,ce^3x^7 - 4\left(7\,cde^2 - 5\,be^3\right)x^5 + 20\left(7\,cd^2e - 5\,bde^2 + 3\,ae^3\right)x^3 + 15\left(7\,cd^3 - 5\,bd^2e + 3\,ade^2 + \left(7\,cd^2e - 5\,bde^2 + 3\,ae^3\right)x}{60\left(e^5x^2 + de^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*x^4/(e*x^2 + d)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*c*e^3*x^7 - 4*(7*c*d*e^2 - 5*b*e^3)*x^5 + 20*(7*c*d^2*e
 - 5*b*d*e^2 + 3*a*e^3)*x^3 + 15*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2
 + (7*c*d^2*e - 5*b*d*e^2 + 3*a*e^3)*x^2)*sqrt(-d/e)*log((e*x^2 -
 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) + 30*(7*c*d^3 - 5*b*d^2*e + 3
*a*d*e^2)*x)/(e^5*x^2 + d*e^4), 1/30*(6*c*e^3*x^7 - 2*(7*c*d*e^2
- 5*b*e^3)*x^5 + 10*(7*c*d^2*e - 5*b*d*e^2 + 3*a*e^3)*x^3 - 15*(7
*c*d^3 - 5*b*d^2*e + 3*a*d*e^2 + (7*c*d^2*e - 5*b*d*e^2 + 3*a*e^3
)*x^2)*sqrt(d/e)*arctan(x/sqrt(d/e)) + 15*(7*c*d^3 - 5*b*d^2*e +
3*a*d*e^2)*x)/(e^5*x^2 + d*e^4)]
```

Sympy [A] time = 5.12089, size = 184, normalized size = 1.22

$$\frac{cx^5}{5e^2} + \frac{x\left(ade^2 - bd^2e + cd^3\right)}{2de^4 + 2e^5x^2} + \frac{\sqrt{-\frac{d}{e^9}}\left(3ae^2 - 5bde + 7cd^2\right)\log\left(-e^4\sqrt{-\frac{d}{e^9}} + x\right)}{4} - \frac{\sqrt{-\frac{d}{e^9}}\left(3ae^2 - 5bde + 7cd^2\right)\log\left(e^4\sqrt{-\frac{d}{e^9}} + x\right)}{4} + \frac{x^3\left(be - 2cd\right)}{3e^3} + \frac{x\left(ae^2 - 2bde + 3cd^2\right)}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**5/(5*e**2) + x*(a*d*e**2 - b*d**2*e + c*d**3)/(2*d*e**4 + 2*
e**5*x**2) + sqrt(-d/e**9)*(3*a*e**2 - 5*b*d*e + 7*c*d**2)*log(-e
**4*sqrt(-d/e**9) + x)/4 - sqrt(-d/e**9)*(3*a*e**2 - 5*b*d*e + 7*
c*d**2)*log(e**4*sqrt(-d/e**9) + x)/4 + x**3*(b*e - 2*c*d)/(3*e**
3) + x*(a*e**2 - 2*b*d*e + 3*c*d**2)/e**4
```

GIAC/XCAS [A] time = 0.270241, size = 169, normalized size = 1.12

$$-\frac{\left(7\,cd^3 - 5\,bd^2e + 3\,ade^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{2\,\sqrt{d}} + \frac{1}{15}\left(3\,cx^5e^8 - 10\,cdx^3e^7 + 5\,bx^3e^8 + 45\,cd^2xe^6 - 30\,bdxe^7 + 15\,axe^8\right)e^{(-10)} + \frac{\left(cd^3x - bd^2xe + adxe^2\right)e^{(-4)}}{2\,(x^2e + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^4/(e*x^2 + d)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2)*arctan(x*e^(1/2)/sqrt(d))*
e^(-9/2)/sqrt(d) + 1/15*(3*c*x^5*e^8 - 10*c*d*x^3*e^7 + 5*b*x^3*e
^8 + 45*c*d^2*x*e^6 - 30*b*d*x*e^7 + 15*a*x*e^8)*e^(-10) + 1/2*(c
*d^3*x - b*d^2*x*e + a*d*x*e^2)*e^(-4)/(x^2*e + d)
```

3.274 $\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2} dx$

Optimal. Leaf size=122

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(5cd^2-e(3bd-ae))}{2\sqrt{d}e^{7/2}}-\frac{x(5cd^2-e(3bd-ae))}{2de^3}+\frac{x^3\left(a+\frac{d(cd-be)}{e^2}\right)}{2d(d+ex^2)}+\frac{cx^3}{3e^2}$$

```
[Out] -((5*c*d^2 - e*(3*b*d - a*e))*x)/(2*d*e^3) + (c*x^3)/(3*e^2) + ((
a + (d*(c*d - b*e))/e^2)*x^3)/(2*d*(d + e*x^2)) + ((5*c*d^2 - e*(
3*b*d - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*Sqrt[d]*e^(7/2))
```

Rubi [A] time = 0.333695, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(5cd^2-e(3bd-ae))}{2\sqrt{d}e^{7/2}}-\frac{x(5cd^2-e(3bd-ae))}{2de^3}+\frac{x^3\left(a+\frac{d(cd-be)}{e^2}\right)}{2d(d+ex^2)}+\frac{cx^3}{3e^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] -((5*c*d^2 - e*(3*b*d - a*e))*x)/(2*d*e^3) + (c*x^3)/(3*e^2) + ((
a + (d*(c*d - b*e))/e^2)*x^3)/(2*d*(d + e*x^2)) + ((5*c*d^2 - e*(
3*b*d - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*Sqrt[d]*e^(7/2))
```

Rubi in Sympy [A] time = 51.7901, size = 95, normalized size = 0.78

$$\frac{cx^3}{3e^2}+\frac{x(be-2cd)}{e^3}-\frac{x(ae^2-bde+cd^2)}{2e^3(d+ex^2)}+\frac{(ae^2-3bde+5cd^2)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**3/(3*e**2) + x*(b*e - 2*c*d)/e**3 - x*(a*e**2 - b*d*e + c*d*
*2)/(2*e**3*(d + e*x**2)) + (a*e**2 - 3*b*d*e + 5*c*d**2)*atan(sq
rt(e)*x/sqrt(d))/(2*sqrt(d)*e**(7/2))
```

Mathematica [A] time = 0.114797, size = 102, normalized size = 0.84

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-3bde+5cd^2\right)}{2\sqrt{d}e^{7/2}}-\frac{x\left(ae^2-bde+cd^2\right)}{2e^3\left(d+ex^2\right)}+\frac{x(be-2cd)}{e^3}+\frac{cx^3}{3e^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] ((-2*c*d + b*e)*x)/e^3 + (c*x^3)/(3*e^2) - ((c*d^2 - b*d*e + a*e^
2)*x)/(2*e^3*(d + e*x^2)) + ((5*c*d^2 - 3*b*d*e + a*e^2)*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/(2*Sqrt[d]*e^(7/2))
```

Maple [A] time = 0.013, size = 141, normalized size = 1.2

$$\frac{cx^3}{3\,e^2}+\frac{bx}{e^2}-2\,\frac{cdx}{e^3}-\frac{ax}{2\,e\,(ex^2+d)}+\frac{bxd}{2\,e^2\,(ex^2+d)}-\frac{cxd^2}{2\,e^3\,(ex^2+d)}+\frac{a}{2\,e}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{3\,bd}{2\,e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{5\,cd^2}{2\,e^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] 1/3*c*x^3/e^2+1/e^2*b*x-2/e^3*c*d*x-1/2/e*x/(e*x^2+d)*a+1/2/e^2*x
/(e*x^2+d)*b*d-1/2/e^3*x/(e*x^2+d)*c*d^2+1/2/e/(d*e)^(1/2)*arctan
(x*e/(d*e)^(1/2))*a-3/2/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b
*d+5/2/e^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*x^2/(e*x^2 + d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.271434, size = 1, normalized size = 0.01

$$\left[\frac{3\left(5\,cd^{3}-3\,bd^{2}e+ade^{2}+\left(5\,cd^{2}e-3\,bde^{2}+ae^{3}\right)x^{2}\right)\log\left(\frac{2\,dex+(ex^{2}-d)\sqrt{-de}}{ex^{2}+d}\right)+2\left(2\,ce^{2}x^{5}-2\left(5\,cde-3\,be^{2}\right)x^{3}-3\left(5\,cd^{2}\right.\right.}{12\left(e^{4}x^{2}+de^{3}\right)\sqrt{-de}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^2/(e*x^2 + d)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(5*c*d^3 - 3*b*d^2*e + a*d*e^2 + (5*c*d^2*e - 3*b*d*e^2
+ a*e^3)*x^2)*log((2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d))
 + 2*(2*c*e^2*x^5 - 2*(5*c*d*e - 3*b*e^2)*x^3 - 3*(5*c*d^2 - 3*b*
d*e + a*e^2)*x)*sqrt(-d*e))/((e^4*x^2 + d*e^3)*sqrt(-d*e)), 1/6*(
3*(5*c*d^3 - 3*b*d^2*e + a*d*e^2 + (5*c*d^2*e - 3*b*d*e^2 + a*e^3
)*x^2)*arctan(sqrt(d*e)*x/d) + (2*c*e^2*x^5 - 2*(5*c*d*e - 3*b*e^
2)*x^3 - 3*(5*c*d^2 - 3*b*d*e + a*e^2)*x)*sqrt(d*e))/((e^4*x^2 +
d*e^3)*sqrt(d*e))]
```

Sympy [A] time = 4.5117, size = 160, normalized size = 1.31

$$\frac{cx^{3}}{3e^{2}}-\frac{x\left(ae^{2}-bde+cd^{2}\right)}{2de^{3}+2e^{4}x^{2}}-\frac{\sqrt{-\frac{1}{de^{7}}}\left(ae^{2}-3bde+5cd^{2}\right)\log\left(-de^{3}\sqrt{-\frac{1}{de^{7}}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{de^{7}}}\left(ae^{2}-3bde+5cd^{2}\right)\log\left(de^{3}\sqrt{-\frac{1}{de^{7}}}+x\right)}{4}+\frac{x\left(be-2cd\right)}{e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**3/(3*e**2) - x*(a*e**2 - b*d*e + c*d**2)/(2*d*e**3 + 2*e**4*
x**2) - sqrt(-1/(d*e**7))*(a*e**2 - 3*b*d*e + 5*c*d**2)*log(-d*e*
*3*sqrt(-1/(d*e**7)) + x)/4 + sqrt(-1/(d*e**7))*(a*e**2 - 3*b*d*e
 + 5*c*d**2)*log(d*e**3*sqrt(-1/(d*e**7)) + x)/4 + x*(b*e - 2*c*d
)/e**3
```

GIAC/XCAS [A] time = 0.270334, size = 123, normalized size = 1.01

$$\frac{\left(5\,cd^2 - 3\,bde + ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{2\,\sqrt{d}} + \frac{1}{3}\left(cx^3e^4 - 6\,cdxe^3 + 3\,bxe^4\right)e^{(-6)} - \frac{\left(cd^2x - bdxe + axe^2\right)e^{(-3)}}{2\,(x^2e + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*x^2/(e*x^2 + d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*c*d^2 - 3*b*d*e + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-7/2
)/sqrt(d) + 1/3*(c*x^3*e^4 - 6*c*d*x*e^3 + 3*b*x*e^4)*e^(-6) - 1/
2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-3)/(x^2*e + d)
```

3.275 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3
*c*d^2 - e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e
^(5/2))
```

Rubi [A] time = 0.14613, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3
*c*d^2 - e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e
^(5/2))
```

Rubi in Sympy [A] time = 32.9286, size = 78, normalized size = 0.94

$$\frac{cx}{e^2}+\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)}+\frac{\left(ae^2+bde-3cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{\frac{3}{2}}e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d*e**2*(d + e*x**2)) +
(a*e**2 + b*d*e - 3*c*d**2)*atan(sqrt(e)*x/sqrt(d))/(2*d**(3/2)*e
**(5/2))
```

Mathematica [A] time = 0.0886695, size = 88, normalized size = 1.06

$$\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-ae^2-bde+3cd^2\right)}{2d^{3/2}e^{5/2}}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - (
(3*c*d^2 - b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)
*e^(5/2))
```

Maple [A] time = 0., size = 118, normalized size = 1.4

$$\frac{cx}{e^2}+\frac{ax}{2\,d\left(ex^2+d\right)}-\frac{bx}{2\,e\left(ex^2+d\right)}+\frac{cdx}{2\,e^2\left(ex^2+d\right)}+\frac{a}{2\,d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{b}{2\,e}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{3\,cd}{2\,e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] c*x/e^2+1/2/d*x/(e*x^2+d)*a-1/2/e*x/(e*x^2+d)*b+1/2/e^2*d*x/(e*x^
2+d)*c+1/2/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+1/2/e/(d*e)^(1
/2)*arctan(x*e/(d*e)^(1/2))*b-3/2/e^2*d/(d*e)^(1/2)*arctan(x*e/(d
*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273728, size = 1, normalized size = 0.01

$$\left[-\frac{\left(3\,cd^{3}-bd^{2}e-ade^{2}+\left(3\,cd^{2}e-bde^{2}-ae^{3}\right)x^{2}\right)\log\left(\frac{2\,dex+\left(ex^{2}-d\right)\sqrt{-de}}{ex^{2}+d}\right)-2\left(2\,cdex^{3}+\left(3\,cd^{2}-bde+ae^{2}\right)x\right)\sqrt{-de}}{4\left(de^{3}x^{2}+d^{2}e^{2}\right)\sqrt{-de}},\right.$$
$$\left.-\frac{\left(3\,cd^{3}-bd^{2}e-ade^{2}+\left(3\,cd^{2}e-bde^{2}-ae^{3}\right)x^{2}\right)\arctan\left(\frac{\sqrt{de}x}{d}\right)-\left(2\,cdex^{3}+\left(3\,cd^{2}-bde+ae^{2}\right)x\right)\sqrt{de}}{2\left(de^{3}x^{2}+d^{2}e^{2}\right)\sqrt{de}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^
3)*x^2)*log((2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d)) - 2*(
2*c*d*e*x^3 + (3*c*d^2 - b*d*e + a*e^2)*x)*sqrt(-d*e))/((d*e^3*x^
2 + d^2*e^2)*sqrt(-d*e)), -1/2*((3*c*d^3 - b*d^2*e - a*d*e^2 + (3
*c*d^2*e - b*d*e^2 - a*e^3)*x^2)*arctan(sqrt(d*e)*x/d) - (2*c*d*e
*x^3 + (3*c*d^2 - b*d*e + a*e^2)*x)*sqrt(d*e))/((d*e^3*x^2 + d^2*
e^2)*sqrt(d*e))]
```

Sympy [A] time = 3.77818, size = 153, normalized size = 1.84

$$\frac{cx}{e^{2}}+\frac{x\left(ae^{2}-bde+cd^{2}\right)}{2d^{2}e^{2}+2de^{3}x^{2}}-\frac{\sqrt{-\frac{1}{d^{3}e^{5}}}\left(ae^{2}+bde-3cd^{2}\right)\log\left(-d^{2}e^{2}\sqrt{-\frac{1}{d^{3}e^{5}}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{d^{3}e^{5}}}\left(ae^{2}+bde-3cd^{2}\right)\log\left(d^{2}e^{2}\sqrt{-\frac{1}{d^{3}e^{5}}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x*
*2) - sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*
e**2*sqrt(-1/(d**3*e**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 +
 b*d*e - 3*c*d**2)*log(d**2*e**2*sqrt(-1/(d**3*e**5)) + x)/4
```

GIAC/XCAS [A] time = 0.270896, size = 101, normalized size = 1.22

$$cxe^{(-2)} - \frac{\left(3\,cd^2 - bde - ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{\left(cd^2x - bdxe + axe^2\right)e^{(-2)}}{2\,(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(
d))*e^(-5/2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/(
(x^2*e + d)*d)
```

3.276 $\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^2} \, dx$

Optimal. Leaf size=86

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(bd-3ae)+cd^2\right)}{2d^{5/2}e^{3/2}}+\frac{x\left(-\frac{ae}{d}+b-\frac{cd}{e}\right)}{2d\left(d+ex^2\right)}-\frac{a}{d^2x}$$

```
[Out] -(a/(d^2*x)) + ((b - (c*d)/e - (a*e)/d)*x)/(2*d*(d + e*x^2)) + ((
c*d^2 + e*(b*d - 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*
e^(3/2))
```

Rubi [A] time = 0.203208, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(bd-3ae)+cd^2\right)}{2d^{5/2}e^{3/2}}+\frac{x\left(-\frac{ae}{d}+b-\frac{cd}{e}\right)}{2d\left(d+ex^2\right)}-\frac{a}{d^2x}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -(a/(d^2*x)) + ((b - (c*d)/e - (a*e)/d)*x)/(2*d*(d + e*x^2)) + ((
c*d^2 + e*(b*d - 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*
e^(3/2))
```

Rubi in Sympy [A] time = 25.0272, size = 80, normalized size = 0.93

$$-\frac{a}{d^2x}-\frac{x\left(ae^2-bde+cd^2\right)}{2d^2e\left(d+ex^2\right)}-\frac{\left(3ae^2-bde-cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{\frac{5}{2}}e^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**2/(e*x**2+d)**2,x)
```

```
[Out] -a/(d**2*x) - x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e*(d + e*x**2))
 - (3*a*e**2 - b*d*e - c*d**2)*atan(sqrt(e)*x/sqrt(d))/(2*d**(5/2
)*e**(3/2))
```

Mathematica [A] time = 0.109278, size = 89, normalized size = 1.03

$$-\frac{x\left(ae^2-bde+cd^2\right)}{2d^2e\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-3ae^2+bde+cd^2\right)}{2d^{5/2}e^{3/2}}-\frac{a}{d^2x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -(a/(d^2*x)) - ((c*d^2 - b*d*e + a*e^2)*x)/(2*d^2*e*(d + e*x^2))
+ ((c*d^2 + b*d*e - 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5
/2)*e^(3/2))
```

Maple [A] time = 0.016, size = 121, normalized size = 1.4

$$-\frac{a}{d^2x}-\frac{exa}{2\,d^2\left(ex^2+d\right)}+\frac{bx}{2\,d\left(ex^2+d\right)}-\frac{cx}{2\,e\left(ex^2+d\right)}-\frac{3\,ae}{2\,d^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{b}{2\,d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{c}{2\,e}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x)
```

```
[Out] -a/d^2/x-1/2/d^2*e*x/(e*x^2+d)*a+1/2/d*x/(e*x^2+d)*b-1/2/e*x/(e*x
^2+d)*c-3/2/d^2*e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+1/2/d/(d*
e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b+1/2/e/(d*e)^(1/2)*arctan(x*e/(
d*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^2*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.267372, size = 1, normalized size = 0.01

$$\left[-\frac{\left(\left(cd^2e+bde^2-3\,ae^3\right)x^3+\left(cd^3+bd^2e-3\,ade^2\right)x\right)\log\left(-\frac{2\,dex-\left(ex^2-d\right)\sqrt{-de}}{ex^2+d}\right)+2\left(2\,ade+\left(cd^2-bde+3\,ae^2\right)x^2\right)\sqrt{-de}}{4\left(d^2e^2x^3+d^3ex\right)\sqrt{-de}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^2*x^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((c*d^2*e + b*d*e^2 - 3*a*e^3)*x^3 + (c*d^3 + b*d^2*e - 3*
a*d*e^2)*x)*log(-(2*d*e*x - (e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d))
+ 2*(2*a*d*e + (c*d^2 - b*d*e + 3*a*e^2)*x^2)*sqrt(-d*e))/((d^2*e
^2*x^3 + d^3*e*x)*sqrt(-d*e)), 1/2*(((c*d^2*e + b*d*e^2 - 3*a*e^3
)*x^3 + (c*d^3 + b*d^2*e - 3*a*d*e^2)*x)*arctan(sqrt(d*e)*x/d) -
(2*a*d*e + (c*d^2 - b*d*e + 3*a*e^2)*x^2)*sqrt(d*e))/((d^2*e^2*x^
3 + d^3*e*x)*sqrt(d*e))]
```

Sympy [A] time = 5.07592, size = 155, normalized size = 1.8

$$\frac{\sqrt{-\frac{1}{d^5e^3}}\left(3ae^2-bde-cd^2\right)\log\left(-d^3e\sqrt{-\frac{1}{d^5e^3}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{d^5e^3}}\left(3ae^2-bde-cd^2\right)\log\left(d^3e\sqrt{-\frac{1}{d^5e^3}}+x\right)}{4}-\frac{2ade+x^2\left(3ae^2-bde+cd^2\right)}{2d^3ex+2d^2e^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**2/(e*x**2+d)**2,x)
```

```
[Out] sqrt(-1/(d**5*e**3))*(3*a*e**2 - b*d*e - c*d**2)*log(-d**3*e*sqrt
(-1/(d**5*e**3)) + x)/4 - sqrt(-1/(d**5*e**3))*(3*a*e**2 - b*d*e
- c*d**2)*log(d**3*e*sqrt(-1/(d**5*e**3)) + x)/4 - (2*a*d*e + x**
2*(3*a*e**2 - b*d*e + c*d**2))/(2*d**3*e*x + 2*d**2*e**2*x**3)
```

GIAC/XCAS [A] time = 0.270593, size = 112, normalized size = 1.3

$$\frac{\left(cd^2+bde-3\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{2\,d^{\frac{5}{2}}}-\frac{\left(cd^2x^2-bdx^2e+3\,ax^2e^2+2\,ade\right)e^{(-1)}}{2\left(x^3e+dx\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^2*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(c*d^2 + b*d*e - 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/
d^(5/2) - 1/2*(c*d^2*x^2 - b*d*x^2*e + 3*a*x^2*e^2 + 2*a*d*e)*e^(
-1)/((x^3*e + d*x)*d^2)
```

3.277 $\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^2}\,dx$

Optimal. Leaf size=106

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(cd^2-e(3bd-5ae)\right)}{2d^{7/2}\sqrt{e}}+\frac{x\left(ae^2-bde+cd^2\right)}{2d^3\left(d+ex^2\right)}-\frac{bd-2ae}{d^3x}-\frac{a}{3d^2x^3}$$

```
[Out] -a/(3*d^2*x^3) - (b*d - 2*a*e)/(d^3*x) + ((c*d^2 - b*d*e + a*e^2)
*x)/(2*d^3*(d + e*x^2)) + ((c*d^2 - e*(3*b*d - 5*a*e))*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/(2*d^(7/2)*Sqrt[e])
```

Rubi [A] time = 0.294092, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(cd^2-e(3bd-5ae)\right)}{2d^{7/2}\sqrt{e}}+\frac{x\left(ae^2-bde+cd^2\right)}{2d^3\left(d+ex^2\right)}-\frac{bd-2ae}{d^3x}-\frac{a}{3d^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x]
```

```
[Out] -a/(3*d^2*x^3) - (b*d - 2*a*e)/(d^3*x) + ((c*d^2 - b*d*e + a*e^2)
*x)/(2*d^3*(d + e*x^2)) + ((c*d^2 - e*(3*b*d - 5*a*e))*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/(2*d^(7/2)*Sqrt[e])
```

Rubi in Sympy [A] time = 38.9661, size = 95, normalized size = 0.9

$$-\frac{a}{3d^2x^3}+\frac{x\left(ae^2-bde+cd^2\right)}{2d^3\left(d+ex^2\right)}+\frac{2ae-bd}{d^3x}+\frac{\left(5ae^2-3bde+cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{\frac{7}{2}}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**4/(e*x**2+d)**2,x)
```

```
[Out] -a/(3*d**2*x**3) + x*(a*e**2 - b*d*e + c*d**2)/(2*d**3*(d + e*x**
2)) + (2*a*e - b*d)/(d**3*x) + (5*a*e**2 - 3*b*d*e + c*d**2)*atan
(sqrt(e)*x/sqrt(d))/(2*d**(7/2)*sqrt(e))
```

Mathematica [A] time = 0.10333, size = 105, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5ae^2-3bde+cd^2\right)}{2d^{7/2}\sqrt{e}}+\frac{x\left(ae^2-bde+cd^2\right)}{2d^3\left(d+ex^2\right)}+\frac{2ae-bd}{d^3x}-\frac{a}{3d^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x]
```

```
[Out] -a/(3*d^2*x^3) + (-(b*d) + 2*a*e)/(d^3*x) + ((c*d^2 - b*d*e + a*e
^2)*x)/(2*d^3*(d + e*x^2)) + ((c*d^2 - 3*b*d*e + 5*a*e^2)*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/(2*d^(7/2)*Sqrt[e])
```

Maple [A] time = 0.018, size = 146, normalized size = 1.4

$$-\frac{a}{3\,d^2x^3}+2\,\frac{ae}{d^3x}-\frac{b}{d^2x}+\frac{axe^2}{2\,d^3\left(ex^2+d\right)}-\frac{bxe}{2\,d^2\left(ex^2+d\right)}+\frac{cx}{2\,d\left(ex^2+d\right)}+\frac{5\,ae^2}{2\,d^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{3\,be}{2\,d^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{c}{2\,d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x)
```

```
[Out] -1/3*a/d^2/x^3+2/d^3/x*a*e-1/d^2/x*b+1/2/d^3*x/(e*x^2+d)*a*e^2-1/
2/d^2*x/(e*x^2+d)*b*e+1/2/d*x/(e*x^2+d)*c+5/2/d^3/(d*e)^(1/2)*arc
tan(x*e/(d*e)^(1/2))*a*e^2-3/2/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(
1/2))*b*e+1/2/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^2*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.281256, size = 1, normalized size = 0.01

$$\left[\frac{3\left(\left(cd^2e-3\,bde^2+5\,ae^3\right)x^5+\left(cd^3-3\,bd^2e+5\,ade^2\right)x^3\right)\log\left(\frac{2\,dex+\left(ex^2-d\right)\sqrt{-de}}{ex^2+d}\right)+2\left(3\left(cd^2-3\,bde+5\,ae^2\right)x^4-2\,ad^2\right.}{12\left(d^3ex^5+d^4x^3\right)\sqrt{-de}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^2*x^4),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((c*d^2*e - 3*b*d*e^2 + 5*a*e^3)*x^5 + (c*d^3 - 3*b*d^2*
e + 5*a*d*e^2)*x^3)*log((2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^2
 + d)) + 2*(3*(c*d^2 - 3*b*d*e + 5*a*e^2)*x^4 - 2*a*d^2 - 2*(3*b*
d^2 - 5*a*d*e)*x^2)*sqrt(-d*e))/((d^3*e*x^5 + d^4*x^3)*sqrt(-d*e)
), 1/6*(3*((c*d^2*e - 3*b*d*e^2 + 5*a*e^3)*x^5 + (c*d^3 - 3*b*d^2
*e + 5*a*d*e^2)*x^3)*arctan(sqrt(d*e)*x/d) + (3*(c*d^2 - 3*b*d*e
+ 5*a*e^2)*x^4 - 2*a*d^2 - 2*(3*b*d^2 - 5*a*d*e)*x^2)*sqrt(d*e))/
((d^3*e*x^5 + d^4*x^3)*sqrt(d*e))]
```

Sympy [A] time = 6.89004, size = 167, normalized size = 1.58

$$-\frac{\sqrt{-\frac{1}{d^7e}}\left(5ae^2-3bde+cd^2\right)\log\left(-d^4\sqrt{-\frac{1}{d^7e}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{d^7e}}\left(5ae^2-3bde+cd^2\right)\log\left(d^4\sqrt{-\frac{1}{d^7e}}+x\right)}{4}+\frac{-2ad^2+x^4\left(15ae^2-9bde+3cd^2\right)+x^2\left(10ade-6bd^2\right)}{6d^4x^3+6d^3ex^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**4/(e*x**2+d)**2,x)
```

```
[Out] -sqrt(-1/(d**7*e))*(5*a*e**2 - 3*b*d*e + c*d**2)*log(-d**4*sqrt(-
1/(d**7*e)) + x)/4 + sqrt(-1/(d**7*e))*(5*a*e**2 - 3*b*d*e + c*d*
*2)*log(d**4*sqrt(-1/(d**7*e)) + x)/4 + (-2*a*d**2 + x**4*(15*a*e
**2 - 9*b*d*e + 3*c*d**2) + x**2*(10*a*d*e - 6*b*d**2))/(6*d**4*x
**3 + 6*d**3*e*x**5)
```

GIAC/XCAS [A] time = 0.270983, size = 127, normalized size = 1.2

$$\frac{\left(cd^2 - 3\,bde + 5\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d^{\frac{7}{2}}} + \frac{cd^2x - bdxe + axe^2}{2\,(x^2e + d)d^3} - \frac{3\,bdx^2 - 6\,ax^2e + ad}{3\,d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^2*x^4),x, algorithm="giac")
```

```
[Out] 1/2*(c*d^2 - 3*b*d*e + 5*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2
)/d^(7/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)/((x^2*e + d)*d^3) -
 1/3*(3*b*d*x^2 - 6*a*x^2*e + a*d)/(d^3*x^3)
```

3.278 $\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^2}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(5bd-7ae)\right)}{2d^{9/2}}-\frac{ex\left(ae^2-bde+cd^2\right)}{2d^4\left(d+ex^2\right)}-\frac{cd^2-e(2bd-3ae)}{d^4x}-\frac{bd-2ae}{3d^3x^3}-\frac{a}{5d^2x^5}$$

```
[Out] -a/(5*d^2*x^5) - (b*d - 2*a*e)/(3*d^3*x^3) - (c*d^2 - e*(2*b*d -
3*a*e))/(d^4*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^4*(d + e*x^2
)) - (Sqrt[e]*(3*c*d^2 - e*(5*b*d - 7*a*e))*ArcTan[(Sqrt[e]*x)/Sq
rt[d]])/(2*d^(9/2))
```

Rubi [A] time = 0.423074, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(5bd-7ae)\right)}{2d^{9/2}}-\frac{ex\left(ae^2-bde+cd^2\right)}{2d^4\left(d+ex^2\right)}-\frac{cd^2-e(2bd-3ae)}{d^4x}-\frac{bd-2ae}{3d^3x^3}-\frac{a}{5d^2x^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x]
```

```
[Out] -a/(5*d^2*x^5) - (b*d - 2*a*e)/(3*d^3*x^3) - (c*d^2 - e*(2*b*d -
3*a*e))/(d^4*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^4*(d + e*x^2
)) - (Sqrt[e]*(3*c*d^2 - e*(5*b*d - 7*a*e))*ArcTan[(Sqrt[e]*x)/Sq
rt[d]])/(2*d^(9/2))
```

Rubi in Sympy [A] time = 66.2802, size = 126, normalized size = 0.93

$$-\frac{a}{5d^2x^5}+\frac{2ae-bd}{3d^3x^3}-\frac{ex\left(ae^2-bde+cd^2\right)}{2d^4\left(d+ex^2\right)}-\frac{3ae^2-2bde+cd^2}{d^4x}-\frac{\sqrt{e}\left(7ae^2-5bde+3cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**6/(e*x**2+d)**2,x)
```

```
[Out] -a/(5*d**2*x**5) + (2*a*e - b*d)/(3*d**3*x**3) - e*x*(a*e**2 - b*
d*e + c*d**2)/(2*d**4*(d + e*x**2)) - (3*a*e**2 - 2*b*d*e + c*d**
2)/(d**4*x) - sqrt(e)*(7*a*e**2 - 5*b*d*e + 3*c*d**2)*atan(sqrt(e
```

```
)*x/sqrt(d))/(2*d**(9/2))
```

Mathematica [A] time = 0.153693, size = 135, normalized size = 0.99

$$-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(7ae^2-5bde+3cd^2\right)}{2d^{9/2}}-\frac{ex\left(ae^2-bde+cd^2\right)}{2d^4\left(d+ex^2\right)}+\frac{-3ae^2+2bde-cd^2}{d^4x}+\frac{2ae-bd}{3d^3x^3}-\frac{a}{5d^2x^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x]
```

```
[Out] -a/(5*d^2*x^5) + (-(b*d) + 2*a*e)/(3*d^3*x^3) + (-(c*d^2) + 2*b*d
*e - 3*a*e^2)/(d^4*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^4*(d +
 e*x^2)) - (Sqrt[e]*(3*c*d^2 - 5*b*d*e + 7*a*e^2)*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(2*d^(9/2))
```

Maple [A] time = 0.021, size = 183, normalized size = 1.4

$$-\frac{a}{5\,d^2x^5}+\frac{2\,ae}{3\,d^3x^3}-\frac{b}{3\,d^2x^3}-3\,\frac{ae^2}{d^4x}+2\,\frac{be}{d^3x}-\frac{c}{d^2x}-\frac{e^3xa}{2\,d^4\left(ex^2+d\right)}+\frac{e^2xb}{2\,d^3\left(ex^2+d\right)}-\frac{cex}{2\,d^2\left(ex^2+d\right)}-\frac{7\,e^3a}{2\,d^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{5\,be^2}{2\,d^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{3\,ce}{2\,d^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x)
```

```
[Out] -1/5*a/d^2/x^5+2/3/d^3/x^3*a*e-1/3/d^2/x^3*b-3/d^4/x*a*e^2+2/d^3/
x*b*e-1/d^2/x*c-1/2*e^3/d^4*x/(e*x^2+d)*a+1/2*e^2/d^3*x/(e*x^2+d)
*b-1/2*e/d^2*x/(e*x^2+d)*c-7/2*e^3/d^4/(d*e)^(1/2)*arctan(x*e/(d*
e)^(1/2))*a+5/2*e^2/d^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b-3/2
*e/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^2*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274425, size = 1, normalized size = 0.01

$$\left[-\frac{30\left(3\,cd^{2}e-5\,bde^{2}+7\,ae^{3}\right)x^{6}+20\left(3\,cd^{3}-5\,bd^{2}e+7\,ade^{2}\right)x^{4}+12\,ad^{3}+4\left(5\,bd^{3}-7\,ad^{2}e\right)x^{2}-15\left(\left(3\,cd^{2}e-5\,bde^{2}+\right.\right.}{60\left(d^{4}ex^{7}+d^{5}x^{5}\right)}\right.$$

$$-\frac{15\left(3\,cd^{2}e-5\,bde^{2}+7\,ae^{3}\right)x^{6}+10\left(3\,cd^{3}-5\,bd^{2}e+7\,ade^{2}\right)x^{4}+6\,ad^{3}+2\left(5\,bd^{3}-7\,ad^{2}e\right)x^{2}+15\left(\left(3\,cd^{2}e-5\,bde^{2}+7\,a\right.\right.}{30\left(d^{4}ex^{7}+d^{5}x^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^2*x^6),x, algorithm="fricas")
```

```
[Out] [-1/60*(30*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*x^6 + 20*(3*c*d^3 -
5*b*d^2*e + 7*a*d*e^2)*x^4 + 12*a*d^3 + 4*(5*b*d^3 - 7*a*d^2*e)*x
^2 - 15*((3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*x^7 + (3*c*d^3 - 5*b*d
^2*e + 7*a*d*e^2)*x^5)*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-e/d) -
 d)/(e*x^2 + d)))/(d^4*e*x^7 + d^5*x^5), -1/30*(15*(3*c*d^2*e - 5
*b*d*e^2 + 7*a*e^3)*x^6 + 10*(3*c*d^3 - 5*b*d^2*e + 7*a*d*e^2)*x^
4 + 6*a*d^3 + 2*(5*b*d^3 - 7*a*d^2*e)*x^2 + 15*((3*c*d^2*e - 5*b*
d*e^2 + 7*a*e^3)*x^7 + (3*c*d^3 - 5*b*d^2*e + 7*a*d*e^2)*x^5)*sqr
t(e/d)*arctan(e*x/(d*sqrt(e/d))))/(d^4*e*x^7 + d^5*x^5)]
```

Sympy [A] time = 9.70136, size = 284, normalized size = 2.09

$$\frac{\sqrt{-\frac{e}{d^{9}}}\left(7ae^{2}-5bde+3cd^{2}\right)\log\left(-\frac{d^{5}\sqrt{-\frac{e}{d^{9}}}\left(7ae^{2}-5bde+3cd^{2}\right)}{7ae^{3}-5bde^{2}+3cd^{2}e}+x\right)}{4}-\frac{\sqrt{-\frac{e}{d^{9}}}\left(7ae^{2}-5bde+3cd^{2}\right)\log\left(\frac{d^{5}\sqrt{-\frac{e}{d^{9}}}\left(7ae^{2}-5bde+3cd^{2}\right)}{7ae^{3}-5bde^{2}+3cd^{2}e}+x\right)}{4}-\frac{6ad^{3}+x^{6}\left(105ae^{3}-75bde^{2}+45cd^{2}e\right)+x^{4}\left(70ade^{2}-50bd^{2}e+30cd^{3}\right)+x^{2}\left(-14ad^{2}e+10bd^{3}\right)}{30d^{5}x^{5}+30d^{4}ex^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**6/(e*x**2+d)**2,x)
```

```
[Out] sqrt(-e/d**9)*(7*a*e**2 - 5*b*d*e + 3*c*d**2)*log(-d**5*sqrt(-e/d
**9)*(7*a*e**2 - 5*b*d*e + 3*c*d**2)/(7*a*e**3 - 5*b*d*e**2 + 3*c
*d**2*e) + x)/4 - sqrt(-e/d**9)*(7*a*e**2 - 5*b*d*e + 3*c*d**2)*l
og(d**5*sqrt(-e/d**9)*(7*a*e**2 - 5*b*d*e + 3*c*d**2)/(7*a*e**3 -
 5*b*d*e**2 + 3*c*d**2*e) + x)/4 - (6*a*d**3 + x**6*(105*a*e**3 -
 75*b*d*e**2 + 45*c*d**2*e) + x**4*(70*a*d*e**2 - 50*b*d**2*e + 3
0*c*d**3) + x**2*(-14*a*d**2*e + 10*b*d**3))/(30*d**5*x**5 + 30*d
**4*e*x**7)
```

GIAC/XCAS [A] time = 0.270252, size = 177, normalized size = 1.3

$$-\frac{\left(3\,cd^2e - 5\,bde^2 + 7\,ae^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d^{\frac{9}{2}}} - \frac{cd^2xe - bdxe^2 + axe^3}{2\,(x^2e + d)d^4} - \frac{15\,cd^2x^4 - 30\,bdx^4e + 45\,ax^4e^2 + 5\,bd^2x^2 - 10\,adx^2e + 3\,ad^2}{15\,d^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^2*x^6),x, algorithm="giac")
```

```
[Out] -1/2*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*arctan(x*e^(1/2)/sqrt(d))*
e^(-1/2)/d^(9/2) - 1/2*(c*d^2*x*e - b*d*x*e^2 + a*x*e^3)/((x^2*e
+ d)*d^4) - 1/15*(15*c*d^2*x^4 - 30*b*d*x^4*e + 45*a*x^4*e^2 + 5*
b*d^2*x^2 - 10*a*d*x^2*e + 3*a*d^2)/(d^4*x^5)
```

3.279 $\int \frac{a+bx^2+cx^4}{x^8(d+ex^2)^2} dx$

Optimal. Leaf size=167

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5cd^2-e(7bd-9ae)\right)}{2d^{11/2}}+\frac{e^2x\left(ae^2-bde+cd^2\right)}{2d^5\left(d+ex^2\right)}+\frac{e\left(2cd^2-e(3bd-4ae)\right)}{d^5x}-\frac{cd^2-e(2bd-3ae)}{3d^4x^3}-\frac{bd-2ae}{5d^3x^5}-\frac{a}{7d^2x^7}$$

```
[Out] -a/(7*d^2*x^7) - (b*d - 2*a*e)/(5*d^3*x^5) - (c*d^2 - e*(2*b*d -
3*a*e))/(3*d^4*x^3) + (e*(2*c*d^2 - e*(3*b*d - 4*a*e)))/(d^5*x) +
 (e^2*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^5*(d + e*x^2)) + (e^(3/2)*(
5*c*d^2 - e*(7*b*d - 9*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(1
1/2))
```

Rubi [A] time = 0.584933, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5cd^2-e(7bd-9ae)\right)}{2d^{11/2}}+\frac{e^2x\left(ae^2-bde+cd^2\right)}{2d^5\left(d+ex^2\right)}+\frac{e\left(2cd^2-e(3bd-4ae)\right)}{d^5x}-\frac{cd^2-e(2bd-3ae)}{3d^4x^3}-\frac{bd-2ae}{5d^3x^5}-\frac{a}{7d^2x^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x]
```

```
[Out] -a/(7*d^2*x^7) - (b*d - 2*a*e)/(5*d^3*x^5) - (c*d^2 - e*(2*b*d -
3*a*e))/(3*d^4*x^3) + (e*(2*c*d^2 - e*(3*b*d - 4*a*e)))/(d^5*x) +
 (e^2*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^5*(d + e*x^2)) + (e^(3/2)*(
5*c*d^2 - e*(7*b*d - 9*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(1
1/2))
```

Rubi in Sympy [A] time = 88.3198, size = 158, normalized size = 0.95

$$-\frac{a}{7d^2x^7}+\frac{2ae-bd}{5d^3x^5}-\frac{3ae^2-2bde+cd^2}{3d^4x^3}+\frac{e^2x\left(ae^2-bde+cd^2\right)}{2d^5\left(d+ex^2\right)}+\frac{e\left(4ae^2-3bde+2cd^2\right)}{d^5x}+\frac{e^{\frac{3}{2}}\left(9ae^2-7bde+5cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**8/(e*x**2+d)**2,x)
```

```
[Out] -a/(7*d**2*x**7) + (2*a*e - b*d)/(5*d**3*x**5) - (3*a*e**2 - 2*b*
d*e + c*d**2)/(3*d**4*x**3) + e**2*x*(a*e**2 - b*d*e + c*d**2)/(2
*d**5*(d + e*x**2)) + e*(4*a*e**2 - 3*b*d*e + 2*c*d**2)/(d**5*x)
+ e**(3/2)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)*atan(sqrt(e)*x/sqrt(d)
)/(2*d**(11/2))
```

Mathematica [A] time = 0.166284, size = 166, normalized size = 0.99

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(9ae^2-7bde+5cd^2\right)}{2d^{11/2}}+\frac{e^2x\left(ae^2-bde+cd^2\right)}{2d^5\left(d+ex^2\right)}+\frac{e\left(4ae^2-3bde+2cd^2\right)}{d^5x}+\frac{-3ae^2+2bde-cd^2}{3d^4x^3}+\frac{2ae-bd}{5d^3x^5}-\frac{a}{7d^2x^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x]
```

```
[Out] -a/(7*d^2*x^7) + (-(b*d) + 2*a*e)/(5*d^3*x^5) + (-(c*d^2) + 2*b*d
*e - 3*a*e^2)/(3*d^4*x^3) + (e*(2*c*d^2 - 3*b*d*e + 4*a*e^2))/(d^
5*x) + (e^2*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^5*(d + e*x^2)) + (e^(
3/2)*(5*c*d^2 - 7*b*d*e + 9*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
2*d^(11/2))
```

Maple [A] time = 0.023, size = 221, normalized size = 1.3

$$-\frac{a}{7\,d^2x^7}+\frac{2\,ae}{5\,d^3x^5}-\frac{b}{5\,d^2x^5}-\frac{ae^2}{d^4x^3}+\frac{2\,be}{3\,d^3x^3}-\frac{c}{3\,d^2x^3}+4\,\frac{e^3a}{d^5x}-3\,\frac{be^2}{d^4x}+2\,\frac{ce}{d^3x}+\frac{e^4xa}{2\,d^5\left(ex^2+d\right)}-\frac{e^3xb}{2\,d^4\left(ex^2+d\right)}+\frac{e^2xc}{2\,d^3\left(ex^2+d\right)}+\frac{9\,ae^4}{2\,d^5}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{7\,be^3}{2\,d^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{5\,e^2c}{2\,d^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x)
```

```
[Out] -1/7*a/d^2/x^7+2/5/d^3/x^5*a*e-1/5/d^2/x^5*b-1/d^4/x^3*a*e^2+2/3/
d^3/x^3*b*e-1/3/d^2/x^3*c+4*e^3/d^5/x*a-3*e^2/d^4/x*b+2*e/d^3/x*c
```

```
+1/2*e^4/d^5*x/(e*x^2+d)*a-1/2*e^3/d^4*x/(e*x^2+d)*b+1/2*e^2/d^3*
x/(e*x^2+d)*c+9/2*e^4/d^5/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a-7
/2*e^3/d^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b+5/2*e^2/d^3/(d*e
)^(1/2)*arctan(x*e/(d*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^2*x^8),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266977, size = 1, normalized size = 0.01

$$\left[\frac{210\left(5\,cd^{2}e^{2}-7\,bde^{3}+9\,ae^{4}\right)x^{8}+140\left(5\,cd^{3}e-7\,bd^{2}e^{2}+9\,ade^{3}\right)x^{6}-60\,ad^{4}-28\left(5\,cd^{4}-7\,bd^{3}e+9\,ad^{2}e^{2}\right)x^{4}-12\left(7\,ba\right.}{420\left(d^{5}ex^{9}+d^{6}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^2*x^8),x, algorithm="fricas")
```

```
[Out] [1/420*(210*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^8 + 140*(5*c*d^
3*e - 7*b*d^2*e^2 + 9*a*d*e^3)*x^6 - 60*a*d^4 - 28*(5*c*d^4 - 7*b
*d^3*e + 9*a*d^2*e^2)*x^4 - 12*(7*b*d^4 - 9*a*d^3*e)*x^2 + 105*((
5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^9 + (5*c*d^3*e - 7*b*d^2*e^2
 + 9*a*d*e^3)*x^7)*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/
(e*x^2 + d)))/(d^5*e*x^9 + d^6*x^7), 1/210*(105*(5*c*d^2*e^2 - 7*
b*d*e^3 + 9*a*e^4)*x^8 + 70*(5*c*d^3*e - 7*b*d^2*e^2 + 9*a*d*e^3)
*x^6 - 30*a*d^4 - 14*(5*c*d^4 - 7*b*d^3*e + 9*a*d^2*e^2)*x^4 - 6*
(7*b*d^4 - 9*a*d^3*e)*x^2 + 105*((5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e
^4)*x^9 + (5*c*d^3*e - 7*b*d^2*e^2 + 9*a*d*e^3)*x^7)*sqrt(e/d)*ar
ctan(e*x/(d*sqrt(e/d))))/(d^5*e*x^9 + d^6*x^7)]
```

Sympy [A] time = 13.0207, size = 328, normalized size = 1.96

$$-\frac{\sqrt{-\frac{e^3}{d^{11}}}\left(9ae^2-7bde+5cd^2\right)\log\left(-\frac{d^6\sqrt{-\frac{e^3}{d^{11}}}\left(9ae^2-7bde+5cd^2\right)}{9ae^4-7bde^3+5cd^2e^2}+x\right)}{4}+\frac{\sqrt{-\frac{e^3}{d^{11}}}\left(9ae^2-7bde+5cd^2\right)\log\left(\frac{d^6\sqrt{-\frac{e^3}{d^{11}}}\left(9ae^2-7bde+5cd^2\right)}{9ae^4-7bde^3+5cd^2e^2}+x\right)}{4}+\frac{-30ad^4+x^8\left(945ae^4-735bde^3+525cd^2e^2\right)+x^6\left(630ade^3-490bd^2e^2+350cd^3e\right)+x^4\left(-126ad^2e^2+98bd^3e-70cd^4\right)+}{210d^6x^7+210d^5ex^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**8/(e*x**2+d)**2,x)
```

```
[Out] -sqrt(-e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)*log(-d**6*sqrt
(-e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)/(9*a*e**4 - 7*b*d*e
**3 + 5*c*d**2*e**2) + x)/4 + sqrt(-e**3/d**11)*(9*a*e**2 - 7*b*d
*e + 5*c*d**2)*log(d**6*sqrt(-e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5
*c*d**2)/(9*a*e**4 - 7*b*d*e**3 + 5*c*d**2*e**2) + x)/4 + (-30*a*
d**4 + x**8*(945*a*e**4 - 735*b*d*e**3 + 525*c*d**2*e**2) + x**6*
(630*a*d*e**3 - 490*b*d**2*e**2 + 350*c*d**3*e) + x**4*(-126*a*d*
*2*e**2 + 98*b*d**3*e - 70*c*d**4) + x**2*(54*a*d**3*e - 42*b*d**
4))/(210*d**6*x**7 + 210*d**5*e*x**9)
```

GIAC/XCAS [A] time = 0.269673, size = 221, normalized size = 1.32

$$\frac{\left(5\,cd^2e^2-7\,bde^3+9\,ae^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d^{\frac{11}{2}}}+\frac{cd^2xe^2-bdxe^3+axe^4}{2\,(x^2e+d)d^5}+\frac{210\,cd^2x^6e-315\,bdx^6e^2-35\,cd^3x^4+420\,ax^6e^3+70\,bd^2x^4e-105\,adx^4e^2-21\,bd^3x^2+42\,ad^2x^2e-15\,ad^3}{105\,d^5x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^2*x^8),x, algorithm="giac")
```

```
[Out] 1/2*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*arctan(x*e^(1/2)/sqrt(d))
*e^(-1/2)/d^(11/2) + 1/2*(c*d^2*x*e^2 - b*d*x*e^3 + a*x*e^4)/((x^
2*e + d)*d^5) + 1/105*(210*c*d^2*x^6*e - 315*b*d*x^6*e^2 - 35*c*d
^3*x^4 + 420*a*x^6*e^3 + 70*b*d^2*x^4*e - 105*a*d*x^4*e^2 - 21*b*
d^3*x^2 + 42*a*d^2*x^2*e - 15*a*d^3)/(d^5*x^7)
```

3.280 $\int \frac{x^6\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=187

$$\frac{dx\left(11cd^2-e(7bd-3ae)\right)}{8e^5\left(d+ex^2\right)}+\frac{x\left(13cd^2-e(7bd-3ae)\right)}{2e^5}-\frac{x^3\left(15cd^2-e(7bd-3ae)\right)}{12de^4}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(15ae^2-35bde+63cd^2\right)}{8e^{11/2}}+\frac{x^7\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}+\frac{cx^5}{5e^3}$$

```
[Out] ((13*c*d^2 - e*(7*b*d - 3*a*e))*x)/(2*e^5) - ((15*c*d^2 - e*(7*b*
d - 3*a*e))*x^3)/(12*d*e^4) + (c*x^5)/(5*e^3) + ((a + (d*(c*d - b
*e))/e^2)*x^7)/(4*d*(d + e*x^2)^2) + (d*(11*c*d^2 - e*(7*b*d - 3*
a*e))*x)/(8*e^5*(d + e*x^2)) - (Sqrt[d]*(63*c*d^2 - 35*b*d*e + 15
*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*e^(11/2))
```

Rubi [A] time = 0.599985, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{dx\left(11cd^2-e(7bd-3ae)\right)}{8e^5\left(d+ex^2\right)}+\frac{x\left(13cd^2-e(7bd-3ae)\right)}{2e^5}-\frac{x^3\left(15cd^2-e(7bd-3ae)\right)}{12de^4}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(15ae^2-35bde+63cd^2\right)}{8e^{11/2}}+\frac{x^7\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}+\frac{cx^5}{5e^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] ((13*c*d^2 - e*(7*b*d - 3*a*e))*x)/(2*e^5) - ((15*c*d^2 - e*(7*b*
d - 3*a*e))*x^3)/(12*d*e^4) + (c*x^5)/(5*e^3) + ((a + (d*(c*d - b
*e))/e^2)*x^7)/(4*d*(d + e*x^2)^2) + (d*(11*c*d^2 - e*(7*b*d - 3*
a*e))*x)/(8*e^5*(d + e*x^2)) - (Sqrt[d]*(63*c*d^2 - 35*b*d*e + 15
*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*e^(11/2))
```

Rubi in Sympy [A] time = 129.758, size = 165, normalized size = 0.88

$$\frac{cx^5}{5e^3}-\frac{\sqrt{d}\left(15ae^2-35bde+63cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8e^{\frac{11}{2}}}-\frac{d^2x\left(ae^2-bde+cd^2\right)}{4e^5\left(d+ex^2\right)^2}+\frac{dx\left(9ae^2-13bde+17cd^2\right)}{8e^5\left(d+ex^2\right)}+\frac{x^3\left(be-3cd\right)}{3e^4}+\frac{x\left(ae^2-3bde+6cd^2\right)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x**5/(5*e**3) - sqrt(d)*(15*a*e**2 - 35*b*d*e + 63*c*d**2)*atan
(sqrt(e)*x/sqrt(d))/(8*e**(11/2)) - d**2*x*(a*e**2 - b*d*e + c*d*
*2)/(4*e**5*(d + e*x**2)**2) + d*x*(9*a*e**2 - 13*b*d*e + 17*c*d*
*2)/(8*e**5*(d + e*x**2)) + x**3*(b*e - 3*c*d)/(3*e**4) + x*(a*e*
*2 - 3*b*d*e + 6*c*d**2)/e**5
```

Mathematica [A] time = 0.190401, size = 170, normalized size = 0.91

$$\frac{x\left(de(9ae-13bd)+17cd^3\right)}{8e^5\left(d+ex^2\right)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5e(3ae-7bd)+63cd^2\right)}{8e^{11/2}}+\frac{x\left(e(ae-3bd)+6cd^2\right)}{e^5}-\frac{x\left(d^2e(ae-bd)+cd^4\right)}{4e^5\left(d+ex^2\right)^2}+\frac{x^3(be-3cd)}{3e^4}+\frac{cx^5}{5e^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] ((6*c*d^2 + e*(-3*b*d + a*e))*x)/e^5 + ((-3*c*d + b*e)*x^3)/(3*e^
4) + (c*x^5)/(5*e^3) - ((c*d^4 + d^2*e*(-(b*d) + a*e))*x)/(4*e^5*
(d + e*x^2)^2) + ((17*c*d^3 + d*e*(-13*b*d + 9*a*e))*x)/(8*e^5*(d
 + e*x^2)) - (Sqrt[d]*(63*c*d^2 + 5*e*(-7*b*d + 3*a*e))*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(8*e^(11/2))
```

Maple [A] time = 0.018, size = 239, normalized size = 1.3

$$\frac{cx^5}{5\,e^3}+\frac{bx^3}{3\,e^3}-\frac{cdx^3}{e^4}+\frac{ax}{e^3}-3\,\frac{bxd}{e^4}+6\,\frac{cd^2x}{e^5}+\frac{9\,dx^3a}{8\,e^2\left(ex^2+d\right)^2}-\frac{13\,d^2x^3b}{8\,e^3\left(ex^2+d\right)^2}+\frac{17\,d^3x^3c}{8\,e^4\left(ex^2+d\right)^2}+\frac{7\,ad^2x}{8\,e^3\left(ex^2+d\right)^2}-\frac{11\,bd^3x}{8\,e^4\left(ex^2+d\right)^2}+\frac{15\,cd^4x}{8\,e^5\left(ex^2+d\right)^2}-\frac{15\,ad}{8\,e^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{35\,bd^2}{8\,e^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{63\,cd^3}{8\,e^5}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x)
```

```
[Out] 1/5*c*x^5/e^3+1/3/e^3*x^3*b-1/e^4*c*d*x^3+1/e^3*a*x-3/e^4*x*b*d+6
/e^5*c*d^2*x+9/8*d/e^2/(e*x^2+d)^2*x^3*a-13/8*d^2/e^3/(e*x^2+d)^2
```

```
*x^3*b+17/8*d^3/e^4/(e*x^2+d)^2*x^3*c+7/8*d^2/e^3/(e*x^2+d)^2*a*x
-11/8*d^3/e^4/(e*x^2+d)^2*b*x+15/8*d^4/e^5/(e*x^2+d)^2*c*x-15/8*d
/e^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+35/8*d^2/e^4/(d*e)^(1/
2)*arctan(x*e/(d*e)^(1/2))*b-63/8*d^3/e^5/(d*e)^(1/2)*arctan(x*e/
(d*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*x^6/(e*x^2 + d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.267524, size = 1, normalized size = 0.01

$$\left[\frac{48\,ce^4x^9 - 16\left(9\,cde^3 - 5\,be^4\right)x^7 + 16\left(63\,cd^2e^2 - 35\,bde^3 + 15\,ae^4\right)x^5 + 50\left(63\,cd^3e - 35\,bd^2e^2 + 15\,ade^3\right)x^3 + 15\left(63\,cd^4 -\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*x^6/(e*x^2 + d)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(48*c*e^4*x^9 - 16*(9*c*d*e^3 - 5*b*e^4)*x^7 + 16*(63*c*d^
2*e^2 - 35*b*d*e^3 + 15*a*e^4)*x^5 + 50*(63*c*d^3*e - 35*b*d^2*e^
2 + 15*a*d*e^3)*x^3 + 15*(63*c*d^4 - 35*b*d^3*e + 15*a*d^2*e^2 +
(63*c*d^2*e^2 - 35*b*d*e^3 + 15*a*e^4)*x^4 + 2*(63*c*d^3*e - 35*b
*d^2*e^2 + 15*a*d*e^3)*x^2)*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-d
/e) - d)/(e*x^2 + d)) + 30*(63*c*d^4 - 35*b*d^3*e + 15*a*d^2*e^2)
*x)/(e^7*x^4 + 2*d*e^6*x^2 + d^2*e^5), 1/120*(24*c*e^4*x^9 - 8*(9
*c*d*e^3 - 5*b*e^4)*x^7 + 8*(63*c*d^2*e^2 - 35*b*d*e^3 + 15*a*e^4
)*x^5 + 25*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^3 - 15*(63*
c*d^4 - 35*b*d^3*e + 15*a*d^2*e^2 + (63*c*d^2*e^2 - 35*b*d*e^3 +
15*a*e^4)*x^4 + 2*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^2)*s
qrt(d/e)*arctan(x/sqrt(d/e)) + 15*(63*c*d^4 - 35*b*d^3*e + 15*a*d
^2*e^2)*x)/(e^7*x^4 + 2*d*e^6*x^2 + d^2*e^5)]
```

Sympy [A] time = 15.8907, size = 233, normalized size = 1.25

$$\frac{cx^5}{5e^3} + \frac{\sqrt{-\frac{d}{e^{11}}}\left(15ae^2 - 35bde + 63cd^2\right)\log\left(-e^5\sqrt{-\frac{d}{e^{11}}} + x\right)}{16} - \frac{\sqrt{-\frac{d}{e^{11}}}\left(15ae^2 - 35bde + 63cd^2\right)\log\left(e^5\sqrt{-\frac{d}{e^{11}}} + x\right)}{16} + \frac{x^3\left(9ade^3 - 13bd^2e^2 + 17cd^3e\right) + x\left(7ad^2e^2 - 11bd^3e + 15cd^4\right)}{8d^2e^5 + 16de^6x^2 + 8e^7x^4} + \frac{x^3\left(be - 3cd\right)}{3e^4} + \frac{x\left(ae^2 - 3bde + 6cd^2\right)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x**5/(5*e**3) + sqrt(-d/e**11)*(15*a*e**2 - 35*b*d*e + 63*c*d**
2)*log(-e**5*sqrt(-d/e**11) + x)/16 - sqrt(-d/e**11)*(15*a*e**2 -
 35*b*d*e + 63*c*d**2)*log(e**5*sqrt(-d/e**11) + x)/16 + (x**3*(9
*a*d*e**3 - 13*b*d**2*e**2 + 17*c*d**3*e) + x*(7*a*d**2*e**2 - 11
*b*d**3*e + 15*c*d**4))/(8*d**2*e**5 + 16*d*e**6*x**2 + 8*e**7*x*
*4) + x**3*(b*e - 3*c*d)/(3*e**4) + x*(a*e**2 - 3*b*d*e + 6*c*d**
2)/e**5
```

GIAC/XCAS [A] time = 0.27167, size = 216, normalized size = 1.16

$$-\frac{\left(63\,cd^3 - 35\,bd^2e + 15\,ade^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{11}{2}\right)}}{8\,\sqrt{d}} + \frac{1}{15}\left(3\,cx^5e^{12} - 15\,cdx^3e^{11} + 5\,bx^3e^{12} + 90\,cd^2xe^{10} - 45\,bdxe^{11} + 15\,axe^{12}\right)e^{(-15)} + \frac{\left(17\,cd^3x^3e - 13\,bd^2x^3e^2 + 15\,cd^4x + 9\,adx^3e^3 - 11\,bd^3xe + 7\,ad^2xe^2\right)e^{(-5)}}{8\left(x^2e + d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^6/(e*x^2 + d)^3,x, algorithm="giac")
```

```
[Out] -1/8*(63*c*d^3 - 35*b*d^2*e + 15*a*d*e^2)*arctan(x*e^(1/2)/sqrt(d
))*e^(-11/2)/sqrt(d) + 1/15*(3*c*x^5*e^12 - 15*c*d*x^3*e^11 + 5*b
*x^3*e^12 + 90*c*d^2*x*e^10 - 45*b*d*x*e^11 + 15*a*x*e^12)*e^(-15
) + 1/8*(17*c*d^3*x^3*e - 13*b*d^2*x^3*e^2 + 15*c*d^4*x + 9*a*d*x
^3*e^3 - 11*b*d^3*x*e + 7*a*d^2*x*e^2)*e^(-5)/(x^2*e + d)^2
```

3.281 $\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=158

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(35cd^2-3e(5bd-ae)\right)}{8\sqrt{d}e^{9/2}}-\frac{x\left(9cd^2-e(5bd-ae)\right)}{8e^4\left(d+ex^2\right)}-\frac{x\left(13cd^2-e(5bd-ae)\right)}{4de^4}+\frac{x^5\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}+\frac{cx^3}{3e^3}$$

```
[Out] -((13*c*d^2 - e*(5*b*d - a*e))*x)/(4*d*e^4) + (c*x^3)/(3*e^3) + (
(a + (d*(c*d - b*e))/e^2)*x^5)/(4*d*(d + e*x^2)^2) - ((9*c*d^2 -
e*(5*b*d - a*e))*x)/(8*e^4*(d + e*x^2)) + ((35*c*d^2 - 3*e*(5*b*d
 - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(9/2))
```

Rubi [A] time = 0.47689, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(35cd^2-3e(5bd-ae)\right)}{8\sqrt{d}e^{9/2}}-\frac{x\left(9cd^2-e(5bd-ae)\right)}{8e^4\left(d+ex^2\right)}-\frac{x\left(13cd^2-e(5bd-ae)\right)}{4de^4}+\frac{x^5\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}+\frac{cx^3}{3e^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] -((13*c*d^2 - e*(5*b*d - a*e))*x)/(4*d*e^4) + (c*x^3)/(3*e^3) + (
(a + (d*(c*d - b*e))/e^2)*x^5)/(4*d*(d + e*x^2)^2) - ((9*c*d^2 -
e*(5*b*d - a*e))*x)/(8*e^4*(d + e*x^2)) + ((35*c*d^2 - 3*e*(5*b*d
 - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(9/2))
```

Rubi in Sympy [A] time = 97.3094, size = 134, normalized size = 0.85

$$\frac{cx^3}{3e^3}+\frac{dx\left(ae^2-bde+cd^2\right)}{4e^4\left(d+ex^2\right)^2}+\frac{x\left(be-3cd\right)}{e^4}-\frac{x\left(5ae^2-9bde+13cd^2\right)}{8e^4\left(d+ex^2\right)}+\frac{\left(3ae^2-15bde+35cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{d}e^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x**3/(3*e**3) + d*x*(a*e**2 - b*d*e + c*d**2)/(4*e**4*(d + e*x*
*2)**2) + x*(b*e - 3*c*d)/e**4 - x*(5*a*e**2 - 9*b*d*e + 13*c*d**
2)/(8*e**4*(d + e*x**2)) + (3*a*e**2 - 15*b*d*e + 35*c*d**2)*atan
(sqrt(e)*x/sqrt(d))/(8*sqrt(d)*e**(9/2))
```

Mathematica [A] time = 0.161904, size = 141, normalized size = 0.89

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3ae^2-15bde+35cd^2\right)}{8\sqrt{d}e^{9/2}}-\frac{x\left(5ae^2-9bde+13cd^2\right)}{8e^4\left(d+ex^2\right)}+\frac{x\left(ade^2-bd^2e+cd^3\right)}{4e^4\left(d+ex^2\right)^2}+\frac{x(be-3cd)}{e^4}+\frac{cx^3}{3e^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] ((-3*c*d + b*e)*x)/e^4 + (c*x^3)/(3*e^3) + ((c*d^3 - b*d^2*e + a*
d*e^2)*x)/(4*e^4*(d + e*x^2)^2) - ((13*c*d^2 - 9*b*d*e + 5*a*e^2)
*x)/(8*e^4*(d + e*x^2)) + ((35*c*d^2 - 15*b*d*e + 3*a*e^2)*ArcTan
[(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(9/2))
```

Maple [A] time = 0.016, size = 202, normalized size = 1.3

$$\frac{cx^3}{3\,e^3}+\frac{bx}{e^3}-3\,\frac{cdx}{e^4}-\frac{5\,x^3a}{8\,e\,(ex^2+d)^2}+\frac{9\,bx^3d}{8\,e^2\,(ex^2+d)^2}-\frac{13\,x^3cd^2}{8\,e^3\,(ex^2+d)^2}-\frac{3\,adx}{8\,e^2\,(ex^2+d)^2}+\frac{7\,xbd^2}{8\,e^3\,(ex^2+d)^2}-\frac{11\,cd^3x}{8\,e^4\,(ex^2+d)^2}+\frac{3\,a}{8\,e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{15\,bd}{8\,e^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{35\,cd^2}{8\,e^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x)
```

```
[Out] 1/3*c*x^3/e^3+1/e^3*b*x-3/e^4*c*d*x-5/8/e/(e*x^2+d)^2*x^3*a+9/8/e
^2/(e*x^2+d)^2*x^3*b*d-13/8/e^3/(e*x^2+d)^2*x^3*c*d^2-3/8/e^2/(e*
x^2+d)^2*d*a*x+7/8/e^3/(e*x^2+d)^2*b*d^2*x-11/8/e^4/(e*x^2+d)^2*d
^3*c*x+3/8/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a-15/8/e^3/(d*
e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b*d+35/8/e^4/(d*e)^(1/2)*arctan(
```

```
x*e/(d*e)^(1/2))*c*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*x^4/(e*x^2 + d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.279843, size = 1, normalized size = 0.01

$$\left[\frac{3\left(35\,cd^4-15\,bd^3e+3\,ad^2e^2+\left(35\,cd^2e^2-15\,bde^3+3\,ae^4\right)x^4+2\left(35\,cd^3e-15\,bd^2e^2+3\,ade^3\right)x^2\right)\log\left(\frac{2\,dex+\left(ex^2-d\right)\sqrt{-de}}{ex^2+d}\right.}{48\left(e^6x^4+2\,de^5x^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*x^4/(e*x^2 + d)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(35*c*d^4 - 15*b*d^3*e + 3*a*d^2*e^2 + (35*c*d^2*e^2 - 1
5*b*d*e^3 + 3*a*e^4)*x^4 + 2*(35*c*d^3*e - 15*b*d^2*e^2 + 3*a*d*e
^3)*x^2)*log((2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d)) + 2*
(8*c*e^3*x^7 - 8*(7*c*d*e^2 - 3*b*e^3)*x^5 - 5*(35*c*d^2*e - 15*b
*d*e^2 + 3*a*e^3)*x^3 - 3*(35*c*d^3 - 15*b*d^2*e + 3*a*d*e^2)*x)*
sqrt(-d*e))/((e^6*x^4 + 2*d*e^5*x^2 + d^2*e^4)*sqrt(-d*e)), 1/24*
(3*(35*c*d^4 - 15*b*d^3*e + 3*a*d^2*e^2 + (35*c*d^2*e^2 - 15*b*d*
e^3 + 3*a*e^4)*x^4 + 2*(35*c*d^3*e - 15*b*d^2*e^2 + 3*a*d*e^3)*x^
2)*arctan(sqrt(d*e)*x/d) + (8*c*e^3*x^7 - 8*(7*c*d*e^2 - 3*b*e^3)
*x^5 - 5*(35*c*d^2*e - 15*b*d*e^2 + 3*a*e^3)*x^3 - 3*(35*c*d^3 -
15*b*d^2*e + 3*a*d*e^2)*x)*sqrt(d*e))/((e^6*x^4 + 2*d*e^5*x^2 + d
^2*e^4)*sqrt(d*e))]
```

Sympy [A] time = 14.4553, size = 211, normalized size = 1.34

$$\frac{cx^3}{3e^3} - \frac{\sqrt{-\frac{1}{de^9}}\left(3ae^2 - 15bde + 35cd^2\right)\log\left(-de^4\sqrt{-\frac{1}{de^9}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{de^9}}\left(3ae^2 - 15bde + 35cd^2\right)\log\left(de^4\sqrt{-\frac{1}{de^9}} + x\right)}{16} - \frac{x^3\left(5ae^3 - 9bde^2 + 13cd^2e\right) + x\left(3ade^2 - 7bd^2e + 11cd^3\right)}{8d^2e^4 + 16de^5x^2 + 8e^6x^4} + \frac{x(be - 3cd)}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x**3/(3*e**3) - sqrt(-1/(d*e**9))*(3*a*e**2 - 15*b*d*e + 35*c*d
**2)*log(-d*e**4*sqrt(-1/(d*e**9)) + x)/16 + sqrt(-1/(d*e**9))*(3
*a*e**2 - 15*b*d*e + 35*c*d**2)*log(d*e**4*sqrt(-1/(d*e**9)) + x)
/16 - (x**3*(5*a*e**3 - 9*b*d*e**2 + 13*c*d**2*e) + x*(3*a*d*e**2
 - 7*b*d**2*e + 11*c*d**3))/(8*d**2*e**4 + 16*d*e**5*x**2 + 8*e**
6*x**4) + x*(b*e - 3*c*d)/e**4
```

GIAC/XCAS [A] time = 0.272715, size = 169, normalized size = 1.07

$$\frac{\left(35\,cd^2 - 15\,bde + 3\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{8\sqrt{d}} + \frac{1}{3}\left(cx^3e^6 - 9\,cdxe^5 + 3\,bxe^6\right)e^{(-9)} - \frac{\left(13\,cd^2x^3e - 9\,bdx^3e^2 + 11\,cd^3x + 5\,ax^3e^3 - 7\,bd^2xe + 3\,adxe^2\right)e^{(-4)}}{8\left(x^2e + d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^4/(e*x^2 + d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*c*d^2 - 15*b*d*e + 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(
-9/2)/sqrt(d) + 1/3*(c*x^3*e^6 - 9*c*d*x*e^5 + 3*b*x*e^6)*e^(-9)
- 1/8*(13*c*d^2*x^3*e - 9*b*d*x^3*e^2 + 11*c*d^3*x + 5*a*x^3*e^3
- 7*b*d^2*x*e + 3*a*d*x*e^2)*e^(-4)/(x^2*e + d)^2
```

3.282 $\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3} \, dx$

Optimal. Leaf size=124

$$\frac{x\left(7cd^2-e(ae+3bd)\right)}{8de^3\left(d+ex^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(15cd^2-e(ae+3bd)\right)}{8d^{3/2}e^{7/2}}+\frac{x^3\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}+\frac{cx}{e^3}$$

```
[Out] (c*x)/e^3 + ((a + (d*(c*d - b*e))/e^2)*x^3)/(4*d*(d + e*x^2)^2) +
 ((7*c*d^2 - e*(3*b*d + a*e))*x)/(8*d*e^3*(d + e*x^2)) - ((15*c*d
^2 - e*(3*b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(
7/2))
```

Rubi [A] time = 0.367433, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{x\left(7cd^2-e(ae+3bd)\right)}{8de^3\left(d+ex^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(15cd^2-e(ae+3bd)\right)}{8d^{3/2}e^{7/2}}+\frac{x^3\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}+\frac{cx}{e^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] (c*x)/e^3 + ((a + (d*(c*d - b*e))/e^2)*x^3)/(4*d*(d + e*x^2)^2) +
 ((7*c*d^2 - e*(3*b*d + a*e))*x)/(8*d*e^3*(d + e*x^2)) - ((15*c*d
^2 - e*(3*b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(
7/2))
```

Rubi in Sympy [A] time = 54.6233, size = 114, normalized size = 0.92

$$\frac{cx}{e^3}-\frac{x\left(ae^2-bde+cd^2\right)}{4e^3\left(d+ex^2\right)^2}+\frac{x\left(ae^2-5bde+9cd^2\right)}{8de^3\left(d+ex^2\right)}+\frac{\left(ae^2+3bde-15cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{\frac{3}{2}}e^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x/e**3 - x*(a*e**2 - b*d*e + c*d**2)/(4*e**3*(d + e*x**2)**2) +
 x*(a*e**2 - 5*b*d*e + 9*c*d**2)/(8*d*e**3*(d + e*x**2)) + (a*e**
2 + 3*b*d*e - 15*c*d**2)*atan(sqrt(e)*x/sqrt(d))/(8*d**(3/2)*e**(
```

```
7/2))
```

Mathematica [A] time = 0.184947, size = 122, normalized size = 0.98

$$\frac{x\left(ae^2-5bde+9cd^2\right)}{8de^3\left(d+ex^2\right)}-\frac{x\left(ae^2-bde+cd^2\right)}{4e^3\left(d+ex^2\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-ae^2-3bde+15cd^2\right)}{8d^{3/2}e^{7/2}}+\frac{cx}{e^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] (c*x)/e^3 - ((c*d^2 - b*d*e + a*e^2)*x)/(4*e^3*(d + e*x^2)^2) + (
(9*c*d^2 - 5*b*d*e + a*e^2)*x)/(8*d*e^3*(d + e*x^2)) - ((15*c*d^2
 - 3*b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(7/
2))
```

Maple [A] time = 0.015, size = 179, normalized size = 1.4

$$\frac{cx}{e^3}+\frac{x^3a}{8\,(ex^2+d)^2\,d}-\frac{5\,bx^3}{8\,e\,(ex^2+d)^2}+\frac{9\,cdx^3}{8\,e^2\,(ex^2+d)^2}-\frac{ax}{8\,e\,(ex^2+d)^2}-\frac{3\,bxd}{8\,e^2\,(ex^2+d)^2}+\frac{7\,cd^2x}{8\,e^3\,(ex^2+d)^2}+\frac{a}{8\,de}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{3\,b}{8\,e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{15\,cd}{8\,e^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x)
```

```
[Out] c*x/e^3+1/8/(e*x^2+d)^2/d*x^3*a-5/8/e/(e*x^2+d)^2*x^3*b+9/8/e^2/(
e*x^2+d)^2*c*d*x^3-1/8/e/(e*x^2+d)^2*a*x-3/8/e^2/(e*x^2+d)^2*x*b*
d+7/8/e^3/(e*x^2+d)^2*c*d^2*x+1/8/e/d/(d*e)^(1/2)*arctan(x*e/(d*e
)^(1/2))*a+3/8/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b-15/8/e^3
*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*x^2/(e*x^2 + d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269027, size = 1, normalized size = 0.01

$$\left[-\frac{\left(15\,cd^4-3\,bd^3e-ad^2e^2+\left(15\,cd^2e^2-3\,bde^3-ae^4\right)x^4+2\left(15\,cd^3e-3\,bd^2e^2-ade^3\right)x^2\right)\log\left(\frac{2\,dex+\left(ex^2-d\right)\sqrt{-de}}{ex^2+d}\right)-2\,(8}{16\left(de^5x^4+2\,d^2e^4x^2+d^3e^3\right)\sqrt{-de}}\right.$$

$$-\frac{\left(15\,cd^4-3\,bd^3e-ad^2e^2+\left(15\,cd^2e^2-3\,bde^3-ae^4\right)x^4+2\left(15\,cd^3e-3\,bd^2e^2-ade^3\right)x^2\right)\arctan\left(\frac{\sqrt{de}x}{d}\right)-\left(8\,cde^2x^5+(2\right.}{8\left(de^5x^4+2\,d^2e^4x^2+d^3e^3\right)\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*x^2/(e*x^2 + d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*((15*c*d^4 - 3*b*d^3*e - a*d^2*e^2 + (15*c*d^2*e^2 - 3*b*d
*e^3 - a*e^4)*x^4 + 2*(15*c*d^3*e - 3*b*d^2*e^2 - a*d*e^3)*x^2)*l
og((2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d)) - 2*(8*c*d*e^2
*x^5 + (25*c*d^2*e - 5*b*d*e^2 + a*e^3)*x^3 + (15*c*d^3 - 3*b*d^2
*e - a*d*e^2)*x)*sqrt(-d*e))/((d*e^5*x^4 + 2*d^2*e^4*x^2 + d^3*e^
3)*sqrt(-d*e)), -1/8*((15*c*d^4 - 3*b*d^3*e - a*d^2*e^2 + (15*c*d
^2*e^2 - 3*b*d*e^3 - a*e^4)*x^4 + 2*(15*c*d^3*e - 3*b*d^2*e^2 - a
*d*e^3)*x^2)*arctan(sqrt(d*e)*x/d) - (8*c*d*e^2*x^5 + (25*c*d^2*e
 - 5*b*d*e^2 + a*e^3)*x^3 + (15*c*d^3 - 3*b*d^2*e - a*d*e^2)*x)*s
qrt(d*e))/((d*e^5*x^4 + 2*d^2*e^4*x^2 + d^3*e^3)*sqrt(d*e))]
```

Sympy [A] time = 11.1834, size = 201, normalized size = 1.62

$$\frac{cx}{e^3}-\frac{\sqrt{-\frac{1}{d^3e^7}}\left(ae^2+3bde-15cd^2\right)\log\left(-d^2e^3\sqrt{-\frac{1}{d^3e^7}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{d^3e^7}}\left(ae^2+3bde-15cd^2\right)\log\left(d^2e^3\sqrt{-\frac{1}{d^3e^7}}+x\right)}{16}+\frac{x^3\left(ae^3-5bde^2+9cd^2e\right)+x\left(-ade^2-3bd^2e+7cd^3\right)}{8d^3e^3+16d^2e^4x^2+8de^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x/e**3 - sqrt(-1/(d**3*e**7))*(a*e**2 + 3*b*d*e - 15*c*d**2)*lo
g(-d**2*e**3*sqrt(-1/(d**3*e**7)) + x)/16 + sqrt(-1/(d**3*e**7))*
(a*e**2 + 3*b*d*e - 15*c*d**2)*log(d**2*e**3*sqrt(-1/(d**3*e**7))
 + x)/16 + (x**3*(a*e**3 - 5*b*d*e**2 + 9*c*d**2*e) + x*(-a*d*e**
2 - 3*b*d**2*e + 7*c*d**3))/(8*d**3*e**3 + 16*d**2*e**4*x**2 + 8*
d*e**5*x**4)
```

GIAC/XCAS [A] time = 0.271772, size = 144, normalized size = 1.16

$$cxe^{(-3)} - \frac{\left(15\,cd^2 - 3\,bde - ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{8\,d^{\frac{3}{2}}} + \frac{\left(9\,cd^2x^3e - 5\,bdx^3e^2 + 7\,cd^3x + ax^3e^3 - 3\,bd^2xe - adxe^2\right)e^{(-3)}}{8\left(x^2e + d\right)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^2/(e*x^2 + d)^3,x, algorithm="giac")
```

```
[Out] c*x*e^(-3) - 1/8*(15*c*d^2 - 3*b*d*e - a*e^2)*arctan(x*e^(1/2)/sq
rt(d))*e^(-7/2)/d^(3/2) + 1/8*(9*c*d^2*x^3*e - 5*b*d*x^3*e^2 + 7*
c*d^3*x + a*x^3*e^3 - 3*b*d^2*x*e - a*d*x*e^2)*e^(-3)/((x^2*e + d
)^2*d)
```

3.283 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=115

$$-\frac{x\left(5cd^2-e(3ae+bd)\right)}{8d^2e^2\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}$$

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(4*d*(d + e*x^2)^2) - ((5*c*d^2 - e
*(b*d + 3*a*e))*x)/(8*d^2*e^2*(d + e*x^2)) + ((3*c*d^2 + e*(b*d +
 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Rubi [A] time = 0.2063, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{x\left(5cd^2-e(3ae+bd)\right)}{8d^2e^2\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(4*d*(d + e*x^2)^2) - ((5*c*d^2 - e
*(b*d + 3*a*e))*x)/(8*d^2*e^2*(d + e*x^2)) + ((3*c*d^2 + e*(b*d +
 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Rubi in Sympy [A] time = 34.0753, size = 110, normalized size = 0.96

$$\frac{x\left(ae^2-bde+cd^2\right)}{4de^2\left(d+ex^2\right)^2}+\frac{x\left(3ae^2+bde-5cd^2\right)}{8d^2e^2\left(d+ex^2\right)}+\frac{\left(3ae^2+bde+3cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{\frac{5}{2}}e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] x*(a*e**2 - b*d*e + c*d**2)/(4*d*e**2*(d + e*x**2)**2) + x*(3*a*e
**2 + b*d*e - 5*c*d**2)/(8*d**2*e**2*(d + e*x**2)) + (3*a*e**2 +
b*d*e + 3*c*d**2)*atan(sqrt(e)*x/sqrt(d))/(8*d**(5/2)*e**(5/2))
```

Mathematica [A] time = 0.16758, size = 110, normalized size = 0.96

$$\frac{x\left(e\left(ae\left(5d+3ex^2\right)+bd\left(ex^2-d\right)\right)-cd^2\left(3d+5ex^2\right)\right)}{8d^2e^2\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] (x*(-(c*d^2*(3*d + 5*e*x^2)) + e*(b*d*(-d + e*x^2) + a*e*(5*d + 3
*e*x^2))))/(8*d^2*e^2*(d + e*x^2)^2) + ((3*c*d^2 + e*(b*d + 3*a*e
))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Maple [A] time = 0., size = 131, normalized size = 1.1

$$\frac{1}{\left(ex^2+d\right)^2}\left(\frac{\left(3\,ae^2+bde-5\,cd^2\right)x^3}{8\,d^2e}+\frac{\left(5\,ae^2-bde-3\,cd^2\right)x}{8\,e^2d}\right)+\frac{3\,a}{8\,d^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{b}{8\,de}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{3\,c}{8\,e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/(e*x^2+d)^3,x)
```

```
[Out] (1/8*(3*a*e^2+b*d*e-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-b*d*e-3*c*d^2
)/e^2/d*x)/(e*x^2+d)^2+3/8/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)
)*a+1/8/d/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b+3/8/e^2/(d*e)^(
1/2)*arctan(x*e/(d*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277568, size = 1, normalized size = 0.01

$$\left[\frac{\left(3\,cd^4 + bd^3e + 3\,ad^2e^2 + \left(3\,cd^2e^2 + bde^3 + 3\,ae^4\right)x^4 + 2\left(3\,cd^3e + bd^2e^2 + 3\,ade^3\right)x^2\right)\log\left(\frac{2\,dex + \left(ex^2 - d\right)\sqrt{-de}}{ex^2 + d}\right) - 2\left(\left(5\,cd^2e\right.\right.}{16\left(d^2e^4x^4 + 2\,d^3e^3x^2 + d^4e^2\right)\sqrt{-de}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^3,x, algorithm="fricas")
```

```
[Out] [1/16*((3*c*d^4 + b*d^3*e + 3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3
+ 3*a*e^4)*x^4 + 2*(3*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*x^2)*log((
2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d)) - 2*((5*c*d^2*e -
b*d*e^2 - 3*a*e^3)*x^3 + (3*c*d^3 + b*d^2*e - 5*a*d*e^2)*x)*sqrt(
-d*e))/((d^2*e^4*x^4 + 2*d^3*e^3*x^2 + d^4*e^2)*sqrt(-d*e)), 1/8*
((3*c*d^4 + b*d^3*e + 3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*
e^4)*x^4 + 2*(3*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*x^2)*arctan(sqrt
(d*e)*x/d) - ((5*c*d^2*e - b*d*e^2 - 3*a*e^3)*x^3 + (3*c*d^3 + b*
d^2*e - 5*a*d*e^2)*x)*sqrt(d*e))/((d^2*e^4*x^4 + 2*d^3*e^3*x^2 +
d^4*e^2)*sqrt(d*e))]
```

Sympy [A] time = 6.63788, size = 196, normalized size = 1.7

$$-\frac{\sqrt{-\frac{1}{d^5e^5}}\left(3ae^2 + bde + 3cd^2\right)\log\left(-d^3e^2\sqrt{-\frac{1}{d^5e^5}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{d^5e^5}}\left(3ae^2 + bde + 3cd^2\right)\log\left(d^3e^2\sqrt{-\frac{1}{d^5e^5}} + x\right)}{16} + \frac{x^3\left(3ae^3 + bde^2 - 5cd^2e\right) + x\left(5ade^2 - bd^2e - 3cd^3\right)}{8d^4e^2 + 16d^3e^3x^2 + 8d^2e^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] -sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*log(-d**3*e**
2*sqrt(-1/(d**5*e**5)) + x)/16 + sqrt(-1/(d**5*e**5))*(3*a*e**2 +
 b*d*e + 3*c*d**2)*log(d**3*e**2*sqrt(-1/(d**5*e**5)) + x)/16 + (
x**3*(3*a*e**3 + b*d*e**2 - 5*c*d**2*e) + x*(5*a*d*e**2 - b*d**2*
e - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e**3*x**2 + 8*d**2*e**4*x**
4)
```

GIAC/XCAS [A] time = 0.271979, size = 136, normalized size = 1.18

$$\frac{\left(3\,cd^2 + bde + 3\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{8\,d^{\frac{5}{2}}} - \frac{\left(5\,cd^2x^3e - bdx^3e^2 + 3\,cd^3x - 3\,ax^3e^3 + bd^2xe - 5\,adxe^2\right)e^{(-2)}}{8\,(x^2e + d)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/(e*x^2 + d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*c*d^2 + b*d*e + 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2
)/d^(5/2) - 1/8*(5*c*d^2*x^3*e - b*d*x^3*e^2 + 3*c*d^3*x - 3*a*x^
3*e^3 + b*d^2*x*e - 5*a*d*x*e^2)*e^(-2)/((x^2*e + d)^2*d^2)
```

3.284 $\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^3}\,dx$

Optimal. Leaf size=124

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3e(bd-5ae)+cd^2\right)}{8d^{7/2}e^{3/2}}+\frac{x\left(e(3bd-7ae)+cd^2\right)}{8d^3e\left(d+ex^2\right)}+\frac{x\left(-\frac{ae}{d}+b-\frac{cd}{e}\right)}{4d\left(d+ex^2\right)^2}-\frac{a}{d^3x}$$

```
[Out] -(a/(d^3*x)) + ((b - (c*d)/e - (a*e)/d)*x)/(4*d*(d + e*x^2)^2) +
((c*d^2 + e*(3*b*d - 7*a*e))*x)/(8*d^3*e*(d + e*x^2)) + ((c*d^2 +
 3*e*(b*d - 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)*e^(3/
2))
```

Rubi [A] time = 0.34954, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3e(bd-5ae)+cd^2\right)}{8d^{7/2}e^{3/2}}+\frac{x\left(e(3bd-7ae)+cd^2\right)}{8d^3e\left(d+ex^2\right)}+\frac{x\left(-\frac{ae}{d}+b-\frac{cd}{e}\right)}{4d\left(d+ex^2\right)^2}-\frac{a}{d^3x}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x]
```

```
[Out] -(a/(d^3*x)) + ((b - (c*d)/e - (a*e)/d)*x)/(4*d*(d + e*x^2)^2) +
((c*d^2 + e*(3*b*d - 7*a*e))*x)/(8*d^3*e*(d + e*x^2)) + ((c*d^2 +
 3*e*(b*d - 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)*e^(3/
2))
```

Rubi in Sympy [A] time = 37.784, size = 116, normalized size = 0.94

$$-\frac{a}{d^3x}-\frac{x\left(ae^2-bde+cd^2\right)}{4d^2e\left(d+ex^2\right)^2}-\frac{x\left(7ae^2-3bde-cd^2\right)}{8d^3e\left(d+ex^2\right)}-\frac{\left(15ae^2-d\left(3be+cd\right)\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{\frac{7}{2}}e^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**2/(e*x**2+d)**3,x)
```

```
[Out] -a/(d**3*x) - x*(a*e**2 - b*d*e + c*d**2)/(4*d**2*e*(d + e*x**2)*
*2) - x*(7*a*e**2 - 3*b*d*e - c*d**2)/(8*d**3*e*(d + e*x**2)) - (
15*a*e**2 - d*(3*b*e + c*d))*atan(sqrt(e)*x/sqrt(d))/(8*d**(7/2)*
```

```
e**(3/2))
```

Mathematica [A] time = 0.255609, size = 124, normalized size = 1.

$$\frac{\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3e(bd-5ae)+cd^2\right)}{e^{3/2}}+\frac{\sqrt{d}\left(dx^2\left(be\left(5d+3ex^2\right)+cd\left(ex^2-d\right)\right)-ae\left(8d^2+25dex^2+15e^2x^4\right)\right)}{ex\left(d+ex^2\right)^2}}{8d^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x]
```

```
[Out] ((Sqrt[d]*(-(a*e*(8*d^2 + 25*d*e*x^2 + 15*e^2*x^4)) + d*x^2*(c*d*
(-d + e*x^2) + b*e*(5*d + 3*e*x^2))))/(e*x*(d + e*x^2)^2) + ((c*d
^2 + 3*e*(b*d - 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2))/(8*
d^(7/2))
```

Maple [A] time = 0.018, size = 182, normalized size = 1.5

$$-\frac{a}{d^3x}-\frac{7\,x^3ae^2}{8\,d^3\,(ex^2+d)^2}+\frac{3\,bx^3e}{8\,d^2\,(ex^2+d)^2}+\frac{cx^3}{8\,d\,(ex^2+d)^2}-\frac{9\,axe}{8\,d^2\,(ex^2+d)^2}+\frac{5\,bx}{8\,d\,(ex^2+d)^2}-\frac{cx}{8\,(ex^2+d)^2\,e}-\frac{15\,ae}{8\,d^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{3\,b}{8\,d^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{c}{8\,de}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x)
```

```
[Out] -a/d^3/x-7/8/d^3/(e*x^2+d)^2*x^3*a*e^2+3/8/d^2/(e*x^2+d)^2*x^3*b*
e+1/8/d/(e*x^2+d)^2*x^3*c-9/8/d^2/(e*x^2+d)^2*x*a*e+5/8/d/(e*x^2+
d)^2*x*b-1/8/(e*x^2+d)^2/e*x*c-15/8/d^3*e/(d*e)^(1/2)*arctan(x*e/
(d*e)^(1/2))*a+3/8/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b+1/8/
d/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^3*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.27154, size = 1, normalized size = 0.01

$$\left[-\frac{\left(\left(cd^2e^2+3\,bde^3-15\,ae^4\right)x^5+2\left(cd^3e+3\,bd^2e^2-15\,ade^3\right)x^3+\left(cd^4+3\,bd^3e-15\,ad^2e^2\right)x\right)\log\left(-\frac{2\,dex-(ex^2-d)\sqrt{-de}}{ex^2+d}\right)\cdot}{16\left(d^3e^3x^5+2\,d^4e^2x^3+d^5ex\right)\sqrt{-de}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^3*x^2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*x^5 + 2*(c*d^3*e + 3*
b*d^2*e^2 - 15*a*d*e^3)*x^3 + (c*d^4 + 3*b*d^3*e - 15*a*d^2*e^2)*
x)*log(-(2*d*e*x - (e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d)) - 2*((c*d
^2*e + 3*b*d*e^2 - 15*a*e^3)*x^4 - 8*a*d^2*e - (c*d^3 - 5*b*d^2*e
 + 25*a*d*e^2)*x^2)*sqrt(-d*e))/((d^3*e^3*x^5 + 2*d^4*e^2*x^3 + d
^5*e*x)*sqrt(-d*e)), 1/8*(((c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*x^5
 + 2*(c*d^3*e + 3*b*d^2*e^2 - 15*a*d*e^3)*x^3 + (c*d^4 + 3*b*d^3*
e - 15*a*d^2*e^2)*x)*arctan(sqrt(d*e)*x/d) + ((c*d^2*e + 3*b*d*e^
2 - 15*a*e^3)*x^4 - 8*a*d^2*e - (c*d^3 - 5*b*d^2*e + 25*a*d*e^2)*
x^2)*sqrt(d*e))/((d^3*e^3*x^5 + 2*d^4*e^2*x^3 + d^5*e*x)*sqrt(d*e
))]
```

Sympy [A] time = 9.16674, size = 202, normalized size = 1.63

$$\frac{\sqrt{-\frac{1}{d^7e^3}}\left(15ae^2-3bde-cd^2\right)\log\left(-d^4e\sqrt{-\frac{1}{d^7e^3}}+x\right)}{16}-\frac{\sqrt{-\frac{1}{d^7e^3}}\left(15ae^2-3bde-cd^2\right)\log\left(d^4e\sqrt{-\frac{1}{d^7e^3}}+x\right)}{16}-\frac{8ad^2e+x^4\left(15ae^3-3bde^2-cd^2e\right)+x^2\left(25ade^2-5bd^2e+cd^3\right)}{8d^5ex+16d^4e^2x^3+8d^3e^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**2/(e*x**2+d)**3,x)
```

```
[Out] sqrt(-1/(d**7*e**3))*(15*a*e**2 - 3*b*d*e - c*d**2)*log(-d**4*e*s
qrt(-1/(d**7*e**3)) + x)/16 - sqrt(-1/(d**7*e**3))*(15*a*e**2 - 3
```

```
*b*d*e - c*d**2)*log(d**4*e*sqrt(-1/(d**7*e**3)) + x)/16 - (8*a*d
**2*e + x**4*(15*a*e**3 - 3*b*d*e**2 - c*d**2*e) + x**2*(25*a*d*e
**2 - 5*b*d**2*e + c*d**3))/(8*d**5*e*x + 16*d**4*e**2*x**3 + 8*d
**3*e**3*x**5)
```

GIAC/XCAS [A] time = 0.272482, size = 149, normalized size = 1.2

$$\frac{\left(cd^2 + 3\,bde - 15\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{8\,d^{\frac{7}{2}}} - \frac{a}{d^3x} + \frac{\left(cd^2x^3e + 3\,bdx^3e^2 - cd^3x - 7\,ax^3e^3 + 5\,bd^2xe - 9\,adxe^2\right)e^{(-1)}}{8\,(x^2e + d)^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^3*x^2),x, algorithm="giac")
```

```
[Out] 1/8*(c*d^2 + 3*b*d*e - 15*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/
2)/d^(7/2) - a/(d^3*x) + 1/8*(c*d^2*x^3*e + 3*b*d*x^3*e^2 - c*d^3
*x - 7*a*x^3*e^3 + 5*b*d^2*x*e - 9*a*d*x*e^2)*e^(-1)/((x^2*e + d)
^2*d^3)
```

3.285 $\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^3}\,dx$

Optimal. Leaf size=142

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(35ae^2-15bde+3cd^2)}{8d^{9/2}\sqrt{e}}+\frac{x\left(3cd^2-e(7bd-11ae)\right)}{8d^4\left(d+ex^2\right)}+\frac{x\left(ae^2-bde+cd^2\right)}{4d^3\left(d+ex^2\right)^2}-\frac{bd-3ae}{d^4x}-\frac{a}{3d^3x^3}$$

```
[Out] -a/(3*d^3*x^3) - (b*d - 3*a*e)/(d^4*x) + ((c*d^2 - b*d*e + a*e^2)
*x)/(4*d^3*(d + e*x^2)^2) + ((3*c*d^2 - e*(7*b*d - 11*a*e))*x)/(8
*d^4*(d + e*x^2)) + ((3*c*d^2 - 15*b*d*e + 35*a*e^2)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(9/2)*Sqrt[e])
```

Rubi [A] time = 0.423639, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(35ae^2-15bde+3cd^2)}{8d^{9/2}\sqrt{e}}+\frac{x\left(3cd^2-e(7bd-11ae)\right)}{8d^4\left(d+ex^2\right)}+\frac{x\left(ae^2-bde+cd^2\right)}{4d^3\left(d+ex^2\right)^2}-\frac{bd-3ae}{d^4x}-\frac{a}{3d^3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x]
```

```
[Out] -a/(3*d^3*x^3) - (b*d - 3*a*e)/(d^4*x) + ((c*d^2 - b*d*e + a*e^2)
*x)/(4*d^3*(d + e*x^2)^2) + ((3*c*d^2 - e*(7*b*d - 11*a*e))*x)/(8
*d^4*(d + e*x^2)) + ((3*c*d^2 - 15*b*d*e + 35*a*e^2)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(9/2)*Sqrt[e])
```

Rubi in Sympy [A] time = 62.8802, size = 133, normalized size = 0.94

$$-\frac{a}{3d^3x^3}+\frac{x\left(ae^2-bde+cd^2\right)}{4d^3\left(d+ex^2\right)^2}+\frac{x\left(11ae^2-7bde+3cd^2\right)}{8d^4\left(d+ex^2\right)}+\frac{3ae-bd}{d^4x}+\frac{\left(35ae^2-15bde+3cd^2\right)\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{\frac{9}{2}}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**4/(e*x**2+d)**3,x)
```

```
[Out] -a/(3*d**3*x**3) + x*(a*e**2 - b*d*e + c*d**2)/(4*d**3*(d + e*x**
2)**2) + x*(11*a*e**2 - 7*b*d*e + 3*c*d**2)/(8*d**4*(d + e*x**2))
 + (3*a*e - b*d)/(d**4*x) + (35*a*e**2 - 15*b*d*e + 3*c*d**2)*ata
n(sqrt(e)*x/sqrt(d))/(8*d**(9/2)*sqrt(e))
```

Mathematica [A] time = 0.145062, size = 141, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(35ae^2-15bde+3cd^2\right)}{8d^{9/2}\sqrt{e}}+\frac{x\left(11ae^2-7bde+3cd^2\right)}{8d^4\left(d+ex^2\right)}+\frac{x\left(ae^2-bde+cd^2\right)}{4d^3\left(d+ex^2\right)^2}+\frac{3ae-bd}{d^4x}-\frac{a}{3d^3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x]
```

```
[Out] -a/(3*d^3*x^3) + (-(b*d) + 3*a*e)/(d^4*x) + ((c*d^2 - b*d*e + a*e
^2)*x)/(4*d^3*(d + e*x^2)^2) + ((3*c*d^2 - 7*b*d*e + 11*a*e^2)*x)
/(8*d^4*(d + e*x^2)) + ((3*c*d^2 - 15*b*d*e + 35*a*e^2)*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(8*d^(9/2)*Sqrt[e])
```

Maple [A] time = 0.022, size = 207, normalized size = 1.5

$$-\frac{a}{3\,d^3x^3}+3\,\frac{ae}{d^4x}-\frac{b}{d^3x}+\frac{11\,x^3ae^3}{8\,d^4\left(ex^2+d\right)^2}-\frac{7\,bx^3e^2}{8\,d^3\left(ex^2+d\right)^2}+\frac{3\,cx^3e}{8\,d^2\left(ex^2+d\right)^2}+\frac{13\,ae^2x}{8\,d^3\left(ex^2+d\right)^2}-\frac{9\,bex}{8\,d^2\left(ex^2+d\right)^2}+\frac{5\,cx}{8\,d\left(ex^2+d\right)^2}+\frac{35\,ae^2}{8\,d^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{15\,be}{8\,d^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{3\,c}{8\,d^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x)
```

```
[Out] -1/3*a/d^3/x^3+3/d^4/x*a*e-1/d^3/x*b+11/8/d^4/(e*x^2+d)^2*x^3*a*e
^3-7/8/d^3/(e*x^2+d)^2*x^3*b*e^2+3/8/d^2/(e*x^2+d)^2*x^3*c*e+13/8
/d^3/(e*x^2+d)^2*e^2*a*x-9/8/d^2/(e*x^2+d)^2*b*e*x+5/8/d/(e*x^2+d
)^2*c*x+35/8/d^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*e^2-15/8/d
^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b*e+3/8/d^2/(d*e)^(1/2)*ar
ctan(x*e/(d*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^3*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277585, size = 1, normalized size = 0.01

$$\left[\frac{3\left(\left(3\,cd^{2}e^{2}-15\,bde^{3}+35\,ae^{4}\right)x^{7}+2\left(3\,cd^{3}e-15\,bd^{2}e^{2}+35\,ade^{3}\right)x^{5}+\left(3\,cd^{4}-15\,bd^{3}e+35\,ad^{2}e^{2}\right)x^{3}\right)\log\left(\frac{2\,dex+\left(ex^{2}-d\right)}{ex^{2}+d}\right.}{48\left(d^{4}e^{2}x^{7}+2\,d^{5}ex^{5}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^3*x^4),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((3*c*d^2*e^2 - 15*b*d*e^3 + 35*a*e^4)*x^7 + 2*(3*c*d^3*
e - 15*b*d^2*e^2 + 35*a*d*e^3)*x^5 + (3*c*d^4 - 15*b*d^3*e + 35*a
*d^2*e^2)*x^3)*log((2*d*e*x + (e*x^2 - d)*sqrt(-d*e))/(e*x^2 + d)
) + 2*(3*(3*c*d^2*e - 15*b*d*e^2 + 35*a*e^3)*x^6 + 5*(3*c*d^3 - 1
5*b*d^2*e + 35*a*d*e^2)*x^4 - 8*a*d^3 - 8*(3*b*d^3 - 7*a*d^2*e)*x
^2)*sqrt(-d*e))/((d^4*e^2*x^7 + 2*d^5*e*x^5 + d^6*x^3)*sqrt(-d*e)
), 1/24*(3*((3*c*d^2*e^2 - 15*b*d*e^3 + 35*a*e^4)*x^7 + 2*(3*c*d^
3*e - 15*b*d^2*e^2 + 35*a*d*e^3)*x^5 + (3*c*d^4 - 15*b*d^3*e + 35
*a*d^2*e^2)*x^3)*arctan(sqrt(d*e)*x/d) + (3*(3*c*d^2*e - 15*b*d*e
^2 + 35*a*e^3)*x^6 + 5*(3*c*d^3 - 15*b*d^2*e + 35*a*d*e^2)*x^4 -
8*a*d^3 - 8*(3*b*d^3 - 7*a*d^2*e)*x^2)*sqrt(d*e))/((d^4*e^2*x^7 +
 2*d^5*e*x^5 + d^6*x^3)*sqrt(d*e))]
```

Sympy [A] time = 12.595, size = 214, normalized size = 1.51

$$-\frac{\sqrt{-\frac{1}{d^{9}e}}\left(35ae^{2}-15bde+3cd^{2}\right)\log\left(-d^{5}\sqrt{-\frac{1}{d^{9}e}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{d^{9}e}}\left(35ae^{2}-15bde+3cd^{2}\right)\log\left(d^{5}\sqrt{-\frac{1}{d^{9}e}}+x\right)}{16}+\frac{-8ad^{3}+x^{6}\left(105ae^{3}-45bde^{2}+9cd^{2}e\right)+x^{4}\left(175ade^{2}-75bd^{2}e+15cd^{3}\right)+x^{2}\left(56ad^{2}e-24bd^{3}\right)}{24d^{6}x^{3}+48d^{5}ex^{5}+24d^{4}e^{2}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**4/(e*x**2+d)**3,x)
```

```
[Out] -sqrt(-1/(d**9*e))*(35*a*e**2 - 15*b*d*e + 3*c*d**2)*log(-d**5*sq
rt(-1/(d**9*e)) + x)/16 + sqrt(-1/(d**9*e))*(35*a*e**2 - 15*b*d*e
 + 3*c*d**2)*log(d**5*sqrt(-1/(d**9*e)) + x)/16 + (-8*a*d**3 + x*
*6*(105*a*e**3 - 45*b*d*e**2 + 9*c*d**2*e) + x**4*(175*a*d*e**2 -
 75*b*d**2*e + 15*c*d**3) + x**2*(56*a*d**2*e - 24*b*d**3))/(24*d
**6*x**3 + 48*d**5*e*x**5 + 24*d**4*e**2*x**7)
```

GIAC/XCAS [A] time = 0.272068, size = 173, normalized size = 1.22

$$\frac{\left(3\,cd^2 - 15\,bde + 35\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{8\,d^{\frac{9}{2}}} + \frac{3\,cd^2x^3e - 7\,bdx^3e^2 + 5\,cd^3x + 11\,ax^3e^3 - 9\,bd^2xe + 13\,adxe^2}{8\,(x^2e + d)^2d^4} - \frac{3\,bdx^2 - 9\,ax^2e + ad}{3\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^3*x^4),x, algorithm="giac")
```

```
[Out] 1/8*(3*c*d^2 - 15*b*d*e + 35*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(
-1/2)/d^(9/2) + 1/8*(3*c*d^2*x^3*e - 7*b*d*x^3*e^2 + 5*c*d^3*x +
11*a*x^3*e^3 - 9*b*d^2*x*e + 13*a*d*x*e^2)/((x^2*e + d)^2*d^4) -
1/3*(3*b*d*x^2 - 9*a*x^2*e + a*d)/(d^4*x^3)
```

3.286 $\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^3}\,dx$

Optimal. Leaf size=171

$$-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(63ae^2-35bde+15cd^2\right)}{8d^{11/2}}-\frac{6ae^2-3bde+cd^2}{d^5x}-\frac{ex\left(7cd^2-e(11bd-15ae)\right)}{8d^5\left(d+ex^2\right)}-\frac{ex\left(ae^2-bde+cd^2\right)}{4d^4\left(d+ex^2\right)^2}-\frac{bd-3ae}{3d^4x^3}-\frac{a}{5d^3x^5}$$

```
[Out] -a/(5*d^3*x^5) - (b*d - 3*a*e)/(3*d^4*x^3) - (c*d^2 - 3*b*d*e + 6
*a*e^2)/(d^5*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(4*d^4*(d + e*x^2
)^2) - (e*(7*c*d^2 - e*(11*b*d - 15*a*e))*x)/(8*d^5*(d + e*x^2))
- (Sqrt[e]*(15*c*d^2 - 35*b*d*e + 63*a*e^2)*ArcTan[(Sqrt[e]*x)/Sq
rt[d]])/(8*d^(11/2))
```

Rubi [A] time = 0.583797, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(63ae^2-35bde+15cd^2\right)}{8d^{11/2}}-\frac{6ae^2-3bde+cd^2}{d^5x}-\frac{ex\left(7cd^2-e(11bd-15ae)\right)}{8d^5\left(d+ex^2\right)}-\frac{ex\left(ae^2-bde+cd^2\right)}{4d^4\left(d+ex^2\right)^2}-\frac{bd-3ae}{3d^4x^3}-\frac{a}{5d^3x^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x]
```

```
[Out] -a/(5*d^3*x^5) - (b*d - 3*a*e)/(3*d^4*x^3) - (c*d^2 - 3*b*d*e + 6
*a*e^2)/(d^5*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(4*d^4*(d + e*x^2
)^2) - (e*(7*c*d^2 - e*(11*b*d - 15*a*e))*x)/(8*d^5*(d + e*x^2))
- (Sqrt[e]*(15*c*d^2 - 35*b*d*e + 63*a*e^2)*ArcTan[(Sqrt[e]*x)/Sq
rt[d]])/(8*d^(11/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**6/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.193211, size = 173, normalized size = 1.01

$$-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(63ae^2-35bde+15cd^2\right)}{8d^{11/2}}+\frac{-6ae^2+3bde-cd^2}{d^5x}-\frac{x\left(15ae^3-11bde^2+7cd^2e\right)}{8d^5\left(d+ex^2\right)}-\frac{ex\left(ae^2-bde+cd^2\right)}{4d^4\left(d+ex^2\right)^2}+\frac{3ae-bd}{3d^4x^3}-\frac{a}{5d^3x^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x]
```

```
[Out] -a/(5*d^3*x^5) + (-(b*d) + 3*a*e)/(3*d^4*x^3) + (-(c*d^2) + 3*b*d
*e - 6*a*e^2)/(d^5*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(4*d^4*(d +
 e*x^2)^2) - ((7*c*d^2*e - 11*b*d*e^2 + 15*a*e^3)*x)/(8*d^5*(d +
e*x^2)) - (Sqrt[e]*(15*c*d^2 - 35*b*d*e + 63*a*e^2)*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(8*d^(11/2))
```

Maple [A] time = 0.023, size = 245, normalized size = 1.4

$$-\frac{a}{5\,d^3x^5}+\frac{ae}{d^4x^3}-\frac{b}{3\,d^3x^3}-6\,\frac{ae^2}{d^5x}+3\,\frac{be}{d^4x}-\frac{c}{d^3x}-\frac{15\,e^4x^3a}{8\,d^5\left(ex^2+d\right)^2}+\frac{11\,e^3x^3b}{8\,d^4\left(ex^2+d\right)^2}-\frac{7\,e^2x^3c}{8\,d^3\left(ex^2+d\right)^2}-\frac{17\,e^3ax}{8\,d^4\left(ex^2+d\right)^2}+\frac{13\,be^2x}{8\,d^3\left(ex^2+d\right)^2}-\frac{9\,cex}{8\,d^2\left(ex^2+d\right)^2}-\frac{63\,e^3a}{8\,d^5}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{35\,be^2}{8\,d^4}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{15\,ce}{8\,d^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x)
```

```
[Out] -1/5*a/d^3/x^5+1/d^4/x^3*a*e-1/3/d^3/x^3*b-6/d^5/x*a*e^2+3/d^4/x*
b*e-1/d^3/x*c-15/8*e^4/d^5/(e*x^2+d)^2*x^3*a+11/8*e^3/d^4/(e*x^2+
d)^2*x^3*b-7/8*e^2/d^3/(e*x^2+d)^2*x^3*c-17/8*e^3/d^4/(e*x^2+d)^2
*a*x+13/8*e^2/d^3/(e*x^2+d)^2*b*x-9/8*e/d^2/(e*x^2+d)^2*c*x-63/8*
e^3/d^5/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+35/8*e^2/d^4/(d*e)^
(1/2)*arctan(x*e/(d*e)^(1/2))*b-15/8*e/d^3/(d*e)^(1/2)*arctan(x*e
/(d*e)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^3*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273341, size = 1, normalized size = 0.01

$$\left[-\frac{30\left(15\,cd^2e^2-35\,bde^3+63\,ae^4\right)x^8+50\left(15\,cd^3e-35\,bd^2e^2+63\,ade^3\right)x^6+48\,ad^4+16\left(15\,cd^4-35\,bd^3e+63\,ad^2e^2\right)x^4+}{}\right.$$

$$-\frac{15\left(15\,cd^2e^2-35\,bde^3+63\,ae^4\right)x^8+25\left(15\,cd^3e-35\,bd^2e^2+63\,ade^3\right)x^6+24\,ad^4+8\left(15\,cd^4-35\,bd^3e+63\,ad^2e^2\right)x^4+8}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^3*x^6),x, algorithm="fricas")
```

```
[Out] [-1/240*(30*(15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*x^8 + 50*(15*c
*d^3*e - 35*b*d^2*e^2 + 63*a*d*e^3)*x^6 + 48*a*d^4 + 16*(15*c*d^4
 - 35*b*d^3*e + 63*a*d^2*e^2)*x^4 + 16*(5*b*d^4 - 9*a*d^3*e)*x^2
- 15*((15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*x^9 + 2*(15*c*d^3*e
- 35*b*d^2*e^2 + 63*a*d*e^3)*x^7 + (15*c*d^4 - 35*b*d^3*e + 63*a*
d^2*e^2)*x^5)*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^
2 + d)))/(d^5*e^2*x^9 + 2*d^6*e*x^7 + d^7*x^5), -1/120*(15*(15*c*
d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*x^8 + 25*(15*c*d^3*e - 35*b*d^2*
e^2 + 63*a*d*e^3)*x^6 + 24*a*d^4 + 8*(15*c*d^4 - 35*b*d^3*e + 63*
a*d^2*e^2)*x^4 + 8*(5*b*d^4 - 9*a*d^3*e)*x^2 + 15*((15*c*d^2*e^2
- 35*b*d*e^3 + 63*a*e^4)*x^9 + 2*(15*c*d^3*e - 35*b*d^2*e^2 + 63*
a*d*e^3)*x^7 + (15*c*d^4 - 35*b*d^3*e + 63*a*d^2*e^2)*x^5)*sqrt(e
/d)*arctan(e*x/(d*sqrt(e/d))))/(d^5*e^2*x^9 + 2*d^6*e*x^7 + d^7*x
^5)]
```

Sympy [A] time = 18.2704, size = 330, normalized size = 1.93

$$\frac{\sqrt{-\frac{e}{d^{11}}}\left(63ae^2-35bde+15cd^2\right)\log\left(-\frac{d^6\sqrt{-\frac{e}{d^{11}}}\left(63ae^2-35bde+15cd^2\right)}{63ae^3-35bde^2+15cd^2e}+x\right)}{16}$$
$$-\frac{\sqrt{-\frac{e}{d^{11}}}\left(63ae^2-35bde+15cd^2\right)\log\left(\frac{d^6\sqrt{-\frac{e}{d^{11}}}\left(63ae^2-35bde+15cd^2\right)}{63ae^3-35bde^2+15cd^2e}+x\right)}{16}$$
$$-\frac{24ad^4+x^8\left(945ae^4-525bde^3+225cd^2e^2\right)+x^6\left(1575ade^3-875bd^2e^2+375cd^3e\right)+x^4\left(504ad^2e^2-280bd^3e+120cd^4\right)+}{120d^7x^5+240d^6ex^7+120d^5e^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**6/(e*x**2+d)**3,x)
```

```
[Out] sqrt(-e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)*log(-d**6*sqrt(
-e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)/(63*a*e**3 - 35*b*d*
e**2 + 15*c*d**2*e) + x)/16 - sqrt(-e/d**11)*(63*a*e**2 - 35*b*d*
e + 15*c*d**2)*log(d**6*sqrt(-e/d**11)*(63*a*e**2 - 35*b*d*e + 15
*c*d**2)/(63*a*e**3 - 35*b*d*e**2 + 15*c*d**2*e) + x)/16 - (24*a*
d**4 + x**8*(945*a*e**4 - 525*b*d*e**3 + 225*c*d**2*e**2) + x**6*
(1575*a*d*e**3 - 875*b*d**2*e**2 + 375*c*d**3*e) + x**4*(504*a*d*
*2*e**2 - 280*b*d**3*e + 120*c*d**4) + x**2*(-72*a*d**3*e + 40*b*
d**4))/(120*d**7*x**5 + 240*d**6*e*x**7 + 120*d**5*e**2*x**9)
```

GIAC/XCAS [A] time = 0.271815, size = 221, normalized size = 1.29

$$-\frac{\left(15\,cd^2e-35\,bde^2+63\,ae^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{8\,d^{\frac{11}{2}}}-\frac{7\,cd^2x^3e^2-11\,bdx^3e^3+9\,cd^3xe+15\,ax^3e^4-13\,bd^2xe^2+17\,adxe^3}{8\,(x^2e+d)^2d^5}-\frac{15\,cd^2x^4-45\,bdx^4e+90\,ax^4e^2+5\,bd^2x^2-15\,adx^2e+3\,ad^2}{15\,d^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/((e*x^2 + d)^3*x^6),x, algorithm="giac")
```

```
[Out] -1/8*(15*c*d^2*e - 35*b*d*e^2 + 63*a*e^3)*arctan(x*e^(1/2)/sqrt(d
))*e^(-1/2)/d^(11/2) - 1/8*(7*c*d^2*x^3*e^2 - 11*b*d*x^3*e^3 + 9*
c*d^3*x*e + 15*a*x^3*e^4 - 13*b*d^2*x*e^2 + 17*a*d*x*e^3)/((x^2*e
 + d)^2*d^5) - 1/15*(15*c*d^2*x^4 - 45*b*d*x^4*e + 90*a*x^4*e^2 +
 5*b*d^2*x^2 - 15*a*d*x^2*e + 3*a*d^2)/(d^5*x^5)
```

3.287 $\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=230

$$-\frac{(a^2ce-ab^2e-2abcd+b^3d)\log(a+bx^2+cx^4)}{4c^3(ae^2-bde+cd^2)} - \frac{(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)} + \frac{d^4\log(d+ex^2)}{2e^3(ae^2-bde+cd^2)} - \frac{x^2(be+cd)}{2c^2e^2} + \frac{x^4}{4ce}$$

```
[Out] -((c*d + b*e)*x^2)/(2*c^2*e^2) + x^4/(4*c*e) - ((b^4*d - 4*a*b^2*
c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e
^2)) + (d^4*Log[d + e*x^2])/(2*e^3*(c*d^2 - b*d*e + a*e^2)) - ((b
^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e)*Log[a + b*x^2 + c*x^4])/(4*
c^3*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.926173, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{(a^2ce-ab^2e-2abcd+b^3d)\log(a+bx^2+cx^4)}{4c^3(ae^2-bde+cd^2)} - \frac{(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)} + \frac{d^4\log(d+ex^2)}{2e^3(ae^2-bde+cd^2)} - \frac{x^2(be+cd)}{2c^2e^2} + \frac{x^4}{4ce}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```


```
[Out] -((c*d + b*e)*x^2)/(2*c^2*e^2) + x^4/(4*c*e) - ((b^4*d - 4*a*b^2*
c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e
^2)) + (d^4*Log[d + e*x^2])/(2*e^3*(c*d^2 - b*d*e + a*e^2)) - ((b
^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e)*Log[a + b*x^2 + c*x^4])/(4*
c^3*(c*d^2 - b*d*e + a*e^2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.410466, size = 228, normalized size = 0.99

$$\frac{1}{4}\left(\frac{\left(-a^2ce+ab^2e+2abcd+b^3(-d)\right)\log\left(a+bx^2+cx^4\right)}{c^3\left(e(ae-bd)+cd^2\right)}-\frac{2\left(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{c^3\sqrt{4ac-b^2}\left(e(bd-ae)-cd^2\right)}+\frac{2d^4\log\left(d+ex^2\right)}{e^3\left(e(ae-bd)+cd^2\right)}-\frac{2x^2(be+cd)}{c^2e^2}+\frac{x^4}{ce}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*(c*d + b*e)*x^2)/(c^2*e^2) + x^4/(c*e) - (2*(b^4*d - 4*a*b^2
*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcTan[(b + 2*c*x^2)/
Sqrt[-b^2 + 4*a*c]])/(c^3*Sqrt[-b^2 + 4*a*c]*(-(c*d^2) + e*(b*d -
 a*e))) + (2*d^4*Log[d + e*x^2])/(e^3*(c*d^2 + e*(-(b*d) + a*e)))
 + ((-(b^3*d) + 2*a*b*c*d + a*b^2*e - a^2*c*e)*Log[a + b*x^2 + c*
x^4])/(c^3*(c*d^2 + e*(-(b*d) + a*e))))/4
```

Maple [B] time = 0.019, size = 538, normalized size = 2.3

$$\frac{x^4}{4\,ce}-\frac{bx^2}{2\,c^2e}-\frac{dx^2}{2\,e^2c}-\frac{\ln\left(cx^4+bx^2+a\right)a^2e}{4\,(ae^2-bde+cd^2)\,c^2}+\frac{\ln\left(cx^4+bx^2+a\right)ab^2e}{4\,c^3\,(ae^2-bde+cd^2)}+\frac{\ln\left(cx^4+bx^2+a\right)abd}{2\,(ae^2-bde+cd^2)\,c^2}$$
$$-\frac{\ln\left(cx^4+bx^2+a\right)b^3d}{4\,c^3\,(ae^2-bde+cd^2)}+\frac{3\,a^2be}{2\,(ae^2-bde+cd^2)\,c^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$+\frac{a^2d}{c\,(ae^2-bde+cd^2)}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$-2\,\frac{ab^2d}{(ae^2-bde+cd^2)\,c^2\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$
$$-\frac{ab^3e}{2\,c^3\,(ae^2-bde+cd^2)}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$+\frac{b^4d}{2\,c^3\,(ae^2-bde+cd^2)}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{d^4\ln\left(ex^2+d\right)}{2\,e^3\,(ae^2-bde+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*x^4/c/e-1/2/c^2/e*x^2*b-1/2*d*x^2/e^2/c-1/4/c^2/(a*e^2-b*d*e+
c*d^2)*ln(c*x^4+b*x^2+a)*a^2*e+1/4/c^3/(a*e^2-b*d*e+c*d^2)*ln(c*x
^4+b*x^2+a)*a*b^2*e+1/2/c^2/(a*e^2-b*d*e+c*d^2)*ln(c*x^4+b*x^2+a)
*a*b*d-1/4/c^3/(a*e^2-b*d*e+c*d^2)*ln(c*x^4+b*x^2+a)*b^3*d+3/2/c^
2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2))*a^2*b*e+1/c/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*ar
ctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*d-2/c^2/(a*e^2-b*d*e+c*d^
2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^2*
d-1/2/c^3/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b
)/(4*a*c-b^2)^(1/2))*b^3*a*e+1/2/c^3/(a*e^2-b*d*e+c*d^2)/(4*a*c-b
^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*d+1/2*d^4*ln(
e*x^2+d)/e^3/(a*e^2-b*d*e+c*d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.301296, size = 319, normalized size = 1.39

$$\frac{d^4 \ln\left(\left|x^2 e + d\right|\right)}{2\,(cd^2e^3 - bde^4 + ae^5)} - \frac{\left(b^3 d - 2\,abcd - ab^2 e + a^2 ce\right)\ln\left(cx^4 + bx^2 + a\right)}{4\,(c^4d^2 - bc^3de + ac^3e^2)} + \frac{\left(b^4 d - 4\,ab^2cd + 2\,a^2c^2d - ab^3e + 3\,a^2bce\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(c^4d^2 - bc^3de + ac^3e^2)\sqrt{-b^2 + 4\,ac}} + \frac{\left(cx^4e - 2\,cdx^2 - 2\,bx^2e\right)e^{(-2)}}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] 1/2*d^4*ln(abs(x^2*e + d))/(c*d^2*e^3 - b*d*e^4 + a*e^5) - 1/4*(b
^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e)*ln(c*x^4 + b*x^2 + a)/(c^4*
d^2 - b*c^3*d*e + a*c^3*e^2) + 1/2*(b^4*d - 4*a*b^2*c*d + 2*a^2*c
^2*d - a*b^3*e + 3*a^2*b*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*
a*c))/((c^4*d^2 - b*c^3*d*e + a*c^3*e^2)*sqrt(-b^2 + 4*a*c)) + 1/
4*(c*x^4*e - 2*c*d*x^2 - 2*b*x^2*e)*e^(-2)/c^2
```

3.288 $\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=189

$$\frac{\left(2a^2ce-ab^2e-3abcd+b^3d\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{\left(-abe-acd+b^2d\right)\log\left(a+bx^2+cx^4\right)}{4c^2\left(ae^2-bde+cd^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(ae^2-bde+cd^2\right)}+\frac{x^2}{2ce}$$

```
[Out] x^2/(2*c*e) + ((b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*ArcTanh[
(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]*(c*d^2
 - b*d*e + a*e^2)) - (d^3*Log[d + e*x^2])/(2*e^2*(c*d^2 - b*d*e +
 a*e^2)) + ((b^2*d - a*c*d - a*b*e)*Log[a + b*x^2 + c*x^4])/(4*c^
2*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.597878, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{\left(2a^2ce-ab^2e-3abcd+b^3d\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{\left(-abe-acd+b^2d\right)\log\left(a+bx^2+cx^4\right)}{4c^2\left(ae^2-bde+cd^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(ae^2-bde+cd^2\right)}+\frac{x^2}{2ce}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] x^2/(2*c*e) + ((b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*ArcTanh[
(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]*(c*d^2
 - b*d*e + a*e^2)) - (d^3*Log[d + e*x^2])/(2*e^2*(c*d^2 - b*d*e +
 a*e^2)) + ((b^2*d - a*c*d - a*b*e)*Log[a + b*x^2 + c*x^4])/(4*c^
2*(c*d^2 - b*d*e + a*e^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(ae^2-bde+cd^2\right)}+\frac{\int^{x^2}\frac{1}{c}\,dx}{2e}+\frac{\left(-abe-acd+b^2d\right)\log\left(a+bx^2+cx^4\right)}{4c^2\left(ae^2-bde+cd^2\right)}+\frac{\left(2a^2ce-ab^2e-3abcd+b^3d\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^2\sqrt{-4ac+b^2}\left(ae^2-bde+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] -d**3*log(d + e*x**2)/(2*e**2*(a*e**2 - b*d*e + c*d**2)) + Integr
al(1/c, (x, x**2))/(2*e) + (-a*b*e - a*c*d + b**2*d)*log(a + b*x*
*2 + c*x**4)/(4*c**2*(a*e**2 - b*d*e + c*d**2)) + (2*a**2*c*e - a
*b**2*e - 3*a*b*c*d + b**3*d)*atanh((b + 2*c*x**2)/sqrt(-4*a*c +
b**2))/(2*c**2*sqrt(-4*a*c + b**2)*(a*e**2 - b*d*e + c*d**2))
```

Mathematica [A] time = 0.311577, size = 186, normalized size = 0.98

$$\frac{2e^2\left(2a^2ce-ab^2e-3abcd+b^3d\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)+\sqrt{4ac-b^2}\left(e\left(e\left(abe+acd+b^2(-d)\right)\log\left(a+bx^2+cx^4\right)-2cx^2\left(ae^2\right.\right.\right.}{4c^2e^2\sqrt{4ac-b^2}\left(e(bd-ae)-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*e^2*(b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*ArcTan[(b + 2*c*
x^2)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(2*c^2*d^3*Log[d +
e*x^2] + e*(-2*c*(c*d^2 - b*d*e + a*e^2)*x^2 + e*(-(b^2*d) + a*c*
d + a*b*e)*Log[a + b*x^2 + c*x^4])))/(4*c^2*Sqrt[-b^2 + 4*a*c]*e^
2*(-(c*d^2) + e*(b*d - a*e)))
```

Maple [B] time = 0.014, size = 408, normalized size = 2.2

$$\frac{x^2}{2\,ce}-\frac{\ln\left(cx^4+bx^2+a\right)abe}{\left(4\,ae^2-4\,bde+4\,cd^2\right)c^2}-\frac{\ln\left(cx^4+bx^2+a\right)ad}{\left(4\,ae^2-4\,bde+4\,cd^2\right)c}+\frac{\ln\left(cx^4+bx^2+a\right)b^2d}{\left(4\,ae^2-4\,bde+4\,cd^2\right)c^2}$$
$$-\frac{a^2e}{\left(ae^2-bde+cd^2\right)c}\arctan\left(\left(2\,cx^2+b\right)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$+\frac{3\,abd}{\left(2\,ae^2-2\,bde+2\,cd^2\right)c}\arctan\left(\left(2\,cx^2+b\right)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$+\frac{ab^2e}{\left(2\,ae^2-2\,bde+2\,cd^2\right)c^2}\arctan\left(\left(2\,cx^2+b\right)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$-\frac{b^3d}{\left(2\,ae^2-2\,bde+2\,cd^2\right)c^2}\arctan\left(\left(2\,cx^2+b\right)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{d^3\ln\left(ex^2+d\right)}{2\,e^2\left(ae^2-bde+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*x^2/c/e-1/4/(a*e^2-b*d*e+c*d^2)/c^2*ln(c*x^4+b*x^2+a)*a*b*e-1
/4/(a*e^2-b*d*e+c*d^2)/c*ln(c*x^4+b*x^2+a)*a*d+1/4/(a*e^2-b*d*e+c
*d^2)/c^2*ln(c*x^4+b*x^2+a)*b^2*d-1/(a*e^2-b*d*e+c*d^2)/c/(4*a*c-
b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*e+3/2/(a*e^2
-b*d*e+c*d^2)/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^
(1/2))*a*b*d+1/2/(a*e^2-b*d*e+c*d^2)/c^2/(4*a*c-b^2)^(1/2)*arctan
((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*a*e-1/2/(a*e^2-b*d*e+c*d^2)/c
^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*d-
1/2*d^3*ln(e*x^2+d)/e^2/(a*e^2-b*d*e+c*d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.301881, size = 262, normalized size = 1.39

$$-\frac{d^3 \ln\left(\left|x^2 e + d\right|\right)}{2\left(cd^2e^2 - bde^3 + ae^4\right)} + \frac{x^2 e^{(-1)}}{2\,c} + \frac{\left(b^2 d - acd - abe\right)\ln\left(cx^4 + bx^2 + a\right)}{4\left(c^3d^2 - bc^2de + ac^2e^2\right)} - \frac{\left(b^3 d - 3\,abcd - ab^2 e + 2\,a^2 ce\right)\arctan\left(\frac{2\,cx^2 + b}{\sqrt{-b^2 + 4\,ac}}\right)}{2\left(c^3d^2 - bc^2de + ac^2e^2\right)\sqrt{-b^2 + 4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] -1/2*d^3*ln(abs(x^2*e + d))/(c*d^2*e^2 - b*d*e^3 + a*e^4) + 1/2*x
^2*e^(-1)/c + 1/4*(b^2*d - a*c*d - a*b*e)*ln(c*x^4 + b*x^2 + a)/(
c^3*d^2 - b*c^2*d*e + a*c^2*e^2) - 1/2*(b^3*d - 3*a*b*c*d - a*b^2
*e + 2*a^2*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((c^3*d^
2 - b*c^2*d*e + a*c^2*e^2)*sqrt(-b^2 + 4*a*c))
```

3.289 $\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=158

$$-\frac{\left(-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}+\frac{d^2\log\left(d+ex^2\right)}{2e\,(ae^2-bde+cd^2)}-\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{4c\,(ae^2-bde+cd^2)}$$

```
[Out] -((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*
c]])/(2*c*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (d^2*Log[d
 + e*x^2])/(2*e*(c*d^2 - b*d*e + a*e^2)) - ((b*d - a*e)*Log[a + b
*x^2 + c*x^4])/(4*c*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.467962, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{\left(-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}+\frac{d^2\log\left(d+ex^2\right)}{2e\,(ae^2-bde+cd^2)}-\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{4c\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*
c]])/(2*c*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (d^2*Log[d
 + e*x^2])/(2*e*(c*d^2 - b*d*e + a*e^2)) - ((b*d - a*e)*Log[a + b
*x^2 + c*x^4])/(4*c*(c*d^2 - b*d*e + a*e^2))
```

Rubi in Sympy [A] time = 66.3865, size = 136, normalized size = 0.86

$$\frac{d^2\log\left(d+ex^2\right)}{2e\,(ae^2-bde+cd^2)}+\frac{(ae-bd)\log\left(a+bx^2+cx^4\right)}{4c\,(ae^2-bde+cd^2)}+\frac{(2acd+b\,(ae-bd))\,\mathrm{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c\sqrt{-4ac+b^2}\,(ae^2-bde+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] d**2*log(d + e*x**2)/(2*e*(a*e**2 - b*d*e + c*d**2)) + (a*e - b*d
)*log(a + b*x**2 + c*x**4)/(4*c*(a*e**2 - b*d*e + c*d**2)) + (2*a
*c*d + b*(a*e - b*d))*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(
```

```
2*c*sqrt(-4*a*c + b**2)*(a*e**2 - b*d*e + c*d**2))
```

Mathematica [A] time = 0.179215, size = 139, normalized size = 0.88

$$-\frac{\sqrt{4ac-b^2}\left(e(bd-ae)\log\left(a+bx^2+cx^4\right)-2cd^2\log\left(d+ex^2\right)\right)+2e\left(abe+2acd+b^2(-d)\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{4ce\sqrt{4ac-b^2}\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(2*e*(-(b^2*d) + 2*a*c*d + a*b*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2
 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(-2*c*d^2*Log[d + e*x^2] + e*(b*d
 - a*e)*Log[a + b*x^2 + c*x^4]))/(4*c*Sqrt[-b^2 + 4*a*c]*e*(c*d^2
 + e*(-(b*d) + a*e)))
```

Maple [A] time = 0.012, size = 289, normalized size = 1.8

$$\frac{\ln\left(cx^4+bx^2+a\right)ae}{(4\,ae^2-4\,bde+4\,cd^2)\,c}-\frac{\ln\left(cx^4+bx^2+a\right)bd}{(4\,ae^2-4\,bde+4\,cd^2)\,c}-\frac{ad}{ae^2-bde+cd^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{abe}{(2\,ae^2-2\,bde+2\,cd^2)\,c}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{b^2d}{(2\,ae^2-2\,bde+2\,cd^2)\,c}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{d^2\ln\left(ex^2+d\right)}{2\,e\,(ae^2-bde+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/(a*e^2-b*d*e+c*d^2)/c*ln(c*x^4+b*x^2+a)*a*e-1/4/(a*e^2-b*d*e+
c*d^2)/c*ln(c*x^4+b*x^2+a)*b*d-1/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^
(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*d-1/2/(a*e^2-b*d*e+
c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b/
c*a*e+1/2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b
)/(4*a*c-b^2)^(1/2))*b^2/c*d+1/2*d^2*ln(e*x^2+d)/e/(a*e^2-b*d*e+c
*d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 56.3409, size = 1, normalized size = 0.01

$$\left[\frac{\left(abe^2-\left(b^2-2\,ac\right)de\right)\log\left(\frac{b^3-4\,abc+2\,\left(b^2c-4\,ac^2\right)x^2+\left(2\,c^2x^4+2\,bcx^2+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(2\,cd^2\log\left(ex^2+d\right)-\left(bde-ae^2\right)\log}{4\,(c^2d^2e-bcde^2+ace^3)\sqrt{b^2-4\,ac}}\right.$$

$$-\frac{2\left(abe^2-\left(b^2-2\,ac\right)de\right)\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-\left(2\,cd^2\log\left(ex^2+d\right)-\left(bde-ae^2\right)\log\left(cx^4+bx^2+a\right)\right)\sqrt{-b^2+4}}{4\,(c^2d^2e-bcde^2+ace^3)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*e^2 - (b^2 - 2*a*c)*d*e)*log((b^3 - 4*a*b*c + 2*(b^2*c
 - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2
- 4*a*c))/(c*x^4 + b*x^2 + a)) + (2*c*d^2*log(e*x^2 + d) - (b*d*e
 - a*e^2)*log(c*x^4 + b*x^2 + a))*sqrt(b^2 - 4*a*c))/((c^2*d^2*e
- b*c*d*e^2 + a*c*e^3)*sqrt(b^2 - 4*a*c)), -1/4*(2*(a*b*e^2 - (b^
2 - 2*a*c)*d*e)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c)) - (2*c*d^2*log(e*x^2 + d) - (b*d*e - a*e^2)*log(c*x^4 + b*
x^2 + a))*sqrt(-b^2 + 4*a*c))/((c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*
sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

[Out] Timed out

GIAC/XCAS [A] time = 0.301517, size = 212, normalized size = 1.34

$$\frac{d^2\ln\left(\left|x^2e+d\right|\right)}{2\left(cd^2e-bde^2+ae^3\right)}-\frac{(bd-ae)\ln\left(cx^4+bx^2+a\right)}{4\left(c^2d^2-bcde+ace^2\right)}+\frac{\left(b^2d-2\,acd-abe\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(c^2d^2-bcde+ace^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] 1/2*d^2*ln(abs(x^2*e + d))/(c*d^2*e - b*d*e^2 + a*e^3) - 1/4*(b*d
 - a*e)*ln(c*x^4 + b*x^2 + a)/(c^2*d^2 - b*c*d*e + a*c*e^2) + 1/2
*(b^2*d - 2*a*c*d - a*b*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c
))/((c^2*d^2 - b*c*d*e + a*c*e^2)*sqrt(-b^2 + 4*a*c))
```

3.290 $\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=132

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}-\frac{d\log\left(d+ex^2\right)}{2\,(ae^2-bde+cd^2)}+\frac{d\log\left(a+bx^2+cx^4\right)}{4\,(ae^2-bde+cd^2)}$$

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*Sqrt[
b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) - (d*Log[d + e*x^2])/(2*(c*
d^2 - b*d*e + a*e^2)) + (d*Log[a + b*x^2 + c*x^4])/(4*(c*d^2 - b*
d*e + a*e^2))
```

Rubi [A] time = 0.330501, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}-\frac{d\log\left(d+ex^2\right)}{2\,(ae^2-bde+cd^2)}+\frac{d\log\left(a+bx^2+cx^4\right)}{4\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*Sqrt[
b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) - (d*Log[d + e*x^2])/(2*(c*
d^2 - b*d*e + a*e^2)) + (d*Log[a + b*x^2 + c*x^4])/(4*(c*d^2 - b*
d*e + a*e^2))
```

Rubi in Sympy [A] time = 47.9577, size = 124, normalized size = 0.94

$$-\frac{d\log\left(d+ex^2\right)}{2\,(ae^2-bde+cd^2)}+\frac{d\log\left(a+bx^2+cx^4\right)}{4\,(ae^2-bde+cd^2)}-\frac{(2ae-bd)\,\mathrm{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2\sqrt{-4ac+b^2}\,(ae^2-bde+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] -d*log(d + e*x**2)/(2*(a*e**2 - b*d*e + c*d**2)) + d*log(a + b*x*
*2 + c*x**4)/(4*(a*e**2 - b*d*e + c*d**2)) - (2*a*e - b*d)*atanh(
(b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*sqrt(-4*a*c + b**2)*(a*e**
```

```
2 - b*d*e + c*d**2))
```

Mathematica [A] time = 0.121788, size = 114, normalized size = 0.86

$$\frac{2(bd-2ae)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)+d\sqrt{4ac-b^2}\left(2\log\left(d+ex^2\right)-\log\left(a+bx^2+cx^4\right)\right)}{4\sqrt{4ac-b^2}\left(e(bd-ae)-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*(b*d - 2*a*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]] + Sqrt[
-b^2 + 4*a*c]*d*(2*Log[d + e*x^2] - Log[a + b*x^2 + c*x^4]))/(4*S
qrt[-b^2 + 4*a*c]*(-(c*d^2) + e*(b*d - a*e)))
```

Maple [A] time = 0.012, size = 176, normalized size = 1.3

$$\frac{d\ln\left(cx^4+bx^2+a\right)}{4\,ae^2-4\,bde+4\,cd^2}+\frac{ae}{ae^2-bde+cd^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{bd}{2\,ae^2-2\,bde+2\,cd^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{d\ln\left(ex^2+d\right)}{2\,ae^2-2\,bde+2\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*d*ln(c*x^4+b*x^2+a)/(a*e^2-b*d*e+c*d^2)+1/(a*e^2-b*d*e+c*d^2)
/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*e-1/2/
(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b
^2)^(1/2))*b*d-1/2*d*ln(e*x^2+d)/(a*e^2-b*d*e+c*d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 18.2972, size = 1, normalized size = 0.01

$$\left[-\frac{(bd-2\,ae)\log\left(-\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x^2-(2\,c^2x^4+2\,bcx^2+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\sqrt{b^2-4\,ac}\left(d\log\left(cx^4+bx^2+a\right)-2\,d\log\left(ex^2\right.\right.}{4\,(cd^2-bde+ae^2)\sqrt{b^2-4\,ac}}\right.$$
$$\left.-\frac{2\,(bd-2\,ae)\arctan\left(-\frac{(2\,cx^2+b)\,\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-\sqrt{-b^2+4\,ac}\left(d\log\left(cx^4+bx^2+a\right)-2\,d\log\left(ex^2+d\right)\right)}{4\,(cd^2-bde+ae^2)\sqrt{-b^2+4\,ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*d - 2*a*e)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^
2 - (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x
^4 + b*x^2 + a)) - sqrt(b^2 - 4*a*c)*(d*log(c*x^4 + b*x^2 + a) -
2*d*log(e*x^2 + d)))/((c*d^2 - b*d*e + a*e^2)*sqrt(b^2 - 4*a*c)),
 -1/4*(2*(b*d - 2*a*e)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(
b^2 - 4*a*c)) - sqrt(-b^2 + 4*a*c)*(d*log(c*x^4 + b*x^2 + a) - 2*
d*log(e*x^2 + d)))/((c*d^2 - b*d*e + a*e^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.301104, size = 180, normalized size = 1.36

$$-\frac{d\ln\left(\left|x^2e+d\right|\right)}{2\,(cd^2e-bde^2+ae^3)}+\frac{d\ln\left(cx^4+bx^2+a\right)}{4\,(cd^2-bde+ae^2)}-\frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(cd^2-bde+ae^2)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] -1/2*d*e*ln(abs(x^2*e + d))/(c*d^2*e - b*d*e^2 + a*e^3) + 1/4*d*l
n(c*x^4 + b*x^2 + a)/(c*d^2 - b*d*e + a*e^2) - 1/2*(b*d - 2*a*e)*
arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((c*d^2 - b*d*e + a*e^2)
*sqrt(-b^2 + 4*a*c))
```

3.291 $\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=133

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}+\frac{e\log\left(d+ex^2\right)}{2\,(ae^2-bde+cd^2)}-\frac{e\log\left(a+bx^2+cx^4\right)}{4\,(ae^2-bde+cd^2)}$$

```
[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*Sqrt
[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (e*Log[d + e*x^2])/(2*(c
*d^2 - b*d*e + a*e^2)) - (e*Log[a + b*x^2 + c*x^4])/(4*(c*d^2 - b
*d*e + a*e^2))
```

Rubi [A] time = 0.25077, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}+\frac{e\log\left(d+ex^2\right)}{2\,(ae^2-bde+cd^2)}-\frac{e\log\left(a+bx^2+cx^4\right)}{4\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*Sqrt
[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (e*Log[d + e*x^2])/(2*(c
*d^2 - b*d*e + a*e^2)) - (e*Log[a + b*x^2 + c*x^4])/(4*(c*d^2 - b
*d*e + a*e^2))
```

Rubi in Sympy [A] time = 47.2823, size = 124, normalized size = 0.93

$$\frac{e\log\left(d+ex^2\right)}{2\,(ae^2-bde+cd^2)}-\frac{e\log\left(a+bx^2+cx^4\right)}{4\,(ae^2-bde+cd^2)}+\frac{(be-2cd)\,\mathrm{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2\sqrt{-4ac+b^2}\,(ae^2-bde+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] e*log(d + e*x**2)/(2*(a*e**2 - b*d*e + c*d**2)) - e*log(a + b*x**
2 + c*x**4)/(4*(a*e**2 - b*d*e + c*d**2)) + (b*e - 2*c*d)*atanh((
b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*sqrt(-4*a*c + b**2)*(a*e**2
```

```
- b*d*e + c*d**2))
```

Mathematica [A] time = 0.11464, size = 112, normalized size = 0.84

$$\frac{(2be-4cd)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)+e\sqrt{4ac-b^2}\left(\log\left(a+bx^2+cx^4\right)-2\log\left(d+ex^2\right)\right)}{4\sqrt{4ac-b^2}\left(e(bd-ae)-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-4*c*d + 2*b*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]] + Sqrt
[-b^2 + 4*a*c]*e*(-2*Log[d + e*x^2] + Log[a + b*x^2 + c*x^4]))/(4
*Sqrt[-b^2 + 4*a*c]*(-(c*d^2) + e*(b*d - a*e)))
```

Maple [A] time = 0.011, size = 176, normalized size = 1.3

$$-\frac{e\ln\left(cx^4+bx^2+a\right)}{4\,ae^2-4\,bde+4\,cd^2}-\frac{be}{2\,ae^2-2\,bde+2\,cd^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{cd}{ae^2-bde+cd^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{e\ln\left(ex^2+d\right)}{2\,ae^2-2\,bde+2\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*e*ln(c*x^4+b*x^2+a)/(a*e^2-b*d*e+c*d^2)-1/2/(a*e^2-b*d*e+c*d
^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*e+1
/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-
b^2)^(1/2))*c*d+1/2*e*ln(e*x^2+d)/(a*e^2-b*d*e+c*d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 12.9931, size = 1, normalized size = 0.01

$$\left[-\frac{(2\,cd-be)\log\left(\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x^2+(2\,c^2x^4+2\,bcx^2+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\sqrt{b^2-4\,ac}\big(e\log\left(cx^4+bx^2+a\right)-2\,e\log\left(ex^2+\right.}{4\,(cd^2-bde+ae^2)\sqrt{b^2-4\,ac}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*c*d - b*e)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2
 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^
4 + b*x^2 + a)) + sqrt(b^2 - 4*a*c)*(e*log(c*x^4 + b*x^2 + a) - 2
*e*log(e*x^2 + d)))/((c*d^2 - b*d*e + a*e^2)*sqrt(b^2 - 4*a*c)),
1/4*(2*(2*c*d - b*e)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^
2 - 4*a*c)) - sqrt(-b^2 + 4*a*c)*(e*log(c*x^4 + b*x^2 + a) - 2*e*
log(e*x^2 + d)))/((c*d^2 - b*d*e + a*e^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.301574, size = 181, normalized size = 1.36

$$-\frac{e\ln\left(cx^4+bx^2+a\right)}{4\,(cd^2-bde+ae^2)}+\frac{e^2\ln\left(\left|x^2e+d\right|\right)}{2\,(cd^2e-bde^2+ae^3)}+\frac{(2\,cd-be)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(cd^2-bde+ae^2)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] -1/4*e*ln(c*x^4 + b*x^2 + a)/(c*d^2 - b*d*e + a*e^2) + 1/2*e^2*ln
(abs(x^2*e + d))/(c*d^2*e - b*d*e^2 + a*e^3) + 1/2*(2*c*d - b*e)*
arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((c*d^2 - b*d*e + a*e^2)
*sqrt(-b^2 + 4*a*c))
```

3.292 $\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=167

$$\frac{(2ace+b^2(-e)+bcd)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}-\frac{e^2\log(d+ex^2)}{2d(ae^2-bde+cd^2)}-\frac{(cd-be)\log(a+bx^2+cx^4)}{4a(ae^2-bde+cd^2)}+\frac{\log(x)}{ad}$$

```
[Out] ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c
]])/(2*a*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + Log[x]/(a*d
) - (e^2*Log[d + e*x^2])/(2*d*(c*d^2 - b*d*e + a*e^2)) - ((c*d -
b*e)*Log[a + b*x^2 + c*x^4])/(4*a*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.571201, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{(2ace+b^2(-e)+bcd)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}-\frac{e^2\log(d+ex^2)}{2d(ae^2-bde+cd^2)}-\frac{(cd-be)\log(a+bx^2+cx^4)}{4a(ae^2-bde+cd^2)}+\frac{\log(x)}{ad}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c
]])/(2*a*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + Log[x]/(a*d
) - (e^2*Log[d + e*x^2])/(2*d*(c*d^2 - b*d*e + a*e^2)) - ((c*d -
b*e)*Log[a + b*x^2 + c*x^4])/(4*a*(c*d^2 - b*d*e + a*e^2))
```

Rubi in Sympy [A] time = 100.157, size = 148, normalized size = 0.89

$$-\frac{e^2\log(d+ex^2)}{2d(ae^2-bde+cd^2)}+\frac{(be-cd)\log(a+bx^2+cx^4)}{4a(ae^2-bde+cd^2)}-\frac{(-2ace+b^2e-bcd)\,\mathrm{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a\sqrt{-4ac+b^2}\,(ae^2-bde+cd^2)}+\frac{\log(x^2)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] -e**2*log(d + e*x**2)/(2*d*(a*e**2 - b*d*e + c*d**2)) + (b*e - c*
d)*log(a + b*x**2 + c*x**4)/(4*a*(a*e**2 - b*d*e + c*d**2)) - (-2
*a*c*e + b**2*e - b*c*d)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2)
```

```
)/(2*a*sqrt(-4*a*c + b**2)*(a*e**2 - b*d*e + c*d**2)) + log(x**2)
/(2*a*d)
```

Mathematica [A] time = 0.591792, size = 242, normalized size = 1.45

$$\frac{4\log(x)\sqrt{b^2-4ac}\left(e(ae-bd)+cd^2\right)-2ae^2\sqrt{b^2-4ac}\log\left(d+ex^2\right)-d\left(cd\sqrt{b^2-4ac}-be\sqrt{b^2-4ac}+2ace+b^2(-e)+bc\right.}{4ad\sqrt{b^2-4ac}\,(e(a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*(c*d^2 + e*(-(b*d) + a*e))*Log[x] - d*(b*c*d
 + c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e - b*Sqrt[b^2 - 4*a*c]*
e)*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2] + d*(b*c*d - c*Sqrt[b^2 -
 4*a*c]*d - b^2*e + 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e)*Log[b + Sqrt
[b^2 - 4*a*c] + 2*c*x^2] - 2*a*Sqrt[b^2 - 4*a*c]*e^2*Log[d + e*x^
2])/(4*a*Sqrt[b^2 - 4*a*c]*d*(c*d^2 + e*(-(b*d) + a*e)))
```

Maple [A] time = 0.016, size = 298, normalized size = 1.8

$$\begin{aligned}&\frac{\ln\left(cx^4+bx^2+a\right)be}{(4\,ae^2-4\,bde+4\,cd^2)\,a}-\frac{c\ln\left(cx^4+bx^2+a\right)d}{(4\,ae^2-4\,bde+4\,cd^2)\,a}\\&-\frac{ce}{ae^2-bde+cd^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}\\&+\frac{b^2e}{(2\,ae^2-2\,bde+2\,cd^2)\,a}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}\\&-\frac{bcd}{(2\,ae^2-2\,bde+2\,cd^2)\,a}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}\\&+\frac{\ln(x)}{ad}-\frac{e^2\ln\left(ex^2+d\right)}{2\,d\,(ae^2-bde+cd^2)}\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/(a*e^2-b*d*e+c*d^2)/a*ln(c*x^4+b*x^2+a)*b*e-1/4/(a*e^2-b*d*e+
c*d^2)/a*c*ln(c*x^4+b*x^2+a)*d-1/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^
(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*e+1/2/(a*e^2-b*d*e+
c*d^2)/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*
b^2*e-1/2/(a*e^2-b*d*e+c*d^2)/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
```

```
+b)/(4*a*c-b^2)^(1/2))*b*c*d+ln(x)/a/d-1/2*e^2*ln(e*x^2+d)/d/(a*e
^2-b*d*e+c*d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.302381, size = 232, normalized size = 1.39

$$-\frac{(cd-be)\ln\left(cx^4+bx^2+a\right)}{4\left(acd^2-abde+a^2e^2\right)}-\frac{e^3\ln\left(\left|x^2e+d\right|\right)}{2\left(cd^3e-bd^2e^2+ade^3\right)}-\frac{\left(bcd-b^2e+2\,ace\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(acd^2-abde+a^2e^2\right)\sqrt{-b^2+4\,ac}}+\frac{\ln\left(x^2\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)*x),x, algorithm="giac")
```

```
[Out] -1/4*(c*d - b*e)*ln(c*x^4 + b*x^2 + a)/(a*c*d^2 - a*b*d*e + a^2*e
^2) - 1/2*e^3*ln(abs(x^2*e + d))/(c*d^3*e - b*d^2*e^2 + a*d*e^3)
- 1/2*(b*c*d - b^2*e + 2*a*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 +
4*a*c))/((a*c*d^2 - a*b*d*e + a^2*e^2)*sqrt(-b^2 + 4*a*c)) + 1/2*
ln(x^2)/(a*d)
```

3.293 $\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=205

$$\frac{(ace+b^2(-e)+bcd)\log(a+bx^2+cx^4)}{4a^2(ae^2-bde+cd^2)}-\frac{(3abce-2ac^2d+b^3(-e)+b^2cd)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}(ae^2-bde+cd^2)}-\frac{\log(x)(ae+bd)}{a^2d^2}+\frac{e^3\log(d+ex^2)}{2d^2(ae^2-bde+cd^2)}-\frac{1}{2adx^2}$$

```
[Out] -1/(2*a*d*x^2) - ((b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*ArcTa
nh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*(c*
d^2 - b*d*e + a*e^2)) - ((b*d + a*e)*Log[x])/(a^2*d^2) + (e^3*Log
[d + e*x^2])/(2*d^2*(c*d^2 - b*d*e + a*e^2)) + ((b*c*d - b^2*e +
a*c*e)*Log[a + b*x^2 + c*x^4])/(4*a^2*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.787316, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{(ace+b^2(-e)+bcd)\log(a+bx^2+cx^4)}{4a^2(ae^2-bde+cd^2)}-\frac{(3abce-2ac^2d+b^3(-e)+b^2cd)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}(ae^2-bde+cd^2)}-\frac{\log(x)(ae+bd)}{a^2d^2}+\frac{e^3\log(d+ex^2)}{2d^2(ae^2-bde+cd^2)}-\frac{1}{2adx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```


```
[Out] -1/(2*a*d*x^2) - ((b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*ArcTa
nh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*(c*
d^2 - b*d*e + a*e^2)) - ((b*d + a*e)*Log[x])/(a^2*d^2) + (e^3*Log
[d + e*x^2])/(2*d^2*(c*d^2 - b*d*e + a*e^2)) + ((b*c*d - b^2*e +
a*c*e)*Log[a + b*x^2 + c*x^4])/(4*a^2*(c*d^2 - b*d*e + a*e^2))
```

Rubi in Sympy [A] time = 157.915, size = 194, normalized size = 0.95

$$\frac{e^3\log(d+ex^2)}{2d^2(ae^2-bde+cd^2)}-\frac{1}{2adx^2}-\frac{(-ace+b^2e-bcd)\log(a+bx^2+cx^4)}{4a^2(ae^2-bde+cd^2)}+\frac{(-3abce+2ac^2d+b^3e-b^2cd)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^2\sqrt{-4ac+b^2}(ae^2-bde+cd^2)}-\frac{(ae+bd)\log(x^2)}{2a^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] e**3*log(d + e*x**2)/(2*d**2*(a*e**2 - b*d*e + c*d**2)) - 1/(2*a*
d*x**2) - (-a*c*e + b**2*e - b*c*d)*log(a + b*x**2 + c*x**4)/(4*a
**2*(a*e**2 - b*d*e + c*d**2)) + (-3*a*b*c*e + 2*a*c**2*d + b**3*
e - b**2*c*d)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*a**2*s
qrt(-4*a*c + b**2)*(a*e**2 - b*d*e + c*d**2)) - (a*e + b*d)*log(x
**2)/(2*a**2*d**2)
```

Mathematica [A] time = 0.574493, size = 331, normalized size = 1.61

$$\frac{1}{4}\left(\frac{\left(b^2\left(e\sqrt{b^2-4ac}-cd\right)-bc\left(d\sqrt{b^2-4ac}+3ae\right)+ac\left(2cd-e\sqrt{b^2-4ac}\right)+b^3e\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{a^2\sqrt{b^2-4ac}\,(e(bd-ae)-cd^2)}\right.$$
$$+\frac{\left(b^2\left(e\sqrt{b^2-4ac}+cd\right)+bc\left(3ae-d\sqrt{b^2-4ac}\right)-ac\left(e\sqrt{b^2-4ac}+2cd\right)+b^3(-e)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{a^2\sqrt{b^2-4ac}\,(e(bd-ae)-cd^2)}$$
$$\left.-\frac{4\log(x)(ae+bd)}{a^2d^2}+\frac{2e^3\log\left(d+ex^2\right)}{d^2e(ae-bd)+cd^4}-\frac{2}{adx^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-2/(a*d*x^2) - (4*(b*d + a*e)*Log[x])/(a^2*d^2) + ((b^3*e - b*c*
(Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e)
 + b^2*(-(c*d) + Sqrt[b^2 - 4*a*c]*e))*Log[b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(a^2*Sqrt[b^2 - 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) +
((-(b^3*e) + b*c*(-(Sqrt[b^2 - 4*a*c]*d) + 3*a*e) + b^2*(c*d + Sq
rt[b^2 - 4*a*c]*e) - a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e))*Log[b + S
qrt[b^2 - 4*a*c] + 2*c*x^2])/(a^2*Sqrt[b^2 - 4*a*c]*(-(c*d^2) + e
*(b*d - a*e))) + (2*e^3*Log[d + e*x^2])/(c*d^4 + d^2*e*(-(b*d) +
a*e)))/4
```

Maple [B] time = 0.021, size = 430, normalized size = 2.1

$$\frac{c\ln\left(cx^{4}+bx^{2}+a\right)e}{\left(4ae^{2}-4bde+4cd^{2}\right)a}-\frac{\ln\left(cx^{4}+bx^{2}+a\right)b^{2}e}{\left(4ae^{2}-4bde+4cd^{2}\right)a^{2}}+\frac{c\ln\left(cx^{4}+bx^{2}+a\right)bd}{\left(4ae^{2}-4bde+4cd^{2}\right)a^{2}}+\frac{3bce}{\left(2ae^{2}-2bde+2cd^{2}\right)a}\arctan\left(\left(2cx^{2}+b\right)\frac{1}{\sqrt{4ac-b^{2}}}\right)\frac{1}{\sqrt{4ac-b^{2}}}-\frac{c^{2}d}{\left(ae^{2}-bde+cd^{2}\right)a}\arctan\left(\left(2cx^{2}+b\right)\frac{1}{\sqrt{4ac-b^{2}}}\right)\frac{1}{\sqrt{4ac-b^{2}}}-\frac{b^{3}e}{\left(2ae^{2}-2bde+2cd^{2}\right)a^{2}}\arctan\left(\left(2cx^{2}+b\right)\frac{1}{\sqrt{4ac-b^{2}}}\right)\frac{1}{\sqrt{4ac-b^{2}}}+\frac{b^{2}cd}{\left(2ae^{2}-2bde+2cd^{2}\right)a^{2}}\arctan\left(\left(2cx^{2}+b\right)\frac{1}{\sqrt{4ac-b^{2}}}\right)\frac{1}{\sqrt{4ac-b^{2}}}-\frac{1}{2adx^{2}}-\frac{\ln\left(x\right)e}{ad^{2}}-\frac{\ln\left(x\right)b}{a^{2}d}+\frac{e^{3}\ln\left(ex^{2}+d\right)}{2d^{2}\left(ae^{2}-bde+cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/(a*e^2-b*d*e+c*d^2)/a*c*ln(c*x^4+b*x^2+a)*e-1/4/(a*e^2-b*d*e+
c*d^2)/a^2*ln(c*x^4+b*x^2+a)*b^2*e+1/4/(a*e^2-b*d*e+c*d^2)/a^2*c*
ln(c*x^4+b*x^2+a)*b*d+3/2/(a*e^2-b*d*e+c*d^2)/a/(4*a*c-b^2)^(1/2)
*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c*e-1/(a*e^2-b*d*e+c*d^2
)/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c^2*d
-1/2/(a*e^2-b*d*e+c*d^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2))*b^3*e+1/2/(a*e^2-b*d*e+c*d^2)/a^2/(4*a*c-b^2)
^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*c*d-1/2/a/d/x^2-
e*ln(x)/a/d^2-1/a^2/d*ln(x)*b+1/2*e^3*ln(e*x^2+d)/d^2/(a*e^2-b*d*
e+c*d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.302586, size = 320, normalized size = 1.56

$$\frac{\left(bcd - b^2e + ace\right)\ln\left(cx^4 + bx^2 + a\right)}{4\left(a^2cd^2 - a^2bde + a^3e^2\right)} + \frac{e^4\ln\left(\left|x^2e + d\right|\right)}{2\left(cd^4e - bd^3e^2 + ad^2e^3\right)} + \frac{\left(b^2cd - 2\,ac^2d - b^3e + 3\,abce\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^2cd^2 - a^2bde + a^3e^2\right)\sqrt{-b^2 + 4\,ac}} - \frac{(bd + ae)\ln\left(x^2\right)}{2\,a^2d^2} + \frac{bdx^2 + ax^2e - ad}{2\,a^2d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^3),x, algorithm="giac")
```

```
[Out] 1/4*(b*c*d - b^2*e + a*c*e)*ln(c*x^4 + b*x^2 + a)/(a^2*c*d^2 - a^
2*b*d*e + a^3*e^2) + 1/2*e^4*ln(abs(x^2*e + d))/(c*d^4*e - b*d^3*
e^2 + a*d^2*e^3) + 1/2*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*
arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^2*c*d^2 - a^2*b*d*e
+ a^3*e^2)*sqrt(-b^2 + 4*a*c)) - 1/2*(b*d + a*e)*ln(x^2)/(a^2*d^2
) + 1/2*(b*d*x^2 + a*x^2*e - a*d)/(a^2*d^2*x^2)
```

3.294 $\int \frac{1}{x^5(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=268

$$\frac{\log(x)\left(abde-a\left(cd^2-ae^2\right)+b^2d^2\right)}{a^3d^3}-\frac{\left(2abce-ac^2d+b^3(-e)+b^2cd\right)\log\left(a+bx^2+cx^4\right)}{4a^3\left(ae^2-bde+cd^2\right)}+\frac{ae+bd}{2a^2d^2x^2}+\frac{\left(-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(ae^2-bde+cd^2\right)}-\frac{1}{4adx^4}$$

```
[Out] -1/(4*a*d*x^4) + (b*d + a*e)/(2*a^2*d^2*x^2) + ((b^3*c*d - 3*a*b*
c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*a^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e
^2)) + ((b^2*d^2 + a*b*d*e - a*(c*d^2 - a*e^2))*Log[x])/(a^3*d^3)
 - (e^4*Log[d + e*x^2])/(2*d^3*(c*d^2 - b*d*e + a*e^2)) - ((b^2*c
*d - a*c^2*d - b^3*e + 2*a*b*c*e)*Log[a + b*x^2 + c*x^4])/(4*a^3*
(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 1.06887, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{\log(x)\left(abde-a\left(cd^2-ae^2\right)+b^2d^2\right)}{a^3d^3}-\frac{\left(2abce-ac^2d+b^3(-e)+b^2cd\right)\log\left(a+bx^2+cx^4\right)}{4a^3\left(ae^2-bde+cd^2\right)}+\frac{ae+bd}{2a^2d^2x^2}+\frac{\left(-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(ae^2-bde+cd^2\right)}-\frac{1}{4adx^4}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```


```
[Out] -1/(4*a*d*x^4) + (b*d + a*e)/(2*a^2*d^2*x^2) + ((b^3*c*d - 3*a*b*
c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*a^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e
^2)) + ((b^2*d^2 + a*b*d*e - a*(c*d^2 - a*e^2))*Log[x])/(a^3*d^3)
 - (e^4*Log[d + e*x^2])/(2*d^3*(c*d^2 - b*d*e + a*e^2)) - ((b^2*c
*d - a*c^2*d - b^3*e + 2*a*b*c*e)*Log[a + b*x^2 + c*x^4])/(4*a^3*
(c*d^2 - b*d*e + a*e^2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.795197, size = 426, normalized size = 1.59

$$\frac{1}{4}\left(\frac{4\log(x)\left(abde+a\left(ae^2-cd^2\right)+b^2d^2\right)}{a^3d^3}-\frac{\left(ac^2\left(d\sqrt{b^2-4ac}+2ae\right)-b^2c\left(d\sqrt{b^2-4ac}+4ae\right)+abc\left(3cd-2e\sqrt{b^2-4ac}\right)+b^3\left(e\sqrt{b^2-4ac}-cd\right)+b^4e\right)\log\left(-\sqrt{b\right.}}{a^3\sqrt{b^2-4ac}\,(e(bd-ae)-cd^2)}-\frac{\left(ac^2\left(d\sqrt{b^2-4ac}-2ae\right)+b^2c\left(4ae-d\sqrt{b^2-4ac}\right)-abc\left(2e\sqrt{b^2-4ac}+3cd\right)+b^3\left(e\sqrt{b^2-4ac}+cd\right)+b^4(-e)\right)\log\left(\sqrt{\right.}}{a^3\sqrt{b^2-4ac}\,(e(bd-ae)-cd^2)}+\frac{2(ae+bd)}{a^2d^2x^2}-\frac{2e^4\log\left(d+ex^2\right)}{d^3e(ae-bd)+cd^5}-\frac{1}{adx^4}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-(1/(a*d*x^4)) + (2*(b*d + a*e))/(a^2*d^2*x^2) + (4*(b^2*d^2 + a
*b*d*e + a*(-(c*d^2) + a*e^2))*Log[x])/(a^3*d^3) - ((b^4*e + a*c^
2*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - b^2*c*(Sqrt[b^2 - 4*a*c]*d + 4*
a*e) + a*b*c*(3*c*d - 2*Sqrt[b^2 - 4*a*c]*e) + b^3*(-(c*d) + Sqrt
[b^2 - 4*a*c]*e))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^3*Sqrt
[b^2 - 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) - ((-(b^4*e) + a*c^2*(S
qrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*c*(-(Sqrt[b^2 - 4*a*c]*d) + 4*a
*e) + b^3*(c*d + Sqrt[b^2 - 4*a*c]*e) - a*b*c*(3*c*d + 2*Sqrt[b^2
 - 4*a*c]*e))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^3*Sqrt[b^2
 - 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) - (2*e^4*Log[d + e*x^2])/(c
*d^5 + d^3*e*(-(b*d) + a*e)))/4
```

Maple [B] time = 0.028, size = 584, normalized size = 2.2

$$-\frac{c\ln\left(cx^4+bx^2+a\right)be}{\left(2\,ae^2-2\,bde+2\,cd^2\right)a^2}+\frac{c^2\ln\left(cx^4+bx^2+a\right)d}{\left(4\,ae^2-4\,bde+4\,cd^2\right)a^2}+\frac{\ln\left(cx^4+bx^2+a\right)b^3e}{\left(4\,ae^2-4\,bde+4\,cd^2\right)a^3}$$
$$-\frac{c\ln\left(cx^4+bx^2+a\right)b^2d}{\left(4\,ae^2-4\,bde+4\,cd^2\right)a^3}+\frac{c^2e}{\left(ae^2-bde+cd^2\right)a}\arctan\left(\left(2\,cx^2+b\right)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$-2\,\frac{b^2ce}{\left(ae^2-bde+cd^2\right)a^2\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$
$$+\frac{3\,bc^2d}{\left(2\,ae^2-2\,bde+2\,cd^2\right)a^2}\arctan\left(\left(2\,cx^2+b\right)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$+\frac{b^4e}{\left(2\,ae^2-2\,bde+2\,cd^2\right)a^3}\arctan\left(\left(2\,cx^2+b\right)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$-\frac{b^3cd}{\left(2\,ae^2-2\,bde+2\,cd^2\right)a^3}\arctan\left(\left(2\,cx^2+b\right)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{1}{4\,adx^4}$$
$$+\frac{e}{2\,ad^2x^2}+\frac{b}{2\,a^2dx^2}+\frac{\ln(x)e^2}{ad^3}+\frac{\ln(x)be}{a^2d^2}-\frac{\ln(x)c}{a^2d}+\frac{\ln(x)b^2}{a^3d}-\frac{e^4\ln\left(ex^2+d\right)}{2\,d^3\left(ae^2-bde+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/(a*e^2-b*d*e+c*d^2)/a^2*c*ln(c*x^4+b*x^2+a)*b*e+1/4/(a*e^2-b
*d*e+c*d^2)/a^2*c^2*ln(c*x^4+b*x^2+a)*d+1/4/(a*e^2-b*d*e+c*d^2)/a
^3*ln(c*x^4+b*x^2+a)*b^3*e-1/4/(a*e^2-b*d*e+c*d^2)/a^3*c*ln(c*x^4
+b*x^2+a)*b^2*d+1/(a*e^2-b*d*e+c*d^2)/a/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x^2+b)/(4*a*c-b^2)^(1/2))*e*c^2-2/(a*e^2-b*d*e+c*d^2)/a^2/(4
*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*c*e+3/2
/(a*e^2-b*d*e+c*d^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*
a*c-b^2)^(1/2))*b*c^2*d+1/2/(a*e^2-b*d*e+c*d^2)/a^3/(4*a*c-b^2)^(
1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*e-1/2/(a*e^2-b*d*e
+c*d^2)/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2
))*b^3*c*d-1/4/a/d/x^4+1/2*e/a/d^2/x^2+1/2/a^2/d/x^2*b+1/d^3/a*ln
(x)*e^2+1/d^2/a^2*ln(x)*b*e-1/d/a^2*ln(x)*c+1/d/a^3*ln(x)*b^2-1/2
*e^4*ln(e*x^2+d)/d^3/(a*e^2-b*d*e+c*d^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.302801, size = 448, normalized size = 1.67

$$-\frac{\left(b^{2} c d - a c^{2} d - b^{3} e + 2 \, a b c e\right) \ln\left(c x^{4} + b x^{2} + a\right)}{4 \, {\left(a^{3} c d^{2} - a^{3} b d e + a^{4} e^{2}\right)}} - \frac{e^{5} \ln\left({\left| x^{2} e + d \right|}\right)}{2 \, {\left(c d^{5} e - b d^{4} e^{2} + a d^{3} e^{3}\right)}} - \frac{{\left(b^{3} c d - 3 \, a b c^{2} d - b^{4} e + 4 \, a b^{2} c e - 2 \, a^{2} c^{2} e\right)} \arctan\left(\frac{2 \, c x^{2} + b}{\sqrt{-b^{2} + 4 \, a c}}\right)}{2 \, {\left(a^{3} c d^{2} - a^{3} b d e + a^{4} e^{2}\right)} \sqrt{-b^{2} + 4 \, a c}} + \frac{{\left(b^{2} d^{2} - a c d^{2} + a b d e + a^{2} e^{2}\right)} \ln\left(x^{2}\right)}{2 \, a^{3} d^{3}} - \frac{3 \, b^{2} d^{2} x^{4} - 3 \, a c d^{2} x^{4} + 3 \, a b d x^{4} e + 3 \, a^{2} x^{4} e^{2} - 2 \, a b d^{2} x^{2} - 2 \, a^{2} d x^{2} e + a^{2} d^{2}}{4 \, a^{3} d^{3} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^5),x, algorithm="giac")
```

```
[Out] -1/4*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*ln(c*x^4 + b*x^2 + a
)/(a^3*c*d^2 - a^3*b*d*e + a^4*e^2) - 1/2*e^5*ln(abs(x^2*e + d))/
```

```
(c*d^5*e - b*d^4*e^2 + a*d^3*e^3) - 1/2*(b^3*c*d - 3*a*b*c^2*d -
b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2
 + 4*a*c))/((a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*sqrt(-b^2 + 4*a*c))
 + 1/2*(b^2*d^2 - a*c*d^2 + a*b*d*e + a^2*e^2)*ln(x^2)/(a^3*d^3)
- 1/4*(3*b^2*d^2*x^4 - 3*a*c*d^2*x^4 + 3*a*b*d*x^4*e + 3*a^2*x^4*
e^2 - 2*a*b*d^2*x^2 - 2*a^2*d*x^2*e + a^2*d^2)/(a^3*d^3*x^4)
```

3.295 $\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=387

$$-\frac{\left(-\frac{3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d}{\sqrt{b^2-4ac}}+a^2ce-ab^2e-2abcd+b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\left(\frac{3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d}{\sqrt{b^2-4ac}}+a^2ce-ab^2e-2abcd+b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\,(ae^2-bde+cd^2)}-\frac{x(be+cd)}{c^2e^2}+\frac{x^3}{3ce}$$

```
[Out] -(((c*d + b*e)*x)/(c^2*e^2)) + x^3/(3*c*e) - ((b^3*d - 2*a*b*c*d
- a*b^2*e + a^2*c*e - (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*
e + 3*a^2*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 -
4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - ((b^3*d - 2*a*b*c*d - a*b^2*e
+ a^2*c*e + (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*
b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c
*d^2 - b*d*e + a*e^2)) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e
^(5/2)*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 8.19544, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-\frac{\left(-\frac{3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d}{\sqrt{b^2-4ac}}+a^2ce-ab^2e-2abcd+b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\left(\frac{3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d}{\sqrt{b^2-4ac}}+a^2ce-ab^2e-2abcd+b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\,(ae^2-bde+cd^2)}-\frac{x(be+cd)}{c^2e^2}+\frac{x^3}{3ce}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((c*d + b*e)*x)/(c^2*e^2)) + x^3/(3*c*e) - ((b^3*d - 2*a*b*c*d
- a*b^2*e + a^2*c*e - (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*
```

```
e + 3*a^2*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 -
4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - ((b^3*d - 2*a*b*c*d - a*b^2*e
+ a^2*c*e + (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*
b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c
*d^2 - b*d*e + a*e^2)) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e
^(5/2)*(c*d^2 - b*d*e + a*e^2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.16463, size = 463, normalized size = 1.2

$$\frac{\left(a^2c\left(e\sqrt{b^2-4ac}-2cd\right)+ab^2\left(4cd-e\sqrt{b^2-4ac}\right)-abc\left(2d\sqrt{b^2-4ac}+3ae\right)+b^3\left(d\sqrt{b^2-4ac}+ae\right)+b^4(-d)\right)\tan^{-1}\left(\right.}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(e(bd-ae)-cd^2)}$$
$$+\frac{\left(a^2c\left(e\sqrt{b^2-4ac}+2cd\right)-ab^2\left(e\sqrt{b^2-4ac}+4cd\right)+abc\left(3ae-2d\sqrt{b^2-4ac}\right)+b^3\left(d\sqrt{b^2-4ac}-ae\right)+b^4d\right)\tan^{-1}\left(\right.}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\,(e(bd-ae)-cd^2)}$$
$$+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\left(ae^2-bde+cd^2\right)}-\frac{x(be+cd)}{c^2e^2}+\frac{x^3}{3ce}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((c*d + b*e)*x)/(c^2*e^2)) + x^3/(3*c*e) + ((-(b^4*d) + b^3*(Sq
rt[b^2 - 4*a*c]*d + a*e) - a*b*c*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)
+ a*b^2*(4*c*d - Sqrt[b^2 - 4*a*c]*e) + a^2*c*(-2*c*d + Sqrt[b^2
- 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c
]]*(-(c*d^2) + e*(b*d - a*e))) + ((b^4*d + b^3*(Sqrt[b^2 - 4*a*c]
*d - a*e) + a*b*c*(-2*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a^2*c*(2*c*d
 + Sqrt[b^2 - 4*a*c]*e) - a*b^2*(4*c*d + Sqrt[b^2 - 4*a*c]*e))*Ar
```

```
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) +
e*(b*d - a*e))) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(5/2)*
(c*d^2 - b*d*e + a*e^2))
```

Maple [B] time = 0.051, size = 1449, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*x^3/c/e-1/c^2/e*b*x-d*x/e^2/c-1/2/c/(a*e^2-b*d*e+c*d^2)*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*a^2*e+1/2/c^2/(a*e^2-b*d*e+c*d^2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*a*b^2*e+1/c/(a*e^2-b*d*e+c*d^2)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*a*b*d-1/2/c^2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*b^3*d-3/2/c/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*b*e-1/(a*e^2-b*d*e+c*d^2)/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*d+1/2/c^2/(a*e^2-
b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^
3*e+2/c/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*a*b^2*d-1/2/c^2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*d+1/2/c/(a*e^2-b*d*e+c*d^2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*e-1/2/c^2/(a*e^2-b*d*e+c*d^2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2*e-1/c/(a*e^2-b*d*e+c*d^2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*d+1/2/c^2/(a*e^2-b*d*e+c*d^2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d-3/2/c/(a*e^2-b*d*e+c*d^2)/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*b*e-1/(a*e^2-
b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a
^2*d+1/2/c^2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*a*b^3*e+2/c/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(
1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2
```

```
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2*d-1/2/c^2/(a*e^2-b*d*e
+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*d+
1/e^2*d^4/(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.296 $\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=323

$$\frac{\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}+\frac{\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}\,(ae^2-bde+cd^2)}+\frac{x}{ce}$$

```
[Out] x/(c*e) + ((b^2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e
+ 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d - a*c*d - a*b*e + (b^3*d -
 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqr
t[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (d^(5/2)*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 3.296, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}+\frac{\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}\,(ae^2-bde+cd^2)}+\frac{x}{ce}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```


```
[Out] x/(c*e) + ((b^2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e
+ 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d - a*c*d - a*b*e + (b^3*d -
 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqr
t[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (d^(5/2)*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 - b*d*e + a*e^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{d^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{\frac{3}{2}}\left(ae^2-bde+cd^2\right)}+\frac{\int\frac{1}{c}\,dx}{e}+\frac{\sqrt{2}\left(2ac\left(ae-bd\right)+b\left(-abe-acd+b^2d\right)+\sqrt{-4ac+b^2}\left(-abe-acd+b^2d\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{2}\left(2ac\left(ae-bd\right)+b\left(-abe-acd+b^2d\right)-\sqrt{-4ac+b^2}\left(-abe-acd+b^2d\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\left(ae^2-bde+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] -d**(5/2)*atan(sqrt(e)*x/sqrt(d))/(e**(3/2)*(a*e**2 - b*d*e + c*d
**2)) + Integral(1/c, x)/e + sqrt(2)*(2*a*c*(a*e - b*d) + b*(-a*b
*e - a*c*d + b**2*d) + sqrt(-4*a*c + b**2)*(-a*b*e - a*c*d + b**2
*d))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*c**
(3/2)*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)*(a*e**2 -
 b*d*e + c*d**2)) - sqrt(2)*(2*a*c*(a*e - b*d) + b*(-a*b*e - a*c*
d + b**2*d) - sqrt(-4*a*c + b**2)*(-a*b*e - a*c*d + b**2*d))*atan
(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*c**(3/2)*sqr
t(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)*(a*e**2 - b*d*e +
c*d**2))
```

Mathematica [A] time = 0.946898, size = 385, normalized size = 1.19

$$\frac{\left(-b^2\left(d\sqrt{b^2-4ac}+ae\right)+ab\left(e\sqrt{b^2-4ac}-3cd\right)+ac\left(d\sqrt{b^2-4ac}+2ae\right)+b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(e(bd-ae)-cd^2\right)}+\frac{\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)+ac\left(2ae-d\sqrt{b^2-4ac}\right)+b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\left(e(ae-bd)+cd^2\right)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(ae^2-bde+cd^2\right)}+\frac{x}{ce}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] x/(c*e) + ((b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b
^2 - 4*a*c]*d + 2*a*e) + a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(
3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*
(b*d - a*e))) + ((b^3*d + b^2*(Sqrt[b^2 - 4*a*c]*d - a*e) + a*c*(
-(Sqrt[b^2 - 4*a*c]*d) + 2*a*e) - a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*
e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2
 + e*(-(b*d) + a*e))) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^
(3/2)*(c*d^2 - b*d*e + a*e^2))
```

Maple [B] time = 0.042, size = 1098, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] x/c/e-1/2/(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*
e-1/2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*d+1/2/(a
*e^2-b*d*e+c*d^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d+1/(a*e^2-b
*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*e
-1/2/(a*e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*a*b^2*e-3/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*a*b*d+1/2/(a*e^2-b*d*e+c*d^2)/c/(-4*a*c+
b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d+1/2/(a*e^2-b*d*e+c*
d^2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*e+1/2/(a*e^2-b*d*e+c*d
^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*d-1/2/(a*e^2-b*d*e+c*d^2)/c
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d+1/(a*e^2-b*d*e+c*d^2)/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*e-1/2/(a*e^2-b
*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
a*b^2*e-3/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*a*b*d+1/2/(a*e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d-1/e*d^3/(a*e^2-b*d*e+c*
```

```
d^2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.297 $\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=280

$$-\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\,(ae^2-bde+cd^2)}$$

```
[Out] -(((b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqr
t[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2))) - ((b*
d - a*e + (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*S
qrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (d^(3/2)*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 1.87799, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```


```
[Out] -(((b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqr
t[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2))) - ((b*
d - a*e + (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*S
qrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (d^(3/2)*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 - b*d*e + a*e^2))
```

Rubi in Sympy [A] time = 113.205, size = 279, normalized size = 1.

$$\frac{d^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{2}\left(2acd+b\left(ae-bd\right)+\sqrt{-4ac+b^2}\left(ae-bd\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{2}\left(2acd+b\left(ae-bd\right)-\sqrt{-4ac+b^2}\left(ae-bd\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\left(ae^2-bde+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] d**(3/2)*atan(sqrt(e)*x/sqrt(d))/(sqrt(e)*(a*e**2 - b*d*e + c*d**
2)) + sqrt(2)*(2*a*c*d + b*(a*e - b*d) + sqrt(-4*a*c + b**2)*(a*e
 - b*d))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2
*sqrt(c)*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)*(a*e**
2 - b*d*e + c*d**2)) - sqrt(2)*(2*a*c*d + b*(a*e - b*d) - sqrt(-4
*a*c + b**2)*(a*e - b*d))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4
*a*c + b**2)))/(2*sqrt(c)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a
*c + b**2)*(a*e**2 - b*d*e + c*d**2))
```

Mathematica [A] time = 0.578539, size = 323, normalized size = 1.15

$$\frac{\left(bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(-ae^2+bde-cd^2\right)}+\frac{\left(bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}-abe-2acd+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\left(-ae^2+bde-cd^2\right)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2
 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c
]]*(-(c*d^2) + b*d*e - a*e^2)) + ((b^2*d - 2*a*c*d + b*Sqrt[b^2 -
 4*a*c]*d - a*b*e - a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*
a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (d
^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 - b*d*e + a*e
```

```
^2))
```

Maple [B] time = 0.034, size = 764, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*e-1/2/(a*e
^2-b*d*e+c*d^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c
*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d+1/2/(a*e^2-b*d*e
+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*e+c/(
a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*a*d-1/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*b^2*d-1/2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*a*e+1/2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*b*d+1/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*a*b*e+c/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*d-1/2/(a*e^2-b*d*e+c*d^2)/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d+d^2/(a*e^2-b
*d*e+c*d^2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.77168, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 -
 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^
4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*
c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a
^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c +
a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*
d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e
 + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3
- a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42
*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 -
 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^
4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c
^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*
a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a
*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-
2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x + sqrt(1/2)*((b
^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (
a^2*b^2 - 4*a^3*c)*d*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^
2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^
2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^
3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)
*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2
)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2
)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3
*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3
*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^
2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3
*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*
d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*
a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2
 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c
^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*
c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^
3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3
- a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a
*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b
^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c
^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^
5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 +
 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*
b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/(
(b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c
- 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*
e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e +
```

```
a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2
*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*
e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^
2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^
3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*
c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^
8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 -
 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*
e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e
^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^
2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2
- 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3
 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^
2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a
^2*b^2*c - 4*a^3*c^2)*e^4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^
2 - a^2*c)*d^2)*x - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3
- 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 - ((b^3
*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4
*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c -
3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*
d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4
*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*
e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(
b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*
c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (
b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*
(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c
^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4
*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2
 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*
a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2
 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^
2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a
*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2
- a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c
^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*
(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4
*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*
b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c
^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (
a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3
*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e
^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^
4))) + sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3
- 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d
^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2
*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*
a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c +
 a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)
*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*
e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3
 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 4
2*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3
- 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a
```

^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*
c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4
*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(
a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*log(
-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x + sqrt(1/2)*((
b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e +
(a^2*b^2 - 4*a^3*c)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c
^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a
^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e
^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2
)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^
2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^
2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(
3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^
3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b
^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^
3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)
*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4
*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^
2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*
c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3
*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a
^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3
 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*
a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*
b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*
c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c
^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5
 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3
*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/
((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c
 - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d
*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e
 + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^
2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3
*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a
^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e
^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b
*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d
^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5
 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5
*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*
e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a
^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2
 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^
3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b
^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (
a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b
^2 - a^2*c)*d^2)*x - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3
 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 + ((b^
3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^
4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c -
 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)
*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^

```
4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3
*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*
(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2
*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 +
(b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4
*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*
c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^
4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^
2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4
*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^
2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b
^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*
a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2
 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*
c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4
*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^
4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2
*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*
c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 +
(a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^
3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*
e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e
^4))) + d*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 +
d)))/(c*d^2 - b*d*e + a*e^2), 1/2*(sqrt(1/2)*(c*d^2 - b*d*e + a*e
^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*
d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e +
(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*
c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 -
a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d
^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 -
4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a
^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3
+ (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 -
 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^
4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*
a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4
*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^
2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b
^2*c - 4*a^3*c^2)*e^4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 -
a^2*c)*d^2)*x + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*
(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 - ((b^3*c^3
 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e +
 (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^
2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^
4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4
 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e -
2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*
c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)
*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*
c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b
^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 -
 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c
^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (
b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^
```

```
4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8
*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c
- 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2
*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^
3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*
d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5
*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3
 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*
c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 -
 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*
b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2
 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 -
 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)))
 - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*
a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 -
 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3
)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*
c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2
*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2
*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e +
2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a
*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^
2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12
*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c
^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2
- 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b
*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^
3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-2*(
2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x - sqrt(1/2)*((b^4
- 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2
*b^2 - 4*a^3*c)*d*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 -
 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b
*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 +
 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^
5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d
^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(
(b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^
4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^
4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c
^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*
c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2
*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5
*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 -
2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)
*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c -
 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b
*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a
^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^
7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*
c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)
*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*
d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*
(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3
*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^
```

```
2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2
*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3
 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) + sqrt(1/2)*(c*d^2 - b*d*e + a*
e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)
*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e +
 (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b
*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 -
 a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*
d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 -
 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*
a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3
 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4
- 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b
^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4
*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 -
4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c
^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*
b^2*c - 4*a^3*c^2)*e^4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 -
 a^2*c)*d^2)*x + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2
*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 + ((b^3*c^
3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e
+ (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a
^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e
^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^
4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e -
 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3
*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6
)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6
*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*
b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2
- 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*
c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 +
(b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c
^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 -
8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c
 - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^
2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a
^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)
*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^
5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^
3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3
*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3
- 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4
*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^
2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2
- 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))
) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3
*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4
- 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^
3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3
*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^
2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^
2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e +
 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 -
```

```
a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a
^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 1
2*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*
c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2
 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*
b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b
^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-2*
(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x - sqrt(1/2)*((b^4
 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^
2*b^2 - 4*a^3*c)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2
- 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*
b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3
+ 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e
^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*
d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/
((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b
^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c
^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*
c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b
*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^
2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^
5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 -
 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3
)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c
- 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*
b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 -
a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c
^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2
*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5
)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)
*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2
*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^
3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b
^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c -
2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^
3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) + 2*d*sqrt(d/e)*arctan(x/sqrt(
d/e)))/(c*d^2 - b*d*e + a*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.298 $\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)} dx$

Optimal. Leaf size=251

$$\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}+\frac{\sqrt{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}-\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{ae^2-bde+cd^2}$$

```
[Out] (Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (Sqrt[c]*(d + (b*d - 2*a*e)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e +
 a*e^2)) - (Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 -
 b*d*e + a*e^2)
```

Rubi [A] time = 0.870546, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}+\frac{\sqrt{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}-\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{ae^2-bde+cd^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (Sqrt[c]*(d + (b*d - 2*a*e)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e +
 a*e^2)) - (Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 -
 b*d*e + a*e^2)
```

Rubi in Sympy [A] time = 105.017, size = 255, normalized size = 1.02

$$-\frac{\sqrt{2}\sqrt{c}\left(2ae-bd-d\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\,(ae^2-bde+cd^2)}+\frac{\sqrt{2}\sqrt{c}\left(2ae-bd+d\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\,(ae^2-bde+cd^2)}-\frac{\sqrt{d}\sqrt{e}\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{ae^2-bde+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] -sqrt(2)*sqrt(c)*(2*a*e - b*d - d*sqrt(-4*a*c + b**2))*atan(sqrt(
2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*sqrt(b + sqrt(-4*a
*c + b**2))*sqrt(-4*a*c + b**2)*(a*e**2 - b*d*e + c*d**2)) + sqrt
(2)*sqrt(c)*(2*a*e - b*d + d*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sq
rt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*sqrt(b - sqrt(-4*a*c +
b**2))*sqrt(-4*a*c + b**2)*(a*e**2 - b*d*e + c*d**2)) - sqrt(d)*s
qrt(e)*atan(sqrt(e)*x/sqrt(d))/(a*e**2 - b*d*e + c*d**2)
```

Mathematica [A] time = 0.993148, size = 277, normalized size = 1.1

$$-\frac{\sqrt{c}\left(d\sqrt{b^2-4ac}+2ae-bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-ae^2+bde-cd^2)}-\frac{\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\,(-ae^2+bde-cd^2)}-\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{ae^2-bde+cd^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a
*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2))) - (S
qrt[c]*(b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqr
t[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) - (Sqrt[d]*S
qrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 - b*d*e + a*e^2)
```

Maple [B] time = 0.028, size = 478, normalized size = 1.9

$$\frac{c\sqrt{2}d}{2\,ae^2-2\,bde+2\,cd^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$-\frac{c\sqrt{2}ae}{ae^2-bde+cd^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}bd}{2\,ae^2-2\,bde+2\,cd^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$-\frac{c\sqrt{2}d}{2\,ae^2-2\,bde+2\,cd^2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$-\frac{c\sqrt{2}ae}{ae^2-bde+cd^2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}bd}{2\,ae^2-2\,bde+2\,cd^2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$-\frac{de}{ae^2-bde+cd^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*c/(a*e^2-b*d*e+c*d^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d-c/(a*e^2
-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*e
+1/2*c/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*b*d-1/2*c/(a*e^2-b*d*e+c*d^2)*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*d-c/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*a*e+1/2*c/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((
```

```
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d-e*d/(a*e^2-b*d*e+c*d^2)/(d*e
)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45329, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*
e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^
3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*
c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2
 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^
7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c
 - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*
b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c
^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 -
4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*
c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2
*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2
- 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x + sqrt(1/2)*((b^2*c
 - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 - (2*(b^2*c^3 - 4*a*c^4
)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 -
4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2
*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*
e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5
)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c
^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^
5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 -
 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10
*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 +
 (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2
```

```
+ ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 -
 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (
a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/
((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b
^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2
 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24
*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5
+ 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 -
 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^
3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c
^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e
^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*
d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*
d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*
b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e
^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*
d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5
*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^
2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b
*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6
- 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^
2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b
^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^
2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x - sqrt(1/2)*((b^2
*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 - (2*(b^2*c^3 - 4*a*c
^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2
- 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 +
 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c
)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c
^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2
*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*
d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4
 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 -
10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7
 + (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^
2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4
 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 +
 (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4
)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3
*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c
^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 -
24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^
5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3
 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*
c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2
*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)
*e^4))) + sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*
c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2
)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^
2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2
*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4
)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b
^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*
a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3
```

```
*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^
6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((
b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a
*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*
b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x + sqrt(1/2)*((b
^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 + (2*(b^2*c^3 - 4*a
*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^
2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3
 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b
*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a
*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b
^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3
)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e
^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4
- 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e
^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*
e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b
^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3
 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e
^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*
(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3
*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2
- 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*
e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b
^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*
a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a
^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*
c)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*
a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c
^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*
a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d
^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c
^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*
(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 4
2*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a
^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*
e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/
((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2
*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^
2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x - sqrt(1/2)*(
(b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 + (2*(b^2*c^3 - 4
*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*
c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e
^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3
*b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4
*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a
*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c
^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4
*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^
4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d
*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*
b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e +
(b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e
^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2
```

```
*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e +
2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b
^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^
2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^
3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3
*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 -
4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8
*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^
3*c)*e^4))) + sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2
+ d)))/(c*d^2 - b*d*e + a*e^2), 1/2*(sqrt(1/2)*(c*d^2 - b*d*e + a
*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)
*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2
)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4
)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d
^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3
- 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e
^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*
(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^
3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a
^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*
a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3
 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 -
 a*c*e^2)*x + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2
*c)*e^3 - (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^
4*e + 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^
3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)
*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2
 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^
7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c
 - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*
b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c
^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 -
4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-
(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^
3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 -
2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*
d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*
c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)
*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 +
8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2
*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*
a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a
^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3
*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c
)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e +
 a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^
3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c
^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e
^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)
*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^
3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5
*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 -
4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*
a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 +
```

(a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x - sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 - (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))) + sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 + (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*

```
c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^
6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 -
 a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c
- 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 -
 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)
*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2
*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))) - sqrt(1/2)*(c*d^2 - b*d
*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*
a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a
^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*
c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*
c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^
2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)
*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^
4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 -
 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^
7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*
c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*
(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2
*d^2 - a*c*e^2)*x - sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 -
 4*a^2*c)*e^3 + (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c
^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 +
2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^
3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d
^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c
^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*
(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 4
2*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a
^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*
e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*
sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 -
 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*
e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt
((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4
*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^
2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (
b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5
 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*
c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2
 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^
2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a
^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))) - 2*sqrt(d*e)*arctan(e
*x/sqrt(d*e)))/(c*d^2 - b*d*e + a*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.299 $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=254

$$-\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\sqrt{c}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2-bde+cd^2)}$$

```
[Out] -((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b
^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2))) - (Sqrt[c]*(e + (2*c*d - b
*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*
e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d
^2 - b*d*e + a*e^2))
```

Rubi [A] time = 1.07816, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\sqrt{c}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b
^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2))) - (Sqrt[c]*(e + (2*c*d - b
*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*
e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d
^2 - b*d*e + a*e^2))
```

Rubi in Sympy [A] time = 102.417, size = 255, normalized size = 1.

$$\frac{\sqrt{2}\sqrt{c}\left(be-2cd-e\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\,(ae^2-bde+cd^2)} - \frac{\sqrt{2}\sqrt{c}\left(be-2cd+e\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\,(ae^2-bde+cd^2)} + \frac{e^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2-bde+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] sqrt(2)*sqrt(c)*(b*e - 2*c*d - e*sqrt(-4*a*c + b**2))*atan(sqrt(2
)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*sqrt(b + sqrt(-4*a*
c + b**2))*sqrt(-4*a*c + b**2)*(a*e**2 - b*d*e + c*d**2)) - sqrt(
2)*sqrt(c)*(b*e - 2*c*d + e*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqr
t(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*sqrt(b - sqrt(-4*a*c + b
**2))*sqrt(-4*a*c + b**2)*(a*e**2 - b*d*e + c*d**2)) + e**(3/2)*a
tan(sqrt(e)*x/sqrt(d))/(sqrt(d)*(a*e**2 - b*d*e + c*d**2))
```

Mathematica [A] time = 0.424378, size = 274, normalized size = 1.08

$$\frac{\sqrt{c}\left(e\sqrt{b^2-4ac}+be-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-ae^2+bde-cd^2)} + \frac{\sqrt{c}\left(e\sqrt{b^2-4ac}-be+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\,(-ae^2+bde-cd^2)} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[c]*(-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (Sqrt[c
]*(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (e^(3/2)*ArcTan
[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Maple [B] time = 0.002, size = 480, normalized size = 1.9

$$-\frac{c\sqrt{2}e}{2\,ae^{2}-2\,bde+2\,cd^{2}}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}+\frac{c\sqrt{2}be}{2\,ae^{2}-2\,bde+2\,cd^{2}}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^{2}}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}-\frac{c^{2}\sqrt{2}d}{ae^{2}-bde+cd^{2}}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^{2}}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}+\frac{c\sqrt{2}e}{2\,ae^{2}-2\,bde+2\,cd^{2}}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}+\frac{c\sqrt{2}be}{2\,ae^{2}-2\,bde+2\,cd^{2}}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^{2}}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}-\frac{c^{2}\sqrt{2}d}{ae^{2}-bde+cd^{2}}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^{2}}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}+\frac{e^{2}}{ae^{2}-bde+cd^{2}}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*c/(a*e^2-b*d*e+c*d^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e+1/2*c/(
a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*b*e-c^2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*d+1/2*c/(a*e^2-b*d*e+c*d^2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*e+1/2*c/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*b*e-c^2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)
```

```
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d+e^2/(a*e^2-b*d*e+c*d^2)/(d*
e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.300 $\int \frac{1}{x^2(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=298

$$-\frac{\sqrt{c}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\sqrt{c}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(ae^2-bde+cd^2)}-\frac{1}{adx}$$

```
[Out] -(1/(a*d*x)) - (Sqrt[c]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e +
a*e^2)) - (Sqrt[c]*(c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2
)) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 - b*d*
e + a*e^2))
```

Rubi [A] time = 1.8006, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{\sqrt{c}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\sqrt{c}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(ae^2-bde+cd^2)}-\frac{1}{adx}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```


```
[Out] -(1/(a*d*x)) - (Sqrt[c]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e +
a*e^2)) - (Sqrt[c]*(c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2
)) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 - b*d*
e + a*e^2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.774324, size = 340, normalized size = 1.14

$$-\frac{\sqrt{c}\left(cd\sqrt{b^2-4ac}-be\sqrt{b^2-4ac}+2ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(e(ae-bd)+cd^2\right)}+\frac{\sqrt{c}\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}+2ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\left(e(ae-bd)+cd^2\right)}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(ae^2-bde+cd^2\right)}-\frac{1}{adx}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*d*x)) - (Sqrt[c]*(b*c*d + c*Sqrt[b^2 - 4*a*c]*d - b^2*e +
2*a*c*e - b*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sq
rt[b^2 - 4*a*c]]*(c*d^2 + e*(-(b*d) + a*e))) + (Sqrt[c]*(b*c*d -
c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e)*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-(b*
d) + a*e))) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d
^2 - b*d*e + a*e^2))
```

Maple [B] time = 0.039, size = 817, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)/a*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e-1/2/
(a*e^2-b*d*e+c*d^2)/a*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d+1/(a*e^2
-b*d*e+c*d^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*e-1/2/(a*e^2-b*d*e+c*d^2)/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2))*b^2*e+1/2/(a*e^2-b*d*e+c*d^2)/a*c^2/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d-1/2/(a*e^2-b*d*e+c*d^2)/a*c*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e+1/2/(a*e^2-b*d*e+c*d^2)/a*c^2
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d+1/(a*e^2-b*d*e+c*d^2)*c^2/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e-1/2/(a*e^2-b*d*e
+c*d^2)/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^
2*e+1/2/(a*e^2-b*d*e+c*d^2)/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*b*d-1/d*e^3/(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*a
rctan(x*e/(d*e)^(1/2))-1/a/d/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.301 $\int \frac{1}{x^4(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=348

$$\frac{\sqrt{c}\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}+\frac{\sqrt{c}\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{ae+bd}{a^2d^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\,(ae^2-bde+cd^2)}-\frac{1}{3adx^3}$$

```
[Out] -1/(3*a*d*x^3) + (b*d + a*e)/(a^2*d^2*x) + (Sqrt[c]*(b*c*d - b^2*
e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2))
 + (Sqrt[c]*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e
 + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]
*(c*d^2 - b*d*e + a*e^2)) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(d^(5/2)*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 3.32743, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{\sqrt{c}\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}+\frac{\sqrt{c}\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{ae+bd}{a^2d^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\,(ae^2-bde+cd^2)}-\frac{1}{3adx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(3*a*d*x^3) + (b*d + a*e)/(a^2*d^2*x) + (Sqrt[c]*(b*c*d - b^2*
e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 -
```

```
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2))
 + (Sqrt[c]*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e
 + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]
*(c*d^2 - b*d*e + a*e^2)) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(d^(5/2)*(c*d^2 - b*d*e + a*e^2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.993453, size = 410, normalized size = 1.18

$$\frac{\sqrt{c}\left(b^2\left(cd-e\sqrt{b^2-4ac}\right)+bc\left(d\sqrt{b^2-4ac}+3ae\right)+ac\left(e\sqrt{b^2-4ac}-2cd\right)+b^3(-e)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(e(ae-bd)+cd^2\right)}+\frac{\sqrt{c}\left(-b^2\left(e\sqrt{b^2-4ac}+cd\right)+bc\left(d\sqrt{b^2-4ac}-3ae\right)+ac\left(e\sqrt{b^2-4ac}+2cd\right)+b^3e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\left(e(ae-bd)+cd^2\right)}+\frac{ae+bd}{a^2d^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(ae^2-bde+cd^2\right)}-\frac{1}{3adx^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(3*a*d*x^3) + (b*d + a*e)/(a^2*d^2*x) + (Sqrt[c]*(-(b^3*e) + b
*c*(Sqrt[b^2 - 4*a*c]*d + 3*a*e) + b^2*(c*d - Sqrt[b^2 - 4*a*c]*e
) + a*c*(-2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sq
rt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-(b*d) + a*e))) + (Sqrt[c]*
(b^3*e + b*c*(Sqrt[b^2 - 4*a*c]*d - 3*a*e) - b^2*(c*d + Sqrt[b^2
- 4*a*c]*e) + a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2 -
4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-(b*d) + a*e))) +
```

```
(e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*(c*d^2 - b*d*e + a
*e^2))
```

Maple [B] time = 0.048, size = 1160, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)/a*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e-1/2/
(a*e^2-b*d*e+c*d^2)/a^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e+1/2/
(a*e^2-b*d*e+c*d^2)/a^2*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d-3/2/
(a*e^2-b*d*e+c*d^2)/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*b*e+1/(a*e^2-b*d*e+c*d^2)/a*c^3/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*d+1/2/(a*e^2-b*d*e+c*d^2)/a^2*c/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*e-1/2/(a*e^2-b*d*e+c*d
^2)/a^2*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d
-1/2/(a*e^2-b*d*e+c*d^2)/a*c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e+
1/2/(a*e^2-b*d*e+c*d^2)/a^2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2
*e-1/2/(a*e^2-b*d*e+c*d^2)/a^2*c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*b*d-3/2/(a*e^2-b*d*e+c*d^2)/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*b*e+1/(a*e^2-b*d*e+c*d^2)/a*c^3/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d+1/2/(a*e^2-b*d*e+c*d^2)
/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*e-1
/2/(a*e^2-b*d*e+c*d^2)/a^2*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b^2*d-1/3/a/d/x^3+e/a/d^2/x+1/x/a^2/d*b+1/d^2*e^
4/(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.302 $\int \frac{1}{\sqrt{fx}(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=866

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}\right)e^{7/4}}{\sqrt{2}d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}+1\right)e^{7/4}}{\sqrt{2}d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}$$
$$-\frac{\log\left(\sqrt{e}\sqrt{f}x+\sqrt{d}\sqrt{f}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)e^{7/4}}{2\sqrt{2}d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}+\frac{\log\left(\sqrt{e}\sqrt{f}x+\sqrt{d}\sqrt{f}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)e^{7/4}}{2\sqrt{2}d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}$$
$$+\frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}$$
$$-\frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}$$
$$+\frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}$$
$$-\frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}$$

```
[Out] (c^(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2^(1/4)*c^(1
/4)*Sqrt[f*x])/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])]])/(2^(1/4
)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e
 + a*e^2)*Sqrt[f]) - (c^(3/4)*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)
*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b + Sqrt[b^2 - 4*a*c])^(1/
4)*Sqrt[f])]])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])
^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (e^(7/4)*ArcTan[1 - (Sq
rt[2]*e^(1/4)*Sqrt[f*x])/(d^(1/4)*Sqrt[f])]])/(Sqrt[2]*d^(3/4)*(c*
d^2 - b*d*e + a*e^2)*Sqrt[f]) + (e^(7/4)*ArcTan[1 + (Sqrt[2]*e^(1
/4)*Sqrt[f*x])/(d^(1/4)*Sqrt[f])]])/(Sqrt[2]*d^(3/4)*(c*d^2 - b*d*
e + a*e^2)*Sqrt[f]) + (c^(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b - Sqrt[b^2 - 4*a*c])^(
1/4)*Sqrt[f])]])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c
])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (c^(3/4)*(2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b
```

```
+ Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*
(-b + Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) -
 (e^(7/4)*Log[Sqrt[d]*Sqrt[f] + Sqrt[e]*Sqrt[f]*x - Sqrt[2]*d^(1/
4)*e^(1/4)*Sqrt[f*x]])/(2*Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)
*Sqrt[f]) + (e^(7/4)*Log[Sqrt[d]*Sqrt[f] + Sqrt[e]*Sqrt[f]*x + Sq
rt[2]*d^(1/4)*e^(1/4)*Sqrt[f*x]])/(2*Sqrt[2]*d^(3/4)*(c*d^2 - b*d
*e + a*e^2)*Sqrt[f])
```

Rubi [A] time = 4.77455, antiderivative size = 866, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}\right)e^{7/4}}{\sqrt{2}d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}+1\right)e^{7/4}}{\sqrt{2}d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}$$
$$-\frac{\log\left(\sqrt{e}\sqrt{f}x+\sqrt{d}\sqrt{f}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)e^{7/4}}{2\sqrt{2}d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}+\frac{\log\left(\sqrt{e}\sqrt{f}x+\sqrt{d}\sqrt{f}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)e^{7/4}}{2\sqrt{2}d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}$$
$$+\frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}$$
$$-\frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}$$
$$+\frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}$$
$$-\frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (c^(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2^(1/4)*c^(1
/4)*Sqrt[f*x])/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4
```

```
)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e
 + a*e^2)*Sqrt[f]) - (c^(3/4)*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)
*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b + Sqrt[b^2 - 4*a*c])^(1/
4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])
^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (e^(7/4)*ArcTan[1 - (Sq
rt[2]*e^(1/4)*Sqrt[f*x])/(d^(1/4)*Sqrt[f])])/(Sqrt[2]*d^(3/4)*(c*
d^2 - b*d*e + a*e^2)*Sqrt[f]) + (e^(7/4)*ArcTan[1 + (Sqrt[2]*e^(1
/4)*Sqrt[f*x])/(d^(1/4)*Sqrt[f])])/(Sqrt[2]*d^(3/4)*(c*d^2 - b*d*
e + a*e^2)*Sqrt[f]) + (c^(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b - Sqrt[b^2 - 4*a*c])^(
1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c
])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (c^(3/4)*(2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b
+ Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*
(-b + Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) -
 (e^(7/4)*Log[Sqrt[d]*Sqrt[f] + Sqrt[e]*Sqrt[f]*x - Sqrt[2]*d^(1/
4)*e^(1/4)*Sqrt[f*x]])/(2*Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)
*Sqrt[f]) + (e^(7/4)*Log[Sqrt[d]*Sqrt[f] + Sqrt[e]*Sqrt[f]*x + Sq
rt[2]*d^(1/4)*e^(1/4)*Sqrt[f*x]])/(2*Sqrt[2]*d^(3/4)*(c*d^2 - b*d
*e + a*e^2)*Sqrt[f])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)/(f*x)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.514124, size = 267, normalized size = 0.31

$$\frac{\sqrt{x}\left(\sqrt{2}e^{7/4}\left(-\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{d}+\sqrt{e}x\right)+\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{d}+\sqrt{e}x\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\right.\right.\right.}{\sqrt[4]{d}}\right.}{4d^{3/4}\sqrt{fx}\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[x]*(Sqrt[2]*e^(7/4)*(-2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x]
)/d^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - Lo
g[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] + Log[Sq
```

```
rt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]) - 2*d^(3/4)
*RootSum[a + b*#1^4 + c*#1^8 & , (-(c*d*Log[Sqrt[x] - #1]) + b*e*
Log[Sqrt[x] - #1] + c*e*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^
7) & ]))/(4*d^(3/4)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[f*x])
```

Maple [C] time = 0.103, size = 336, normalized size = 0.4

$$\frac{e^2\sqrt{2}}{4f(ae^2-bde+cd^2)d}\sqrt[4]{\frac{df^2}{e}}\ln\left(1\left(fx+\sqrt[4]{\frac{df^2}{e}}\sqrt{fx}\sqrt{2}+\sqrt{\frac{df^2}{e}}\right)\left(fx-\sqrt[4]{\frac{df^2}{e}}\sqrt{fx}\sqrt{2}+\sqrt{\frac{df^2}{e}}\right)^{-1}\right)$$
$$+\frac{e^2\sqrt{2}}{2f(ae^2-bde+cd^2)d}\sqrt[4]{\frac{df^2}{e}}\arctan\left(\sqrt{2}\sqrt{fx}\frac{1}{\sqrt[4]{\frac{df^2}{e}}}+1\right)$$
$$+\frac{e^2\sqrt{2}}{2f(ae^2-bde+cd^2)d}\sqrt[4]{\frac{df^2}{e}}\arctan\left(\sqrt{2}\sqrt{fx}\frac{1}{\sqrt[4]{\frac{df^2}{e}}}-1\right)$$
$$+\frac{f}{2ae^2-2bde+2cd^2}\sum_{\_R=RootOf(c\_Z^8+bf^2\_Z^4+af^4)}\frac{-\_R^4ce-bef^2+cdf^2}{2\_R^7c+\_R^3bf^2}\ln\left(\sqrt{fx}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x)
```

```
[Out] 1/4/f*e^2/(a*e^2-b*d*e+c*d^2)*(d*f^2/e)^(1/4)/d*2^(1/2)*ln((f*x+(
d*f^2/e)^(1/4)*(f*x)^(1/2)*2^(1/2)+(d*f^2/e)^(1/2))/(f*x-(d*f^2/e
)^(1/4)*(f*x)^(1/2)*2^(1/2)+(d*f^2/e)^(1/2)))+1/2/f*e^2/(a*e^2-b*
d*e+c*d^2)*(d*f^2/e)^(1/4)/d*2^(1/2)*arctan(2^(1/2)/(d*f^2/e)^(1/
4)*(f*x)^(1/2)+1)+1/2/f*e^2/(a*e^2-b*d*e+c*d^2)*(d*f^2/e)^(1/4)/d
*2^(1/2)*arctan(2^(1/2)/(d*f^2/e)^(1/4)*(f*x)^(1/2)-1)+1/2*f/(a*e
^2-b*d*e+c*d^2)*sum((-_R^4*c*e-b*e*f^2+c*d*f^2)/(2*_R^7*c+_R^3*b*
f^2)*ln((f*x)^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b*f^2+a*f^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)/(f*x)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)(ex^2 + d)\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)), x)
```

3.303 $\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

Optimal. Leaf size=272

$$-\frac{\left(-8c^2de(bd-ae)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}e^4}+\frac{\sqrt{a+bx^2+cx^4}\left((2cd-be)(be+4cd)-2cex^2(be+2cd)\right)}{16c^2e^3}+\frac{d^2\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^4}+\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce}$$

```
[Out] (((2*c*d - b*e)*(4*c*d + b*e) - 2*c*e*(2*c*d + b*e)*x^2)*Sqrt[a +
 b*x^2 + c*x^4])/(16*c^2*e^3) + (a + b*x^2 + c*x^4)^(3/2)/(6*c*e)
 - ((16*c^3*d^3 - b^3*e^3 - 2*b*c*e^2*(b*d - 2*a*e) - 8*c^2*d*e*(
b*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x
^4])])/(32*c^(5/2)*e^4) + (d^2*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^4)
```

Rubi [A] time = 1.16697, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$-\frac{\left(-8c^2de(bd-ae)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}e^4}+\frac{\sqrt{a+bx^2+cx^4}\left((2cd-be)(be+4cd)-2cex^2(be+2cd)\right)}{16c^2e^3}+\frac{d^2\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^4}+\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```


```
[Out] (((2*c*d - b*e)*(4*c*d + b*e) - 2*c*e*(2*c*d + b*e)*x^2)*Sqrt[a +
 b*x^2 + c*x^4])/(16*c^2*e^3) + (a + b*x^2 + c*x^4)^(3/2)/(6*c*e)
 - ((16*c^3*d^3 - b^3*e^3 - 2*b*c*e^2*(b*d - 2*a*e) - 8*c^2*d*e*(
b*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x
^4])])/(32*c^(5/2)*e^4) + (d^2*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^4)
```

Rubi in Sympy [A] time = 104.923, size = 350, normalized size = 1.29

$$-\frac{b\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{16c^2e}+\frac{b\left(-4ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{\frac{5}{2}}e}+\frac{d^2\sqrt{a+bx^2+cx^4}}{2e^3}-\frac{d^2\sqrt{ae^2-bde+cd^2}\operatorname{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^4}-\frac{d\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8ce^2}+\frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{6ce}+\frac{d^2(be-2cd)\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}e^4}+\frac{d\left(-4ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{\frac{3}{2}}e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] -b*(b + 2*c*x**2)*sqrt(a + b*x**2 + c*x**4)/(16*c**2*e) + b*(-4*a
*c + b**2)*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x*
*4)))/(32*c**(5/2)*e) + d**2*sqrt(a + b*x**2 + c*x**4)/(2*e**3) -
 d**2*sqrt(a*e**2 - b*d*e + c*d**2)*atanh((2*a*e - b*d + x**2*(b*
e - 2*c*d))/(2*sqrt(a + b*x**2 + c*x**4)*sqrt(a*e**2 - b*d*e + c*
d**2)))/(2*e**4) - d*(b + 2*c*x**2)*sqrt(a + b*x**2 + c*x**4)/(8*
c*e**2) + (a + b*x**2 + c*x**4)**(3/2)/(6*c*e) + d**2*(b*e - 2*c*
d)*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(4
*sqrt(c)*e**4) + d*(-4*a*c + b**2)*atanh((b + 2*c*x**2)/(2*sqrt(c
)*sqrt(a + b*x**2 + c*x**4)))/(16*c**(3/2)*e**2)
```

Mathematica [A] time = 1.02148, size = 301, normalized size = 1.11

$$-3\left(8c^2de(ae-bd)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3\right)\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)+2\sqrt{c}\left(e\sqrt{a+bx^2+cx^4}\left(2ce\left(\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] (48*c^(5/2)*d^2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Log[d + e*x^2] - 3
*(16*c^3*d^3 - b^3*e^3 - 2*b*c*e^2*(b*d - 2*a*e) + 8*c^2*d*e*(-(b
*d) + a*e))*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]
+ 2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2*e^2 + 2*c*e*(-3*b*
d + 4*a*e + b*e*x^2) + 4*c^2*(6*d^2 - 3*d*e*x^2 + 2*e^2*x^4)) - 2
4*c^2*d^2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Log[-(b*d) + 2*a*e - 2*c
*d*x^2 + b*e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 +
 c*x^4]]))/(96*c^(5/2)*e^4)
```

Maple [B] time = 0.03, size = 1049, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x)
```

```
[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)/c/e-1/8/e*b/c*(c*x^4+b*x^2+a)^(1/2)*x^2
-1/16/e*b^2/c^2*(c*x^4+b*x^2+a)^(1/2)-1/8/e*b/c^(3/2)*ln((1/2*b+c
*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a+1/32/e*b^3/c^(5/2)*ln((1/2
*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/4/e^2*d*(c*x^4+b*x^2+a
)^(1/2)*x^2-1/8/e^2*d/c*(c*x^4+b*x^2+a)^(1/2)*b-1/4/e^2*d/c^(1/2)
*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a+1/16/e^2*d/c^(
3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b^2+1/2*d^2/
e^3*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)+1/4*d^2/e^3*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(
(x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2))/c^(1/2)*b-1/2*d^3/e^4*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(
1/2)+((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))*c^(1/2)-1/2*d^2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln
((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^
2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a+1/2*d^3/e^4/((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x
^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c
*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b-1/2*
d^4/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)
/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((
x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2))/(x^2+d/e))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)*x^5/(e*x^2 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)*x^5/(e*x^2 + d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2+a}\,x^5}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)*x^5/(e*x^2 + d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*x^5/(e*x^2 + d), x)
```

3.304 $\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

Optimal. Leaf size=208

$$\frac{\left(-4ce(bd-ae)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}e^3}-\frac{d\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^3}-\frac{\sqrt{a+bx^2+cx^4}\left(-be+4cd-2cex^2\right)}{8ce^2}$$

```
[Out] -((4*c*d - b*e - 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c*e^2) +
((8*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - a*e))*ArcTanh[(b + 2*c*x^2)/
(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(3/2)*e^3) - (d*Sqrt[
c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/
(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^3)
```

Rubi [A] time = 0.713101, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$\frac{\left(-4ce(bd-ae)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}e^3}-\frac{d\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^3}-\frac{\sqrt{a+bx^2+cx^4}\left(-be+4cd-2cex^2\right)}{8ce^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] -((4*c*d - b*e - 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c*e^2) +
((8*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - a*e))*ArcTanh[(b + 2*c*x^2)/
(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(3/2)*e^3) - (d*Sqrt[
c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/
(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^3)
```

Rubi in Sympy [A] time = 68.1067, size = 190, normalized size = 0.91

$$\frac{d\sqrt{ae^2-bde+cd^2}\,\operatorname{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^3}+\frac{\sqrt{a+bx^2+cx^4}\left(\frac{be}{2}-2cd+cex^2\right)}{4ce^2}-\frac{\left(b^2e^2-8c^2d^2-4ce\left(ae-bd\right)\right)\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{\frac{3}{2}}e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] d*sqrt(a*e**2 - b*d*e + c*d**2)*atanh((2*a*e - b*d + x**2*(b*e -
2*c*d))/(2*sqrt(a + b*x**2 + c*x**4)*sqrt(a*e**2 - b*d*e + c*d**2
)))/(2*e**3) + sqrt(a + b*x**2 + c*x**4)*(b*e/2 - 2*c*d + c*e*x**
2)/(4*c*e**2) - (b**2*e**2 - 8*c**2*d**2 - 4*c*e*(a*e - b*d))*ata
nh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(16*c**(
3/2)*e**3)
```

Mathematica [A] time = 0.557578, size = 237, normalized size = 1.14

$$\frac{(4ce(ae-bd)-b^2e^2+8c^2d^2)\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)-8c^{3/2}d\log\left(d+ex^2\right)\sqrt{e(ae-bd)+cd^2}+2\sqrt{c}\left(4cd\sqrt{a\right.}{16c^{3/}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] (-8*c^(3/2)*d*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Log[d + e*x^2] + (8*
c^2*d^2 - b^2*e^2 + 4*c*e*(-(b*d) + a*e))*Log[b + 2*c*x^2 + 2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4]] + 2*Sqrt[c]*(e*(-4*c*d + b*e + 2*c*
e*x^2)*Sqrt[a + b*x^2 + c*x^4] + 4*c*d*Sqrt[c*d^2 - b*d*e + a*e^2
]*Log[-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2 + 2*Sqrt[c*d^2 - b*d*e
 + a*e^2]*Sqrt[a + b*x^2 + c*x^4]]))/(16*c^(3/2)*e^3)
```

Maple [B] time = 0.015, size = 887, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x)
```

```
[Out] 1/4/e*(c*x^4+b*x^2+a)^(1/2)*x^2+1/8/e/c*(c*x^4+b*x^2+a)^(1/2)*b+1
/4/e/c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a-1/
16/e/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b^2-
1/2*d/e^2*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)-1/4*d/e^2*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/
2)+((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))/c^(1/2)*b+1/2*d^2/e^3*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))
/c^(1/2)+((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^
```

```
2)/e^2)^(1/2))*c^(1/2)+1/2*d/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a-1/2*d^2/e^3/((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*
(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2
*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b+1/
2*d^3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^
2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2))/(x^2+d/e))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)*x^3/(e*x^2 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 18.5122, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)*x^3/(e*x^2 + d),x, algorithm="fricas")
```

```
[Out] [1/32*(8*sqrt(c*d^2 - b*d*e + a*e^2)*c^(3/2)*d*log(-((8*c^2*d^2 -
 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^
2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*
x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c
*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + 4*(2
*c*e^2*x^2 - 4*c*d*e + b*e^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(c) + (
8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*log(-4*sqrt(c*x^4 + b*
x^2 + a)*(2*c^2*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c
)*sqrt(c)))/(c^(3/2)*e^3), 1/32*(16*sqrt(-c*d^2 + b*d*e - a*e^2)*
c^(3/2)*d*arctan(-1/2*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(sqrt(c*x
^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2))) + 4*(2*c*e^2*x^2 -
 4*c*d*e + b*e^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(c) + (8*c^2*d^2 -
4*b*c*d*e - (b^2 - 4*a*c)*e^2)*log(-4*sqrt(c*x^4 + b*x^2 + a)*(2*
c^2*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)))/
(c^(3/2)*e^3), 1/16*(4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(-c)*c*d*l
```

```
og(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e
+ 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b
^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d
*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x
^2 + d^2)) + 2*(2*c*e^2*x^2 - 4*c*d*e + b*e^2)*sqrt(c*x^4 + b*x^2
 + a)*sqrt(-c) + (8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*arct
an(1/2*(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a)*c)))/(sqrt
(-c)*c*e^3), 1/16*(8*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(-c)*c*d*ar
ctan(-1/2*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(sqrt(c*x^4 + b*x^2 +
 a)*sqrt(-c*d^2 + b*d*e - a*e^2))) + 2*(2*c*e^2*x^2 - 4*c*d*e + b
*e^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c) + (8*c^2*d^2 - 4*b*c*d*e -
 (b^2 - 4*a*c)*e^2)*arctan(1/2*(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4
 + b*x^2 + a)*c)))/(sqrt(-c)*c*e^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x^3/(e*x^2 + d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.305 $\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

Optimal. Leaf size=168

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}e^2}+\frac{\sqrt{a+bx^2+cx^4}}{2e}$$

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*e) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x
^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*e^2) + (Sqrt
[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)
/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^2
)
```

Rubi [A] time = 0.494487, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}e^2}+\frac{\sqrt{a+bx^2+cx^4}}{2e}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*e) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x
^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*e^2) + (Sqrt
[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)
/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^2
)
```

Rubi in Sympy [A] time = 61.5061, size = 150, normalized size = 0.89

$$\frac{\sqrt{a+bx^2+cx^4}}{2e}-\frac{\sqrt{ae^2-bde+cd^2}\,\text{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2}+\frac{(be-2cd)\,\text{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] sqrt(a + b*x**2 + c*x**4)/(2*e) - sqrt(a*e**2 - b*d*e + c*d**2)*a
tanh((2*a*e - b*d + x**2*(b*e - 2*c*d))/(2*sqrt(a + b*x**2 + c*x*
*4)*sqrt(a*e**2 - b*d*e + c*d**2)))/(2*e**2) + (b*e - 2*c*d)*atan
h((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(4*sqrt(c
)*e**2)
```

Mathematica [A] time = 0.245834, size = 199, normalized size = 1.18

$$\frac{\log\left(d+ex^2\right)\sqrt{ae^2-bde+cd^2}}{2e^2} - \frac{\sqrt{ae^2-bde+cd^2}\log\left(2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex^2-2cdx^2\right)}{2e^2} + \frac{(be-2cd)\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{4\sqrt{c}e^2} + \frac{\sqrt{a+bx^2+cx^4}}{2e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*e) + (Sqrt[c*d^2 - b*d*e + a*e^2]*Log[
d + e*x^2])/(2*e^2) + ((-2*c*d + b*e)*Log[b + 2*c*x^2 + 2*Sqrt[c]
*Sqrt[a + b*x^2 + c*x^4]])/(4*Sqrt[c]*e^2) - (Sqrt[c*d^2 - b*d*e
+ a*e^2]*Log[-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2 + 2*Sqrt[c*d^2
- b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]])/(2*e^2)
```

Maple [B] time = 0.009, size = 757, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x)
```

```
[Out] 1/2/e*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)+1/4/e*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+((x^2
+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))
/c^(1/2)*b-1/2/e^2*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+((x
^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
))*c^(1/2)*d-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b
```

```
*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2))/(x^2+d/e))*a+1/2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^
2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*d-1/2/e^3/((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e
*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-
2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c*d
^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)*x/(e*x^2 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.2625, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)*x/(e*x^2 + d),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c)*e - (2*c*d - b*e)*log(-4*
sqrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^
2 + b^2 + 4*a*c)*sqrt(c)) + 2*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c)
*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*
e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3
*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b
*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e
*x^2 + d^2)))/(sqrt(c)*e^2), 1/4*(2*sqrt(c*x^4 + b*x^2 + a)*sqrt(
-c)*e - (2*c*d - b*e)*arctan(1/2*(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x
^4 + b*x^2 + a)*c)) + sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(-c)*log(-(
(8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a
^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 +
4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e +
a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 +
d^2)))/(sqrt(-c)*e^2), 1/8*(4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c)*e -
 4*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c)*arctan(-1/2*((2*c*d - b*e
```

```
)*x^2 + b*d - 2*a*e)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e
 - a*e^2))) - (2*c*d - b*e)*log(-4*sqrt(c*x^4 + b*x^2 + a)*(2*c^2
*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)))/(sq
rt(c)*e^2), 1/4*(2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c)*e - 2*sqrt(-c
*d^2 + b*d*e - a*e^2)*sqrt(-c)*arctan(-1/2*((2*c*d - b*e)*x^2 + b
*d - 2*a*e)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)
)) - (2*c*d - b*e)*arctan(1/2*(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4
+ b*x^2 + a)*c)))/(sqrt(-c)*e^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{a + bx^2 + cx^4}}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(x*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x/(e*x^2 + d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.306 $\int \frac{\sqrt{a+bx^2+cx^4}}{x(d+ex^2)}\,dx$

Optimal. Leaf size=186

$$-\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2de}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2e}$$

```
[Out] -(Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4
])])/(2*d) + (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b
*x^2 + c*x^4])])/(2*e) - (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*
d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqr
t[a + b*x^2 + c*x^4])])/(2*d*e)
```

Rubi [A] time = 0.614251, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$-\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2de}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2 + c*x^4]/(x*(d + e*x^2)),x]
```

```
[Out] -(Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4
])])/(2*d) + (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b
*x^2 + c*x^4])])/(2*e) - (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*
d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqr
t[a + b*x^2 + c*x^4])])/(2*d*e)
```

Rubi in Sympy [A] time = 72.8193, size = 162, normalized size = 0.87

$$-\frac{\sqrt{a}\,\text{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\sqrt{c}\,\text{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2e}+\frac{\sqrt{ae^2-bde+cd^2}\,\text{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(1/2)/x/(e*x**2+d),x)
```

```
[Out] -sqrt(a)*atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**4
)))/(2*d) + sqrt(c)*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x*
*2 + c*x**4)))/(2*e) + sqrt(a*e**2 - b*d*e + c*d**2)*atanh((2*a*e
 - b*d + x**2*(b*e - 2*c*d))/(2*sqrt(a + b*x**2 + c*x**4)*sqrt(a*
e**2 - b*d*e + c*d**2)))/(2*d*e)
```

Mathematica [A] time = 0.793336, size = 214, normalized size = 1.15

$$\frac{-\log\left(d+ex^2\right)\sqrt{ae^2-bde+cd^2}+\sqrt{ae^2-bde+cd^2}\log\left(2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex^2-2cdx^2\right)+}{2de}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2 + c*x^4]/(x*(d + e*x^2)),x]
```

```
[Out] (Sqrt[a]*e*Log[x^2] - Sqrt[c*d^2 - b*d*e + a*e^2]*Log[d + e*x^2]
- Sqrt[a]*e*Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]]
+ Sqrt[c]*d*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]
+ Sqrt[c*d^2 - b*d*e + a*e^2]*Log[-(b*d) + 2*a*e - 2*c*d*x^2 + b*
e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]])/(
2*d*e)
```

Maple [B] time = 0.016, size = 851, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x)
```

```
[Out] 1/2/d*(c*x^4+b*x^2+a)^(1/2)+1/4/d*b*ln((1/2*b+c*x^2)/c^(1/2)+(c*x
^4+b*x^2+a)^(1/2))/c^(1/2)-1/2/d*a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*
(c*x^4+b*x^2+a)^(1/2))/x^2)-1/2/d*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x
^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/4/d*ln((1/2*(b*e-2*c*d)/
e+c*(x^2+d/e))/c^(1/2)+((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b+1/2/e*ln((1/2*(b*e-2*c*d)/
e+c*(x^2+d/e))/c^(1/2)+((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)+1/2/d/((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2
```

```
*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^
2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a-1/2/e/((a*e^2
-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d
)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b
*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*
b+1/2*d/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*
d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2))/(x^2+d/e))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{(ex^2 + d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x), x)
```

Fricas [A] time = 73.2405, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(c)*d*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 +
 b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + sqrt(a)*e*log(-((b^2
 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*
a)*sqrt(a) + 8*a^2)/x^4) + sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c
^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e
^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*
c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^
2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)
))/(d*e), 1/4*(2*sqrt(-c)*d*arctan(1/2*(2*c*x^2 + b)/(sqrt(c*x^4
+ b*x^2 + a)*sqrt(-c))) + sqrt(a)*e*log(-((b^2 + 4*a*c)*x^4 + 8*a
*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)
/x^4) + sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e
+ (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*
d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqr
t(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x
^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)))/(d*e), 1/4*(sqrt
```

```
(c)*d*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a
)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + sqrt(a)*e*log(-((b^2 + 4*a*c)*
x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a)
 + 8*a^2)/x^4) + 2*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*((2*c
*d - b*e)*x^2 + b*d - 2*a*e)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2
 + b*d*e - a*e^2))))/(d*e), 1/4*(2*sqrt(-c)*d*arctan(1/2*(2*c*x^2
 + b)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(-c))) + sqrt(a)*e*log(-((b^2
+ 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a
)*sqrt(a) + 8*a^2)/x^4) + 2*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-
1/2*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(sqrt(c*x^4 + b*x^2 + a)*sq
rt(-c*d^2 + b*d*e - a*e^2))))/(d*e), -1/4*(2*sqrt(-a)*e*arctan(1/
2*(b*x^2 + 2*a)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(-a))) - sqrt(c)*d*l
og(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*
x^2 + b)*sqrt(c) - 4*a*c) - sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*
c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*
e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a
*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e
^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2
)))/(d*e), -1/4*(2*sqrt(-a)*e*arctan(1/2*(b*x^2 + 2*a)/(sqrt(c*x^
4 + b*x^2 + a)*sqrt(-a))) - 2*sqrt(-c)*d*arctan(1/2*(2*c*x^2 + b)
/(sqrt(c*x^4 + b*x^2 + a)*sqrt(-c))) - sqrt(c*d^2 - b*d*e + a*e^2
)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d
*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (
3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 -
b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*
e*x^2 + d^2)))/(d*e), -1/4*(2*sqrt(-a)*e*arctan(1/2*(b*x^2 + 2*a)
/(sqrt(c*x^4 + b*x^2 + a)*sqrt(-a))) - sqrt(c)*d*log(-8*c^2*x^4 -
 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c
) - 4*a*c) - 2*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*((2*c*d -
 b*e)*x^2 + b*d - 2*a*e)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b
*d*e - a*e^2))))/(d*e), -1/2*(sqrt(-a)*e*arctan(1/2*(b*x^2 + 2*a)
/(sqrt(c*x^4 + b*x^2 + a)*sqrt(-a))) - sqrt(-c)*d*arctan(1/2*(2*c
*x^2 + b)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(-c))) - sqrt(-c*d^2 + b*d
*e - a*e^2)*arctan(-1/2*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(sqrt(c
*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2))))/(d*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x\,(d + ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(1/2)/x/(e*x**2+d),x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/(x*(d + e*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{(ex^2 + d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x), x)
```

3.307 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^3(d+ex^2)}\,dx$

Optimal. Leaf size=361

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2}+\frac{\sqrt{a}e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}-\frac{be\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d^2}-\frac{\sqrt{a+bx^2+cx^4}}{2dx^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2d}$$

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*d*x^2) - (b*ArcTanh[(2*a + b*x^2)/(2*
Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[a]*d) + (Sqrt[a]*e*Arc
Tanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2)
+ (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(2*d) - (b*e*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^
2 + c*x^4])])/(4*Sqrt[c]*d^2) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x
^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d^2) + (Sqrt
[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)
/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2
)
```

Rubi [A] time = 1.1858, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2}+\frac{\sqrt{a}e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}-\frac{be\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d^2}-\frac{\sqrt{a+bx^2+cx^4}}{2dx^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2 + c*x^4]/(x^3*(d + e*x^2)),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*d*x^2) - (b*ArcTanh[(2*a + b*x^2)/(2*
Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[a]*d) + (Sqrt[a]*e*Arc
Tanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2)
+ (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
```

```
4])])/(2*d) - (b*e*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^
2 + c*x^4])])/(4*Sqrt[c]*d^2) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x
^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d^2) + (Sqrt
[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)
/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2
)
```

Rubi in Sympy [A] time = 125.688, size = 326, normalized size = 0.9

$$\frac{\sqrt{a}e\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}-\frac{be\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d^2}+\frac{\sqrt{c}\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2d}-\frac{\sqrt{a+bx^2+cx^4}}{2dx^2}-\frac{\sqrt{ae^2-bde+cd^2}\operatorname{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2}+\frac{(be-2cd)\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d^2}-\frac{b\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(1/2)/x**3/(e*x**2+d),x)
```

```
[Out] sqrt(a)*e*atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**
4)))/(2*d**2) - b*e*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x*
*2 + c*x**4)))/(4*sqrt(c)*d**2) + sqrt(c)*atanh((b + 2*c*x**2)/(2
*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(2*d) - sqrt(a + b*x**2 + c*
x**4)/(2*d*x**2) - sqrt(a*e**2 - b*d*e + c*d**2)*atanh((2*a*e - b
*d + x**2*(b*e - 2*c*d))/(2*sqrt(a + b*x**2 + c*x**4)*sqrt(a*e**2
 - b*d*e + c*d**2)))/(2*d**2) + (b*e - 2*c*d)*atanh((b + 2*c*x**2
)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(4*sqrt(c)*d**2) - b*ata
nh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**4)))/(4*sqrt(
a)*d)
```

Mathematica [A] time = 1.17068, size = 244, normalized size = 0.68

$$\frac{2\log\left(d+ex^2\right)\sqrt{ae^2-bde+cd^2}-2\sqrt{ae^2-bde+cd^2}\log\left(2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex^2-2cdx^2\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2 + c*x^4]/(x^3*(d + e*x^2)),x]
```

```
[Out] ((-2*d*Sqrt[a + b*x^2 + c*x^4])/x^2 + ((b*d - 2*a*e)*Log[x^2])/Sq
rt[a] + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Log[d + e*x^2] - (b*d*Log[2
*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]])/Sqrt[a] + 2*Sqrt
[a]*e*Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]] - 2*Sq
rt[c*d^2 - b*d*e + a*e^2]*Log[-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^
2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]])/(4*d^
2)
```

Maple [B] time = 0.017, size = 1009, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x)
```

```
[Out] -1/2/d/a/x^2*(c*x^4+b*x^2+a)^(3/2)+1/2/d*b/a*(c*x^4+b*x^2+a)^(1/2
)-1/4/d*b/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/
x^2)+1/2/d*c/a*(c*x^4+b*x^2+a)^(1/2)*x^2+1/2/d*c^(1/2)*ln((1/2*b+
c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/2/d^2*e*((x^2+d/e)^2*c+(b
*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/4/d^2*e*ln
((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+((x^2+d/e)^2*c+(b*e-2*c*
d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b-1/2/d*ln
((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+((x^2+d/e)^2*c+(b*e-2*c*
d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)-1/2/d^2*e/
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*
e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)
^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2
+d/e))*a+1/2/d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e
+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))/(x^2+d/e))*b-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(
(2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2
-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c-1/2/d^2*e*(c*x^4+b*x^2+a)^
(1/2)-1/4/d^2*e*b*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
/c^(1/2)+1/2/d^2*e*a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a
)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{(ex^2 + d)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x^3),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x^3), x)
```

Fricas [A] time = 0.477808, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x^3),x, algorithm="fricas")

[Out] [1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(a)*x^2*log(-((8*c^2*d^2
- 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b
^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)
*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*
c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - (b*
d - 2*a*e)*x^2*log(-(4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2)
+ ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4) - 4*sqrt(
c*x^4 + b*x^2 + a)*sqrt(a)*d)/(sqrt(a)*d^2*x^2), -1/8*(4*sqrt(-c*
d^2 + b*d*e - a*e^2)*sqrt(a)*x^2*arctan(-1/2*((2*c*d - b*e)*x^2 +
 b*d - 2*a*e)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^
2))) + (b*d - 2*a*e)*x^2*log(-(4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2
 + 2*a^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4)
 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(a)*d)/(sqrt(a)*d^2*x^2), -1/4*(
(b*d - 2*a*e)*x^2*arctan(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 +
 b*x^2 + a)*a)) - sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(-a)*x^2*log(-(
(8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a
^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 +
4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e +
a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 +
d^2)) + 2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-a)*d)/(sqrt(-a)*d^2*x^2),
 -1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(-a)*x^2*arctan(-1/2*((
2*c*d - b*e)*x^2 + b*d - 2*a*e)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*
d^2 + b*d*e - a*e^2))) + (b*d - 2*a*e)*x^2*arctan(1/2*(b*x^2 + 2*
a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)*a)) + 2*sqrt(c*x^4 + b*x^2 +
 a)*sqrt(-a)*d)/(sqrt(-a)*d^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^3 (d + ex^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(1/2)/x**3/(e*x**2+d),x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/(x**3*(d + e*x**2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x^3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.308 $\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

Optimal. Leaf size=424

$$-\frac{1}{60}\left(13-6x^2\right)\sqrt{2x^4+2x^2+1}x+\frac{109\sqrt{2x^4+2x^2+1}x}{60\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{3}{16}\sqrt{15}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{\left(263\sqrt{2}-70\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{60\,2^{3/4}\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}-\frac{109\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{60\,2^{3/4}\sqrt{2x^4+2x^2+1}}+\frac{15\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{16\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

```
[Out] -(x*(13 - 6*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/60 + (109*x*Sqrt[1 + 2*
x^2 + 2*x^4])/(60*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (3*Sqrt[15]*ArcTan
[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/16 - (109*(1 + Sqrt[2]*x
^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*Arc
Tan[2^(1/4)*x], (2 - Sqrt[2])/4])/(60*2^(3/4)*Sqrt[1 + 2*x^2 + 2*
x^4]) + ((-70 + 263*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - S
qrt[2])/4])/(60*2^(3/4)*(-2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
 + (15*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(
1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(
1/4)*x], (2 - Sqrt[2])/4])/(16*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2
*x^2 + 2*x^4])
```

Rubi [A] time = 0.996114, antiderivative size = 632, normalized size of antiderivative = 1.49, number

of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$

$$\frac{1}{30}\left(3x^2+1\right)\sqrt{2x^4+2x^2+1}x+\frac{109\sqrt{2x^4+2x^2+1}x}{60\sqrt{2}\left(\sqrt{2}x^2+1\right)} - \frac{1}{4}\sqrt{2x^4+2x^2+1}x+\frac{3}{16}\sqrt{15}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right) - \frac{\left(1+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\ 2^{3/4}\sqrt{2x^4+2x^2+1}} - \frac{139\left(1-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{240\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} - \frac{45\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}} - \frac{109\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{60\ 2^{3/4}\sqrt{2x^4+2x^2+1}} + \frac{15\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{16\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(x^4*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

```
[Out] -(x*Sqrt[1 + 2*x^2 + 2*x^4])/4 + (x*(1 + 3*x^2)*Sqrt[1 + 2*x^2 +
2*x^4])/30 + (109*x*Sqrt[1 + 2*x^2 + 2*x^4])/(60*Sqrt[2]*(1 + Sqr
t[2]*x^2)) + (3*Sqrt[15]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*
x^4]])/16 - (109*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/
(60*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (45*(1 + Sqrt[2]*x^2)*Sqrt
[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1
/4)*x], (2 - Sqrt[2])/4])/(8*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x
^2 + 2*x^4]) - (139*(1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x
^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(240*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((1 + Sq
rt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^
2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(4*2^(3/4)
*Sqrt[1 + 2*x^2 + 2*x^4]) + (15*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*S
qrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*
Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(16*2^(3/4)*(
2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 80.4071, size = 571, normalized size = 1.35

$$\frac{x\left(6x^{2}+2\right)\sqrt{2x^{4}+2x^{2}+1}}{60}-\frac{x\sqrt{2x^{4}+2x^{2}+1}}{4}+\frac{109\sqrt{2}x\sqrt{2x^{4}+2x^{2}+1}}{120\left(\sqrt{2}x^{2}+1\right)}-\frac{109\sqrt[4]{2}\sqrt{\frac{2x^{4}+2x^{2}+1}{\left(\sqrt{2}x^{2}+1\right)^{2}}}\left(\sqrt{2}x^{2}+1\right)E\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{120\sqrt{2x^{4}+2x^{2}+1}}-\frac{\sqrt[4]{2}\sqrt{\frac{2x^{4}+2x^{2}+1}{\left(\sqrt{2}x^{2}+1\right)^{2}}}\left(2+2\sqrt{2}\right)\left(\sqrt{2}x^{2}+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{16\sqrt{2x^{4}+2x^{2}+1}}+\frac{139\sqrt[4]{2}\sqrt{\frac{2x^{4}+2x^{2}+1}{\left(\sqrt{2}x^{2}+1\right)^{2}}}\left(-2\sqrt{2}+4\right)\left(\sqrt{2}x^{2}+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{960\sqrt{2x^{4}+2x^{2}+1}}-\frac{45\sqrt[4]{2}\sqrt{\frac{2x^{4}+2x^{2}+1}{\left(\sqrt{2}x^{2}+1\right)^{2}}}\left(\sqrt{2}x^{2}+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{16\left(-3\sqrt{2}+2\right)\sqrt{2x^{4}+2x^{2}+1}}+\frac{15\cdot 2^{\frac{3}{4}}\sqrt{\frac{2x^{4}+2x^{2}+1}{\left(\sqrt{2}x^{2}+1\right)^{2}}}\left(2+3\sqrt{2}\right)\left(\sqrt{2}x^{2}+1\right)\left(-\frac{11\sqrt{2}}{24}+\frac{1}{2};2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{64\left(-3\sqrt{2}+2\right)\sqrt{2x^{4}+2x^{2}+1}}+\frac{3\sqrt{15}\operatorname{atan}\left(\frac{\sqrt{15}x}{3\sqrt{2x^{4}+2x^{2}+1}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
```

```
[Out] x*(6*x**2 + 2)*sqrt(2*x**4 + 2*x**2 + 1)/60 - x*sqrt(2*x**4 + 2*x
**2 + 1)/4 + 109*sqrt(2)*x*sqrt(2*x**4 + 2*x**2 + 1)/(120*(sqrt(2
)*x**2 + 1)) - 109*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x
**2 + 1)**2)*(sqrt(2)*x**2 + 1)*elliptic_e(2*atan(2**(1/4)*x), -s
qrt(2)/4 + 1/2)/(120*sqrt(2*x**4 + 2*x**2 + 1)) - 2**(1/4)*sqrt((
2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(2 + 2*sqrt(2))*(sqrt
(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(1
6*sqrt(2*x**4 + 2*x**2 + 1)) + 139*2**(1/4)*sqrt((2*x**4 + 2*x**2
 + 1)/(sqrt(2)*x**2 + 1)**2)*(-2*sqrt(2) + 4)*(sqrt(2)*x**2 + 1)*
elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(960*sqrt(2*x**4
 + 2*x**2 + 1)) - 45*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)
*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x),
-sqrt(2)/4 + 1/2)/(16*(-3*sqrt(2) + 2)*sqrt(2*x**4 + 2*x**2 + 1))
 + 15*2**(3/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*
(2 + 3*sqrt(2))*(sqrt(2)*x**2 + 1)*elliptic_pi(-11*sqrt(2)/24 + 1
/2, 2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(64*(-3*sqrt(2) + 2)*sq
```

```
rt(2*x**4 + 2*x**2 + 1)) + 3*sqrt(15)*atan(sqrt(15)*x/(3*sqrt(2*x
**4 + 2*x**2 + 1)))/16
```

Mathematica [C] time = 0.214539, size = 209, normalized size = 0.49

$$\frac{48x^7 - 56x^5 - 80x^3 - (199 - 417i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right) - 218i\sqrt{1-i}\sqrt{1+(1-i)x^2}}{240\sqrt{2x^4+}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

```
[Out] (-52*x - 80*x^3 - 56*x^5 + 48*x^7 - (218*I)*Sqrt[1 - I]*Sqrt[1 +
(1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I
]*x], I] - (199 - 417*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1
 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] + 225*(1 -
 I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[
1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(240*Sqrt[1 + 2*x^2 + 2*
x^4])
```

Maple [C] time = 0.093, size = 528, normalized size = 1.3

$$-\frac{13x}{60}\sqrt{2x^4+2x^2+1}$$
$$-\frac{8\,EllipticF\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{15\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\left(\frac{13}{60}-\frac{13i}{60}\right)\left(EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)-EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)\right)}{\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2}}$$
$$+\frac{x^3}{10}\sqrt{2x^4+2x^2+1}$$
$$-\frac{9\,EllipticF\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{4\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\frac{9i}{8}EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{9\,EllipticE\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{8\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\frac{9i}{8}EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{15}{8\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\,EllipticPi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x)
```

```
[Out] -13/60*x*(2*x^4+2*x^2+1)^(1/2)-8/15/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1
/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^
(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(13/60-13/60*I)/(-1+I)^(1/2)*(1+
(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(Ellip
ticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)
^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+1/10*x^3*(2*x^4+2*x^2+1)^(1/2)
-9/4/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4
+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2
))+9/8*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2
*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^
(1/2))+9/8/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/
(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*
2^(1/2))-9/8*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1
/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/
2*I*2^(1/2))+15/8/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)
^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,
(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}x^4}{2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}x^4}{2\,x^2+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{2x^4+2x^2+1}}{2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
```

```
[Out] Integral(x**4*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}x^4}{2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3), x)
```

3.309 $\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

Optimal. Leaf size=417

$$-\frac{7\sqrt{2x^4+2x^2+1}x}{6\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{1}{6}\sqrt{2x^4+2x^2+1}x-\frac{1}{8}\sqrt{15}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{\left(17\sqrt{2}-4\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{6\,2^{3/4}\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}+\frac{7\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{6\,2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{5\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/6 - (7*x*Sqrt[1 + 2*x^2 + 2*x^4])/(6*
Sqrt[2]*(1 + Sqrt[2]*x^2)) - (Sqrt[15]*ArcTan[(Sqrt[5/3]*x)/Sqrt[
1 + 2*x^2 + 2*x^4]])/8 + (7*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2
*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sq
rt[2])/4])/(6*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((-4 + 17*Sqrt[2
])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2
]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(6*2^(3/4)*(-2
 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - (5*(3 + Sqrt[2])*(1 + Sq
rt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elliptic
Pi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(
8*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.811266, antiderivative size = 604, normalized size of antiderivative = 1.45, number

of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241

$$-\frac{7\sqrt{2x^4+2x^2+1}x}{6\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{1}{6}\sqrt{2x^4+2x^2+1}x-\frac{1}{8}\sqrt{15}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{\left(1+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{6\ 2^{3/4}\sqrt{2x^4+2x^2+1}}+\frac{3\left(1-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{15\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}+\frac{7\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{6\ 2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{5\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(x^2*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/6 - (7*x*Sqrt[1 + 2*x^2 + 2*x^4])/(6*
Sqrt[2]*(1 + Sqrt[2]*x^2)) - (Sqrt[15]*ArcTan[(Sqrt[5/3]*x)/Sqrt[
1 + 2*x^2 + 2*x^4]])/8 + (7*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2
*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sq
rt[2])/4])/(6*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (15*(1 + Sqrt[2]
*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*A
rcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(4*2^(3/4)*(2 - 3*Sqrt[2])*Sq
rt[1 + 2*x^2 + 2*x^4]) + (3*(1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[
(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/
4)*x], (2 - Sqrt[2])/4])/(8*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + ((
1 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(6*2
^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (5*(3 + Sqrt[2])*(1 + Sqrt[2]*x
^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12
- 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(3/
4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 74.9102, size = 544, normalized size = 1.3

$$\frac{x\sqrt{2x^4+2x^2+1}}{6}-\frac{7\sqrt{2}x\sqrt{2x^4+2x^2+1}}{12\left(\sqrt{2}x^2+1\right)}+\frac{7\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)E\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{12\sqrt{2x^4+2x^2+1}}+\frac{15\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{8\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}-\frac{3\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(-2\sqrt{2}+4\right)\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{32\sqrt{2x^4+2x^2+1}}+\frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(2+2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{24\sqrt{2x^4+2x^2+1}}-\frac{5\cdot 2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(2+3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\left(-\frac{11\sqrt{2}}{24}+\frac{1}{2};2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{32\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{15}\operatorname{atan}\left(\frac{\sqrt{15}x}{3\sqrt{2x^4+2x^2+1}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
```

```
[Out] x*sqrt(2*x**4 + 2*x**2 + 1)/6 - 7*sqrt(2)*x*sqrt(2*x**4 + 2*x**2
+ 1)/(12*(sqrt(2)*x**2 + 1)) + 7*2**(1/4)*sqrt((2*x**4 + 2*x**2 +
 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*elliptic_e(2*atan(2
**(1/4)*x), -sqrt(2)/4 + 1/2)/(12*sqrt(2*x**4 + 2*x**2 + 1)) + 15
*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt
(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(8
*(-3*sqrt(2) + 2)*sqrt(2*x**4 + 2*x**2 + 1)) - 3*2**(1/4)*sqrt((2
*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(-2*sqrt(2) + 4)*(sqrt
(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(3
2*sqrt(2*x**4 + 2*x**2 + 1)) + 2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1
)/(sqrt(2)*x**2 + 1)**2)*(2 + 2*sqrt(2))*(sqrt(2)*x**2 + 1)*ellip
tic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(24*sqrt(2*x**4 + 2*x
**2 + 1)) - 5*2**(3/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 +
 1)**2)*(2 + 3*sqrt(2))*(sqrt(2)*x**2 + 1)*elliptic_pi(-11*sqrt(2
)/24 + 1/2, 2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(32*(-3*sqrt(2)
 + 2)*sqrt(2*x**4 + 2*x**2 + 1)) - sqrt(15)*atan(sqrt(15)*x/(3*sq
rt(2*x**4 + 2*x**2 + 1)))/8
```

Mathematica [C] time = 0.199715, size = 204, normalized size = 0.49

$$\frac{8x^5+8x^3+(13-27i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)+14i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}}{24\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

```
[Out] (4*x + 8*x^3 + 8*x^5 + (14*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*S
qrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (13
 - 27*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*
EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 15*(1 - I)^(3/2)*Sqrt[1
+ (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcS
inh[Sqrt[1 - I]*x], I])/(24*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.012, size = 509, normalized size = 1.2

$$\frac{x}{6}\sqrt{2x^4+2x^2+1}+\frac{EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{3\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\left(\frac{1}{6}-\frac{i}{6}\right)\left(EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)-EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)\right)}{\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+}}$$
$$+\frac{3\,EllipticF\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{2\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\frac{3i}{4}EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{3\,EllipticE\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{4\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\frac{3i}{4}EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{5}{4\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}EllipticPi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x)
```

```
[Out] 1/6*x*(2*x^4+2*x^2+1)^(1/2)+1/3/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*
(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2
),1/2*2^(1/2)+1/2*I*2^(1/2))+(-1/6+1/6*I)/(-1+I)^(1/2)*(1+(1-I)*x
^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*
(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),
1/2*2^(1/2)+1/2*I*2^(1/2)))+3/2/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)
*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/
2),1/2*2^(1/2)+1/2*I*2^(1/2))-3/4*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(
1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)
^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-3/4/(-1+I)^(1/2)*(-I*x^2+x^2+1)
^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+
I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+3/4*I/(-1+I)^(1/2)*(-I*x^2+x^
2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*
(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-5/4/(-1+I)^(1/2)*(-I*x^2+
x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi
(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}x^2}{2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}x^2}{2\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{2x^4+2x^2+1}}{2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
```

```
[Out] Integral(x**2*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}x^2}{2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3), x)
```

3.310 $\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

Optimal. Leaf size=381

$$\frac{\sqrt{2x^4+2x^2+1}\,x}{\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{1}{4}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$+\frac{2^{3/4}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{5\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/(Sqrt[2]*(1 + Sqrt[2]*x^2)) + (Sqrt[5
/3]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/4 - ((1 + Sqrt
[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[
2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2^(3/4)*Sqrt[1 + 2*x^2 +
2*x^4]) + (2^(3/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1
+ Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4]
)/((-2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt[2])*(
1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*El
lipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])
/4])/(12*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.399423, antiderivative size = 483, normalized size of antiderivative = 1.27, number

of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{\sqrt{2x^4+2x^2+1}x}{\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{1}{4}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right) - \frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} - \frac{5\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}} - \frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2^{3/4}\sqrt{2x^4+2x^2+1}} + \frac{5\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[Sqrt[1 + 2*x^2 + 2*x^4]/(3 + 2*x^2),x]
```

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/(Sqrt[2]*(1 + Sqrt[2]*x^2)) + (Sqrt[5
/3]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/4 - ((1 + Sqrt
[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[
2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2^(3/4)*Sqrt[1 + 2*x^2 +
2*x^4]) - (5*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2*2
^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - ((1 - Sqrt[2])*
(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*E
llipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(4*2^(1/4)*Sqrt[1
 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 +
 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])
/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(3/4)*(2 - 3*Sq
rt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 45.0291, size = 432, normalized size = 1.13

$$\frac{\sqrt{2}x\sqrt{2x^4+2x^2+1}}{2\left(\sqrt{2}x^2+1\right)}-\frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)E\left(2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{2\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(-2\sqrt{2}+4\right)\left(\sqrt{2}x^2+1\right)F\left(2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{16\sqrt{2x^4+2x^2+1}}$$
$$-\frac{5\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{4\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}$$
$$+\frac{5\cdot 2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(2+3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\left(-\frac{11\sqrt{2}}{24}+\frac{1}{2};2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{48\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\sqrt{15}\,\mathrm{atan}\left(\frac{\sqrt{15}x}{3\sqrt{2x^4+2x^2+1}}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
```

```
[Out] sqrt(2)*x*sqrt(2*x**4 + 2*x**2 + 1)/(2*(sqrt(2)*x**2 + 1)) - 2**(
1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x
**2 + 1)*elliptic_e(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(2*sqrt
(2*x**4 + 2*x**2 + 1)) + 2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqr
t(2)*x**2 + 1)**2)*(-2*sqrt(2) + 4)*(sqrt(2)*x**2 + 1)*elliptic_f
(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(16*sqrt(2*x**4 + 2*x**2 +
 1)) - 5*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**
2)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 +
 1/2)/(4*(-3*sqrt(2) + 2)*sqrt(2*x**4 + 2*x**2 + 1)) + 5*2**(3/4)
*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(2 + 3*sqrt(2)
)*(sqrt(2)*x**2 + 1)*elliptic_pi(-11*sqrt(2)/24 + 1/2, 2*atan(2**
(1/4)*x), -sqrt(2)/4 + 1/2)/(48*(-3*sqrt(2) + 2)*sqrt(2*x**4 + 2*
x**2 + 1)) + sqrt(15)*atan(sqrt(15)*x/(3*sqrt(2*x**4 + 2*x**2 + 1
)))/12
```

Mathematica [C] time = 0.101229, size = 127, normalized size = 0.33

$$-\frac{\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left(-(3+6i)F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)+(3+3i)E\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)+5i\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\right.\right.\right.}{6\sqrt{1-i}\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(3 + 2*x^2),x]
```

```
[Out] -(Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*((3 + 3*I)*Elliptic
E[I*ArcSinh[Sqrt[1 - I]*x], I] - (3 + 6*I)*EllipticF[I*ArcSinh[Sq
rt[1 - I]*x], I] + (5*I)*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 -
 I]*x], I]))/(6*Sqrt[1 - I]*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.007, size = 341, normalized size = 0.9

$$
\begin{aligned}
&-\frac{EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}\\
&+\frac{\frac{i}{2}EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}\\
&+\frac{EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{2\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}\\
&-\frac{\frac{i}{2}EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}\\
&+\frac{5}{6\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}EllipticPi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x)
```

```
[Out] -1/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2
*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))
+1/2*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x
^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1
/2))+1/2/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2
*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^
(1/2))-1/2*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2
)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*
I*2^(1/2))+5/6/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1
/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1
-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^4+2x^2+1}}{2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
```

```
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3), x)
```

3.311 $\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)}\,dx$

Optimal. Leaf size=412

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}x}{3\left(\sqrt{2}x^2+1\right)}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$+\frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{2x^4+2x^2+1}}$$
$$-\frac{5\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{18\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(3*x) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^
4])/(3*(1 + Sqrt[2]*x^2)) - (Sqrt[5/3]*ArcTan[(Sqrt[5/3]*x)/Sqrt[
1 + 2*x^2 + 2*x^4]])/6 - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x
^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(3*Sqrt[1 + 2*x^2 + 2*x^4]) + (2^(1/4)*(1 + Sqrt
[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[
2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*(-2 + 3*Sqrt[2])*Sqrt[1
 + 2*x^2 + 2*x^4]) - (5*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 +
 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])
/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(18*2^(3/4)*(2 - 3*Sq
rt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.490612, antiderivative size = 509, normalized size of antiderivative = 1.24, number

of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}x}{3\left(\sqrt{2}x^2+1\right)}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$+\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{6\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{5\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{2x^4+2x^2+1}}$$
$$-\frac{5\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{18\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[Sqrt[1 + 2*x^2 + 2*x^4]/(x^2*(3 + 2*x^2)),x]
```

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(3*x) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^
4])/(3*(1 + Sqrt[2]*x^2)) - (Sqrt[5/3]*ArcTan[(Sqrt[5/3]*x)/Sqrt[
1 + 2*x^2 + 2*x^4]])/6 - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x
^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(3*Sqrt[1 + 2*x^2 + 2*x^4]) + (5*(1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcT
an[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[
1 + 2*x^2 + 2*x^4]) + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 +
2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x]
, (2 - Sqrt[2])/4])/(6*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (5*(3 +
 Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]
*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2
 - Sqrt[2])/4])/(18*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^
4])
```

Rubi in Sympy [A] time = 53.1953, size = 450, normalized size = 1.09

$$\frac{\sqrt{2}x\sqrt{2x^4+2x^2+1}}{3\left(\sqrt{2}x^2+1\right)} - \frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)E\left(2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{3\sqrt{2x^4+2x^2+1}}$$
$$+\frac{5\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{6\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(4+6\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)F\left(2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{24\sqrt{2x^4+2x^2+1}}$$
$$-\frac{5\cdot 2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(2+3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\left(-\frac{11\sqrt{2}}{24}+\frac{1}{2};2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{72\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\sqrt{15}\,\mathrm{atan}\left(\frac{\sqrt{15}x}{3\sqrt{2x^4+2x^2+1}}\right)}{18}-\frac{\sqrt{2x^4+2x^2+1}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**4+2*x**2+1)**(1/2)/x**2/(2*x**2+3),x)
```

```
[Out] sqrt(2)*x*sqrt(2*x**4 + 2*x**2 + 1)/(3*(sqrt(2)*x**2 + 1)) - 2**(
1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x
**2 + 1)*elliptic_e(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(3*sqrt
(2*x**4 + 2*x**2 + 1)) + 5*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(s
qrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4
)*x), -sqrt(2)/4 + 1/2)/(6*(-3*sqrt(2) + 2)*sqrt(2*x**4 + 2*x**2
+ 1)) + 2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2
)*(4 + 6*sqrt(2))*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x
), -sqrt(2)/4 + 1/2)/(24*sqrt(2*x**4 + 2*x**2 + 1)) - 5*2**(3/4)*
sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(2 + 3*sqrt(2))
*(sqrt(2)*x**2 + 1)*elliptic_pi(-11*sqrt(2)/24 + 1/2, 2*atan(2**(
1/4)*x), -sqrt(2)/4 + 1/2)/(72*(-3*sqrt(2) + 2)*sqrt(2*x**4 + 2*x
**2 + 1)) - sqrt(15)*atan(sqrt(15)*x/(3*sqrt(2*x**4 + 2*x**2 + 1)
))/18 - sqrt(2*x**4 + 2*x**2 + 1)/(3*x)
```

Mathematica [C] time = 0.190419, size = 208, normalized size = 0.5

$$\frac{-12x^4-12x^2+(9-3i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}xF\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)-6i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x}}{18x\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(x^2*(3 + 2*x^2)),x]
```

```
[Out] (-6 - 12*x^2 - 12*x^4 - (6*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]
*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (
9 - 3*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2
]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 5*(1 - I)^(3/2)*x*Sqrt
[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*A
rcSinh[Sqrt[1 - I]*x], I])/(18*x*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C]   time = 0.021, size = 511, normalized size = 1.2

$$
\begin{aligned}
&-\frac{1}{3x}\sqrt{2x^4+2x^2+1} \\
&+\frac{2\,\mathit{EllipticF}\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{3\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+1}} \\
&-\frac{\left(\frac{2}{3}-\frac{2i}{3}\right)\left(\mathit{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)-\mathit{EllipticE}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)\right)}{\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+}} \\
&+\frac{2\,\mathit{EllipticF}\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{3\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}} \\
&-\frac{\frac{i}{3}\mathit{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}} \\
&-\frac{\mathit{EllipticE}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{3\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}} \\
&+\frac{\frac{i}{3}\mathit{EllipticE}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}} \\
&-\frac{5}{9\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\mathit{EllipticPi}\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2x^4+2x^2+1}}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x)
```

```
[Out] -1/3*(2*x^4+2*x^2+1)^(1/2)/x+2/3/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)
*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/
2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-2/3+2/3*I)/(-1+I)^(1/2)*(1+(1-I)*
x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x
*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2)
,1/2*2^(1/2)+1/2*I*2^(1/2)))+2/3/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2
)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1
/2),1/2*2^(1/2)+1/2*I*2^(1/2))-1/3*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^
(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I
```

```
)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-1/3/(-1+I)^(1/2)*(-I*x^2+x^2+1
)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1
+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+1/3*I/(-1+I)^(1/2)*(-I*x^2+x
^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x
*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-5/9/(-1+I)^(1/2)*(-I*x^2
+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticP
i(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^4+3\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^4 + 3*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^4+2x^2+1}}{x^2\,(2x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(1/2)/x**2/(2*x**2+3),x)
```

```
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(x**2*(2*x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4 + 2\,x^2 + 1}}{(2\,x^2 + 3)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^2), x)
```

3.312 $\int \frac{\sqrt{1+2x^2+2x^4}}{x^4(3+2x^2)}\,dx$

Optimal. Leaf size=373

$$\frac{1}{9}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{5\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}-\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{5\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{27\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}$$

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(9*x^3) + (Sqrt[5/3]*ArcTan[(Sqrt[5/3]*x
)/Sqrt[1 + 2*x^2 + 2*x^4]])/9 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^
2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2
 - Sqrt[2])/4])/(9*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (5*2^(1/4)*
(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*E
llipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(9*(2 - 3*Sqrt[2]
)*Sqrt[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*S
qrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*
Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(27*2^(3/4)*(
2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.392102, antiderivative size = 373, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172

$$\frac{1}{9}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{5\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}-\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{5\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{27\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + 2*x^2 + 2*x^4]/(x^4*(3 + 2*x^2)),x]
```

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(9*x^3) + (Sqrt[5/3]*ArcTan[(Sqrt[5/3]*x
)/Sqrt[1 + 2*x^2 + 2*x^4]])/9 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^
2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2
 - Sqrt[2])/4])/(9*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (5*2^(1/4)*
(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*E
llipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(9*(2 - 3*Sqrt[2]
)*Sqrt[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*S
qrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*
Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(27*2^(3/4)*(
2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 37.1327, size = 330, normalized size = 0.88

$$-\frac{2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{18\sqrt{2x^4+2x^2+1}}-\frac{5\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{9\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}+\frac{5\cdot 2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(2+3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\left(-\frac{11\sqrt{2}}{24}+\frac{1}{2};2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{108\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{15}\operatorname{atan}\left(\frac{\sqrt{15}x}{3\sqrt{2x^4+2x^2+1}}\right)}{27}-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**4+2*x**2+1)**(1/2)/x**4/(2*x**2+3),x)
```

```
[Out] -2**(3/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt
(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(1
8*sqrt(2*x**4 + 2*x**2 + 1)) - 5*2**(1/4)*sqrt((2*x**4 + 2*x**2 +
 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2
**(1/4)*x), -sqrt(2)/4 + 1/2)/(9*(-3*sqrt(2) + 2)*sqrt(2*x**4 + 2
*x**2 + 1)) + 5*2**(3/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2
 + 1)**2)*(2 + 3*sqrt(2))*(sqrt(2)*x**2 + 1)*elliptic_pi(-11*sqrt
(2)/24 + 1/2, 2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(108*(-3*sqrt
(2) + 2)*sqrt(2*x**4 + 2*x**2 + 1)) + sqrt(15)*atan(sqrt(15)*x/(3
*sqrt(2*x**4 + 2*x**2 + 1)))/27 - sqrt(2*x**4 + 2*x**2 + 1)/(9*x*
*3)
```

Mathematica [C] time = 0.165498, size = 154, normalized size = 0.41

$$-\frac{6x^4+6x^2+3(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}x^3F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)-5(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}x^3}{27x^3\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(x^4*(3 + 2*x^2)),x]
```

```
[Out] -(3 + 6*x^2 + 6*x^4 + 3*(1 - I)^(3/2)*x^3*Sqrt[1 + (1 - I)*x^2]*S
qrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 5*(
1 - I)^(3/2)*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*Elli
pticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(27*x^3*Sqrt[1 +
2*x^2 + 2*x^4])
```

Maple [C] time = 0.024, size = 448, normalized size = 1.2

$$-\frac{1}{9x^3}\sqrt{2x^4+2x^2+1}$$
$$+\frac{\left(\frac{2}{9}-\frac{2i}{9}\right)\left(EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)-EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)\right)}{\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+}}$$
$$-\frac{4\,EllipticF\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{9\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\frac{2i}{9}EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{2\,EllipticE\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{9\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\frac{2i}{9}EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{10}{27\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}EllipticPi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x)
```

```
[Out] -1/9*(2*x^4+2*x^2+1)^(1/2)/x^3+(2/9-2/9*I)/(-1+I)^(1/2)*(1+(1-I)*
x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x
```

```
*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2)
,1/2*2^(1/2)+1/2*I*2^(1/2)))-4/9/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2
)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1
/2),1/2*2^(1/2)+1/2*I*2^(1/2))+2/9*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^
(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I
)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+2/9/(-1+I)^(1/2)*(-I*x^2+x^2+1
)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1
+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-2/9*I/(-1+I)^(1/2)*(-I*x^2+x
^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x
*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+10/27/(-1+I)^(1/2)*(-I*x
^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ellipti
cPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^6+3\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^6 + 3*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^4+2x^2+1}}{x^4\,(2x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**4+2*x**2+1)**(1/2)/x**4/(2*x**2+3),x)
```

```
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(x**4*(2*x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4 + 2\,x^2 + 1}}{(2\,x^2 + 3)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^4), x)
```

3.313 $\int \frac{\sqrt{1+2x^2+2x^4}}{x^6(3+2x^2)}\,dx$

Optimal. Leaf size=647

$$\frac{4\sqrt{2}\sqrt{2x^4+2x^2+1}x}{45\left(\sqrt{2}x^2+1\right)}-\frac{4\sqrt{2x^4+2x^2+1}}{45x}-\frac{2}{27}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$-\frac{\sqrt[4]{2}\left(19-2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{135\sqrt{2x^4+2x^2+1}}$$
$$+\frac{10\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{27\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$+\frac{5\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{27\sqrt{2x^4+2x^2+1}}$$
$$-\frac{4\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{45\sqrt{2x^4+2x^2+1}}$$
$$-\frac{5\sqrt[4]{2}\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{81\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\sqrt{2x^4+2x^2+1}}{15x^5}+\frac{4\sqrt{2x^4+2x^2+1}}{135x^3}$$

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(15*x^5) + (4*Sqrt[1 + 2*x^2 + 2*x^4])/(
135*x^3) - (4*Sqrt[1 + 2*x^2 + 2*x^4])/(45*x) + (4*Sqrt[2]*x*Sqrt
[1 + 2*x^2 + 2*x^4])/(45*(1 + Sqrt[2]*x^2)) - (2*Sqrt[5/3]*ArcTan
[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/27 - (4*2^(1/4)*(1 + Sqr
t[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE
[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(45*Sqrt[1 + 2*x^2 + 2*x^
4]) + (5*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/
(27*Sqrt[1 + 2*x^2 + 2*x^4]) + (10*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt
[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1
/4)*x], (2 - Sqrt[2])/4])/(27*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*
x^4]) - (2^(1/4)*(19 - 2*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x
^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(135*Sqrt[1 + 2*x^2 + 2*x^4]) - (5*2^(1/4)*(3 +
Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*
x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2
- Sqrt[2])/4])/(81*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 1.03856, antiderivative size = 647, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$

$$\frac{4\sqrt{2}\sqrt{2x^4+2x^2+1}x}{45\left(\sqrt{2}x^2+1\right)}-\frac{4\sqrt{2x^4+2x^2+1}}{45x}-\frac{2}{27}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$-\frac{\sqrt[4]{2}\left(19-2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{135\sqrt{2x^4+2x^2+1}}$$
$$+\frac{10\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{27\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$+\frac{5\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{27\sqrt{2x^4+2x^2+1}}$$
$$-\frac{4\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{45\sqrt{2x^4+2x^2+1}}$$
$$-\frac{5\sqrt[4]{2}\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{81\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\sqrt{2x^4+2x^2+1}}{15x^5}+\frac{4\sqrt{2x^4+2x^2+1}}{135x^3}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[Sqrt[1 + 2*x^2 + 2*x^4]/(x^6*(3 + 2*x^2)),x]
```

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(15*x^5) + (4*Sqrt[1 + 2*x^2 + 2*x^4])/(
135*x^3) - (4*Sqrt[1 + 2*x^2 + 2*x^4])/(45*x) + (4*Sqrt[2]*x*Sqrt
[1 + 2*x^2 + 2*x^4])/(45*(1 + Sqrt[2]*x^2)) - (2*Sqrt[5/3]*ArcTan
[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/27 - (4*2^(1/4)*(1 + Sqr
t[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE
[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(45*Sqrt[1 + 2*x^2 + 2*x^
4]) + (5*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/
(27*Sqrt[1 + 2*x^2 + 2*x^4]) + (10*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt
[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1
/4)*x], (2 - Sqrt[2])/4])/(27*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*
x^4]) - (2^(1/4)*(19 - 2*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x
^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(135*Sqrt[1 + 2*x^2 + 2*x^4]) - (5*2^(1/4)*(3 +
Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*
x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2
```

```
- Sqrt[2])/4])/(81*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 95.4528, size = 581, normalized size = 0.9

$$\frac{4\sqrt{2}x\sqrt{2x^4+2x^2+1}}{45\left(\sqrt{2}x^2+1\right)} - \frac{4\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)E\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{45\sqrt{2x^4+2x^2+1}}$$
$$+ \frac{10\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{27\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}$$
$$- \frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(-8\sqrt{2}+76\right)\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{540\sqrt{2x^4+2x^2+1}}$$
$$+ \frac{5\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{27\sqrt{2x^4+2x^2+1}}$$
$$- \frac{5\cdot 2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(2+3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\left(-\frac{11\sqrt{2}}{24}+\frac{1}{2}; 2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{162\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}$$
$$- \frac{2\sqrt{15}\operatorname{atan}\left(\frac{\sqrt{15}x}{3\sqrt{2x^4+2x^2+1}}\right)}{81} - \frac{4\sqrt{2x^4+2x^2+1}}{45x} + \frac{4\sqrt{2x^4+2x^2+1}}{135x^3} - \frac{\sqrt{2x^4+2x^2+1}}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**4+2*x**2+1)**(1/2)/x**6/(2*x**2+3),x)
```

```
[Out] 4*sqrt(2)*x*sqrt(2*x**4 + 2*x**2 + 1)/(45*(sqrt(2)*x**2 + 1)) - 4
*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt
(2)*x**2 + 1)*elliptic_e(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(4
5*sqrt(2*x**4 + 2*x**2 + 1)) + 10*2**(1/4)*sqrt((2*x**4 + 2*x**2
+ 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(
2**(1/4)*x), -sqrt(2)/4 + 1/2)/(27*(-3*sqrt(2) + 2)*sqrt(2*x**4 +
 2*x**2 + 1)) - 2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2
 + 1)**2)*(-8*sqrt(2) + 76)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(
2**(1/4)*x), -sqrt(2)/4 + 1/2)/(540*sqrt(2*x**4 + 2*x**2 + 1)) +
5*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqr
t(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(
27*sqrt(2*x**4 + 2*x**2 + 1)) - 5*2**(3/4)*sqrt((2*x**4 + 2*x**2
+ 1)/(sqrt(2)*x**2 + 1)**2)*(2 + 3*sqrt(2))*(sqrt(2)*x**2 + 1)*el
liptic_pi(-11*sqrt(2)/24 + 1/2, 2*atan(2**(1/4)*x), -sqrt(2)/4 +
1/2)/(162*(-3*sqrt(2) + 2)*sqrt(2*x**4 + 2*x**2 + 1)) - 2*sqrt(15
)*atan(sqrt(15)*x/(3*sqrt(2*x**4 + 2*x**2 + 1)))/81 - 4*sqrt(2*x*
```

```
*4 + 2*x**2 + 1)/(45*x) + 4*sqrt(2*x**4 + 2*x**2 + 1)/(135*x**3)
- sqrt(2*x**4 + 2*x**2 + 1)/(15*x**5)
```

Mathematica [C] time = 0.236431, size = 224, normalized size = 0.35

$$-\frac{72x^8+48x^6+66x^4+42x^2-(12+24i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}x^5F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)+36i\sqrt{1-i}\sqrt{1+(1}{405x^5\sqrt{}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(x^6*(3 + 2*x^2)),x]
```

```
[Out] -(27 + 42*x^2 + 66*x^4 + 48*x^6 + 72*x^8 + (36*I)*Sqrt[1 - I]*x^5
*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[
Sqrt[1 - I]*x], I] - (12 + 24*I)*Sqrt[1 - I]*x^5*Sqrt[1 + (1 - I)
*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I
] + 50*(1 - I)^(3/2)*x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x
^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(405*x^5*
Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.028, size = 549, normalized size = 0.9

$$-\frac{1}{15x^5}\sqrt{2x^4+2x^2+1}+\frac{4}{135x^3}\sqrt{2x^4+2x^2+1}-\frac{4}{45x}\sqrt{2x^4+2x^2+1}$$
$$-\frac{4\,EllipticF\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{45\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\left(\frac{32}{135}-\frac{32i}{135}\right)\left(EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)-EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)\right)}{\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x}}$$
$$+\frac{8\,EllipticF\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{27\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\frac{4i}{27}EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{4\,EllipticE\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{27\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\frac{4i}{27}EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{20}{81\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}EllipticPi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x)
```

```
[Out] -1/15*(2*x^4+2*x^2+1)^(1/2)/x^5+4/135*(2*x^4+2*x^2+1)^(1/2)/x^3-4
/45*(2*x^4+2*x^2+1)^(1/2)/x-4/45/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)
*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/
2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-32/135+32/135*I)/(-1+I)^(1/2)*(1+
(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(Ellip
ticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)
^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+8/27/(-1+I)^(1/2)*(-I*x^2+x^2+
1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-
1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-4/27*I/(-1+I)^(1/2)*(-I*x^2
+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF
(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-4/27/(-1+I)^(1/2)*(-I*
x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ellipt
icE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+4/27*I/(-1+I)^(1/2)
*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*E
llipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-20/81/(-1+I)^(
1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/
2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^6),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^8+3\,x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^8 + 3*x^6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^4 + 2x^2 + 1}}{x^6 (2x^2 + 3)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**4+2*x**2+1)**(1/2)/x**6/(2*x**2+3),x)
```

```
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(x**6*(2*x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4 + 2\,x^2 + 1}}{(2\,x^2 + 3)x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^6),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^6), x)
```

3.314 $\int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx$

Optimal. Leaf size=482

$$-\frac{\left(16bc^2e^3\left(3a^2e^2-3abde+b^2d^2\right)+6b^3ce^4(bd-4ae)-384c^4d^3e(bd-ae)+96c^3de^2(bd-ae)^2+3b^5e^5+256c^5d^5\right)\tanh^{-1}}{512c^{7/2}e^6}$$
$$+\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(-3b^2e^2-6cex^2(be+2cd)-6bcde+16c^2d^2\right)}{96c^2e^3}$$
$$+\frac{\sqrt{a+bx^2+cx^4}\left(-2cex^2\left(-8c^2de(2bd-3ae)-6bce^2(bd-2ae)-3b^3e^3+32c^3d^3\right)+6b^2ce^3(bd-2ae)-32c^3d^2e(5bd-4a}{256c^3e^5}$$
$$+\frac{d^2\left(ae^2-bde+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^6}+\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10ce}$$

```
[Out] ((128*c^4*d^4 + 3*b^4*e^4 - 32*c^3*d^2*e*(5*b*d - 4*a*e) + 8*b*c^
2*d*e^2*(2*b*d - 3*a*e) + 6*b^2*c*e^3*(b*d - 2*a*e) - 2*c*e*(32*c
^3*d^3 - 3*b^3*e^3 - 8*c^2*d*e*(2*b*d - 3*a*e) - 6*b*c*e^2*(b*d -
 2*a*e))*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*c^3*e^5) + ((16*c^2*d
^2 - 6*b*c*d*e - 3*b^2*e^2 - 6*c*e*(2*c*d + b*e)*x^2)*(a + b*x^2
+ c*x^4)^(3/2))/(96*c^2*e^3) + (a + b*x^2 + c*x^4)^(5/2)/(10*c*e)
 - ((256*c^5*d^5 + 3*b^5*e^5 + 6*b^3*c*e^4*(b*d - 4*a*e) - 384*c^
4*d^3*e*(b*d - a*e) + 96*c^3*d*e^2*(b*d - a*e)^2 + 16*b*c^2*e^3*(
b^2*d^2 - 3*a*b*d*e + 3*a^2*e^2))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c
]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(7/2)*e^6) + (d^2*(c*d^2 - b*
d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^6)
```

Rubi [A] time = 2.15979, antiderivative size = 482, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241

$$-\frac{\left(16bc^2e^3\left(3a^2e^2-3abde+b^2d^2\right)+6b^3ce^4(bd-4ae)-384c^4d^3e(bd-ae)+96c^3de^2(bd-ae)^2+3b^5e^5+256c^5d^5\right)\tanh^{-1}}{512c^{7/2}e^6}$$
$$+\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(-3b^2e^2-6cex^2(be+2cd)-6bcde+16c^2d^2\right)}{96c^2e^3}$$
$$+\frac{\sqrt{a+bx^2+cx^4}\left(-2cex^2\left(-8c^2de(2bd-3ae)-6bce^2(bd-2ae)-3b^3e^3+32c^3d^3\right)+6b^2ce^3(bd-2ae)-32c^3d^2e(5bd-4a}{256c^3e^5}$$
$$+\frac{d^2\left(ae^2-bde+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^6}+\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10ce}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] ((128*c^4*d^4 + 3*b^4*e^4 - 32*c^3*d^2*e*(5*b*d - 4*a*e) + 8*b*c^
2*d*e^2*(2*b*d - 3*a*e) + 6*b^2*c*e^3*(b*d - 2*a*e) - 2*c*e*(32*c
^3*d^3 - 3*b^3*e^3 - 8*c^2*d*e*(2*b*d - 3*a*e) - 6*b*c*e^2*(b*d -
 2*a*e))*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*c^3*e^5) + ((16*c^2*d
^2 - 6*b*c*d*e - 3*b^2*e^2 - 6*c*e*(2*c*d + b*e)*x^2)*(a + b*x^2
+ c*x^4)^(3/2))/(96*c^2*e^3) + (a + b*x^2 + c*x^4)^(5/2)/(10*c*e)
 - ((256*c^5*d^5 + 3*b^5*e^5 + 6*b^3*c*e^4*(b*d - 4*a*e) - 384*c^
4*d^3*e*(b*d - a*e) + 96*c^3*d*e^2*(b*d - a*e)^2 + 16*b*c^2*e^3*(
b^2*d^2 - 3*a*b*d*e + 3*a^2*e^2))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c
]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(7/2)*e^6) + (d^2*(c*d^2 - b*
d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^6)
```

Rubi in Sympy [A] time = 178.164, size = 551, normalized size = 1.14

$$-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{32c^2e}+\frac{3b\left(b+2cx^2\right)\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}{256c^3e} - \frac{3b\left(-4ac+b^2\right)^2\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{\frac{7}{2}}e}+\frac{d^2\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{6e^3} - \frac{d^2\left(ae^2-bde+cd^2\right)^{\frac{3}{2}}\operatorname{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^6} + \frac{d^2\sqrt{a+bx^2+cx^4}\left(4ace^2+\frac{b^2e^2}{2}-5bcde+4c^2d^2+cex^2\left(be-2cd\right)\right)}{8ce^5} - \frac{d\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{16ce^2}+\frac{\left(a+bx^2+cx^4\right)^{\frac{5}{2}}}{10ce} + \frac{3d\left(b+2cx^2\right)\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}{128c^2e^2} - \frac{d^2\left(be-2cd\right)\left(-12ace^2+b^2e^2+8bcde-8c^2d^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{\frac{3}{2}}e^6} - \frac{3d\left(-4ac+b^2\right)^2\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{\frac{5}{2}}e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] -b*(b + 2*c*x**2)*(a + b*x**2 + c*x**4)**(3/2)/(32*c**2*e) + 3*b*
(b + 2*c*x**2)*(-4*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)/(256*c**
3*e) - 3*b*(-4*a*c + b**2)**2*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqr
t(a + b*x**2 + c*x**4)))/(512*c**(7/2)*e) + d**2*(a + b*x**2 + c*
x**4)**(3/2)/(6*e**3) - d**2*(a*e**2 - b*d*e + c*d**2)**(3/2)*ata
nh((2*a*e - b*d + x**2*(b*e - 2*c*d))/(2*sqrt(a + b*x**2 + c*x**4
```

```
)*sqrt(a*e**2 - b*d*e + c*d**2)))/(2*e**6) + d**2*sqrt(a + b*x**2
 + c*x**4)*(4*a*c*e**2 + b**2*e**2/2 - 5*b*c*d*e + 4*c**2*d**2 +
c*e*x**2*(b*e - 2*c*d))/(8*c*e**5) - d*(b + 2*c*x**2)*(a + b*x**2
 + c*x**4)**(3/2)/(16*c*e**2) + (a + b*x**2 + c*x**4)**(5/2)/(10*
c*e) + 3*d*(b + 2*c*x**2)*(-4*a*c + b**2)*sqrt(a + b*x**2 + c*x**
4)/(128*c**2*e**2) - d**2*(b*e - 2*c*d)*(-12*a*c*e**2 + b**2*e**2
 + 8*b*c*d*e - 8*c**2*d**2)*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(
a + b*x**2 + c*x**4)))/(32*c**(3/2)*e**6) - 3*d*(-4*a*c + b**2)**
2*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(25
6*c**(5/2)*e**2)
```

Mathematica [A] time = 1.10718, size = 492, normalized size = 1.02

$$\frac{\frac{2e\sqrt{a+bx^2+cx^4}\left(12c^2e^2\left(32a^2e^2+2abe\left(7ex^2-25d\right)+b^2\left(20d^2-5dex^2+2e^2x^4\right)\right)-30b^2ce^3\left(10ae-3bd+bex^2\right)-16c^3e\left(ae\left(-160d^2+75dex^2-48e^2x^4\right)+b\left(150d^3-70a\right.\right.\right.}{c^3}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] ((2*e*Sqrt[a + b*x^2 + c*x^4]*(45*b^4*e^4 - 30*b^2*c*e^3*(-3*b*d
+ 10*a*e + b*e*x^2) + 32*c^4*(60*d^4 - 30*d^3*e*x^2 + 20*d^2*e^2*
x^4 - 15*d*e^3*x^6 + 12*e^4*x^8) + 12*c^2*e^2*(32*a^2*e^2 + 2*a*b
*e*(-25*d + 7*e*x^2) + b^2*(20*d^2 - 5*d*e*x^2 + 2*e^2*x^4)) - 16
*c^3*e*(a*e*(-160*d^2 + 75*d*e*x^2 - 48*e^2*x^4) + b*(150*d^3 - 7
0*d^2*e*x^2 + 45*d*e^2*x^4 - 33*e^3*x^6))))/c^3 + 3840*d^2*(c*d^2
 + e*(-(b*d) + a*e))^(3/2)*Log[d + e*x^2] - (15*(256*c^5*d^5 + 3*
b^5*e^5 + 6*b^3*c*e^4*(b*d - 4*a*e) - 384*c^4*d^3*e*(b*d - a*e) +
 96*c^3*d*e^2*(b*d - a*e)^2 + 16*b*c^2*e^3*(b^2*d^2 - 3*a*b*d*e +
 3*a^2*e^2))*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]
)/c^(7/2) - 3840*d^2*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*Log[-(b*d)
+ 2*a*e - 2*c*d*x^2 + b*e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqr
t[a + b*x^2 + c*x^4]])/(7680*e^6)
```

Maple [B] time = 0.045, size = 2068, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x)
```

```
[Out] d^3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)
/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((
```

```
x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2))/(x^2+d/e))*a*b-d^4/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*
(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*c+d^5/e^6/((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e
)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*
(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*c+1/10/e*a
^2/c*(c*x^4+b*x^2+a)^(1/2)+1/10/e*c*x^8*(c*x^4+b*x^2+a)^(1/2)+11/
80/e*b*x^6*(c*x^4+b*x^2+a)^(1/2)+3/256/e/c^3*b^4*(c*x^4+b*x^2+a)^
(1/2)-3/512/e/c^(7/2)*b^5*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a
)^(1/2))+1/5/e*a*x^4*(c*x^4+b*x^2+a)^(1/2)-5/8*d^3/e^4*b*(c*x^4+b
*x^2+a)^(1/2)+2/3*d^2/e^3*(c*x^4+b*x^2+a)^(1/2)*a+1/2*d^4/e^5*c*(
c*x^4+b*x^2+a)^(1/2)-1/2*d^5/e^6*c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)
+(c*x^4+b*x^2+a)^(1/2))-1/128/e/c^2*b^3*x^2*(c*x^4+b*x^2+a)^(1/2)
+3/64/e/c^(5/2)*b^3*a*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1
/2))+7/160/e/c*b*a*x^2*(c*x^4+b*x^2+a)^(1/2)-1/64/e^2*d/c*b^2*x^2
*(c*x^4+b*x^2+a)^(1/2)+3/32/e^2*d/c^(3/2)*b^2*a*ln((1/2*b+c*x^2)/
c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-5/32/e^2*d/c*b*a*(c*x^4+b*x^2+a)^(
1/2)+3/8*d^2/e^3*a/c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+
a)^(1/2))*b-5/64/e/c^2*b^2*a*(c*x^4+b*x^2+a)^(1/2)-3/16/e^2*d*a^2
*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-1/8/e^2*
d*c*x^6*(c*x^4+b*x^2+a)^(1/2)-3/32/e*a^2*b/c^(3/2)*ln((1/2*b+c*x^
2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/160/e/c*b^2*x^4*(c*x^4+b*x^2+
a)^(1/2)+7/24*d^2/e^3*x^2*(c*x^4+b*x^2+a)^(1/2)*b-1/4*d^3/e^4*x^2
*c*(c*x^4+b*x^2+a)^(1/2)+1/16*d^2/e^3*b^2/c*(c*x^4+b*x^2+a)^(1/2)
-1/32*d^2/e^3*b^3/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a
)^(1/2))-3/16*d^3/e^4*b^2/c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4
+b*x^2+a)^(1/2))-3/4*d^3/e^4*a*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(
c*x^4+b*x^2+a)^(1/2))+3/4*d^4/e^5*b*c^(1/2)*ln((1/2*b+c*x^2)/c^(1
/2)+(c*x^4+b*x^2+a)^(1/2))+1/6*d^2/e^3*c*x^4*(c*x^4+b*x^2+a)^(1/2
)-1/2*d^2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+
c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2))/(x^2+d/e))*a^2-1/2*d^4/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)
+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b^2-1/2*d^6/e^7/((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c
*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+
(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e)
)*c^2-3/16/e^2*d*b*x^4*(c*x^4+b*x^2+a)^(1/2)+3/128/e^2*d/c^2*b^3*
(c*x^4+b*x^2+a)^(1/2)-3/256/e^2*d/c^(5/2)*b^4*ln((1/2*b+c*x^2)/c^
(1/2)+(c*x^4+b*x^2+a)^(1/2))-5/16/e^2*d*a*x^2*(c*x^4+b*x^2+a)^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^5/(e*x^2 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^5/(e*x^2 + d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} x^5}{ex^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^5/(e*x^2 + d),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^5/(e*x^2 + d), x)
```

3.315 $\int \frac{x^3\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx$

Optimal. Leaf size=360

$$\frac{\left(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}e^5}$$
$$-\frac{\sqrt{a+bx^2+cx^4}\left(-2cex^2\left(-4ce(2bd-3ae)-3b^2e^2+16c^2d^2\right)-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)+3b^3e^3+64c^3d^3\right)}{128c^2e^4}$$
$$-\frac{d\left(ae^2-bde+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^5}-\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(-3be+8cd-6cex^2\right)}{48ce^2}$$

```
[Out] -((64*c^3*d^3 + 3*b^3*e^3 - 16*c^2*d*e*(5*b*d - 4*a*e) + 4*b*c*e^
2*(2*b*d - 3*a*e) - 2*c*e*(16*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d
- 3*a*e))*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*c^2*e^4) - ((8*c*d -
 3*b*e - 6*c*e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(48*c*e^2) + ((128
*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*d - 3*a*e) - 192*c^3*d^2*e*
(b*d - a*e) + 48*c^2*e^2*(b*d - a*e)^2)*ArcTanh[(b + 2*c*x^2)/(2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(256*c^(5/2)*e^5) - (d*(c*d^2
- b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/
(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^5)
```

Rubi [A] time = 1.49269, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$\frac{\left(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}e^5}$$
$$-\frac{\sqrt{a+bx^2+cx^4}\left(-2cex^2\left(-4ce(2bd-3ae)-3b^2e^2+16c^2d^2\right)-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)+3b^3e^3+64c^3d^3\right)}{128c^2e^4}$$
$$-\frac{d\left(ae^2-bde+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^5}-\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(-3be+8cd-6cex^2\right)}{48ce^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```


```
[Out] -((64*c^3*d^3 + 3*b^3*e^3 - 16*c^2*d*e*(5*b*d - 4*a*e) + 4*b*c*e^
2*(2*b*d - 3*a*e) - 2*c*e*(16*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d
- 3*a*e))*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*c^2*e^4) - ((8*c*d -
 3*b*e - 6*c*e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(48*c*e^2) + ((128
*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*d - 3*a*e) - 192*c^3*d^2*e*
(b*d - a*e) + 48*c^2*e^2*(b*d - a*e)^2)*ArcTanh[(b + 2*c*x^2)/(2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(256*c^(5/2)*e^5) - (d*(c*d^2
```

```
- b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/
(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^5)
```

Rubi in Sympy [A] time = 127.517, size = 388, normalized size = 1.08

$$\frac{d\left(ae^2-bde+cd^2\right)^{\frac{3}{2}}\operatorname{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^5}+\frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\left(\frac{3be}{2}-4cd+3cex^2\right)}{24ce^2}$$
$$-\frac{\sqrt{a+bx^2+cx^4}\left(-3abce^3+16ac^2de^2+\frac{3b^3e^3}{4}+2b^2cde^2-20bc^2d^2e+16c^3d^3+\frac{cex^2(3b^2e^2-16c^2d^2-4ce(3ae-2bd))}{2}\right)}{32c^2e^4}$$
$$+\frac{\left(-cde\left(be-2cd\right)\left(4ace+b\left(3be-8cd\right)\right)+\left(\frac{b^2e^2}{4}-2c^2d^2-ce\left(ae-bd\right)\right)\left(3b^2e^2-16c^2d^2-4ce\left(3ae-2bd\right)\right)\right)\operatorname{atanh}\left(\frac{b}{2\sqrt{c}\sqrt{}}\right.}{64c^{\frac{5}{2}}e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] d*(a*e**2 - b*d*e + c*d**2)**(3/2)*atanh((2*a*e - b*d + x**2*(b*e
 - 2*c*d))/(2*sqrt(a + b*x**2 + c*x**4)*sqrt(a*e**2 - b*d*e + c*d
**2)))/(2*e**5) + (a + b*x**2 + c*x**4)**(3/2)*(3*b*e/2 - 4*c*d +
 3*c*e*x**2)/(24*c*e**2) - sqrt(a + b*x**2 + c*x**4)*(-3*a*b*c*e*
*3 + 16*a*c**2*d*e**2 + 3*b**3*e**3/4 + 2*b**2*c*d*e**2 - 20*b*c*
*2*d**2*e + 16*c**3*d**3 + c*e*x**2*(3*b**2*e**2 - 16*c**2*d**2 -
 4*c*e*(3*a*e - 2*b*d))/2)/(32*c**2*e**4) + (-c*d*e*(b*e - 2*c*d)
*(4*a*c*e + b*(3*b*e - 8*c*d)) + (b**2*e**2/4 - 2*c**2*d**2 - c*e
*(a*e - b*d))*(3*b**2*e**2 - 16*c**2*d**2 - 4*c*e*(3*a*e - 2*b*d)
))*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(6
4*c**(5/2)*e**5)
```

Mathematica [A] time = 0.473627, size = 376, normalized size = 1.04

$$\frac{-2\sqrt{c}e\sqrt{a+bx^2+cx^4}\left(-8c^2e\left(ae\left(15ex^2-32d\right)+b\left(30d^2-14dex^2+9e^2x^4\right)\right)-6bce^2\left(10ae-4bd+bex^2\right)+9b^3e^3+16c^3\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] (-2*Sqrt[c]*e*Sqrt[a + b*x^2 + c*x^4]*(9*b^3*e^3 - 6*b*c*e^2*(-4*
b*d + 10*a*e + b*e*x^2) + 16*c^3*(12*d^3 - 6*d^2*e*x^2 + 4*d*e^2*
x^4 - 3*e^3*x^6) - 8*c^2*e*(a*e*(-32*d + 15*e*x^2) + b*(30*d^2 -
14*d*e*x^2 + 9*e^2*x^4))) - 384*c^(5/2)*d*(c*d^2 + e*(-(b*d) + a*
```

```
e))^(3/2)*Log[d + e*x^2] + 3*(128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e
^3*(b*d - 3*a*e) - 192*c^3*d^2*e*(b*d - a*e) + 48*c^2*e^2*(b*d -
a*e)^2)*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]] + 38
4*c^(5/2)*d*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*Log[-(b*d) + 2*a*e -
 2*c*d*x^2 + b*e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x
^2 + c*x^4]])/(768*c^(5/2)*e^5)
```

Maple [B] time = 0.017, size = 1696, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x)
```

```
[Out] 3/16/e*b*x^4*(c*x^4+b*x^2+a)^(1/2)-3/128/e/c^2*b^3*(c*x^4+b*x^2+a
)^(1/2)+3/256/e/c^(5/2)*b^4*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2
+a)^(1/2))+5/16/e*a*x^2*(c*x^4+b*x^2+a)^(1/2)+3/16/e*a^2*ln((1/2*
b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+1/8/e*c*x^6*(c*x^
4+b*x^2+a)^(1/2)+5/8*d^2/e^3*b*(c*x^4+b*x^2+a)^(1/2)-2/3*d/e^2*(c
*x^4+b*x^2+a)^(1/2)*a-1/2*d^3/e^4*c*(c*x^4+b*x^2+a)^(1/2)+1/2*d^4
/e^5*c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-3/4*
d^3/e^4*b*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
-1/6*d/e^2*c*x^4*(c*x^4+b*x^2+a)^(1/2)-7/24*d/e^2*x^2*(c*x^4+b*x^
2+a)^(1/2)*b-1/16*d/e^2*b^2/c*(c*x^4+b*x^2+a)^(1/2)+1/32*d/e^2*b^
3/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/16*d^
2/e^3*b^2/c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
+3/4*d^2/e^3*a*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(
1/2))-3/32/e/c^(3/2)*b^2*a*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+
a)^(1/2))+5/32/e/c*b*a*(c*x^4+b*x^2+a)^(1/2)+1/64/e/c*b^2*x^2*(c*
x^4+b*x^2+a)^(1/2)+1/2*d^3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln
((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^
2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b^2+1/2*d^5/e^6/((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*
(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2
*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c^2+
1/4*d^2/e^3*x^2*c*(c*x^4+b*x^2+a)^(1/2)-3/8*d/e^2*a/c^(1/2)*ln((1
/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b-d^2/e^3/((a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(
x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*
c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*b+d
^3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/
e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x
^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
))/(x^2+d/e))*a*c-d^4/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(
a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*c+1/2*d/e^2/((a*e^2-b*d*e+c*d^
```

```
2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/
e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e
*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^3/(e*x^2 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^3/(e*x^2 + d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x^3/(e*x^2 + d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.316 $\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2} dx$

Optimal. Leaf size=269

$$\frac{\sqrt{a+bx^2+cx^4}\left(-2ce(5bd-4ae)+b^2e^2-2cex^2(2cd-be)+8c^2d^2\right)}{16ce^3} - \frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}e^4} + \frac{\left(ae^2-bde+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^4} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6e}$$

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*
e)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c*e^3) + (a + b*x^2 + c*x^4)
^(3/2)/(6*e) - ((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d
 - 3*a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(32*c^(3/2)*e^4) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(
b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*S
qrt[a + b*x^2 + c*x^4])])/(2*e^4)
```

Rubi [A] time = 1.00379, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259

$$\frac{\sqrt{a+bx^2+cx^4}\left(-2ce(5bd-4ae)+b^2e^2-2cex^2(2cd-be)+8c^2d^2\right)}{16ce^3} - \frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}e^4} + \frac{\left(ae^2-bde+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^4} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6e}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```


```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*
e)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c*e^3) + (a + b*x^2 + c*x^4)
^(3/2)/(6*e) - ((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d
 - 3*a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(32*c^(3/2)*e^4) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(
b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*S
qrt[a + b*x^2 + c*x^4])])/(2*e^4)
```

Rubi in Sympy [A] time = 109.583, size = 253, normalized size = 0.94

$$\frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{6e} - \frac{\left(ae^2-bde+cd^2\right)^{\frac{3}{2}}\operatorname{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^4} + \frac{\sqrt{a+bx^2+cx^4}\left(4ace^2+\frac{b^2e^2}{2}-5bcde+4c^2d^2+cex^2(be-2cd)\right)}{8ce^3} - \frac{(be-2cd)\left(-12ace^2+b^2e^2+8bcde-8c^2d^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{\frac{3}{2}}e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] (a + b*x**2 + c*x**4)**(3/2)/(6*e) - (a*e**2 - b*d*e + c*d**2)**(
3/2)*atanh((2*a*e - b*d + x**2*(b*e - 2*c*d))/(2*sqrt(a + b*x**2
+ c*x**4)*sqrt(a*e**2 - b*d*e + c*d**2)))/(2*e**4) + sqrt(a + b*x
**2 + c*x**4)*(4*a*c*e**2 + b**2*e**2/2 - 5*b*c*d*e + 4*c**2*d**2
 + c*e*x**2*(b*e - 2*c*d))/(8*c*e**3) - (b*e - 2*c*d)*(-12*a*c*e*
*2 + b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*atanh((b + 2*c*x**2)/(2
*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(32*c**(3/2)*e**4)
```

Mathematica [A] time = 0.416168, size = 276, normalized size = 1.03

$$\frac{2e\sqrt{a+bx^2+cx^4}\left(2ce\left(16ae-15bd+7bex^2\right)+3b^2e^2+4c^2\left(6d^2-3dex^2+2e^2x^4\right)\right)}{c} - \frac{3(2cd-be)\left(4ce(3ae-2bd)-b^2e^2+8c^2d^2\right)\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{c^{3/2}} - 4$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] ((2*e*Sqrt[a + b*x^2 + c*x^4]*(3*b^2*e^2 + 2*c*e*(-15*b*d + 16*a*
e + 7*b*e*x^2) + 4*c^2*(6*d^2 - 3*d*e*x^2 + 2*e^2*x^4)))/c + 48*(
c*d^2 + e*(-(b*d) + a*e))^(3/2)*Log[d + e*x^2] - (3*(2*c*d - b*e)
*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*Log[b + 2*c*x^2 +
 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/c^(3/2) - 48*(c*d^2 + e*(-(b
*d) + a*e))^(3/2)*Log[-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2 + 2*Sq
rt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]])/(96*e^4)
```

Maple [B] time = 0.011, size = 1411, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x)
```

```
[Out] 7/24/e*x^2*(c*x^4+b*x^2+a)^(1/2)*b-1/4/e^2*x^2*c*(c*x^4+b*x^2+a)^
(1/2)*d+1/16/e*b^2/c*(c*x^4+b*x^2+a)^(1/2)-5/8/e^2*b*(c*x^4+b*x^2
+a)^(1/2)*d-1/32/e*b^3/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*
x^2+a)^(1/2))-3/16/e^2*b^2/c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^
4+b*x^2+a)^(1/2))*d+3/8/e*a/c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x
^4+b*x^2+a)^(1/2))*b-3/4/e^2*a*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(
c*x^4+b*x^2+a)^(1/2))*d+2/3/e*(c*x^4+b*x^2+a)^(1/2)*a+1/2/e^3*c*(
c*x^4+b*x^2+a)^(1/2)*d^2+3/4/e^3*b*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/
2)+(c*x^4+b*x^2+a)^(1/2))*d^2+1/6/e*c*x^4*(c*x^4+b*x^2+a)^(1/2)-1
/2/e^4*c^(3/2)*d^3*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2)
)-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)
/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((
x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2))/(x^2+d/e))*a^2+1/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a
*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*
e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*b*d-1/e^3/((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+
2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x
^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*c*d^2-1/2/e^
3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(
b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/
e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x
^2+d/e))*b^2*d^2+1/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e
^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+
c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*c*d^3-1/2/e^5/((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e
)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*
(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c^2*d^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^(3/2)*x/(e*x^2 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x/(e*x^2 + d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x/(e*x^2 + d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.317 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)}\,dx$

Optimal. Leaf size=350

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\left(-12cde(bd-ae)+be^2(3bd-4ae)+8c^2d^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}de^3}-\frac{\sqrt{a+bx^2+cx^4}\left(-e(5bd-4ae)+4cd^2-2cdex^2\right)}{8de^2}-\frac{\left(ae^2-bde+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2de^3}+\frac{a\sqrt{a+bx^2+cx^4}}{2d}+\frac{ab\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d}$$

```
[Out] (a*Sqrt[a + b*x^2 + c*x^4])/(2*d) - ((4*c*d^2 - e*(5*b*d - 4*a*e)
 - 2*c*d*e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*d*e^2) - (a^(3/2)*Arc
Tanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d) +
(a*b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/
(4*Sqrt[c]*d) + ((8*c^2*d^3 + b*e^2*(3*b*d - 4*a*e) - 12*c*d*e*(b
*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(16*Sqrt[c]*d*e^3) - ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh
[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x^2 + c*x^4])])/(2*d*e^3)
```

Rubi [A] time = 1.27014, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\left(-12cde(bd-ae)+be^2(3bd-4ae)+8c^2d^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}de^3}-\frac{\sqrt{a+bx^2+cx^4}\left(-e(5bd-4ae)+4cd^2-2cdex^2\right)}{8de^2}-\frac{\left(ae^2-bde+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2de^3}+\frac{a\sqrt{a+bx^2+cx^4}}{2d}+\frac{ab\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x]
```

```
[Out] (a*Sqrt[a + b*x^2 + c*x^4])/(2*d) - ((4*c*d^2 - e*(5*b*d - 4*a*e)
 - 2*c*d*e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*d*e^2) - (a^(3/2)*Arc
Tanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d) +
(a*b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/
(4*Sqrt[c]*d) + ((8*c^2*d^3 + b*e^2*(3*b*d - 4*a*e) - 12*c*d*e*(b
*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(16*Sqrt[c]*d*e^3) - ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh
[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x^2 + c*x^4])])/(2*d*e^3)
```

Rubi in Sympy [A] time = 130.837, size = 332, normalized size = 0.95

$$-\frac{a^{\frac{3}{2}}\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{ab\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d}+\frac{a\sqrt{a+bx^2+cx^4}}{2d}-\frac{\sqrt{a+bx^2+cx^4}\left(2ae^2-\frac{5bde}{2}+2cd^2-cdex^2\right)}{4de^2}+\frac{\left(ae^2-bde+cd^2\right)^{\frac{3}{2}}\operatorname{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2de^3}+\frac{\left(-4abe^3+12acde^2+3b^2de^2-12bcd^2e+8c^2d^3\right)\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(3/2)/x/(e*x**2+d),x)
```

```
[Out] -a**(3/2)*atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**
4)))/(2*d) + a*b*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2
+ c*x**4)))/(4*sqrt(c)*d) + a*sqrt(a + b*x**2 + c*x**4)/(2*d) - s
qrt(a + b*x**2 + c*x**4)*(2*a*e**2 - 5*b*d*e/2 + 2*c*d**2 - c*d*e
*x**2)/(4*d*e**2) + (a*e**2 - b*d*e + c*d**2)**(3/2)*atanh((2*a*e
 - b*d + x**2*(b*e - 2*c*d))/(2*sqrt(a + b*x**2 + c*x**4)*sqrt(a*
e**2 - b*d*e + c*d**2)))/(2*d*e**3) + (-4*a*b*e**3 + 12*a*c*d*e**
2 + 3*b**2*d*e**2 - 12*b*c*d**2*e + 8*c**2*d**3)*atanh((b + 2*c*x
**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(16*sqrt(c)*d*e**3)
```

Mathematica [A] time = 2.25722, size = 292, normalized size = 0.83

$$\frac{-\frac{8a^{3/2}e^3\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)}{d}+\frac{8a^{3/2}e^3\log(x^2)}{d}+\frac{(12ce(ae-bd)+3b^2e^2+8c^2d^2)\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{\sqrt{c}}+\frac{8\left(e(ae-bd)+cd^2\right)^{3/2}\log(\cdots}{\cdots}}{16e^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x]
```

```
[Out] (2*e*(-4*c*d + 5*b*e + 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4] + (8*a^
(3/2)*e^3*Log[x^2])/d - (8*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*Log[d
 + e*x^2])/d - (8*a^(3/2)*e^3*Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a
+ b*x^2 + c*x^4]])/d + ((8*c^2*d^2 + 3*b^2*e^2 + 12*c*e*(-(b*d) +
 a*e))*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/Sqrt
[c] + (8*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*Log[-(b*d) + 2*a*e - 2*
c*d*x^2 + b*e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2
+ c*x^4]])/d)/(16*e^3)
```

Maple [B] time = 0.017, size = 1270, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x)
```

```
[Out] 3/16/e*b^2/c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2)
)+3/4/e*a*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
-1/2*d/e^2*c*(c*x^4+b*x^2+a)^(1/2)+1/2*d^2/e^3*c^(3/2)*ln((1/2*b+
c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/2/d/((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+
2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x
^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a^2+1/4/e*x^2*
c*(c*x^4+b*x^2+a)^(1/2)-3/4*d/e^2*b*c^(1/2)*ln((1/2*b+c*x^2)/c^(1
/2)+(c*x^4+b*x^2+a)^(1/2))-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln
((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^
2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*b+1/2*d/e^2/((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x
^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c
*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b^2+1/
2*d^3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^
2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2))/(x^2+d/e))*c^2+d/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*
(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c
```

```
*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*c-d^2/e^3/((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e
)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*
(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*c+5/8/e*b*
(c*x^4+b*x^2+a)^(1/2)-1/2/d*a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^
4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{(ex^2 + d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x\left(d + ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(3/2)/x/(e*x**2+d),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/(x*(d + e*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{(ex^2 + d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x), x)
```

3.318 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx$

Optimal. Leaf size=562

$$\frac{a^{3/2}e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}+\frac{be\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2}+\frac{\sqrt{a+bx^2+cx^4}\left(-2ce(5bd-4ae)+b^2e^2-2cex^2(2cd-be)+8c^2d^2\right)}{16cd^2e}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2e^2}-\frac{e\left(8ac+b^2+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16cd^2}+\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}d}+\frac{\left(ae^2-bde+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2e^2}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{2dx^2}+\frac{3\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8d}-\frac{3\sqrt{a}b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4d}$$

```
[Out] (3*(3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*d) - (e*(b^2 + 8*a
*c + 2*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c*d^2) + ((8*c^2*d^2
 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x^2)*Sqr
t[a + b*x^2 + c*x^4])/(16*c*d^2*e) - (a + b*x^2 + c*x^4)^(3/2)/(2
*d*x^2) - (3*Sqrt[a]*b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a +
b*x^2 + c*x^4])])/(4*d) + (a^(3/2)*e*ArcTanh[(2*a + b*x^2)/(2*Sqr
t[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2) + (3*(b^2 + 4*a*c)*ArcTan
h[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*Sqrt[c]
*d) + (b*(b^2 - 12*a*c)*e*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
 + b*x^2 + c*x^4])])/(32*c^(3/2)*d^2) - ((2*c*d - b*e)*(8*c^2*d^2
 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(3/2)*d^2*e^2) + ((c*d^2 - b
*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*
Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2*e^2
)
```

Rubi [A] time = 2.19164, antiderivative size = 562, normalized size of antiderivative = 1., number of

steps used = 24, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.31$

$$\frac{a^{3/2}e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}+\frac{be\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2}+\frac{\sqrt{a+bx^2+cx^4}\left(-2ce(5bd-4ae)+b^2e^2-2cex^2(2cd-be)+8c^2d^2\right)}{16cd^2e}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2e^2}-\frac{e\left(8ac+b^2+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16cd^2}+\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}d}+\frac{\left(ae^2-bde+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2e^2}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{2dx^2}+\frac{3\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8d}-\frac{3\sqrt{a}b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x]
```

```
[Out] (3*(3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*d) - (e*(b^2 + 8*a
*c + 2*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c*d^2) + ((8*c^2*d^2
 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x^2)*Sqr
t[a + b*x^2 + c*x^4])/(16*c*d^2*e) - (a + b*x^2 + c*x^4)^(3/2)/(2
*d*x^2) - (3*Sqrt[a]*b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a +
b*x^2 + c*x^4])])/(4*d) + (a^(3/2)*e*ArcTanh[(2*a + b*x^2)/(2*Sqr
t[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2) + (3*(b^2 + 4*a*c)*ArcTan
h[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*Sqrt[c]
*d) + (b*(b^2 - 12*a*c)*e*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
 + b*x^2 + c*x^4])])/(32*c^(3/2)*d^2) - ((2*c*d - b*e)*(8*c^2*d^2
 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(3/2)*d^2*e^2) + ((c*d^2 - b
*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*
Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2*e^2
)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(3/2)/x**3/(e*x**2+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.76583, size = 284, normalized size = 0.51

$$\frac{1}{2}\sqrt{a+bx^2+cx^4}\left(\frac{c}{e}-\frac{a}{dx^2}\right) - \frac{2\left(e(ae-bd)+cd^2\right)^{3/2}\log\left(2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex^2-2cdx^2\right)-2\log\left(d+ex^2\right)\left(e(ae-bd)\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x]
```

```
[Out] ((c/e - a/(d*x^2))*Sqrt[a + b*x^2 + c*x^4])/2 - (Sqrt[a]*e^2*(-3*
b*d + 2*a*e)*Log[x^2] - 2*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*Log[d
+ e*x^2] - Sqrt[a]*e^2*(-3*b*d + 2*a*e)*Log[2*a + b*x^2 + 2*Sqrt[
a]*Sqrt[a + b*x^2 + c*x^4]] + Sqrt[c]*d^2*(2*c*d - 3*b*e)*Log[b +
 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]] + 2*(c*d^2 + e*(-(b
*d) + a*e))^(3/2)*Log[-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2 + 2*Sq
rt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]])/(4*d^2*e^2)
```

Maple [B] time = 0.018, size = 1207, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x)
```

```
[Out] 3/4/e*b*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1
/2*d/e^2*c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-
1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e
^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^
2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)/(x^2+d/e))*b^2-1/2/d*a/x^2*(c*x^4+b*x^2+a)^(1/2)-3/4/d*a^(1/2)*
b*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-1/2/d^2*e/(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e
-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^
2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+
d/e))*a^2+1/d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+
c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2))/(x^2+d/e))*a*b-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((
2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e
```

```
+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*c-1/2*d^2/e^3/((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x
^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c
*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c^2+d/
e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2
+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+
d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
(x^2+d/e))*b*c+1/2/e*c*(c*x^4+b*x^2+a)^(1/2)+1/2/d^2*e*a^(3/2)*ln
((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{(ex^2 + d)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x^3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x^3\left(d + ex^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**3/(e*x**2+d),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/(x**3*(d + e*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{(ex^2 + d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x^3),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x^3), x)
```

3.319 $\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx$

Optimal. Leaf size=463

$$-\frac{1}{14}\left(2x^4+2x^2+1\right)^{3/2}x-\frac{2211\sqrt{2x^4+2x^2+1}\,x}{140\sqrt{2}\left(\sqrt{2}x^2+1\right)}$$
$$-\frac{213}{140}\sqrt{2x^4+2x^2+1}\,x+\frac{17}{16}\sqrt{51}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$-\frac{3\left(514+2717\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{140\ 2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$+\frac{2211\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{140\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{289\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12+11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{16\ 2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{27}{70}\sqrt{2x^4+2x^2+1}\,x^3$$

```
[Out] (-213*x*Sqrt[1 + 2*x^2 + 2*x^4])/140 - (27*x^3*Sqrt[1 + 2*x^2 + 2
*x^4])/70 - (2211*x*Sqrt[1 + 2*x^2 + 2*x^4])/(140*Sqrt[2]*(1 + Sq
rt[2]*x^2)) - (x*(1 + 2*x^2 + 2*x^4)^(3/2))/14 + (17*Sqrt[51]*Arc
Tanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/16 + (2211*(1 + Sqr
t[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE
[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(140*2^(3/4)*Sqrt[1 + 2*x
^2 + 2*x^4]) - (3*(514 + 2717*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1
+ 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*
x], (2 - Sqrt[2])/4])/(140*2^(3/4)*(2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2
 + 2*x^4]) - (289*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*Sqrt[2])/24, 2
*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(16*2^(3/4)*(2 + 3*Sqrt[2])
*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 1.1918, antiderivative size = 888, normalized size of antiderivative = 1.92, number

of steps used = 18, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$

$$
\begin{aligned}
&-\frac{1}{14}x\left(2x^4+2x^2+1\right)^{3/2}-\frac{3}{35}x\left(x^2+2\right)\sqrt{2x^4+2x^2+1}-\frac{3}{20}x\left(2x^2+9\right)\sqrt{2x^4+2x^2+1} \\
&-\frac{6\sqrt{2}x\sqrt{2x^4+2x^2+1}}{35\left(\sqrt{2}x^2+1\right)}-\frac{309x\sqrt{2x^4+2x^2+1}}{20\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{17}{16}\sqrt{51}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right) \\
&+\frac{6\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{35\sqrt{2x^4+2x^2+1}} \\
&+\frac{309\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{20\,2^{3/4}\sqrt{2x^4+2x^2+1}} \\
&-\frac{3\left(9+8\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{20\,2^{3/4}\sqrt{2x^4+2x^2+1}} \\
&+\frac{867\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\,2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}} \\
&-\frac{3\left(3+2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{70\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} \\
&-\frac{51\left(5+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{16\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} \\
&-\frac{289\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12+11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{16\,2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}
\end{aligned}
$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(x^2*(1 + 2*x^2 + 2*x^4)^(3/2))/(3 - 2*x^2),x]
```

```
[Out] (-3*x*(2 + x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/35 - (3*x*(9 + 2*x^2)*Sq
rt[1 + 2*x^2 + 2*x^4])/20 - (309*x*Sqrt[1 + 2*x^2 + 2*x^4])/(20*S
qrt[2]*(1 + Sqrt[2]*x^2)) - (6*Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])
/(35*(1 + Sqrt[2]*x^2)) - (x*(1 + 2*x^2 + 2*x^4)^(3/2))/14 + (17*
Sqrt[51]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/16 + (3
09*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2
]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(20*2^(3/4)*Sq
rt[1 + 2*x^2 + 2*x^4]) + (6*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2
*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x],
 (2 - Sqrt[2])/4])/(35*Sqrt[1 + 2*x^2 + 2*x^4]) - (51*(5 + Sqrt[2
])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2
```

```
]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(16*2^(1/4)*Sq
rt[1 + 2*x^2 + 2*x^4]) - (3*(3 + 2*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(
1/4)*x], (2 - Sqrt[2])/4])/(70*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) +
 (867*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2
)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(3/4)*
(2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - (3*(9 + 8*Sqrt[2])*(1
+ Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elli
pticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(20*2^(3/4)*Sqrt[1 +
 2*x^2 + 2*x^4]) - (289*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 +
 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*Sqrt[2])
/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(16*2^(3/4)*(2 + 3*Sq
rt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 134.069, size = 796, normalized size = 1.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(2*x**4+2*x**2+1)**(3/2)/(-2*x**2+3),x)
```

```
[Out] -x*(6*x**2 + 2)*sqrt(2*x**4 + 2*x**2 + 1)/20 - x*(12*x**2 + 24)*s
qrt(2*x**4 + 2*x**2 + 1)/140 - x*(2*x**4 + 2*x**2 + 1)**(3/2)/14
- 5*x*sqrt(2*x**4 + 2*x**2 + 1)/4 - 2211*sqrt(2)*x*sqrt(2*x**4 +
2*x**2 + 1)/(280*(sqrt(2)*x**2 + 1)) + 2211*2**(1/4)*sqrt((2*x**4
 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*elliptic
_e(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(280*sqrt(2*x**4 + 2*x**
2 + 1)) - 51*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 +
1)**2)*(4 + 10*sqrt(2))*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(
1/4)*x), -sqrt(2)/4 + 1/2)/(64*sqrt(2*x**4 + 2*x**2 + 1)) - 5*2**
(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(2 + 2*sq
rt(2))*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)
/4 + 1/2)/(16*sqrt(2*x**4 + 2*x**2 + 1)) - 2**(1/4)*sqrt((2*x**4
+ 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(48 + 36*sqrt(2))*(sqrt(2)*x
**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(560*sq
rt(2*x**4 + 2*x**2 + 1)) - 2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(s
qrt(2)*x**2 + 1)**2)*(-2*sqrt(2) + 4)*(sqrt(2)*x**2 + 1)*elliptic
_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(80*sqrt(2*x**4 + 2*x**2
 + 1)) + 867*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 +
1)**2)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)
/4 + 1/2)/(16*(2 + 3*sqrt(2))*sqrt(2*x**4 + 2*x**2 + 1)) + 289*2*
*(3/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(-3*sqrt
(2) + 2)*(sqrt(2)*x**2 + 1)*elliptic_pi(1/2 + 11*sqrt(2)/24, 2*at
an(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(64*(2 + 3*sqrt(2))*sqrt(2*x**4
 + 2*x**2 + 1)) + 17*sqrt(51)*atanh(sqrt(51)*x/(3*sqrt(2*x**4 + 2
*x**2 + 1)))/16
```

Mathematica [C] time = 0.213384, size = 214, normalized size = 0.46

$$\frac{-160x^9-752x^7-2456x^5-2080x^3-(9669-5247i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)+4422i\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(1 + 2*x^2 + 2*x^4)^(3/2))/(3 - 2*x^2),x]
```

```
[Out] (-892*x - 2080*x^3 - 2456*x^5 - 752*x^7 - 160*x^9 + (4422*I)*Sqrt
[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*A
rcSinh[Sqrt[1 - I]*x], I] - (9669 - 5247*I)*Sqrt[1 - I]*Sqrt[1 +
(1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I
]*x], I] + 10115*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1
+ I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(5
60*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.032, size = 547, normalized size = 1.2

$$-\frac{x^5}{7}\sqrt{2x^4+2x^2+1}-\frac{37x^3}{70}\sqrt{2x^4+2x^2+1}-\frac{223x}{140}\sqrt{2x^4+2x^2+1}$$
$$-\frac{9\,EllipticF\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{35\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\left(\frac{6}{35}-\frac{6i}{35}\right)\left(EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)-EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)\right)}{\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2}}$$
$$-\frac{531\,EllipticF\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{20\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\frac{309i}{40}EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{309\,EllipticE\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{40\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\frac{309i}{40}EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{289}{8\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}EllipticPi\left(x\sqrt{-1+i},-\frac{1}{3}-\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x)
```

```
[Out] -1/7*x^5*(2*x^4+2*x^2+1)^(1/2)-37/70*x^3*(2*x^4+2*x^2+1)^(1/2)-22
3/140*x*(2*x^4+2*x^2+1)^(1/2)-9/35/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/
2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(
1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(6/35-6/35*I)/(-1+I)^(1/2)*(1+(1-
I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(Elliptic
F(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1
/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-531/20/(-1+I)^(1/2)*(-I*x^2+x^2+1
)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1
+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-309/40*I/(-1+I)^(1/2)*(-I*x^
2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Elliptic
F(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-309/40/(-1+I)^(1/2)*(
-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ell
ipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+309/40*I/(-1+I)^
(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1
/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+289/8/(-1
+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1
)^(1/2)*EllipticPi(x*(-1+I)^(1/2),-1/3-1/3*I,(-1-I)^(1/2)/(-1+I)^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}x^2}{2\,x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)*x^2/(2*x^2 - 3),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)*x^2/(2*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(2\,x^6+2\,x^4+x^2\right)\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)*x^2/(2*x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^6 + 2*x^4 + x^2)*sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 -
3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx-\int \frac{2x^6\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(2*x**4+2*x**2+1)**(3/2)/(-2*x**2+3),x)
```

```
[Out] -Integral(x**2*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**4*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(
2*x**6*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}x^2}{2\,x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)*x^2/(2*x^2 - 3),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)*x^2/(2*x^2 - 3), x)
```

3.320 $\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx$

Optimal. Leaf size=428

$$-\frac{1}{10}\left(2x^2+9\right)\sqrt{2x^4+2x^2+1}\,x-\frac{103\sqrt{2x^4+2x^2+1}\,x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{17}{8}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$-\frac{\left(66+383\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\,2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$+\frac{103\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\,2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{289\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12+11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{24\,2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

```
[Out] -(x*(9 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/10 - (103*x*Sqrt[1 + 2*x
^2 + 2*x^4])/(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (17*Sqrt[17/3]*ArcT
anh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/8 + (103*(1 + Sqrt[2
]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*
ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[1 + 2*x^2 +
 2*x^4]) - ((66 + 383*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 -
 Sqrt[2])/4])/(10*2^(3/4)*(2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]
) - (289*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)
/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2
^(1/4)*x], (2 - Sqrt[2])/4])/(24*2^(3/4)*(2 + 3*Sqrt[2])*Sqrt[1 +
 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.653905, antiderivative size = 615, normalized size of antiderivative = 1.44, number

of steps used = 11, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{1}{10}\left(2x^2+9\right)\sqrt{2x^4+2x^2+1}x-\frac{103\sqrt{2x^4+2x^2+1}x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{17}{8}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$-\frac{\left(9+8\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{289\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\ 2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{17\left(5+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{103\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{289\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12+11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{24\ 2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(3 - 2*x^2),x]
```

```
[Out] -(x*(9 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/10 - (103*x*Sqrt[1 + 2*x
^2 + 2*x^4])/(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (17*Sqrt[17/3]*ArcT
anh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/8 + (103*(1 + Sqrt[2
]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*
ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[1 + 2*x^2 +
 2*x^4]) - (17*(5 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - S
qrt[2])/4])/(8*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (289*(1 + Sqrt[
2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2
*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(4*2^(3/4)*(2 + 3*Sqrt[2])*
Sqrt[1 + 2*x^2 + 2*x^4]) - ((9 + 8*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(
1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) -
 (289*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1
 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(1
/4)*x], (2 - Sqrt[2])/4])/(24*2^(3/4)*(2 + 3*Sqrt[2])*Sqrt[1 + 2*
x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 97.2721, size = 663, normalized size = 1.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**4+2*x**2+1)**(3/2)/(-2*x**2+3),x)
```

```
[Out] -x*(6*x**2 + 2)*sqrt(2*x**4 + 2*x**2 + 1)/30 - 5*x*sqrt(2*x**4 +
2*x**2 + 1)/6 - 103*sqrt(2)*x*sqrt(2*x**4 + 2*x**2 + 1)/(20*(sqrt
(2)*x**2 + 1)) + 103*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)
*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*elliptic_e(2*atan(2**(1/4)*x),
-sqrt(2)/4 + 1/2)/(20*sqrt(2*x**4 + 2*x**2 + 1)) - 17*2**(1/4)*sq
rt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(4 + 10*sqrt(2))*
(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/
2)/(32*sqrt(2*x**4 + 2*x**2 + 1)) - 5*2**(1/4)*sqrt((2*x**4 + 2*x
**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(2 + 2*sqrt(2))*(sqrt(2)*x**2 + 1
)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(24*sqrt(2*x**
4 + 2*x**2 + 1)) - 2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x
**2 + 1)**2)*(-2*sqrt(2) + 4)*(sqrt(2)*x**2 + 1)*elliptic_f(2*ata
n(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(120*sqrt(2*x**4 + 2*x**2 + 1))
+ 289*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*
(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/
2)/(8*(2 + 3*sqrt(2))*sqrt(2*x**4 + 2*x**2 + 1)) + 289*2**(3/4)*s
qrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(-3*sqrt(2) + 2)
*(sqrt(2)*x**2 + 1)*elliptic_pi(1/2 + 11*sqrt(2)/24, 2*atan(2**(1
/4)*x), -sqrt(2)/4 + 1/2)/(96*(2 + 3*sqrt(2))*sqrt(2*x**4 + 2*x**
2 + 1)) + 17*sqrt(51)*atanh(sqrt(51)*x/(3*sqrt(2*x**4 + 2*x**2 +
1)))/24
```

Mathematica [C] time = 0.198059, size = 209, normalized size = 0.49

$$\frac{-48x^7 - 264x^5 - 240x^3 - (1371 - 753i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right) + 618i\sqrt{1-i}\sqrt{1+(1-}{120\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(3 - 2*x^2),x]
```

```
[Out] (-108*x - 240*x^3 - 264*x^5 - 48*x^7 + (618*I)*Sqrt[1 - I]*Sqrt[1
 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1
- I]*x], I] - (1371 - 753*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sq
rt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] + 1445
*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*Ellipt
icPi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(120*Sqrt[1 + 2*x^
2 + 2*x^4])
```

Maple [C] time = 0.009, size = 377, normalized size = 0.9

$$-\frac{x^3}{5}\sqrt{2x^4+2x^2+1}-\frac{9x}{10}\sqrt{2x^4+2x^2+1}$$
$$-\frac{177\,EllipticF\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{10\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\frac{103i}{20}EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{103\,EllipticE\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{20\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\frac{103i}{20}EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{289}{12\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}EllipticPi\left(x\sqrt{-1+i},-\frac{1}{3}-\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x)
```

```
[Out] -1/5*x^3*(2*x^4+2*x^2+1)^(1/2)-9/10*x*(2*x^4+2*x^2+1)^(1/2)-177/1
0/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*
x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-
103/20*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2
*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^
(1/2))-103/20/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/
2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2
*I*2^(1/2))+103/20*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2
+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1
/2)+1/2*I*2^(1/2))+289/12/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^
2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),-1
/3-1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}{2x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{2\,x^2-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx - \int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx - \int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**4+2*x**2+1)**(3/2)/(-2*x**2+3),x)
```

```
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2
*x**2*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**
4*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{2\,x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3), x)
```

3.321 $\int \frac{(1+2x^2+2x^4)^{3/2}}{x^2(3-2x^2)}\,dx$

Optimal. Leaf size=735

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{3\left(\sqrt{2}x^2+1\right)}-\frac{17\sqrt{2x^4+2x^2+1}\,x}{3\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{\left(x^2+1\right)\sqrt{2x^4+2x^2+1}}{3x}+\frac{17}{12}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{289\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{6\,2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}-\frac{17\left(5+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\,2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{2x^4+2x^2+1}}+\frac{17\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\,2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{289\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12+11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{36\,2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

```
[Out] -((1 + x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/(3*x) - (17*x*Sqrt[1 + 2*x^2
 + 2*x^4])/(3*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (Sqrt[2]*x*Sqrt[1 + 2*
x^2 + 2*x^4])/(3*(1 + Sqrt[2]*x^2)) + (17*Sqrt[17/3]*ArcTanh[(Sqr
t[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/12 + (17*(1 + Sqrt[2]*x^2)*S
qrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2
^(1/4)*x], (2 - Sqrt[2])/4])/(3*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4])
- (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2
]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*Sqr
t[1 + 2*x^2 + 2*x^4]) + ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^
4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[
2])/4])/(3*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (17*(5 + Sqrt[2])*(
1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*El
lipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(1/4)*Sqrt[1
 + 2*x^2 + 2*x^4]) + (289*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x
^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt
```

```
[2])/4])/(6*2^(3/4)*(2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - (2
89*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)
*x], (2 - Sqrt[2])/4])/(36*2^(3/4)*(2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2
 + 2*x^4])
```

Rubi [A] time = 0.758078, antiderivative size = 735, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}x}{3\left(\sqrt{2}x^2+1\right)}-\frac{17\sqrt{2x^4+2x^2+1}x}{3\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{\left(x^2+1\right)\sqrt{2x^4+2x^2+1}}{3x}+\frac{17}{12}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{289\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{6\,2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}-\frac{17\left(5+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\,2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{2x^4+2x^2+1}}+\frac{17\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\,2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{289\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12+11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{36\,2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^2*(3 - 2*x^2)),x]
```

```
[Out] -((1 + x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/(3*x) - (17*x*Sqrt[1 + 2*x^2
 + 2*x^4])/(3*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (Sqrt[2]*x*Sqrt[1 + 2*
x^2 + 2*x^4])/(3*(1 + Sqrt[2]*x^2)) + (17*Sqrt[17/3]*ArcTanh[(Sqr
t[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/12 + (17*(1 + Sqrt[2]*x^2)*S
```

```
qrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2
^(1/4)*x], (2 - Sqrt[2])/4])/(3*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4])
- (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2
]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*Sqr
t[1 + 2*x^2 + 2*x^4]) + ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^
4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[
2])/4])/(3*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (17*(5 + Sqrt[2])*(
1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*El
lipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(1/4)*Sqrt[1
 + 2*x^2 + 2*x^4]) + (289*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x
^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt
[2])/4])/(6*2^(3/4)*(2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - (2
89*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)
*x], (2 - Sqrt[2])/4])/(36*2^(3/4)*(2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2
 + 2*x^4])
```

Rubi in Sympy [A] time = 62.9018, size = 544, normalized size = 0.74

$$-\frac{5\sqrt{2}x\sqrt{2x^4+2x^2+1}}{2\left(\sqrt{2}x^2+1\right)}+\frac{5\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)E\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{2\sqrt{2x^4+2x^2+1}}$$
$$-\frac{17\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(4+10\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{48\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{6\sqrt{2x^4+2x^2+1}}$$
$$+\frac{289\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{12\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$+\frac{289\cdot 2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(-3\sqrt{2}+2\right)\left(\sqrt{2}x^2+1\right)\left(\frac{1}{2}+\frac{11\sqrt{2}}{24};2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{144\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$+\frac{17\sqrt{51}\operatorname{atanh}\left(\frac{\sqrt{51}x}{3\sqrt{2x^4+2x^2+1}}\right)}{36}-\frac{\left(6x^2+6\right)\sqrt{2x^4+2x^2+1}}{18x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**4+2*x**2+1)**(3/2)/x**2/(-2*x**2+3),x)
```

```
[Out] -5*sqrt(2)*x*sqrt(2*x**4 + 2*x**2 + 1)/(2*(sqrt(2)*x**2 + 1)) + 5
*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt
```

```
(2)*x**2 + 1)*elliptic_e(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(2
*sqrt(2*x**4 + 2*x**2 + 1)) - 17*2**(1/4)*sqrt((2*x**4 + 2*x**2 +
 1)/(sqrt(2)*x**2 + 1)**2)*(4 + 10*sqrt(2))*(sqrt(2)*x**2 + 1)*el
liptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(48*sqrt(2*x**4 +
2*x**2 + 1)) + 2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2
+ 1)**2)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(
2)/4 + 1/2)/(6*sqrt(2*x**4 + 2*x**2 + 1)) + 289*2**(1/4)*sqrt((2*
x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*elli
ptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(12*(2 + 3*sqrt(2))*
sqrt(2*x**4 + 2*x**2 + 1)) + 289*2**(3/4)*sqrt((2*x**4 + 2*x**2 +
 1)/(sqrt(2)*x**2 + 1)**2)*(-3*sqrt(2) + 2)*(sqrt(2)*x**2 + 1)*el
liptic_pi(1/2 + 11*sqrt(2)/24, 2*atan(2**(1/4)*x), -sqrt(2)/4 + 1
/2)/(144*(2 + 3*sqrt(2))*sqrt(2*x**4 + 2*x**2 + 1)) + 17*sqrt(51)
*atanh(sqrt(51)*x/(3*sqrt(2*x**4 + 2*x**2 + 1)))/36 - (6*x**2 + 6
)*sqrt(2*x**4 + 2*x**2 + 1)/(18*x)
```

Mathematica [C] time = 0.195391, size = 213, normalized size = 0.29

$$\frac{-24x^6-48x^4-36x^2-(255-165i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}xF\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)+90i\sqrt{1-i}\sqrt{1+(1-i)x}}{36x\sqrt{2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^2*(3 - 2*x^2)),x]
```

```
[Out] (-12 - 36*x^2 - 48*x^4 - 24*x^6 + (90*I)*Sqrt[1 - I]*x*Sqrt[1 + (
1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]
*x], I] - (255 - 165*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[
1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] + 289*(1
- I)^(3/2)*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*Elliptic
Pi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(36*x*Sqrt[1 + 2*x^2
 + 2*x^4])
```

Maple [C] time = 0.021, size = 528, normalized size = 0.7

$$-\frac{1}{3x}\sqrt{2x^4+2x^2+1}-\frac{x}{3}\sqrt{2x^4+2x^2+1}$$
$$+\frac{16\,EllipticF\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{15\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\left(\frac{14}{15}-\frac{14i}{15}\right)\left(EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)-EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)\right)}{\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2}}$$
$$-\frac{59\,EllipticF\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{5\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\frac{103i}{30}EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{103\,EllipticE\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{30\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\frac{103i}{30}EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{289}{18\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}EllipticPi\left(x\sqrt{-1+i},-\frac{1}{3}-\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x)
```

```
[Out] -1/3*(2*x^4+2*x^2+1)^(1/2)/x-1/3*x*(2*x^4+2*x^2+1)^(1/2)+16/15/(-
1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1
)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-14/
15+14/15*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/
(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I
*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-59
/5/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2
*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))
-103/30*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(
2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2
^(1/2))-103/30/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1
/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/
2*I*2^(1/2))+103/30*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^
2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(
1/2)+1/2*I*2^(1/2))+289/18/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x
^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),-
1/3-1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{(2\,x^2-3)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{2\,x^4-3\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^2),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^4 - 3*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{2x^4+2x^2+1}}{2x^4-3x^2}\,dx-\int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^4-3x^2}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^4-3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**4+2*x**2+1)**(3/2)/x**2/(-2*x**2+3),x)
```

```
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**4 - 3*x**2), x) - Integ
ral(2*x**2*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**4 - 3*x**2), x) - Inte
gral(2*x**4*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**4 - 3*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{(2\,x^2-3)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^2), x)
```

3.322 $\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4(3-2x^2)}\,dx$

Optimal. Leaf size=638

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{9\left(\sqrt{2}x^2+1\right)}-\frac{2\sqrt{2x^4+2x^2+1}}{x}+\frac{17}{18}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$+\frac{\sqrt[4]{2}\left(9+5\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\sqrt{2x^4+2x^2+1}}$$
$$+\frac{289\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\,2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{17\left(5+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{18\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\sqrt{2x^4+2x^2+1}}$$
$$-\frac{289\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12+11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{54\,2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\left(1-8x^2\right)\sqrt{2x^4+2x^2+1}}{9x^3}$$

```
[Out] (-2*Sqrt[1 + 2*x^2 + 2*x^4])/x - ((1 - 8*x^2)*Sqrt[1 + 2*x^2 + 2*
x^4])/(9*x^3) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(9*(1 + Sqrt[
2]*x^2)) + (17*Sqrt[17/3]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 +
 2*x^4]])/18 - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4
)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2
])/4])/(9*Sqrt[1 + 2*x^2 + 2*x^4]) - (17*(5 + Sqrt[2])*(1 + Sqrt[
2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2
*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(18*2^(1/4)*Sqrt[1 + 2*x^2
+ 2*x^4]) + (289*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/
(9*2^(3/4)*(2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + (2^(1/4)*(9
 + 5*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(9*
Sqrt[1 + 2*x^2 + 2*x^4]) - (289*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*S
qrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*
Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(54*2^(3/4)*(
2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.798781, antiderivative size = 638, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}x}{9\left(\sqrt{2}x^2+1\right)}-\frac{2\sqrt{2x^4+2x^2+1}}{x}+\frac{17}{18}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{\sqrt[4]{2}\left(9+5\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\sqrt{2x^4+2x^2+1}}+\frac{289\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\,2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}-\frac{17\left(5+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{18\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\sqrt{2x^4+2x^2+1}}-\frac{289\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12+11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{54\,2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}-\frac{\left(1-8x^2\right)\sqrt{2x^4+2x^2+1}}{9x^3}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^4*(3 - 2*x^2)),x]
```

```
[Out] (-2*Sqrt[1 + 2*x^2 + 2*x^4])/x - ((1 - 8*x^2)*Sqrt[1 + 2*x^2 + 2*
x^4])/(9*x^3) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(9*(1 + Sqrt[
2]*x^2)) + (17*Sqrt[17/3]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 +
 2*x^4]])/18 - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4
)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2
])/4])/(9*Sqrt[1 + 2*x^2 + 2*x^4]) - (17*(5 + Sqrt[2])*(1 + Sqrt[
2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2
*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(18*2^(1/4)*Sqrt[1 + 2*x^2
+ 2*x^4]) + (289*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/
(9*2^(3/4)*(2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + (2^(1/4)*(9
 + 5*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(9*
Sqrt[1 + 2*x^2 + 2*x^4]) - (289*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*S
qrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*
Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(54*2^(3/4)*(
```

```
2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 67.2975, size = 570, normalized size = 0.89

$$\frac{\sqrt{2}x\sqrt{2x^4+2x^2+1}}{9\left(\sqrt{2}x^2+1\right)} - \frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)E\left(2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{9\sqrt{2x^4+2x^2+1}} - \frac{17\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(4+10\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)F\left(2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{72\sqrt{2x^4+2x^2+1}} + \frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(60\sqrt{2}+108\right)\left(\sqrt{2}x^2+1\right)F\left(2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{108\sqrt{2x^4+2x^2+1}} + \frac{289\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{18\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}} + \frac{289\cdot 2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(-3\sqrt{2}+2\right)\left(\sqrt{2}x^2+1\right)\left(\frac{1}{2}+\frac{11\sqrt{2}}{24};2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{216\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}} + \frac{17\sqrt{51}\,\mathrm{atanh}\left(\frac{\sqrt{51}x}{3\sqrt{2x^4+2x^2+1}}\right)}{54} - \frac{2\sqrt{2x^4+2x^2+1}}{x} - \frac{\left(-24x^2+3\right)\sqrt{2x^4+2x^2+1}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**4+2*x**2+1)**(3/2)/x**4/(-2*x**2+3),x)
```

```
[Out] sqrt(2)*x*sqrt(2*x**4 + 2*x**2 + 1)/(9*(sqrt(2)*x**2 + 1)) - 2**(
1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x
**2 + 1)*elliptic_e(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(9*sqrt
(2*x**4 + 2*x**2 + 1)) - 17*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(
sqrt(2)*x**2 + 1)**2)*(4 + 10*sqrt(2))*(sqrt(2)*x**2 + 1)*ellipti
c_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(72*sqrt(2*x**4 + 2*x**
2 + 1)) + 2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)*
*2)*(60*sqrt(2) + 108)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1
/4)*x), -sqrt(2)/4 + 1/2)/(108*sqrt(2*x**4 + 2*x**2 + 1)) + 289*2
**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2
)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(18*
(2 + 3*sqrt(2))*sqrt(2*x**4 + 2*x**2 + 1)) + 289*2**(3/4)*sqrt((2
*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(-3*sqrt(2) + 2)*(sqrt
(2)*x**2 + 1)*elliptic_pi(1/2 + 11*sqrt(2)/24, 2*atan(2**(1/4)*x)
, -sqrt(2)/4 + 1/2)/(216*(2 + 3*sqrt(2))*sqrt(2*x**4 + 2*x**2 + 1
)) + 17*sqrt(51)*atanh(sqrt(51)*x/(3*sqrt(2*x**4 + 2*x**2 + 1)))/
```

```
54 - 2*sqrt(2*x**4 + 2*x**2 + 1)/x - (-24*x**2 + 3)*sqrt(2*x**4 +
 2*x**2 + 1)/(27*x**3)
```

Mathematica [C] time = 0.205584, size = 219, normalized size = 0.34

$$\frac{-120x^6-132x^4-72x^2-(195-201i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}x^3F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)-6i\sqrt{1-i}\sqrt{1+(1-}{54x^3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^4*(3 - 2*x^2)),x]
```

```
[Out] (-6 - 72*x^2 - 132*x^4 - 120*x^6 - (6*I)*Sqrt[1 - I]*x^3*Sqrt[1 +
 (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 -
I]*x], I] - (195 - 201*I)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x^2]*S
qrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] + 289
*(1 - I)^(3/2)*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*El
lipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(54*x^3*Sqrt[1
 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.024, size = 530, normalized size = 0.8

$$-\frac{1}{9x^3}\sqrt{2x^4+2x^2+1}-\frac{10}{9x}\sqrt{2x^4+2x^2+1}$$
$$+\frac{44\,EllipticF\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{15\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\left(\frac{12}{5}-\frac{12i}{5}\right)\left(EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)-EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)\right)}{\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2}}$$
$$-\frac{118\,EllipticF\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{15\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\frac{103i}{45}EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{103\,EllipticE\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{45\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\frac{103i}{45}EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{289}{27\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}EllipticPi\left(x\sqrt{-1+i},-\frac{1}{3}-\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x)
```

```
[Out] -1/9*(2*x^4+2*x^2+1)^(1/2)/x^3-10/9*(2*x^4+2*x^2+1)^(1/2)/x+44/15
/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^
2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-
12/5+12/5*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)
/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*
I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-1
18/15/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^
4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/
2))-103/45*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2
)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*
I*2^(1/2))-103/45/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)
^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)
+1/2*I*2^(1/2))+103/45*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2
+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*
2^(1/2)+1/2*I*2^(1/2))+289/27/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(
I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2
),-1/3-1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{(2\,x^2-3)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^4),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{2\,x^6-3\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^4),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^6 - 3*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{2x^4+2x^2+1}}{2x^6-3x^4}\,dx-\int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^6-3x^4}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^6-3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**4+2*x**2+1)**(3/2)/x**4/(-2*x**2+3),x)
```

```
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**6 - 3*x**4), x) - Integ
ral(2*x**2*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**6 - 3*x**4), x) - Inte
gral(2*x**4*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**6 - 3*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{(2\,x^2-3)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^4),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^4), x)
```

3.323 $\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx$

Optimal. Leaf size=665

$$\frac{262\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{135\left(\sqrt{2}x^2+1\right)}-\frac{262\sqrt{2x^4+2x^2+1}}{135x}+\frac{17}{27}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$+\frac{2^{3/4}\left(37+23\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{135\sqrt{2x^4+2x^2+1}}$$
$$+\frac{289\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{27\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{17\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{27\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{262\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{135\sqrt{2x^4+2x^2+1}}$$
$$-\frac{289\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12+11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{81\,2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\left(40x^2+3\right)\sqrt{2x^4+2x^2+1}}{45x^5}+\frac{74\sqrt{2x^4+2x^2+1}}{135x^3}$$

```
[Out] (74*Sqrt[1 + 2*x^2 + 2*x^4])/(135*x^3) - (262*Sqrt[1 + 2*x^2 + 2*
x^4])/(135*x) - ((3 + 40*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/(45*x^5) +
 (262*Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(135*(1 + Sqrt[2]*x^2))
+ (17*Sqrt[17/3]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])
/27 - (262*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1
+ Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4]
)/(135*Sqrt[1 + 2*x^2 + 2*x^4]) - (17*(3 - Sqrt[2])*(1 + Sqrt[2]*
x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*Ar
cTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(27*2^(1/4)*Sqrt[1 + 2*x^2 + 2
*x^4]) + (289*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/
(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])
/4])/(27*(2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + (2^(3/4)*(37
+ 23*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(13
5*Sqrt[1 + 2*x^2 + 2*x^4]) - (289*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)
*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 1
1*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(81*2^(3/4)
*(2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.957133, antiderivative size = 665, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$

$$\frac{262\sqrt{2}\sqrt{2x^4+2x^2+1}x}{135\left(\sqrt{2}x^2+1\right)}-\frac{262\sqrt{2x^4+2x^2+1}}{135x}+\frac{17}{27}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$+\frac{2^{3/4}\left(37+23\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{135\sqrt{2x^4+2x^2+1}}$$
$$+\frac{289\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{27\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{17\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{27\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{262\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{135\sqrt{2x^4+2x^2+1}}$$
$$-\frac{289\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12+11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{81\,2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\left(40x^2+3\right)\sqrt{2x^4+2x^2+1}}{45x^5}+\frac{74\sqrt{2x^4+2x^2+1}}{135x^3}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^6*(3 - 2*x^2)),x]
```

```
[Out] (74*Sqrt[1 + 2*x^2 + 2*x^4])/(135*x^3) - (262*Sqrt[1 + 2*x^2 + 2*
x^4])/(135*x) - ((3 + 40*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/(45*x^5) +
 (262*Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(135*(1 + Sqrt[2]*x^2))
+ (17*Sqrt[17/3]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])
/27 - (262*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1
+ Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4]
)/(135*Sqrt[1 + 2*x^2 + 2*x^4]) - (17*(3 - Sqrt[2])*(1 + Sqrt[2]*
x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*Ar
cTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(27*2^(1/4)*Sqrt[1 + 2*x^2 + 2
*x^4]) + (289*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/
(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])
/4])/(27*(2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + (2^(3/4)*(37
+ 23*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(13
5*Sqrt[1 + 2*x^2 + 2*x^4]) - (289*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)
```

```
*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 1
1*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(81*2^(3/4)
*(2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 86.9038, size = 597, normalized size = 0.9

$$\frac{262\sqrt{2}x\sqrt{2x^4+2x^2+1}}{135\left(\sqrt{2}x^2+1\right)} - \frac{262\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)E\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{135\sqrt{2x^4+2x^2+1}}$$
$$+\frac{17\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(-6\sqrt{2}+4\right)\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{108\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(184+148\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{540\sqrt{2x^4+2x^2+1}}$$
$$+\frac{289\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{27\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$+\frac{289\cdot 2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(-3\sqrt{2}+2\right)\left(\sqrt{2}x^2+1\right)\Pi\left(\frac{1}{2}+\frac{11\sqrt{2}}{24};2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{324\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$+\frac{17\sqrt{51}\operatorname{atanh}\left(\frac{\sqrt{51}x}{3\sqrt{2x^4+2x^2+1}}\right)}{81} - \frac{262\sqrt{2x^4+2x^2+1}}{135x}$$
$$+\frac{74\sqrt{2x^4+2x^2+1}}{135x^3} - \frac{\left(40x^2+3\right)\sqrt{2x^4+2x^2+1}}{45x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**4+2*x**2+1)**(3/2)/x**6/(-2*x**2+3),x)
```

```
[Out] 262*sqrt(2)*x*sqrt(2*x**4 + 2*x**2 + 1)/(135*(sqrt(2)*x**2 + 1))
- 262*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*
(sqrt(2)*x**2 + 1)*elliptic_e(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/
2)/(135*sqrt(2*x**4 + 2*x**2 + 1)) + 17*2**(1/4)*sqrt((2*x**4 + 2
*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(-6*sqrt(2) + 4)*(sqrt(2)*x**2
+ 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(108*sqrt(2
*x**4 + 2*x**2 + 1)) + 2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(
2)*x**2 + 1)**2)*(184 + 148*sqrt(2))*(sqrt(2)*x**2 + 1)*elliptic_
f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(540*sqrt(2*x**4 + 2*x**2
 + 1)) + 289*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 +
1)**2)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)
/4 + 1/2)/(27*(2 + 3*sqrt(2))*sqrt(2*x**4 + 2*x**2 + 1)) + 289*2*
```

```
*(3/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(-3*sqrt
(2) + 2)*(sqrt(2)*x**2 + 1)*elliptic_pi(1/2 + 11*sqrt(2)/24, 2*at
an(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(324*(2 + 3*sqrt(2))*sqrt(2*x**
4 + 2*x**2 + 1)) + 17*sqrt(51)*atanh(sqrt(51)*x/(3*sqrt(2*x**4 +
2*x**2 + 1)))/81 - 262*sqrt(2*x**4 + 2*x**2 + 1)/(135*x) + 74*sqr
t(2*x**4 + 2*x**2 + 1)/(135*x**3) - (40*x**2 + 3)*sqrt(2*x**4 + 2
*x**2 + 1)/(45*x**5)
```

Mathematica [C] time = 0.245698, size = 224, normalized size = 0.34

$$-\frac{1572x^8 + 1848x^6 + 1116x^4 + 192x^2 + (543 - 1329i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}x^5F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right) + 786i\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^6*(3 - 2*x^2)),x]
```

```
[Out] -(27 + 192*x^2 + 1116*x^4 + 1848*x^6 + 1572*x^8 + (786*I)*Sqrt[1
- I]*x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*
ArcSinh[Sqrt[1 - I]*x], I] + (543 - 1329*I)*Sqrt[1 - I]*x^5*Sqrt[
1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1
 - I]*x], I] - 1445*(1 - I)^(3/2)*x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[
1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x],
 I])/(405*x^5*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.027, size = 549, normalized size = 0.8

$$-\frac{1}{15\,x^5}\sqrt{2\,x^4+2\,x^2+1}-\frac{46}{135\,x^3}\sqrt{2\,x^4+2\,x^2+1}-\frac{262}{135\,x}\sqrt{2\,x^4+2\,x^2+1}$$
$$+\frac{184\,EllipticF\left(x\sqrt{-1+i},1/2\,\sqrt{2}+i/2\sqrt{2}\right)}{45\,\sqrt{-1+i}}\sqrt{1+(1-i)\,x^2}\sqrt{1+(1+i)\,x^2}\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}$$
$$-\frac{\left(\frac{52}{15}-\frac{52\,i}{15}\right)\left(EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)-EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)\right)}{\sqrt{-1+i}}\sqrt{1+(1-i)\,x^2}\sqrt{1+(1+i)\,x^2}\frac{1}{\sqrt{2\,x^4+2\,x^2}}$$
$$-\frac{236\,EllipticF\left(x\sqrt{-1+i},1/2\,\sqrt{2}+i/2\sqrt{2}\right)}{45\,\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}$$
$$-\frac{\frac{206\,i}{135}EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}$$
$$-\frac{206\,EllipticE\left(x\sqrt{-1+i},1/2\,\sqrt{2}+i/2\sqrt{2}\right)}{135\,\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}$$
$$+\frac{\frac{206\,i}{135}EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}$$
$$+\frac{578}{81\,\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}EllipticPi\left(x\sqrt{-1+i},-\frac{1}{3}-\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x)
```

```
[Out] -1/15*(2*x^4+2*x^2+1)^(1/2)/x^5-46/135*(2*x^4+2*x^2+1)^(1/2)/x^3-
262/135*(2*x^4+2*x^2+1)^(1/2)/x+184/45/(-1+I)^(1/2)*(1+(1-I)*x^2)
^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+
I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-52/15+52/15*I)/(-1+I)^(1/2)
*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(E
llipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-
1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-236/45/(-1+I)^(1/2)*(-I*x^
2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Elliptic
F(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-206/135*I/(-1+I)^(1/2
)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*
EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-206/135/(-1+I
)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^
(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+206/135
*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2
*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))
+578/81/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*
x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),-1/3-1/3*I,(-1-I)^(1
/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{(2\,x^2-3)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^6),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{2\,x^8-3\,x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^6),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^8 - 3*x^6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{2x^4+2x^2+1}}{2x^8-3x^6}\,dx-\int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^8-3x^6}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^8-3x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(3/2)/x**6/(-2*x**2+3),x)
```

```
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**8 - 3*x**6), x) - Integ
ral(2*x**2*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**8 - 3*x**6), x) - Inte
gral(2*x**4*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**8 - 3*x**6), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{(2\,x^2-3)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^6),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^6), x)
```

3.324 $\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=173

$$-\frac{(be+2cd)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2}+\frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2\sqrt{ae^2-bde+cd^2}}+\frac{\sqrt{a+bx^2+cx^4}}{2ce}$$

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c*e) - ((2*c*d + b*e)*ArcTanh[(b + 2*c
*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2)*e^2) + (d^
2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e
 + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^2*Sqrt[c*d^2 - b*d*e +
a*e^2])
```

Rubi [A] time = 0.756937, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$-\frac{(be+2cd)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2}+\frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2\sqrt{ae^2-bde+cd^2}}+\frac{\sqrt{a+bx^2+cx^4}}{2ce}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c*e) - ((2*c*d + b*e)*ArcTanh[(b + 2*c
*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2)*e^2) + (d^
2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e
 + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^2*Sqrt[c*d^2 - b*d*e +
a*e^2])
```

Rubi in Sympy [A] time = 62.1153, size = 190, normalized size = 1.1

$$-\frac{b\,\mathrm{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{\frac{3}{2}}e}-\frac{d^2\,\mathrm{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2\sqrt{ae^2-bde+cd^2}}+\frac{\sqrt{a+bx^2+cx^4}}{2ce}-\frac{d\,\mathrm{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -b*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(4
*c**(3/2)*e) - d**2*atanh((2*a*e - b*d + x**2*(b*e - 2*c*d))/(2*s
qrt(a + b*x**2 + c*x**4)*sqrt(a*e**2 - b*d*e + c*d**2)))/(2*e**2*
sqrt(a*e**2 - b*d*e + c*d**2)) + sqrt(a + b*x**2 + c*x**4)/(2*c*e
) - d*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))
/(2*sqrt(c)*e**2)
```

Mathematica [A] time = 0.75862, size = 196, normalized size = 1.13

$$\frac{-\frac{(be+2cd)\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{c^{3/2}}-\frac{2d^2\log\left(2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex^2-2cdx^2\right)}{\sqrt{ae^2-bde+cd^2}}+\frac{2d^2\log(d+ex^2)}{\sqrt{e(ae-bd)+cd^2}}+\frac{2e\sqrt{a+bx^2+cx^4}}{c}}{4e^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((2*e*Sqrt[a + b*x^2 + c*x^4])/c + (2*d^2*Log[d + e*x^2])/Sqrt[c*
d^2 + e*(-(b*d) + a*e)] - ((2*c*d + b*e)*Log[b + 2*c*x^2 + 2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4]])/c^(3/2) - (2*d^2*Log[-(b*d) + 2*a*e
 - 2*c*d*x^2 + b*e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b
*x^2 + c*x^4]])/Sqrt[c*d^2 - b*d*e + a*e^2])/(4*e^2)
```

Maple [A] time = 0.026, size = 267, normalized size = 1.5

$$\frac{1}{2\,ce}\sqrt{cx^4+bx^2+a}-\frac{b}{4\,e}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}$$
$$-\frac{d}{2\,e^2}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)\frac{1}{\sqrt{c}}$$
$$-\frac{d^2}{2\,e^3}\ln\left(1\left(2\,\frac{ae^2-bde+cd^2}{e^2}+\frac{be-2\,cd}{e}\left(x^2+\frac{d}{e}\right)+2\,\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{be-2\,cd}{e}\left(x^2+\frac{d}{e}\right)+\frac{ae^2-bd}{e^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*(c*x^4+b*x^2+a)^(1/2)/c/e-1/4/e*b/c^(3/2)*ln((1/2*b+c*x^2)/c^
(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/2/e^2*d*ln((1/2*b+c*x^2)/c^(1/2)+(
c*x^4+b*x^2+a)^(1/2))/c^(1/2)-1/2*d^2/e^3/((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2
+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 21.3666, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/8*(2*c^(3/2)*d^2*log(-(4*(b*c*d^3 + 3*a*b*d*e^2 - 2*a^2*e^3 -
(b^2 + 2*a*c)*d^2*e + (2*c^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 +
 2*a*c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a) + ((8*c^2*d^2 - 8*b*c
*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*
a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2)*s
qrt(c*d^2 - b*d*e + a*e^2))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + 4*sqrt
(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c)*e + sqrt(
c*d^2 - b*d*e + a*e^2)*(2*c*d + b*e)*log(4*sqrt(c*x^4 + b*x^2 + a
)*(2*c^2*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(
c)))/(sqrt(c*d^2 - b*d*e + a*e^2)*c^(3/2)*e^2), -1/8*(4*c^(3/2)*d
^2*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 +
b*d - 2*a*e)/(sqrt(c*x^4 + b*x^2 + a)*(c*d^2 - b*d*e + a*e^2))) -
 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c)*e
 - sqrt(-c*d^2 + b*d*e - a*e^2)*(2*c*d + b*e)*log(4*sqrt(c*x^4 +
b*x^2 + a)*(2*c^2*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a
*c)*sqrt(c)))/(sqrt(-c*d^2 + b*d*e - a*e^2)*c^(3/2)*e^2), 1/4*(sq
rt(-c)*c*d^2*log(-(4*(b*c*d^3 + 3*a*b*d*e^2 - 2*a^2*e^3 - (b^2 +
2*a*c)*d^2*e + (2*c^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)
*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a) + ((8*c^2*d^2 - 8*b*c*d*e +
(b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^
2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2)*sqrt(c*d
^2 - b*d*e + a*e^2))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + 2*sqrt(c*x^4
+ b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(-c)*e - sqrt(c*d^2
- b*d*e + a*e^2)*(2*c*d + b*e)*arctan(1/2*(2*c*x^2 + b)*sqrt(-c)/
(sqrt(c*x^4 + b*x^2 + a)*c)))/(sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(-
c)*c*e^2), -1/4*(2*sqrt(-c)*c*d^2*arctan(-1/2*sqrt(-c*d^2 + b*d*e
 - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(sqrt(c*x^4 + b*x^2 +
 a)*(c*d^2 - b*d*e + a*e^2))) - 2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c
```

```
*d^2 + b*d*e - a*e^2)*sqrt(-c)*e + sqrt(-c*d^2 + b*d*e - a*e^2)*(
2*c*d + b*e)*arctan(1/2*(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4 + b*x^
2 + a)*c)))/(sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(-c)*c*e^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] integrate(x^5/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

3.325 $\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=137

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}e}-\frac{d\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e\sqrt{ae^2-bde+cd^2}}$$

```
[Out] ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqr
t[c]*e) - (d*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*
d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e*Sqrt[c*d^2 -
 b*d*e + a*e^2])
```

Rubi [A] time = 0.445254, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}e}-\frac{d\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqr
t[c]*e) - (d*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*
d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e*Sqrt[c*d^2 -
 b*d*e + a*e^2])
```

Rubi in Sympy [A] time = 44.9327, size = 121, normalized size = 0.88

$$\frac{d\,\mathrm{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e\sqrt{ae^2-bde+cd^2}}+\frac{\mathrm{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] d*atanh((2*a*e - b*d + x**2*(b*e - 2*c*d))/(2*sqrt(a + b*x**2 + c
*x**4)*sqrt(a*e**2 - b*d*e + c*d**2)))/(2*e*sqrt(a*e**2 - b*d*e +
 c*d**2)) + atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x
```

```
**4)))/(2*sqrt(c)*e)
```

Mathematica [A] time = 0.283517, size = 161, normalized size = 1.18

$$-\frac{\frac{d\log\left(d+ex^2\right)}{\sqrt{ae^2-bde+cd^2}}-\frac{d\log\left(2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex^2-2cdx^2\right)}{\sqrt{ae^2-bde+cd^2}}-\frac{\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{\sqrt{c}}}{2e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -((d*Log[d + e*x^2])/Sqrt[c*d^2 - b*d*e + a*e^2] - Log[b + 2*c*x^
2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]/Sqrt[c] - (d*Log[-(b*d) +
2*a*e - 2*c*d*x^2 + b*e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[
a + b*x^2 + c*x^4]])/Sqrt[c*d^2 - b*d*e + a*e^2])/(2*e)
```

Maple [A] time = 0.014, size = 204, normalized size = 1.5

$$\frac{1}{2\,e}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)\frac{1}{\sqrt{c}}$$
$$+\frac{d}{2\,e^2}\ln\left(1\left(2\,\frac{ae^2-bde+cd^2}{e^2}+\frac{be-2\,cd}{e}\left(x^2+\frac{d}{e}\right)+2\,\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{be-2\,cd}{e}\left(x^2+\frac{d}{e}\right)+\frac{ae^2-bde}{e^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2/e*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+1/2
*d/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/
e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x
^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
))/(x^2+d/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42537, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(c)*d*log((4*(b*c*d^3 + 3*a*b*d*e^2 - 2*a^2*e^3 - (b^2
+ 2*a*c)*d^2*e + (2*c^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*
c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a) - ((8*c^2*d^2 - 8*b*c*d*e
+ (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*
d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2)*sqrt(c
*d^2 - b*d*e + a*e^2))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + sqrt(c*d^2
- b*d*e + a*e^2)*log(-4*sqrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 + b*c)
 - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)))/(sqrt(c*d^2 -
b*d*e + a*e^2)*sqrt(c)*e), 1/4*(2*sqrt(c)*d*arctan(-1/2*sqrt(-c*d
^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(sqrt(c*x^4
 + b*x^2 + a)*(c*d^2 - b*d*e + a*e^2))) + sqrt(-c*d^2 + b*d*e - a
*e^2)*log(-4*sqrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 + b*c) - (8*c^2*x
^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)))/(sqrt(-c*d^2 + b*d*e - a*
e^2)*sqrt(c)*e), 1/4*(sqrt(-c)*d*log((4*(b*c*d^3 + 3*a*b*d*e^2 -
2*a^2*e^3 - (b^2 + 2*a*c)*d^2*e + (2*c^2*d^3 - 3*b*c*d^2*e - a*b*
e^3 + (b^2 + 2*a*c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a) - ((8*c^2
*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2
 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)
*d*e)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2))/(e^2*x^4 + 2*d*e*x^2 + d^
2)) + 2*sqrt(c*d^2 - b*d*e + a*e^2)*arctan(1/2*(2*c*x^2 + b)*sqrt
(-c)/(sqrt(c*x^4 + b*x^2 + a)*c)))/(sqrt(c*d^2 - b*d*e + a*e^2)*s
qrt(-c)*e), 1/2*(sqrt(-c)*d*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e
^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(sqrt(c*x^4 + b*x^2 + a)*(c
*d^2 - b*d*e + a*e^2))) + sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(1/2
*(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a)*c)))/(sqrt(-c*d^
2 + b*d*e - a*e^2)*sqrt(-c)*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.326 $\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=86

$$\frac{\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\sqrt{ae^2-bde+cd^2}}$$

```
[Out] ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e +
 a*e^2]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rubi [A] time = 0.244218, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e +
 a*e^2]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rubi in Sympy [A] time = 30.9928, size = 78, normalized size = 0.91

$$-\frac{\operatorname{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\sqrt{ae^2-bde+cd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -atanh((2*a*e - b*d + x**2*(b*e - 2*c*d))/(2*sqrt(a + b*x**2 + c*
x**4)*sqrt(a*e**2 - b*d*e + c*d**2)))/(2*sqrt(a*e**2 - b*d*e + c*
d**2))
```

Mathematica [A] time = 0.0907648, size = 96, normalized size = 1.12

$$\frac{\log\left(d+ex^2\right)-\log\left(2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex^2-2cdx^2\right)}{2\sqrt{e(ae-bd)+cd^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Log[d + e*x^2] - Log[-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2 + 2*Sq
rt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]])/(2*Sqrt[c*d^2
 + e*(-(b*d) + a*e)])
```

Maple [B] time = 0.008, size = 165, normalized size = 1.9

$$-\frac{1}{2e}\ln\left(1\left(2\frac{ae^2-bde+cd^2}{e^2}+\frac{be-2cd}{e}\left(x^2+\frac{d}{e}\right)+2\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{be-2cd}{e}\left(x^2+\frac{d}{e}\right)+\frac{ae^2-bde}{e^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/
e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x
^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
))/(x^2+d/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.359839, size = 1, normalized size = 0.01

$$\left[\frac{\log\left(-\frac{4\left(bcd^3+3\,abde^2-2\,a^2e^3-\left(b^2+2\,ac\right)d^2e+\left(2\,c^2d^3-3\,bcd^2e-abe^3+\left(b^2+2\,ac\right)de^2\right)x^2\right)\sqrt{cx^4+bx^2+a}+\left(\left(8\,c^2d^2-8\,bcde+\left(b^2+4\,ac\right)e^2\right)x^4-8\,abde+8\,a^2e\right.}{e^2x^4+2\,dex^2+d^2}\right.}{4\sqrt{cd^2-bde+ae^2}}\right.$$

$$\left.-\frac{\arctan\left(-\frac{\sqrt{-cd^2+bde-ae^2}\left((2\,cd-be)x^2+bd-2\,ae\right)}{2\sqrt{cx^4+bx^2+a}(cd^2-bde+ae^2)}\right)}{2\sqrt{-cd^2+bde-ae^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/4*log(-(4*(b*c*d^3 + 3*a*b*d*e^2 - 2*a^2*e^3 - (b^2 + 2*a*c)*d
^2*e + (2*c^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*
x^2)*sqrt(c*x^4 + b*x^2 + a) + ((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4
*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4
*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2)*sqrt(c*d^2 - b*d
*e + a*e^2))/(e^2*x^4 + 2*d*e*x^2 + d^2))/sqrt(c*d^2 - b*d*e + a*
e^2), -1/2*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e
)*x^2 + b*d - 2*a*e)/(sqrt(c*x^4 + b*x^2 + a)*(c*d^2 - b*d*e + a*
e^2)))/sqrt(-c*d^2 + b*d*e - a*e^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [A] time = 0.287555, size = 101, normalized size = 1.17

$$\frac{\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/s
qrt(-c*d^2 + b*d*e - a*e^2))/sqrt(-c*d^2 + b*d*e - a*e^2)
```

3.327 $\int \frac{1}{x(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=138

$$-\frac{e\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d\sqrt{ae^2-bde+cd^2}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d}$$

```
[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sq
rt[a]*d) - (e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c
*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*Sqrt[c*d^2
- b*d*e + a*e^2])
```

Rubi [A] time = 0.520403, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$-\frac{e\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d\sqrt{ae^2-bde+cd^2}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sq
rt[a]*d) - (e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c
*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*Sqrt[c*d^2
- b*d*e + a*e^2])
```

Rubi in Sympy [A] time = 53.6563, size = 121, normalized size = 0.88

$$\frac{e\,\text{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d\sqrt{ae^2-bde+cd^2}}-\frac{\text{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] e*atanh((2*a*e - b*d + x**2*(b*e - 2*c*d))/(2*sqrt(a + b*x**2 + c
*x**4)*sqrt(a*e**2 - b*d*e + c*d**2)))/(2*d*sqrt(a*e**2 - b*d*e +
 c*d**2)) - atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x
```

```
**4)))/(2*sqrt(a)*d)
```

Mathematica [A] time = 0.942169, size = 174, normalized size = 1.26

$$\frac{-\frac{e\log(d+ex^2)}{\sqrt{ae^2-bde+cd^2}}+\frac{e\log\left(2\sqrt{a+x^2(b+cx^2)}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex^2-2cdx^2\right)}{\sqrt{ae^2-bde+cd^2}}-\frac{\log\left(2\sqrt{a}\sqrt{a+x^2(b+cx^2)}+2a+bx^2\right)}{\sqrt{a}}+\frac{\log(x^2)}{\sqrt{a}}}{2d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Log[x^2]/Sqrt[a] - (e*Log[d + e*x^2])/Sqrt[c*d^2 - b*d*e + a*e^2
] - Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + x^2*(b + c*x^2)]]/Sqrt[a
] + (e*Log[-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2 + 2*Sqrt[c*d^2 -
b*d*e + a*e^2]*Sqrt[a + x^2*(b + c*x^2)]])/Sqrt[c*d^2 - b*d*e + a
*e^2])/(2*d)
```

Maple [A] time = 0.017, size = 207, normalized size = 1.5

$$-\frac{1}{2\,d}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$
$$+\frac{1}{2\,d}\ln\left(1\left(2\,\frac{ae^2-bde+cd^2}{e^2}+\frac{be-2\,cd}{e}\left(x^2+\frac{d}{e}\right)+2\,\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{be-2\,cd}{e}\left(x^2+\frac{d}{e}\right)+\frac{ae^2-bde}{e^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2/d/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2
)+1/2/d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)
/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((
x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2))/(x^2+d/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}(ex^2+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x), x)
```

Fricas [A] time = 0.486653, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*e*log((4*(b*c*d^3 + 3*a*b*d*e^2 - 2*a^2*e^3 - (b^2
+ 2*a*c)*d^2*e + (2*c^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*
c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a) - ((8*c^2*d^2 - 8*b*c*d*e
+ (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*
d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2)*sqrt(c
*d^2 - b*d*e + a*e^2))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + sqrt(c*d^2
- b*d*e + a*e^2)*log((4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2)
 - ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4))/(sqrt(c
*d^2 - b*d*e + a*e^2)*sqrt(a)*d), 1/4*(2*sqrt(a)*e*arctan(-1/2*sq
rt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(sqr
t(c*x^4 + b*x^2 + a)*(c*d^2 - b*d*e + a*e^2))) + sqrt(-c*d^2 + b*
d*e - a*e^2)*log((4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2) - (
(b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4))/(sqrt(-c*d^
2 + b*d*e - a*e^2)*sqrt(a)*d), 1/4*(sqrt(-a)*e*log((4*(b*c*d^3 +
3*a*b*d*e^2 - 2*a^2*e^3 - (b^2 + 2*a*c)*d^2*e + (2*c^2*d^3 - 3*b*
c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2
+ a) - ((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d
*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (
3*b^2 + 4*a*c)*d*e)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2))/(e^2*x^4 +
2*d*e*x^2 + d^2)) - 2*sqrt(c*d^2 - b*d*e + a*e^2)*arctan(1/2*(b*x
^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)*a)))/(sqrt(c*d^2 - b*
d*e + a*e^2)*sqrt(-a)*d), 1/2*(sqrt(-a)*e*arctan(-1/2*sqrt(-c*d^2
 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(sqrt(c*x^4 +
 b*x^2 + a)*(c*d^2 - b*d*e + a*e^2))) - sqrt(-c*d^2 + b*d*e - a*e
^2)*arctan(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)*a)
))/(sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(-a)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.328 $\int \frac{1}{x^3(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=218

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d}+\frac{e^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2\sqrt{ae^2-bde+cd^2}}+\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d^2}-\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}$$

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*a*d*x^2) + (b*ArcTanh[(2*a + b*x^2)/(
2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2)*d) + (e*ArcTanh[(
2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a]*d^2
) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2
- b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2*Sqrt[c*d^2 - b
*d*e + a*e^2])
```

Rubi [A] time = 0.701368, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d}+\frac{e^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2\sqrt{ae^2-bde+cd^2}}+\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d^2}-\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*a*d*x^2) + (b*ArcTanh[(2*a + b*x^2)/(
2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2)*d) + (e*ArcTanh[(
2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a]*d^2
) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2
- b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2*Sqrt[c*d^2 - b
*d*e + a*e^2])
```

Rubi in Sympy [A] time = 64.3852, size = 194, normalized size = 0.89

$$-\frac{e^2\operatorname{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2\sqrt{ae^2-bde+cd^2}}-\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}+\frac{e\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d^2}+\frac{b\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -e**2*atanh((2*a*e - b*d + x**2*(b*e - 2*c*d))/(2*sqrt(a + b*x**2
 + c*x**4)*sqrt(a*e**2 - b*d*e + c*d**2)))/(2*d**2*sqrt(a*e**2 -
b*d*e + c*d**2)) - sqrt(a + b*x**2 + c*x**4)/(2*a*d*x**2) + e*ata
nh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**4)))/(2*sqrt(
a)*d**2) + b*atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*
x**4)))/(4*a**(3/2)*d)
```

Mathematica [A] time = 1.85177, size = 256, normalized size = 1.17

$$\frac{\frac{2a^{3/2}e^2\log\left(d+ex^2\right)}{\sqrt{ae^2-bde+cd^2}}-\frac{2a^{3/2}e^2\log\left(2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex^2-2cdx^2\right)}{\sqrt{ae^2-bde+cd^2}}-\frac{2\sqrt{a}d\sqrt{a+bx^2+cx^4}}{x^2}+bd\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2\cdots\right.}{4a^{3/2}d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-2*Sqrt[a]*d*Sqrt[a + b*x^2 + c*x^4])/x^2 - (b*d + 2*a*e)*Log[x
^2] + (2*a^(3/2)*e^2*Log[d + e*x^2])/Sqrt[c*d^2 - b*d*e + a*e^2]
+ b*d*Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]] + 2*a*
e*Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]] - (2*a^(3/
2)*e^2*Log[-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2 + 2*Sqrt[c*d^2 -
b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]])/Sqrt[c*d^2 - b*d*e + a*e
^2])/(4*a^(3/2)*d^2)
```

Maple [A] time = 0.019, size = 276, normalized size = 1.3

$$-\frac{1}{2\,adx^2}\sqrt{cx^4+bx^2+a}+\frac{b}{4\,d}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{3}{2}}$$
$$-\frac{e}{2\,d^2}\ln\left(1\left(2\,\frac{ae^2-bde+cd^2}{e^2}+\frac{be-2\,cd}{e}\left(x^2+\frac{d}{e}\right)+2\,\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\sqrt{\left(x^2+\frac{d}{e}\right)^2c+\frac{be-2\,cd}{e}\left(x^2+\frac{d}{e}\right)+\frac{ae^2-bd}{e^2}}\right.\right.$$
$$+\frac{e}{2\,d^2}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*(c*x^4+b*x^2+a)^(1/2)/a/d/x^2+1/4/d*b/a^(3/2)*ln((2*a+b*x^2+
2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-1/2/d^2*e/((a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d
/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/
e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))+1/2/d^2*e/
a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}(ex^2+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^3), x)
```

Fricas [A] time = 0.950815, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^3),x, algorithm="fricas")
```

```
[Out] [1/8*(2*a^(3/2)*e^2*x^2*log(-(4*(b*c*d^3 + 3*a*b*d*e^2 - 2*a^2*e^
3 - (b^2 + 2*a*c)*d^2*e + (2*c^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b
```

```
^2 + 2*a*c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a) + ((8*c^2*d^2 - 8
*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2
+ 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^
2)*sqrt(c*d^2 - b*d*e + a*e^2))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + sq
rt(c*d^2 - b*d*e + a*e^2)*(b*d + 2*a*e)*x^2*log(-(4*sqrt(c*x^4 +
b*x^2 + a)*(a*b*x^2 + 2*a^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8
*a^2)*sqrt(a))/x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*
e + a*e^2)*sqrt(a)*d)/(sqrt(c*d^2 - b*d*e + a*e^2)*a^(3/2)*d^2*x^
2), -1/8*(4*a^(3/2)*e^2*x^2*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e
^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(sqrt(c*x^4 + b*x^2 + a)*(c
*d^2 - b*d*e + a*e^2))) - sqrt(-c*d^2 + b*d*e - a*e^2)*(b*d + 2*a
*e)*x^2*log(-(4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2) + ((b^2
 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4) + 4*sqrt(c*x^4 +
 b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(a)*d)/(sqrt(-c*d^2
+ b*d*e - a*e^2)*a^(3/2)*d^2*x^2), 1/4*(sqrt(-a)*a*e^2*x^2*log(-(
4*(b*c*d^3 + 3*a*b*d*e^2 - 2*a^2*e^3 - (b^2 + 2*a*c)*d^2*e + (2*c
^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*x^2)*sqrt(c
*x^4 + b*x^2 + a) + ((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*
x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 +
4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2)
)/(e^2*x^4 + 2*d*e*x^2 + d^2)) + sqrt(c*d^2 - b*d*e + a*e^2)*(b*d
 + 2*a*e)*x^2*arctan(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x
^2 + a)*a)) - 2*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^
2)*sqrt(-a)*d)/(sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(-a)*a*d^2*x^2),
-1/4*(2*sqrt(-a)*a*e^2*x^2*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^
2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(sqrt(c*x^4 + b*x^2 + a)*(c*
d^2 - b*d*e + a*e^2))) - sqrt(-c*d^2 + b*d*e - a*e^2)*(b*d + 2*a*
e)*x^2*arctan(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)
*a)) + 2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*sqr
t(-a)*d)/(sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(-a)*a*d^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral(1/(x**3*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^3),x, algorithm="giac")

[Out] Exception raised: RuntimeError
```

3.329 $\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=398

$$\frac{\sqrt{2x^4+2x^2+1}x}{2\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{3}{8}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{\left(\left(\sqrt{2}-6\right)x^2-3\sqrt{2}+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\,2^{3/4}\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}-\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\,2^{3/4}\sqrt{2x^4+2x^2+1}}+\frac{3\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/(2*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (3*Sq
rt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/8 - ((1 +
Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellipt
icE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2*2^(3/4)*Sqrt[1 + 2*
x^2 + 2*x^4]) - ((1 - 3*Sqrt[2] + (-6 + Sqrt[2])*x^2)*Sqrt[(1 + 2
*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x],
 (2 - Sqrt[2])/4])/(2*2^(3/4)*(-2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2
*x^4]) + (3*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x
^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTa
n[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1
 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.462023, antiderivative size = 488, normalized size of antiderivative = 1.23, number

of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$\frac{\sqrt{2x^4+2x^2+1}x}{2\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{3}{8}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$-\frac{\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{9\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\,2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{3\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^4/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/(2*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (3*Sq
rt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/8 - ((1 +
Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellipt
icE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2*2^(3/4)*Sqrt[1 + 2*
x^2 + 2*x^4]) - (9*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1
+ Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4]
)/(4*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - ((3 - Sqr
t[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2
)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(1/4)*
Sqrt[1 + 2*x^2 + 2*x^4]) + (3*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sq
rt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(3/4)*(2 -
 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 43.7509, size = 507, normalized size = 1.27

$$
\begin{aligned}
&\frac{\sqrt{2}x\sqrt{2x^4+2x^2+1}}{4\left(\sqrt{2}x^2+1\right)}-\frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)E\left(2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{4\sqrt{2x^4+2x^2+1}}\\
&-\frac{3\cdot 2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{16\sqrt{2x^4+2x^2+1}}\\
&+\frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{8\sqrt{2x^4+2x^2+1}}\\
&-\frac{9\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{8\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}\\
&+\frac{3\cdot 2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(2+3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\left(-\frac{11\sqrt{2}}{24}+\frac{1}{2};2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{32\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}\\
&+\frac{3\sqrt{15}\,\mathrm{atan}\left(\frac{\sqrt{15}x}{3\sqrt{2x^4+2x^2+1}}\right)}{40}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] sqrt(2)*x*sqrt(2*x**4 + 2*x**2 + 1)/(4*(sqrt(2)*x**2 + 1)) - 2**(
1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x
**2 + 1)*elliptic_e(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(4*sqrt
(2*x**4 + 2*x**2 + 1)) - 3*2**(3/4)*sqrt((2*x**4 + 2*x**2 + 1)/(s
qrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4
)*x), -sqrt(2)/4 + 1/2)/(16*sqrt(2*x**4 + 2*x**2 + 1)) + 2**(1/4)
*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2
+ 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(8*sqrt(2*x
**4 + 2*x**2 + 1)) - 9*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(
2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x)
, -sqrt(2)/4 + 1/2)/(8*(-3*sqrt(2) + 2)*sqrt(2*x**4 + 2*x**2 + 1)
) + 3*2**(3/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*
(2 + 3*sqrt(2))*(sqrt(2)*x**2 + 1)*elliptic_pi(-11*sqrt(2)/24 + 1
/2, 2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(32*(-3*sqrt(2) + 2)*sq
rt(2*x**4 + 2*x**2 + 1)) + 3*sqrt(15)*atan(sqrt(15)*x/(3*sqrt(2*x
**4 + 2*x**2 + 1)))/40
```

Mathematica [C] time = 0.142511, size = 127, normalized size = 0.32

$$-\frac{\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left(-(1+4i)F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)+(1+i)E\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)+3i\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\right.\right.\right.}{4\sqrt{1-i}\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] -(Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*((1 + I)*EllipticE[
I*ArcSinh[Sqrt[1 - I]*x], I] - (1 + 4*I)*EllipticF[I*ArcSinh[Sqrt
[1 - I]*x], I] + (3*I)*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I
]*x], I]))/(4*Sqrt[1 - I]*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.03, size = 222, normalized size = 0.6

$$-\frac{3\,EllipticF\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{4\sqrt{-1+i}}\sqrt{1+(1-i)\,x^2}\sqrt{1+(1+i)\,x^2}\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}$$
$$-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\left(EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)-EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)\right)}{\sqrt{-1+i}}\sqrt{1+(1-i)\,x^2}\sqrt{1+(1+i)\,x^2}\frac{1}{\sqrt{2\,x^4+2\,x^2+}}$$
$$+\frac{3}{4\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}EllipticPi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x)
```

```
[Out] -3/4/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+
2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)
)+(-1/4+1/4*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/
2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/
2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))
+3/4/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4
+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/
(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)),x, algorithm="fricas")
```

```
[Out] integral(x^4/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(2x^2+3)\sqrt{2x^4+2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4/((2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

3.330 $\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=280

$$\frac{\sqrt{\frac{3}{10}}\left(3-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{2\left(2-3\sqrt{2}\right)}+\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\sqrt[4]{2}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}-\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

```
[Out] (Sqrt[3/10]*(3 - Sqrt[2])*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2
*x^4]])/(2*(2 - 3*Sqrt[2])) + ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 -
 Sqrt[2])/4])/(2*2^(1/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
 - ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
 Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4
)*x], (2 - Sqrt[2])/4])/(4*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2
 + 2*x^4])
```

Rubi [A] time = 0.395794, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{\sqrt{\frac{3}{10}}\left(3-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{2\left(2-3\sqrt{2}\right)}+\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\sqrt[4]{2}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}-\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```


```
[Out] (Sqrt[3/10]*(3 - Sqrt[2])*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2
*x^4]])/(2*(2 - 3*Sqrt[2])) + ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 -
 Sqrt[2])/4])/(2*2^(1/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
 - ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
 Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4
```

```
)*x], (2 - Sqrt[2])/4])/(4*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2
 + 2*x^4])
```

Rubi in Sympy [A] time = 29.1523, size = 308, normalized size = 1.1

$$\frac{3\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{4\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}+\frac{2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{8\sqrt{2x^4+2x^2+1}}-\frac{2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(2+3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\left(-\frac{11\sqrt{2}}{24}+\frac{1}{2};2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{16\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{15}\operatorname{atan}\left(\frac{\sqrt{15}x}{3\sqrt{2x^4+2x^2+1}}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] 3*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqr
t(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(
4*(-3*sqrt(2) + 2)*sqrt(2*x**4 + 2*x**2 + 1)) + 2**(3/4)*sqrt((2*
x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*elli
ptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(8*sqrt(2*x**4 + 2*x
**2 + 1)) - 2**(3/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1
)**2)*(2 + 3*sqrt(2))*(sqrt(2)*x**2 + 1)*elliptic_pi(-11*sqrt(2)/
24 + 1/2, 2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(16*(-3*sqrt(2) +
 2)*sqrt(2*x**4 + 2*x**2 + 1)) - sqrt(15)*atan(sqrt(15)*x/(3*sqrt
(2*x**4 + 2*x**2 + 1)))/20
```

Mathematica [C] time = 0.0950225, size = 99, normalized size = 0.35

$$\frac{(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left(F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)-\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)\right)}{4\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] ((1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*(Ellip
ticF[I*ArcSinh[Sqrt[1 - I]*x], I] - EllipticPi[1/3 + I/3, I*ArcSi
nh[Sqrt[1 - I]*x], I]))/(4*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.011, size = 134, normalized size = 0.5

$$\frac{EllipticF\left(x\sqrt{-1+i}, \frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{2\sqrt{-1+i}}\sqrt{1+(1-i)\,x^2}\sqrt{1+(1+i)\,x^2}\frac{1}{\sqrt{2\,x^4+2\,x^2+1}} - \frac{1}{2\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}EllipticPi\left(x\sqrt{-1+i}, \frac{1}{3}+\frac{i}{3}, \frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x)
```

```
[Out] 1/2/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2
*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))
-1/2/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4
+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/
(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)),x, algorithm="fricas")

[Out] integral(x^2/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)

[Out] Integral(x**2/((2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)),x, algorithm="giac")

[Out] integrate(x^2/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

3.331 $\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=256

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{2\sqrt{15}}-\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}+\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{6\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

```
[Out] ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]]/(2*Sqrt[15]) - ((1
+ Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elli
pticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2^(3/4)*(2 - 3*Sqrt
[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*
Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11
*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(6*2^(3/4)*(
2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.192689, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{2\sqrt{15}}-\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}+\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{6\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]]/(2*Sqrt[15]) - ((1
+ Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elli
pticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2^(3/4)*(2 - 3*Sqrt
[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*
Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11
*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(6*2^(3/4)*(
2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 15.2345, size = 226, normalized size = 0.88

$$-\frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{2\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}+\frac{2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(2+3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\left(-\frac{11\sqrt{2}}{24}+\frac{1}{2};2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{24\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{15}\operatorname{atan}\left(\frac{\sqrt{15}x}{3\sqrt{2x^4+2x^2+1}}\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] -2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt
(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(2
*(-3*sqrt(2) + 2)*sqrt(2*x**4 + 2*x**2 + 1)) + 2**(3/4)*sqrt((2*x
**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(2 + 3*sqrt(2))*(sqrt(2)
*x**2 + 1)*elliptic_pi(-11*sqrt(2)/24 + 1/2, 2*atan(2**(1/4)*x),
-sqrt(2)/4 + 1/2)/(24*(-3*sqrt(2) + 2)*sqrt(2*x**4 + 2*x**2 + 1))
 + sqrt(15)*atan(sqrt(15)*x/(3*sqrt(2*x**4 + 2*x**2 + 1)))/30
```

Mathematica [C] time = 0.0366582, size = 80, normalized size = 0.31

$$-\frac{i\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)}{3\sqrt{1-i}\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] ((-I/3)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/
3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(Sqrt[1 - I]*Sqrt[1 + 2*x^
2 + 2*x^4])
```

Maple [C] time = 0.007, size = 70, normalized size = 0.3

$$\frac{1}{3\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}EllipticPi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x)
```

```
[Out] 1/3/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+
2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(
-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+2x^2+1}(2x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2x^4+2x^2+1}(2x^2+3)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/((2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2 x^4 + 2 x^2 + 1}(2 x^2 + 3)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

3.332 $\int \frac{1}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=500

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}x}{3\left(\sqrt{2}x^2+1\right)}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{3\sqrt{15}}+\frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\mid\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}+\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\mid\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\,2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\mid\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{2x^4+2x^2+1}}-\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)\mid\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(3*x) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^
4])/(3*(1 + Sqrt[2]*x^2)) - ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 +
 2*x^4]]/(3*Sqrt[15]) - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^
2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2
 - Sqrt[2])/4])/(3*Sqrt[1 + 2*x^2 + 2*x^4]) + ((1 + Sqrt[2]*x^2)*
Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[
2^(1/4)*x], (2 - Sqrt[2])/4])/(3*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4])
 + (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[
2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*(2
 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - ((3 + Sqrt[2])*(1 + Sqrt
[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi
[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(9*
2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.62178, antiderivative size = 500, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{3\left(\sqrt{2}x^2+1\right)}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{3\sqrt{15}}$$
$$+\frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\,2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/(x^2*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(3*x) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^
4])/(3*(1 + Sqrt[2]*x^2)) - ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 +
 2*x^4]]/(3*Sqrt[15]) - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^
2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2
 - Sqrt[2])/4])/(3*Sqrt[1 + 2*x^2 + 2*x^4]) + ((1 + Sqrt[2]*x^2)*
Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[
2^(1/4)*x], (2 - Sqrt[2])/4])/(3*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4])
 + (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[
2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*(2
 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - ((3 + Sqrt[2])*(1 + Sqrt
[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi
[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(9*
2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 47.3494, size = 439, normalized size = 0.88

$$\frac{\sqrt{2}x\sqrt{2x^4+2x^2+1}}{3\left(\sqrt{2}x^2+1\right)}-\frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)E\left(2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{3\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{3\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{6\sqrt{2x^4+2x^2+1}}$$
$$-\frac{2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(2+3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\left(-\frac{11\sqrt{2}}{24}+\frac{1}{2};2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{36\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\sqrt{15}\,\mathrm{atan}\left(\frac{\sqrt{15}x}{3\sqrt{2x^4+2x^2+1}}\right)}{45}-\frac{\sqrt{2x^4+2x^2+1}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] sqrt(2)*x*sqrt(2*x**4 + 2*x**2 + 1)/(3*(sqrt(2)*x**2 + 1)) - 2**(
1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x
**2 + 1)*elliptic_e(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(3*sqrt
(2*x**4 + 2*x**2 + 1)) + 2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqr
t(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*
x), -sqrt(2)/4 + 1/2)/(3*(-3*sqrt(2) + 2)*sqrt(2*x**4 + 2*x**2 +
1)) + 2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*
(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/
2)/(6*sqrt(2*x**4 + 2*x**2 + 1)) - 2**(3/4)*sqrt((2*x**4 + 2*x**2
 + 1)/(sqrt(2)*x**2 + 1)**2)*(2 + 3*sqrt(2))*(sqrt(2)*x**2 + 1)*e
lliptic_pi(-11*sqrt(2)/24 + 1/2, 2*atan(2**(1/4)*x), -sqrt(2)/4 +
 1/2)/(36*(-3*sqrt(2) + 2)*sqrt(2*x**4 + 2*x**2 + 1)) - sqrt(15)*
atan(sqrt(15)*x/(3*sqrt(2*x**4 + 2*x**2 + 1)))/45 - sqrt(2*x**4 +
 2*x**2 + 1)/(3*x)
```

Mathematica [C] time = 0.266046, size = 147, normalized size = 0.29

$$-\frac{i\left(\sqrt{1-i}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left(-3F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)+3E\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)-(1+i)\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\right.\right.\right.}{9x\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] ((-I/9)*((-3*I)*(1 + 2*x^2 + 2*x^4) + Sqrt[1 - I]*x*Sqrt[1 + (1 -
 I)*x^2]*Sqrt[1 + (1 + I)*x^2]*(3*EllipticE[I*ArcSinh[Sqrt[1 - I]
*x], I] - 3*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - (1 + I)*Elli
pticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])))/(x*Sqrt[1 + 2*x
^2 + 2*x^4])
```

Maple [C] time = 0.022, size = 178, normalized size = 0.4

$$-\frac{1}{3x}\sqrt{2x^4+2x^2+1}-\frac{\left(\frac{1}{3}-\frac{i}{3}\right)\left(EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)-EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)\right)}{\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+}}$$
$$-\frac{2}{9\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}EllipticPi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x)
```

```
[Out] -1/3*(2*x^4+2*x^2+1)^(1/2)/x+(-1/3+1/3*I)/(-1+I)^(1/2)*(1+(1-I)*x
^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*
(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),
1/2*2^(1/2)+1/2*I*2^(1/2)))-2/9/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)
*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1
/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+2x^2+1}(2x^2+3)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(2\,x^4+3\,x^2)\sqrt{2\,x^4+2\,x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/((2*x^4 + 3*x^2)*sqrt(2*x^4 + 2*x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\,(2x^2+3)\,\sqrt{2x^4+2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^2), x)
```

3.333 $\int \frac{1}{x^4(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=622

$$-\frac{2\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{3\left(\sqrt{2}x^2+1\right)}+\frac{2\sqrt{2x^4+2x^2+1}}{3x}+\frac{2\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{9\sqrt{15}}$$
$$-\frac{\left(1+2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{2\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\sqrt{2x^4+2x^2+1}}$$
$$+\frac{2\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\sqrt[4]{2}\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{27\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}$$

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(9*x^3) + (2*Sqrt[1 + 2*x^2 + 2*x^4])/(3
*x) - (2*Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1 + Sqrt[2]*x^2))
 + (2*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(9*Sqrt[15])
 + (2*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*
Sqrt[1 + 2*x^2 + 2*x^4]) - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2
*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x],
 (2 - Sqrt[2])/4])/(9*Sqrt[1 + 2*x^2 + 2*x^4]) - (2*2^(1/4)*(1 +
Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellipt
icF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(9*(2 - 3*Sqrt[2])*Sqr
t[1 + 2*x^2 + 2*x^4]) - ((1 + 2*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(
1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4
)*x], (2 - Sqrt[2])/4])/(9*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (2^
(1/4)*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1
 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1
/4)*x], (2 - Sqrt[2])/4])/(27*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*
x^4])
```

Rubi [A] time = 0.936202, antiderivative size = 622, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31

$$-\frac{2\sqrt{2}\sqrt{2x^4+2x^2+1}x}{3\left(\sqrt{2}x^2+1\right)}+\frac{2\sqrt{2x^4+2x^2+1}}{3x}+\frac{2\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{9\sqrt{15}}$$
$$-\frac{\left(1+2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{2\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\sqrt{2x^4+2x^2+1}}$$
$$+\frac{2\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\sqrt[4]{2}\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{27\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/(x^4*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(9*x^3) + (2*Sqrt[1 + 2*x^2 + 2*x^4])/(3
*x) - (2*Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1 + Sqrt[2]*x^2))
 + (2*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(9*Sqrt[15])
 + (2*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*
Sqrt[1 + 2*x^2 + 2*x^4]) - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2
*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x],
 (2 - Sqrt[2])/4])/(9*Sqrt[1 + 2*x^2 + 2*x^4]) - (2*2^(1/4)*(1 +
Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellipt
icF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(9*(2 - 3*Sqrt[2])*Sqr
t[1 + 2*x^2 + 2*x^4]) - ((1 + 2*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(
1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4
)*x], (2 - Sqrt[2])/4])/(9*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (2^
(1/4)*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1
```

```
 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1
/4)*x], (2 - Sqrt[2])/4])/(27*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*
x^4])
```

Rubi in Sympy [A] time = 69.8287, size = 556, normalized size = 0.89

$$-\frac{2\sqrt{2}x\sqrt{2x^4+2x^2+1}}{3\left(\sqrt{2}x^2+1\right)}+\frac{2\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)E\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{3\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(2\sqrt{2}+8\right)\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{36\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{9\sqrt{2x^4+2x^2+1}}$$
$$-\frac{2\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{9\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}$$
$$+\frac{2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(2+3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\left(-\frac{11\sqrt{2}}{24}+\frac{1}{2};2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{54\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}$$
$$+\frac{2\sqrt{15}\operatorname{atan}\left(\frac{\sqrt{15}x}{3\sqrt{2x^4+2x^2+1}}\right)}{135}+\frac{2\sqrt{2x^4+2x^2+1}}{3x}-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] -2*sqrt(2)*x*sqrt(2*x**4 + 2*x**2 + 1)/(3*(sqrt(2)*x**2 + 1)) + 2
*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt
(2)*x**2 + 1)*elliptic_e(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(3
*sqrt(2*x**4 + 2*x**2 + 1)) - 2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)
/(sqrt(2)*x**2 + 1)**2)*(2*sqrt(2) + 8)*(sqrt(2)*x**2 + 1)*ellipt
ic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(36*sqrt(2*x**4 + 2*x*
*2 + 1)) - 2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)
**2)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4
 + 1/2)/(9*sqrt(2*x**4 + 2*x**2 + 1)) - 2*2**(1/4)*sqrt((2*x**4 +
 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*elliptic_f
(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(9*(-3*sqrt(2) + 2)*sqrt(2
*x**4 + 2*x**2 + 1)) + 2**(3/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(
2)*x**2 + 1)**2)*(2 + 3*sqrt(2))*(sqrt(2)*x**2 + 1)*elliptic_pi(-
11*sqrt(2)/24 + 1/2, 2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(54*(-
```

```
3*sqrt(2) + 2)*sqrt(2*x**4 + 2*x**2 + 1)) + 2*sqrt(15)*atan(sqrt(
15)*x/(3*sqrt(2*x**4 + 2*x**2 + 1)))/135 + 2*sqrt(2*x**4 + 2*x**2
 + 1)/(3*x) - sqrt(2*x**4 + 2*x**2 + 1)/(9*x**3)
```

Mathematica [C] time = 0.176342, size = 219, normalized size = 0.35

$$\frac{36x^6+30x^4+12x^2-(3+15i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}x^3F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)+18i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1}}{27x^3\sqrt{2x^4+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] (-3 + 12*x^2 + 30*x^4 + 36*x^6 + (18*I)*Sqrt[1 - I]*x^3*Sqrt[1 +
(1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I
]*x], I] - (3 + 15*I)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[
1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] + 2*(1 -
I)^(3/2)*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*Elliptic
Pi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(27*x^3*Sqrt[1 + 2*x^
2 + 2*x^4])
```

Maple [C] time = 0.023, size = 260, normalized size = 0.4

$$-\frac{1}{9\,x^3}\sqrt{2\,x^4+2\,x^2+1}+\frac{2}{3\,x}\sqrt{2\,x^4+2\,x^2+1}$$
$$-\frac{2\,EllipticF\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{9\sqrt{-1+i}}\sqrt{1+(1-i)\,x^2}\sqrt{1+(1+i)\,x^2}\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}$$
$$+\frac{\left(\frac{2}{3}-\frac{2\,i}{3}\right)\left(EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)-EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)\right)}{\sqrt{-1+i}}\sqrt{1+(1-i)\,x^2}\sqrt{1+(1+i)\,x^2}\frac{1}{\sqrt{2\,x^4+2\,x^2+}}$$
$$+\frac{4}{27\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}EllipticPi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x)
```

```
[Out] -1/9*(2*x^4+2*x^2+1)^(1/2)/x^3+2/3*(2*x^4+2*x^2+1)^(1/2)/x-2/9/(-
1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1
)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(2/3-
2/3*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^
4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1
```

```
/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+4/27/(-
1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+
1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{(2\,x^6+3\,x^4)\sqrt{2\,x^4+2\,x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^4),x, algorithm="fricas")
```

```
[Out] integral(1/((2*x^6 + 3*x^4)*sqrt(2*x^4 + 2*x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\,(2x^2+3)\,\sqrt{2x^4+2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^4),x, algorithm="giac")

[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^4), x)
```

3.334 $\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=236

$$\frac{x^2\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}e}-\frac{d^3\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e\left(ae^2-bde+cd^2\right)^{3/2}}$$

```
[Out] (a*(b^2*d - 2*a*c*d - a*b*e) + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a
^2*c*e)*x^2)/(c*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*
x^2 + c*x^4]) + ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 +
 c*x^4])]/(2*c^(3/2)*e) - (d^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*
e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])
/(2*e*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rubi [A] time = 1.11349, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$

$$\frac{x^2\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}e}-\frac{d^3\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e\left(ae^2-bde+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```


```
[Out] (a*(b^2*d - 2*a*c*d - a*b*e) + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a
^2*c*e)*x^2)/(c*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*
x^2 + c*x^4]) + ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 +
 c*x^4])]/(2*c^(3/2)*e) - (d^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*
e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])
/(2*e*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rubi in Sympy [A] time = 134.65, size = 348, normalized size = 1.47

$$-\frac{b\sqrt{a+bx^2+cx^4}}{ce\,(-4ac+b^2)}+\frac{d^3\operatorname{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e\,(ae^2-bde+cd^2)^{\frac{3}{2}}}-\frac{d^3\left(-2ace+b^2e-bcd+cx^2\,(be-2cd)\right)}{e^3\,(-4ac+b^2)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)}-\frac{d^2\left(2b+4cx^2\right)}{2e^3\,(-4ac+b^2)\sqrt{a+bx^2+cx^4}}-\frac{d\left(4a+2bx^2\right)}{2e^2\,(-4ac+b^2)\sqrt{a+bx^2+cx^4}}+\frac{x^2\left(2a+bx^2\right)}{e\,(-4ac+b^2)\sqrt{a+bx^2+cx^4}}+\frac{\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{\frac{3}{2}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -b*sqrt(a + b*x**2 + c*x**4)/(c*e*(-4*a*c + b**2)) + d**3*atanh((
2*a*e - b*d + x**2*(b*e - 2*c*d))/(2*sqrt(a + b*x**2 + c*x**4)*sq
rt(a*e**2 - b*d*e + c*d**2)))/(2*e*(a*e**2 - b*d*e + c*d**2)**(3/
2)) - d**3*(-2*a*c*e + b**2*e - b*c*d + c*x**2*(b*e - 2*c*d))/(e*
*3*(-4*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)*(a*e**2 - b*d*e + c*
d**2)) - d**2*(2*b + 4*c*x**2)/(2*e**3*(-4*a*c + b**2)*sqrt(a + b
*x**2 + c*x**4)) - d*(4*a + 2*b*x**2)/(2*e**2*(-4*a*c + b**2)*sqr
t(a + b*x**2 + c*x**4)) + x**2*(2*a + b*x**2)/(e*(-4*a*c + b**2)*
sqrt(a + b*x**2 + c*x**4)) + atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt
(a + b*x**2 + c*x**4)))/(2*c**(3/2)*e)
```

Mathematica [A] time = 0.634432, size = 271, normalized size = 1.15

$$\frac{1}{2}\left(\frac{2\left(a^2\left(be+2c\left(d-ex^2\right)\right)+ab\left(-bd+bex^2+3cdx^2\right)+b^3(-d)x^2\right)}{c\,(4ac-b^2)\sqrt{a+bx^2+cx^4}\,(e(ae-bd)+cd^2)}+\frac{\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{c^{3/2}e}+\frac{d^3\log\left(2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex^2-2cdx^2\right)}{e\left(e(ae-bd)+cd^2\right)^{3/2}}-\frac{d^3\log\left(d+ex^2\right)}{e\left(e(ae-bd)+cd^2\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] ((2*(-(b^3*d*x^2) + a*b*(-(b*d) + 3*c*d*x^2 + b*e*x^2) + a^2*(b*e
 + 2*c*(d - e*x^2))))/(c*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e)
```

```
)*Sqrt[a + b*x^2 + c*x^4]) - (d^3*Log[d + e*x^2])/(e*(c*d^2 + e*(
-(b*d) + a*e))^(3/2)) + Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^
2 + c*x^4]]/(c^(3/2)*e) + (d^3*Log[-(b*d) + 2*a*e - 2*c*d*x^2 + b
*e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]])/
(e*(c*d^2 + e*(-(b*d) + a*e))^(3/2)))/2
```

Maple [B] time = 0.08, size = 720, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/2/e*x^2/c/(c*x^4+b*x^2+a)^(1/2)+1/4/e*b/c^2/(c*x^4+b*x^2+a)^(1
/2)+1/2/e*b^2/c/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2+1/4/e*b^3/c
^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)+1/2/e/c^(3/2)*ln((1/2*b+c*x^
2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+d^2/e^3*(2*c*x^2+b)/(4*a*c-b^2)
/(c*x^4+b*x^2+a)^(1/2)+d/e^2/(c*x^4+b*x^2+a)^(1/2)*(b*x^2+2*a)/(4
*a*c-b^2)+2*d^3/e^3*c/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(-4*a*c+b^
2)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*((x^2-1/2/c*(-b+(-4*a*c
+b^2)^(1/2)))^2*c+(-4*a*c+b^2)^(1/2)*(x^2-1/2/c*(-b+(-4*a*c+b^2)^
(1/2))))^(1/2)-2*d^3/e^3*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(-4*a
*c+b^2)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*((x^2+1/2*(b+(-4*a
*c+b^2)^(1/2))/c)^2*c-(-4*a*c+b^2)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)
^(1/2))/c))^(1/2)-2*d^3/e^2*c/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(e
*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln
((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^
2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{(cx^4 + bx^2 + a)^{\frac{3}{2}}(ex^2 + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

Fricas [A] time = 69.96, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^2*b*e^2 - (a*b^2 - 2*a^2*c)*d*e - ((b^3 - 3*a*b*c)*d*
e - (a*b^2 - 2*a^2*c)*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^
2 - b*d*e + a*e^2)*sqrt(c) - (((b^2*c^2 - 4*a*c^3)*d^2 - (b^3*c -
 4*a*b*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^4 + (a*b^2*c - 4*a
^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2 +
 ((b^3*c - 4*a*b*c^2)*d^2 - (b^4 - 4*a*b^2*c)*d*e + (a*b^3 - 4*a^
2*b*c)*e^2)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-4*sqrt(c*x^4 +
b*x^2 + a)*(2*c^2*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a
*c)*sqrt(c)) - ((b^2*c^2 - 4*a*c^3)*d^3*x^4 + (b^3*c - 4*a*b*c^2)
*d^3*x^2 + (a*b^2*c - 4*a^2*c^2)*d^3)*sqrt(c)*log((4*(b*c*d^3 + 3
*a*b*d*e^2 - 2*a^2*e^3 - (b^2 + 2*a*c)*d^2*e + (2*c^2*d^3 - 3*b*c
*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 +
 a) - ((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*
e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3
*b^2 + 4*a*c)*d*e)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2))/(e^2*x^4 + 2
*d*e*x^2 + d^2)))/((((b^2*c^3 - 4*a*c^4)*d^2*e - (b^3*c^2 - 4*a*b
*c^3)*d*e^2 + (a*b^2*c^2 - 4*a^2*c^3)*e^3)*x^4 + (a*b^2*c^2 - 4*a
^2*c^3)*d^2*e - (a*b^3*c - 4*a^2*b*c^2)*d*e^2 + (a^2*b^2*c - 4*a^
3*c^2)*e^3 + ((b^3*c^2 - 4*a*b*c^3)*d^2*e - (b^4*c - 4*a*b^2*c^2)
*d*e^2 + (a*b^3*c - 4*a^2*b*c^2)*e^3)*x^2)*sqrt(c*d^2 - b*d*e + a
*e^2)*sqrt(c)), -1/4*(4*(a^2*b*e^2 - (a*b^2 - 2*a^2*c)*d*e - ((b^
3 - 3*a*b*c)*d*e - (a*b^2 - 2*a^2*c)*e^2)*x^2)*sqrt(c*x^4 + b*x^2
 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c) - 2*((b^2*c^2 - 4*a*c^
3)*d^3*x^4 + (b^3*c - 4*a*b*c^2)*d^3*x^2 + (a*b^2*c - 4*a^2*c^2)*
d^3)*sqrt(c)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b
*e)*x^2 + b*d - 2*a*e)/(sqrt(c*x^4 + b*x^2 + a)*(c*d^2 - b*d*e +
a*e^2))) - (((b^2*c^2 - 4*a*c^3)*d^2 - (b^3*c - 4*a*b*c^2)*d*e +
(a*b^2*c - 4*a^2*c^2)*e^2)*x^4 + (a*b^2*c - 4*a^2*c^2)*d^2 - (a*b
^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2 + ((b^3*c - 4*a*b*c
^2)*d^2 - (b^4 - 4*a*b^2*c)*d*e + (a*b^3 - 4*a^2*b*c)*e^2)*x^2)*s
qrt(-c*d^2 + b*d*e - a*e^2)*log(-4*sqrt(c*x^4 + b*x^2 + a)*(2*c^2
*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)))/(((
(b^2*c^3 - 4*a*c^4)*d^2*e - (b^3*c^2 - 4*a*b*c^3)*d*e^2 + (a*b^2*
c^2 - 4*a^2*c^3)*e^3)*x^4 + (a*b^2*c^2 - 4*a^2*c^3)*d^2*e - (a*b^
3*c - 4*a^2*b*c^2)*d*e^2 + (a^2*b^2*c - 4*a^3*c^2)*e^3 + ((b^3*c^
2 - 4*a*b*c^3)*d^2*e - (b^4*c - 4*a*b^2*c^2)*d*e^2 + (a*b^3*c - 4
*a^2*b*c^2)*e^3)*x^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c)), -1/4
*(4*(a^2*b*e^2 - (a*b^2 - 2*a^2*c)*d*e - ((b^3 - 3*a*b*c)*d*e - (
a*b^2 - 2*a^2*c)*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b
*d*e + a*e^2)*sqrt(-c) - 2*(((b^2*c^2 - 4*a*c^3)*d^2 - (b^3*c - 4
*a*b*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^4 + (a*b^2*c - 4*a^2
*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2 + (
(b^3*c - 4*a*b*c^2)*d^2 - (b^4 - 4*a*b^2*c)*d*e + (a*b^3 - 4*a^2*
b*c)*e^2)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2)*arctan(1/2*(2*c*x^2 +
```

```
b)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a)*c)) - ((b^2*c^2 - 4*a*c^3)*d
^3*x^4 + (b^3*c - 4*a*b*c^2)*d^3*x^2 + (a*b^2*c - 4*a^2*c^2)*d^3)
*sqrt(-c)*log((4*(b*c*d^3 + 3*a*b*d*e^2 - 2*a^2*e^3 - (b^2 + 2*a*
c)*d^2*e + (2*c^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e
^2)*x^2)*sqrt(c*x^4 + b*x^2 + a) - ((8*c^2*d^2 - 8*b*c*d*e + (b^2
 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 +
2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2)*sqrt(c*d^2 -
 b*d*e + a*e^2))/(e^2*x^4 + 2*d*e*x^2 + d^2)))/((((b^2*c^3 - 4*a*
c^4)*d^2*e - (b^3*c^2 - 4*a*b*c^3)*d*e^2 + (a*b^2*c^2 - 4*a^2*c^3
)*e^3)*x^4 + (a*b^2*c^2 - 4*a^2*c^3)*d^2*e - (a*b^3*c - 4*a^2*b*c
^2)*d*e^2 + (a^2*b^2*c - 4*a^3*c^2)*e^3 + ((b^3*c^2 - 4*a*b*c^3)*
d^2*e - (b^4*c - 4*a*b^2*c^2)*d*e^2 + (a*b^3*c - 4*a^2*b*c^2)*e^3
)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(-c)), -1/2*(2*(a^2*b*e^2
- (a*b^2 - 2*a^2*c)*d*e - ((b^3 - 3*a*b*c)*d*e - (a*b^2 - 2*a^2*c
)*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*
sqrt(-c) - ((b^2*c^2 - 4*a*c^3)*d^3*x^4 + (b^3*c - 4*a*b*c^2)*d^3
*x^2 + (a*b^2*c - 4*a^2*c^2)*d^3)*sqrt(-c)*arctan(-1/2*sqrt(-c*d^
2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(sqrt(c*x^4
+ b*x^2 + a)*(c*d^2 - b*d*e + a*e^2))) - (((b^2*c^2 - 4*a*c^3)*d^
2 - (b^3*c - 4*a*b*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^4 + (a
*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*
a^3*c)*e^2 + ((b^3*c - 4*a*b*c^2)*d^2 - (b^4 - 4*a*b^2*c)*d*e + (
a*b^3 - 4*a^2*b*c)*e^2)*x^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(
1/2*(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a)*c)))/((((b^2*
c^3 - 4*a*c^4)*d^2*e - (b^3*c^2 - 4*a*b*c^3)*d*e^2 + (a*b^2*c^2 -
 4*a^2*c^3)*e^3)*x^4 + (a*b^2*c^2 - 4*a^2*c^3)*d^2*e - (a*b^3*c -
 4*a^2*b*c^2)*d*e^2 + (a^2*b^2*c - 4*a^3*c^2)*e^3 + ((b^3*c^2 - 4
*a*b*c^3)*d^2*e - (b^4*c - 4*a*b^2*c^2)*d*e^2 + (a*b^3*c - 4*a^2*
b*c^2)*e^3)*x^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(-c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{(d+ex^2)\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**7/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] integrate(x^7/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

3.335 $\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=167

$$\frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{x^2\left(-abe-2acd+b^2d\right)+a(bd-2ae)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}$$

```
[Out] -((a*(b*d - 2*a*e) + (b^2*d - 2*a*c*d - a*b*e)*x^2)/((b^2 - 4*a*c
)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (d^2*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x^2 + c*x^4])])/(2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rubi [A] time = 0.6344, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172

$$\frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{x^2\left(-abe-2acd+b^2d\right)+a(bd-2ae)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((a*(b*d - 2*a*e) + (b^2*d - 2*a*c*d - a*b*e)*x^2)/((b^2 - 4*a*c
)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (d^2*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x^2 + c*x^4])])/(2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rubi in Sympy [A] time = 105.888, size = 235, normalized size = 1.41

$$-\frac{d^2\,\mathrm{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{\frac{3}{2}}}+\frac{d^2\left(-2ace+b^2e-bcd+cx^2\left(be-2cd\right)\right)}{e^2\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{d\left(2b+4cx^2\right)}{2e^2\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}+\frac{4a+2bx^2}{2e\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -d**2*atanh((2*a*e - b*d + x**2*(b*e - 2*c*d))/(2*sqrt(a + b*x**2
 + c*x**4)*sqrt(a*e**2 - b*d*e + c*d**2)))/(2*(a*e**2 - b*d*e + c
*d**2)**(3/2)) + d**2*(-2*a*c*e + b**2*e - b*c*d + c*x**2*(b*e -
2*c*d))/(e**2*(-4*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)*(a*e**2 -
 b*d*e + c*d**2)) + d*(2*b + 4*c*x**2)/(2*e**2*(-4*a*c + b**2)*sq
rt(a + b*x**2 + c*x**4)) + (4*a + 2*b*x**2)/(2*e*(-4*a*c + b**2)*
sqrt(a + b*x**2 + c*x**4))
```

Mathematica [A] time = 0.735774, size = 204, normalized size = 1.22

$$\frac{1}{2}\left(\frac{2\left(-2a^2e+ab\left(d-ex^2\right)-2acdx^2+b^2dx^2\right)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(e(bd-ae)-cd^2)}-\frac{d^2\log\left(2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex^2-2cdx^2\right)}{(e(ae-bd)+cd^2)^{3/2}}+\frac{d^2\log\left(d+ex^2\right)}{(e(ae-bd)+cd^2)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] ((2*(-2*a^2*e + b^2*d*x^2 - 2*a*c*d*x^2 + a*b*(d - e*x^2)))/((b^2
 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*Sqrt[a + b*x^2 + c*x^4]) + (
d^2*Log[d + e*x^2])/(c*d^2 + e*(-(b*d) + a*e))^(3/2) - (d^2*Log[-
(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^
2]*Sqrt[a + b*x^2 + c*x^4]])/(c*d^2 + e*(-(b*d) + a*e))^(3/2))/2
```

Maple [B] time = 0.019, size = 613, normalized size = 3.7

$$-{\frac{b{x}^{2}}{e\left(4\,ac-{b}^{2}\right)}{\frac{1}{\sqrt{c{x}^{4}+b{x}^{2}+a}}}}-2\,{\frac{a}{e\sqrt{c{x}^{4}+b{x}^{2}+a}\left(4\,ac-{b}^{2}\right)}}$$
$$-2\,{\frac{cd{x}^{2}}{{e}^{2}\left(4\,ac-{b}^{2}\right)\sqrt{c{x}^{4}+b{x}^{2}+a}}}-{\frac{bd}{{e}^{2}\left(4\,ac-{b}^{2}\right)}{\frac{1}{\sqrt{c{x}^{4}+b{x}^{2}+a}}}}$$
$$-2\,{\frac{c{d}^{2}}{{e}^{2}\left(e\sqrt{-4\,ac+{b}^{2}}-be+2\,cd\right)\left(-4\,ac+{b}^{2}\right)}\sqrt{\left({x}^{2}-1/2\,{\frac{-b+\sqrt{-4\,ac+{b}^{2}}}{c}}\right)^{2}c+\sqrt{-4\,ac+{b}^{2}}\left({x}^{2}-1/2\,{\frac{-b+\sqrt{-4\,ac+b}}{c}}\right.}}$$
$$+2\,{\frac{c{d}^{2}}{{e}^{2}\left(e\sqrt{-4\,ac+{b}^{2}}+be-2\,cd\right)\left(-4\,ac+{b}^{2}\right)}\sqrt{\left({x}^{2}+1/2\,{\frac{b+\sqrt{-4\,ac+{b}^{2}}}{c}}\right)^{2}c-\sqrt{-4\,ac+{b}^{2}}\left({x}^{2}+1/2\,{\frac{b+\sqrt{-4\,ac+{b}^{2}}}{c}}\right)\left(\right.}}$$
$$+2\,{\frac{c{d}^{2}}{e\left(e\sqrt{-4\,ac+{b}^{2}}-be+2\,cd\right)\left(e\sqrt{-4\,ac+{b}^{2}}+be-2\,cd\right)}}\ln\left(1\left(2\,{\frac{a{e}^{2}-bde+c{d}^{2}}{{e}^{2}}}+{\frac{be-2\,cd}{e}}\left({x}^{2}+{\frac{d}{e}}\right)+2\,\sqrt{{\frac{a{e}^{2}-bde}{{e}^{2}}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/e/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)*x^2*b-2/e/(c*x^4+b*x^2+a)^
(1/2)/(4*a*c-b^2)*a-2/e^2*d/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2
*c-1/e^2*d/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*b-2*d^2/e^2*c/(e*(-4
*a*c+b^2)^(1/2)-b*e+2*c*d)/(-4*a*c+b^2)/(x^2+1/2*b/c-1/2/c*(-4*a*
c+b^2)^(1/2))*((x^2-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))^2*c+(-4*a*c+b^
2)^(1/2)*(x^2-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)+2*d^2/e^2*c/(
e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(-4*a*c+b^2)/(x^2+1/2/c*(-4*a*c+b
^2)^(1/2)+1/2*b/c)*((x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*c-(-4*a*
c+b^2)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)+2*d^2/e*c/
(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)
/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b
*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e
)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^
2+d/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(cx^4 + bx^2 + a)^{\frac{3}{2}}(ex^2 + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] integrate(x^5/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

Fricas [A] time = 0.633866, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(c*x^4 + b*x^2 + a)*(a*b*d - 2*a^2*e - (a*b*e - (b^2
 - 2*a*c)*d)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2) - ((b^2*c - 4*a*c^2
)*d^2*x^4 + (b^3 - 4*a*b*c)*d^2*x^2 + (a*b^2 - 4*a^2*c)*d^2)*log(
-(4*(b*c*d^3 + 3*a*b*d*e^2 - 2*a^2*e^3 - (b^2 + 2*a*c)*d^2*e + (2
*c^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*x^2)*sqrt
```

```
(c*x^4 + b*x^2 + a) + ((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2
)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
+ 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2)*sqrt(c*d^2 - b*d*e + a*e^
2))/(e^2*x^4 + 2*d*e*x^2 + d^2)))/((((b^2*c^2 - 4*a*c^3)*d^2 - (b
^3*c - 4*a*b*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^4 + (a*b^2*c
 - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)
*e^2 + ((b^3*c - 4*a*b*c^2)*d^2 - (b^4 - 4*a*b^2*c)*d*e + (a*b^3
- 4*a^2*b*c)*e^2)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2)), -1/2*(2*sqrt
(c*x^4 + b*x^2 + a)*(a*b*d - 2*a^2*e - (a*b*e - (b^2 - 2*a*c)*d)*
x^2)*sqrt(-c*d^2 + b*d*e - a*e^2) + ((b^2*c - 4*a*c^2)*d^2*x^4 +
(b^3 - 4*a*b*c)*d^2*x^2 + (a*b^2 - 4*a^2*c)*d^2)*arctan(-1/2*sqrt
(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(sqrt(
c*x^4 + b*x^2 + a)*(c*d^2 - b*d*e + a*e^2))))/((((b^2*c^2 - 4*a*c
^3)*d^2 - (b^3*c - 4*a*b*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^
4 + (a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^
2 - 4*a^3*c)*e^2 + ((b^3*c - 4*a*b*c^2)*d^2 - (b^4 - 4*a*b^2*c)*d
*e + (a*b^3 - 4*a^2*b*c)*e^2)*x^2)*sqrt(-c*d^2 + b*d*e - a*e^2))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(d+ex^2)\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**5/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.351573, size = 536, normalized size = 3.21

$$\frac{d^2\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{(cd^2-bde+ae^2)\sqrt{-cd^2+bde-ae^2}} - \frac{\frac{\left(b^4cd^3-6\,ab^2c^2d^3+8\,a^2c^3d^3-b^5d^2e+5\,ab^3cd^2e-4\,a^2bc^2d^2e+2\,ab^4de^2-10\,a^2b^2cde^2+8\,a^3c^2de^2-a^2b^3e^3+4\,a^3bce^3\right)x^2}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4} + \frac{ab^3cd^3-4\,a^2bc^2d^3-ab^4d^2e+2\,a^2}{}}{32\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] d^2*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*
d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c
```

```
*d^2 + b*d*e - a*e^2)) - 1/32*((b^4*c*d^3 - 6*a*b^2*c^2*d^3 + 8*a
^2*c^3*d^3 - b^5*d^2*e + 5*a*b^3*c*d^2*e - 4*a^2*b*c^2*d^2*e + 2*
a*b^4*d*e^2 - 10*a^2*b^2*c*d*e^2 + 8*a^3*c^2*d*e^2 - a^2*b^3*e^3
+ 4*a^3*b*c*e^3)*x^2/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4) + (
a*b^3*c*d^3 - 4*a^2*b*c^2*d^3 - a*b^4*d^2*e + 2*a^2*b^2*c*d^2*e +
 8*a^3*c^2*d^2*e + 3*a^2*b^3*d*e^2 - 12*a^3*b*c*d*e^2 - 2*a^3*b^2
*e^3 + 8*a^4*c*e^3)/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4))/sqr
t(c*x^4 + b*x^2 + a)
```

3.336 $\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=159

$$\frac{cx^2(bd-2ae)+a(2cd-be)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)}-\frac{de\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\,(ae^2-bde+cd^2)^{3/2}}$$

```
[Out] (a*(2*c*d - b*e) + c*(b*d - 2*a*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 - b
*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - (d*e*ArcTanh[(b*d - 2*a*
e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*
x^2 + c*x^4])])/(2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rubi [A] time = 0.521329, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172

$$\frac{cx^2(bd-2ae)+a(2cd-be)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)}-\frac{de\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\,(ae^2-bde+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (a*(2*c*d - b*e) + c*(b*d - 2*a*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 - b
*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - (d*e*ArcTanh[(b*d - 2*a*
e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*
x^2 + c*x^4])])/(2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rubi in Sympy [A] time = 64.4128, size = 143, normalized size = 0.9

$$\frac{de\,\mathrm{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\,(ae^2-bde+cd^2)^{\frac{3}{2}}}-\frac{a\,(be-2cd)+cx^2\,(2ae-bd)}{(-4ac+b^2)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] d*e*atanh((2*a*e - b*d + x**2*(b*e - 2*c*d))/(2*sqrt(a + b*x**2 +
 c*x**4)*sqrt(a*e**2 - b*d*e + c*d**2)))/(2*(a*e**2 - b*d*e + c*d
**2)**(3/2)) - (a*(b*e - 2*c*d) + c*x**2*(2*a*e - b*d))/((-4*a*c
```

```
+ b**2)*sqrt(a + b*x**2 + c*x**4)*(a*e**2 - b*d*e + c*d**2))
```

Mathematica [A] time = 0.510719, size = 225, normalized size = 1.42

$$\frac{-de\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\log\left(2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex^2-2cdx^2\right)+de\left(b^2-4ac\right)\sqrt{a+bx}}{2\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}\left(e(ae-bd)+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (2*Sqrt[c*d^2 - b*d*e + a*e^2]*(-2*a*c*d + a*b*e - b*c*d*x^2 + 2*
a*c*e*x^2) + (b^2 - 4*a*c)*d*e*Sqrt[a + b*x^2 + c*x^4]*Log[d + e*
x^2] - (b^2 - 4*a*c)*d*e*Sqrt[a + b*x^2 + c*x^4]*Log[-(b*d) + 2*a
*e - 2*c*d*x^2 + b*e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a +
 b*x^2 + c*x^4]])/(2*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^(3
/2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [B] time = 0.016, size = 506, normalized size = 3.2

$$\frac{2\,cx^2+b}{e\left(4\,ac-b^2\right)}\frac{1}{\sqrt{cx^4+bx^2+a}}$$
$$+2\,\frac{cd}{e\left(e\sqrt{-4\,ac+b^2}-be+2\,cd\right)\left(-4\,ac+b^2\right)}\sqrt{\left(x^2-1/2\,\frac{-b+\sqrt{-4\,ac+b^2}}{c}\right)^2c+\sqrt{-4\,ac+b^2}\left(x^2-1/2\,\frac{-b+\sqrt{-4\,ac+b^2}}{c}\right.}$$
$$-2\,\frac{cd}{e\left(e\sqrt{-4\,ac+b^2}+be-2\,cd\right)\left(-4\,ac+b^2\right)}\sqrt{\left(x^2+1/2\,\frac{b+\sqrt{-4\,ac+b^2}}{c}\right)^2c-\sqrt{-4\,ac+b^2}\left(x^2+1/2\,\frac{b+\sqrt{-4\,ac+b^2}}{c}\right)}\left(\right.$$
$$-2\,\frac{cd}{\left(e\sqrt{-4\,ac+b^2}-be+2\,cd\right)\left(e\sqrt{-4\,ac+b^2}+be-2\,cd\right)}\ln\left(1\left(2\,\frac{ae^2-bde+cd^2}{e^2}+\frac{be-2\,cd}{e}\left(x^2+\frac{d}{e}\right)+2\sqrt{\frac{ae^2-bde}{e^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/e*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)+2*d/e*c/(e*(-4*
a*c+b^2)^(1/2)-b*e+2*c*d)/(-4*a*c+b^2)/(x^2+1/2*b/c-1/2/c*(-4*a*c
+b^2)^(1/2))*((x^2-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))^2*c+(-4*a*c+b^2
)^(1/2)*(x^2-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)-2*d/e*c/(e*(-4
*a*c+b^2)^(1/2)+b*e-2*c*d)/(-4*a*c+b^2)/(x^2+1/2/c*(-4*a*c+b^2)^(
```

```
1/2)+1/2*b/c)*((x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*c-(-4*a*c+b^2
)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)-2*d*c/(e*(-4*a*
c+b^2)^(1/2)-b*e+2*c*d)/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/((a*e^2-
b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)
/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*
e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(cx^4 + bx^2 + a)^{\frac{3}{2}}(ex^2 + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

Fricas [A] time = 0.620768, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*x^4 + b*x^2 + a)*(2*a*c*d - a*b*e + (b*c*d - 2*a*c
*e)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2*c - 4*a*c^2)*d*e*x^4
 + (b^3 - 4*a*b*c)*d*e*x^2 + (a*b^2 - 4*a^2*c)*d*e)*log((4*(b*c*d
^3 + 3*a*b*d*e^2 - 2*a^2*e^3 - (b^2 + 2*a*c)*d^2*e + (2*c^2*d^3 -
 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*x^2)*sqrt(c*x^4 + b
*x^2 + a) - ((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*
a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^
2 - (3*b^2 + 4*a*c)*d*e)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2))/(e^2*x
^4 + 2*d*e*x^2 + d^2)))/((((b^2*c^2 - 4*a*c^3)*d^2 - (b^3*c - 4*a
*b*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^4 + (a*b^2*c - 4*a^2*c
^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2 + ((b
^3*c - 4*a*b*c^2)*d^2 - (b^4 - 4*a*b^2*c)*d*e + (a*b^3 - 4*a^2*b*
c)*e^2)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2)), 1/2*(2*sqrt(c*x^4 + b*
x^2 + a)*(2*a*c*d - a*b*e + (b*c*d - 2*a*c*e)*x^2)*sqrt(-c*d^2 +
b*d*e - a*e^2) + ((b^2*c - 4*a*c^2)*d*e*x^4 + (b^3 - 4*a*b*c)*d*e
*x^2 + (a*b^2 - 4*a^2*c)*d*e)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a
*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(sqrt(c*x^4 + b*x^2 + a)*
(c*d^2 - b*d*e + a*e^2))))/((((b^2*c^2 - 4*a*c^3)*d^2 - (b^3*c -
```

```
4*a*b*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^4 + (a*b^2*c - 4*a^
2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2 +
((b^3*c - 4*a*b*c^2)*d^2 - (b^4 - 4*a*b^2*c)*d*e + (a*b^3 - 4*a^2
*b*c)*e^2)*x^2)*sqrt(-c*d^2 + b*d*e - a*e^2))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(d+ex^2)\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**3/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.31888, size = 595, normalized size = 3.74

$$-\frac{d\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)e}{(cd^2-bde+ae^2)\sqrt{-cd^2+bde-ae^2}}+\frac{\frac{\left(bc^2d^3-b^2cd^2e-2\,ac^2d^2e+3\,abcde^2-2\,a^2ce^3\right)x^2}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}+\frac{2\,ac^2d^3\cdots}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b\cdots}}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] -d*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d
)/sqrt(-c*d^2 + b*d*e - a*e^2))*e/((c*d^2 - b*d*e + a*e^2)*sqrt(-
c*d^2 + b*d*e - a*e^2)) + ((b*c^2*d^3 - b^2*c*d^2*e - 2*a*c^2*d^2
*e + 3*a*b*c*d*e^2 - 2*a^2*c*e^3)*x^2/(b^2*c^2*d^4 - 4*a*c^3*d^4
- 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e
^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^
2*e^4 - 4*a^3*c*e^4) + (2*a*c^2*d^3 - 3*a*b*c*d^2*e + a*b^2*d*e^2
 + 2*a^2*c*d*e^2 - a^2*b*e^3)/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*
c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a
^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 -
4*a^3*c*e^4))/sqrt(c*x^4 + b*x^2 + a)
```

3.337 $\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=166

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2ace+b^2(-e)+cx^2(2cd-be)+bcd}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}$$

```
[Out] -((b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2)/((b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (e^2*ArcTanh[
(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*
Sqrt[a + b*x^2 + c*x^4])])/(2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rubi [A] time = 0.436966, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2ace+b^2(-e)+cx^2(2cd-be)+bcd}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2)/((b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (e^2*ArcTanh[
(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*
Sqrt[a + b*x^2 + c*x^4])])/(2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rubi in Sympy [A] time = 61.4212, size = 150, normalized size = 0.9

$$-\frac{e^2\,\mathrm{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{\frac{3}{2}}}+\frac{-2ace+b^2e-bcd+cx^2\left(be-2cd\right)}{\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -e**2*atanh((2*a*e - b*d + x**2*(b*e - 2*c*d))/(2*sqrt(a + b*x**2
 + c*x**4)*sqrt(a*e**2 - b*d*e + c*d**2)))/(2*(a*e**2 - b*d*e + c
*d**2)**(3/2)) + (-2*a*c*e + b**2*e - b*c*d + c*x**2*(b*e - 2*c*d
```

```
))/((-4*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)*(a*e**2 - b*d*e + c
*d**2))
```

Mathematica [A] time = 0.618395, size = 201, normalized size = 1.21

$$\frac{2c\left(ae+cdx^2\right)+b^2(-e)+bc\left(d-ex^2\right)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(e(bd-ae)-cd^2)}+\frac{e^2\log\left(d+ex^2\right)}{2\left(e(ae-bd)+cd^2\right)^{3/2}}-\frac{e^2\log\left(2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex^2-2cdx^2\right)}{2\left(e(ae-bd)+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-(b^2*e) + 2*c*(a*e + c*d*x^2) + b*c*(d - e*x^2))/((b^2 - 4*a*c)
*(-(c*d^2) + e*(b*d - a*e))*Sqrt[a + b*x^2 + c*x^4]) + (e^2*Log[d
 + e*x^2])/(2*(c*d^2 + e*(-(b*d) + a*e))^(3/2)) - (e^2*Log[-(b*d)
 + 2*a*e - 2*c*d*x^2 + b*e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sq
rt[a + b*x^2 + c*x^4]])/(2*(c*d^2 + e*(-(b*d) + a*e))^(3/2))
```

Maple [B] time = 0.011, size = 454, normalized size = 2.7

$$-2\,\frac{c}{\left(e\sqrt{-4\,ac+b^2}-be+2\,cd\right)\left(-4\,ac+b^2\right)}\sqrt{\left(x^2-1/2\,\frac{-b+\sqrt{-4\,ac+b^2}}{c}\right)^2c+\sqrt{-4\,ac+b^2}\left(x^2-1/2\,\frac{-b+\sqrt{-4\,ac+b^2}}{c}\right)}$$
$$+2\,\frac{c}{\left(e\sqrt{-4\,ac+b^2}+be-2\,cd\right)\left(-4\,ac+b^2\right)}\sqrt{\left(x^2+1/2\,\frac{b+\sqrt{-4\,ac+b^2}}{c}\right)^2c-\sqrt{-4\,ac+b^2}\left(x^2+1/2\,\frac{b+\sqrt{-4\,ac+b^2}}{c}\right)\left(x^2\right.}$$
$$+2\,\frac{ce}{\left(e\sqrt{-4\,ac+b^2}-be+2\,cd\right)\left(e\sqrt{-4\,ac+b^2}+be-2\,cd\right)}\ln\left(1\left(2\,\frac{ae^2-bde+cd^2}{e^2}+\frac{be-2\,cd}{e}\left(x^2+\frac{d}{e}\right)+2\sqrt{\frac{ae^2-bde}{e^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -2*c/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(-4*a*c+b^2)/(x^2-1/2/c*(-b
+(-4*a*c+b^2)^(1/2)))*((x^2-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))^2*c+(-
4*a*c+b^2)^(1/2)*(x^2-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)+2*c/(
e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(-4*a*c+b^2)/(x^2+1/2*(b+(-4*a*c+
b^2)^(1/2))/c)*((x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*c-(-4*a*c+b^
```

```
2)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)+2*c*e/(e*(-4*a
*c+b^2)^(1/2)-b*e+2*c*d)/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/((a*e^2
-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d
)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b
*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(cx^4 + bx^2 + a)^{\frac{3}{2}}(ex^2 + d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] integrate(x/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

Fricas [A] time = 0.632573, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(c*x^4 + b*x^2 + a)*(b*c*d + (2*c^2*d - b*c*e)*x^2 -
 (b^2 - 2*a*c)*e)*sqrt(c*d^2 - b*d*e + a*e^2) - ((b^2*c - 4*a*c^2
)*e^2*x^4 + (b^3 - 4*a*b*c)*e^2*x^2 + (a*b^2 - 4*a^2*c)*e^2)*log(
-(4*(b*c*d^3 + 3*a*b*d*e^2 - 2*a^2*e^3 - (b^2 + 2*a*c)*d^2*e + (2
*c^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*x^2)*sqrt
(c*x^4 + b*x^2 + a) + ((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2
)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
+ 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2)*sqrt(c*d^2 - b*d*e + a*e^
2))/(e^2*x^4 + 2*d*e*x^2 + d^2)))/((((b^2*c^2 - 4*a*c^3)*d^2 - (b
^3*c - 4*a*b*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^4 + (a*b^2*c
 - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)
*e^2 + ((b^3*c - 4*a*b*c^2)*d^2 - (b^4 - 4*a*b^2*c)*d*e + (a*b^3
- 4*a^2*b*c)*e^2)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2)), -1/2*(2*sqrt
(c*x^4 + b*x^2 + a)*(b*c*d + (2*c^2*d - b*c*e)*x^2 - (b^2 - 2*a*c
)*e)*sqrt(-c*d^2 + b*d*e - a*e^2) + ((b^2*c - 4*a*c^2)*e^2*x^4 +
(b^3 - 4*a*b*c)*e^2*x^2 + (a*b^2 - 4*a^2*c)*e^2)*arctan(-1/2*sqrt
(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(sqrt(
c*x^4 + b*x^2 + a)*(c*d^2 - b*d*e + a*e^2))))/((((b^2*c^2 - 4*a*c
^3)*d^2 - (b^3*c - 4*a*b*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^
4 + (a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^
```

```
2 - 4*a^3*c)*e^2 + ((b^3*c - 4*a*b*c^2)*d^2 - (b^4 - 4*a*b^2*c)*d
*e + (a*b^3 - 4*a^2*b*c)*e^2)*x^2)*sqrt(-c*d^2 + b*d*e - a*e^2))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(d+ex^2)\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.320088, size = 613, normalized size = 3.69

$$-\frac{\frac{(2\,c^3d^3-3\,bc^2d^2e+b^2cde^2+2\,ac^2de^2-abce^3)\,x^2}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}+\frac{bc^2d^3-2\,b^2cd\cdots}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+\cdots}}{\sqrt{cx^4+bx^2+a}}+\frac{\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)e^2}{(cd^2-bde+ae^2)\sqrt{-cd^2+bde-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] -((2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2 + 2*a*c^2*d*e^2 - a*b*
c*e^3)*x^2/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2
*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*
a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c
^2*d^3 - 2*b^2*c*d^2*e + 2*a*c^2*d^2*e + b^3*d*e^2 - a*b*c*d*e^2
- a*b^2*e^3 + 2*a^2*c*e^3)/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d
^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*
c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a
^3*c*e^4))/sqrt(c*x^4 + b*x^2 + a) + arctan(-((sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))
*e^2/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2))
```

3.338 $\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=266

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d}+\frac{e\left(2ace+b^2(-e)+cx^2(2cd-be)+bcd\right)}{d\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{-2ac+b^2+bcx^2}{ad\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{e^3\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d\left(ae^2-bde+cd^2\right)^{3/2}}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x^2 + c*x^4
]) + (e*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2))/((b^2 -
4*a*c)*d*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - ArcTa
nh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*a^(3/2)*
d) - (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2
 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*(c*d^2 - b*d*e
+ a*e^2)^(3/2))
```

Rubi [A] time = 0.923201, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d}+\frac{e\left(2ace+b^2(-e)+cx^2(2cd-be)+bcd\right)}{d\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{-2ac+b^2+bcx^2}{ad\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{e^3\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d\left(ae^2-bde+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```


```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x^2 + c*x^4
]) + (e*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2))/((b^2 -
4*a*c)*d*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - ArcTa
nh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*a^(3/2)*
d) - (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2
 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*(c*d^2 - b*d*e
+ a*e^2)^(3/2))
```

Rubi in Sympy [A] time = 111.14, size = 238, normalized size = 0.89

$$\frac{e^3 \operatorname{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d\left(ae^2-bde+cd^2\right)^{\frac{3}{2}}}-\frac{e\left(-2ace+b^2e-bcd+cx^2\left(be-2cd\right)\right)}{d\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{-2ac+b^2+bcx^2}{ad\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}-\frac{\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] e**3*atanh((2*a*e - b*d + x**2*(b*e - 2*c*d))/(2*sqrt(a + b*x**2
+ c*x**4)*sqrt(a*e**2 - b*d*e + c*d**2)))/(2*d*(a*e**2 - b*d*e +
c*d**2)**(3/2)) - e*(-2*a*c*e + b**2*e - b*c*d + c*x**2*(b*e - 2*
c*d))/(d*(-4*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)*(a*e**2 - b*d*
e + c*d**2)) + (-2*a*c + b**2 + b*c*x**2)/(a*d*(-4*a*c + b**2)*sq
rt(a + b*x**2 + c*x**4)) - atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a
 + b*x**2 + c*x**4)))/(2*a**(3/2)*d)
```

Mathematica [A] time = 3.09718, size = 378, normalized size = 1.42

$$\frac{\frac{-\frac{a^{3/2}e^3\log\left(d+ex^2\right)}{\sqrt{ae^2-bde+cd^2}}+\frac{a^{3/2}e^3\log\left(2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex^2-2cdx^2\right)}{\sqrt{ae^2-bde+cd^2}}+\log\left(x^2\right)\left(e(ae-bd)+cd^2\right)-cd^2\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)+bde\log\left(2\sqrt{a}\sqrt{\ldots}\right.}{d}}{2a^{3/2}\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] ((2*Sqrt[a]*(-(b^3*e) + b*c*(3*a*e + c*d*x^2) + b^2*c*(d - e*x^2)
 - 2*a*c^2*(d - e*x^2)))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
+ ((c*d^2 + e*(-(b*d) + a*e))*Log[x^2] - (a^(3/2)*e^3*Log[d + e*x
^2])/Sqrt[c*d^2 - b*d*e + a*e^2] - c*d^2*Log[2*a + b*x^2 + 2*Sqrt
[a]*Sqrt[a + b*x^2 + c*x^4]] + b*d*e*Log[2*a + b*x^2 + 2*Sqrt[a]*
Sqrt[a + b*x^2 + c*x^4]] - a*e^2*Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt
[a + b*x^2 + c*x^4]] + (a^(3/2)*e^3*Log[-(b*d) + 2*a*e - 2*c*d*x^
2 + b*e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^
4]])/Sqrt[c*d^2 - b*d*e + a*e^2])/d)/(2*a^(3/2)*(c*d^2 + e*(-(b*d
) + a*e)))
```

Maple [B] time = 0.018, size = 612, normalized size = 2.3

$$\frac{1}{2\,ad}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{bcx^2}{ad\,(4\,ac-b^2)}\frac{1}{\sqrt{cx^4+bx^2+a}}$$
$$-\frac{b^2}{2\,ad\,(4\,ac-b^2)}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{1}{2\,d}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{3}{2}}$$
$$+2\,\frac{ce}{d\left(e\sqrt{-4\,ac+b^2}-be+2\,cd\right)(-4\,ac+b^2)}\sqrt{\left(x^2-1/2\,\frac{-b+\sqrt{-4\,ac+b^2}}{c}\right)^2c+\sqrt{-4\,ac+b^2}\left(x^2-1/2\,\frac{-b+\sqrt{-4\,ac+b^2}}{c}\right.}$$
$$-2\,\frac{ce}{d\left(e\sqrt{-4\,ac+b^2}+be-2\,cd\right)(-4\,ac+b^2)}\sqrt{\left(x^2+1/2\,\frac{b+\sqrt{-4\,ac+b^2}}{c}\right)^2c-\sqrt{-4\,ac+b^2}\left(x^2+1/2\,\frac{b+\sqrt{-4\,ac+b^2}}{c}\right)\left(\right.}$$
$$-2\,\frac{e^2c}{d\left(e\sqrt{-4\,ac+b^2}-be+2\,cd\right)\left(e\sqrt{-4\,ac+b^2}+be-2\,cd\right)}\ln\left(1\left(2\,\frac{ae^2-bde+cd^2}{e^2}+\frac{be-2\,cd}{e}\left(x^2+\frac{d}{e}\right)+2\,\sqrt{\frac{ae^2-bd}{e^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/2/d/a/(c*x^4+b*x^2+a)^(1/2)-1/d*b/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)
^(1/2)*x^2*c-1/2/d*b^2/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)-1/2/d/
a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+2/d*e
*c/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(-4*a*c+b^2)/(x^2+1/2*b/c-1/2
/c*(-4*a*c+b^2)^(1/2))*((x^2-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))^2*c+(
-4*a*c+b^2)^(1/2)*(x^2-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/d*
e*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(-4*a*c+b^2)/(x^2+1/2/c*(-4*
a*c+b^2)^(1/2)+1/2*b/c)*((x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*c-(
-4*a*c+b^2)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)-2/d*e
^2*c/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(e*(-4*a*c+b^2)^(1/2)+b*e-2
*c*d)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e
^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x^
2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)/(x^2+d/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x), x)
```

Fricas [A] time = 3.51356, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*x^4 + b*x^2 + a)*((b^2*c - 2*a*c^2)*d^2 - (b^3 - 3
*a*b*c)*d*e + (b*c^2*d^2 - (b^2*c - 2*a*c^2)*d*e)*x^2)*sqrt(c*d^2
 - b*d*e + a*e^2)*sqrt(a) + ((a*b^2*c - 4*a^2*c^2)*e^3*x^4 + (a*b
^3 - 4*a^2*b*c)*e^3*x^2 + (a^2*b^2 - 4*a^3*c)*e^3)*sqrt(a)*log((4
*(b*c*d^3 + 3*a*b*d*e^2 - 2*a^2*e^3 - (b^2 + 2*a*c)*d^2*e + (2*c^
2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*x^2)*sqrt(c*
x^4 + b*x^2 + a) - ((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x
^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4
*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2))
/(e^2*x^4 + 2*d*e*x^2 + d^2)) + (((b^2*c^2 - 4*a*c^3)*d^2 - (b^3*
c - 4*a*b*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^4 + (a*b^2*c -
4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^
2 + ((b^3*c - 4*a*b*c^2)*d^2 - (b^4 - 4*a*b^2*c)*d*e + (a*b^3 - 4
*a^2*b*c)*e^2)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log((4*sqrt(c*x^4
 + b*x^2 + a)*(a*b*x^2 + 2*a^2) - ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2
+ 8*a^2)*sqrt(a))/x^4))/((((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c
 - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*e^2)*x^4 + (a^2
*b^2*c - 4*a^3*c^2)*d^3 - (a^2*b^3 - 4*a^3*b*c)*d^2*e + (a^3*b^2
- 4*a^4*c)*d*e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d^3 - (a*b^4 - 4*a^2*
b^2*c)*d^2*e + (a^2*b^3 - 4*a^3*b*c)*d*e^2)*x^2)*sqrt(c*d^2 - b*d
*e + a*e^2)*sqrt(a)), 1/4*(4*sqrt(c*x^4 + b*x^2 + a)*((b^2*c - 2*
a*c^2)*d^2 - (b^3 - 3*a*b*c)*d*e + (b*c^2*d^2 - (b^2*c - 2*a*c^2)
*d*e)*x^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(a) + 2*((a*b^2*c - 4
*a^2*c^2)*e^3*x^4 + (a*b^3 - 4*a^2*b*c)*e^3*x^2 + (a^2*b^2 - 4*a^
3*c)*e^3)*sqrt(a)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*
d - b*e)*x^2 + b*d - 2*a*e)/(sqrt(c*x^4 + b*x^2 + a)*(c*d^2 - b*d
*e + a*e^2))) + (((b^2*c^2 - 4*a*c^3)*d^2 - (b^3*c - 4*a*b*c^2)*d
*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^4 + (a*b^2*c - 4*a^2*c^2)*d^2 -
 (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2 + ((b^3*c - 4*
a*b*c^2)*d^2 - (b^4 - 4*a*b^2*c)*d*e + (a*b^3 - 4*a^2*b*c)*e^2)*x
^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*log((4*sqrt(c*x^4 + b*x^2 + a)*(
a*b*x^2 + 2*a^2) - ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a
))/x^4))/((((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)
*d^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*e^2)*x^4 + (a^2*b^2*c - 4*a^3*
c^2)*d^3 - (a^2*b^3 - 4*a^3*b*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*d*e^
2 + ((a*b^3*c - 4*a^2*b*c^2)*d^3 - (a*b^4 - 4*a^2*b^2*c)*d^2*e +
(a^2*b^3 - 4*a^3*b*c)*d*e^2)*x^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*sq
rt(a)), 1/4*(4*sqrt(c*x^4 + b*x^2 + a)*((b^2*c - 2*a*c^2)*d^2 - (
b^3 - 3*a*b*c)*d*e + (b*c^2*d^2 - (b^2*c - 2*a*c^2)*d*e)*x^2)*sqr
t(c*d^2 - b*d*e + a*e^2)*sqrt(-a) - 2*(((b^2*c^2 - 4*a*c^3)*d^2 -
```

```
 (b^3*c - 4*a*b*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^4 + (a*b^
2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3
*c)*e^2 + ((b^3*c - 4*a*b*c^2)*d^2 - (b^4 - 4*a*b^2*c)*d*e + (a*b
^3 - 4*a^2*b*c)*e^2)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2)*arctan(1/2*
(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)*a)) + ((a*b^2*c -
 4*a^2*c^2)*e^3*x^4 + (a*b^3 - 4*a^2*b*c)*e^3*x^2 + (a^2*b^2 - 4*
a^3*c)*e^3)*sqrt(-a)*log((4*(b*c*d^3 + 3*a*b*d*e^2 - 2*a^2*e^3 -
(b^2 + 2*a*c)*d^2*e + (2*c^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 +
 2*a*c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a) - ((8*c^2*d^2 - 8*b*c
*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*
a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2)*s
qrt(c*d^2 - b*d*e + a*e^2))/(e^2*x^4 + 2*d*e*x^2 + d^2)))/((((a*b
^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^
2*c - 4*a^3*c^2)*d*e^2)*x^4 + (a^2*b^2*c - 4*a^3*c^2)*d^3 - (a^2*
b^3 - 4*a^3*b*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*d*e^2 + ((a*b^3*c -
4*a^2*b*c^2)*d^3 - (a*b^4 - 4*a^2*b^2*c)*d^2*e + (a^2*b^3 - 4*a^3
*b*c)*d*e^2)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(-a)), 1/2*(2*s
qrt(c*x^4 + b*x^2 + a)*((b^2*c - 2*a*c^2)*d^2 - (b^3 - 3*a*b*c)*d
*e + (b*c^2*d^2 - (b^2*c - 2*a*c^2)*d*e)*x^2)*sqrt(-c*d^2 + b*d*e
 - a*e^2)*sqrt(-a) + ((a*b^2*c - 4*a^2*c^2)*e^3*x^4 + (a*b^3 - 4*
a^2*b*c)*e^3*x^2 + (a^2*b^2 - 4*a^3*c)*e^3)*sqrt(-a)*arctan(-1/2*
sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(s
qrt(c*x^4 + b*x^2 + a)*(c*d^2 - b*d*e + a*e^2))) - (((b^2*c^2 - 4
*a*c^3)*d^2 - (b^3*c - 4*a*b*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2
)*x^4 + (a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^
2*b^2 - 4*a^3*c)*e^2 + ((b^3*c - 4*a*b*c^2)*d^2 - (b^4 - 4*a*b^2*
c)*d*e + (a*b^3 - 4*a^2*b*c)*e^2)*x^2)*sqrt(-c*d^2 + b*d*e - a*e^
2)*arctan(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)*a))
)/((((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e
+ (a^2*b^2*c - 4*a^3*c^2)*d*e^2)*x^4 + (a^2*b^2*c - 4*a^3*c^2)*d^
3 - (a^2*b^3 - 4*a^3*b*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*d*e^2 + ((a
*b^3*c - 4*a^2*b*c^2)*d^3 - (a*b^4 - 4*a^2*b^2*c)*d^2*e + (a^2*b^
3 - 4*a^3*b*c)*d*e^2)*x^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(-a))
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,(d+ex^2)\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.339 $\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=419

$$\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d^2}+\frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}d}-\frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{2a^2dx^2(b^2-4ac)}-\frac{e^2\left(2ace+b^2(-e)+cx^2(2cd-be)+bcd\right)}{d^2(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)}-\frac{e\left(-2ac+b^2+bcx^2\right)}{ad^2(b^2-4ac)\sqrt{a+bx^2+cx^4}}+\frac{-2ac+b^2+bcx^2}{adx^2(b^2-4ac)\sqrt{a+bx^2+cx^4}}+\frac{e^4\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2(ae^2-bde+cd^2)^{3/2}}$$

```
[Out] -((e*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*d^2*Sqrt[a + b*x^2
 + c*x^4])) + (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*d*x^2*Sqrt
[a + b*x^2 + c*x^4]) - (e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d -
 b*e)*x^2))/((b^2 - 4*a*c)*d^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b
*x^2 + c*x^4]) - ((3*b^2 - 8*a*c)*Sqrt[a + b*x^2 + c*x^4])/(2*a^2
*(b^2 - 4*a*c)*d*x^2) + (3*b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqr
t[a + b*x^2 + c*x^4])])/(4*a^(5/2)*d) + (e*ArcTanh[(2*a + b*x^2)/
(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*a^(3/2)*d^2) + (e^4*ArcT
anh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2*(c*d^2 - b*d*e + a*e^2)^(3/
2))
```

Rubi [A] time = 1.34419, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d^2}+\frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}d}-\frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{2a^2dx^2(b^2-4ac)}-\frac{e^2\left(2ace+b^2(-e)+cx^2(2cd-be)+bcd\right)}{d^2(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)}-\frac{e\left(-2ac+b^2+bcx^2\right)}{ad^2(b^2-4ac)\sqrt{a+bx^2+cx^4}}+\frac{-2ac+b^2+bcx^2}{adx^2(b^2-4ac)\sqrt{a+bx^2+cx^4}}+\frac{e^4\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2(ae^2-bde+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((e*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*d^2*Sqrt[a + b*x^2
 + c*x^4])) + (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*d*x^2*Sqrt
[a + b*x^2 + c*x^4]) - (e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d -
 b*e)*x^2))/((b^2 - 4*a*c)*d^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b
```

```
*x^2 + c*x^4]) - ((3*b^2 - 8*a*c)*Sqrt[a + b*x^2 + c*x^4])/(2*a^2
*(b^2 - 4*a*c)*d*x^2) + (3*b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqr
t[a + b*x^2 + c*x^4])])/(4*a^(5/2)*d) + (e*ArcTanh[(2*a + b*x^2)/
(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*a^(3/2)*d^2) + (e^4*ArcT
anh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2*(c*d^2 - b*d*e + a*e^2)^(3/
2))
```

Rubi in Sympy [A] time = 148.256, size = 384, normalized size = 0.92

$$-\frac{e^4 \operatorname{atanh}\left(\frac{2ae-bd+x^2(be-2cd)}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2\left(ae^2-bde+cd^2\right)^{\frac{3}{2}}}+\frac{e^2\left(-2ace+b^2e-bcd+cx^2\left(be-2cd\right)\right)}{d^2\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{-2ac+b^2+bcx^2}{adx^2\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}-\frac{e\left(-2ac+b^2+bcx^2\right)}{ad^2\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}-\frac{\left(-8ac+3b^2\right)\sqrt{a+bx^2+cx^4}}{2a^2dx^2\left(-4ac+b^2\right)}+\frac{e\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{\frac{3}{2}}d^2}+\frac{3b\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -e**4*atanh((2*a*e - b*d + x**2*(b*e - 2*c*d))/(2*sqrt(a + b*x**2
 + c*x**4)*sqrt(a*e**2 - b*d*e + c*d**2)))/(2*d**2*(a*e**2 - b*d*
e + c*d**2)**(3/2)) + e**2*(-2*a*c*e + b**2*e - b*c*d + c*x**2*(b
*e - 2*c*d))/(d**2*(-4*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)*(a*e
**2 - b*d*e + c*d**2)) + (-2*a*c + b**2 + b*c*x**2)/(a*d*x**2*(-4
*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)) - e*(-2*a*c + b**2 + b*c*
x**2)/(a*d**2*(-4*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)) - (-8*a*
c + 3*b**2)*sqrt(a + b*x**2 + c*x**4)/(2*a**2*d*x**2*(-4*a*c + b*
*2)) + e*atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**4
)))/(2*a**(3/2)*d**2) + 3*b*atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(
a + b*x**2 + c*x**4)))/(4*a**(5/2)*d)
```

Mathematica [A] time = 2.93003, size = 349, normalized size = 0.83

$$\frac{1}{4}\left(\frac{(2ae+3bd)\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)}{a^{5/2}d^2}-\frac{\log\left(x^2\right)(2ae+3bd)}{a^{5/2}d^2}-\frac{4\left(-b^2c\left(4ae+cdx^2\right)+3abc^2\left(d-ex^2\right)+2ac^2\left(ae+cdx^2\right)+b^4e+b^3c\left(ex^2-d\right)\right)}{a^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(e(bd-ae)-cd^2\right)}-\frac{2\sqrt{a+bx^2+cx^4}}{a^2dx^2}+\frac{2e^4\log\left(d+ex^2\right)}{d^2\left(e(ae-bd)+cd^2\right)^{3/2}}-\frac{2e^4\log\left(2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex^2-2cdx^2\right)}{d^2\left(e(ae-bd)+cd^2\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] ((-2*Sqrt[a + b*x^2 + c*x^4])/(a^2*d*x^2) - (4*(b^4*e + 2*a*c^2*(
a*e + c*d*x^2) - b^2*c*(4*a*e + c*d*x^2) + 3*a*b*c^2*(d - e*x^2)
+ b^3*c*(-d + e*x^2)))/(a^2*(b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*
e))*Sqrt[a + b*x^2 + c*x^4]) - ((3*b*d + 2*a*e)*Log[x^2])/(a^(5/2
)*d^2) + (2*e^4*Log[d + e*x^2])/(d^2*(c*d^2 + e*(-(b*d) + a*e))^(
3/2)) + ((3*b*d + 2*a*e)*Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x
^2 + c*x^4]])/(a^(5/2)*d^2) - (2*e^4*Log[-(b*d) + 2*a*e - 2*c*d*x
^2 + b*e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x
^4]])/(d^2*(c*d^2 + e*(-(b*d) + a*e))^(3/2)))/4
```

Maple [B] time = 0.019, size = 863, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/2/d/a/x^2/(c*x^4+b*x^2+a)^(1/2)-3/4/d*b/a^2/(c*x^4+b*x^2+a)^(1
/2)+3/2/d*b^2/a^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*c*x^2+3/4/d*b
^3/a^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)+3/4/d*b/a^(5/2)*ln((2*a+
b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-4/d*c^2/a/(4*a*c-b^2)
/(c*x^4+b*x^2+a)^(1/2)*x^2-2/d*c/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1
/2)*b-2/d^2*e^2*c/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(-4*a*c+b^2)/(
x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*((x^2-1/2/c*(-b+(-4*a*c+b^2
)^(1/2)))^2*c+(-4*a*c+b^2)^(1/2)*(x^2-1/2/c*(-b+(-4*a*c+b^2)^(1/2
))))^(1/2)+2/d^2*e^2*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(-4*a*c+b
```

```
^2)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*((x^2+1/2*(b+(-4*a*c+b
^2)^(1/2))/c)^2*c-(-4*a*c+b^2)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)^(1/
2))/c))^(1/2)+2/d^2*e^3*c/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(e*(-4
*a*c+b^2)^(1/2)+b*e-2*c*d)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*
(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*((x^2+d/e)^2*c+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))-1/2/d^2*e/a/(c*x^4+b*x^2+a)^(1/
2)+1/d^2*e*b/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2*c+1/2/d^2*e*
b^2/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)+1/2/d^2*e/a^(3/2)*ln((2*a
+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)^{\frac{3}{2}}(ex^2 + d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x^3), x)
```

Fricas [A] time = 8.48454, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(((3*b^2*c^2 - 8*a*c^3)*d^3 - (3*b^3*c - 10*a*b*c^2)*d^2
*e + (a*b^2*c - 4*a^2*c^2)*d*e^2)*x^4 + (a*b^2*c - 4*a^2*c^2)*d^3
 - (a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 + ((3*b^
3*c - 10*a*b*c^2)*d^3 - (3*b^4 - 12*a*b^2*c + 4*a^2*c^2)*d^2*e +
(a*b^3 - 4*a^2*b*c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^
2 - b*d*e + a*e^2)*sqrt(a) - 2*((a^2*b^2*c - 4*a^3*c^2)*e^4*x^6 +
 (a^2*b^3 - 4*a^3*b*c)*e^4*x^4 + (a^3*b^2 - 4*a^4*c)*e^4*x^2)*sqr
t(a)*log(-(4*(b*c*d^3 + 3*a*b*d*e^2 - 2*a^2*e^3 - (b^2 + 2*a*c)*d
^2*e + (2*c^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*
x^2)*sqrt(c*x^4 + b*x^2 + a) + ((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4
*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4
*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2)*sqrt(c*d^2 - b*d
*e + a*e^2))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - ((3*(b^3*c^2 - 4*a*b*
c^3)*d^3 - (3*b^4*c - 14*a*b^2*c^2 + 8*a^2*c^3)*d^2*e + (a*b^3*c
- 4*a^2*b*c^2)*d*e^2 + 2*(a^2*b^2*c - 4*a^3*c^2)*e^3)*x^6 + (3*(b
```

```
^4*c - 4*a*b^2*c^2)*d^3 - (3*b^5 - 14*a*b^3*c + 8*a^2*b*c^2)*d^2*
e + (a*b^4 - 4*a^2*b^2*c)*d*e^2 + 2*(a^2*b^3 - 4*a^3*b*c)*e^3)*x^
4 + (3*(a*b^3*c - 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 14*a^2*b^2*c + 8*
a^3*c^2)*d^2*e + (a^2*b^3 - 4*a^3*b*c)*d*e^2 + 2*(a^3*b^2 - 4*a^4
*c)*e^3)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-(4*sqrt(c*x^4 + b*
x^2 + a)*(a*b*x^2 + 2*a^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a
^2)*sqrt(a))/x^4))/((((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - (a^2*b^3*c
 - 4*a^3*b*c^2)*d^3*e + (a^3*b^2*c - 4*a^4*c^2)*d^2*e^2)*x^6 + ((a
^2*b^3*c - 4*a^3*b*c^2)*d^4 - (a^2*b^4 - 4*a^3*b^2*c)*d^3*e + (a^
3*b^3 - 4*a^4*b*c)*d^2*e^2)*x^4 + ((a^3*b^2*c - 4*a^4*c^2)*d^4 -
(a^3*b^3 - 4*a^4*b*c)*d^3*e + (a^4*b^2 - 4*a^5*c)*d^2*e^2)*x^2)*s
qrt(c*d^2 - b*d*e + a*e^2)*sqrt(a)), -1/8*(4*(((3*b^2*c^2 - 8*a*c
^3)*d^3 - (3*b^3*c - 10*a*b*c^2)*d^2*e + (a*b^2*c - 4*a^2*c^2)*d*
e^2)*x^4 + (a*b^2*c - 4*a^2*c^2)*d^3 - (a*b^3 - 4*a^2*b*c)*d^2*e
 + (a^2*b^2 - 4*a^3*c)*d*e^2 + ((3*b^3*c - 10*a*b*c^2)*d^3 - (3*b^
4 - 12*a*b^2*c + 4*a^2*c^2)*d^2*e + (a*b^3 - 4*a^2*b*c)*d*e^2)*x^
2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(a) +
 4*((a^2*b^2*c - 4*a^3*c^2)*e^4*x^6 + (a^2*b^3 - 4*a^3*b*c)*e^4*x
^4 + (a^3*b^2 - 4*a^4*c)*e^4*x^2)*sqrt(a)*arctan(-1/2*sqrt(-c*d^2
 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(sqrt(c*x^4 +
 b*x^2 + a)*(c*d^2 - b*d*e + a*e^2))) - ((3*(b^3*c^2 - 4*a*b*c^3)
*d^3 - (3*b^4*c - 14*a*b^2*c^2 + 8*a^2*c^3)*d^2*e + (a*b^3*c - 4*
a^2*b*c^2)*d*e^2 + 2*(a^2*b^2*c - 4*a^3*c^2)*e^3)*x^6 + (3*(b^4*c
 - 4*a*b^2*c^2)*d^3 - (3*b^5 - 14*a*b^3*c + 8*a^2*b*c^2)*d^2*e +
(a*b^4 - 4*a^2*b^2*c)*d*e^2 + 2*(a^2*b^3 - 4*a^3*b*c)*e^3)*x^4 +
(3*(a*b^3*c - 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 14*a^2*b^2*c + 8*a^3*
c^2)*d^2*e + (a^2*b^3 - 4*a^3*b*c)*d*e^2 + 2*(a^3*b^2 - 4*a^4*c)*
e^3)*x^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*log(-(4*sqrt(c*x^4 + b*x^2
 + a)*(a*b*x^2 + 2*a^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)
*sqrt(a))/x^4))/((((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - (a^2*b^3*c - 4
*a^3*b*c^2)*d^3*e + (a^3*b^2*c - 4*a^4*c^2)*d^2*e^2)*x^6 + ((a^2*
b^3*c - 4*a^3*b*c^2)*d^4 - (a^2*b^4 - 4*a^3*b^2*c)*d^3*e + (a^3*b
^3 - 4*a^4*b*c)*d^2*e^2)*x^4 + ((a^3*b^2*c - 4*a^4*c^2)*d^4 - (a^
3*b^3 - 4*a^4*b*c)*d^3*e + (a^4*b^2 - 4*a^5*c)*d^2*e^2)*x^2)*sqrt
(-c*d^2 + b*d*e - a*e^2)*sqrt(a)), -1/4*(2*(((3*b^2*c^2 - 8*a*c^3
)*d^3 - (3*b^3*c - 10*a*b*c^2)*d^2*e + (a*b^2*c - 4*a^2*c^2)*d*e^
2)*x^4 + (a*b^2*c - 4*a^2*c^2)*d^3 - (a*b^3 - 4*a^2*b*c)*d^2*e +
(a^2*b^2 - 4*a^3*c)*d*e^2 + ((3*b^3*c - 10*a*b*c^2)*d^3 - (3*b^4
- 12*a*b^2*c + 4*a^2*c^2)*d^2*e + (a*b^3 - 4*a^2*b*c)*d*e^2)*x^2)
*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(-a) - (
(3*(b^3*c^2 - 4*a*b*c^3)*d^3 - (3*b^4*c - 14*a*b^2*c^2 + 8*a^2*c^
3)*d^2*e + (a*b^3*c - 4*a^2*b*c^2)*d*e^2 + 2*(a^2*b^2*c - 4*a^3*c
^2)*e^3)*x^6 + (3*(b^4*c - 4*a*b^2*c^2)*d^3 - (3*b^5 - 14*a*b^3*c
 + 8*a^2*b*c^2)*d^2*e + (a*b^4 - 4*a^2*b^2*c)*d*e^2 + 2*(a^2*b^3
- 4*a^3*b*c)*e^3)*x^4 + (3*(a*b^3*c - 4*a^2*b*c^2)*d^3 - (3*a*b^4
 - 14*a^2*b^2*c + 8*a^3*c^2)*d^2*e + (a^2*b^3 - 4*a^3*b*c)*d*e^2
+ 2*(a^3*b^2 - 4*a^4*c)*e^3)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2)*arc
tan(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)*a)) - ((a
^2*b^2*c - 4*a^3*c^2)*e^4*x^6 + (a^2*b^3 - 4*a^3*b*c)*e^4*x^4 + (
a^3*b^2 - 4*a^4*c)*e^4*x^2)*sqrt(-a)*log(-(4*(b*c*d^3 + 3*a*b*d*e
^2 - 2*a^2*e^3 - (b^2 + 2*a*c)*d^2*e + (2*c^2*d^3 - 3*b*c*d^2*e -
 a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a) + ((
8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^
2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4
```

```
*a*c)*d*e)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2))/(e^2*x^4 + 2*d*e*x^2
 + d^2)))/((((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - (a^2*b^3*c - 4*a^3*b
*c^2)*d^3*e + (a^3*b^2*c - 4*a^4*c^2)*d^2*e^2)*x^6 + ((a^2*b^3*c
- 4*a^3*b*c^2)*d^4 - (a^2*b^4 - 4*a^3*b^2*c)*d^3*e + (a^3*b^3 - 4
*a^4*b*c)*d^2*e^2)*x^4 + ((a^3*b^2*c - 4*a^4*c^2)*d^4 - (a^3*b^3
- 4*a^4*b*c)*d^3*e + (a^4*b^2 - 4*a^5*c)*d^2*e^2)*x^2)*sqrt(c*d^2
 - b*d*e + a*e^2)*sqrt(-a)), -1/4*(2*(((3*b^2*c^2 - 8*a*c^3)*d^3
- (3*b^3*c - 10*a*b*c^2)*d^2*e + (a*b^2*c - 4*a^2*c^2)*d*e^2)*x^4
 + (a*b^2*c - 4*a^2*c^2)*d^3 - (a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b
^2 - 4*a^3*c)*d*e^2 + ((3*b^3*c - 10*a*b*c^2)*d^3 - (3*b^4 - 12*a
*b^2*c + 4*a^2*c^2)*d^2*e + (a*b^3 - 4*a^2*b*c)*d*e^2)*x^2)*sqrt(
c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(-a) + 2*((a^
2*b^2*c - 4*a^3*c^2)*e^4*x^6 + (a^2*b^3 - 4*a^3*b*c)*e^4*x^4 + (a
^3*b^2 - 4*a^4*c)*e^4*x^2)*sqrt(-a)*arctan(-1/2*sqrt(-c*d^2 + b*d
*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(sqrt(c*x^4 + b*x^2
 + a)*(c*d^2 - b*d*e + a*e^2))) - ((3*(b^3*c^2 - 4*a*b*c^3)*d^3 -
 (3*b^4*c - 14*a*b^2*c^2 + 8*a^2*c^3)*d^2*e + (a*b^3*c - 4*a^2*b*
c^2)*d*e^2 + 2*(a^2*b^2*c - 4*a^3*c^2)*e^3)*x^6 + (3*(b^4*c - 4*a
*b^2*c^2)*d^3 - (3*b^5 - 14*a*b^3*c + 8*a^2*b*c^2)*d^2*e + (a*b^4
 - 4*a^2*b^2*c)*d*e^2 + 2*(a^2*b^3 - 4*a^3*b*c)*e^3)*x^4 + (3*(a*
b^3*c - 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 14*a^2*b^2*c + 8*a^3*c^2)*d
^2*e + (a^2*b^3 - 4*a^3*b*c)*d*e^2 + 2*(a^3*b^2 - 4*a^4*c)*e^3)*x
^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(1/2*(b*x^2 + 2*a)*sqrt(-a
)/(sqrt(c*x^4 + b*x^2 + a)*a)))/((((a^2*b^2*c^2 - 4*a^3*c^3)*d^4
- (a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^3*b^2*c - 4*a^4*c^2)*d^2*e
^2)*x^6 + ((a^2*b^3*c - 4*a^3*b*c^2)*d^4 - (a^2*b^4 - 4*a^3*b^2*c
)*d^3*e + (a^3*b^3 - 4*a^4*b*c)*d^2*e^2)*x^4 + ((a^3*b^2*c - 4*a^
4*c^2)*d^4 - (a^3*b^3 - 4*a^4*b*c)*d^3*e + (a^4*b^2 - 4*a^5*c)*d^
2*e^2)*x^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(-a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (d + ex^2) (a + bx^2 + cx^4)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x^3),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.340 $\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=449

$$\frac{\sqrt{2x^4+2x^2+1}\,x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{1}{20}\sqrt{2x^4+2x^2+1}\,x+\frac{27}{80}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$+\frac{\left(7\sqrt{2}-2\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\,2^{3/4}\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\,2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{27\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{80\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\left(1-2x^2\right)x^3}{20\sqrt{2x^4+2x^2+1}}$$

```
[Out] (x^3*(1 - 2*x^2))/(20*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*Sqrt[1 + 2*x^
2 + 2*x^4])/20 + (x*Sqrt[1 + 2*x^2 + 2*x^4])/(10*Sqrt[2]*(1 + Sqr
t[2]*x^2)) + (27*Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 +
2*x^4]])/80 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sq
rt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(1
0*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + ((-2 + 7*Sqrt[2])*(1 + Sqrt[
2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2
*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(3/4)*(-2 + 3*Sqrt[2])
*Sqrt[1 + 2*x^2 + 2*x^4]) + (27*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*S
qrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*
Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(80*2^(3/4)*(
2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.99187, antiderivative size = 636, normalized size of antiderivative = 1.42, number

of steps used = 13, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.31$

$$\frac{\sqrt{2x^4+2x^2+1}x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{1}{20}\sqrt{2x^4+2x^2+1}x+\frac{27}{80}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$-\frac{\left(7+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{40\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{9\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{80\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{81\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{40\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{27\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{80\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\left(1-2x^2\right)x^3}{20\sqrt{2x^4+2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^8/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (x^3*(1 - 2*x^2))/(20*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*Sqrt[1 + 2*x^
2 + 2*x^4])/20 + (x*Sqrt[1 + 2*x^2 + 2*x^4])/(10*Sqrt[2]*(1 + Sqr
t[2]*x^2)) + (27*Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 +
2*x^4]])/80 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sq
rt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(1
0*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (81*(1 + Sqrt[2]*x^2)*Sqrt[(
1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4
)*x], (2 - Sqrt[2])/4])/(40*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^
2 + 2*x^4]) - (9*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 -
 Sqrt[2])/4])/(80*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((7 + Sqrt[2
])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2
]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(40*2^(3/4)*Sq
rt[1 + 2*x^2 + 2*x^4]) + (27*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt
[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqr
t[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(80*2^(3/4)*(2 -
 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 84.5409, size = 643, normalized size = 1.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] x**3*(-4*x**2 + 2)/(40*sqrt(2*x**4 + 2*x**2 + 1)) + x*sqrt(2*x**4
 + 2*x**2 + 1)/20 + sqrt(2)*x*sqrt(2*x**4 + 2*x**2 + 1)/(20*(sqrt
(2)*x**2 + 1)) - 2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**
2 + 1)**2)*(sqrt(2)*x**2 + 1)*elliptic_e(2*atan(2**(1/4)*x), -sqr
t(2)/4 + 1/2)/(20*sqrt(2*x**4 + 2*x**2 + 1)) - 27*2**(3/4)*sqrt((
2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*el
liptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(160*sqrt(2*x**4 +
 2*x**2 + 1)) - 2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2
 + 1)**2)*(12*sqrt(2) + 84)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(
2**(1/4)*x), -sqrt(2)/4 + 1/2)/(960*sqrt(2*x**4 + 2*x**2 + 1)) +
9*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqr
t(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(
80*sqrt(2*x**4 + 2*x**2 + 1)) - 81*2**(1/4)*sqrt((2*x**4 + 2*x**2
 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan
(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(80*(-3*sqrt(2) + 2)*sqrt(2*x**4
+ 2*x**2 + 1)) + 27*2**(3/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*
x**2 + 1)**2)*(2 + 3*sqrt(2))*(sqrt(2)*x**2 + 1)*elliptic_pi(-11*
sqrt(2)/24 + 1/2, 2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(320*(-3*
sqrt(2) + 2)*sqrt(2*x**4 + 2*x**2 + 1)) + 27*sqrt(15)*atan(sqrt(1
5)*x/(3*sqrt(2*x**4 + 2*x**2 + 1)))/400
```

Mathematica [C] time = 0.203012, size = 199, normalized size = 0.44

$$\frac{12x^3-(29-33i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)-4i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\,\mathrm{si}\right.}{80\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (4*x + 12*x^3 - (4*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 +
(1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (29 - 33*I)
*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*Elliptic
F[I*ArcSinh[Sqrt[1 - I]*x], I] + 27*(1 - I)^(3/2)*Sqrt[1 + (1 - I
)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt
[1 - I]*x], I])/(80*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.052, size = 603, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x)
```

```
[Out] 27/16*x^3/(2*x^4+2*x^2+1)^(1/2)-11/4/(-1+I)^(1/2)*(1+(1-I)*x^2)^(
1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)
^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(47/32-47/32*I)/(-1+I)^(1/2)*(1
+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(Elli
pticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I
)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-9/2*(-1/4*x^3-1/8*x)/(2*x^4+2
*x^2+1)^(1/2)+3*(1/8*x^3+1/8*x)/(2*x^4+2*x^2+1)^(1/2)+1/8*x/(2*x^
4+2*x^2+1)^(1/2)-81/4*(3/20*x^3+1/20*x)/(2*x^4+2*x^2+1)^(1/2)+81/
160/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+
2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)
)+243/160*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)
/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I
*2^(1/2))+243/160/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)
^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)
+1/2*I*2^(1/2))-243/160*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^
2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2
*2^(1/2)+1/2*I*2^(1/2))+27/40/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(
I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2
),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^8}{(4\,x^6+10\,x^4+8\,x^2+3)\sqrt{2\,x^4+2\,x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)),x, algorithm="fricas")
```

```
[Out] integral(x^8/((4*x^6 + 10*x^4 + 8*x^2 + 3)*sqrt(2*x^4 + 2*x^2 + 1
)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{(2x^2+3)\,(2x^4+2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**8/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)),x, algorithm="giac")
```

```
[Out] integrate(x^8/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.341 $\int \frac{x^6}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=423

$$\frac{\sqrt{2x^4+2x^2+1}\,x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{\left(1-2x^2\right)x}{20\sqrt{2x^4+2x^2+1}}-\frac{9}{40}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{\left(\sqrt[4]{2}+2^{3/4}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}-\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\,2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{9\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{40\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

```
[Out] (x*(1 - 2*x^2))/(20*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*Sqrt[1 + 2*x^2
+ 2*x^4])/(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) - (9*Sqrt[3/5]*ArcTan[(S
qrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/40 - ((1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(
1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) -
 ((2^(1/4) + 2^(3/4))*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/
(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])
/4])/(8*(-2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - (9*(3 + Sqrt[
2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^
2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqr
t[2])/4])/(40*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.82538, antiderivative size = 536, normalized size of antiderivative = 1.27, number

of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$\frac{\sqrt{2x^4+2x^2+1}x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{\left(1-2x^2\right)x}{20\sqrt{2x^4+2x^2+1}}+\frac{9\sqrt{\frac{3}{10}}\left(3-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{20\left(2-3\sqrt{2}\right)}-\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{40\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{9\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{20\sqrt[4]{2}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}-\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\,2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{9\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{40\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^6/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (x*(1 - 2*x^2))/(20*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*Sqrt[1 + 2*x^2
+ 2*x^4])/(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (9*Sqrt[3/10]*(3 - Sqr
t[2])*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(20*(2 - 3*S
qrt[2])) - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[
2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2
^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (9*(1 + Sqrt[2]*x^2)*Sqrt[(1 +
2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x]
, (2 - Sqrt[2])/4])/(20*2^(1/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 +
2*x^4]) - ((1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^
4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[
2])/4])/(40*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (9*(3 + Sqrt[2])*(
1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*El
lipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])
/4])/(40*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 65.212, size = 541, normalized size = 1.28

$$\frac{x\left(-4x^2+2\right)}{40\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{2}x\sqrt{2x^4+2x^2+1}}{20\left(\sqrt{2}x^2+1\right)}-\frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)E\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{20\sqrt{2x^4+2x^2+1}}$$
$$+\frac{27\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{40\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(-2\sqrt{2}+4\right)\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{160\sqrt{2x^4+2x^2+1}}$$
$$+\frac{9\cdot 2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{80\sqrt{2x^4+2x^2+1}}$$
$$-\frac{9\cdot 2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(2+3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\left(-\frac{11\sqrt{2}}{24}+\frac{1}{2};2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{160\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{9\sqrt{15}\operatorname{atan}\left(\frac{\sqrt{15}x}{3\sqrt{2x^4+2x^2+1}}\right)}{200}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] x*(-4*x**2 + 2)/(40*sqrt(2*x**4 + 2*x**2 + 1)) + sqrt(2)*x*sqrt(2
*x**4 + 2*x**2 + 1)/(20*(sqrt(2)*x**2 + 1)) - 2**(1/4)*sqrt((2*x*
*4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*ellipt
ic_e(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(20*sqrt(2*x**4 + 2*x*
*2 + 1)) + 27*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 +
 1)**2)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2
)/4 + 1/2)/(40*(-3*sqrt(2) + 2)*sqrt(2*x**4 + 2*x**2 + 1)) + 2**(
1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(-2*sqrt(2
) + 4)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)
/4 + 1/2)/(160*sqrt(2*x**4 + 2*x**2 + 1)) + 9*2**(3/4)*sqrt((2*x*
*4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*ellipt
ic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(80*sqrt(2*x**4 + 2*x*
*2 + 1)) - 9*2**(3/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 +
1)**2)*(2 + 3*sqrt(2))*(sqrt(2)*x**2 + 1)*elliptic_pi(-11*sqrt(2)
/24 + 1/2, 2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(160*(-3*sqrt(2)
 + 2)*sqrt(2*x**4 + 2*x**2 + 1)) - 9*sqrt(15)*atan(sqrt(15)*x/(3*
sqrt(2*x**4 + 2*x**2 + 1)))/200
```

Mathematica [C] time = 0.18348, size = 199, normalized size = 0.47

$$\frac{-4x^3+(8-6i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)-2i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\sinh\right.}{40\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (2*x - 4*x^3 - (2*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (
1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (8 - 6*I)*Sq
rt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I
*ArcSinh[Sqrt[1 - I]*x], I] - 9*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^
2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 -
 I]*x], I])/(40*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.015, size = 586, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x)
```

```
[Out] -9/8*x^3/(2*x^4+2*x^2+1)^(1/2)+7/4/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/
2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(
1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-17/16+17/16*I)/(-1+I)^(1/2)*(1+
(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(Ellip
ticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)
^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+3*(-1/4*x^3-1/8*x)/(2*x^4+2*x^
2+1)^(1/2)-2*(1/8*x^3+1/8*x)/(2*x^4+2*x^2+1)^(1/2)+27/2*(3/20*x^3
+1/20*x)/(2*x^4+2*x^2+1)^(1/2)-27/80/(-1+I)^(1/2)*(-I*x^2+x^2+1)^
(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I
)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-81/80*I/(-1+I)^(1/2)*(-I*x^2+x
^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x
*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-81/80/(-1+I)^(1/2)*(-I*x
^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ellipti
cE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+81/80*I/(-1+I)^(1/2)
*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*E
llipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-9/20/(-1+I)^(1
/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2
)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(2x^4+2x^2+1)^{\frac{3}{2}}(2x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{(4x^6+10x^4+8x^2+3)\sqrt{2x^4+2x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)),x, algorithm="fricas")
```

```
[Out] integral(x^6/((4*x^6 + 10*x^4 + 8*x^2 + 3)*sqrt(2*x^4 + 2*x^2 + 1
)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(2x^2+3)(2x^4+2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**6/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(2x^4+2x^2+1)^{\frac{3}{2}}(2x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)),x, algorithm="giac")
```

```
[Out] integrate(x^6/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.342 $\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=422

$$\frac{\sqrt{2x^4+2x^2+1}\,x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{\left(x^2+2\right)x}{10\sqrt{2x^4+2x^2+1}}+\frac{3}{20}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$+\frac{\left(2+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\,2^{3/4}\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\,2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{3\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{20\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

```
[Out] -(x*(2 + x^2))/(10*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*Sqrt[1 + 2*x^2 +
 2*x^4])/(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (3*Sqrt[3/5]*ArcTan[(Sq
rt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/20 - ((1 + Sqrt[2]*x^2)*Sqrt
[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1
/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) +
((2 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sq
rt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(4
*2^(3/4)*(-2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + (3*(3 + Sqrt
[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)
^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sq
rt[2])/4])/(20*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.581038, antiderivative size = 514, normalized size of antiderivative = 1.22, number

of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$\frac{\sqrt{2x^4+2x^2+1}x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{\left(x^2+2\right)x}{10\sqrt{2x^4+2x^2+1}}+\frac{3}{20}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{20\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{9\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{3\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{20\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^4/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -(x*(2 + x^2))/(10*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*Sqrt[1 + 2*x^2 +
 2*x^4])/(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (3*Sqrt[3/5]*ArcTan[(Sq
rt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/20 - ((1 + Sqrt[2]*x^2)*Sqrt
[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1
/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) -
(9*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2
]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*(2
 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + ((1 - Sqrt[2])*(1 + Sqrt
[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[
2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(20*2^(3/4)*Sqrt[1 + 2*x^2
 + 2*x^4]) + (3*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*A
rcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(20*2^(3/4)*(2 - 3*Sqrt[2])*S
qrt[1 + 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 47.4503, size = 459, normalized size = 1.09

$$-\frac{x\left(4x^{2}+8\right)}{40\sqrt{2x^{4}+2x^{2}+1}}+\frac{\sqrt{2}x\sqrt{2x^{4}+2x^{2}+1}}{20\left(\sqrt{2}x^{2}+1\right)}-\frac{\sqrt[4]{2}\sqrt{\frac{2x^{4}+2x^{2}+1}{\left(\sqrt{2}x^{2}+1\right)^{2}}}\left(\sqrt{2}x^{2}+1\right)E\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{20\sqrt{2x^{4}+2x^{2}+1}}+\frac{\sqrt[4]{2}\sqrt{\frac{2x^{4}+2x^{2}+1}{\left(\sqrt{2}x^{2}+1\right)^{2}}}\left(-4\sqrt{2}+4\right)\left(\sqrt{2}x^{2}+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{160\sqrt{2x^{4}+2x^{2}+1}}-\frac{9\sqrt[4]{2}\sqrt{\frac{2x^{4}+2x^{2}+1}{\left(\sqrt{2}x^{2}+1\right)^{2}}}\left(\sqrt{2}x^{2}+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{20\left(-3\sqrt{2}+2\right)\sqrt{2x^{4}+2x^{2}+1}}+\frac{3\cdot2^{\frac{3}{4}}\sqrt{\frac{2x^{4}+2x^{2}+1}{\left(\sqrt{2}x^{2}+1\right)^{2}}}\left(2+3\sqrt{2}\right)\left(\sqrt{2}x^{2}+1\right)\left(-\frac{11\sqrt{2}}{24}+\frac{1}{2};2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{80\left(-3\sqrt{2}+2\right)\sqrt{2x^{4}+2x^{2}+1}}+\frac{3\sqrt{15}\operatorname{atan}\left(\frac{\sqrt{15}x}{3\sqrt{2x^{4}+2x^{2}+1}}\right)}{100}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] -x*(4*x**2 + 8)/(40*sqrt(2*x**4 + 2*x**2 + 1)) + sqrt(2)*x*sqrt(2
*x**4 + 2*x**2 + 1)/(20*(sqrt(2)*x**2 + 1)) - 2**(1/4)*sqrt((2*x*
*4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*ellipt
ic_e(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(20*sqrt(2*x**4 + 2*x*
*2 + 1)) + 2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)
**2)*(-4*sqrt(2) + 4)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/
4)*x), -sqrt(2)/4 + 1/2)/(160*sqrt(2*x**4 + 2*x**2 + 1)) - 9*2**(
1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x
**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(20*(-3
*sqrt(2) + 2)*sqrt(2*x**4 + 2*x**2 + 1)) + 3*2**(3/4)*sqrt((2*x**
4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(2 + 3*sqrt(2))*(sqrt(2)*x
**2 + 1)*elliptic_pi(-11*sqrt(2)/24 + 1/2, 2*atan(2**(1/4)*x), -s
qrt(2)/4 + 1/2)/(80*(-3*sqrt(2) + 2)*sqrt(2*x**4 + 2*x**2 + 1)) +
 3*sqrt(15)*atan(sqrt(15)*x/(3*sqrt(2*x**4 + 2*x**2 + 1)))/100
```

Mathematica [C] time = 0.217251, size = 199, normalized size = 0.47

$$-\frac{2x^{3}+(1-2i)\sqrt{1-i}\sqrt{1+(1-i)x^{2}}\sqrt{1+(1+i)x^{2}}F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)+i\sqrt{1-i}\sqrt{1+(1-i)x^{2}}\sqrt{1+(1+i)x^{2}}E\left(i\sinh\right.}{20\sqrt{2x^{4}+2x^{2}+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -(4*x + 2*x^3 + I*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 +
 I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (1 - 2*I)*Sqrt[
1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*Ar
cSinh[Sqrt[1 - I]*x], I] - 3*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*
Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]
*x], I])/(20*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.013, size = 561, normalized size = 1.3

$$\frac{3x^3}{4}\frac{1}{\sqrt{2x^4+2x^2+1}}-\frac{EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\left(\frac{5}{8}-\frac{5i}{8}\right)\left(EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)-EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)\right)}{\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-2\frac{-1/4x^3-x/8}{\sqrt{2x^4+2x^2+1}}-9\frac{1}{\sqrt{2x^4+2x^2+1}}\left(\frac{3x^3}{20}+x/20\right)$$
$$+\frac{9\,EllipticF\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{40\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\frac{27i}{40}EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{27\,EllipticE\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{40\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\frac{27i}{40}EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{3}{10\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}EllipticPi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x)
```

```
[Out] 3/4*x^3/(2*x^4+2*x^2+1)^(1/2)-1/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*
(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2
),1/2*2^(1/2)+1/2*I*2^(1/2))+(5/8-5/8*I)/(-1+I)^(1/2)*(1+(1-I)*x^
2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(
-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1
/2*2^(1/2)+1/2*I*2^(1/2)))-2*(-1/4*x^3-1/8*x)/(2*x^4+2*x^2+1)^(1/
```

```
2)-9*(3/20*x^3+1/20*x)/(2*x^4+2*x^2+1)^(1/2)+9/40/(-1+I)^(1/2)*(-
I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Elli
pticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+27/40*I/(-1+I)^(1
/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2
)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+27/40/(-1+I
)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^
(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-27/40*I
/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x
^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+3
/10/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+
2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(
-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{(4\,x^6+10\,x^4+8\,x^2+3)\sqrt{2\,x^4+2\,x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)),x, algorithm="fricas")
```

```
[Out] integral(x^4/((4*x^6 + 10*x^4 + 8*x^2 + 3)*sqrt(2*x^4 + 2*x^2 + 1
)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(2x^2+3)\,(2x^4+2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**4/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)),x, algorithm="giac")
```

```
[Out] integrate(x^4/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.343 $\int \frac{x^2}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=423

$$-\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}x}{5\left(\sqrt{2}x^2+1\right)}+\frac{\left(4x^2+3\right)x}{10\sqrt{2x^4+2x^2+1}}-\frac{1}{10}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{\left(\sqrt[4]{2}+2^{3/4}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}+\frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{5\sqrt{2x^4+2x^2+1}}-\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

```
[Out] (x*(3 + 4*x^2))/(10*Sqrt[1 + 2*x^2 + 2*x^4]) - (Sqrt[2]*x*Sqrt[1
+ 2*x^2 + 2*x^4])/(5*(1 + Sqrt[2]*x^2)) - (Sqrt[3/5]*ArcTan[(Sqrt
[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/10 + (2^(1/4)*(1 + Sqrt[2]*x^2
)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTa
n[2^(1/4)*x], (2 - Sqrt[2])/4])/(5*Sqrt[1 + 2*x^2 + 2*x^4]) - ((2
^(1/4) + 2^(3/4))*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
 Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])
/(4*(-2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - ((3 + Sqrt[2])*(1
 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(10*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.564333, antiderivative size = 516, normalized size of antiderivative = 1.22, number

of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$

$$-\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}x}{5\left(\sqrt{2}x^2+1\right)}+\frac{\left(4x^2+3\right)x}{10\sqrt{2x^4+2x^2+1}}-\frac{1}{10}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$
$$-\frac{\left(1+2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{20\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{3\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{5\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{5\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^2/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (x*(3 + 4*x^2))/(10*Sqrt[1 + 2*x^2 + 2*x^4]) - (Sqrt[2]*x*Sqrt[1
+ 2*x^2 + 2*x^4])/(5*(1 + Sqrt[2]*x^2)) - (Sqrt[3/5]*ArcTan[(Sqrt
[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/10 + (2^(1/4)*(1 + Sqrt[2]*x^2
)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTa
n[2^(1/4)*x], (2 - Sqrt[2])/4])/(5*Sqrt[1 + 2*x^2 + 2*x^4]) + (3*
(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*E
llipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(5*2^(3/4)*(2 - 3
*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - ((1 + 2*Sqrt[2])*(1 + Sqrt[2
]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*
ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(20*2^(1/4)*Sqrt[1 + 2*x^2 +
 2*x^4]) - ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x
^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTa
n[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[
1 + 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 50.597, size = 456, normalized size = 1.08

$$\frac{x\left(16x^{2}+12\right)}{40\sqrt{2x^{4}+2x^{2}+1}}-\frac{\sqrt{2}x\sqrt{2x^{4}+2x^{2}+1}}{5\left(\sqrt{2}x^{2}+1\right)}+\frac{\sqrt[4]{2}\sqrt{\frac{2x^{4}+2x^{2}+1}{\left(\sqrt{2}x^{2}+1\right)^{2}}}\left(\sqrt{2}x^{2}+1\right)E\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{5\sqrt{2x^{4}+2x^{2}+1}}$$
$$-\frac{\sqrt[4]{2}\sqrt{\frac{2x^{4}+2x^{2}+1}{\left(\sqrt{2}x^{2}+1\right)^{2}}}\left(4\sqrt{2}+16\right)\left(\sqrt{2}x^{2}+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{160\sqrt{2x^{4}+2x^{2}+1}}$$
$$+\frac{3\sqrt[4]{2}\sqrt{\frac{2x^{4}+2x^{2}+1}{\left(\sqrt{2}x^{2}+1\right)^{2}}}\left(\sqrt{2}x^{2}+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{10\left(-3\sqrt{2}+2\right)\sqrt{2x^{4}+2x^{2}+1}}$$
$$-\frac{2^{\frac{3}{4}}\sqrt{\frac{2x^{4}+2x^{2}+1}{\left(\sqrt{2}x^{2}+1\right)^{2}}}\left(2+3\sqrt{2}\right)\left(\sqrt{2}x^{2}+1\right)\left(-\frac{11\sqrt{2}}{24}+\frac{1}{2};2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{40\left(-3\sqrt{2}+2\right)\sqrt{2x^{4}+2x^{2}+1}}$$
$$-\frac{\sqrt{15}\operatorname{atan}\left(\frac{\sqrt{15}x}{3\sqrt{2x^{4}+2x^{2}+1}}\right)}{50}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] x*(16*x**2 + 12)/(40*sqrt(2*x**4 + 2*x**2 + 1)) - sqrt(2)*x*sqrt(
2*x**4 + 2*x**2 + 1)/(5*(sqrt(2)*x**2 + 1)) + 2**(1/4)*sqrt((2*x*
*4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*ellipt
ic_e(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(5*sqrt(2*x**4 + 2*x**
2 + 1)) - 2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)*
*2)*(4*sqrt(2) + 16)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4
)*x), -sqrt(2)/4 + 1/2)/(160*sqrt(2*x**4 + 2*x**2 + 1)) + 3*2**(1
/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x*
*2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(10*(-3*
sqrt(2) + 2)*sqrt(2*x**4 + 2*x**2 + 1)) - 2**(3/4)*sqrt((2*x**4 +
 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(2 + 3*sqrt(2))*(sqrt(2)*x**2
 + 1)*elliptic_pi(-11*sqrt(2)/24 + 1/2, 2*atan(2**(1/4)*x), -sqrt
(2)/4 + 1/2)/(40*(-3*sqrt(2) + 2)*sqrt(2*x**4 + 2*x**2 + 1)) - sq
rt(15)*atan(sqrt(15)*x/(3*sqrt(2*x**4 + 2*x**2 + 1)))/50
```

Mathematica [C] time = 0.181865, size = 199, normalized size = 0.47

$$\frac{8x^{3}-(1+3i)\sqrt{1-i}\sqrt{1+(1-i)x^{2}}\sqrt{1+(1+i)x^{2}}F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)+4i\sqrt{1-i}\sqrt{1+(1-i)x^{2}}\sqrt{1+(1+i)x^{2}}E\left(i\sinh^{-}\right.}{20\sqrt{2x^{4}+2x^{2}+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (6*x + 8*x^3 + (4*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (
1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (1 + 3*I)*Sq
rt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I
*ArcSinh[Sqrt[1 - I]*x], I] - 2*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^
2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 -
 I]*x], I])/(20*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.013, size = 536, normalized size = 1.3

$$-\frac{x^3}{2}\frac{1}{\sqrt{2x^4+2x^2+1}}+\frac{EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{2\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\left(EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)-EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)\right)}{\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+}}$$
$$+6\frac{1}{\sqrt{2x^4+2x^2+1}}\left(\frac{3x^3}{20}+x/20\right)$$
$$-\frac{3\,EllipticF\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{20\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\frac{9i}{20}EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{9\,EllipticE\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{20\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\frac{9i}{20}EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{1}{5\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}EllipticPi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*x^3/(2*x^4+2*x^2+1)^(1/2)+1/2/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/
2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(
1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-1/4+1/4*I)/(-1+I)^(1/2)*(1+(1-I
)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF
(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/
2),1/2*2^(1/2)+1/2*I*2^(1/2)))+6*(3/20*x^3+1/20*x)/(2*x^4+2*x^2+1
)^(1/2)-3/20/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2
)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*
```

```
I*2^(1/2))-9/20*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)
^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)
+1/2*I*2^(1/2))-9/20/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2
+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1
/2)+1/2*I*2^(1/2))+9/20*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^
2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2
*2^(1/2)+1/2*I*2^(1/2))-1/5/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*
x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),
1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(2x^4+2x^2+1)^{\frac{3}{2}}(2x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{(4x^6+10x^4+8x^2+3)\sqrt{2x^4+2x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)),x, algorithm="fricas")
```

```
[Out] integral(x^2/((4*x^6 + 10*x^4 + 8*x^2 + 3)*sqrt(2*x^4 + 2*x^2 + 1
)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(2x^2+3)(2x^4+2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)

[Out] Integral(x**2/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(2x^4 + 2x^2 + 1)^{\frac{3}{2}}(2x^2 + 3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)),x, algorithm="giac")

[Out] integrate(x^2/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.344 $\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=422

$$\frac{3\sqrt{2x^4+2x^2+1}\,x}{5\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{\left(3x^2+1\right)x}{5\sqrt{2x^4+2x^2+1}}+\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{5\sqrt{15}}$$
$$+\frac{\left(2+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\,2^{3/4}\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{3\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{5\,2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{15\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

```
[Out] -(x*(1 + 3*x^2))/(5*Sqrt[1 + 2*x^2 + 2*x^4]) + (3*x*Sqrt[1 + 2*x^
2 + 2*x^4])/(5*Sqrt[2]*(1 + Sqrt[2]*x^2)) + ArcTan[(Sqrt[5/3]*x)/
Sqrt[1 + 2*x^2 + 2*x^4]]/(5*Sqrt[15]) - (3*(1 + Sqrt[2]*x^2)*Sqrt
[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1
/4)*x], (2 - Sqrt[2])/4])/(5*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (
(2 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2*
2^(3/4)*(-2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + ((3 + Sqrt[2]
)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]
*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[
2])/4])/(15*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.509002, antiderivative size = 514, normalized size of antiderivative = 1.22, number

of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{3\sqrt{2x^4+2x^2+1}x}{5\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{\left(3x^2+1\right)x}{5\sqrt{2x^4+2x^2+1}}+\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{5\sqrt{15}}$$
$$+\frac{\left(3+2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{5\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{3\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{5\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{15\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -(x*(1 + 3*x^2))/(5*Sqrt[1 + 2*x^2 + 2*x^4]) + (3*x*Sqrt[1 + 2*x^
2 + 2*x^4])/(5*Sqrt[2]*(1 + Sqrt[2]*x^2)) + ArcTan[(Sqrt[5/3]*x)/
Sqrt[1 + 2*x^2 + 2*x^4]]/(5*Sqrt[15]) - (3*(1 + Sqrt[2]*x^2)*Sqrt
[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1
/4)*x], (2 - Sqrt[2])/4])/(5*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (
2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x
^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(5*(2 - 3
*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + ((3 + 2*Sqrt[2])*(1 + Sqrt[2
]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*
ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[1 + 2*x^2 +
 2*x^4]) + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x
^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTa
n[2^(1/4)*x], (2 - Sqrt[2])/4])/(15*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[
1 + 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 96.2821, size = 530, normalized size = 1.26

$$-\frac{x\left(24x^2+8\right)}{40\sqrt{2x^4+2x^2+1}}+\frac{3\sqrt{2}x\sqrt{2x^4+2x^2+1}}{10\left(\sqrt{2}x^2+1\right)}-\frac{3\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)E\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{10\sqrt{2x^4+2x^2+1}}-\frac{\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{5\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}+\frac{2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{10\sqrt{2x^4+2x^2+1}}+\frac{3\sqrt[4]{2}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{20\sqrt{2x^4+2x^2+1}}+\frac{2^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(2+3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\left(-\frac{11\sqrt{2}}{24}+\frac{1}{2};2\operatorname{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{\sqrt{2}}{4}+\frac{1}{2}\right)}{60\left(-3\sqrt{2}+2\right)\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{15}\operatorname{atan}\left(\frac{\sqrt{15}x}{3\sqrt{2x^4+2x^2+1}}\right)}{75}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] -x*(24*x**2 + 8)/(40*sqrt(2*x**4 + 2*x**2 + 1)) + 3*sqrt(2)*x*sqr
t(2*x**4 + 2*x**2 + 1)/(10*(sqrt(2)*x**2 + 1)) - 3*2**(1/4)*sqrt(
(2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*e
lliptic_e(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(10*sqrt(2*x**4 +
 2*x**2 + 1)) - 2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2
 + 1)**2)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt
(2)/4 + 1/2)/(5*(-3*sqrt(2) + 2)*sqrt(2*x**4 + 2*x**2 + 1)) + 2**
(3/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*
x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(10*sq
rt(2*x**4 + 2*x**2 + 1)) + 3*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/
(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1
/4)*x), -sqrt(2)/4 + 1/2)/(20*sqrt(2*x**4 + 2*x**2 + 1)) + 2**(3/
4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(2 + 3*sqrt(
2))*(sqrt(2)*x**2 + 1)*elliptic_pi(-11*sqrt(2)/24 + 1/2, 2*atan(2
**(1/4)*x), -sqrt(2)/4 + 1/2)/(60*(-3*sqrt(2) + 2)*sqrt(2*x**4 +
2*x**2 + 1)) + sqrt(15)*atan(sqrt(15)*x/(3*sqrt(2*x**4 + 2*x**2 +
 1)))/75
```

Mathematica [C] time = 0.161755, size = 199, normalized size = 0.47

$$\frac{-18x^3+(6+3i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)-9i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\sin\right.}{30\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (-6*x - 18*x^3 - (9*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 +
 (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (6 + 3*I)*
Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF
[I*ArcSinh[Sqrt[1 - I]*x], I] + 2*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*
x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1
 - I]*x], I])/(30*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.009, size = 366, normalized size = 0.9

$$
\begin{aligned}
&-4\frac{1}{\sqrt{2x^4+2x^2+1}}\left(\frac{3x^3}{20}+x/20\right) \\
&+\frac{\mathit{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{10\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}} \\
&+\frac{\frac{3i}{10}\mathit{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}} \\
&+\frac{3\,\mathit{EllipticE}\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{10\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}} \\
&-\frac{\frac{3i}{10}\mathit{EllipticE}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}} \\
&+\frac{2}{15\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\mathit{EllipticPi}\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2x^4+2x^2+1}}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x)
```

```
[Out] -4*(3/20*x^3+1/20*x)/(2*x^4+2*x^2+1)^(1/2)+1/10/(-1+I)^(1/2)*(-I*
x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ellipt
icF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+3/10*I/(-1+I)^(1/2)
*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*E
```

```
llipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+3/10/(-1+I)^(1
/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2
)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-3/10*I/(-1+
I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)
^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+2/15/(
-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2
+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)),x, algorithm="maxima")

[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(4\,x^6+10\,x^4+8\,x^2+3)\sqrt{2\,x^4+2\,x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)),x, algorithm="fricas")

[Out] integral(1/((4*x^6 + 10*x^4 + 8*x^2 + 3)*sqrt(2*x^4 + 2*x^2 + 1))
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2+3)\,(2x^4+2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(1/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)),x, algorithm="giac")
```

```
[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.345 $\int \frac{1}{x^2(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=468

$$\frac{2\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{15\left(\sqrt{2}x^2+1\right)}+\frac{2\left(3x^2+1\right)x}{15\sqrt{2x^4+2x^2+1}}-\frac{x}{3\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2x^4+2x^2+1}}{3x}$$
$$-\frac{2\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{15\sqrt{15}}+\frac{\left(3\sqrt{2}-7\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\,2^{3/4}\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{2\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{15\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\sqrt[4]{2}\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{45\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

```
[Out] -x/(3*Sqrt[1 + 2*x^2 + 2*x^4]) + (2*x*(1 + 3*x^2))/(15*Sqrt[1 + 2
*x^2 + 2*x^4]) - Sqrt[1 + 2*x^2 + 2*x^4]/(3*x) + (2*Sqrt[2]*x*Sqr
t[1 + 2*x^2 + 2*x^4])/(15*(1 + Sqrt[2]*x^2)) - (2*ArcTan[(Sqrt[5/
3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(15*Sqrt[15]) - (2*2^(1/4)*(1 + S
qrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellipti
cE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(15*Sqrt[1 + 2*x^2 + 2*
x^4]) + ((-7 + 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x
^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt
[2])/4])/(3*2^(3/4)*(-2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - (
2^(1/4)*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/
(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^
(1/4)*x], (2 - Sqrt[2])/4])/(45*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 +
2*x^4])
```

Rubi [A] time = 1.07878, antiderivative size = 657, normalized size of antiderivative = 1.4, number

of steps used = 14, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31

$$\frac{2\sqrt{2}\sqrt{2x^4+2x^2+1}x}{15\left(\sqrt{2}x^2+1\right)}+\frac{2\left(3x^2+1\right)x}{15\sqrt{2x^4+2x^2+1}}-\frac{x}{3\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2x^4+2x^2+1}}{3x}$$
$$-\frac{2\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{15\sqrt{15}}-\frac{\left(3+2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{15\,2^{3/4}\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{6\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$
$$+\frac{2\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{15\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$
$$-\frac{2\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{15\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\sqrt[4]{2}\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{45\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/(x^2*(3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -x/(3*Sqrt[1 + 2*x^2 + 2*x^4]) + (2*x*(1 + 3*x^2))/(15*Sqrt[1 + 2
*x^2 + 2*x^4]) - Sqrt[1 + 2*x^2 + 2*x^4]/(3*x) + (2*Sqrt[2]*x*Sqr
t[1 + 2*x^2 + 2*x^4])/(15*(1 + Sqrt[2]*x^2)) - (2*ArcTan[(Sqrt[5/
3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(15*Sqrt[15]) - (2*2^(1/4)*(1 + S
qrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellipti
cE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(15*Sqrt[1 + 2*x^2 + 2*
x^4]) + (2*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1
+ Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4]
)/(15*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - ((1 - Sqrt[2])*(
1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*El
lipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(6*2^(1/4)*Sqrt[1
+ 2*x^2 + 2*x^4]) - ((3 + 2*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 +
2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x]
, (2 - Sqrt[2])/4])/(15*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (2^(1/
4)*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)
*x], (2 - Sqrt[2])/4])/(45*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4
])
```

Rubi in Sympy [A] time = 133.09, size = 658, normalized size = 1.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] x*(24*x**2 + 8)/(60*sqrt(2*x**4 + 2*x**2 + 1)) - x/(3*sqrt(2*x**4
 + 2*x**2 + 1)) + 2*sqrt(2)*x*sqrt(2*x**4 + 2*x**2 + 1)/(15*(sqrt
(2)*x**2 + 1)) - 2*2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x
**2 + 1)**2)*(sqrt(2)*x**2 + 1)*elliptic_e(2*atan(2**(1/4)*x), -s
qrt(2)/4 + 1/2)/(15*sqrt(2*x**4 + 2*x**2 + 1)) - 2**(1/4)*sqrt((2
*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*ell
iptic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(10*sqrt(2*x**4 + 2
*x**2 + 1)) - 2**(3/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 +
 1)**2)*(sqrt(2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -sqrt(2
)/4 + 1/2)/(15*sqrt(2*x**4 + 2*x**2 + 1)) + 2*2**(1/4)*sqrt((2*x*
*4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)**2)*(sqrt(2)*x**2 + 1)*ellipt
ic_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(15*(-3*sqrt(2) + 2)*s
qrt(2*x**4 + 2*x**2 + 1)) + 2**(1/4)*sqrt((2*x**4 + 2*x**2 + 1)/(
sqrt(2)*x**2 + 1)**2)*(-4*sqrt(2) + 8)*(sqrt(2)*x**2 + 1)*ellipti
c_f(2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(48*sqrt(2*x**4 + 2*x**
2 + 1)) - 2**(3/4)*sqrt((2*x**4 + 2*x**2 + 1)/(sqrt(2)*x**2 + 1)*
*2)*(2 + 3*sqrt(2))*(sqrt(2)*x**2 + 1)*elliptic_pi(-11*sqrt(2)/24
 + 1/2, 2*atan(2**(1/4)*x), -sqrt(2)/4 + 1/2)/(90*(-3*sqrt(2) + 2
)*sqrt(2*x**4 + 2*x**2 + 1)) - 2*sqrt(15)*atan(sqrt(15)*x/(3*sqrt
(2*x**4 + 2*x**2 + 1)))/225 - sqrt(2*x**4 + 2*x**2 + 1)/(3*x)
```

Mathematica [C] time = 0.23665, size = 211, normalized size = 0.45

$$\frac{-(27-39i)\sqrt{1-i}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}F\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)-12i\sqrt{1-i}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\sinh\right.}{90x\sqrt{2x^4+2x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] ((-12*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2
]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (27 - 39*I)*Sqrt[1 - I
]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSi
nh[Sqrt[1 - I]*x], I] - 2*(15 + 39*x^2 + 12*x^4 + 2*(1 - I)^(3/2)
*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I
/3, I*ArcSinh[Sqrt[1 - I]*x], I]))/(90*x*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.024, size = 553, normalized size = 1.2

$$-\frac{1}{3x}\sqrt{2x^4+2x^2+1}-\frac{x}{3}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{3\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\left(\frac{1}{3}-\frac{i}{3}\right)\left(EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)-EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)\right)}{\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+}}$$
$$+\frac{8}{3}\left(\frac{3x^3}{20}+\frac{x}{20}\right)\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{15\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{\frac{i}{5}EllipticF\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{5\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$+\frac{\frac{i}{5}EllipticE\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}$$
$$-\frac{4}{45\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}EllipticPi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x)
```

```
[Out] -1/3*(2*x^4+2*x^2+1)^(1/2)/x-1/3*x/(2*x^4+2*x^2+1)^(1/2)-1/3/(-1+
I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^
(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-1/3+1
/3*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4
+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/
2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+8/3*(3/2
0*x^3+1/20*x)/(2*x^4+2*x^2+1)^(1/2)-1/15/(-1+I)^(1/2)*(-I*x^2+x^2
+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(
-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-1/5*I/(-1+I)^(1/2)*(-I*x^2
+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF
(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-1/5/(-1+I)^(1/2)*(-I*x
^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ellipti
cE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+1/5*I/(-1+I)^(1/2)*(
-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ell
ipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-4/45/(-1+I)^(1/2
)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*
EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^4+2x^2+1)^{\frac{3}{2}}(2x^2+3)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(4x^8+10x^6+8x^4+3x^2)\sqrt{2x^4+2x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/((4*x^8 + 10*x^6 + 8*x^4 + 3*x^2)*sqrt(2*x^4 + 2*x^2 +
 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2(2x^2+3)(2x^4+2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^4+2x^2+1)^{\frac{3}{2}}(2x^2+3)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)*x^2), x)
```

3.346 $\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=406

$$-\frac{\left(-\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^3(-e)+b^2cd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^3(-e)+b^2cd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{c^3}-\frac{\left(d+ex^2\right)^{3/2}(be+cd)}{3c^2e^2}+\frac{\left(d+ex^2\right)^{5/2}}{5ce^2}$$

```
[Out] ((b^2 - a*c)*Sqrt[d + e*x^2])/c^3 - ((c*d + b*e)*(d + e*x^2)^(3/2
))/(3*c^2*e^2) + (d + e*x^2)^(5/2)/(5*c*e^2) - ((b^2*c*d - a*c^2*
d - b^3*e + 2*a*b*c*e - (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*
c*e - 2*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sq
rt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]
*c^(7/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b^2*c*d - a
*c^2*d - b^3*e + 2*a*b*c*e + (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a
*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[
c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sq
rt[2]*c^(7/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 17.8635, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172

$$-\frac{\left(-\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^3(-e)+b^2cd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^3(-e)+b^2cd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{c^3}-\frac{\left(d+ex^2\right)^{3/2}(be+cd)}{3c^2e^2}+\frac{\left(d+ex^2\right)^{5/2}}{5ce^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^7*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b^2 - a*c)*Sqrt[d + e*x^2])/c^3 - ((c*d + b*e)*(d + e*x^2)^(3/2
))/(3*c^2*e^2) + (d + e*x^2)^(5/2)/(5*c*e^2) - ((b^2*c*d - a*c^2*
d - b^3*e + 2*a*b*c*e - (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*
c*e - 2*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sq
rt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]
*c^(7/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b^2*c*d - a
*c^2*d - b^3*e + 2*a*b*c*e + (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a
*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[
c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sq
rt[2]*c^(7/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.51953, size = 476, normalized size = 1.17

$$\frac{\sqrt{d+ex^2}\left(-5ce\left(3ae+b\left(d+ex^2\right)\right)+15b^2e^2+c^2\left(-2d^2+dex^2+3e^2x^4\right)\right)}{15c^3e^2}+\frac{\left(ac^2\left(d\sqrt{b^2-4ac}-2ae\right)+b^2c\left(4ae-d\sqrt{b^2-4ac}\right)-abc\left(2e\sqrt{b^2-4ac}+3cd\right)+b^3\left(e\sqrt{b^2-4ac}+cd\right)+b^4(-e)\right)\tanh^{-1}}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{\left(ac^2\left(d\sqrt{b^2-4ac}+2ae\right)-b^2c\left(d\sqrt{b^2-4ac}+4ae\right)+abc\left(3cd-2e\sqrt{b^2-4ac}\right)+b^3\left(e\sqrt{b^2-4ac}-cd\right)+b^4e\right)\tanh^{-1}\left(-\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^7*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[d + e*x^2]*(15*b^2*e^2 + c^2*(-2*d^2 + d*e*x^2 + 3*e^2*x^4)
 - 5*c*e*(3*a*e + b*(d + e*x^2))))/(15*c^3*e^2) + ((-(b^4*e) + a*
c^2*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*c*(-(Sqrt[b^2 - 4*a*c]*d)
 + 4*a*e) + b^3*(c*d + Sqrt[b^2 - 4*a*c]*e) - a*b*c*(3*c*d + 2*Sq
rt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqr
t[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2
- 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + ((b^4*e + a*
c^2*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - b^2*c*(Sqrt[b^2 - 4*a*c]*d +
4*a*e) + a*b*c*(3*c*d - 2*Sqrt[b^2 - 4*a*c]*e) + b^3*(-(c*d) + Sq
rt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqr
t[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2
- 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Maple [C] time = 0.084, size = 496, normalized size = 1.2

$$\frac{x^2}{5ce}\left(ex^2+d\right)^{\frac{3}{2}}-\frac{2d}{15e^2c}\left(ex^2+d\right)^{\frac{3}{2}}-\frac{b}{3c^2e}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{ax}{2c^2}\sqrt{e}-\frac{xb^2}{2c^3}\sqrt{e}-\frac{a}{2c^2}\sqrt{ex^2+d}+\frac{b^2}{2c^3}\sqrt{ex^2+d} -\frac{1}{4c^3}\sum_{\_R=RootOf\left(c\_Z^8+(4be-4cd)\_Z^6+(16ae^2-8bde+6cd^2)\_Z^4+(4bd^2e-4cd^3)\_Z^2+cd^4\right)}\frac{\left(-2abce+ac^2d+b^3e-b^2cd\right)\_R^6+\left(-4a^2ce^2+\right.}{} -\frac{ad}{2c^2}\left(\sqrt{ex^2+d}-x\sqrt{e}\right)^{-1}+\frac{b^2d}{2c^3}\left(\sqrt{ex^2+d}-x\sqrt{e}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/5/c*x^2*(e*x^2+d)^(3/2)/e-2/15/c*d/e^2*(e*x^2+d)^(3/2)-1/3/c^2*
b*(e*x^2+d)^(3/2)/e+1/2/c^2*e^(1/2)*x*a-1/2/c^3*e^(1/2)*x*b^2-1/2
/c^2*(e*x^2+d)^(1/2)*a+1/2/c^3*(e*x^2+d)^(1/2)*b^2-1/4/c^3*sum(((
-2*a*b*c*e+a*c^2*d+b^3*e-b^2*c*d)*_R^6+(-4*a^2*c*e^2+4*a*b^2*e^2+
2*a*b*c*d*e-3*a*c^2*d^2-3*b^3*d*e+3*b^2*c*d^2)*_R^4+d*(4*a^2*c*e^
2-4*a*b^2*e^2-2*a*b*c*d*e+3*a*c^2*d^2+3*b^3*d*e-3*b^2*c*d^2)*_R^2
+2*a*b*c*d^3*e-a*c^2*d^4-b^3*d^3*e+b^2*c*d^4)/(_R^7*c+3*_R^5*b*e-
3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c
*d^3)*ln((e*x^2+d)^(1/2)-x*e^(1/2)-_R),_R=RootOf(c*_Z^8+(4*b*e-4*
c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^
2+c*d^4))-1/2/c^2*d/((e*x^2+d)^(1/2)-x*e^(1/2))*a+1/2/c^3*d/((e*x
^2+d)^(1/2)-x*e^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ex^2+d}\,x^7}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)*x^7/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x^7/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)*x^7/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7 \sqrt{d + ex^2}}{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**7*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)*x^7/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.347 $\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=324

$$\frac{\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{b\sqrt{d+ex^2}}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3ce}$$

```
[Out] -((b*Sqrt[d + e*x^2])/c^2) + (d + e*x^2)^(3/2)/(3*c*e) + ((b*c*d
- b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[
b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b
 - Sqrt[b^2 - 4*a*c])*e]) + ((b*c*d - b^2*e + a*c*e + (b^2*c*d -
2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
]])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 6.78648, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{b\sqrt{d+ex^2}}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3ce}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*Sqrt[d + e*x^2])/c^2) + (d + e*x^2)^(3/2)/(3*c*e) + ((b*c*d
- b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[
b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b
 - Sqrt[b^2 - 4*a*c])*e]) + ((b*c*d - b^2*e + a*c*e + (b^2*c*d -
2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
]])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.51065, size = 383, normalized size = 1.18

$$\frac{\frac{3\sqrt{2}\left(-b^2\left(e\sqrt{b^2-4ac}+cd\right)+bc\left(d\sqrt{b^2-4ac}-3ae\right)+ac\left(e\sqrt{b^2-4ac}+2cd\right)+b^3e\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{3\sqrt{2}\left(b^2\left(e\sqrt{b^2-4ac}-cd\right)-bc\left(d\sqrt{b^2-4ac}+3ae\right)\cdots}{\sqrt{b^2-4ac}\sqrt{\cdots}}}{6c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((2*Sqrt[c]*Sqrt[d + e*x^2]*(-3*b*e + c*(d + e*x^2)))/e + (3*Sqrt
[2]*(b^3*e + b*c*(Sqrt[b^2 - 4*a*c]*d - 3*a*e) - b^2*(c*d + Sqrt[
b^2 - 4*a*c]*e) + a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqr
t[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c
]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e
]) - (3*Sqrt[2]*(b^3*e - b*c*(Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a*c*
(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b^2*(-(c*d) + Sqrt[b^2 - 4*a*c]*e
))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e]))/(6*c^(5/2))
```

Maple [C] time = 0.038, size = 332, normalized size = 1.

$$\frac{1}{3ce}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{bx}{2c^2}\sqrt{e}-\frac{b}{2c^2}\sqrt{ex^2+d}$$
$$-\frac{1}{4c^2}\sum_{\_R=RootOf\left(c\_Z^8+(4be-4cd)\_Z^6+(16ae^2-8bde+6cd^2)\_Z^4+(4bd^2e-4cd^3)\_Z^2+cd^4\right)}\frac{\left(ace-b^2e+bcd\right)\_R^6+\left(-4abe^2+acde+3b^2d\right.}{\_R^7c+3\_R^5be-3\_R^5}$$
$$-\frac{bd}{2c^2}\left(\sqrt{ex^2+d}-x\sqrt{e}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*(e*x^2+d)^(3/2)/c/e+1/2/c^2*b*x*e^(1/2)-1/2*b*(e*x^2+d)^(1/2)
/c^2-1/4/c^2*sum(((a*c*e-b^2*e+b*c*d)*_R^6+(-4*a*b*e^2+a*c*d*e+3*
b^2*d*e-3*b*c*d^2)*_R^4+d*(4*a*b*e^2-a*c*d*e-3*b^2*d*e+3*b*c*d^2)
*_R^2-a*c*d^3*e+b^2*d^3*e-c*d^4*b)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+
8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln((e
*x^2+d)^(1/2)-x*e^(1/2)-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(
16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+c*d^4))-1
/2/c^2*b*d/((e*x^2+d)^(1/2)-x*e^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ex^2+d}x^5}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x^2 + d)*x^5/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x^5/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x^2 + d)*x^5/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**5*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x^2 + d)*x^5/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.348 $\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=292

$$\frac{\left(-\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{d+ex^2}}{c}$$

```
[Out] Sqrt[d + e*x^2]/c + ((b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*c]
*(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b*c*d - b^2*e + 2
*a*c*e + Sqrt[b^2 - 4*a*c]*(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*
Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[
2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e])
```

Rubi [A] time = 8.74498, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172

$$\frac{\left(-\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{d+ex^2}}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Sqrt[d + e*x^2]/c + ((b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*c]
*(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*
```

```
d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b*c*d - b^2*e + 2
*a*c*e + Sqrt[b^2 - 4*a*c]*(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*
Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[
2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.536858, size = 310, normalized size = 1.06

$$\frac{\frac{\sqrt{2}\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{\sqrt{2}\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}-2ace+b^2e-bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\cdots}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[d + e*x^2] + (Sqrt[2]*(b*c*d - c*Sqrt[b^2 - 4*a*c
]*d - b^2*e + 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*S
qrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])
/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (
Sqrt[2]*(-(b*c*d) - c*Sqrt[b^2 - 4*a*c]*d + b^2*e - 2*a*c*e + b*S
qrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqr
t[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(2*c^(3/2))
```

Maple [C] time = 0.031, size = 275, normalized size = 0.9

$$-\frac{x}{2c}\sqrt{e}+\frac{1}{2c}\sqrt{ex^2+d}+\frac{1}{4c}\sum_{\_R=RootOf\left(c\_Z^8+(4be-4cd)\_Z^6+(16ae^2-8bde+6cd^2)\_Z^4+(4bd^2e-4cd^3)\_Z^2+cd^4\right)}\frac{(-be+cd)\_R^6+\left(-4ae^2+3bde-3cd^2\right)\_R^4+d}{\_R^7c+3\_R^5be-3\_R^5cd+8\_R^3ae^2-4\_R^3}+\frac{d}{2c}\left(\sqrt{ex^2+d}-x\sqrt{e}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/c*x*e^(1/2)+1/2*(e*x^2+d)^(1/2)/c+1/4/c*sum(((-b*e+c*d)*_R^6
+(-4*a*e^2+3*b*d*e-3*c*d^2)*_R^4+d*(4*a*e^2-3*b*d*e+3*c*d^2)*_R^2
+b*d^3*e-c*d^4)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3
*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-x*e^(
1/2)-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*
c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+c*d^4))+1/2/c*d/((e*x^2+d)^(
1/2)-x*e^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ex^2+d}x^3}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x^2 + d)*x^3/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x^3/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 53.8713, size = 3287, normalized size = 11.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x^2 + d)*x^3/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e +
(b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e +
```

```
(b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3
- 4*a*c^4))*log((2*a*b^2*c*d^2 - 2*a*b^3*d*e + 2*(a^2*b^2 - a^3*c
)*e^2 + (a*b^2*c*d*e - (a*b^3 - a^2*b*c)*e^2)*x^2 + 2*sqrt(1/2)*s
qrt(e*x^2 + d)*((b^4*c - 4*a*b^2*c^2)*d - (b^5 - 5*a*b^3*c + 4*a^
2*b*c^2)*e - (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt((b^2*c^2*d^
2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b
^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e
 + (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*
e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c
^3 - 4*a*c^4)) - ((a*b^2*c^3 - 4*a^2*c^4)*e*x^2 + 2*(a*b^2*c^3 -
4*a^2*c^4)*d)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4
- 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/x^2) - sqrt(1/2
)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*
a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b
^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*l
og((2*a*b^2*c*d^2 - 2*a*b^3*d*e + 2*(a^2*b^2 - a^3*c)*e^2 + (a*b^
2*c*d*e - (a*b^3 - a^2*b*c)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d
)*((b^4*c - 4*a*b^2*c^2)*d - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e -
(b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt((b^2*c^2*d^2 - 2*(b^3*c
- a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*
c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 -
 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*
a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)
) - ((a*b^2*c^3 - 4*a^2*c^4)*e*x^2 + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)
*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c +
 a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/x^2) + sqrt(1/2)*c*sqrt(((b^
2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqrt((
b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^
2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a*b^2*c
*d^2 - 2*a*b^3*d*e + 2*(a^2*b^2 - a^3*c)*e^2 + (a*b^2*c*d*e - (a*
b^3 - a^2*b*c)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c - 4
*a*b^2*c^2)*d - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e + (b^4*c^3 - 6*
a*b^2*c^4 + 8*a^2*c^5)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*
e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt((
(b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqr
t((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2
*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + ((a*b^2*c
^3 - 4*a^2*c^4)*e*x^2 + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*sqrt((b^2*c^
2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2
)/(b^2*c^6 - 4*a*c^7)))/x^2) - sqrt(1/2)*c*sqrt(((b^2*c - 2*a*c^2
)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 -
 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*
c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a*b^2*c*d^2 - 2*a*b^
3*d*e + 2*(a^2*b^2 - a^3*c)*e^2 + (a*b^2*c*d*e - (a*b^3 - a^2*b*c
)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c - 4*a*b^2*c^2)*d
 - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e + (b^4*c^3 - 6*a*b^2*c^4 + 8
*a^2*c^5)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*
a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*
c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^
2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b
^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + ((a*b^2*c^3 - 4*a^2*c^
4)*e*x^2 + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*sqrt((b^2*c^2*d^2 - 2*(b^
3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 -
4*a*c^7)))/x^2) + 4*sqrt(e*x^2 + d))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**3*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)*x^3/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.349 $\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=202

$$\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -((Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[
c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sq
rt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sqrt[2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]
)
```

Rubi [A] time = 0.891763, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148

$$\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```


```
[Out] -((Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[
c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sq
rt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sqrt[2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]
)
```

Rubi in Sympy [A] time = 75.7342, size = 192, normalized size = 0.95

$$-\frac{\sqrt{2}\sqrt{be-2cd-e\sqrt{-4ac+b^2}}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{be-2cd-e\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\sqrt{be-2cd+e\sqrt{-4ac+b^2}}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{be-2cd+e\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] -sqrt(2)*sqrt(b*e - 2*c*d - e*sqrt(-4*a*c + b**2))*atan(sqrt(2)*s
qrt(c)*sqrt(d + e*x**2)/sqrt(b*e - 2*c*d - e*sqrt(-4*a*c + b**2))
)/(2*sqrt(c)*sqrt(-4*a*c + b**2)) + sqrt(2)*sqrt(b*e - 2*c*d + e*
sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*sqrt(d + e*x**2)/sqrt(b
*e - 2*c*d + e*sqrt(-4*a*c + b**2)))/(2*sqrt(c)*sqrt(-4*a*c + b**
2))
```

Mathematica [A] time = 0.407126, size = 179, normalized size = 0.89

$$\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)-\sqrt{e\sqrt{b^2-4ac}-be+2cd}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*ArcTanh[(Sqrt[2]*Sqrt[
c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]]) + S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*S
qrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2
]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [C] time = 0.019, size = 177, normalized size = 0.9

$$\frac{e}{4}\sum_{\_R=RootOf\left(c\_Z^8+(4be-4cd)\_Z^6+(16ae^2-8bde+6cd^2)\_Z^4+(4bd^2e-4cd^3)\_Z^2+cd^4\right)}\frac{\_R^6+\_R^4d-\_R^2d^2-d^3}{\_R^7c+3\_R^5be-3\_R^5cd+8\_R^3ae^2-4\_R^3bde+3\_}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*e*sum((_R^6+_R^4*d-_R^2*d^2-d^3)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*
d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln(
(e*x^2+d)^(1/2)-x*e^(1/2)-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6
+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}x}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x^2 + d)*x/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 11.6173, size = 1465, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x^2 + d)*x/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(1/2)*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^
2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log((b*e^2*x^2 + 2*b*d*e -
2*a*e^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e + (b^3*c -
 4*a*b*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))*sqrt((2*c*d - b*e + (b
^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))
 + ((b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*c^2)*d)*sqrt(e^2/(b^
2*c^2 - 4*a*c^3)))/x^2) + 1/4*sqrt(1/2)*sqrt((2*c*d - b*e + (b^2*
c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*lo
```

```
g((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((
b^2 - 4*a*c)*e + (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)
))*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*
c^3)))/(b^2*c - 4*a*c^2)) + ((b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c -
 4*a*c^2)*d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/x^2) - 1/4*sqrt(1/2)*
sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3
)))/(b^2*c - 4*a*c^2))*log((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 + 2*sqr
t(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e - (b^3*c - 4*a*b*c^2)*sqr
t(e^2/(b^2*c^2 - 4*a*c^3)))*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)
*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) - ((b^2*c - 4*
a*c^2)*e*x^2 + 2*(b^2*c - 4*a*c^2)*d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3
)))/x^2) + 1/4*sqrt(1/2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sq
rt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log((b*e^2*x^2 +
2*b*d*e - 2*a*e^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e
- (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))*sqrt((2*c*d
- b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c -
 4*a*c^2)) - ((b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*c^2)*d)*sq
rt(e^2/(b^2*c^2 - 4*a*c^3)))/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x^2 + d)*x/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.350 $\int \frac{\sqrt{d+ex^2}}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=281

$$\frac{\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{c}\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a}$$

```
[Out] -((Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) + (Sqrt[c]*(b*d +
 Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e
*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[
b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c]*
(b*d - Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt
[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a
*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 2.87735, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$\frac{\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{c}\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d + e*x^2]/(x*(a + b*x^2 + c*x^4)),x]
```



```
[Out] -((Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) + (Sqrt[c]*(b*d +
 Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e
```

```
*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[
b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c]*
(b*d - Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt
[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a
*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**(1/2)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.19482, size = 269, normalized size = 0.96

$$\frac{\frac{\sqrt{2}\left(\frac{c\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{c\left(d\sqrt{b^2-4ac}+2ae-bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}-2\sqrt{d}\log\left(\sqrt{d}\sqrt{d+ex^2}+d\right)+2\sqrt{d}\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d + e*x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((Sqrt[2]*((c*(b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e
]])/Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e] + (c*(-(b*d) + Sqrt[
b^2 - 4*a*c]*d + 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])
/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/Sqrt[2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e]))/(Sqrt[c]*Sqrt[b^2 - 4*a*c]) + 2*Sqrt[d]*Log[x
] - 2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/(2*a)
```

Maple [C] time = 0.034, size = 294, normalized size = 1.1

$$-\frac{1}{a}\sqrt{d}\ln\left(\frac{1}{x}\left(2d+2\sqrt{d}\sqrt{ex^2+d}\right)\right)+\frac{1}{2a}\sqrt{ex^2+d}+\frac{x}{2a}\sqrt{e}$$
$$-\frac{1}{4a}\sum_{\_R=RootOf\left(c\_Z^8+(4be-4cd)\_Z^6+(16ae^2-8bde+6cd^2)\_Z^4+(4bd^2e-4cd^3)\_Z^2+cd^4\right)}\frac{\_R^6cd+\left(-4ae^2+4bde-3cd^2\right)\_R^4+d\left(4ae^2\right.}{\_R^7c+3\_R^5be-3\_R^5cd+8\_R^3ae^2-4\_R^3bde}$$
$$-\frac{d}{2a}\left(\sqrt{ex^2+d}-x\sqrt{e}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a*d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)+1/2/a*(e*x^2+d
)^(1/2)+1/2/a*x*e^(1/2)-1/4/a*sum((_R^6*c*d+(-4*a*e^2+4*b*d*e-3*c
*d^2)*_R^4+d*(4*a*e^2-4*b*d*e+3*c*d^2)*_R^2-c*d^4)/(_R^7*c+3*_R^5
*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e
-_R*c*d^3)*ln((e*x^2+d)^(1/2)-x*e^(1/2)-_R),_R=RootOf(c*_Z^8+(4*b
*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3
)*_Z^2+c*d^4))-1/2/a*d/((e*x^2+d)^(1/2)-x*e^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x), x)
```

Fricas [A] time = 92.386, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*
a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))
```

```
/(a^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 - 4*a*b*d*e + 2*a^2*e^2 + (
b^2*d*e - a*b*e^2)*x^2 + 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(e*x
^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))
*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d
^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*
c)) - ((a^2*b^2 - 4*a^3*c)*e*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt(
(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2) - sqrt
(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt
((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 -
 4*a^3*c))*log(-(2*b^2*d^2 - 4*a*b*d*e + 2*a^2*e^2 + (b^2*d*e - a
*b*e^2)*x^2 - 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(e*x^2 + d)*sqr
t((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-(a*b
*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*
d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - ((a^2
*b^2 - 4*a^3*c)*e*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2 -
2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2) - sqrt(1/2)*a*sqr
t(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 -
 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*
log(-(2*b^2*d^2 - 4*a*b*d*e + 2*a^2*e^2 + (b^2*d*e - a*b*e^2)*x^2
 + 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(e*x^2 + d)*sqrt((b^2*d^2
- 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 -
 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e
^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) + ((a^2*b^2 - 4*a^
3*c)*e*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a*b*d*e +
 a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2) + sqrt(1/2)*a*sqrt(-(a*b*e -
 (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e
+ a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*b^2
*d^2 - 4*a*b*d*e + 2*a^2*e^2 + (b^2*d*e - a*b*e^2)*x^2 - 4*sqrt(1
/2)*(a^3*b^2 - 4*a^4*c)*sqrt(e*x^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e
 + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d -
 (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^
2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) + ((a^2*b^2 - 4*a^3*c)*e*x^2
+ 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(
a^4*b^2 - 4*a^5*c)))/x^2) - 2*sqrt(d)*log(-(e*x^2 - 2*sqrt(e*x^2
+ d)*sqrt(d) + 2*d)/x^2))/a, -1/4*(sqrt(1/2)*a*sqrt(-(a*b*e - (b^
2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^
2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2
 - 4*a*b*d*e + 2*a^2*e^2 + (b^2*d*e - a*b*e^2)*x^2 + 4*sqrt(1/2)*
(a^3*b^2 - 4*a^4*c)*sqrt(e*x^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a
^2*e^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^
2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 -
4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - ((a^2*b^2 - 4*a^3*c)*e*x^2 + 2*
(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*
b^2 - 4*a^5*c)))/x^2) - sqrt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*
d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4
*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 - 4*a*b*d*
e + 2*a^2*e^2 + (b^2*d*e - a*b*e^2)*x^2 - 4*sqrt(1/2)*(a^3*b^2 -
4*a^4*c)*sqrt(e*x^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^
4*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a
^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/
(a^2*b^2 - 4*a^3*c)) - ((a^2*b^2 - 4*a^3*c)*e*x^2 + 2*(a^2*b^2 -
4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5
*c)))/x^2) - sqrt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^
2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^
```

```
5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 - 4*a*b*d*e + 2*a^2*e
^2 + (b^2*d*e - a*b*e^2)*x^2 + 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*sq
rt(e*x^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a
^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt(
(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 -
4*a^3*c)) + ((a^2*b^2 - 4*a^3*c)*e*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d)
*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2)
+ sqrt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c
)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2
*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 - 4*a*b*d*e + 2*a^2*e^2 + (b^2*d
*e - a*b*e^2)*x^2 - 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(e*x^2 +
d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*sqrt
(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 -
2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) +
 ((a^2*b^2 - 4*a^3*c)*e*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*
d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2) + 4*sqrt(-d
)*arctan(d/(sqrt(e*x^2 + d)*sqrt(-d))))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d + ex^2}}{x\,(a + bx^2 + cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**(1/2)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(x*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] Timed out
```

3.351 $\int \frac{\sqrt{d+ex^2}}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=382

$$-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}\left(-b\left(d\sqrt{b^2-4ac}+ae\right)-a\left(2cd-e\sqrt{b^2-4ac}\right)+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2\sqrt{d}}-\frac{\sqrt{d+ex^2}}{2ax^2}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a\sqrt{d}}$$

```
[Out] -Sqrt[d + e*x^2]/(2*a*x^2) + (e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])
/(2*a*Sqrt[d]) + ((b*d - a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(
a^2*Sqrt[d]) - (Sqrt[c]*(b^2*d - 2*a*c*d - a*b*e + Sqrt[b^2 - 4*a
*c]*(b*d - a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c
]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[c]*(b^2*d - b*
(Sqrt[b^2 - 4*a*c]*d + a*e) - a*(2*c*d - Sqrt[b^2 - 4*a*c]*e))*Ar
cTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 8.18623, antiderivative size = 370, normalized size of antiderivative = 0.97, number

of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$
$$+\frac{\sqrt{c}\left(-\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$
$$+\frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2\sqrt{d}}-\frac{\sqrt{d+ex^2}}{2ax^2}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d + e*x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(2*a*x^2) + (e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])
/(2*a*Sqrt[d]) + ((b*d - a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(
a^2*Sqrt[d]) - (Sqrt[c]*(b^2*d - 2*a*c*d - a*b*e + Sqrt[b^2 - 4*a
*c]*(b*d - a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c
]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[c]*(b^2*d - 2*
a*c*d - a*b*e - Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sqrt[2]*S
qrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])
/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**(1/2)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.70371, size = 351, normalized size = 0.92

$$\frac{\frac{\sqrt{2}\left(\frac{c\left(-bd\sqrt{b^2-4ac}+ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{c\left(bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}+\frac{(2bd-\cdots}{\cdots}}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d + e*x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((a*Sqrt[d + e*x^2])/x^2) + (Sqrt[2]*((c*(-(b^2*d) + 2*a*c*d -
b*Sqrt[b^2 - 4*a*c]*d + a*b*e + a*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(S
qrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a
*c]*e]])/Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e] - (c*(-(b^2*d)
+ 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2 - 4*a*c]*e
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(S
qrt[c]*Sqrt[b^2 - 4*a*c]) + ((-2*b*d + a*e)*Log[x])/Sqrt[d] + ((2
*b*d - a*e)*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/Sqrt[d])/(2*a^2)
```

Maple [C] time = 0.038, size = 401, normalized size = 1.1

$$-\frac{1}{2\,adx^2}\left(ex^2+d\right)^{\frac{3}{2}}-\frac{e}{2\,a}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{ex^2+d}\right)\right)\frac{1}{\sqrt{d}}+\frac{e}{2\,ad}\sqrt{ex^2+d}-\frac{bx}{2\,a^2}\sqrt{e}-\frac{b}{2\,a^2}\sqrt{ex^2+d}$$
$$+\frac{1}{4\,a^2}\sum_{\_R=RootOf\left(c\_Z^8+(4\,be-4\,cd)\_Z^6+(16\,ae^2-8\,bde+6\,cd^2)\_Z^4+(4\,bd^2e-4\,cd^3)\_Z^2+cd^4\right)}\frac{c\,(-ae+bd)\,\_R^6+\left(-4\,abe^2-acde+4\,b^2de-3\,b\cdots\right.}{\_R^7c+3\,\_R^5be-3\,\_R^5cd+8\,\_\cdots}$$
$$+\frac{bd}{2\,a^2}\left(\sqrt{ex^2+d}-x\sqrt{e}\right)^{-1}+\frac{b}{a^2}\sqrt{d}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{ex^2+d}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/a/d/x^2*(e*x^2+d)^(3/2)-1/2/a/d^(1/2)*e*ln((2*d+2*d^(1/2)*(e
*x^2+d)^(1/2))/x)+1/2/a/d*e*(e*x^2+d)^(1/2)-1/2/a^2*e^(1/2)*x*b-1
/2/a^2*(e*x^2+d)^(1/2)*b+1/4/a^2*sum((c*(-a*e+b*d)*_R^6+(-4*a*b*e
^2-a*c*d*e+4*b^2*d*e-3*b*c*d^2)*_R^4+d*(4*a*b*e^2+a*c*d*e-4*b^2*d
*e+3*b*c*d^2)*_R^2+a*c*d^3*e-c*d^4*b)/(_R^7*c+3*_R^5*b*e-3*_R^5*c
*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln
((e*x^2+d)^(1/2)-x*e^(1/2)-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^
6+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+c*d^4)
)+1/2/a^2*b*d/((e*x^2+d)^(1/2)-x*e^(1/2))+b/a^2*d^(1/2)*ln((2*d+2
```

```
*d^(1/2)*(e*x^2+d)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(x**3*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] Timed out
```

3.352 $\int \frac{\sqrt{d+ex^2}}{x^5(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=552

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{a^3\sqrt{d}}+\frac{\sqrt{c}\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-4ac}-2ae\right)+b^3d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{c}\left(-b^2\left(d\sqrt{b^2-4ac}+ae\right)-ab\left(3cd-e\sqrt{b^2-4ac}\right)+ac\left(d\sqrt{b^2-4ac}+2ae\right)+b^3d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a^2d^{3/2}}+\frac{\sqrt{d+ex^2}(bd-ae)}{2a^2dx^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8ad^{3/2}}+\frac{3e\sqrt{d+ex^2}}{8adx^2}-\frac{\sqrt{d+ex^2}}{4ax^4}$$

```
[Out] -Sqrt[d + e*x^2]/(4*a*x^4) + (3*e*Sqrt[d + e*x^2])/(8*a*d*x^2) +
((b*d - a*e)*Sqrt[d + e*x^2])/(2*a^2*d*x^2) - (3*e^2*ArcTanh[Sqrt
[d + e*x^2]/Sqrt[d]])/(8*a*d^(3/2)) - (e*(b*d - a*e)*ArcTanh[Sqrt
[d + e*x^2]/Sqrt[d]])/(2*a^2*d^(3/2)) - ((b^2*d - a*c*d - a*b*e)*
ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(a^3*Sqrt[d]) + (Sqrt[c]*(b^3*d
 - a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(Sqrt[b^2 - 4*a*c]*d -
 a*e) - a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[
c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sq
rt[2]*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e]) - (Sqrt[c]*(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sq
rt[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3*c*d - Sqrt[b^2 - 4*a*c]*e))*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e]])/(Sqrt[2]*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 9.29025, antiderivative size = 552, normalized size of antiderivative = 1., number of

steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{a^3\sqrt{d}}+\frac{\sqrt{c}\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-4ac}-2ae\right)+b^3d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{c}\left(-b^2\left(d\sqrt{b^2-4ac}+ae\right)-ab\left(3cd-e\sqrt{b^2-4ac}\right)+ac\left(d\sqrt{b^2-4ac}+2ae\right)+b^3d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a^2d^{3/2}}+\frac{\sqrt{d+ex^2}(bd-ae)}{2a^2dx^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8ad^{3/2}}+\frac{3e\sqrt{d+ex^2}}{8adx^2}-\frac{\sqrt{d+ex^2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d + e*x^2]/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(4*a*x^4) + (3*e*Sqrt[d + e*x^2])/(8*a*d*x^2) +
((b*d - a*e)*Sqrt[d + e*x^2])/(2*a^2*d*x^2) - (3*e^2*ArcTanh[Sqrt
[d + e*x^2]/Sqrt[d]])/(8*a*d^(3/2)) - (e*(b*d - a*e)*ArcTanh[Sqrt
[d + e*x^2]/Sqrt[d]])/(2*a^2*d^(3/2)) - ((b^2*d - a*c*d - a*b*e)*
ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(a^3*Sqrt[d]) + (Sqrt[c]*(b^3*d
 - a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(Sqrt[b^2 - 4*a*c]*d -
 a*e) - a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[
c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sq
rt[2]*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e]) - (Sqrt[c]*(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sq
rt[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3*c*d - Sqrt[b^2 - 4*a*c]*e))*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e]])/(Sqrt[2]*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**(1/2)/x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.95803, size = 468, normalized size = 0.85

$$\frac{\frac{\log\left(\sqrt{d}\sqrt{d+ex^2}+d\right)\left(4abde+a\left(ae^2+8cd^2\right)-8b^2d^2\right)}{d^{3/2}}-\frac{\log(x)\left(4abde+a\left(ae^2+8cd^2\right)-8b^2d^2\right)}{d^{3/2}}-\frac{4\sqrt{2}\left(\frac{c\left(b^2\left(ae-d\sqrt{b^2-4ac}\right)+ab\left(e\sqrt{b^2-4ac}+3cd\right)+ac\left(d\sqrt{b^2-4ac}-2ae\right)\right.}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d + e*x^2]/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((a*Sqrt[d + e*x^2]*(4*b*d*x^2 - a*(2*d + e*x^2)))/(d*x^4) - (4*S
qrt[2]*((c*(-(b^3*d) + a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(-
(Sqrt[b^2 - 4*a*c]*d) + a*e) + a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))
*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqr
t[b^2 - 4*a*c]*e]])/Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e] + (c
*(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c
]*d + 2*a*e) + a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[
2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(Sqrt[c]*Sqrt[b^2
- 4*a*c]) - ((-8*b^2*d^2 + 4*a*b*d*e + a*(8*c*d^2 + a*e^2))*Log[x
])/d^(3/2) + ((-8*b^2*d^2 + 4*a*b*d*e + a*(8*c*d^2 + a*e^2))*Log[
d + Sqrt[d]*Sqrt[d + e*x^2]])/d^(3/2))/(8*a^3)
```

Maple [C] time = 0.047, size = 655, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4/a/d/x^4*(e*x^2+d)^(3/2)+1/8/a/d^2*e/x^2*(e*x^2+d)^(3/2)+1/8/
a/d^(3/2)*e^2*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)-1/8/a/d^2*e^2
*(e*x^2+d)^(1/2)+1/a^2*d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))
/x)*c-1/a^3*d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)*b^2-1/2
/a^2*(e*x^2+d)^(1/2)*c+1/2/a^3*(e*x^2+d)^(1/2)*b^2-1/2/a^2*e^(1/2
)*x*c+1/2/a^3*e^(1/2)*x*b^2+1/4/a^3*sum((c*(a*b*e+a*c*d-b^2*d)*_R
^6+(-4*a^2*c*e^2+4*a*b^2*e^2+5*a*b*c*d*e-3*a*c^2*d^2-4*b^3*d*e+3*
b^2*c*d^2)*_R^4+d*(4*a^2*c*e^2-4*a*b^2*e^2-5*a*b*c*d*e+3*a*c^2*d^
```

```
2+4*b^3*d*e-3*b^2*c*d^2)*_R^2-a*b*c*d^3*e-a*c^2*d^4+b^2*c*d^4)/(_
R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^
2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-x*e^(1/2)-_R),_R=RootOf
(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d
^2*e-4*c*d^3)*_Z^2+c*d^4))+1/2/a^2*d/((e*x^2+d)^(1/2)-x*e^(1/2))*
c-1/2/a^3*d/((e*x^2+d)^(1/2)-x*e^(1/2))*b^2+1/2*b/a^2/d/x^2*(e*x^
2+d)^(3/2)+1/2*b/a^2/d^(1/2)*e*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))
/x)-1/2*b/a^2/d*e*(e*x^2+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^5),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^5), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^5),x, algorithm="giac")
```

```
[Out] Timed out
```

3.353 $\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=390

$$-\frac{\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{(cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}}+\frac{x\sqrt{d+ex^2}}{2c}$$

```
[Out] (x*Sqrt[d + e*x^2])/(2*c) - ((b*c*d - b^2*e + a*c*e - (b^2*c*d -
2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*
c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*
Sqrt[d + e*x^2])])/(c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e]) - ((b*c*d - b^2*e + a*c*e + (b^2*c*d
- 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]
]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e]) + ((c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x
)/Sqrt[d + e*x^2]])/(2*c^2*Sqrt[e])
```

Rubi [A] time = 6.52865, antiderivative size = 390, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$-\frac{\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{(cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}}+\frac{x\sqrt{d+ex^2}}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c) - ((b*c*d - b^2*e + a*c*e - (b^2*c*d -
2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*
c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*
Sqrt[d + e*x^2])])/(c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e]) - ((b*c*d - b^2*e + a*c*e + (b^2*c*d
- 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]
]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e]) + ((c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x
)/Sqrt[d + e*x^2]])/(2*c^2*Sqrt[e])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.808581, size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x^4*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[(x^4*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4), x]
```

Maple [C] time = 0.048, size = 290, normalized size = 0.7

$$\frac{x}{2c}\sqrt{ex^2+d}+\frac{d}{2c}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)\frac{1}{\sqrt{e}} +\frac{1}{2c^2}\sqrt{e}\sum_{\_R=RootOf\left(c\_Z^4+(4be-4cd)\_Z^3+(16ae^2-8bde+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z+cd^4\right)}\frac{\left(ace-b^2e+bcd\right)\_R^2+2\left(-2abe^2+acde+b^2\right.}{\_R^3c+3\_R^2be-3\_R^2cd+8\_Rae^2-4} +\frac{b}{c^2}\sqrt{e}\ln\left(\sqrt{ex^2+d}-x\sqrt{e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*x*(e*x^2+d)^(1/2)/c+1/2/c*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1
/2))+1/2/c^2*e^(1/2)*sum(((a*c*e-b^2*e+b*c*d)*_R^2+2*(-2*a*b*e^2+
a*c*d*e+b^2*d*e-b*c*d^2)*_R+a*c*d^2*e-b^2*d^2*e+b*c*d^3)/(_R^3*c+
3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*
d^3)*ln(((e*x^2+d)^(1/2)-x*e^(1/2))^2-_R),_R=RootOf(c*_Z^4+(4*b*e
-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*
_Z+c*d^4))+1/c^2*e^(1/2)*b*ln((e*x^2+d)^(1/2)-x*e^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}x^4}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)*x^4/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x^4/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 25.8277, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)*x^4/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(1/2)*c^2*sqrt(e)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 -
 4*a*b^2*c + 2*a^2*c^2)*e + (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 -
2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3
)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9
)))/(b^2*c^4 - 4*a*c^5))*log(((a*b^2*c^4 - 4*a^2*c^5)*d*x^2*sqrt(
((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 +
 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c
^8 - 4*a*c^9)) + 2*(a^2*b^2*c - a^3*c^2)*d^2 - 2*(a^2*b^3 - 2*a^3
*b*c)*d*e - ((a*b^3*c - a^2*b*c^2)*d^2 - (a*b^4 + 2*a^2*b^2*c - 4
*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c)*e^2)*x^2 + 2*sqrt(1/2)*sq
rt(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x*sqrt(((b^4*c
^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*
b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*
a*c^9)) - ((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4
*c + 8*a^2*b^2*c^2)*e)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4
*a*b^2*c + 2*a^2*c^2)*e + (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*
a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*
d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9))
)/(b^2*c^4 - 4*a*c^5)))/x^2) - sqrt(1/2)*c^2*sqrt(e)*sqrt(-((b^3*
c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e + (b^2*c^4 - 4
*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c -
3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2
)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(((a*b^2*c^4
 - 4*a^2*c^5)*d*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 -
 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4
*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) + 2*(a^2*b^2*c - a^3*c^2)
*d^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d*e - ((a*b^3*c - a^2*b*c^2)*d^2 -
 (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c)*
e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5 +
8*a^2*c^6)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5
*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^
2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) - ((b^5*c - 5*a*b^3*c^2 + 4*a^2*
b*c^3)*d - (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*e)*x)*sqrt(-((b^3*c
- 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e + (b^2*c^4 - 4*a
*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*
a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*
e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) + sqrt(1/2)
*c^2*sqrt(e)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*
a^2*c^2)*e - (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a
^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 -
```

```
4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 -
4*a*c^5))*log(-((a*b^2*c^4 - 4*a^2*c^5)*d*x^2*sqrt(((b^4*c^2 - 2*
a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*
d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9))
 - 2*(a^2*b^2*c - a^3*c^2)*d^2 + 2*(a^2*b^3 - 2*a^3*b*c)*d*e + ((
a*b^3*c - a^2*b*c^2)*d^2 - (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e
+ 4*(a^2*b^3 - 2*a^3*b*c)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*
((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x*sqrt(((b^4*c^2 - 2*a*b^2*c
^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (
b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) + ((b^
5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*a^2*b^2
*c^2)*e)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a
^2*c^2)*e - (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^
2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4
*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4
*a*c^5)))/x^2) - sqrt(1/2)*c^2*sqrt(e)*sqrt(-((b^3*c - 3*a*b*c^2)
*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 - 4*a*c^5)*sqrt((
(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 +
2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^
8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-((a*b^2*c^4 - 4*a^2*c^5)
*d*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3
*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)
*e^2)/(b^2*c^8 - 4*a*c^9)) - 2*(a^2*b^2*c - a^3*c^2)*d^2 + 2*(a^2
*b^3 - 2*a^3*b*c)*d*e + ((a*b^3*c - a^2*b*c^2)*d^2 - (a*b^4 + 2*a
^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c)*e^2)*x^2 - 2*
sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x*
sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*
c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(
b^2*c^8 - 4*a*c^9)) + ((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b
^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*e)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*
d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 - 4*a*c^5)*sqrt(((
b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2
*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8
 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) + 2*sqrt(e*x^2 + d)*c*sq
rt(e)*x - (c*d - 2*b*e)*log(2*sqrt(e*x^2 + d)*e*x - (2*e*x^2 + d)
*sqrt(e)))/(c^2*sqrt(e)), 1/4*(sqrt(1/2)*c^2*sqrt(-e)*sqrt(-((b^3
*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e + (b^2*c^4 -
4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c -
 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^
2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(((a*b^2*c^
4 - 4*a^2*c^5)*d*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2
- 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c +
4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) + 2*(a^2*b^2*c - a^3*c^2
)*d^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d*e - ((a*b^3*c - a^2*b*c^2)*d^2
- (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c)
*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5 +
 8*a^2*c^6)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^
5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b
^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) - ((b^5*c - 5*a*b^3*c^2 + 4*a^2
*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*e)*x)*sqrt(-((b^3*c
 - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e + (b^2*c^4 - 4*
a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3
*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)
*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) - sqrt(1/2
```

```
)*c^2*sqrt(-e)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c +
2*a^2*c^2)*e + (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 +
 a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6
- 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4
- 4*a*c^5))*log(((a*b^2*c^4 - 4*a^2*c^5)*d*x^2*sqrt(((b^4*c^2 - 2
*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)
*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)
) + 2*(a^2*b^2*c - a^3*c^2)*d^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d*e - (
(a*b^3*c - a^2*b*c^2)*d^2 - (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e
 + 4*(a^2*b^3 - 2*a^3*b*c)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)
*((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x*sqrt(((b^4*c^2 - 2*a*b^2*
c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e +
(b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) - ((b
^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*a^2*b^
2*c^2)*e)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*
a^2*c^2)*e + (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a
^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 -
4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 -
4*a*c^5)))/x^2) + sqrt(1/2)*c^2*sqrt(-e)*sqrt(-((b^3*c - 3*a*b*c^
2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 - 4*a*c^5)*sqrt
(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2
+ 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*
c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-((a*b^2*c^4 - 4*a^2*c^
5)*d*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c -
 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^
2)*e^2)/(b^2*c^8 - 4*a*c^9)) - 2*(a^2*b^2*c - a^3*c^2)*d^2 + 2*(a
^2*b^3 - 2*a^3*b*c)*d*e + ((a*b^3*c - a^2*b*c^2)*d^2 - (a*b^4 + 2
*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b^3 - 2*a^3*b*c)*e^2)*x^2 +
2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*
x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^
3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)
/(b^2*c^8 - 4*a*c^9)) + ((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d -
(b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*e)*x)*sqrt(-((b^3*c - 3*a*b*c^2
)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 - 4*a*c^5)*sqrt(
((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 +
 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c
^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) - sqrt(1/2)*c^2*sqrt(-
e)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e
 - (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^
2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c
+ 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*
log(-((a*b^2*c^4 - 4*a^2*c^5)*d*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3
+ a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6
 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) - 2*(a^2*
b^2*c - a^3*c^2)*d^2 + 2*(a^2*b^3 - 2*a^3*b*c)*d*e + ((a*b^3*c -
a^2*b*c^2)*d^2 - (a*b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*e + 4*(a^2*b
^3 - 2*a^3*b*c)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c^4
- 6*a*b^2*c^5 + 8*a^2*c^6)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c
^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*
b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) + ((b^5*c - 5*a*
b^3*c^2 + 4*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*e)*x
)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e
- (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2
 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c +
```

```
 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/
x^2) + 2*sqrt(e*x^2 + d)*c*sqrt(-e)*x + 2*(c*d - 2*b*e)*arctan(sq
rt(-e)*x/sqrt(e*x^2 + d)))/(c^2*sqrt(-e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**4*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x^2 + d)*x^4/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.354 $\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=324

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c}$$

```
[Out] ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 -
 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((c*d - b*e + (b*c*d - b^2*e
 + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/
(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]) + (Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/c
```

Rubi [A] time = 3.46182, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 -
```

```
 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((c*d - b*e + (b*c*d - b^2*e
 + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/
(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]) + (Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/c
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.587051, size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x^2*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[(x^2*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4), x]
```

Maple [C] time = 0.033, size = 224, normalized size = 0.7

$$\frac{1}{2c}\sqrt{e}\sum_{\_R=RootOf\left(c\_Z^4+(4be-4cd)\_Z^3+(16ae^2-8bde+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z+cd^4\right)}\frac{(be-cd)\_R^2+2\left(2ae^2-bde+cd^2\right)\_R}{\_R^3c+3\_R^2be-3\_R^2cd+8\_Rae^2-4\_Rbde+}$$
$$-\frac{1}{c}\sqrt{e}\ln\left(\sqrt{ex^2+d}-x\sqrt{e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*e^(1/2)/c*sum(((b*e-c*d)*_R^2+2*(2*a*e^2-b*d*e+c*d^2)*_R+b*d^
2*e-c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*
_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-x*e^(1/2))^2-_R),_R=R
ootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(
4*b*d^2*e-4*c*d^3)*_Z+c*d^4))-e^(1/2)/c*ln((e*x^2+d)^(1/2)-x*e^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}x^2}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)*x^2/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x^2/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 3.86322, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)*x^2/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*
a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))
/(b^2*c^2 - 4*a*c^3))*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d
^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*
b*d*e - (b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 + 2*sqrt(1/
2)*sqrt(e*x^2 + d)*((b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c
*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - ((b^2*c - 4*a*c^2)*d - (b^
3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*
a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))
/(b^2*c^2 - 4*a*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2
*a*c)*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2
)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(-((b^2*c^2 - 4*a
*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c
^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*a*
c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2 - 4*a*b*c^3)*
x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - ((b
^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2
*a*c)*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2
)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) + sqrt(1/2)*c*
```

```
sqrt(-(b*c*d - (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^
2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3
))*log(((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2
*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 + 2*a*b*d*e + (b*c*d^2 + 4
*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(
(b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2
*c^4 - 4*a*c^5)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*s
qrt(-(b*c*d - (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2
 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)
))/x^2) - sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - (b^2*c^2 -
 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5
)))/(b^2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2
*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 + 2*
a*b*d*e + (b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(
1/2)*sqrt(e*x^2 + d)*((b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b
*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + ((b^2*c - 4*a*c^2)*d - (
b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - (b^2*c^2 -
4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)
))/(b^2*c^2 - 4*a*c^3)))/x^2) - 2*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e
*x^2 + d)*sqrt(e)*x - d))/c, -1/4*(sqrt(1/2)*c*sqrt(-(b*c*d - (b^
2 - 2*a*c)*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^
2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(-((b^2*c^2
- 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 -
4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d^2 + 4*a*b*e^2 - (b^2 +
 4*a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2 - 4*a*b*
c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5))
- ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^
2 - 2*a*c)*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^
2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) - sqrt(1/
2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*a*c^3)*sqrt((c
^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*
a*c^3))*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e
 + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d
^2 + 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2
+ d)*((b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2
)/(b^2*c^4 - 4*a*c^5)) - ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e
)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*a*c^3)*sqrt((c
^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*
a*c^3)))/x^2) + sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - (b^2
*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4
*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*x^2*sqr
t((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^
2 + 2*a*b*d*e + (b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 + 2
*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2
 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + ((b^2*c - 4*a*c^2)
*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - (b^2*
c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*
a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c*d -
(b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e +
 b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(((b^2*c^
2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4
- 4*a*c^5)) - 2*a*c*d^2 + 2*a*b*d*e + (b*c*d^2 + 4*a*b*e^2 - (b^2
 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2 - 4*a*
b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)
```

```
) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (
b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e +
b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) - 4*sqr
t(-e)*arctan(e*x/(sqrt(e*x^2 + d)*sqrt(-e))))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**2*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x^2 + d)*x^2/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.355 $\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=240

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTan[(Sqrt[2*c*d - (b
- Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d +
e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTan[(Sqrt[2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2
])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.925522, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d + e*x^2]/(a + b*x^2 + c*x^4),x]
```


```
[Out] (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTan[(Sqrt[2*c*d - (b
- Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d +
e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTan[(Sqrt[2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2
])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 103.247, size = 219, normalized size = 0.91

$$\frac{\sqrt{be-2cd+e\sqrt{-4ac+b^2}}\operatorname{atanh}\left(\frac{x\sqrt{be-2cd+e\sqrt{-4ac+b^2}}}{\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}} - \frac{\sqrt{be-2cd-e\sqrt{-4ac+b^2}}\operatorname{atanh}\left(\frac{x\sqrt{be-2cd-e\sqrt{-4ac+b^2}}}{\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] sqrt(b*e - 2*c*d + e*sqrt(-4*a*c + b**2))*atanh(x*sqrt(b*e - 2*c*
d + e*sqrt(-4*a*c + b**2))/(sqrt(b + sqrt(-4*a*c + b**2))*sqrt(d
+ e*x**2)))/(sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) -
 sqrt(b*e - 2*c*d - e*sqrt(-4*a*c + b**2))*atanh(x*sqrt(b*e - 2*c
*d - e*sqrt(-4*a*c + b**2))/(sqrt(b - sqrt(-4*a*c + b**2))*sqrt(d
 + e*x**2)))/(sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.125985, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[d + e*x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[Sqrt[d + e*x^2]/(a + b*x^2 + c*x^4), x]
```

Maple [C] time = 0.02, size = 161, normalized size = 0.7

$$-\frac{1}{2}e^{\frac{3}{2}}\sum_{\_R=RootOf\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+(16\,ae^2-8\,bde+6\,cd^2)\_Z^2+(4\,bd^2e-4\,cd^3)\_Z+cd^4\right)}\frac{\_R^2+2\_Rd+d^2}{\_R^3c+3\_R^2be-3\_R^2cd+8\_Rae^2-4\_Rbde+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*e^(3/2)*sum((_R^2+2*_R*d+d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+
8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/
2)-x*e^(1/2))^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2
-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x^2 + d)/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 1.04514, size = 1330, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x^2 + d)/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^
2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c))*log(-((a*b^2 - 4*a^2*c)*d*s
qrt(d^2/(a^2*b^2 - 4*a^3*c))*x^2 + 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c
)*sqrt(e*x^2 + d)*sqrt(d^2/(a^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*
a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4
*a^2*c)) - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2)/x^2) - 1/4*sqrt(1/2)*
sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4*a^3*
c)))/(a*b^2 - 4*a^2*c))*log(-((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b
^2 - 4*a^3*c))*x^2 - 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 +
 d)*sqrt(d^2/(a^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e + (a*b^2 -
 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) - 2*a
*d^2 + (b*d^2 - 4*a*d*e)*x^2)/x^2) + 1/4*sqrt(1/2)*sqrt(-(b*d - 2
*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4*a^3*c)))/(a*b^2 -
4*a^2*c))*log(((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 - 4*a^3*c))*
x^2 + 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/(a
^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(
d^2/(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) + 2*a*d^2 - (b*d^2 -
 4*a*d*e)*x^2)/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - (a*b^2 -
```

```
 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c))*log((
(a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 - 4*a^3*c))*x^2 - 4*sqrt(1/
2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/(a^2*b^2 - 4*a^3*
c))*x*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 -
4*a^3*c)))/(a*b^2 - 4*a^2*c)) + 2*a*d^2 - (b*d^2 - 4*a*d*e)*x^2)/
x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d + ex^2}}{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x^2 + d)/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.356 $\int \frac{\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=291

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{ax}$$

```
[Out] -(Sqrt[d + e*x^2]/(a*x)) - (c*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - S
qrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c
]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (c*(d - (b*d - 2*a*
e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 1.63786, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d + e*x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[d + e*x^2]/(a*x)) - (c*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - S
qrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c
]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (c*(d - (b*d - 2*a*
e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**(1/2)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.595705, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[d + e*x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[Sqrt[d + e*x^2]/(x^2*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.038, size = 272, normalized size = 0.9

$$-\frac{1}{adx}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{ex}{ad}\sqrt{ex^2+d}+\frac{1}{a}\sqrt{e}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)$$
$$+\frac{1}{2a}\sqrt{e}\sum_{\_R=RootOf\left(c\_Z^4+(4be-4cd)\_Z^3+(16ae^2-8bde+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z+cd^4\right)}\frac{\_R^2cd+2\left(-2ae^2+2bde-cd^2\right)}{\_R^3c+3\_R^2be-3\_R^2cd+8\_Rae^2-4\_Rbde}$$
$$+\frac{1}{a}\sqrt{e}\ln\left(\sqrt{ex^2+d}-x\sqrt{e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a/d/x*(e*x^2+d)^(3/2)+1/a/d*e*x*(e*x^2+d)^(1/2)+1/a*e^(1/2)*ln
(x*e^(1/2)+(e*x^2+d)^(1/2))+1/2/a*e^(1/2)*sum((_R^2*c*d+2*(-2*a*e
^2+2*b*d*e-c*d^2)*_R+c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*
e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-x*e^
(1/2))^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*
e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))+1/a*e^(1/2)*ln((e*
x^2+d)^(1/2)-x*e^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}}{(cx^4 + bx^2 + a)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [A] time = 2.27264, size = 3243, normalized size = 11.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*a*x*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*
e + (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^
2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*
b^2 - 4*a^4*c))*log((2*a^2*b*c*d*e + (a^3*b^2*c - 4*a^4*c^2)*d*x^
2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3
- a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) - 2*(a*b^2*c - a^2*c^2)*d^2
+ (4*a^2*b*c*e^2 + (b^3*c - a*b*c^2)*d^2 - (5*a*b^2*c - 4*a^2*c^2
)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^4*b^3 - 4*a^5*b*c)*x
*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 -
 a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) - ((a*b^4 - 5*a^2*b^2*c + 4*a
^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e)*x)*sqrt(-((b^3 - 3*a*b*c)*d
- (a*b^2 - 2*a^2*c)*e + (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (
b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) - sqrt(1/2)*a*x*sqrt(-(
(b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b^2 - 4*a^4*c)*sqr
t((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2
*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a^2*
b*c*d*e + (a^3*b^2*c - 4*a^4*c^2)*d*x^2*sqrt((a^2*b^2*e^2 + (b^4
- 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 -
4*a^7*c)) - 2*(a*b^2*c - a^2*c^2)*d^2 + (4*a^2*b*c*e^2 + (b^3*c -
 a*b*c^2)*d^2 - (5*a*b^2*c - 4*a^2*c^2)*d*e)*x^2 - 2*sqrt(1/2)*sq
rt(e*x^2 + d)*((a^4*b^3 - 4*a^5*b*c)*x*sqrt((a^2*b^2*e^2 + (b^4 -
 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4
*a^7*c)) - ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^
3*b*c)*e)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^
3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*
d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4
*a^4*c)))/x^2) - sqrt(1/2)*a*x*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2
```

```
- 2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a
*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7
*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a^2*b*c*d*e - (a^3*b^2*c - 4*a^
4*c^2)*d*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2
- 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) - 2*(a*b^2*c - a^
2*c^2)*d^2 + (4*a^2*b*c*e^2 + (b^3*c - a*b*c^2)*d^2 - (5*a*b^2*c
- 4*a^2*c^2)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^4*b^3 - 4
*a^5*b*c)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 -
 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) + ((a*b^4 - 5*a^2*
b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e)*x)*sqrt(-((b^3 -
3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*
b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d
*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) + sqrt(1/2)*
a*x*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - (a^3*b^2 - 4
*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(
a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*
log((2*a^2*b*c*d*e - (a^3*b^2*c - 4*a^4*c^2)*d*x^2*sqrt((a^2*b^2*
e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/
(a^6*b^2 - 4*a^7*c)) - 2*(a*b^2*c - a^2*c^2)*d^2 + (4*a^2*b*c*e^2
 + (b^3*c - a*b*c^2)*d^2 - (5*a*b^2*c - 4*a^2*c^2)*d*e)*x^2 - 2*s
qrt(1/2)*sqrt(e*x^2 + d)*((a^4*b^3 - 4*a^5*b*c)*x*sqrt((a^2*b^2*e
^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(
a^6*b^2 - 4*a^7*c)) + ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2
*b^3 - 4*a^3*b*c)*e)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2
*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c
+ a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(
a^3*b^2 - 4*a^4*c)))/x^2) + 4*sqrt(e*x^2 + d))/(a*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**(1/2)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(x**2*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.357 $\int \frac{\sqrt{d+ex^2}}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=373

$$\frac{c\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{d+ex^2}(bd-ae)}{a^2dx}+\frac{2e\sqrt{d+ex^2}}{3adx}-\frac{\sqrt{d+ex^2}}{3ax^3}$$

```
[Out] -Sqrt[d + e*x^2]/(3*a*x^3) + (2*e*Sqrt[d + e*x^2])/(3*a*d*x) + ((
b*d - a*e)*Sqrt[d + e*x^2])/(a^2*d*x) + (c*(b*d - a*e + (b^2*d -
2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2
])])/(a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) + (c*(b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(S
qrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b + Sqrt[
b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 5.63495, antiderivative size = 373, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$\frac{c\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{d+ex^2}(bd-ae)}{a^2dx}+\frac{2e\sqrt{d+ex^2}}{3adx}-\frac{\sqrt{d+ex^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d + e*x^2]/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(3*a*x^3) + (2*e*Sqrt[d + e*x^2])/(3*a*d*x) + ((
b*d - a*e)*Sqrt[d + e*x^2])/(a^2*d*x) + (c*(b*d - a*e + (b^2*d -
2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2
])])/(a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) + (c*(b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(S
qrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b + Sqrt[
b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**(1/2)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.781711, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}}{x^4\left(a+bx^2+cx^4\right)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[d + e*x^2]/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[Sqrt[d + e*x^2]/(x^4*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.042, size = 322, normalized size = 0.9

$$-\frac{1}{3\,adx^{3}}\left(ex^{2}+d\right)^{\frac{3}{2}}+\frac{1}{2\,a^{2}}\sqrt{e}\sum_{\_R=RootOf\left(c\_Z^{4}+(4\,be-4\,cd)\_Z^{3}+\left(16\,ae^{2}-8\,bde+6\,cd^{2}\right)\_Z^{2}+\left(4\,bd^{2}e-4\,cd^{3}\right)\_Z+cd^{4}\right)}\frac{c\left(ae-bd\right)\_R^{2}+2\left(2\,abe^{2}+acde-2\,b^{2}de+b\right.}{\_R^{3}c+3\,\_R^{2}be-3\,\_R^{2}cd+8\,\_R\,ae^{2}-4\,\_R\,bde}$$

$$-\frac{b}{a^{2}}\sqrt{e}\ln\left(\sqrt{ex^{2}+d}-x\sqrt{e}\right)+\frac{b}{a^{2}dx}\left(ex^{2}+d\right)^{\frac{3}{2}}-\frac{bex}{a^{2}d}\sqrt{ex^{2}+d}-\frac{b}{a^{2}}\sqrt{e}\ln\left(x\sqrt{e}+\sqrt{ex^{2}+d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/3/a/d/x^3*(e*x^2+d)^(3/2)+1/2/a^2*e^(1/2)*sum((c*(a*e-b*d)*_R^
2+2*(2*a*b*e^2+a*c*d*e-2*b^2*d*e+b*c*d^2)*_R+a*c*d^2*e-b*c*d^3)/(
_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d
^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-x*e^(1/2))^2-_R),_R=RootOf(c*_Z^4
+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*
c*d^3)*_Z+c*d^4))-1/a^2*e^(1/2)*b*ln((e*x^2+d)^(1/2)-x*e^(1/2))+b
/a^2/d/x*(e*x^2+d)^(3/2)-b/a^2/d*e*x*(e*x^2+d)^(1/2)-b/a^2*e^(1/2
)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ex^{2}+d}}{(cx^{4}+bx^{2}+a)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^4), x)
```

Fricas [A] time = 9.84009, size = 5528, normalized size = 14.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^4),x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2
)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*a^6*c)*s
qrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e
+ (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11
*c)))/(a^5*b^2 - 4*a^6*c))*log(((a^5*b^2*c^2 - 4*a^6*c^3)*d*x^2*s
qrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e
+ (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11
*c)) + 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 - 2*(a^2*b^3*c
^2 - 2*a^3*b*c^3)*d*e - ((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2
- (5*a*b^4*c^2 - 14*a^2*b^2*c^3 + 4*a^3*c^4)*d*e + 4*(a^2*b^3*c^2
 - 2*a^3*b*c^3)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^6*b^4
- 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a
^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^
2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + ((a*b^7 - 7*a^2*b^5*c + 13*a
^3*b^3*c^2 - 4*a^4*b*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*
c^2)*e)*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*
a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 -
 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^
3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 -
4*a^6*c)))/x^2) - 3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c +
 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2
- 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^
3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4
*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*
b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(((a^5*b^2*c^2 - 4*a^6*
c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^
3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4
*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*
b^2 - 4*a^11*c)) + 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 -
2*(a^2*b^3*c^2 - 2*a^3*b*c^3)*d*e - ((b^5*c^2 - 3*a*b^3*c^3 + a^2
*b*c^4)*d^2 - (5*a*b^4*c^2 - 14*a^2*b^2*c^3 + 4*a^3*c^4)*d*e + 4*
(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d
)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt(((b^8 - 6*a*b^6*c +
 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2
*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*
c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + ((a*b^7 - 7*a^2*
b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c
+ 8*a^4*b^2*c^2)*e)*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d -
 (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*a^6*c)*sqrt((
(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2
- 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^
2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))
/(a^5*b^2 - 4*a^6*c)))/x^2) + 3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 -
 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e
 + (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3
*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)
*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(-((a^5*b^2
*c^2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^
```

```
3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2
)*e^2)/(a^10*b^2 - 4*a^11*c)) - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^
3*c^4)*d^2 + 2*(a^2*b^3*c^2 - 2*a^3*b*c^3)*d*e + ((b^5*c^2 - 3*a*
b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c^2 - 14*a^2*b^2*c^3 + 4*a^3*
c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x^2 + 2*sqrt(1/2)*s
qrt(e*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt(((b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(
a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6
 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) - ((a
*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b*c^3)*d - (a^2*b^6 -
 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a
^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e + (a^5*b^2 - 4*
a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c
^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2
- 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)))/x^2) - 3*sqrt(1/2)*a^2*d*x^3*
sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c +
 2*a^3*c^2)*e + (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*
c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4
*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*l
og(-((a^5*b^2*c^2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*
a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5
*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c +
4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) - 2*(a*b^4*c^2 - 3*a^2
*b^2*c^3 + a^3*c^4)*d^2 + 2*(a^2*b^3*c^2 - 2*a^3*b*c^3)*d*e + ((b
^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c^2 - 14*a^2*b^2
*c^3 + 4*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x^2 -
2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*
x*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c
^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d
*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a
^11*c)) - ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b*c^3)*d
 - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)*x)*sqrt(-((b^5 - 5*
a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e +
(a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^
2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^
2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)))/x^2) + 4*((3*b*d
 - a*e)*x^2 - a*d)*sqrt(e*x^2 + d))/(a^2*d*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d + ex^2}}{x^4 (a + bx^2 + cx^4)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**(1/2)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(x**4*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.358 $\int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=512

$$-\frac{\sqrt{d+ex^2}\left(-abe-acd+b^2d\right)}{a^3dx}-\frac{2e\sqrt{d+ex^2}(bd-ae)}{3a^2d^2x}+\frac{\sqrt{d+ex^2}(bd-ae)}{3a^2dx^3}-\frac{c\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{8e^2\sqrt{d+ex^2}}{15ad^2x}-\frac{\sqrt{d+ex^2}}{5ax^5}+\frac{4e\sqrt{d+ex^2}}{15adx^3}$$

```
[Out] -Sqrt[d + e*x^2]/(5*a*x^5) + (4*e*Sqrt[d + e*x^2])/(15*a*d*x^3) +
 ((b*d - a*e)*Sqrt[d + e*x^2])/(3*a^2*d*x^3) - (8*e^2*Sqrt[d + e*
x^2])/(15*a*d^2*x) - (2*e*(b*d - a*e)*Sqrt[d + e*x^2])/(3*a^2*d^2
*x) - ((b^2*d - a*c*d - a*b*e)*Sqrt[d + e*x^2])/(a^3*d*x) - (c*(b
^2*d - a*c*d - a*b*e + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^3*Sqrt[b
- Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (
c*(b^2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c
*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^3*Sqr
t[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 12.8262, antiderivative size = 512, normalized size of antiderivative = 1., number of

steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$-\frac{\sqrt{d+ex^2}\left(-abe-acd+b^2d\right)}{a^3dx}-\frac{2e\sqrt{d+ex^2}(bd-ae)}{3a^2d^2x}+\frac{\sqrt{d+ex^2}(bd-ae)}{3a^2dx^3}-\frac{c\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{8e^2\sqrt{d+ex^2}}{15ad^2x}-\frac{\sqrt{d+ex^2}}{5ax^5}+\frac{4e\sqrt{d+ex^2}}{15adx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d + e*x^2]/(x^6*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(5*a*x^5) + (4*e*Sqrt[d + e*x^2])/(15*a*d*x^3) +
 ((b*d - a*e)*Sqrt[d + e*x^2])/(3*a^2*d*x^3) - (8*e^2*Sqrt[d + e*
x^2])/(15*a*d^2*x) - (2*e*(b*d - a*e)*Sqrt[d + e*x^2])/(3*a^2*d^2
*x) - ((b^2*d - a*c*d - a*b*e)*Sqrt[d + e*x^2])/(a^3*d*x) - (c*(b
^2*d - a*c*d - a*b*e + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^3*Sqrt[b
- Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (
c*(b^2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c
*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^3*Sqr
t[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**(1/2)/x**6/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.0065, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}}{x^6\left(a+bx^2+cx^4\right)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[d + e*x^2]/(x^6*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[Sqrt[d + e*x^2]/(x^6*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.051, size = 503, normalized size = 1.

$$-\frac{1}{5\,adx^5}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{2\,e}{15\,ad^2x^3}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{c}{a^2dx}\left(ex^2+d\right)^{\frac{3}{2}}-\frac{b^2}{a^3dx}\left(ex^2+d\right)^{\frac{3}{2}}$$
$$-\frac{cex}{a^2d}\sqrt{ex^2+d}+\frac{b^2ex}{a^3d}\sqrt{ex^2+d}-\frac{c}{a^2}\sqrt{e}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)+\frac{b^2}{a^3}\sqrt{e}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)$$
$$-\frac{1}{2\,a^3}\sqrt{e}\sum_{\_R=RootOf\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+(16\,ae^2-8\,bde+6\,cd^2)\_Z^2+(4\,bd^2e-4\,cd^3)\_Z+cd^4\right)}\frac{c\left(abe+acd-b^2d\right)\_R^2+2\left(-2\,a^2ce^2+2\,ab^2e\right.}{\_R^3c+3\,\_R^2be-3\,\_R^2ca}$$
$$-\frac{c}{a^2}\sqrt{e}\ln\left(\sqrt{ex^2+d}-x\sqrt{e}\right)+\frac{b^2}{a^3}\sqrt{e}\ln\left(\sqrt{ex^2+d}-x\sqrt{e}\right)+\frac{b}{3\,a^2dx^3}\left(ex^2+d\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/5/a/d/x^5*(e*x^2+d)^(3/2)+2/15/a*e/d^2/x^3*(e*x^2+d)^(3/2)+1/a
^2/d/x*(e*x^2+d)^(3/2)*c-1/a^3/d/x*(e*x^2+d)^(3/2)*b^2-1/a^2/d*e*
x*(e*x^2+d)^(1/2)*c+1/a^3/d*e*x*(e*x^2+d)^(1/2)*b^2-1/a^2*e^(1/2)
*ln(x*e^(1/2)+(e*x^2+d)^(1/2))*c+1/a^3*e^(1/2)*ln(x*e^(1/2)+(e*x^
2+d)^(1/2))*b^2-1/2/a^3*e^(1/2)*sum((c*(a*b*e+a*c*d-b^2*d)*_R^2+2
*(-2*a^2*c*e^2+2*a*b^2*e^2+3*a*b*c*d*e-a*c^2*d^2-2*b^3*d*e+b^2*c*
d^2)*_R+a*b*c*d^2*e+a*c^2*d^3-b^2*c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^
2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+
d)^(1/2)-x*e^(1/2))^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16
*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))-1/a^2
*e^(1/2)*ln((e*x^2+d)^(1/2)-x*e^(1/2))*c+1/a^3*e^(1/2)*ln((e*x^2+
d)^(1/2)-x*e^(1/2))*b^2+1/3*b/a^2/d/x^3*(e*x^2+d)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}}{(cx^4 + bx^2 + a)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^6),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^6), x)
```

Fricas [A] time = 12.1491, size = 7794, normalized size = 15.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^6),x, algorithm="fricas")
```

```
[Out] -1/60*(15*sqrt(1/2)*a^3*d^2*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*
b^3*c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 -
 2*a^4*c^3)*e - (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 3
7*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
+ a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^
4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3
*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a
^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*log(-((a^7*b^2*c^3 - 4
*a^8*c^4)*d*x^2*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a
^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(
a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b
^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*
c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c))
 + 2*(a*b^6*c^3 - 5*a^2*b^4*c^4 + 6*a^3*b^2*c^5 - a^4*c^6)*d^2 -
2*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e - ((b^7*c^3 - 5
*a*b^5*c^4 + 6*a^2*b^3*c^5 - a^3*b*c^6)*d^2 - (5*a*b^6*c^3 - 24*a
^2*b^4*c^4 + 27*a^3*b^2*c^5 - 4*a^4*c^6)*d*e + 4*(a^2*b^5*c^3 - 4
*a^3*b^3*c^4 + 3*a^4*b*c^5)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d
)*((a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*x*sqrt(((b^12 - 10*a*b
^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5
*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^
2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^1
0 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4
)*e^2)/(a^14*b^2 - 4*a^15*c)) + ((a*b^10 - 10*a^2*b^8*c + 35*a^3*
b^6*c^2 - 51*a^4*b^4*c^3 + 29*a^5*b^2*c^4 - 4*a^6*c^5)*d - (a^2*b
^9 - 9*a^3*b^7*c + 27*a^4*b^5*c^2 - 31*a^5*b^3*c^3 + 12*a^6*b*c^4
)*e)*x)*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d
 - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e - (a^7*b^2
```

```
 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b
^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^
11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c
^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2
- 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)))/(a
^7*b^2 - 4*a^8*c)))/x^2) - 15*sqrt(1/2)*a^3*d^2*x^5*sqrt(-((b^7 -
 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4
*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e - (a^7*b^2 - 4*a^8*c)*sqrt(((b^
12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c
^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29
*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e
 + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*
a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*lo
g(-((a^7*b^2*c^3 - 4*a^8*c^4)*d*x^2*sqrt(((b^12 - 10*a*b^10*c + 3
7*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
+ a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^
4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3
*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a
^14*b^2 - 4*a^15*c)) + 2*(a*b^6*c^3 - 5*a^2*b^4*c^4 + 6*a^3*b^2*c
^5 - a^4*c^6)*d^2 - 2*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)
*d*e - ((b^7*c^3 - 5*a*b^5*c^4 + 6*a^2*b^3*c^5 - a^3*b*c^6)*d^2 -
 (5*a*b^6*c^3 - 24*a^2*b^4*c^4 + 27*a^3*b^2*c^5 - 4*a^4*c^6)*d*e
+ 4*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*e^2)*x^2 - 2*sqrt
(1/2)*sqrt(e*x^2 + d)*((a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*x*
sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*
a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b
^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b
*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4
*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)) + ((a*b^10 - 10
*a^2*b^8*c + 35*a^3*b^6*c^2 - 51*a^4*b^4*c^3 + 29*a^5*b^2*c^4 - 4
*a^6*c^5)*d - (a^2*b^9 - 9*a^3*b^7*c + 27*a^4*b^5*c^2 - 31*a^5*b^
3*c^3 + 12*a^6*b*c^4)*e)*x)*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*
c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a
^4*c^3)*e - (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^
2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^
6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^
5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8
*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*
b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c)))/x^2) + 15*sqrt(1/2)*a^3*d
^2*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d
- (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*b^2
- 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^
6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^1
1 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^
4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 -
 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)))/(a^
7*b^2 - 4*a^8*c))*log(((a^7*b^2*c^3 - 4*a^8*c^4)*d*x^2*sqrt(((b^1
2 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^
4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*
a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e
+ (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a
^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)) - 2*(a*b^6*c^3 - 5*a^2*b^
4*c^4 + 6*a^3*b^2*c^5 - a^4*c^6)*d^2 + 2*(a^2*b^5*c^3 - 4*a^3*b^3
*c^4 + 3*a^4*b*c^5)*d*e + ((b^7*c^3 - 5*a*b^5*c^4 + 6*a^2*b^3*c^5
```

```
 - a^3*b*c^6)*d^2 - (5*a*b^6*c^3 - 24*a^2*b^4*c^4 + 27*a^3*b^2*c^
5 - 4*a^4*c^6)*d*e + 4*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5
)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^8*b^5 - 7*a^9*b^3*c
+ 12*a^10*b*c^2)*x*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 6
2*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 -
2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^
5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b
^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*
c)) - ((a*b^10 - 10*a^2*b^8*c + 35*a^3*b^6*c^2 - 51*a^4*b^4*c^3 +
 29*a^5*b^2*c^4 - 4*a^6*c^5)*d - (a^2*b^9 - 9*a^3*b^7*c + 27*a^4*
b^5*c^2 - 31*a^5*b^3*c^3 + 12*a^6*b*c^4)*e)*x)*sqrt(-((b^7 - 7*a*
b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c +
9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 -
10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*
b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a
^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b
^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c)))/x^2) -
 15*sqrt(1/2)*a^3*d^2*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^
2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4
*c^3)*e + (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*
b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*
c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*
c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c
 + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^
2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*log(((a^7*b^2*c^3 - 4*a^8*c^
4)*d*x^2*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*
c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11
- 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4
- 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 2
4*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)) - 2*(a
*b^6*c^3 - 5*a^2*b^4*c^4 + 6*a^3*b^2*c^5 - a^4*c^6)*d^2 + 2*(a^2*
b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + ((b^7*c^3 - 5*a*b^5*
c^4 + 6*a^2*b^3*c^5 - a^3*b*c^6)*d^2 - (5*a*b^6*c^3 - 24*a^2*b^4*
c^4 + 27*a^3*b^2*c^5 - 4*a^4*c^6)*d*e + 4*(a^2*b^5*c^3 - 4*a^3*b^
3*c^4 + 3*a^4*b*c^5)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^8
*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*x*sqrt(((b^12 - 10*a*b^10*c +
 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^
5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*
a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a
^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/
(a^14*b^2 - 4*a^15*c)) - ((a*b^10 - 10*a^2*b^8*c + 35*a^3*b^6*c^2
 - 51*a^4*b^4*c^3 + 29*a^5*b^2*c^4 - 4*a^6*c^5)*d - (a^2*b^9 - 9*
a^3*b^7*c + 27*a^4*b^5*c^2 - 31*a^5*b^3*c^3 + 12*a^6*b*c^4)*e)*x)
*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a*b
^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*b^2 - 4*a^
8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
+ 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*
a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*
a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^
5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2
- 4*a^8*c)))/x^2) - 4*((5*a*b*d*e + 2*a^2*e^2 - 15*(b^2 - a*c)*d^
2)*x^4 - 3*a^2*d^2 + (5*a*b*d^2 - a^2*d*e)*x^2)*sqrt(e*x^2 + d))/
(a^3*d^2*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**(1/2)/x**6/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.359 $\int \frac{x^3\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=460

$$\frac{\left(bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)-b^2e\left(e\sqrt{b^2-4ac}+2cd\right)+b^3e^2\right)\tanh^{-}}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

$$-\frac{\left(bc\left(cd^2-e\left(2d\sqrt{b^2-4ac}+3ae\right)\right)-c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}+4ae\right)\right)-b^2e\left(2cd-e\sqrt{b^2-4ac}\right)+b^3e^2\right)\tanh}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

$$+\frac{\sqrt{d+ex^2}(cd-be)}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3c}$$

```
[Out] ((c*d - b*e)*Sqrt[d + e*x^2])/c^2 + (d + e*x^2)^(3/2)/(3*c) + ((b
^3*e^2 - b^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(a*Sqrt[b^2 - 4*
a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) + b*c*(c*d^2 + e*(2
*Sqrt[b^2 - 4*a*c]*d - 3*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(5/
2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (
(b^3*e^2 - b^2*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b*c*(c*d^2 - e*(
2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) - c*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*
d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
 + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(
5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 12.0647, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{\left(bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)-b^2e\left(e\sqrt{b^2-4ac}+2cd\right)+b^3e^2\right)\tanh^{-}}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

$$-\frac{\left(bc\left(cd^2-e\left(2d\sqrt{b^2-4ac}+3ae\right)\right)-c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}+4ae\right)\right)-b^2e\left(2cd-e\sqrt{b^2-4ac}\right)+b^3e^2\right)\tanh}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

$$+\frac{\sqrt{d+ex^2}(cd-be)}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d - b*e)*Sqrt[d + e*x^2])/c^2 + (d + e*x^2)^(3/2)/(3*c) + ((b
^3*e^2 - b^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(a*Sqrt[b^2 - 4*
a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) + b*c*(c*d^2 + e*(2
*Sqrt[b^2 - 4*a*c]*d - 3*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(5/
2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (
(b^3*e^2 - b^2*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b*c*(c*d^2 - e*(
2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) - c*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*
d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
 + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(
5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.70559, size = 448, normalized size = 0.97

$$-\frac{\frac{3\sqrt{2}\left(-bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(cd\left(d\sqrt{b^2-4ac}-4ae\right)-ae^2\sqrt{b^2-4ac}\right)+b^2e\left(e\sqrt{b^2-4ac}+2cd\right)+b^3\left(-e^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{3\sqrt{2}\left(bc\left(c\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[d + e*x^2]*(4*c*d - 3*b*e + c*e*x^2) - (3*Sqrt[2]
*(-(b^3*e^2) + b^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(-(a*Sqrt[
b^2 - 4*a*c]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) - b*c*(c*d
^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 3*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt
[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) - (3*Sqrt
```

```
[2]*(b^3*e^2 + b^2*e*(-2*c*d + Sqrt[b^2 - 4*a*c]*e) + b*c*(c*d^2
- e*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) + c*(-(a*Sqrt[b^2 - 4*a*c]*e
^2) + c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c
]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqr
t[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(6*c^(5/
2))
```

Maple [C] time = 0.041, size = 490, normalized size = 1.1

$$-\frac{x^3}{6c}e^{\frac{3}{2}}+\frac{ex^2}{8c}\sqrt{ex^2+d}-\frac{3\,dx}{4c}\sqrt{e}+\frac{1}{24\,c}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{bx}{2\,c^2}e^{\frac{3}{2}}-\frac{be}{2\,c^2}\sqrt{ex^2+d}+\frac{5\,d}{8\,c}\sqrt{ex^2+d}$$
$$+\frac{1}{4\,c^2}\sum_{\_R=RootOf\left(c\_Z^8+(4\,be-4\,cd)\_Z^6+(16\,ae^2-8\,bde+6\,cd^2)\_Z^4+(4\,bd^2e-4\,cd^3)\_Z^2+cd^4\right)}\frac{\left(-ace^2+b^2e^2-2\,bcde+c^2d^2\right)\_R^6+\left(4\,abe^3-5\right.}{}$$
$$-\frac{bde}{2\,c^2}\left(\sqrt{ex^2+d}-x\sqrt{e}\right)^{-1}+\frac{5\,d^2}{8\,c}\left(\sqrt{ex^2+d}-x\sqrt{e}\right)^{-1}+\frac{d^3}{24\,c}\left(\sqrt{ex^2+d}-x\sqrt{e}\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/6/c*e^(3/2)*x^3+1/8/c*e*(e*x^2+d)^(1/2)*x^2-3/4/c*e^(1/2)*x*d+
1/24*(e*x^2+d)^(3/2)/c+1/2/c^2*e^(3/2)*x*b-1/2/c^2*(e*x^2+d)^(1/2
)*b*e+5/8/c*(e*x^2+d)^(1/2)*d+1/4/c^2*sum(((-a*c*e^2+b^2*e^2-2*b*
c*d*e+c^2*d^2)*_R^6+(4*a*b*e^3-5*a*c*d*e^2-3*b^2*d*e^2+6*b*c*d^2*
e-3*c^2*d^3)*_R^4+d*(-4*a*b*e^3+5*a*c*d*e^2+3*b^2*d*e^2-6*b*c*d^2
*e+3*c^2*d^3)*_R^2+a*c*d^3*e^2-b^2*d^3*e^2+2*b*c*d^4*e-c^2*d^5)/(
_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d
^2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-x*e^(1/2)-_R),_R=RootO
f(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*
d^2*e-4*c*d^3)*_Z^2+c*d^4))-1/2/c^2*d/((e*x^2+d)^(1/2)-x*e^(1/2))
*b*e+5/8/c*d^2/((e*x^2+d)^(1/2)-x*e^(1/2))+1/24/c*d^3/((e*x^2+d)^
(1/2)-x*e^(1/2))^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^2+d\right)^{\frac{3}{2}}x^3}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)*x^3/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*x^3/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)*x^3/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(d + ex^2\right)^{\frac{3}{2}}}{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**3*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)*x^3/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.360 $\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=327

$$-\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{e\sqrt{d+ex^2}}{c}$$

```
[Out] (e*Sqrt[d + e*x^2])/c - ((2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e
^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sq
rt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/
(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4
*a*c])*e]) + ((2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*
(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[
d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^
(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 3.19187, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185

$$-\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{e\sqrt{d+ex^2}}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e*Sqrt[d + e*x^2])/c - ((2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e
^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sq
rt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/
(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4
*a*c])*e]) + ((2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*
(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[
d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^
(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.761663, size = 326, normalized size = 1.

$$\frac{\frac{\sqrt{2}\left(2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}-b\right)-2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{\sqrt{2}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tanh^{-1}\left(\right.}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*e*Sqrt[d + e*x^2] + (Sqrt[2]*(-2*c^2*d^2 + b*(-b + Sqr
t[b^2 - 4*a*c])*e^2 + 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*Ar
cTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b
^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 -
 4*a*c])*e]) + (Sqrt[2]*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^
2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqr
t[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(2*c^
(3/2))
```

Maple [C] time = 0.028, size = 279, normalized size = 0.9

$$-\frac{x}{2c}e^{\frac{3}{2}}+\frac{e}{2c}\sqrt{ex^2+d}+\frac{e}{4c}\sum_{\_R=RootOf\left(c\_Z^8+(4be-4cd)\_Z^6+(16ae^2-8bde+6cd^2)\_Z^4+(4bd^2e-4cd^3)\_Z^2+cd^4\right)}\frac{(-be+2cd)\_R^6+\left(-4ae^2+3bde-2cd^2\right)\_R^4+ c}{\_R^7c+3\_R^5be-3\_R^5cd+8\_R^3ae^2-4\_R}+\frac{de}{2c}\left(\sqrt{ex^2+d}-x\sqrt{e}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*e^(3/2)/c*x+1/2*e*(e*x^2+d)^(1/2)/c+1/4*e/c*sum(((-b*e+2*c*d
)*_R^6+(-4*a*e^2+3*b*d*e-2*c*d^2)*_R^4+d*(4*a*e^2-3*b*d*e+2*c*d^2
)*_R^2+b*d^3*e-2*c*d^4)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^
2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/
2)-x*e^(1/2)-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*
b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+c*d^4))+1/2*e/c*d/((
e*x^2+d)^(1/2)-x*e^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}x}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)*x/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*x/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 129.552, size = 5999, normalized size = 18.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)*x/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*
a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*
c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4
- 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b
^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(6*b*c^3*d^5*e - 6*
(2*b^2*c^2 + a*c^3)*d^4*e^2 + 8*(b^3*c + 2*a*b*c^2)*d^3*e^3 - 2*(
b^4 + 6*a*b^2*c + 2*a^2*c^2)*d^2*e^4 + 2*(2*a*b^3 + a^2*b*c)*d*e^
5 - 2*(a^2*b^2 - a^3*c)*e^6 + (3*b*c^3*d^4*e^2 - 6*b^2*c^2*d^3*e^
3 + 2*(2*b^3*c + a*b*c^2)*d^2*e^4 - (b^4 + 2*a*b^2*c)*d*e^5 + (a*
b^3 - a^2*b*c)*e^6)*x^2 + 2*sqrt(1/2)*(3*(b^2*c^3 - 4*a*c^4)*d^3*
e - 6*(b^3*c^2 - 4*a*b*c^3)*d^2*e^2 + (4*b^4*c - 17*a*b^2*c^2 + 4
*a^2*c^3)*d*e^3 - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e^4 + ((b^3*c^4
 - 4*a*b*c^5)*d - (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*e)*sqrt((9*
c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4
- 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b
^2*c^6 - 4*a*c^7)))*sqrt(e*x^2 + d)*sqrt((2*c^3*d^3 - 3*b*c^2*d^2
*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 -
 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 -
 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c
+ a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + (2*(
b^2*c^4 - 4*a*c^5)*d^3 - 2*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 2*(a*b^2
*c^3 - 4*a^2*c^4)*d*e^2 + ((b^2*c^4 - 4*a*c^5)*d^2*e - (b^3*c^3 -
 4*a*b*c^4)*d*e^2 + (a*b^2*c^3 - 4*a^2*c^4)*e^3)*x^2)*sqrt((9*c^4
*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6
*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*
c^6 - 4*a*c^7)))/x^2) - sqrt(1/2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2
*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 -
 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 -
 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c
+ a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(
6*b*c^3*d^5*e - 6*(2*b^2*c^2 + a*c^3)*d^4*e^2 + 8*(b^3*c + 2*a*b*
c^2)*d^3*e^3 - 2*(b^4 + 6*a*b^2*c + 2*a^2*c^2)*d^2*e^4 + 2*(2*a*b
^3 + a^2*b*c)*d*e^5 - 2*(a^2*b^2 - a^3*c)*e^6 + (3*b*c^3*d^4*e^2
- 6*b^2*c^2*d^3*e^3 + 2*(2*b^3*c + a*b*c^2)*d^2*e^4 - (b^4 + 2*a*
b^2*c)*d*e^5 + (a*b^3 - a^2*b*c)*e^6)*x^2 - 2*sqrt(1/2)*(3*(b^2*c
^3 - 4*a*c^4)*d^3*e - 6*(b^3*c^2 - 4*a*b*c^3)*d^2*e^2 + (4*b^4*c
- 17*a*b^2*c^2 + 4*a^2*c^3)*d*e^3 - (b^5 - 5*a*b^3*c + 4*a^2*b*c^
2)*e^4 + ((b^3*c^4 - 4*a*b*c^5)*d - (b^4*c^3 - 6*a*b^2*c^4 + 8*a^
2*c^5)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 -
 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c
+ a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*sqrt(e*x^2 + d)*sqrt((2*c^3
*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c
)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^
3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 +
 (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3
- 4*a*c^4)) + (2*(b^2*c^4 - 4*a*c^5)*d^3 - 2*(b^3*c^3 - 4*a*b*c^4
)*d^2*e + 2*(a*b^2*c^3 - 4*a^2*c^4)*d*e^2 + ((b^2*c^4 - 4*a*c^5)*
d^2*e - (b^3*c^3 - 4*a*b*c^4)*d*e^2 + (a*b^2*c^3 - 4*a^2*c^4)*e^3
)*x^2)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*
a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a
^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/x^2) + sqrt(1/2)*c*sqrt((2*c^3
*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c
)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^
3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 +
```

```
(b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3
- 4*a*c^4))*log(-(6*b*c^3*d^5*e - 6*(2*b^2*c^2 + a*c^3)*d^4*e^2 +
 8*(b^3*c + 2*a*b*c^2)*d^3*e^3 - 2*(b^4 + 6*a*b^2*c + 2*a^2*c^2)*
d^2*e^4 + 2*(2*a*b^3 + a^2*b*c)*d*e^5 - 2*(a^2*b^2 - a^3*c)*e^6 +
 (3*b*c^3*d^4*e^2 - 6*b^2*c^2*d^3*e^3 + 2*(2*b^3*c + a*b*c^2)*d^2
*e^4 - (b^4 + 2*a*b^2*c)*d*e^5 + (a*b^3 - a^2*b*c)*e^6)*x^2 + 2*s
qrt(1/2)*(3*(b^2*c^3 - 4*a*c^4)*d^3*e - 6*(b^3*c^2 - 4*a*b*c^3)*d
^2*e^2 + (4*b^4*c - 17*a*b^2*c^2 + 4*a^2*c^3)*d*e^3 - (b^5 - 5*a*
b^3*c + 4*a^2*b*c^2)*e^4 - ((b^3*c^4 - 4*a*b*c^5)*d - (b^4*c^3 -
6*a*b^2*c^4 + 8*a^2*c^5)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^
3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 +
 (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*sqrt(e*x^
2 + d)*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^
2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2
 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c
- a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*
a*c^7)))/(b^2*c^3 - 4*a*c^4)) - (2*(b^2*c^4 - 4*a*c^5)*d^3 - 2*(b
^3*c^3 - 4*a*b*c^4)*d^2*e + 2*(a*b^2*c^3 - 4*a^2*c^4)*d*e^2 + ((b
^2*c^4 - 4*a*c^5)*d^2*e - (b^3*c^3 - 4*a*b*c^4)*d*e^2 + (a*b^2*c^
3 - 4*a^2*c^4)*e^3)*x^2)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 +
 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b
^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/x^2) - sqrt(
1/2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^
2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2
 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c
- a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*
a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(6*b*c^3*d^5*e - 6*(2*b^2*c^2
+ a*c^3)*d^4*e^2 + 8*(b^3*c + 2*a*b*c^2)*d^3*e^3 - 2*(b^4 + 6*a*b
^2*c + 2*a^2*c^2)*d^2*e^4 + 2*(2*a*b^3 + a^2*b*c)*d*e^5 - 2*(a^2*
b^2 - a^3*c)*e^6 + (3*b*c^3*d^4*e^2 - 6*b^2*c^2*d^3*e^3 + 2*(2*b^
3*c + a*b*c^2)*d^2*e^4 - (b^4 + 2*a*b^2*c)*d*e^5 + (a*b^3 - a^2*b
*c)*e^6)*x^2 - 2*sqrt(1/2)*(3*(b^2*c^3 - 4*a*c^4)*d^3*e - 6*(b^3*
c^2 - 4*a*b*c^3)*d^2*e^2 + (4*b^4*c - 17*a*b^2*c^2 + 4*a^2*c^3)*d
*e^3 - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e^4 - ((b^3*c^4 - 4*a*b*c^
5)*d - (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*e)*sqrt((9*c^4*d^4*e^2
 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c
- a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*
a*c^7)))*sqrt(e*x^2 + d)*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2
*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*s
qrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d
^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*
e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - (2*(b^2*c^4 - 4
*a*c^5)*d^3 - 2*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 2*(a*b^2*c^3 - 4*a^
2*c^4)*d*e^2 + ((b^2*c^4 - 4*a*c^5)*d^2*e - (b^3*c^3 - 4*a*b*c^4)
*d*e^2 + (a*b^2*c^3 - 4*a^2*c^4)*e^3)*x^2)*sqrt((9*c^4*d^4*e^2 -
18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a
*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c
^7)))/x^2) - 4*sqrt(e*x^2 + d)*e)/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x(d+ex^2)^{\frac{3}{2}}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^(3/2)*x/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.361 $\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=346

$$-\frac{\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$
$$-\frac{\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$
$$-\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a}$$

```
[Out] -((d^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) - ((a*Sqrt[b^2 -
4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2
))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sq
rt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((a*Sqrt[b^2 - 4*a*c]*e^2 -
c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 + a*e^2))*ArcTanh[(S
qrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + S
qrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 3.90841, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$

$$-\frac{\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$
$$-\frac{\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$
$$-\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) - ((a*Sqrt[b^2 -
4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2
))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sq
rt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((a*Sqrt[b^2 - 4*a*c]*e^2 -
c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 + a*e^2))*ArcTanh[(S
qrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + S
qrt[b^2 - 4*a*c])*e])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**(3/2)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.84176, size = 333, normalized size = 0.96

$$-\frac{\frac{\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\left(cd\left(d\sqrt{b^2-4ac}-4ae\right)-ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{\cdots}}{\sqrt{e\sqrt{b^2-4ac}\cdots}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}}$$

$$-\frac{d^{3/2}\log\left(\sqrt{d}\sqrt{d+ex^2}+d\right)}{a}+\frac{d^{3/2}\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((-(((-(a*Sqrt[b^2 - 4*a*c]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d - 4*
a*e) + b*(c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2
])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/Sqrt[2*c*d + (-b + S
qrt[b^2 - 4*a*c])*e]) + ((a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2
```

```
 - 4*a*c]*d + 4*a*e) + b*(c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c
]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b^2
- 4*a*c])) + (d^(3/2)*Log[x])/a - (d^(3/2)*Log[d + Sqrt[d]*Sqrt[d
 + e*x^2]])/a
```

Maple [C] time = 0.039, size = 388, normalized size = 1.1

$$\frac{7}{24a}\left(ex^{2}+d\right)^{\frac{3}{2}}-\frac{1}{a}d^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2d+2\sqrt{d}\sqrt{ex^{2}+d}\right)\right)+\frac{3d}{8a}\sqrt{ex^{2}+d}+\frac{x^{3}}{6a}e^{\frac{3}{2}}-\frac{ex^{2}}{8a}\sqrt{ex^{2}+d}+\frac{3dx}{4a}\sqrt{e}-\frac{1}{4a}\sum_{\_R=RootOf\left(c\_Z^{8}+(4be-4cd)\_Z^{6}+(16ae^{2}-8bde+6cd^{2})\_Z^{4}+(4bd^{2}e-4cd^{3})\_Z^{2}+cd^{4}\right)}\frac{\left(-ae^{2}+cd^{2}\right)\_R^{6}+d\left(-5ae^{2}+4bde-3cd^{2}\right)\_R^{4}}{\_R^{7}c+3\_R^{5}be-3\_R^{5}cd+8\_R^{3}ae^{2}-4}-\frac{5d^{2}}{8a}\left(\sqrt{ex^{2}+d}-x\sqrt{e}\right)^{-1}-\frac{d^{3}}{24a}\left(\sqrt{ex^{2}+d}-x\sqrt{e}\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] 7/24/a*(e*x^2+d)^(3/2)-1/a*d^(3/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1
/2))/x)+3/8/a*(e*x^2+d)^(1/2)*d+1/6/a*e^(3/2)*x^3-1/8/a*e*(e*x^2+
d)^(1/2)*x^2+3/4/a*e^(1/2)*x*d-1/4/a*sum(((-a*e^2+c*d^2)*_R^6+d*(
-5*a*e^2+4*b*d*e-3*c*d^2)*_R^4+d^2*(5*a*e^2-4*b*d*e+3*c*d^2)*_R^2
+a*d^3*e^2-c*d^5)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R
^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-x*e
^(1/2)-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+
6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+c*d^4))-5/8/a*d^2/((e*x^2+
d)^(1/2)-x*e^(1/2))-1/24/a*d^3/((e*x^2+d)^(1/2)-x*e^(1/2))^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^{2}+d\right)^{\frac{3}{2}}}{\left(cx^{4}+bx^{2}+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex^{2}\right)^{\frac{3}{2}}}{x\left(a + bx^{2} + cx^{4}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((d + e*x**2)**(3/2)/(x*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] Timed out
```

3.362 $\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=417

$$-\frac{\sqrt{c}\left(-2a\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)+bd\left(d\sqrt{b^2-4ac}-2ae\right)+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}\left(-2a\left(cd^2-e\left(d\sqrt{b^2-4ac}+ae\right)\right)-bd\left(d\sqrt{b^2-4ac}+2ae\right)+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{d}(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2}-\frac{d\sqrt{d+ex^2}}{2ax^2}+\frac{\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a}$$

```
[Out] -(d*Sqrt[d + e*x^2])/(2*a*x^2) + (Sqrt[d]*e*ArcTanh[Sqrt[d + e*x^
2]/Sqrt[d]])/(2*a) + (Sqrt[d]*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x^
2]/Sqrt[d]])/a^2 - (Sqrt[c]*(b^2*d^2 + b*d*(Sqrt[b^2 - 4*a*c]*d -
 2*a*e) - 2*a*(c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTanh[(S
qrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e]) + (Sqrt[c]*(b^2*d^2 - b*d*(Sqrt[b^2 - 4*a*c]*d +
2*a*e) - 2*a*(c*d^2 - e*(Sqrt[b^2 - 4*a*c]*d + a*e)))*ArcTanh[(Sq
rt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e])
```

Rubi [A] time = 7.54212, antiderivative size = 416, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$-\frac{\sqrt{c}\left(bd\left(d\sqrt{b^2-4ac}-2ae\right)-2ae\left(d\sqrt{b^2-4ac}-ae\right)-2acd^2+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}\left(-bd\left(d\sqrt{b^2-4ac}+2ae\right)+2ae\left(d\sqrt{b^2-4ac}+ae\right)-2acd^2+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{d}(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2}-\frac{d\sqrt{d+ex^2}}{2ax^2}+\frac{\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(d*Sqrt[d + e*x^2])/(2*a*x^2) + (Sqrt[d]*e*ArcTanh[Sqrt[d + e*x^
2]/Sqrt[d]])/(2*a) + (Sqrt[d]*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x^
2]/Sqrt[d]])/a^2 - (Sqrt[c]*(b^2*d^2 - 2*a*c*d^2 + b*d*(Sqrt[b^2
- 4*a*c]*d - 2*a*e) - 2*a*e*(Sqrt[b^2 - 4*a*c]*d - a*e))*ArcTanh[
(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*
a*c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e]) + (Sqrt[c]*(b^2*d^2 - 2*a*c*d^2 + 2*a*e*(Sqrt[b
^2 - 4*a*c]*d + a*e) - b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*ArcTanh
[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**(3/2)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.01722, size = 382, normalized size = 0.92

$$\frac{\frac{\sqrt{2}\left(\frac{c\left(2a\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)+bd\left(2ae-d\sqrt{b^2-4ac}\right)-b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{c\left(bd\left(d\sqrt{b^2-4ac}+2ae\right)-2ae\left(d\sqrt{b^2-4ac}+ae\right)+2acd^2-b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4\right.}}}\right.}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right.}{\sqrt{c}\sqrt{b^2-4ac}}}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((a*d*Sqrt[d + e*x^2])/x^2) + (Sqrt[2]*((c*(-(b^2*d^2) + b*d*(-
(Sqrt[b^2 - 4*a*c]*d) + 2*a*e) + 2*a*(c*d^2 + e*(Sqrt[b^2 - 4*a*c
]*d - a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d
 - b*e + Sqrt[b^2 - 4*a*c]*e]])/Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a
*c])*e] - (c*(-(b^2*d^2) + 2*a*c*d^2 - 2*a*e*(Sqrt[b^2 - 4*a*c]*d
 + a*e) + b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*ArcTanh[(Sqrt[2]*Sqr
t[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(Sqrt[c]*Sqrt[b^2 - 4*a*
c]) - Sqrt[d]*(2*b*d - 3*a*e)*Log[x] + Sqrt[d]*(2*b*d - 3*a*e)*Lo
g[d + Sqrt[d]*Sqrt[d + e*x^2]])/(2*a^2)
```

Maple [C] time = 0.046, size = 555, normalized size = 1.3

$$-\frac{1}{2\,adx^2}\left(ex^2+d\right)^{\frac{5}{2}}+\frac{e}{2\,ad}\left(ex^2+d\right)^{\frac{3}{2}}-\frac{3\,e}{2\,a}\sqrt{d}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{ex^2+d}\right)\right)$$
$$+\frac{e}{a}\sqrt{ex^2+d}-\frac{bx^3}{6\,a^2}e^{\frac{3}{2}}+\frac{x^2be}{8\,a^2}\sqrt{ex^2+d}-\frac{3\,bdx}{4\,a^2}\sqrt{e}-\frac{7\,b}{24\,a^2}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{x}{2\,a}e^{\frac{3}{2}}-\frac{3\,bd}{8\,a^2}\sqrt{ex^2+d}$$
$$+\frac{1}{4\,a^2}\sum_{\_R=RootOf\left(c\_Z^8+(4\,be-4\,cd)\_Z^6+(16\,ae^2-8\,bde+6\,cd^2)\_Z^4+(4\,bd^2e-4\,cd^3)\_Z^2+cd^4\right)}\frac{cd\,(-2\,ae+bd)\_R^6+\left(4\,a^2e^3-8\,abde^2+2\,acd^2e\right.}{\_R^7c+3\,\_R^5be}$$
$$-\frac{de}{2\,a}\left(\sqrt{ex^2+d}-x\sqrt{e}\right)^{-1}+\frac{5\,bd^2}{8\,a^2}\left(\sqrt{ex^2+d}-x\sqrt{e}\right)^{-1}$$
$$+\frac{bd^3}{24\,a^2}\left(\sqrt{ex^2+d}-x\sqrt{e}\right)^{-3}+\frac{b}{a^2}d^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{ex^2+d}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/a/d/x^2*(e*x^2+d)^(5/2)+1/2/a/d*e*(e*x^2+d)^(3/2)-3/2/a*d^(1
/2)*e*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)+1/a*e*(e*x^2+d)^(1/2)
-1/6/a^2*e^(3/2)*x^3*b+1/8/a^2*e*(e*x^2+d)^(1/2)*x^2*b-3/4/a^2*e^
```

```
(1/2)*x*b*d-7/24/a^2*(e*x^2+d)^(3/2)*b+1/2/a*e^(3/2)*x-3/8/a^2*(e
*x^2+d)^(1/2)*b*d+1/4/a^2*sum((c*d*(-2*a*e+b*d)*_R^6+(4*a^2*e^3-8
*a*b*d*e^2+2*a*c*d^2*e+4*b^2*d^2*e-3*b*c*d^3)*_R^4+d*(-4*a^2*e^3+
8*a*b*d*e^2-2*a*c*d^2*e-4*b^2*d^2*e+3*b*c*d^3)*_R^2+2*a*c*d^4*e-b
*c*d^5)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3
*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-x*e^(1/2)-_R)
,_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*_
Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+c*d^4))-1/2/a*d/((e*x^2+d)^(1/2)-x*e
^(1/2))*e+5/8/a^2*d^2/((e*x^2+d)^(1/2)-x*e^(1/2))*b+1/24/a^2*b*d^
3/((e*x^2+d)^(1/2)-x*e^(1/2))^3+b/a^2*d^(3/2)*ln((2*d+2*d^(1/2)*(
e*x^2+d)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^{\frac{3}{2}}}{(cx^4+bx^2+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] Timed out
```

3.363 $\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=595

$$-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{\sqrt{b^2-4ac}+b}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)}{2c^3}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)}{2c^3}+\frac{x\sqrt{d+ex^2}(3cd-4be)}{8c^2}+\frac{d(3cd-4be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8c^2\sqrt{e}}+\frac{x\left(d+ex^2\right)^{3/2}}{4c}$$

```
[Out] ((3*c*d - 4*b*e)*x*Sqrt[d + e*x^2])/(8*c^2) + (x*(d + e*x^2)^(3/2
))/(4*c) - (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*c*d - b^2*
e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[
b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c^3*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(b*c*d -
b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(S
qrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c^3*Sqrt[b + Sqr
t[b^2 - 4*a*c]]) + (d*(3*c*d - 4*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d
+ e*x^2]])/(8*c^2*Sqrt[e]) - (Sqrt[e]*(b*c*d - b^2*e + a*c*e - (b
^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
h[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^3) - (Sqrt[e]*(b*c*d - b^2*e
 + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4
*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^3)
```

Rubi [A] time = 7.25336, antiderivative size = 595, normalized size of antiderivative = 1., number of

steps used = 17, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.31$

$$-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{\sqrt{b^2-4ac}+b}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)}{2c^3}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)}{2c^3}+\frac{x\sqrt{d+ex^2}(3cd-4be)}{8c^2}+\frac{d(3cd-4be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8c^2\sqrt{e}}+\frac{x\left(d+ex^2\right)^{3/2}}{4c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((3*c*d - 4*b*e)*x*Sqrt[d + e*x^2])/(8*c^2) + (x*(d + e*x^2)^(3/2
))/(4*c) - (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*c*d - b^2*
e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[
b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c^3*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(b*c*d -
b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(S
qrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c^3*Sqrt[b + Sqr
t[b^2 - 4*a*c]]) + (d*(3*c*d - 4*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d
+ e*x^2]])/(8*c^2*Sqrt[e]) - (Sqrt[e]*(b*c*d - b^2*e + a*c*e - (b
^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
h[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^3) - (Sqrt[e]*(b*c*d - b^2*e
 + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4
*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^3)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.24665, size = 0, normalized size = 0.

$$\int \frac{x^4 \left(d + ex^2\right)^{3/2}}{a + bx^2 + cx^4} \, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[(x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x]
```

Maple [C] time = 0.048, size = 516, normalized size = 0.9

$$\frac{x}{4\,c}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{3\,dx}{8\,c}\sqrt{ex^2+d}+\frac{3\,d^2}{8\,c}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)\frac{1}{\sqrt{e}}+\frac{bx^2}{4\,c^2}e^{\frac{3}{2}}-\frac{bex}{4\,c^2}\sqrt{ex^2+d}+\frac{bd}{8\,c^2}\sqrt{e}$$
$$-\frac{1}{2\,c^3}\sqrt{e}\sum_{\_R=RootOf\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+(16\,ae^2-8\,bde+6\,cd^2)\_Z^2+(4\,bd^2e-4\,cd^3)\_Z+cd^4\right)}\frac{\left(2\,abe^2c-2\,ac^2de-b^3e^2+2\,b^2dec-bc^2d^2\right)\_}{}$$
$$+\frac{a}{c^2}e^{\frac{3}{2}}\ln\left(\sqrt{ex^2+d}-x\sqrt{e}\right)-\frac{b^2}{c^3}e^{\frac{3}{2}}\ln\left(\sqrt{ex^2+d}-x\sqrt{e}\right)$$
$$+\frac{3\,bd}{2\,c^2}\sqrt{e}\ln\left(\sqrt{ex^2+d}-x\sqrt{e}\right)-\frac{bd^2}{8\,c^2}\sqrt{e}\left(\sqrt{ex^2+d}-x\sqrt{e}\right)^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*x*(e*x^2+d)^(3/2)/c+3/8/c*d*x*(e*x^2+d)^(1/2)+3/8/c*d^2/e^(1/
2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))+1/4/c^2*e^(3/2)*b*x^2-1/4/c^2*e*
b*(e*x^2+d)^(1/2)*x+1/8/c^2*e^(1/2)*b*d-1/2/c^3*e^(1/2)*sum(((2*a
*b*c*e^2-2*a*c^2*d*e-b^3*e^2+2*b^2*c*d*e-b*c^2*d^2)*_R^2+2*(2*a^2
*c*e^3-2*a*b^2*e^3+2*a*b*c*d*e^2+b^3*d*e^2-2*b^2*c*d^2*e+b*c^2*d^
3)*_R+2*a*b*c*d^2*e^2-2*a*c^2*d^3*e-b^3*d^2*e^2+2*b^2*c*d^3*e-c^2
*d^4*b)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*
c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-x*e^(1/2))^2-_R),_R=Root
Of(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b
*d^2*e-4*c*d^3)*_Z+c*d^4))+1/c^2*e^(3/2)*ln((e*x^2+d)^(1/2)-x*e^(
1/2))*a-1/c^3*e^(3/2)*ln((e*x^2+d)^(1/2)-x*e^(1/2))*b^2+3/2/c^2*e
^(1/2)*ln((e*x^2+d)^(1/2)-x*e^(1/2))*b*d-1/8/c^2*e^(1/2)*b*d^2/((
```

```
e*x^2+d)^(1/2)-x*e^(1/2))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^{\frac{3}{2}}x^4}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)*x^4/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*x^4/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)*x^4/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\left(d+ex^2\right)^{\frac{3}{2}}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**4*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^(3/2)*x^4/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.364 $\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=491

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2c^2}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2c^2}+\frac{ex\sqrt{d+ex^2}}{2c}+\frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c}$$

```
[Out] (e*x*Sqrt[d + e*x^2])/(2*c) + (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e]*(c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[
b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c^2*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d - b*e + (b*c*
d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e
*x^2])])/(2*c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (d*Sqrt[e]*ArcTanh
[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c) + (Sqrt[e]*(c*d - b*e - (b*c
*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt
[d + e*x^2]])/(2*c^2) + (Sqrt[e]*(c*d - b*e + (b*c*d - b^2*e + 2*
a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(
2*c^2)
```

Rubi [A] time = 3.99574, antiderivative size = 491, normalized size of antiderivative = 1., number of

steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2c^2}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2c^2}+\frac{ex\sqrt{d+ex^2}}{2c}+\frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e*x*Sqrt[d + e*x^2])/(2*c) + (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e]*(c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[
b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c^2*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d - b*e + (b*c*
d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e
*x^2])])/(2*c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (d*Sqrt[e]*ArcTanh
[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c) + (Sqrt[e]*(c*d - b*e - (b*c
*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt
[d + e*x^2]])/(2*c^2) + (Sqrt[e]*(c*d - b*e + (b*c*d - b^2*e + 2*
a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(
2*c^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.953538, size = 0, normalized size = 0.

$$\int \frac{x^2 \left(d + ex^2\right)^{3/2}}{a + bx^2 + cx^4}\, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[(x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x]
```

Maple [C] time = 0.042, size = 382, normalized size = 0.8

$$-\frac{x^2}{4c}e^{\frac{3}{2}} + \frac{ex}{4c}\sqrt{ex^2+d} - \frac{d}{8c}\sqrt{e} + \frac{1}{2c^2}\sqrt{e}\sum_{\_R=RootOf\left(c\_Z^4+(4be-4cd)\_Z^3+(16ae^2-8bde+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z+cd^4\right)} \frac{\left(ace^2-b^2e^2+2bcde-c^2d^2\right)\_R^2+2\left(-2abe^3\right.}{\_R^3c+3\_R^2be-3\_R} + \frac{d^2}{8c}\sqrt{e}\left(\sqrt{ex^2+d}-x\sqrt{e}\right)^{-2} + \frac{b}{c^2}e^{\frac{3}{2}}\ln\left(\sqrt{ex^2+d}-x\sqrt{e}\right) - \frac{3d}{2c}\sqrt{e}\ln\left(\sqrt{ex^2+d}-x\sqrt{e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*e^(3/2)/c*x^2+1/4*e*x*(e*x^2+d)^(1/2)/c-1/8*e^(1/2)/c*d+1/2*
e^(1/2)/c^2*sum(((a*c*e^2-b^2*e^2+2*b*c*d*e-c^2*d^2)*_R^2+2*(-2*a
*b*e^3+3*a*c*d*e^2+b^2*d*e^2-2*b*c*d^2*e+c^2*d^3)*_R+a*c*d^2*e^2-
b^2*d^2*e^2+2*b*c*d^3*e-c^2*d^4)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*
_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)
-x*e^(1/2))^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8
*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))+1/8*e^(1/2)/c
*d^2/((e*x^2+d)^(1/2)-x*e^(1/2))^2+e^(3/2)/c^2*ln((e*x^2+d)^(1/2)
-x*e^(1/2))*b-3/2*e^(1/2)/c*ln((e*x^2+d)^(1/2)-x*e^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}} x^2}{cx^4+bx^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)*x^2/(c*x^4 + b*x^2 + a),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^(3/2)*x^2/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 99.5536, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)*x^2/(c*x^4 + b*x^2 + a),x, algorithm="fricas")

[Out] [-1/4*(sqrt(1/2)*c^2*sqrt(-(b*c^3*d^3 - 3*(b^2*c^2 - 2*a*c^3)*d^2
*e + 3*(b^3*c - 3*a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*
e^3 + (b^2*c^4 - 4*a*c^5)*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^
2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 +
3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a
*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)
*e^6)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log((2*a*c^4*d^6
 - 8*a*b*c^3*d^5*e + 4*(3*a*b^2*c^2 - a^2*c^3)*d^4*e^2 - 4*(2*a*b
^3*c - a^2*b*c^2)*d^3*e^3 + 2*(a*b^4 + a^2*b^2*c - 3*a^3*c^2)*d^2
*e^4 - 2*(a^2*b^3 - 2*a^3*b*c)*d*e^5 + ((b^2*c^5 - 4*a*c^6)*d^3 -
 (b^3*c^4 - 4*a*b*c^5)*d^2*e + (a*b^2*c^4 - 4*a^2*c^5)*d*e^2)*x^2
*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2
- 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c
^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d
*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)
) - (b*c^4*d^6 - 4*(b^2*c^3 + a*c^4)*d^5*e + 2*(3*b^3*c^2 + 7*a*b
*c^3)*d^4*e^2 - 2*(2*b^4*c + 11*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^3 +
(b^5 + 17*a*b^3*c - 11*a^2*b*c^2)*d^2*e^4 - (5*a*b^4 + 2*a^2*b^2*
c - 12*a^3*c^2)*d*e^5 + 4*(a^2*b^3 - 2*a^3*b*c)*e^6)*x^2 + 2*sqrt
(1/2)*sqrt(e*x^2 + d)*(((b^3*c^5 - 4*a*b*c^6)*d - (b^4*c^4 - 6*a*
b^2*c^5 + 8*a^2*c^6)*e)*x*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^
2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 +
3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a
*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)
*e^6)/(b^2*c^8 - 4*a*c^9)) - ((b^2*c^4 - 4*a*c^5)*d^4 - 4*(b^3*c^
3 - 4*a*b*c^4)*d^3*e + 3*(2*b^4*c^2 - 9*a*b^2*c^3 + 4*a^2*c^4)*d^
2*e^2 - (4*b^5*c - 21*a*b^3*c^2 + 20*a^2*b*c^3)*d*e^3 + (b^6 - 6*
a*b^4*c + 8*a^2*b^2*c^2)*e^4)*x)*sqrt(-(b*c^3*d^3 - 3*(b^2*c^2 -
2*a*c^3)*d^2*e + 3*(b^3*c - 3*a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c +
 2*a^2*c^2)*e^3 + (b^2*c^4 - 4*a*c^5)*sqrt((c^6*d^6 - 6*b*c^5*d^5
*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4
)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*
(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*
a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2
) - sqrt(1/2)*c^2*sqrt(-(b*c^3*d^3 - 3*(b^2*c^2 - 2*a*c^3)*d^2*e
+ 3*(b^3*c - 3*a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3
 + (b^2*c^4 - 4*a*c^5)*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c
```

```
^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(
5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^
3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^
6)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log((2*a*c^4*d^6 -
8*a*b*c^3*d^5*e + 4*(3*a*b^2*c^2 - a^2*c^3)*d^4*e^2 - 4*(2*a*b^3*
c - a^2*b*c^2)*d^3*e^3 + 2*(a*b^4 + a^2*b^2*c - 3*a^3*c^2)*d^2*e^
4 - 2*(a^2*b^3 - 2*a^3*b*c)*d*e^5 + ((b^2*c^5 - 4*a*c^6)*d^3 - (b
^3*c^4 - 4*a*b*c^5)*d^2*e + (a*b^2*c^4 - 4*a^2*c^5)*d*e^2)*x^2*sq
rt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2
*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3
+ 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^
5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)) -
 (b*c^4*d^6 - 4*(b^2*c^3 + a*c^4)*d^5*e + 2*(3*b^3*c^2 + 7*a*b*c^
3)*d^4*e^2 - 2*(2*b^4*c + 11*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^3 + (b^
5 + 17*a*b^3*c - 11*a^2*b*c^2)*d^2*e^4 - (5*a*b^4 + 2*a^2*b^2*c -
 12*a^3*c^2)*d*e^5 + 4*(a^2*b^3 - 2*a^3*b*c)*e^6)*x^2 - 2*sqrt(1/
2)*sqrt(e*x^2 + d)*(((b^3*c^5 - 4*a*b*c^6)*d - (b^4*c^4 - 6*a*b^2
*c^5 + 8*a^2*c^6)*e)*x*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c
^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(
5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^
3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^
6)/(b^2*c^8 - 4*a*c^9)) - ((b^2*c^4 - 4*a*c^5)*d^4 - 4*(b^3*c^3 -
 4*a*b*c^4)*d^3*e + 3*(2*b^4*c^2 - 9*a*b^2*c^3 + 4*a^2*c^4)*d^2*e
^2 - (4*b^5*c - 21*a*b^3*c^2 + 20*a^2*b*c^3)*d*e^3 + (b^6 - 6*a*b
^4*c + 8*a^2*b^2*c^2)*e^4)*x)*sqrt(-(b*c^3*d^3 - 3*(b^2*c^2 - 2*a
*c^3)*d^2*e + 3*(b^3*c - 3*a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*
a^2*c^2)*e^3 + (b^2*c^4 - 4*a*c^5)*sqrt((c^6*d^6 - 6*b*c^5*d^5*e
+ 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d
^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^
5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2
*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) -
 sqrt(1/2)*c^2*sqrt(-(b*c^3*d^3 - 3*(b^2*c^2 - 2*a*c^3)*d^2*e + 3
*(b^3*c - 3*a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3 -
(b^2*c^4 - 4*a*c^5)*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4
- 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b
^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c
^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/
(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log((2*a*c^4*d^6 - 8*a
*b*c^3*d^5*e + 4*(3*a*b^2*c^2 - a^2*c^3)*d^4*e^2 - 4*(2*a*b^3*c -
 a^2*b*c^2)*d^3*e^3 + 2*(a*b^4 + a^2*b^2*c - 3*a^3*c^2)*d^2*e^4 -
 2*(a^2*b^3 - 2*a^3*b*c)*d*e^5 - ((b^2*c^5 - 4*a*c^6)*d^3 - (b^3*
c^4 - 4*a*b*c^5)*d^2*e + (a*b^2*c^4 - 4*a^2*c^5)*d*e^2)*x^2*sqrt(
(c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(1
0*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3
*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 +
 (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)) - (b
*c^4*d^6 - 4*(b^2*c^3 + a*c^4)*d^5*e + 2*(3*b^3*c^2 + 7*a*b*c^3)*
d^4*e^2 - 2*(2*b^4*c + 11*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^3 + (b^5 +
 17*a*b^3*c - 11*a^2*b*c^2)*d^2*e^4 - (5*a*b^4 + 2*a^2*b^2*c - 12
*a^3*c^2)*d*e^5 + 4*(a^2*b^3 - 2*a^3*b*c)*e^6)*x^2 + 2*sqrt(1/2)*
sqrt(e*x^2 + d)*(((b^3*c^5 - 4*a*b*c^6)*d - (b^4*c^4 - 6*a*b^2*c^
5 + 8*a^2*c^6)*e)*x*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4
- 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b
^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c
```

```
^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/
(b^2*c^8 - 4*a*c^9)) + ((b^2*c^4 - 4*a*c^5)*d^4 - 4*(b^3*c^3 - 4*
a*b*c^4)*d^3*e + 3*(2*b^4*c^2 - 9*a*b^2*c^3 + 4*a^2*c^4)*d^2*e^2
- (4*b^5*c - 21*a*b^3*c^2 + 20*a^2*b*c^3)*d*e^3 + (b^6 - 6*a*b^4*
c + 8*a^2*b^2*c^2)*e^4)*x)*sqrt(-(b*c^3*d^3 - 3*(b^2*c^2 - 2*a*c^
3)*d^2*e + 3*(b^3*c - 3*a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2
*c^2)*e^3 - (b^2*c^4 - 4*a*c^5)*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3
*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*
e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c
 - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^
2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) + sq
rt(1/2)*c^2*sqrt(-(b*c^3*d^3 - 3*(b^2*c^2 - 2*a*c^3)*d^2*e + 3*(b
^3*c - 3*a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3 - (b^
2*c^4 - 4*a*c^5)*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2
*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*
c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2
+ 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^
2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log((2*a*c^4*d^6 - 8*a*b*
c^3*d^5*e + 4*(3*a*b^2*c^2 - a^2*c^3)*d^4*e^2 - 4*(2*a*b^3*c - a^
2*b*c^2)*d^3*e^3 + 2*(a*b^4 + a^2*b^2*c - 3*a^3*c^2)*d^2*e^4 - 2*
(a^2*b^3 - 2*a^3*b*c)*d*e^5 - ((b^2*c^5 - 4*a*c^6)*d^3 - (b^3*c^4
 - 4*a*b*c^5)*d^2*e + (a*b^2*c^4 - 4*a^2*c^5)*d*e^2)*x^2*sqrt((c^
6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b
^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^
2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b
^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)) - (b*c^
4*d^6 - 4*(b^2*c^3 + a*c^4)*d^5*e + 2*(3*b^3*c^2 + 7*a*b*c^3)*d^4
*e^2 - 2*(2*b^4*c + 11*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^3 + (b^5 + 17
*a*b^3*c - 11*a^2*b*c^2)*d^2*e^4 - (5*a*b^4 + 2*a^2*b^2*c - 12*a^
3*c^2)*d*e^5 + 4*(a^2*b^3 - 2*a^3*b*c)*e^6)*x^2 - 2*sqrt(1/2)*sqr
t(e*x^2 + d)*(((b^3*c^5 - 4*a*b*c^6)*d - (b^4*c^4 - 6*a*b^2*c^5 +
 8*a^2*c^6)*e)*x*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2
*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*
c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2
+ 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^
2*c^8 - 4*a*c^9)) + ((b^2*c^4 - 4*a*c^5)*d^4 - 4*(b^3*c^3 - 4*a*b
*c^4)*d^3*e + 3*(2*b^4*c^2 - 9*a*b^2*c^3 + 4*a^2*c^4)*d^2*e^2 - (
4*b^5*c - 21*a*b^3*c^2 + 20*a^2*b*c^3)*d*e^3 + (b^6 - 6*a*b^4*c +
 8*a^2*b^2*c^2)*e^4)*x)*sqrt(-(b*c^3*d^3 - 3*(b^2*c^2 - 2*a*c^3)*
d^2*e + 3*(b^3*c - 3*a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^
2)*e^3 - (b^2*c^4 - 4*a*c^5)*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5
*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3
 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c -
3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c
^2)*e^6)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) - 2*sqr
t(e*x^2 + d)*c*e*x + (3*c*d - 2*b*e)*sqrt(e)*log(-2*e*x^2 + 2*sqr
t(e*x^2 + d)*sqrt(e)*x - d))/c^2, -1/4*(sqrt(1/2)*c^2*sqrt(-(b*c^
3*d^3 - 3*(b^2*c^2 - 2*a*c^3)*d^2*e + 3*(b^3*c - 3*a*b*c^2)*d*e^2
 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3 + (b^2*c^4 - 4*a*c^5)*sqrt((
c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10
*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*
a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 +
(b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)))/(b^2
*c^4 - 4*a*c^5))*log((2*a*c^4*d^6 - 8*a*b*c^3*d^5*e + 4*(3*a*b^2*
```

c^2 - a^2*c^3)*d^4*e^2 - 4*(2*a*b^3*c - a^2*b*c^2)*d^3*e^3 + 2*(a
*b^4 + a^2*b^2*c - 3*a^3*c^2)*d^2*e^4 - 2*(a^2*b^3 - 2*a^3*b*c)*d
*e^5 + ((b^2*c^5 - 4*a*c^6)*d^3 - (b^3*c^4 - 4*a*b*c^5)*d^2*e + (
a*b^2*c^4 - 4*a^2*c^5)*d*e^2)*x^2*sqrt((c^6*d^6 - 6*b*c^5*d^5*e +
 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^
3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5
*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*
b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)) - (b*c^4*d^6 - 4*(b^2*c^3 + a*
c^4)*d^5*e + 2*(3*b^3*c^2 + 7*a*b*c^3)*d^4*e^2 - 2*(2*b^4*c + 11*
a*b^2*c^2 - 4*a^2*c^3)*d^3*e^3 + (b^5 + 17*a*b^3*c - 11*a^2*b*c^2
)*d^2*e^4 - (5*a*b^4 + 2*a^2*b^2*c - 12*a^3*c^2)*d*e^5 + 4*(a^2*b
^3 - 2*a^3*b*c)*e^6)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^3*c^5
 - 4*a*b*c^6)*d - (b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*e)*x*sqrt((
c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10
*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*
a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 +
(b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)) - ((b
^2*c^4 - 4*a*c^5)*d^4 - 4*(b^3*c^3 - 4*a*b*c^4)*d^3*e + 3*(2*b^4*
c^2 - 9*a*b^2*c^3 + 4*a^2*c^4)*d^2*e^2 - (4*b^5*c - 21*a*b^3*c^2
+ 20*a^2*b*c^3)*d*e^3 + (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*e^4)*x)
*sqrt(-(b*c^3*d^3 - 3*(b^2*c^2 - 2*a*c^3)*d^2*e + 3*(b^3*c - 3*a*
b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3 + (b^2*c^4 - 4*a
*c^5)*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4
e^2 - 2(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*
b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c
^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a
*c^9)))/(b^2*c^4 - 4*a*c^5)))/x^2) - sqrt(1/2)*c^2*sqrt(-(b*c^3*d
^3 - 3*(b^2*c^2 - 2*a*c^3)*d^2*e + 3*(b^3*c - 3*a*b*c^2)*d*e^2 -
(b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3 + (b^2*c^4 - 4*a*c^5)*sqrt((c^6
*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^
3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2
*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^
6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^
4 - 4*a*c^5))*log((2*a*c^4*d^6 - 8*a*b*c^3*d^5*e + 4*(3*a*b^2*c^2
 - a^2*c^3)*d^4*e^2 - 4*(2*a*b^3*c - a^2*b*c^2)*d^3*e^3 + 2*(a*b^
4 + a^2*b^2*c - 3*a^3*c^2)*d^2*e^4 - 2*(a^2*b^3 - 2*a^3*b*c)*d*e^
5 + ((b^2*c^5 - 4*a*c^6)*d^3 - (b^3*c^4 - 4*a*b*c^5)*d^2*e + (a*b
^2*c^4 - 4*a^2*c^5)*d*e^2)*x^2*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*
(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e
^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c
- 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2
*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)) - (b*c^4*d^6 - 4*(b^2*c^3 + a*c^4
)*d^5*e + 2*(3*b^3*c^2 + 7*a*b*c^3)*d^4*e^2 - 2*(2*b^4*c + 11*a*b
^2*c^2 - 4*a^2*c^3)*d^3*e^3 + (b^5 + 17*a*b^3*c - 11*a^2*b*c^2)*d
^2*e^4 - (5*a*b^4 + 2*a^2*b^2*c - 12*a^3*c^2)*d*e^5 + 4*(a^2*b^3
- 2*a^3*b*c)*e^6)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^3*c^5 -
4*a*b*c^6)*d - (b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*e)*x*sqrt((c^6
*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^
3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2
*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^
6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)) - ((b^2*
c^4 - 4*a*c^5)*d^4 - 4*(b^3*c^3 - 4*a*b*c^4)*d^3*e + 3*(2*b^4*c^2
 - 9*a*b^2*c^3 + 4*a^2*c^4)*d^2*e^2 - (4*b^5*c - 21*a*b^3*c^2 + 2
0*a^2*b*c^3)*d*e^3 + (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*e^4)*x)*sq

```
rt(-(b*c^3*d^3 - 3*(b^2*c^2 - 2*a*c^3)*d^2*e + 3*(b^3*c - 3*a*b*c
^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3 + (b^2*c^4 - 4*a*c^
5)*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^
2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2
*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)
*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^
9)))/(b^2*c^4 - 4*a*c^5)))/x^2) - sqrt(1/2)*c^2*sqrt(-(b*c^3*d^3
- 3*(b^2*c^2 - 2*a*c^3)*d^2*e + 3*(b^3*c - 3*a*b*c^2)*d*e^2 - (b^
4 - 4*a*b^2*c + 2*a^2*c^2)*e^3 - (b^2*c^4 - 4*a*c^5)*sqrt((c^6*d^
6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c
^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^
4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 -
 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 -
 4*a*c^5))*log((2*a*c^4*d^6 - 8*a*b*c^3*d^5*e + 4*(3*a*b^2*c^2 -
a^2*c^3)*d^4*e^2 - 4*(2*a*b^3*c - a^2*b*c^2)*d^3*e^3 + 2*(a*b^4 +
 a^2*b^2*c - 3*a^3*c^2)*d^2*e^4 - 2*(a^2*b^3 - 2*a^3*b*c)*d*e^5 -
 ((b^2*c^5 - 4*a*c^6)*d^3 - (b^3*c^4 - 4*a*b*c^5)*d^2*e + (a*b^2*
c^4 - 4*a^2*c^5)*d*e^2)*x^2*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*
b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3
+ 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3
*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^
2)*e^6)/(b^2*c^8 - 4*a*c^9)) - (b*c^4*d^6 - 4*(b^2*c^3 + a*c^4)*d
^5*e + 2*(3*b^3*c^2 + 7*a*b*c^3)*d^4*e^2 - 2*(2*b^4*c + 11*a*b^2*
c^2 - 4*a^2*c^3)*d^3*e^3 + (b^5 + 17*a*b^3*c - 11*a^2*b*c^2)*d^2*
e^4 - (5*a*b^4 + 2*a^2*b^2*c - 12*a^3*c^2)*d*e^5 + 4*(a^2*b^3 - 2
*a^3*b*c)*e^6)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^3*c^5 - 4*a
*b*c^6)*d - (b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*e)*x*sqrt((c^6*d^
6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c
^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^
4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 -
 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)) + ((b^2*c^4
 - 4*a*c^5)*d^4 - 4*(b^3*c^3 - 4*a*b*c^4)*d^3*e + 3*(2*b^4*c^2 -
9*a*b^2*c^3 + 4*a^2*c^4)*d^2*e^2 - (4*b^5*c - 21*a*b^3*c^2 + 20*a
^2*b*c^3)*d*e^3 + (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*e^4)*x)*sqrt(
-(b*c^3*d^3 - 3*(b^2*c^2 - 2*a*c^3)*d^2*e + 3*(b^3*c - 3*a*b*c^2)
*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3 - (b^2*c^4 - 4*a*c^5)*
sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 -
 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^
3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*
e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9))
)/(b^2*c^4 - 4*a*c^5)))/x^2) + sqrt(1/2)*c^2*sqrt(-(b*c^3*d^3 - 3
*(b^2*c^2 - 2*a*c^3)*d^2*e + 3*(b^3*c - 3*a*b*c^2)*d*e^2 - (b^4 -
 4*a*b^2*c + 2*a^2*c^2)*e^3 - (b^2*c^4 - 4*a*c^5)*sqrt((c^6*d^6 -
 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3
- 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*
d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*
a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*
a*c^5))*log((2*a*c^4*d^6 - 8*a*b*c^3*d^5*e + 4*(3*a*b^2*c^2 - a^2
*c^3)*d^4*e^2 - 4*(2*a*b^3*c - a^2*b*c^2)*d^3*e^3 + 2*(a*b^4 + a^
2*b^2*c - 3*a^3*c^2)*d^2*e^4 - 2*(a^2*b^3 - 2*a^3*b*c)*d*e^5 - ((
b^2*c^5 - 4*a*c^6)*d^3 - (b^3*c^4 - 4*a*b*c^5)*d^2*e + (a*b^2*c^4
 - 4*a^2*c^5)*d*e^2)*x^2*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2
*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3
*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*
```

```
b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*
e^6)/(b^2*c^8 - 4*a*c^9)) - (b*c^4*d^6 - 4*(b^2*c^3 + a*c^4)*d^5*
e + 2*(3*b^3*c^2 + 7*a*b*c^3)*d^4*e^2 - 2*(2*b^4*c + 11*a*b^2*c^2
 - 4*a^2*c^3)*d^3*e^3 + (b^5 + 17*a*b^3*c - 11*a^2*b*c^2)*d^2*e^4
 - (5*a*b^4 + 2*a^2*b^2*c - 12*a^3*c^2)*d*e^5 + 4*(a^2*b^3 - 2*a^
3*b*c)*e^6)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^3*c^5 - 4*a*b*
c^6)*d - (b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*e)*x*sqrt((c^6*d^6 -
 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3
- 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*
d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*
a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)) + ((b^2*c^4 -
4*a*c^5)*d^4 - 4*(b^3*c^3 - 4*a*b*c^4)*d^3*e + 3*(2*b^4*c^2 - 9*a
*b^2*c^3 + 4*a^2*c^4)*d^2*e^2 - (4*b^5*c - 21*a*b^3*c^2 + 20*a^2*
b*c^3)*d*e^3 + (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*e^4)*x)*sqrt(-(b
*c^3*d^3 - 3*(b^2*c^2 - 2*a*c^3)*d^2*e + 3*(b^3*c - 3*a*b*c^2)*d*
e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3 - (b^2*c^4 - 4*a*c^5)*sqr
t((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*
(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 +
 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5
 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)))/(
b^2*c^4 - 4*a*c^5)))/x^2) - 2*sqrt(e*x^2 + d)*c*e*x - 2*(3*c*d -
2*b*e)*sqrt(-e)*arctan(e*x/(sqrt(e*x^2 + d)*sqrt(-e))))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(d + ex^2\right)^{\frac{3}{2}}}{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**2*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^(3/2)*x^2/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.365 $\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=487

$$\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}{2c\sqrt{b^2-4ac}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}{2c\sqrt{b^2-4ac}}$$

```
[Out] ((2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d - Sqrt[b
^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b
^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e]) - ((2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2
*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e
*x^2])])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[e]*(3*c*d - (b - Sqrt[b
^2 - 4*a*c])*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c*Sqrt[b
^2 - 4*a*c]) - (Sqrt[e]*(3*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTa
nh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 4.22557, antiderivative size = 487, normalized size of antiderivative = 1., number of

steps used = 13, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$

$$\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}{2c\sqrt{b^2-4ac}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}{2c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d - Sqrt[b
^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(c*Sqrt[b
^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e]) - ((2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2
*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e
*x^2])]])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[e]*(3*c*d - (b - Sqrt[b
^2 - 4*a*c])*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c*Sqrt[b
^2 - 4*a*c]) - (Sqrt[e]*(3*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTa
nh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c*Sqrt[b^2 - 4*a*c])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.672079, size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[(d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4), x]
```

Maple [C] time = 0.029, size = 217, normalized size = 0.5

$$\frac{1}{2\,c}e^{\frac{3}{2}}\sum_{\_R=RootOf\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+(16\,ae^2-8\,bde+6\,cd^2)\_Z^2+(4\,bd^2e-4\,cd^3)\_Z+cd^4\right)}\frac{(be-2\,cd)\,\_R^2+2\,e\,(2\,ae-bd)\,\_R+bc}{\_R^3c+3\,\_R^2be-3\,\_R^2cd+8\,\_R\,ae^2-4\,\_R\,bde+}$$
$$-\frac{1}{c}e^{\frac{3}{2}}\ln\left(\sqrt{ex^2+d}-x\sqrt{e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*e^(3/2)/c*sum(((b*e-2*c*d)*_R^2+2*e*(2*a*e-b*d)*_R+b*d^2*e-2*
c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c
*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-x*e^(1/2))^2-_R),_R=RootO
f(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*
d^2*e-4*c*d^3)*_Z+c*d^4))-e^(3/2)/c*ln((e*x^2+d)^(1/2)-x*e^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^{\frac{3}{2}}}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 27.1317, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^
2 - (a*b^2 - 2*a^2*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6
 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^
2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2
 - 4*a^2*c^3))*log((2*a*c^3*d^6 - 2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4
*e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*b*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*
d^2*e^4 + ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3
)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((c^4*d^6 - 6*
a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c
*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2
*a*b*c^2*d^4*e^2 - 4*a^3*b*e^6 - (b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a
*b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*b^3 + 19*a^2*b*c)*d^2*e^4 + (5*a
^2*b^2 + 12*a^3*c)*d*e^5)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(
a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e)*x*sqr
t((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*
e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) +
 ((a*b^2*c^2 - 4*a^2*c^3)*d^3*e - 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3
 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e
 + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3
)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2
*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^
5)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c^2*d^
3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 - (a*b^
2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^
3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b
^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log((2*a*c^3*d^6 -
 2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4*e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*
b*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*d^2*e^4 + ((a*b^2*c^3 - 4*a^2*c^4
)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^
3)*d*e^2)*x^2*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3
 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^
4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*a*b*c^2*d^4*e^2 - 4*a^3*b*e^6 -
(b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*
b^3 + 19*a^2*b*c)*d^2*e^4 + (5*a^2*b^2 + 12*a^3*c)*d*e^5)*x^2 - 2
*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2
*b^3*c^2 - 4*a^3*b*c^3)*e)*x*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*
a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e
^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) + ((a*b^2*c^2 - 4*a^2*c^3)*d^3*e -
 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*
sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2
*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2
 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*
```

```
b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^
2) - sqrt(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2
 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6
- 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2
*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2
- 4*a^2*c^3))*log((2*a*c^3*d^6 - 2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4*
e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*b*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*d
^2*e^4 - ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)
*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((c^4*d^6 - 6*a
*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*
d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*
a*b*c^2*d^4*e^2 - 4*a^3*b*e^6 - (b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*
b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*b^3 + 19*a^2*b*c)*d^2*e^4 + (5*a^
2*b^2 + 12*a^3*c)*d*e^5)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a
^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e)*x*sqrt
((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e
^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) -
((a*b^2*c^2 - 4*a^2*c^3)*d^3*e - 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3
+ (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e
+ 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)
*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*
d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5
)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^2) + sqrt(1/2)*c*sqrt(-(b*c^2*d^3
 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2
*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3
*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^
2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log((2*a*c^3*d^6 -
2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4*e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*b
*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*d^2*e^4 - ((a*b^2*c^3 - 4*a^2*c^4)
*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3
)*d*e^2)*x^2*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3
+ 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4
 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*a*b*c^2*d^4*e^2 - 4*a^3*b*e^6 - (
b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*b
^3 + 19*a^2*b*c)*d^2*e^4 + (5*a^2*b^2 + 12*a^3*c)*d*e^5)*x^2 - 2*
sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*
b^3*c^2 - 4*a^3*b*c^3)*e)*x*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a
*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^
6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - ((a*b^2*c^2 - 4*a^2*c^3)*d^3*e -
3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*s
qrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*
c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2
+ 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b
^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^2
) + 2*e^(3/2)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d))/c,
 1/4*(sqrt(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^
2 - (a*b^2 - 2*a^2*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6
 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^
2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2
 - 4*a^2*c^3))*log((2*a*c^3*d^6 - 2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4
*e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*b*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*
d^2*e^4 + ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3
)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((c^4*d^6 - 6*
a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c
```

```
*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2
*a*b*c^2*d^4*e^2 - 4*a^3*b*e^6 - (b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a
*b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*b^3 + 19*a^2*b*c)*d^2*e^4 + (5*a
^2*b^2 + 12*a^3*c)*d*e^5)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(
a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e)*x*sqr
t((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*
e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) +
 ((a*b^2*c^2 - 4*a^2*c^3)*d^3*e - 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3
 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e
 + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3
)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2
*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^
5)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c^2*d^
3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 - (a*b^
2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^
3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b
^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log((2*a*c^3*d^6 -
 2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4*e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*
b*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*d^2*e^4 + ((a*b^2*c^3 - 4*a^2*c^4
)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^
3)*d*e^2)*x^2*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3
 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^
4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*a*b*c^2*d^4*e^2 - 4*a^3*b*e^6 -
(b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*
b^3 + 19*a^2*b*c)*d^2*e^4 + (5*a^2*b^2 + 12*a^3*c)*d*e^5)*x^2 - 2
*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2
*b^3*c^2 - 4*a^3*b*c^3)*e)*x*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*
a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e
^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) + ((a*b^2*c^2 - 4*a^2*c^3)*d^3*e -
 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*
sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2
*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2
 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*
b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^
2) - sqrt(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2
 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6
- 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2
*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2
- 4*a^2*c^3))*log((2*a*c^3*d^6 - 2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4*
e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*b*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*d
^2*e^4 - ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)
*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((c^4*d^6 - 6*a
*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*
d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*
a*b*c^2*d^4*e^2 - 4*a^3*b*e^6 - (b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*
b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*b^3 + 19*a^2*b*c)*d^2*e^4 + (5*a^
2*b^2 + 12*a^3*c)*d*e^5)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a
^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e)*x*sqrt
((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e
^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)) -
((a*b^2*c^2 - 4*a^2*c^3)*d^3*e - 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3
+ (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e
+ 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)
*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*
```

```
d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5
)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^2) + sqrt(1/2)*c*sqrt(-(b*c^2*d^3
 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2
*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3
*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^
2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log((2*a*c^3*d^6 -
2*a*b*c^2*d^5*e - 4*a^2*c^2*d^4*e^2 + 8*a^2*b*c*d^3*e^3 + 2*a^3*b
*d*e^5 - 2*(a^2*b^2 + 3*a^3*c)*d^2*e^4 - ((a*b^2*c^3 - 4*a^2*c^4)
*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3
)*d*e^2)*x^2*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3
+ 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4
 - 4*a^3*c^5)) - (b*c^3*d^6 + 2*a*b*c^2*d^4*e^2 - 4*a^3*b*e^6 - (
b^2*c^2 + 4*a*c^3)*d^5*e + 4*(a*b^2*c + 2*a^2*c^2)*d^3*e^3 - (a*b
^3 + 19*a^2*b*c)*d^2*e^4 + (5*a^2*b^2 + 12*a^3*c)*d*e^5)*x^2 - 2*
sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^3 - 4*a^3*c^4)*d - (a^2*
b^3*c^2 - 4*a^3*b*c^3)*e)*x*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a
*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^
6)/(a^2*b^2*c^4 - 4*a^3*c^5)) - ((a*b^2*c^2 - 4*a^2*c^3)*d^3*e -
3*(a^2*b^2*c - 4*a^3*c^2)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x)*s
qrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*
c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2
+ 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b
^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3)))/x^2
) + 4*sqrt(-e)*e*arctan(e*x/(sqrt(e*x^2 + d)*sqrt(-e))))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex^2)^{\frac{3}{2}}}{a + bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^(3/2)/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

[Out] Timed out

3.366 $\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=260

$$-\frac{\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{3/2}}+\frac{\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{3/2}}-\frac{d\sqrt{d+ex^2}}{ax}$$

```
[Out] -((d*Sqrt[d + e*x^2])/(a*x)) - ((2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e)^(3/2)*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt
[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*(b
- Sqrt[b^2 - 4*a*c])^(3/2)) + ((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)^(3/2)*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[
b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*(b +
 Sqrt[b^2 - 4*a*c])^(3/2))
```

Rubi [A] time = 2.16023, antiderivative size = 432, normalized size of antiderivative = 1.66, number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276

$$-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{2a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{2a}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{2a}-\frac{d\sqrt{d+ex^2}}{ax}+\frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d*Sqrt[d + e*x^2])/(a*x)) - (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2])])/(2*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[
b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]
) + (d*Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/a - (Sqrt[e]
*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d
 + e*x^2]])/(2*a) - (Sqrt[e]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c]
)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*a)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**(3/2)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.736161, size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[(d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.043, size = 360, normalized size = 1.4

$$-\frac{1}{adx}\left(ex^2+d\right)^{\frac{5}{2}}+\frac{ex}{ad}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{5\,ex}{4\,a}\sqrt{ex^2+d}+\frac{3\,d}{2\,a}\sqrt{e}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)+\frac{x^2}{4\,a}e^{\frac{3}{2}}+\frac{d}{8\,a}\sqrt{e}$$
$$-\frac{1}{2\,a}\sqrt{e}\sum_{\_R=RootOf\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+(16\,ae^2-8\,bde+6\,cd^2)\_Z^2+(4\,bd^2e-4\,cd^3)\_Z+cd^4\right)}\frac{\left(ae^2-cd^2\right)\_R^2+2\,d\left(3\,ae^2-2\,bde+cd^2\right.}{\_R^3c+3\,\_R^2be-3\,\_R^2cd+8\,\_R\,ae^2-4\,\_R\,bde}$$
$$-\frac{d^2}{8\,a}\sqrt{e}\left(\sqrt{ex^2+d}-x\sqrt{e}\right)^{-2}+\frac{3\,d}{2\,a}\sqrt{e}\ln\left(\sqrt{ex^2+d}-x\sqrt{e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a/d/x*(e*x^2+d)^(5/2)+1/a/d*e*x*(e*x^2+d)^(3/2)+5/4/a*e*x*(e*x
^2+d)^(1/2)+3/2/a*d*e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))+1/4/a*e
^(3/2)*x^2+1/8/a*e^(1/2)*d-1/2/a*e^(1/2)*sum(((a*e^2-c*d^2)*_R^2+
2*d*(3*a*e^2-2*b*d*e+c*d^2)*_R+a*d^2*e^2-c*d^4)/(_R^3*c+3*_R^2*b*
e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln((
(e*x^2+d)^(1/2)-x*e^(1/2))^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_
Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4)
)-1/8/a*e^(1/2)*d^2/((e*x^2+d)^(1/2)-x*e^(1/2))^2+3/2/a*e^(1/2)*d
*ln((e*x^2+d)^(1/2)-x*e^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}}{(cx^4+bx^2+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [A] time = 12.3863, size = 5480, normalized size = 21.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a
*b*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(
-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*
d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2
)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(12*a^3*b*d^3*e
^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2*(a*b^3 + 2*a^2
*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 + ((a^3*b^2*c - 4*a^4
*c^2)*d^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d*e
^2)*x^2*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*
c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a
```

^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) + (27*a^3*b*d^2*e^4 - 12*a^4*
d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^4 + 6*a*b^2*c - 4*a^2*c^2)*d^5
e + 2(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*a^2*b^2 + 4*a^3*c)*d
^3*e^3)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 - 4*a^5*b*c)
d - 2(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d
^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^
5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) - ((a
*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d^3
e + 3(a^3*b^2 - 4*a^4*c)*d^2*e^2)*x)*sqrt(-(3*a^2*b*d*e^2 - 2*a
^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e + (a^3*b
^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*
a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2
 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/
x^2) - sqrt(1/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*
a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt
(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)
d^6 + 6(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^
2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(12*a^3*b*d^3*
e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2*(a*b^3 + 2*a^
2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 + ((a^3*b^2*c - 4*a^
4*c^2)*d^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d*
e^2)*x^2*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2
*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*
a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) + (27*a^3*b*d^2*e^4 - 12*a^4
*d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^4 + 6*a*b^2*c - 4*a^2*c^2)*d^
5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*a^2*b^2 + 4*a^3*c)*
d^3*e^3)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 - 4*a^5*b*c
)*d - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*
d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d
^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) - ((
a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d^
3*e + 3*(a^3*b^2 - 4*a^4*c)*d^2*e^2)*x)*sqrt(-(3*a^2*b*d*e^2 - 2*
a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e + (a^3*
b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2
*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^
2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))
/x^2) - sqrt(1/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3
*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e - (a^3*b^2 - 4*a^4*c)*sqr
t(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2
)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e
^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(12*a^3*b*d^3
*e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2*(a*b^3 + 2*a
^2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 - ((a^3*b^2*c - 4*a
^4*c^2)*d^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d
*e^2)*x^2*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^
2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2
*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) + (27*a^3*b*d^2*e^4 - 12*a^
4*d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^4 + 6*a*b^2*c - 4*a^2*c^2)*d
^5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*a^2*b^2 + 4*a^3*c)
*d^3*e^3)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 - 4*a^5*b*
c)*d - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4
*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*
d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) + (
(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d

```
^3*e + 3*(a^3*b^2 - 4*a^4*c)*d^2*e^2)*x)*sqrt(-(3*a^2*b*d*e^2 - 2
*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e - (a^3
*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 -
2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b
^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))
)/x^2) + sqrt(1/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 -
3*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e - (a^3*b^2 - 4*a^4*c)*sq
rt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^
2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*
e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(12*a^3*b*d^
3*e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2*(a*b^3 + 2*
a^2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 - ((a^3*b^2*c - 4*
a^4*c^2)*d^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*
d*e^2)*x^2*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b
^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 -
2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) + (27*a^3*b*d^2*e^4 - 12*a
^4*d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^4 + 6*a*b^2*c - 4*a^2*c^2)*
d^5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*a^2*b^2 + 4*a^3*c
)*d^3*e^3)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 - 4*a^5*b
*c)*d - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^
4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)
*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) +
((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*
d^3*e + 3*(a^3*b^2 - 4*a^4*c)*d^2*e^2)*x)*sqrt(-(3*a^2*b*d*e^2 -
2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e - (a^
3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 -
 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*
b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)
))/x^2) + 4*sqrt(e*x^2 + d)*d)/(a*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex^2\right)^{\frac{3}{2}}}{x^2 \left(a + bx^2 + cx^4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**(3/2)/x**2/(c*x**4+b*x**2+a),x)

[Out] Integral((d + e*x**2)**(3/2)/(x**2*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.367 $\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=523

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{2a^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{2a^2}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{2a^2}+\frac{\sqrt{d+ex^2}(bd-ae)}{a^2x}-\frac{\sqrt{e}(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a^2}-\frac{\left(d+ex^2\right)^{3/2}}{3ax^3}$$

```
[Out] ((b*d - a*e)*Sqrt[d + e*x^2])/(a^2*x) - (d + e*x^2)^(3/2)/(3*a*x^
3) + (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e + (b^2*d
 - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b -
Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*
x^2])])/(2*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)
/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a^2*Sqrt[b +
Sqrt[b^2 - 4*a*c]]) - (Sqrt[e]*(b*d - a*e)*ArcTanh[(Sqrt[e]*x)/Sq
rt[d + e*x^2]])/a^2 + (Sqrt[e]*(b*d - a*e - (b^2*d - 2*a*c*d - a*
b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*
a^2) + (Sqrt[e]*(b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 -
 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*a^2)
```

Rubi [A] time = 5.56902, antiderivative size = 523, normalized size of antiderivative = 1., number of

steps used = 19, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a^2\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{2a^2\sqrt{\sqrt{b^2-4ac}+b}}$$
$$+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{2a^2}$$
$$+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{2a^2}$$
$$+\frac{\sqrt{d+ex^2}(bd-ae)}{a^2x}-\frac{\sqrt{e}(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a^2}-\frac{\left(d+ex^2\right)^{3/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*d - a*e)*Sqrt[d + e*x^2])/(a^2*x) - (d + e*x^2)^(3/2)/(3*a*x^
3) + (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e + (b^2*d
 - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b -
Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*
x^2])])/(2*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)
/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a^2*Sqrt[b +
Sqrt[b^2 - 4*a*c]]) - (Sqrt[e]*(b*d - a*e)*ArcTanh[(Sqrt[e]*x)/Sq
rt[d + e*x^2]])/a^2 + (Sqrt[e]*(b*d - a*e - (b^2*d - 2*a*c*d - a*
b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*
a^2) + (Sqrt[e]*(b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 -
 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*a^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**(3/2)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.911807, size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[(d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.047, size = 511, normalized size = 1.

$$-\frac{1}{3\,adx^3}\left(ex^2+d\right)^{\frac{5}{2}}-\frac{2\,e}{3\,ad^2x}\left(ex^2+d\right)^{\frac{5}{2}}+\frac{2\,e^2x}{3\,ad^2}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{e^2x}{ad}\sqrt{ex^2+d}$$
$$+\frac{1}{a}e^{\frac{3}{2}}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)-\frac{bx^2}{4\,a^2}e^{\frac{3}{2}}-\frac{5\,exb}{4\,a^2}\sqrt{ex^2+d}-\frac{bd}{8\,a^2}\sqrt{e}$$
$$+\frac{1}{2\,a^2}\sqrt{e}\sum_{\_R=RootOf\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+(16\,ae^2-8\,bde+6\,cd^2)\_Z^2+(4\,bd^2e-4\,cd^3)\_Z+cd^4\right)}\frac{cd\,(2\,ae-bd)\,\_R^2+2\,\left(-2\,a^2e^3+4\,abde^2-2\,b\right.}{\_R^3c+3\,\_R^2be-3\,\_R^2cd+8\,\_Rae^2-4\,\_R}$$
$$+\frac{1}{a}e^{\frac{3}{2}}\ln\left(\sqrt{ex^2+d}-x\sqrt{e}\right)-\frac{3\,bd}{2\,a^2}\sqrt{e}\ln\left(\sqrt{ex^2+d}-x\sqrt{e}\right)+\frac{bd^2}{8\,a^2}\sqrt{e}\left(\sqrt{ex^2+d}-x\sqrt{e}\right)^{-2}$$
$$+\frac{b}{a^2dx}\left(ex^2+d\right)^{\frac{5}{2}}-\frac{exb}{a^2d}\left(ex^2+d\right)^{\frac{3}{2}}-\frac{3\,bd}{2\,a^2}\sqrt{e}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/3/a/d/x^3*(e*x^2+d)^(5/2)-2/3/a/d^2*e/x*(e*x^2+d)^(5/2)+2/3/a/
d^2*e^2*x*(e*x^2+d)^(3/2)+1/a/d*e^2*x*(e*x^2+d)^(1/2)+1/a*e^(3/2)
*ln(x*e^(1/2)+(e*x^2+d)^(1/2))-1/4/a^2*e^(3/2)*x^2*b-5/4/a^2*e*(e
*x^2+d)^(1/2)*x*b-1/8/a^2*e^(1/2)*b*d+1/2/a^2*e^(1/2)*sum((c*d*(2
*a*e-b*d)*_R^2+2*(-2*a^2*e^3+4*a*b*d*e^2-2*b^2*d^2*e+b*c*d^3)*_R+
2*a*c*d^3*e-c*d^4*b)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_
R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-x*e^(1/2))^
2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d
^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))+1/a*e^(3/2)*ln((e*x^2+d)^
(1/2)-x*e^(1/2))-3/2/a^2*e^(1/2)*ln((e*x^2+d)^(1/2)-x*e^(1/2))*b*
d+1/8/a^2*e^(1/2)*b*d^2/((e*x^2+d)^(1/2)-x*e^(1/2))^2+b/a^2/d/x*(
e*x^2+d)^(5/2)-b/a^2/d*e*x*(e*x^2+d)^(3/2)-3/2*b/a^2*d*e^(1/2)*ln
```

```
(x*e^(1/2)+(e*x^2+d)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}}{(cx^4+bx^2+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^4),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^4), x)
```

Fricas [A] time = 31.0104, size = 10571, normalized size = 20.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^4),x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(1/2)*a^2*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*
d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*
a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 + (a^5*b^2 - 4*a^6*c)*sq
rt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^
4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2
*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*
d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a
^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6
*c))*log((2*a^5*b*c*d*e^5 - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^
4)*d^6 + 2*(a*b^5*c - 5*a^3*b*c^3)*d^5*e - 4*(2*a^2*b^4*c - 3*a^3
*b^2*c^2 - a^4*c^3)*d^4*e^2 + 4*(3*a^3*b^3*c - 4*a^4*b*c^2)*d^3*e
^3 - 2*(4*a^4*b^2*c - 3*a^5*c^2)*d^2*e^4 + ((a^5*b^2*c^2 - 4*a^6*
c^3)*d^3 - (a^5*b^3*c - 4*a^6*b*c^2)*d^2*e + (a^6*b^2*c - 4*a^7*c
^2)*d*e^2)*x^2*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a
^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 2
0*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c
 + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^
2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c))
+ (4*a^5*b*c*e^6 + (b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^6 - (b^6
*c + 4*a*b^4*c^2 - 17*a^2*b^2*c^3 + 4*a^3*c^4)*d^5*e + 2*(4*a*b^5
*c - 3*a^2*b^3*c^2 - 11*a^3*b*c^3)*d^4*e^2 - 2*(11*a^2*b^4*c - 16
```

*a^3*b^2*c^2 - 4*a^4*c^3)*d^3*e^3 + 7*(4*a^3*b^3*c - 5*a^4*b*c^2)
*d^2*e^4 - (17*a^4*b^2*c - 12*a^5*c^2)*d*e^5)*x^2 + 2*sqrt(1/2)*s
qrt(e*x^2 + d)*(((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*d - (a^7*b^3
 - 4*a^8*b*c)*e)*x*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*
b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c +
 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c
 + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b
^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^
6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*
c)) - ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b*c^3)*d^4 -
 (4*a^2*b^6 - 25*a^3*b^4*c + 37*a^4*b^2*c^2 - 4*a^5*c^3)*d^3*e +
3*(2*a^3*b^5 - 11*a^4*b^3*c + 12*a^5*b*c^2)*d^2*e^2 - (4*a^4*b^4
- 19*a^5*b^2*c + 12*a^6*c^2)*d*e^3 + (a^5*b^3 - 4*a^6*b*c)*e^4)*x
)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b
^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^
2 - 2*a^4*c)*e^3 + (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a
*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^
2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(1
0*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 -
 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)
/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)))/x^2) - 3*sqrt(1/2)
*a^2*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 -
4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 -
(a^3*b^2 - 2*a^4*c)*e^3 + (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 +
 (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6
 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e +
3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2
 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^
4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)
*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log((2*a^5*b
*c*d*e^5 - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^6 + 2*(a*b^5
*c - 5*a^3*b*c^3)*d^5*e - 4*(2*a^2*b^4*c - 3*a^3*b^2*c^2 - a^4*c^
3)*d^4*e^2 + 4*(3*a^3*b^3*c - 4*a^4*b*c^2)*d^3*e^3 - 2*(4*a^4*b^2
*c - 3*a^5*c^2)*d^2*e^4 + ((a^5*b^2*c^2 - 4*a^6*c^3)*d^3 - (a^5*b
^3*c - 4*a^6*b*c^2)*d^2*e + (a^6*b^2*c - 4*a^7*c^2)*d*e^2)*x^2*sq
rt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^
4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2
*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*
d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a
^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)) + (4*a^5*b*c*e^6
+ (b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^6 - (b^6*c + 4*a*b^4*c^2
- 17*a^2*b^2*c^3 + 4*a^3*c^4)*d^5*e + 2*(4*a*b^5*c - 3*a^2*b^3*c^
2 - 11*a^3*b*c^3)*d^4*e^2 - 2*(11*a^2*b^4*c - 16*a^3*b^2*c^2 - 4*
a^4*c^3)*d^3*e^3 + 7*(4*a^3*b^3*c - 5*a^4*b*c^2)*d^2*e^4 - (17*a^
4*b^2*c - 12*a^5*c^2)*d*e^5)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((
(a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*d - (a^7*b^3 - 4*a^8*b*c)*e)*
x*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b
^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 -
2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2
 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c
^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 -
6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)) - ((a*b^7 - 7

```
*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b*c^3)*d^4 - (4*a^2*b^6 - 25*
a^3*b^4*c + 37*a^4*b^2*c^2 - 4*a^5*c^3)*d^3*e + 3*(2*a^3*b^5 - 11
*a^4*b^3*c + 12*a^5*b*c^2)*d^2*e^2 - (4*a^4*b^4 - 19*a^5*b^2*c +
12*a^6*c^2)*d*e^3 + (a^5*b^3 - 4*a^6*b*c)*e^4)*x)*sqrt(-((b^5 - 5
*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)
*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3
+ (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*
c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^
4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^
4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3
*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^
11*c)))/(a^5*b^2 - 4*a^6*c)))/x^2) - 3*sqrt(1/2)*a^2*x^3*sqrt(-((
b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a
^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*
c)*e^3 - (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a
^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20
*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5
- 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^
2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2
 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log((2*a^5*b*c*d*e^5 - 2*(a*b
^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^6 + 2*(a*b^5*c - 5*a^3*b*c^3)
*d^5*e - 4*(2*a^2*b^4*c - 3*a^3*b^2*c^2 - a^4*c^3)*d^4*e^2 + 4*(3
*a^3*b^3*c - 4*a^4*b*c^2)*d^3*e^3 - 2*(4*a^4*b^2*c - 3*a^5*c^2)*d
^2*e^4 - ((a^5*b^2*c^2 - 4*a^6*c^3)*d^3 - (a^5*b^3*c - 4*a^6*b*c^
2)*d^2*e + (a^6*b^2*c - 4*a^7*c^2)*d*e^2)*x^2*sqrt((a^6*b^2*e^6 +
 (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6
 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e +
3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2
 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^
4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)
*d*e^5)/(a^10*b^2 - 4*a^11*c)) + (4*a^5*b*c*e^6 + (b^5*c^2 - 3*a*
b^3*c^3 + a^2*b*c^4)*d^6 - (b^6*c + 4*a*b^4*c^2 - 17*a^2*b^2*c^3
+ 4*a^3*c^4)*d^5*e + 2*(4*a*b^5*c - 3*a^2*b^3*c^2 - 11*a^3*b*c^3)
*d^4*e^2 - 2*(11*a^2*b^4*c - 16*a^3*b^2*c^2 - 4*a^4*c^3)*d^3*e^3
+ 7*(4*a^3*b^3*c - 5*a^4*b*c^2)*d^2*e^4 - (17*a^4*b^2*c - 12*a^5*
c^2)*d*e^5)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^6*b^4 - 6*a^7*
b^2*c + 8*a^8*c^2)*d - (a^7*b^3 - 4*a^8*b*c)*e)*x*sqrt((a^6*b^2*e
^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*
e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4
*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(
5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*
b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)) + ((a*b^7 - 7*a^2*b^5*c + 13*a
^3*b^3*c^2 - 4*a^4*b*c^3)*d^4 - (4*a^2*b^6 - 25*a^3*b^4*c + 37*a^
4*b^2*c^2 - 4*a^5*c^3)*d^3*e + 3*(2*a^3*b^5 - 11*a^4*b^3*c + 12*a
^5*b*c^2)*d^2*e^2 - (4*a^4*b^4 - 19*a^5*b^2*c + 12*a^6*c^2)*d*e^3
 + (a^5*b^3 - 4*a^6*b*c)*e^4)*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*
b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b
^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 - (a^5*b^2 - 4*a^
6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2
 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*
```

```
c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*
b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4
 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2
- 4*a^6*c)))/x^2) + 3*sqrt(1/2)*a^2*x^3*sqrt(-((b^5 - 5*a*b^3*c +
 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3
*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 - (a^5*b^2
 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*
b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a
^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c +
19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*
d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a
^5*b^2 - 4*a^6*c))*log((2*a^5*b*c*d*e^5 - 2*(a*b^4*c^2 - 3*a^2*b^
2*c^3 + a^3*c^4)*d^6 + 2*(a*b^5*c - 5*a^3*b*c^3)*d^5*e - 4*(2*a^2
*b^4*c - 3*a^3*b^2*c^2 - a^4*c^3)*d^4*e^2 + 4*(3*a^3*b^3*c - 4*a^
4*b*c^2)*d^3*e^3 - 2*(4*a^4*b^2*c - 3*a^5*c^2)*d^2*e^4 - ((a^5*b^
2*c^2 - 4*a^6*c^3)*d^3 - (a^5*b^3*c - 4*a^6*b*c^2)*d^2*e + (a^6*b
^2*c - 4*a^7*c^2)*d*e^2)*x^2*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a
^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20
*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5
- 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^
2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2
 - 4*a^11*c)) + (4*a^5*b*c*e^6 + (b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c
^4)*d^6 - (b^6*c + 4*a*b^4*c^2 - 17*a^2*b^2*c^3 + 4*a^3*c^4)*d^5*
e + 2*(4*a*b^5*c - 3*a^2*b^3*c^2 - 11*a^3*b*c^3)*d^4*e^2 - 2*(11*
a^2*b^4*c - 16*a^3*b^2*c^2 - 4*a^4*c^3)*d^3*e^3 + 7*(4*a^3*b^3*c
- 5*a^4*b*c^2)*d^2*e^4 - (17*a^4*b^2*c - 12*a^5*c^2)*d*e^5)*x^2 -
 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2
)*d - (a^7*b^3 - 4*a^8*b*c)*e)*x*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 -
 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6
- 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*
b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^
5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10
*b^2 - 4*a^11*c)) + ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^
4*b*c^3)*d^4 - (4*a^2*b^6 - 25*a^3*b^4*c + 37*a^4*b^2*c^2 - 4*a^5
*c^3)*d^3*e + 3*(2*a^3*b^5 - 11*a^4*b^3*c + 12*a^5*b*c^2)*d^2*e^2
 - (4*a^4*b^4 - 19*a^5*b^2*c + 12*a^6*c^2)*d*e^3 + (a^5*b^3 - 4*a
^6*b*c)*e^4)*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a
*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d
*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 - (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^
2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c
^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d
^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*
d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 +
3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a
^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)))/x^2)
 + 4*((3*b*d - 4*a*e)*x^2 - a*d)*sqrt(e*x^2 + d))/(a^2*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**(3/2)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.368 $\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=281

$$\frac{\left(-\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{5/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{b\sqrt{1-x^2}}{c^2}-\frac{\left(1-x^2\right)^{3/2}}{3c}$$

```
[Out] -((b*Sqrt[1 - x^2])/c^2) - (1 - x^2)^(3/2)/(3*c) + ((b^2 - a*c +
b*c - (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + ((b^2
- a*c + b*c + (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c]
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 14.2304, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172

$$\frac{\left(-\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{5/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{b\sqrt{1-x^2}}{c^2}-\frac{\left(1-x^2\right)^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```


```
[Out] -((b*Sqrt[1 - x^2])/c^2) - (1 - x^2)^(3/2)/(3*c) + ((b^2 - a*c +
b*c - (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + ((b^2
- a*c + b*c + (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c]
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.786754, size = 354, normalized size = 1.26

$$-\frac{\frac{3\sqrt{2}\left(b^2\left(\sqrt{b^2-4ac}+c\right)+bc\left(\sqrt{b^2-4ac}-3a\right)-ac\left(\sqrt{b^2-4ac}+2c\right)+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}-b-2c}}-\frac{3\sqrt{2}\left(b^2\left(\sqrt{b^2-4ac}-c\right)+bc\left(\sqrt{b^2-4ac}+3a\right)+ac\left(2c-\sqrt{b^2-4ac}\cdots\right.\right.}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}-b-2c}}\cdots}{6c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-6*b*Sqrt[c]*Sqrt[1 - x^2] - 2*c^(3/2)*(1 - x^2)^(3/2) - (3*Sqrt
[2]*(b^3 + b*c*(-3*a + Sqrt[b^2 - 4*a*c]) + b^2*(c + Sqrt[b^2 - 4
*a*c]) - a*c*(2*c + Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*S
qrt[1 - x^2])/Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*
a*c]*Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*(-b^3 + a*c
*(2*c - Sqrt[b^2 - 4*a*c]) + b*c*(3*a + Sqrt[b^2 - 4*a*c]) + b^2*
(-c + Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/
Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b -
 2*c + Sqrt[b^2 - 4*a*c]]))/(6*c^(5/2))
```

Maple [B] time = 0.119, size = 2134, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/3*(-x^2+1)^(3/2)/c+4/c*a^2/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*
c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*
((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2
```

```
*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*
(-4*a*c+b^2)^(1/2)-2/c^2*a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b
^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-
x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^
2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2
*(-4*a*c+b^2)^(1/2)-2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^
2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x
^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2
-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*
a*c+b^2)^(1/2)*b-8/c*a^2/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2
)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^
2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-
2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b-8*a
^2/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+
b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+
2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a
-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))+2/c^2*a/(8*a*c-2*b^2)/(4*a*
c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2
)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2
*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2
)-2*a*b)^(1/2))*b^3+2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^
2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x
^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2
-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2+
4/c*a^2/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4
*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/
x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(
1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)-2/
c^2*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a
*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^
2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/
2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2*(-4*a*c+b^2)^(1/2)-
2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a
*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^
2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/
2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)*b+8/
c*a^2/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a
*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^
2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/
2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b+8*a^2/(8*a*c-2*b^2)/(
4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^
(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1
/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)
^(1/2)-2*a*b)^(1/2))-2/c^2*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c
+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*
((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2
*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*
b^3-2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(
-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^
2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)
^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2-2/c^2*b/(2/x^2-
2/x^2*(-x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}x^5}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)*x^5/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^5/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 5.48239, size = 5154, normalized size = 18.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)*x^5/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c*x^6 - 6*(b + 2*c)*x^4 + 12*(b + c)*x^2 - 3*sqrt(1/2)*(3*
c^2*x^2 - 4*c^2 - (c^2*x^2 - 4*c^2)*sqrt(-x^2 + 1))*sqrt((b^5 + 2
*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c - (b^2*c^5
 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^
3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*
c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a
*c^6))*log(-(2*a^3*b^4 + (a^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8
+ (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*
a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7
)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^5 - 2*a^4*b)*c^2 + (a^2*b^5 +
(a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^4)*c)*x^2 - 2*(3*a^4
*b^2 - a^3*b^3)*c + sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*
c^7)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b
^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2
 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + (b^8 + 4*(a^4 -
2*a^3*b)*c^4 - (17*a^3*b^2 - 14*a^2*b^3)*c^3 + (20*a^2*b^4 - 7*a*
b^5)*c^2 - (8*a*b^6 - b^7)*c)*x^2)*sqrt((b^5 + 2*a^2*c^3 + (5*a^2
*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt(
(b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3
 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6
- b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b
^4 + (a^5 - 2*a^4*b)*c^2 - (3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2 + 1
))/x^2) + 3*sqrt(1/2)*(3*c^2*x^2 - 4*c^2 - (c^2*x^2 - 4*c^2)*sqrt
(-x^2 + 1))*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*
a*b^3 - b^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b +
 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a
```

```
^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4
*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*a^3*b^4 + (a^2*b^2*c^5 -
4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3
*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^
6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^5 - 2
*a^4*b)*c^2 + (a^2*b^5 + (a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a
^2*b^4)*c)*x^2 - 2*(3*a^4*b^2 - a^3*b^3)*c - sqrt(1/2)*((b^5*c^5
- 7*a*b^3*c^6 + 12*a^2*b*c^7)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*
a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*
b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*
c^11)) + (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a^3*b^2 - 14*a^2*b^3)
*c^3 + (20*a^2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*x^2)*sqrt(
(b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c -
(b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 -
 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5
 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c
^5 - 4*a*c^6)) - 2*(a^3*b^4 + (a^5 - 2*a^4*b)*c^2 - (3*a^4*b^2 -
a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) + 3*sqrt(1/2)*(3*c^2*x^2 - 4*c^2
 - (c^2*x^2 - 4*c^2)*sqrt(-x^2 + 1))*sqrt((b^5 + 2*a^2*c^3 + (5*a
^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*c^5 - 4*a*c^6)*sqr
t((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b
^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^
6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*
a^3*b^4 - (a^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*
b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (1
1*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10
- 4*a*c^11)) + 2*(a^5 - 2*a^4*b)*c^2 + (a^2*b^5 + (a^4*b - 2*a^3*
b^2)*c^2 - (3*a^3*b^3 - a^2*b^4)*c)*x^2 - 2*(3*a^4*b^2 - a^3*b^3)
*c + sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*x^2*sqrt((
b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3
+ 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 -
 b^7)*c)/(b^2*c^10 - 4*a*c^11)) - (b^8 + 4*(a^4 - 2*a^3*b)*c^4 -
(17*a^3*b^2 - 14*a^2*b^3)*c^3 + (20*a^2*b^4 - 7*a*b^5)*c^2 - (8*a
*b^6 - b^7)*c)*x^2)*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c
^2 - (5*a*b^3 - b^4)*c + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4
*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3
 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*
c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 + (a^5 - 2*a
^4*b)*c^2 - (3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) - 3*sqr
t(1/2)*(3*c^2*x^2 - 4*c^2 - (c^2*x^2 - 4*c^2)*sqrt(-x^2 + 1))*sqr
t((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c
+ (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4
 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b
^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2
*c^5 - 4*a*c^6))*log(-(2*a^3*b^4 - (a^2*b^2*c^5 - 4*a^3*c^6)*x^2*
sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^
2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a
*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^5 - 2*a^4*b)*c^2 + (
a^2*b^5 + (a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^4)*c)*x^2
- 2*(3*a^4*b^2 - a^3*b^3)*c - sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 +
 12*a^2*b*c^7)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 -
2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5
+ b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) - (b^8 +
 4*(a^4 - 2*a^3*b)*c^4 - (17*a^3*b^2 - 14*a^2*b^3)*c^3 + (20*a^2*
```

```
b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*x^2)*sqrt((b^5 + 2*a^2*c^
3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*c^5 - 4*a*
c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 -
 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2
*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))
- 2*(a^3*b^4 + (a^5 - 2*a^4*b)*c^2 - (3*a^4*b^2 - a^3*b^3)*c)*sqr
t(-x^2 + 1))/x^2) + 6*(c*x^4 - 2*(b + c)*x^2)*sqrt(-x^2 + 1))/(3*
c^2*x^2 - 4*c^2 - (c^2*x^2 - 4*c^2)*sqrt(-x^2 + 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)*x^5/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.369 $\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=229

$$-\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{\sqrt{1-x^2}}{c}$$

```
[Out] Sqrt[1 - x^2]/c - ((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c]
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]])
- ((b + c + (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[
2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*c^(3/2)*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 3.55968, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172

$$-\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{\sqrt{1-x^2}}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```


```
[Out] Sqrt[1 - x^2]/c - ((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c]
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]])
- ((b + c + (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[
2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*c^(3/2)*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 152.828, size = 243, normalized size = 1.06

$$\frac{\sqrt{-x^2+1}}{c}+\frac{\sqrt{2}\left(2c(a+b+c)-(b+c)(b+2c)-(b+c)\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{-x^2+1}}{\sqrt{b+2c+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{-4ac+b^2}\sqrt{b+2c+\sqrt{-4ac+b^2}}}-\frac{\sqrt{2}\left(2c(a+b+c)-(b+c)(b+2c)+(b+c)\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{-x^2+1}}{\sqrt{b+2c-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{-4ac+b^2}\sqrt{b+2c-\sqrt{-4ac+b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] sqrt(-x**2 + 1)/c + sqrt(2)*(2*c*(a + b + c) - (b + c)*(b + 2*c)
- (b + c)*sqrt(-4*a*c + b**2))*atanh(sqrt(2)*sqrt(c)*sqrt(-x**2 +
 1)/sqrt(b + 2*c + sqrt(-4*a*c + b**2)))/(2*c**(3/2)*sqrt(-4*a*c
+ b**2)*sqrt(b + 2*c + sqrt(-4*a*c + b**2))) - sqrt(2)*(2*c*(a +
b + c) - (b + c)*(b + 2*c) + (b + c)*sqrt(-4*a*c + b**2))*atanh(s
qrt(2)*sqrt(c)*sqrt(-x**2 + 1)/sqrt(b + 2*c - sqrt(-4*a*c + b**2)
))/(2*c**(3/2)*sqrt(-4*a*c + b**2)*sqrt(b + 2*c - sqrt(-4*a*c + b
**2)))
```

Mathematica [A] time = 0.482476, size = 279, normalized size = 1.22

$$-\frac{\left(b\sqrt{b^2-4ac}+c\sqrt{b^2-4ac}+2ac-b^2-bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(b\sqrt{b^2-4ac}+c\sqrt{b^2-4ac}-2ac+b^2+bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{\sqrt{1-x^2}}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Sqrt[1 - x^2]/c - ((-b^2 + 2*a*c - b*c + b*Sqrt[b^2 - 4*a*c] + c*
Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b
 + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*
Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*c + b*Sqrt
[b^2 - 4*a*c] + c*Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqr
t[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*
Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Maple [B] time = 0.063, size = 1223, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a
*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2
*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2
)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)*b+4*a
/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^
2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*
(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2
*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)+8*a^2/(8*a
*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1
/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a
*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-
4*a*c+b^2)^(1/2)-2*a*b)^(1/2))-2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2
*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(
1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(
4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^
(1/2))*b^2+2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*
a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1
/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a
*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)
^(1/2)*b+4*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*
b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-
1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b
^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/
2)-8*a^2/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-
4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2
/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^
(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))+2/c*a/(8*a*c-2*b^2)/
(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)
^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(
1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2
)^(1/2)-2*a*b)^(1/2))*b^2+2/c/(1/x^2*(-x^2+1)-2/x^2*(-x^2+1)^(1/2
)+1/x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}\,x^3}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)*x^3/(c*x^4 + b*x^2 + a),x, algorithm="maxima")

[Out] integrate(sqrt(-x^2 + 1)*x^3/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 2.02675, size = 2858, normalized size = 12.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)*x^3/(c*x^4 + b*x^2 + a),x, algorithm="fricas")

[Out] 1/2*(sqrt(1/2)*(sqrt(-x^2 + 1)*c - c)*sqrt((b^3 - 2*a*c^2 - (3*a*
b - b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*
c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4
))*log((2*a^2*b^2 + (a*b^2*c^3 - 4*a^2*c^4)*x^2*sqrt((b^4 + (a^2
- 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (a
*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c + sqrt(1/2)*((b
^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b +
b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (b^5 + 4*(a^
2*b - a*b^2)*c^2 - (5*a*b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 -
(3*a*b - b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b +
b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*
a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2 + 1))/x^2) - sq
rt(1/2)*(sqrt(-x^2 + 1)*c - c)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2
)*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2
*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(
(2*a^2*b^2 + (a*b^2*c^3 - 4*a^2*c^4)*x^2*sqrt((b^4 + (a^2 - 2*a*b
 + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (a*b^3 -
(a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c - sqrt(1/2)*((b^4*c^3
- 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^
2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (b^5 + 4*(a^2*b - a
*b^2)*c^2 - (5*a*b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b
- b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^
2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))
 - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)
*(sqrt(-x^2 + 1)*c - c)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (
b^2*c^3 - 4*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2
 - b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*
b^2 - (a*b^2*c^3 - 4*a^2*c^4)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)
*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (a*b^3 - (a^2*b
- a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c + sqrt(1/2)*((b^4*c^3 - 6*a*b
^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(
a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) - (b^5 + 4*(a^2*b - a*b^2)*c
^2 - (5*a*b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*
c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(
a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a
^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*(sqrt(
-x^2 + 1)*c - c)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3
```

```
 - 4*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)
*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 - (
a*b^2*c^3 - 4*a^2*c^4)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 -
2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (a*b^3 - (a^2*b - a*b^2
)*c)*x^2 - 2*(a^3 - a^2*b)*c - sqrt(1/2)*((b^4*c^3 - 6*a*b^2*c^4
+ 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 -
 b^3)*c)/(b^2*c^6 - 4*a*c^7)) - (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5
*a*b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^
2*c^3 - 4*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 -
 b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2
- (a^3 - a^2*b)*c)*sqrt(-x^2 + 1))/x^2) - 2*x^2)/(sqrt(-x^2 + 1)*
c - c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**3*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)*x^3/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.370 $\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=182

$$\frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -((Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqr
t[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*
Sqrt[b^2 - 4*a*c])) + (Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*ArcTanh[
(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.619204, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148

$$\frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqr
t[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*
Sqrt[b^2 - 4*a*c])) + (Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*ArcTanh[
(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rubi in Sympy [A] time = 57.6882, size = 168, normalized size = 0.92

$$-\frac{\sqrt{2}\sqrt{b+2c-\sqrt{-4ac+b^2}}\,\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{-x^2+1}}{\sqrt{b+2c-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\sqrt{b+2c+\sqrt{-4ac+b^2}}\,\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{-x^2+1}}{\sqrt{b+2c+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] -sqrt(2)*sqrt(b + 2*c - sqrt(-4*a*c + b**2))*atanh(sqrt(2)*sqrt(c
)*sqrt(-x**2 + 1)/sqrt(b + 2*c - sqrt(-4*a*c + b**2)))/(2*sqrt(c)
*sqrt(-4*a*c + b**2)) + sqrt(2)*sqrt(b + 2*c + sqrt(-4*a*c + b**2
))*atanh(sqrt(2)*sqrt(c)*sqrt(-x**2 + 1)/sqrt(b + 2*c + sqrt(-4*a
*c + b**2)))/(2*sqrt(c)*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.260093, size = 169, normalized size = 0.93

$$\frac{\sqrt{-\sqrt{b^2-4ac}-b-2c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}\right)-\sqrt{\sqrt{b^2-4ac}-b-2c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[
1 - x^2])/Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]] - Sqrt[-b - 2*c + S
qrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b
- 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.052, size = 1167, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -2*a/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+
b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+
2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a
-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)-1/(4*a*c
-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-
2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b
^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*
c+b^2)^(1/2)-2*a*b)^(1/2))*b*(-4*a*c+b^2)^(1/2)+4*a/(4*a*c-b^2)/(
4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^
(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/
2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^
(1/2)-2*a*b)^(1/2))*c-1/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(
1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1
)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(
-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2-2*a/
```

```
(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^
(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-
4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b
*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)-1/(4*a*c-b^2
)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*
b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)
^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b
^2)^(1/2)-2*a*b)^(1/2))*b*(-4*a*c+b^2)^(1/2)-4*a/(4*a*c-b^2)/(4*a
*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/
2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)
-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1
/2)-2*a*b)^(1/2))*c+1/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/
2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)
^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-
4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}x}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)*x/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 0.937263, size = 1176, normalized size = 6.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)*x/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt((b + 2*c - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4
*a*c^3))/(b^2*c - 4*a*c^2))*log((b*x^2 + (b^2*c - 4*a*c^2)*x^2/sq
rt(b^2*c^2 - 4*a*c^3) + sqrt(1/2)*((b^2 - 4*a*c)*x^2 + (b^3*c - 4
*a*b*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3))*sqrt((b + 2*c - (b^2*c - 4
*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2)) - 2*sqrt(-x^2
 + 1)*a + 2*a)/x^2) + 1/2*sqrt(1/2)*sqrt((b + 2*c - (b^2*c - 4*a*
c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log((b*x^2 + (b^
2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3) - sqrt(1/2)*((b^2 - 4*
a*c)*x^2 + (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3))*sqrt(
(b + 2*c - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*
```

```
a*c^2)) - 2*sqrt(-x^2 + 1)*a + 2*a)/x^2) - 1/2*sqrt(1/2)*sqrt((b
+ 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c
^2))*log((b*x^2 - (b^2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3) +
 sqrt(1/2)*((b^2 - 4*a*c)*x^2 - (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*
c^2 - 4*a*c^3))*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 -
4*a*c^3))/(b^2*c - 4*a*c^2)) - 2*sqrt(-x^2 + 1)*a + 2*a)/x^2) + 1
/2*sqrt(1/2)*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a
*c^3))/(b^2*c - 4*a*c^2))*log((b*x^2 - (b^2*c - 4*a*c^2)*x^2/sqrt
(b^2*c^2 - 4*a*c^3) - sqrt(1/2)*((b^2 - 4*a*c)*x^2 - (b^3*c - 4*a
*b*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3))*sqrt((b + 2*c + (b^2*c - 4*a
*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2)) - 2*sqrt(-x^2 +
 1)*a + 2*a)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)

[Out] Integral(x*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)*x/(c*x^4 + b*x^2 + a),x, algorithm="giac")

[Out] Timed out
```

3.371 $\int \frac{\sqrt{1-x^2}}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=241

$$\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b+2c}} - \frac{\sqrt{c}\left(-\sqrt{b^2-4ac}+2a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b+2c}} - \frac{\tanh^{-1}\left(\sqrt{1-x^2}\right)}{a}$$

```
[Out] -(ArcTanh[Sqrt[1 - x^2]]/a) + (Sqrt[c]*(2*a + b + Sqrt[b^2 - 4*a*
c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c - Sqrt[b
^2 - 4*a*c]]) - (Sqrt[c]*(2*a + b - Sqrt[b^2 - 4*a*c])*ArcTanh[(S
qrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 3.56932, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$

$$\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b+2c}} - \frac{\sqrt{c}\left(-\sqrt{b^2-4ac}+2a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b+2c}} - \frac{\tanh^{-1}\left(\sqrt{1-x^2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(ArcTanh[Sqrt[1 - x^2]]/a) + (Sqrt[c]*(2*a + b + Sqrt[b^2 - 4*a*
c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c - Sqrt[b
^2 - 4*a*c]]) - (Sqrt[c]*(2*a + b - Sqrt[b^2 - 4*a*c])*ArcTanh[(S
qrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 143.468, size = 218, normalized size = 0.9

$$-\frac{\sqrt{2}\sqrt{c}\left(2a+b-\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{-x^2+1}}{\sqrt{b+2c+\sqrt{-4ac+b^2}}}\right)}{2a\sqrt{-4ac+b^2}\sqrt{b+2c+\sqrt{-4ac+b^2}}}+\frac{\sqrt{2}\sqrt{c}\left(2a+b+\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{-x^2+1}}{\sqrt{b+2c-\sqrt{-4ac+b^2}}}\right)}{2a\sqrt{-4ac+b^2}\sqrt{b+2c-\sqrt{-4ac+b^2}}}-\frac{\operatorname{atanh}\left(\sqrt{-x^2+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)**(1/2)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] -sqrt(2)*sqrt(c)*(2*a + b - sqrt(-4*a*c + b**2))*atanh(sqrt(2)*sq
rt(c)*sqrt(-x**2 + 1)/sqrt(b + 2*c + sqrt(-4*a*c + b**2)))/(2*a*s
qrt(-4*a*c + b**2)*sqrt(b + 2*c + sqrt(-4*a*c + b**2))) + sqrt(2)
*sqrt(c)*(2*a + b + sqrt(-4*a*c + b**2))*atanh(sqrt(2)*sqrt(c)*sq
rt(-x**2 + 1)/sqrt(b + 2*c - sqrt(-4*a*c + b**2)))/(2*a*sqrt(-4*a
*c + b**2)*sqrt(b + 2*c - sqrt(-4*a*c + b**2))) - atanh(sqrt(-x**
2 + 1))/a
```

Mathematica [A] time = 0.675903, size = 234, normalized size = 0.97

$$-\frac{\sqrt{c}\left(\frac{\left(\sqrt{b^2-4ac}-2a-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}+\frac{\left(\sqrt{b^2-4ac}+2a+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}}-\frac{\log\left(\sqrt{1-x^2}+1\right)}{a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(((-2*a - b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]])/Sqrt[-b -
2*c - Sqrt[b^2 - 4*a*c]] + ((2*a + b + Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]
]])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]))/(Sqrt[2]*a*Sqrt[b^2 - 4*
a*c])) + Log[x]/a - Log[1 + Sqrt[1 - x^2]]/a
```

Maple [B] time = 0.071, size = 2099, normalized size = 8.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/a*(-x^2+1)^(1/2)-1/a*arctanh(1/(-x^2+1)^(1/2))+1/(4*a*c-b^2)/(4
*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(
1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2
)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(
1/2)-2*a*b)^(1/2))*b*(-4*a*c+b^2)^(1/2)-2/(4*a*c-b^2)/(4*a*c-2*b^
2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arct
an(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b
)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*
b)^(1/2))*c*(-4*a*c+b^2)^(1/2)+1/a/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4
*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*
(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*
c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2
))*b^2*(-4*a*c+b^2)^(1/2)-4*a/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+
b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((
-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b
^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*c+
1/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2
)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(
-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*
b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2-4/(4*a*c-b^2)/(4*a*c-2*b^2
-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arcta
n(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)
/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b
)^(1/2))*b*c+1/a/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-
2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)
-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+
b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^3+1/(4*a*c-b^
2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a
*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2
)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+
b^2)^(1/2)-2*a*b)^(1/2))*b*(-4*a*c+b^2)^(1/2)-2/(4*a*c-b^2)/(4*a*
c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2
)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-
2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/
2)-2*a*b)^(1/2))*c*(-4*a*c+b^2)^(1/2)+1/a/(4*a*c-b^2)/(4*a*c-2*b^
2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arct
an(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*
b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a
*b)^(1/2))*b^2*(-4*a*c+b^2)^(1/2)+4*a/(4*a*c-b^2)/(4*a*c-2*b^2+2*
(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1
/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(
4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^
(1/2))*c-1/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-
4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2
```

```
/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^
(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2+4/(4*a*c-b^2)/(4
*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(
1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/
2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^
(1/2)-2*a*b)^(1/2))*b*c-1/a/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^
2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-
x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^
2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^3
-2/a/(2/x^2-2/x^2*(-x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{(cx^4+bx^2+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x), x)
```

Fricas [A] time = 3.94833, size = 1663, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sq
rt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))
*log((2*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2
)/(a^4*b^2 - 4*a^5*c))*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3
*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a
^3*c)) + (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^
2 - 4*a^5*c)) + (a*b + b^2)*x^2 + 2*a^2 + 2*a*b - 2*(a^2 + a*b)*s
qrt(-x^2 + 1))/x^2) - sqrt(1/2)*a*sqrt((a*b + b^2 - 2*a*c + (a^2*
b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^
2*b^2 - 4*a^3*c))*log(-(2*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*x^2*sqrt(
(a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a*b + b^2 - 2*a*c
+ (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c
)))/(a^2*b^2 - 4*a^3*c)) - (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*
a*b + b^2)/(a^4*b^2 - 4*a^5*c)) - (a*b + b^2)*x^2 - 2*a^2 - 2*a*b
 + 2*(a^2 + a*b)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*a*sqrt((a*b + b
```

```
^2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^
2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*sqrt(1/2)*(a^3*b^2 -
4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((
a*b + b^2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/
(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) + (a^2*b^2 - 4*a^3*c)*
x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)) - (a*b + b^2)*x
^2 - 2*a^2 - 2*a*b + 2*(a^2 + a*b)*sqrt(-x^2 + 1))/x^2) - sqrt(1/
2)*a*sqrt((a*b + b^2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*
a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log((2*sqrt
(1/2)*(a^3*b^2 - 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 -
 4*a^5*c))*sqrt((a*b + b^2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^
2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - (a^
2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)
) + (a*b + b^2)*x^2 + 2*a^2 + 2*a*b - 2*(a^2 + a*b)*sqrt(-x^2 + 1
))/x^2) + 2*log((sqrt(-x^2 + 1) - 1)/x))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}}{x(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**(1/2)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] Timed out
```

3.372 $\int \frac{\sqrt{1-x^2}}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=290

$$-\frac{\sqrt{c}\left(\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a^2\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{c}\left(-\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{(a+2b)\tanh^{-1}\left(\sqrt{1-x^2}\right)}{2a^2}-\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(\sqrt{1-x^2}+1\right)}$$

```
[Out] -1/(4*a*(1 - Sqrt[1 - x^2])) + 1/(4*a*(1 + Sqrt[1 - x^2])) + ((a
+ 2*b)*ArcTanh[Sqrt[1 - x^2]])/(2*a^2) - (Sqrt[c]*(a + b + (b^2 +
 a*(b - 2*c))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1
- x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b +
 2*c - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(a + b - (b^2 + a*(b - 2*c)
)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt
[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + 2*c + Sqrt[
b^2 - 4*a*c]])
```

Rubi [A] time = 5.23677, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$

$$-\frac{\sqrt{c}\left(\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a^2\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{c}\left(-\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{(a+2b)\tanh^{-1}\left(\sqrt{1-x^2}\right)}{2a^2}-\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(\sqrt{1-x^2}+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(4*a*(1 - Sqrt[1 - x^2])) + 1/(4*a*(1 + Sqrt[1 - x^2])) + ((a
+ 2*b)*ArcTanh[Sqrt[1 - x^2]])/(2*a^2) - (Sqrt[c]*(a + b + (b^2 +
```

```
 a*(b - 2*c))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1
- x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b +
 2*c - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(a + b - (b^2 + a*(b - 2*c)
)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt
[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + 2*c + Sqrt[
b^2 - 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)**(1/2)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.00805, size = 298, normalized size = 1.03

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(\frac{\left(b\left(\sqrt{b^2-4ac}-b\right)+a\left(\sqrt{b^2-4ac}-b+2c\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}+\frac{\left(b\left(\sqrt{b^2-4ac}+b\right)+a\left(\sqrt{b^2-4ac}+b-2c\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{b^2-4ac}}+(a+2b)\log\left(\sqrt{1-x^2}+1\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((a*Sqrt[1 - x^2])/x^2) + (Sqrt[2]*Sqrt[c]*(((b*(-b + Sqrt[b^2
- 4*a*c]) + a*(-b + 2*c + Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqr
t[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]])/Sqrt[-b
- 2*c - Sqrt[b^2 - 4*a*c]] + ((b*(b + Sqrt[b^2 - 4*a*c]) + a*(b -
 2*c + Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])
/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/Sqrt[-b - 2*c + Sqrt[b^2 -
4*a*c]]))/Sqrt[b^2 - 4*a*c] - (a + 2*b)*Log[x] + (a + 2*b)*Log[1
+ Sqrt[1 - x^2]])/(2*a^2)
```

Maple [B] time = 0.087, size = 2770, normalized size = 9.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/a/x^2*(-x^2+1)^(3/2)-1/2/a*(-x^2+1)^(1/2)+1/2/a*arctanh(1/(-
x^2+1)^(1/2))+2/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2
*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-
1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b
^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*c*(-4*a*c+b^2)^(
1/2)-1/a/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*
a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^
2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/
2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2*(-4*a*c+b^2)^(1/2)+
3/a/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b
^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2
*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-
2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)*b*c-1/a^2
/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)
^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-
4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b
*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)*b^3+4/(4*a*c
-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-
2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b
^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*
c+b^2)^(1/2)-2*a*b)^(1/2))*b*c-4/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a
*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2
*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-
2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))
*c^2-1/a/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*
a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^
2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/
2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^3+5/a/(4*a*c-b^2)/(4*
a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1
/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)
+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1
/2)-2*a*b)^(1/2))*b^2*c-1/a^2/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+
b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((
-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b
^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^
4+2/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b
^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+
2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a
+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*c*(-4*a*c+b^2)^(1/2)-1/a/(4
*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1
/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*
a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(
-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2*(-4*a*c+b^2)^(1/2)+3/a/(4*a*c
-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-
2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+
b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a
*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)*b*c-1/a^2/(4*a*c-b
^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*
a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^
```

```
2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c
+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)*b^3-4/(4*a*c-b^2)/(4
*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(
1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/
2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^
(1/2)-2*a*b)^(1/2))*b*c+4/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)
^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^
2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+
2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*c^2+1
/a/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^
2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2
*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+
2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^3-5/a/(4*a*c-b^2)/(4*a*c-2
*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*a
rctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a
-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-
2*a*b)^(1/2))*b^2*c+1/a^2/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)
^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^
2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+
2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^4+2
/a^2*b/(2/x^2-2/x^2*(-x^2+1)^(1/2))-b/a^2*(-x^2+1)^(1/2)+b/a^2*ar
ctanh(1/(-x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{(cx^4+bx^2+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^3), x)
```

Fricas [A] time = 11.3494, size = 4031, normalized size = 13.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*x^2 - sqrt(1/2)*(a^2*x^4 + 2*sqrt(-x^2 + 1)*a^2*x^2 - 2*
a^2*x^2)*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c -
(a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*
```

```
b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^
2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(((a^4*b^2*c - 4*a^5*c^2)*
x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c
^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) +
2*(a^3 + 2*a^2*b)*c^2 + ((a^2*b + 2*a*b^2)*c^2 - (a*b^3 + b^4)*c)
*x^2 - 2*(a^2*b^2 + a*b^3)*c + sqrt(1/2)*((a^5*b^3 - 4*a^6*b*c)*x
^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^
2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) + (
a^2*b^4 + a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)
*c)*x^2)*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c -
(a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*
b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^
2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 2*((a^3 + 2*a^2*b)*c^2 - (a
^2*b^2 + a*b^3)*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*(a^2*x^4 + 2*
sqrt(-x^2 + 1)*a^2*x^2 - 2*a^2*x^2)*sqrt((a*b^3 + b^4 + 2*a^2*c^2
 - (3*a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*
a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^
2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*lo
g(((a^4*b^2*c - 4*a^5*c^2)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a
^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)
*c)/(a^8*b^2 - 4*a^9*c)) + 2*(a^3 + 2*a^2*b)*c^2 + ((a^2*b + 2*a*
b^2)*c^2 - (a*b^3 + b^4)*c)*x^2 - 2*(a^2*b^2 + a*b^3)*c - sqrt(1/
2)*((a^5*b^3 - 4*a^6*b*c)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^
4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*
c)/(a^8*b^2 - 4*a^9*c)) + (a^2*b^4 + a*b^5 + 4*(a^4 + 2*a^3*b)*c^
2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*sqrt((a*b^3 + b^4 + 2*a^2*c^2
 - (3*a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*
a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^
2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) -
2*((a^3 + 2*a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*sqrt(-x^2 + 1))/x^2
) - sqrt(1/2)*(a^2*x^4 + 2*sqrt(-x^2 + 1)*a^2*x^2 - 2*a^2*x^2)*sq
rt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a^4*b^2 -
4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b
^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c
)))/(a^4*b^2 - 4*a^5*c))*log(-((a^4*b^2*c - 4*a^5*c^2)*x^2*sqrt((
a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^
3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - 2*(a^3 + 2
*a^2*b)*c^2 - ((a^2*b + 2*a*b^2)*c^2 - (a*b^3 + b^4)*c)*x^2 + 2*(
a^2*b^2 + a*b^3)*c + sqrt(1/2)*((a^5*b^3 - 4*a^6*b*c)*x^2*sqrt((a
^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3
*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - (a^2*b^4 +
a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*s
qrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a^4*b^2 -
 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*
b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*
c)))/(a^4*b^2 - 4*a^5*c)) + 2*((a^3 + 2*a^2*b)*c^2 - (a^2*b^2 + a
*b^3)*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*(a^2*x^4 + 2*sqrt(-x^2
+ 1)*a^2*x^2 - 2*a^2*x^2)*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*
b + 4*a*b^2)*c + (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^
6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*
a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-((a^4*b
^2*c - 4*a^5*c^2)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^
3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*
b^2 - 4*a^9*c)) - 2*(a^3 + 2*a^2*b)*c^2 - ((a^2*b + 2*a*b^2)*c^2
```

```
- (a*b^3 + b^4)*c)*x^2 + 2*(a^2*b^2 + a*b^3)*c - sqrt(1/2)*((a^5*
b^3 - 4*a^6*b*c)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3
*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b
^2 - 4*a^9*c)) - (a^2*b^4 + a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^
3*b^2 + 6*a^2*b^3)*c)*x^2)*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2
*b + 4*a*b^2)*c + (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b
^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2
*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 2*((a^3 +
 2*a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*sqrt(-x^2 + 1))/x^2) - ((a +
 2*b)*x^4 + 2*sqrt(-x^2 + 1)*(a + 2*b)*x^2 - 2*(a + 2*b)*x^2)*log
((sqrt(-x^2 + 1) - 1)/x) - (a*x^2 - 2*a)*sqrt(-x^2 + 1) - 2*a)/(a
^2*x^4 + 2*sqrt(-x^2 + 1)*a^2*x^2 - 2*a^2*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**(1/2)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] Timed out
```

3.373 $\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=325

$$-\frac{\left(-\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{(2b+c)\sin^{-1}(x)}{2c^2}+\frac{\sqrt{1-x^2}x}{2c}$$

```
[Out] (x*Sqrt[1 - x^2])/(2*c) + ((2*b + c)*ArcSin[x])/(2*c^2) - ((b^2 -
 a*c + b*c - (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[1 - x^2])])/(c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[
b + 2*c - Sqrt[b^2 - 4*a*c]]) - ((b^2 - a*c + b*c + (b^3 - 3*a*b*
c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c + Sq
rt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])
/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c
]])
```

Rubi [A] time = 11.6874, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$-\frac{\left(-\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{(2b+c)\sin^{-1}(x)}{2c^2}+\frac{\sqrt{1-x^2}x}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```


```
[Out] (x*Sqrt[1 - x^2])/(2*c) + ((2*b + c)*ArcSin[x])/(2*c^2) - ((b^2 -
 a*c + b*c - (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[1 - x^2])])/(c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[
b + 2*c - Sqrt[b^2 - 4*a*c]]) - ((b^2 - a*c + b*c + (b^3 - 3*a*b*
c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c + Sq
rt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])
/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c
```

```
]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.657819, size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x^4*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[(x^4*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4), x]
```

Maple [C] time = 0.045, size = 222, normalized size = 0.7

$$\frac{x}{2\,c}\sqrt{-x^2+1}+\frac{\arcsin(x)}{2\,c} + \frac{1}{4\,c^2}\sum_{\_R=RootOf\left(a\_Z^8+(4\,a+4\,b)\_Z^6+(6\,a+8\,b+16\,c)\_Z^4+(4\,a+4\,b)\_Z^2+a\right)} \frac{a\,(b+c)\,\_R^6+\left(3\,ab-ac+4\,b^2+4\,bc\right)\,\_R^4+\left(3\,ab-ac+4\,b^2+\right.}{\_R^7a+3\,\_R^5a+3\,\_R^5b+3\,\_R^3a+4\,\_R^3b+8\,\_R^3c+\_} - 2\,\frac{b}{c^2}\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*x*(-x^2+1)^(1/2)/c+1/2*arcsin(x)/c+1/4/c^2*sum((a*(b+c)*_R^6+
(3*a*b-a*c+4*b^2+4*b*c)*_R^4+(3*a*b-a*c+4*b^2+4*b*c)*_R^2+a*b+a*c
```

```
)/(_R^7*a+3*_R^5*a+3*_R^5*b+3*_R^3*a+4*_R^3*b+8*_R^3*c+_R*a+_R*b)
*ln(((-x^2+1)^(1/2)-1)/x-_R),_R=RootOf(a*_Z^8+(4*a+4*b)*_Z^6+(6*a
+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^2+a))-2/c^2*b*arctan(((-x^2+1)^(1/2)
-1)/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}x^4}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)*x^4/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^4/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 1.77755, size = 4086, normalized size = 12.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)*x^4/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*c*x^3 + sqrt(1/2)*(c^2*x^2 + 2*sqrt(-x^2 + 1)*c^2 - 2*c^2
)*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c + (b^2*c^4
 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^
2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*
c^9)))/(b^2*c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a^2*
b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2
)*c + sqrt(1/2)*((b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 -
 (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^6 + 4*a^2*b*c^3 + (8*a^
2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x - ((b^4*c^4 - 6*a*b^2
*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 + 8*a
^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2
*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c
^9)))*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c + (b^2
*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (
4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 -
4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b
 - a^2*b^2)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*(c^2*x^2 + 2*sqrt
(-x^2 + 1)*c^2 - 2*c^2)*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b
^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*
b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b
^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-(2*a^2*b^3
```

```
 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*x^2
 - 2*(2*a^3*b - a^2*b^2)*c - sqrt(1/2)*((b^6 + 4*a^2*b*c^3 + (8*a^
2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^6
 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x
 - ((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x - (b^4*c
^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b
 - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^
5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4
*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*
a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4
 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^
3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x^2) + sqrt(
1/2)*(c^2*x^2 + 2*sqrt(-x^2 + 1)*c^2 - 2*c^2)*sqrt(-(b^4 + (2*a^2
 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^6
 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4
)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a
*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^
3*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c + sqrt(1/2)*((b^6
 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*s
qrt(-x^2 + 1)*x - (b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2
 - (6*a*b^4 - b^5)*c)*x + ((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqr
t(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x)*sqrt((b^6
+ a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)
*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4 + (2
*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c - (b^2*c^4 - 4*a*c^5)*sqrt(
(b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 +
 b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 -
 4*a*c^5)) - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-
x^2 + 1))/x^2) - sqrt(1/2)*(c^2*x^2 + 2*sqrt(-x^2 + 1)*c^2 - 2*c^
2)*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c - (b^2*c^
4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a
^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a
*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a^2
*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^
2)*c - sqrt(1/2)*((b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2
 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^6 + 4*a^2*b*c^3 + (8*a
^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x + ((b^4*c^4 - 6*a*b^
2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 + 8*
a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^
2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*
c^9)))*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c - (b^
2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 +
(4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 -
 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*
b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x^2) - 2*c*x + 2*((2*b + c)*x^2 +
 2*sqrt(-x^2 + 1)*(2*b + c) - 4*b - 2*c)*arctan((sqrt(-x^2 + 1) -
 1)/x) - (c*x^3 - 2*c*x)*sqrt(-x^2 + 1))/(c^2*x^2 + 2*sqrt(-x^2 +
 1)*c^2 - 2*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**4*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)*x^4/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.374 $\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=263

$$\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\sin^{-1}(x)}{c}$$

```
[Out] -(ArcSin[x]/c) + ((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[1 - x^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b
+ 2*c - Sqrt[b^2 - 4*a*c]]) + ((b + c + (b^2 - 2*a*c + b*c)/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[
b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(c*Sqrt[b + Sqrt[b^2 - 4*
a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 4.31308, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\sin^{-1}(x)}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(ArcSin[x]/c) + ((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[1 - x^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b
+ 2*c - Sqrt[b^2 - 4*a*c]]) + ((b + c + (b^2 - 2*a*c + b*c)/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[
b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(c*Sqrt[b + Sqrt[b^2 - 4*
a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.485673, size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x^2*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[(x^2*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4), x]
```

Maple [C] time = 0.03, size = 175, normalized size = 0.7

$$-\frac{1}{4c}\sum_{\_R=RootOf\left(a\_Z^8+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)}\frac{\_R^6a+(4c+3a+4b)\_R^4+(4c+3a+4b)\_R^2+a}{\_R^7a+3\_R^5a+3\_R^5b+3\_R^3a+4\_R^3b+8\_R^3c+\_Ra+\_Rb}$$
$$+2\frac{1}{c}\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4/c*sum((_R^6*a+(4*c+3*a+4*b)*_R^4+(4*c+3*a+4*b)*_R^2+a)/(_R^7
*a+3*_R^5*a+3*_R^5*b+3*_R^3*a+4*_R^3*b+8*_R^3*c+_R*a+_R*b)*ln(((-
x^2+1)^(1/2)-1)/x-_R),_R=RootOf(a*_Z^8+(4*a+4*b)*_Z^6+(6*a+8*b+16
*c)*_Z^4+(4*a+4*b)*_Z^2+a))+2/c*arctan(((-x^2+1)^(1/2)-1)/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}x^2}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)*x^2/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^2/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 0.780424, size = 1931, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)*x^2/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3)*
sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3
))*log(-(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c + sqrt(1/2)*((b^3 - 4*
a*c^2 - (4*a*b - b^2)*c)*sqrt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a
*b - b^2)*c)*x - ((b^3*c^2 - 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c
^2 - 4*a*b*c^3)*x)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))
*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c
 + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)) + 2*(a*b + a*c
)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c + (
b^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))
/(b^2*c^2 - 4*a*c^3))*log(-(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c - s
qrt(1/2)*((b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*sqrt(-x^2 + 1)*x - (b
^3 - 4*a*c^2 - (4*a*b - b^2)*c)*x - ((b^3*c^2 - 4*a*b*c^3)*sqrt(-
x^2 + 1)*x - (b^3*c^2 - 4*a*b*c^3)*x)*sqrt((b^2 + 2*b*c + c^2)/(b
^2*c^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3
)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c
^3)) + 2*(a*b + a*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*c*sqrt(-(b^
2 - (2*a - b)*c - (b^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b
^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(-(2*(a*b + a*c)*x^2
- 2*a*b - 2*a*c + sqrt(1/2)*((b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*sq
rt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*x + ((b^3*c^2
- 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a*b*c^3)*x)*sqrt((b^
2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a - b)*c -
 (b^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)
))/(b^2*c^2 - 4*a*c^3)) + 2*(a*b + a*c)*sqrt(-x^2 + 1))/x^2) - sq
rt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c - (b^2*c^2 - 4*a*c^3)*sqrt((b^
2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(-
(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c - sqrt(1/2)*((b^3 - 4*a*c^2 -
```

```
(4*a*b - b^2)*c)*sqrt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a*b - b^2
)*c)*x + ((b^3*c^2 - 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a
*b*c^3)*x)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(-(
b^2 - (2*a - b)*c - (b^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/
(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)) + 2*(a*b + a*c)*sqrt(-
x^2 + 1))/x^2) - 4*arctan((sqrt(-x^2 + 1) - 1)/x))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**2*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)*x^2/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.375 $\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=220

$$\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b + 2*c - Sqrt[b^
2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(Sqr
t[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[b + 2*c + Sqr
t[b^2 - 4*a*c]]*ArcTan[(Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqr
t[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.725753, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b + 2*c - Sqrt[b^
2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(Sqr
t[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[b + 2*c + Sqr
t[b^2 - 4*a*c]]*ArcTan[(Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqr
t[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 78.4928, size = 196, normalized size = 0.89

$$-\frac{\sqrt{b+2c+\sqrt{-4ac+b^2}}\,\mathrm{atan}\left(\frac{x\sqrt{b+2c+\sqrt{-4ac+b^2}}}{\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-x^2+1}}\right)}{\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{b+2c-\sqrt{-4ac+b^2}}\,\mathrm{atan}\left(\frac{x\sqrt{b+2c-\sqrt{-4ac+b^2}}}{\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-x^2+1}}\right)}{\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] -sqrt(b + 2*c + sqrt(-4*a*c + b**2))*atan(x*sqrt(b + 2*c + sqrt(-
4*a*c + b**2))/(sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-x**2 + 1)))/(
sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) + sqrt(b + 2*c
 - sqrt(-4*a*c + b**2))*atan(x*sqrt(b + 2*c - sqrt(-4*a*c + b**2)
)/(sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-x**2 + 1)))/(sqrt(b - sqrt
(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.100479, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[1 - x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[Sqrt[1 - x^2]/(a + b*x^2 + c*x^4), x]
```

Maple [C] time = 0.016, size = 130, normalized size = 0.6

$$-\frac{1}{4}\sum_{\_R=RootOf\left(a\_Z^8+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)}\frac{\_R^6-\_R^4-\_R^2+1}{\_R^7a+3\_R^5a+3\_R^5b+3\_R^3a+4\_R^3b+8\_R^3c+\_Ra+\_Rb}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*sum((_R^6-_R^4-_R^2+1)/(_R^7*a+3*_R^5*a+3*_R^5*b+3*_R^3*a+4*
_R^3*b+8*_R^3*c+_R*a+_R*b)*ln(((-x^2+1)^(1/2)-1)/x-_R),_R=RootOf(
a*_Z^8+(4*a+4*b)*_Z^6+(6*a+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 0.378485, size = 1025, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4
*a^3*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 + sqrt(1/2)*((a*b^2 - 4*a^2
*c)*sqrt(-x^2 + 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b + (a*b
^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^
2*b^2 - 4*a^3*c) + sqrt(-x^2 + 1) - 1)/x^2) - 1/2*sqrt(1/2)*sqrt(
-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4
*a^2*c))*log(-(x^2 - sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt(-x^2 + 1)*
x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(
a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) +
sqrt(-x^2 + 1) - 1)/x^2) - 1/2*sqrt(1/2)*sqrt(-(2*a + b - (a*b^2
- 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(-(x^2
+ sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt(-x^2 + 1)*x - (a*b^2 - 4*a^2*
c)*x)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))
/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2 + 1) - 1)
/x^2) + 1/2*sqrt(1/2)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt(a^2
*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 - sqrt(1/2)*((a*b^2
 - 4*a^2*c)*sqrt(-x^2 + 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a +
b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))
/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2 + 1) - 1)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/(c*x^4 + b*x^2 + a),x, algorithm="giac")

[Out] Timed out
```

3.376 $\int \frac{\sqrt{1-x^2}}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=265

$$-\frac{c\left(\frac{2a+b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{1-x^2}}{ax}$$

```
[Out] -(Sqrt[1 - x^2]/(a*x)) - (c*(1 + (2*a + b)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*
a*c]]*Sqrt[1 - x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*
c - Sqrt[b^2 - 4*a*c]]) - (c*(1 - (2*a + b)/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4
*a*c]]*Sqrt[1 - x^2])])/(a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2
*c + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.84776, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$-\frac{c\left(\frac{2a+b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{1-x^2}}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[1 - x^2]/(a*x)) - (c*(1 + (2*a + b)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*
a*c]]*Sqrt[1 - x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*
c - Sqrt[b^2 - 4*a*c]]) - (c*(1 - (2*a + b)/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4
*a*c]]*Sqrt[1 - x^2])])/(a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2
*c + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 154.285, size = 248, normalized size = 0.94

$$\frac{c\left(2a+b-\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{x\sqrt{b+2c+\sqrt{-4ac+b^2}}}{\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-x^2+1}}\right)}{a\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\sqrt{b+2c+\sqrt{-4ac+b^2}}} - \frac{c\left(2a+b+\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{x\sqrt{b+2c-\sqrt{-4ac+b^2}}}{\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-x^2+1}}\right)}{a\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\sqrt{b+2c-\sqrt{-4ac+b^2}}} - \frac{\sqrt{-x^2+1}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)**(1/2)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] c*(2*a + b - sqrt(-4*a*c + b**2))*atan(x*sqrt(b + 2*c + sqrt(-4*a
*c + b**2))/(sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-x**2 + 1)))/(a*s
qrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)*sqrt(b + 2*c + s
qrt(-4*a*c + b**2))) - c*(2*a + b + sqrt(-4*a*c + b**2))*atan(x*s
qrt(b + 2*c - sqrt(-4*a*c + b**2))/(sqrt(b - sqrt(-4*a*c + b**2))
*sqrt(-x**2 + 1)))/(a*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c +
 b**2)*sqrt(b + 2*c - sqrt(-4*a*c + b**2))) - sqrt(-x**2 + 1)/(a*
x)
```

Mathematica [A] time = 0.493337, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1-x^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[1 - x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[Sqrt[1 - x^2]/(x^2*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.035, size = 217, normalized size = 0.8

$$-\frac{1}{ax}\left(-x^2+1\right)^{\frac{3}{2}} - \frac{x}{a}\sqrt{-x^2+1} - \frac{\arcsin(x)}{a} + \frac{1}{4a}\sum_{\_R=RootOf\left(a\_Z^8+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)} \frac{(a+b)\_R^6+(3a+3b+4c)\_R^4+(3a+3b+4c)\_R^2+a+b}{\_R^7a+3\_R^5a+3\_R^5b+3\_R^3a+4\_R^3b+8\_R^3c+\_Ra+\_Rb} - 2\frac{1}{a}\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a/x*(-x^2+1)^(3/2)-1/a*x*(-x^2+1)^(1/2)-1/a*arcsin(x)+1/4/a*su
m(((a+b)*_R^6+(3*a+3*b+4*c)*_R^4+(3*a+3*b+4*c)*_R^2+a+b)/(_R^7*a+
3*_R^5*a+3*_R^5*b+3*_R^3*a+4*_R^3*b+8*_R^3*c+_R*a+_R*b)*ln(((-x^2
+1)^(1/2)-1)/x-_R),_R=RootOf(a*_Z^8+(4*a+4*b)*_Z^6+(6*a+8*b+16*c)
*_Z^4+(4*a+4*b)*_Z^2+a))-2/a*arctan(((-x^2+1)^(1/2)-1)/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{(cx^4+bx^2+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [A] time = 0.487556, size = 2804, normalized size = 10.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*(sqrt(-x^2 + 1)*a*x - a*x)*sqrt(-(a*b^2 + b^3 - (2
*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b
^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^
2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c)*x^2 - 2*(a
*b + b^2)*c + sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*
a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b
+ 5*a*b^2)*c)*x - ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*
b^3 - 4*a^4*b*c)*x)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(
a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a
^2 + 3*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4
 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2
- 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^2) - sq
rt(1/2)*(sqrt(-x^2 + 1)*a*x - a*x)*sqrt(-(a*b^2 + b^3 - (2*a^2 +
3*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^
```

```
2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a
^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c)*x^2 - 2*(a*b + b^
2)*c - sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^2)*
c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b
^2)*c)*x - ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4
*a^4*b*c)*x)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b +
 a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*
a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*
c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4
*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)
*(sqrt(-x^2 + 1)*a*x - a*x)*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*
c - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 -
 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*
log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c)*x^2 - 2*(a*b + b^2)*c +
sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^2)*c)*sqrt
(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^2)*c)*
x + ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*
c)*x)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)
*c)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c
- (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2
*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)) -
2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*(sqrt(
-x^2 + 1)*a*x - a*x)*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^
3*b^2 - 4*a^4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2
*b + a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*
a*c^2 - 2*(a*c^2 - (a*b + b^2)*c)*x^2 - 2*(a*b + b^2)*c - sqrt(1/
2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^2)*c)*sqrt(-x^2 +
 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^2)*c)*x + ((a
^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x)*s
qrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^
6*b^2 - 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*
b^2 - 4*a^4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b
 + a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^
2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^2) + 2*x^2 + 2*sqrt(-x^2 + 1
) - 2)/(sqrt(-x^2 + 1)*a*x - a*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**(1/2)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

<u>GIAC/XCAS</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.377 $\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx$

Optimal. Leaf size=96

$$\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}x}{\sqrt{1-x^2}}\right)-\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}x}{\sqrt{1-x^2}}\right)-\sin^{-1}(x)$$

```
[Out] -ArcSin[x] + Sqrt[(2 + Sqrt[5])/5]*ArcTan[(Sqrt[(1 + Sqrt[5])/2]*
x)/Sqrt[1 - x^2]] - Sqrt[(-2 + Sqrt[5])/5]*ArcTanh[(Sqrt[(-1 + Sq
rt[5])/2]*x)/Sqrt[1 - x^2]]
```

Rubi [A] time = 0.41985, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}x}{\sqrt{1-x^2}}\right)-\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}x}{\sqrt{1-x^2}}\right)-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*Sqrt[1 - x^2])/(-1 + x^2 + x^4),x]
```

```
[Out] -ArcSin[x] + Sqrt[(2 + Sqrt[5])/5]*ArcTan[(Sqrt[(1 + Sqrt[5])/2]*
x)/Sqrt[1 - x^2]] - Sqrt[(-2 + Sqrt[5])/5]*ArcTanh[(Sqrt[(-1 + Sq
rt[5])/2]*x)/Sqrt[1 - x^2]]
```

Rubi in Sympy [A] time = 59.1373, size = 128, normalized size = 1.33

$$-\operatorname{asin}(x)+\frac{\left(\frac{4\sqrt{5}}{5}+2\right)\operatorname{atan}\left(\frac{x\sqrt{\sqrt{5}+3}}{\sqrt{1+\sqrt{5}}\sqrt{-x^2+1}}\right)}{\sqrt{1+\sqrt{5}}\sqrt{\sqrt{5}+3}}-\frac{\left(-\frac{4\sqrt{5}}{5}+2\right)\operatorname{atanh}\left(\frac{x\sqrt{-\sqrt{5}+3}}{\sqrt{-1+\sqrt{5}}\sqrt{-x^2+1}}\right)}{\sqrt{-1+\sqrt{5}}\sqrt{-\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(-x**2+1)**(1/2)/(x**4+x**2-1),x)
```

```
[Out] -asin(x) + (4*sqrt(5)/5 + 2)*atan(x*sqrt(sqrt(5) + 3)/(sqrt(1 + s
qrt(5))*sqrt(-x**2 + 1)))/(sqrt(1 + sqrt(5))*sqrt(sqrt(5) + 3)) -
```

```
 (-4*sqrt(5)/5 + 2)*atanh(x*sqrt(-sqrt(5) + 3)/(sqrt(-1 + sqrt(5)
)*sqrt(-x**2 + 1)))/(sqrt(-1 + sqrt(5))*sqrt(-sqrt(5) + 3))
```

Mathematica [A] time = 0.264258, size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x^2*Sqrt[1 - x^2])/(-1 + x^2 + x^4),x]
```

```
[Out] Integrate[(x^2*Sqrt[1 - x^2])/(-1 + x^2 + x^4), x]
```

Maple [B] time = 0.101, size = 160, normalized size = 1.7

$$-\frac{\sqrt{5}}{5\sqrt{2+\sqrt{5}}}\mathit{Artanh}\left(\frac{1}{x\sqrt{2+\sqrt{5}}}\left(\sqrt{-x^2+1}-1\right)\right) - \frac{\sqrt{5}}{5\sqrt{-2+\sqrt{5}}}\arctan\left(\frac{1}{x\sqrt{-2+\sqrt{5}}}\left(\sqrt{-x^2+1}-1\right)\right) - \frac{\sqrt{2+\sqrt{5}}\sqrt{5}}{5}\arctan\left(\frac{1}{x\sqrt{2+\sqrt{5}}}\left(\sqrt{-x^2+1}-1\right)\right) + \frac{\sqrt{-2+\sqrt{5}}\sqrt{5}}{5}\mathit{Artanh}\left(\frac{1}{x\sqrt{-2+\sqrt{5}}}\left(\sqrt{-x^2+1}-1\right)\right) + 2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x)
```

```
[Out] -1/5*5^(1/2)/(2+5^(1/2))^(1/2)*arctanh(((-x^2+1)^(1/2)-1)/x/(2+5^
(1/2))^(1/2))-1/5*5^(1/2)/(-2+5^(1/2))^(1/2)*arctan(((-x^2+1)^(1/
2)-1)/x/(-2+5^(1/2))^(1/2))-1/5*(2+5^(1/2))^(1/2)*5^(1/2)*arctan(
((-x^2+1)^(1/2)-1)/x/(2+5^(1/2))^(1/2))+1/5*(-2+5^(1/2))^(1/2)*5^
(1/2)*arctanh(((-x^2+1)^(1/2)-1)/x/(-2+5^(1/2))^(1/2))+2*arctan((
(-x^2+1)^(1/2)-1)/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}x^2}{x^4+x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)*x^2/(x^4 + x^2 - 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^2/(x^4 + x^2 - 1), x)
```

Fricas [A] time = 0.298838, size = 475, normalized size = 4.95

$$\frac{2}{5}\sqrt{\sqrt{5}\left(2\sqrt{5}+5\right)}\arctan\left(\frac{\left(\sqrt{-x^2+1}\left(\sqrt{5}x-x\right)-\sqrt{5}x+x\right)\sqrt{\sqrt{5}\left(2\sqrt{5}+5\right)}}{\sqrt{2}x^2\sqrt{-\frac{\sqrt{5}\left(5x^4-10x^2-\sqrt{5}(x^4-4x^2+4)+2\left(5x^2-\sqrt{5}(x^2-2)\right)\sqrt{-x^2+1}\right)}{x^4}}-2\sqrt{5}(x^2-1)-2\sqrt{5}\sqrt{-x^2+1}}\right)$$
$$+\frac{1}{10}\sqrt{-\sqrt{5}\left(2\sqrt{5}-5\right)}\log\left(-\frac{2\sqrt{5}(x^2-1)+\left(\sqrt{-x^2+1}\left(\sqrt{5}x+x\right)-\sqrt{5}x-x\right)\sqrt{-\sqrt{5}\left(2\sqrt{5}-5\right)}+2\sqrt{5}\sqrt{-x^2+1}}{x^2}\right)$$
$$-\frac{1}{10}\sqrt{-\sqrt{5}\left(2\sqrt{5}-5\right)}\log\left(-\frac{2\sqrt{5}(x^2-1)-\left(\sqrt{-x^2+1}\left(\sqrt{5}x+x\right)-\sqrt{5}x-x\right)\sqrt{-\sqrt{5}\left(2\sqrt{5}-5\right)}+2\sqrt{5}\sqrt{-x^2+1}}{x^2}\right)$$
$$+2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)*x^2/(x^4 + x^2 - 1),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(sqrt(5)*(2*sqrt(5) + 5))*arctan((sqrt(-x^2 + 1)*(sqrt(5)
*x - x) - sqrt(5)*x + x)*sqrt(sqrt(5)*(2*sqrt(5) + 5))/(sqrt(2)*x
^2*sqrt(-sqrt(5)*(5*x^4 - 10*x^2 - sqrt(5)*(x^4 - 4*x^2 + 4) + 2*
(5*x^2 - sqrt(5)*(x^2 - 2))*sqrt(-x^2 + 1))/x^4) - 2*sqrt(5)*(x^2
 - 1) - 2*sqrt(5)*sqrt(-x^2 + 1))) + 1/10*sqrt(-sqrt(5)*(2*sqrt(5
) - 5))*log(-(2*sqrt(5)*(x^2 - 1) + (sqrt(-x^2 + 1)*(sqrt(5)*x +
x) - sqrt(5)*x - x)*sqrt(-sqrt(5)*(2*sqrt(5) - 5)) + 2*sqrt(5)*sq
rt(-x^2 + 1))/x^2) - 1/10*sqrt(-sqrt(5)*(2*sqrt(5) - 5))*log(-(2*
sqrt(5)*(x^2 - 1) - (sqrt(-x^2 + 1)*(sqrt(5)*x + x) - sqrt(5)*x -
 x)*sqrt(-sqrt(5)*(2*sqrt(5) - 5)) + 2*sqrt(5)*sqrt(-x^2 + 1))/x^
2) + 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-(x-1)(x+1)}}{x^4+x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(-x**2+1)**(1/2)/(x**4+x**2-1),x)
```

```
[Out] Integral(x**2*sqrt(-(x - 1)*(x + 1))/(x**4 + x**2 - 1), x)
```

GIAC/XCAS [A] time = 0.341728, size = 282, normalized size = 2.94

$$-\frac{1}{2}\pi\operatorname{sign}(x)-\frac{1}{5}\sqrt{5\sqrt{5}+10}\arctan\left(-\frac{\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}}{\sqrt{2\sqrt{5}+2}}\right)-\frac{1}{10}\sqrt{5\sqrt{5}-10}\ln\left(\left|\sqrt{2\sqrt{5}-2}-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}\right|\right)+\frac{1}{10}\sqrt{5\sqrt{5}-10}\ln\left(\left|-\sqrt{2\sqrt{5}-2}-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}\right|\right)-\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)*x^2/(x^4 + x^2 - 1),x, algorithm="giac")
```

```
[Out] -1/2*pi*sign(x) - 1/5*sqrt(5*sqrt(5) + 10)*arctan(-(x/(sqrt(-x^2
+ 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt(2*sqrt(5) + 2)) - 1/10*s
qrt(5*sqrt(5) - 10)*ln(abs(sqrt(2*sqrt(5) - 2) - x/(sqrt(-x^2 + 1
) - 1) + (sqrt(-x^2 + 1) - 1)/x)) + 1/10*sqrt(5*sqrt(5) - 10)*ln(
abs(-sqrt(2*sqrt(5) - 2) - x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 +
1) - 1)/x)) - arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqr
t(-x^2 + 1) - 1))
```

3.378 $\int \frac{x^8}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=479

$$-\frac{\left(-\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\left(b^2-ac\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^3\sqrt{e}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2e^{3/2}}-\frac{bx\sqrt{d+ex^2}}{2c^2e}+\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8ce^{5/2}}-\frac{3dx\sqrt{d+ex^2}}{8ce^2}+\frac{x^3\sqrt{d+ex^2}}{4ce}$$

```
[Out] (-3*d*x*Sqrt[d + e*x^2])/(8*c*e^2) - (b*x*Sqrt[d + e*x^2])/(2*c^2
*e) + (x^3*Sqrt[d + e*x^2])/(4*c*e) - ((b^3 - 2*a*b*c - (b^4 - 4*
a*b^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e
*x^2])])/(c^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e]) - ((b^3 - 2*a*b*c + (b^4 - 4*a*b^2*c + 2*a^2*c^
2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^3*Sqrt
[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
+ (3*d^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*c*e^(5/2)) + (b
*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^2*e^(3/2)) + ((b^2
- a*c)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(c^3*Sqrt[e])
```

Rubi [A] time = 3.98145, antiderivative size = 479, normalized size of antiderivative = 1., number of

steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$-\frac{\left(-\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\left(b^2-ac\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^3\sqrt{e}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2e^{3/2}}-\frac{bx\sqrt{d+ex^2}}{2c^2e}+\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8ce^{5/2}}-\frac{3dx\sqrt{d+ex^2}}{8ce^2}+\frac{x^3\sqrt{d+ex^2}}{4ce}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-3*d*x*Sqrt[d + e*x^2])/(8*c*e^2) - (b*x*Sqrt[d + e*x^2])/(2*c^2
*e) + (x^3*Sqrt[d + e*x^2])/(4*c*e) - ((b^3 - 2*a*b*c - (b^4 - 4*
a*b^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e
*x^2])])/(c^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e]) - ((b^3 - 2*a*b*c + (b^4 - 4*a*b^2*c + 2*a^2*c^
2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^3*Sqrt
[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
+ (3*d^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*c*e^(5/2)) + (b
*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^2*e^(3/2)) + ((b^2
- a*c)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(c^3*Sqrt[e])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.897436, size = 0, normalized size = 0.

$$\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[x^8/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[x^8/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.04, size = 377, normalized size = 0.8

$$\frac{b^2}{c^3}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)\frac{1}{\sqrt{e}}+\frac{x^3}{4\,ce}\sqrt{ex^2+d}-\frac{3\,dx}{8\,e^2c}\sqrt{ex^2+d}+\frac{3\,d^2}{8\,c}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{5}{2}}$$
$$-\frac{a}{c^2}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)\frac{1}{\sqrt{e}}-\frac{bx}{2\,c^2e}\sqrt{ex^2+d}+\frac{bd}{2\,c^2}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{3}{2}}$$
$$-\frac{1}{2\,c^3}\sqrt{e}\sum_{\_R=RootOf\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+(16\,ae^2-8\,bde+6\,cd^2)\_Z^2+(4\,bd^2e-4\,cd^3)\_Z+cd^4\right)}\frac{b\left(2\,ac-b^2\right)\_R^2+2\,\left(2\,a^2ce-2\,ab^2e-2\,abcd\right.}{\_R^3c+3\,\_R^2be-3\,\_R^2cd+8\,\_R\,ae^2-4\,\_R\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] 1/c^3*b^2*ln(x*e^(1/2)+(e*x^2+d)^(1/2))/e^(1/2)+1/4*x^3*(e*x^2+d)
^(1/2)/c/e-3/8*d*x*(e*x^2+d)^(1/2)/e^2/c+3/8/c*d^2/e^(5/2)*ln(x*e
^(1/2)+(e*x^2+d)^(1/2))-1/c^2*a*ln(x*e^(1/2)+(e*x^2+d)^(1/2))/e^(
1/2)-1/2*b*x*(e*x^2+d)^(1/2)/c^2/e+1/2/c^2*b*d/e^(3/2)*ln(x*e^(1/
2)+(e*x^2+d)^(1/2))-1/2/c^3*e^(1/2)*sum((b*(2*a*c-b^2)*_R^2+2*(2*
a^2*c*e-2*a*b^2*e-2*a*b*c*d+b^3*d)*_R+2*a*b*c*d^2-b^3*d^2)/(_R^3*
c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-
c*d^3)*ln(((e*x^2+d)^(1/2)-x*e^(1/2))^2-_R),_R=RootOf(c*_Z^4+(4*b
*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3
)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{(cx^4+bx^2+a)\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)),x, algorithm="maxima")

[Out] integrate(x^8/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\sqrt{d + ex^2}\,(a + bx^2 + cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)

[Out] Integral(x**8/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)),x, algorithm="giac")

[Out] Timed out
```

3.379 $\int \frac{x^6}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=366

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2ce^{3/2}}+\frac{x\sqrt{d+ex^2}}{2ce}$$

```
[Out] (x*Sqrt[d + e*x^2])/(2*c*e) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x
)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b - S
qrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((b^
2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqr
t[d + e*x^2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e]) - (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]
])/(2*c*e^(3/2)) - (b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(c^2*
Sqrt[e])
```

Rubi [A] time = 2.59874, antiderivative size = 366, normalized size of antiderivative = 1., number of

steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2ce^{3/2}}+\frac{x\sqrt{d+ex^2}}{2ce}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c*e) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x
)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b - S
qrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((b^
2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqr
t[d + e*x^2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e]) - (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]
])/(2*c*e^(3/2)) - (b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(c^2*
Sqrt[e])
```

Rubi in Sympy [A] time = 179.501, size = 359, normalized size = 0.98

$$-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}}-\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2ce^{\frac{3}{2}}}+\frac{x\sqrt{d+ex^2}}{2ce}+\frac{\left(b\left(-3ac+b^2\right)+\sqrt{-4ac+b^2}\left(-ac+b^2\right)\right)\mathrm{atanh}\left(\frac{x\sqrt{be-2cd+e\sqrt{-4ac+b^2}}}{\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\sqrt{be-2cd+e\sqrt{-4ac+b^2}}}-\frac{\left(b\left(-3ac+b^2\right)-\sqrt{-4ac+b^2}\left(-ac+b^2\right)\right)\mathrm{atanh}\left(\frac{x\sqrt{be-2cd-e\sqrt{-4ac+b^2}}}{\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\sqrt{be-2cd-e\sqrt{-4ac+b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] -b*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(c**2*sqrt(e)) - d*atanh(sqr
t(e)*x/sqrt(d + e*x**2))/(2*c*e**(3/2)) + x*sqrt(d + e*x**2)/(2*c
*e) + (b*(-3*a*c + b**2) + sqrt(-4*a*c + b**2)*(-a*c + b**2))*ata
nh(x*sqrt(b*e - 2*c*d + e*sqrt(-4*a*c + b**2))/(sqrt(b + sqrt(-4*
a*c + b**2))*sqrt(d + e*x**2)))/(c**2*sqrt(b + sqrt(-4*a*c + b**2
))*sqrt(-4*a*c + b**2)*sqrt(b*e - 2*c*d + e*sqrt(-4*a*c + b**2)))
 - (b*(-3*a*c + b**2) - sqrt(-4*a*c + b**2)*(-a*c + b**2))*atanh(
x*sqrt(b*e - 2*c*d - e*sqrt(-4*a*c + b**2))/(sqrt(b - sqrt(-4*a*c
 + b**2))*sqrt(d + e*x**2)))/(c**2*sqrt(b - sqrt(-4*a*c + b**2))*
sqrt(-4*a*c + b**2)*sqrt(b*e - 2*c*d - e*sqrt(-4*a*c + b**2)))
```

Mathematica [A] time = 0.699283, size = 0, normalized size = 0.

$$\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[x^6/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[x^6/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.035, size = 269, normalized size = 0.7

$$\frac{x}{2\,ce}\sqrt{ex^2+d}-\frac{d}{2\,c}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{3}{2}}-\frac{b}{c^2}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)\frac{1}{\sqrt{e}}$$
$$+\frac{1}{2\,c^2}\sqrt{e}\sum_{\_R=RootOf\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+(16\,ae^2-8\,bde+6\,cd^2)\_Z^2+(4\,bd^2e-4\,cd^3)\_Z+cd^4\right)}\frac{\left(ac-b^2\right)\_R^2+2\,\left(-2\,abe-acd+b^2d\right)}{\_R^3c+3\,\_R^2be-3\,\_R^2cd+8\,\_R\,ae^2-4\,\_R\,bde}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] 1/2*x*(e*x^2+d)^(1/2)/c/e-1/2/c*d/e^(3/2)*ln(x*e^(1/2)+(e*x^2+d)^
(1/2))-1/c^2*b*ln(x*e^(1/2)+(e*x^2+d)^(1/2))/e^(1/2)+1/2/c^2*e^(1
/2)*sum(((a*c-b^2)*_R^2+2*(-2*a*b*e-a*c*d+b^2*d)*_R+a*c*d^2-b^2*d
^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^
2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-x*e^(1/2))^2-_R),_R=RootOf(c
*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2
```

```
*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(cx^4 + bx^2 + a)\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

Fricas [A] time = 89.8996, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(1/2)*c^2*e^(3/2)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^
2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e + ((b^2*c^5 - 4*a*c^6)
*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)*s
qrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e
+ (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/((b^2*c^10 - 4*a*c
^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*a*b^2*c^
9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^
2*b^2*c^8 - 4*a^3*c^9)*e^4)))/((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4
 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2))*log((((a^2*b^2*
c^5 - 4*a^3*c^6)*d^3 - (a^2*b^3*c^4 - 4*a^3*b*c^5)*d^2*e + (a^3*b
^2*c^4 - 4*a^4*c^5)*d*e^2)*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^
4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7
*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*
b^2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^1
0)*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^
3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)) + 2*
(a^3*b^4 - 3*a^4*b^2*c + a^5*c^2)*d^2 - 2*(a^4*b^3 - 2*a^5*b*c)*d
*e - ((a^2*b^5 - 3*a^3*b^3*c + a^4*b*c^2)*d^2 - (5*a^3*b^4 - 14*a
^4*b^2*c + 4*a^5*c^2)*d*e + 4*(a^4*b^3 - 2*a^5*b*c)*e^2)*x^2 + 2*
sqrt(1/2)*sqrt(e*x^2 + d)*(((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7
)*d^3 - (b^6*c^4 - 6*a*b^4*c^5 + 6*a^2*b^2*c^6 + 8*a^3*c^7)*d^2*e
 + (2*a*b^5*c^4 - 13*a^2*b^3*c^5 + 20*a^3*b*c^6)*d*e^2 - (a^2*b^4
```

```
*c^4 - 6*a^3*b^2*c^5 + 8*a^4*c^6)*e^3)*x*sqrt(((b^8 - 6*a*b^6*c +
 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2
*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*
c + 4*a^4*b^2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 -
 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2
 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*
e^4)) - ((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a
^4*c^4)*d^2 - (2*a*b^7 - 14*a^2*b^5*c + 27*a^3*b^3*c^2 - 12*a^4*b
*c^3)*d*e + (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e^2)*x)*sqrt(
-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^
3*c^2)*e + ((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e +
 (a*b^2*c^4 - 4*a^2*c^5)*e^2)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*
a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b
^2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10
)*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3
*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)))/((b^
2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4
*a^2*c^5)*e^2)))/x^2) - sqrt(1/2)*c^2*e^(3/2)*sqrt(-((b^5 - 5*a*b
^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e + ((b
^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 -
4*a^2*c^5)*e^2)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b
^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 -
2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(
(b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b^4
*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b
*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)))/((b^2*c^5 - 4*a*c^
6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)
)*log((((a^2*b^2*c^5 - 4*a^3*c^6)*d^3 - (a^2*b^3*c^4 - 4*a^3*b*c^
5)*d^2*e + (a^3*b^2*c^4 - 4*a^4*c^5)*d*e^2)*x^2*sqrt(((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7
- 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a
^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^
3*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*
d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^
3*c^9)*e^4)) + 2*(a^3*b^4 - 3*a^4*b^2*c + a^5*c^2)*d^2 - 2*(a^4*b
^3 - 2*a^5*b*c)*d*e - ((a^2*b^5 - 3*a^3*b^3*c + a^4*b*c^2)*d^2 -
(5*a^3*b^4 - 14*a^4*b^2*c + 4*a^5*c^2)*d*e + 4*(a^4*b^3 - 2*a^5*b
*c)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^5*c^5 - 7*a*b^3*c
^6 + 12*a^2*b*c^7)*d^3 - (b^6*c^4 - 6*a*b^4*c^5 + 6*a^2*b^2*c^6 +
 8*a^3*c^7)*d^2*e + (2*a*b^5*c^4 - 13*a^2*b^3*c^5 + 20*a^3*b*c^6)
*d*e^2 - (a^2*b^4*c^4 - 6*a^3*b^2*c^5 + 8*a^4*c^6)*e^3)*x*sqrt(((
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 -
 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2
*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d
^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*
a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*
c^8 - 4*a^3*c^9)*e^4)) - ((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*
a^3*b^2*c^3 + 4*a^4*c^4)*d^2 - (2*a*b^7 - 14*a^2*b^5*c + 27*a^3*b
^3*c^2 - 12*a^4*b*c^3)*d*e + (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c
^2)*e^2)*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4
*a^2*b^2*c + 2*a^3*c^2)*e + ((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 -
 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)*sqrt(((b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 -
```

```
 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^
3*b^4*c + 4*a^4*b^2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3
*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d
^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3
*c^9)*e^4)))/((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e
 + (a*b^2*c^4 - 4*a^2*c^5)*e^2)))/x^2) + sqrt(1/2)*c^2*e^(3/2)*sq
rt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2
*a^3*c^2)*e - ((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*
e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*
b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c +
 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^
4*b^2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c
^10)*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*
b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)))/(
(b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4
 - 4*a^2*c^5)*e^2))*log(-(((a^2*b^2*c^5 - 4*a^3*c^6)*d^3 - (a^2*b^
3*c^4 - 4*a^3*b*c^5)*d^2*e + (a^3*b^2*c^4 - 4*a^4*c^5)*d*e^2)*x^2
*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^
4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*
e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/((b^2*c^10 - 4*a
*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*a*b^2*
c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (
a^2*b^2*c^8 - 4*a^3*c^9)*e^4)) - 2*(a^3*b^4 - 3*a^4*b^2*c + a^5*c
^2)*d^2 + 2*(a^4*b^3 - 2*a^5*b*c)*d*e + ((a^2*b^5 - 3*a^3*b^3*c +
 a^4*b*c^2)*d^2 - (5*a^3*b^4 - 14*a^4*b^2*c + 4*a^5*c^2)*d*e + 4*
(a^4*b^3 - 2*a^5*b*c)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b
^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*d^3 - (b^6*c^4 - 6*a*b^4*c^5
 + 6*a^2*b^2*c^6 + 8*a^3*c^7)*d^2*e + (2*a*b^5*c^4 - 13*a^2*b^3*c
^5 + 20*a^3*b*c^6)*d*e^2 - (a^2*b^4*c^4 - 6*a^3*b^2*c^5 + 8*a^4*c
^6)*e^3)*x*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^
3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4
*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/((b^2*
c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8
 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)
*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)) + ((b^8 - 8*a*b^6*c + 20
*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*d^2 - (2*a*b^7 - 14*a^
2*b^5*c + 27*a^3*b^3*c^2 - 12*a^4*b*c^3)*d*e + (a^2*b^6 - 6*a^3*b
^4*c + 8*a^4*b^2*c^2)*e^2)*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c
^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - ((b^2*c^5 - 4*a*c^6
)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)*
sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4
)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e
 + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/((b^2*c^10 - 4*a*
c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*a*b^2*c
^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a
^2*b^2*c^8 - 4*a^3*c^9)*e^4)))/((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^
4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)))/x^2) - sqrt(1
/2)*c^2*e^(3/2)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4
 - 4*a^2*b^2*c + 2*a^3*c^2)*e - ((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c
^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)*sqrt(((b^8 - 6
*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b
^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 -
4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*
(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^1
```

```
0)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4
*a^3*c^9)*e^4)))/((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)
*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2))*log(-(((a^2*b^2*c^5 - 4*a^3*
c^6)*d^3 - (a^2*b^3*c^4 - 4*a^3*b*c^5)*d^2*e + (a^3*b^2*c^4 - 4*a
^4*c^5)*d*e^2)*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2
 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2
)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (
b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^
2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)) - 2*(a^3*b^4 - 3
*a^4*b^2*c + a^5*c^2)*d^2 + 2*(a^4*b^3 - 2*a^5*b*c)*d*e + ((a^2*b
^5 - 3*a^3*b^3*c + a^4*b*c^2)*d^2 - (5*a^3*b^4 - 14*a^4*b^2*c + 4
*a^5*c^2)*d*e + 4*(a^4*b^3 - 2*a^5*b*c)*e^2)*x^2 - 2*sqrt(1/2)*sq
rt(e*x^2 + d)*(((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*d^3 - (b^6
*c^4 - 6*a*b^4*c^5 + 6*a^2*b^2*c^6 + 8*a^3*c^7)*d^2*e + (2*a*b^5*
c^4 - 13*a^2*b^3*c^5 + 20*a^3*b*c^6)*d*e^2 - (a^2*b^4*c^4 - 6*a^3
*b^2*c^5 + 8*a^4*c^6)*e^3)*x*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a
^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^
2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)
*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*
c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)) + ((b^
8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*d^2
- (2*a*b^7 - 14*a^2*b^5*c + 27*a^3*b^3*c^2 - 12*a^4*b*c^3)*d*e +
(a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e^2)*x)*sqrt(-((b^5 - 5*a
*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (
(b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4
- 4*a^2*c^5)*e^2)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2
- 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)
/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b
^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2
*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)))/((b^2*c^5 - 4*a*
c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^
2)))/x^2) - 2*sqrt(e*x^2 + d)*c*sqrt(e)*x - (c*d + 2*b*e)*log(2*s
qrt(e*x^2 + d)*e*x - (2*e*x^2 + d)*sqrt(e)))/(c^2*e^(3/2)), -1/4*
(sqrt(1/2)*c^2*sqrt(-e)*e*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*
d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e + ((b^2*c^5 - 4*a*c^6)*d^
2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)*sqrt
(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^
2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (
a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11
)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 -
 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b
^2*c^8 - 4*a^3*c^9)*e^4)))/((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 -
4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2))*log((((a^2*b^2*c^5
 - 4*a^3*c^6)*d^3 - (a^2*b^3*c^4 - 4*a^3*b*c^5)*d^2*e + (a^3*b^2*
c^4 - 4*a^4*c^5)*d*e^2)*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^
3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2
*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*
d^3*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c
^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)) + 2*(a^
3*b^4 - 3*a^4*b^2*c + a^5*c^2)*d^2 - 2*(a^4*b^3 - 2*a^5*b*c)*d*e
```

```
- ((a^2*b^5 - 3*a^3*b^3*c + a^4*b*c^2)*d^2 - (5*a^3*b^4 - 14*a^4*
b^2*c + 4*a^5*c^2)*d*e + 4*(a^4*b^3 - 2*a^5*b*c)*e^2)*x^2 + 2*sqr
t(1/2)*sqrt(e*x^2 + d)*(((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*d
^3 - (b^6*c^4 - 6*a*b^4*c^5 + 6*a^2*b^2*c^6 + 8*a^3*c^7)*d^2*e +
(2*a*b^5*c^4 - 13*a^2*b^3*c^5 + 20*a^3*b*c^6)*d*e^2 - (a^2*b^4*c^
4 - 6*a^3*b^2*c^5 + 8*a^4*c^6)*e^3)*x*sqrt(((b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^
5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c +
 4*a^4*b^2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*
a*b*c^10)*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 -
2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4
)) - ((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*
c^4)*d^2 - (2*a*b^7 - 14*a^2*b^5*c + 27*a^3*b^3*c^2 - 12*a^4*b*c^
3)*d*e + (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e^2)*x)*sqrt(-((
b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c
^2)*e + ((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a
*b^2*c^4 - 4*a^2*c^5)*e^2)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^
2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3
*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*
c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d
^3*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^
8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)))/((b^2*c
^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^
2*c^5)*e^2)))/x^2) - sqrt(1/2)*c^2*sqrt(-e)*e*sqrt(-((b^5 - 5*a*b
^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e + ((b
^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 -
4*a^2*c^5)*e^2)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b
^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 -
2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(
(b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b^4
*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b
*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)))/((b^2*c^5 - 4*a*c^
6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)
)*log((((a^2*b^2*c^5 - 4*a^3*c^6)*d^3 - (a^2*b^3*c^4 - 4*a^3*b*c^
5)*d^2*e + (a^3*b^2*c^4 - 4*a^4*c^5)*d*e^2)*x^2*sqrt(((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7
- 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a
^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^
3*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*
d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^
3*c^9)*e^4)) + 2*(a^3*b^4 - 3*a^4*b^2*c + a^5*c^2)*d^2 - 2*(a^4*b
^3 - 2*a^5*b*c)*d*e - ((a^2*b^5 - 3*a^3*b^3*c + a^4*b*c^2)*d^2 -
(5*a^3*b^4 - 14*a^4*b^2*c + 4*a^5*c^2)*d*e + 4*(a^4*b^3 - 2*a^5*b
*c)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^5*c^5 - 7*a*b^3*c
^6 + 12*a^2*b*c^7)*d^3 - (b^6*c^4 - 6*a*b^4*c^5 + 6*a^2*b^2*c^6 +
 8*a^3*c^7)*d^2*e + (2*a*b^5*c^4 - 13*a^2*b^3*c^5 + 20*a^3*b*c^6)
*d*e^2 - (a^2*b^4*c^4 - 6*a^3*b^2*c^5 + 8*a^4*c^6)*e^3)*x*sqrt(((
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 -
 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2
*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d
^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*
a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*
c^8 - 4*a^3*c^9)*e^4)) - ((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*
a^3*b^2*c^3 + 4*a^4*c^4)*d^2 - (2*a*b^7 - 14*a^2*b^5*c + 27*a^3*b
```

^3*c^2 - 12*a^4*b*c^3)*d*e + (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c ^2)*e^2)*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4 *a^2*b^2*c + 2*a^3*c^2)*e + ((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)*sqrt(((b^8 - 6*a*b ^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^ 3*b^4*c + 4*a^4*b^2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3 *c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d ^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3 *c^9)*e^4)))/((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)))/x^2) + sqrt(1/2)*c^2*sqrt(-e)*e *sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - ((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5) *d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)*sqrt(((b^8 - 6*a*b^6*c + 11*a ^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5* c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4 *a^4*b^2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a* b*c^10)*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2* (a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)))/((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c ^4 - 4*a^2*c^5)*e^2))*log(-(((a^2*b^2*c^5 - 4*a^3*c^6)*d^3 - (a^2 *b^3*c^4 - 4*a^3*b*c^5)*d^2*e + (a^3*b^2*c^4 - 4*a^4*c^5)*d*e^2)* x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4 *c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3) *d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*a*b ^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)) - 2*(a^3*b^4 - 3*a^4*b^2*c + a^ 5*c^2)*d^2 + 2*(a^4*b^3 - 2*a^5*b*c)*d*e + ((a^2*b^5 - 3*a^3*b^3* c + a^4*b*c^2)*d^2 - (5*a^3*b^4 - 14*a^4*b^2*c + 4*a^5*c^2)*d*e + 4*(a^4*b^3 - 2*a^5*b*c)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*d^3 - (b^6*c^4 - 6*a*b^4* c^5 + 6*a^2*b^2*c^6 + 8*a^3*c^7)*d^2*e + (2*a*b^5*c^4 - 13*a^2*b^ 3*c^5 + 20*a^3*b*c^6)*d*e^2 - (a^2*b^4*c^4 - 6*a^3*b^2*c^5 + 8*a^ 4*c^6)*e^3)*x*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2 *c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2* a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/((b ^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c ^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c ^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)) + ((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*d^2 - (2*a*b^7 - 14 *a^2*b^5*c + 27*a^3*b^3*c^2 - 12*a^4*b*c^3)*d*e + (a^2*b^6 - 6*a^ 3*b^4*c + 8*a^4*b^2*c^2)*e^2)*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2* b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - ((b^2*c^5 - 4*a* c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^ 2)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4* c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)* d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/((b^2*c^10 - 4 *a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*a*b^ 2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)))/((b^2*c^5 - 4*a*c^6)*d^2 - (b^3 *c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)))/x^2) - sqr t(1/2)*c^2*sqrt(-e)*e*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - ((b^2*c^5 - 4*a*c^6)*d^2 -

```
(b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)*sqrt(((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 -
2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*
b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^
4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*a
^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c
^8 - 4*a^3*c^9)*e^4)))/((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*
b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2))*log(-(((a^2*b^2*c^5 -
4*a^3*c^6)*d^3 - (a^2*b^3*c^4 - 4*a^3*b*c^5)*d^2*e + (a^3*b^2*c^4
 - 4*a^4*c^5)*d*e^2)*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
- 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b
^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^
2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3
*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8
- 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)) - 2*(a^3*b
^4 - 3*a^4*b^2*c + a^5*c^2)*d^2 + 2*(a^4*b^3 - 2*a^5*b*c)*d*e + (
(a^2*b^5 - 3*a^3*b^3*c + a^4*b*c^2)*d^2 - (5*a^3*b^4 - 14*a^4*b^2
*c + 4*a^5*c^2)*d*e + 4*(a^4*b^3 - 2*a^5*b*c)*e^2)*x^2 - 2*sqrt(1
/2)*sqrt(e*x^2 + d)*(((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*d^3
- (b^6*c^4 - 6*a*b^4*c^5 + 6*a^2*b^2*c^6 + 8*a^3*c^7)*d^2*e + (2*
a*b^5*c^4 - 13*a^2*b^3*c^5 + 20*a^3*b*c^6)*d*e^2 - (a^2*b^4*c^4 -
 6*a^3*b^2*c^5 + 8*a^4*c^6)*e^3)*x*sqrt(((b^8 - 6*a*b^6*c + 11*a^
2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c
 + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*
a^4*b^2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b
*c^10)*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(
a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4))
+ ((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4
)*d^2 - (2*a*b^7 - 14*a^2*b^5*c + 27*a^3*b^3*c^2 - 12*a^4*b*c^3)*
d*e + (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e^2)*x)*sqrt(-((b^5
 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)
*e - ((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^
2*c^4 - 4*a^2*c^5)*e^2)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^
3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2
)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*
e + (b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 -
 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)))/((b^2*c^5
- 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c
^5)*e^2)))/x^2) - 2*sqrt(e*x^2 + d)*c*sqrt(-e)*x + 2*(c*d + 2*b*e
)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(c^2*sqrt(-e)*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**6/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.380 $\int \frac{x^4}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=298

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

```
[Out] -(((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b
- Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d +
e*x^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b
^2 - 4*a*c])*e])) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + ArcTanh[(Sqrt[e]*x)/Sqrt[d +
 e*x^2]]/(c*Sqrt[e])
```

Rubi [A] time = 1.75847, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b
- Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d +
e*x^2])]])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b
^2 - 4*a*c])*e])) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[d + e*x^2])]])/(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + ArcTanh[(Sqrt[e]*x)/Sqrt[d +
 e*x^2]]/(c*Sqrt[e])
```

Rubi in Sympy [A] time = 173.891, size = 292, normalized size = 0.98

$$-\frac{\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{x\sqrt{be-2cd+e\sqrt{-4ac+b^2}}}{\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{d+ex^2}}\right)}{c\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\sqrt{be-2cd+e\sqrt{-4ac+b^2}}}+\frac{\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{x\sqrt{be-2cd-e\sqrt{-4ac+b^2}}}{\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\sqrt{be-2cd-e\sqrt{-4ac+b^2}}}+\frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] -(-2*a*c + b**2 + b*sqrt(-4*a*c + b**2))*atanh(x*sqrt(b*e - 2*c*d
 + e*sqrt(-4*a*c + b**2))/(sqrt(b + sqrt(-4*a*c + b**2))*sqrt(d +
 e*x**2)))/(c*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)*s
qrt(b*e - 2*c*d + e*sqrt(-4*a*c + b**2))) + (-2*a*c + b**2 - b*sq
rt(-4*a*c + b**2))*atanh(x*sqrt(b*e - 2*c*d - e*sqrt(-4*a*c + b**
2))/(sqrt(b - sqrt(-4*a*c + b**2))*sqrt(d + e*x**2)))/(c*sqrt(b -
 sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)*sqrt(b*e - 2*c*d - e*sq
rt(-4*a*c + b**2))) + atanh(sqrt(e)*x/sqrt(d + e*x**2))/(c*sqrt(e
))
```

Mathematica [A] time = 0.517109, size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[x^4/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[x^4/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.028, size = 200, normalized size = 0.7

$$\frac{1}{c}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)\frac{1}{\sqrt{e}}+\frac{1}{2c}\sqrt{e}\sum_{\_R=RootOf\left(c\_Z^4+(4be-4cd)\_Z^3+(16ae^2-8bde+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z+cd^4\right)}\frac{\_R^2b+2\left(2ae-bd\right)\_R+b}{\_R^3c+3\_R^2be-3\_R^2cd+8\_Rae^2-4\_Rbde}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] 1/c*ln(x*e^(1/2)+(e*x^2+d)^(1/2))/e^(1/2)+1/2/c*e^(1/2)*sum((_R^2
*b+2*(2*a*e-b*d)*_R+b*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e
^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-x*e^(
1/2))^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e
+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{(cx^4+bx^2+a)\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

Fricas [A] time = 15.2985, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(1/2)*c*sqrt(e)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a
^2*c)*e - ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e +
(a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c
```

```
 + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d
^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a
^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^
4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b
*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e + ((a*
b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2
*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b
^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^
7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 -
 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^
2*c^4 - 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2
 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 + 2*sqr
t(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3
- (b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*
a^2*b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)
*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3
 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^
6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3
*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) + ((b
^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^
3*c^2)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)
*d - (a*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 -
4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 +
 (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2
*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 -
2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*
e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 -
 (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2)
- sqrt(1/2)*c*sqrt(e)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c
)*e - ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b
^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a
^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 -
 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c
^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 -
4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3
)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e + ((a*b^2*
c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2
*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c
 + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d
^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a
^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^
4 - 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (
a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 - 2*sqrt(1/
2)*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b
^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*
b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*s
qrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a
^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d
^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4
 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) + ((b^5 -
 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^
2)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d -
 (a*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*
b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^
```

4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6
 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*
b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3
+ (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^
3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) - sq
rt(1/2)*c*sqrt(e)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e
+ ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c
^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c
^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(
b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*
d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^
3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*
e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e - ((a*b^2*c^3
 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2
 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a
^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 -
 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c
^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 -
4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^
3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 + 2*sqrt(1/2)*s
qrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c
^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*
c^3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt(
(a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b
*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e
 + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4
*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - ((b^5 - 5*a
*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d
*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*
b^2 - 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^
3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 -
2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4
*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*
c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a
^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^
2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) + sqrt(1
/2)*c*sqrt(e)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + ((
b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 -
 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*
d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*
c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*
e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^
5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e +
(a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e - ((a*b^2*c^3 - 4*
a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4
*a^3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c
^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(
b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*
d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^
3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^3 -
a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(
e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2 -
 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3
+ 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a^2

```
*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*
d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (
b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2
*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - ((b^5 - 5*a*b^3
*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e +
 (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2
- 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d
*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*
b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c
^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5
- 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b
^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 -
4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) + 2*log(-2*s
qrt(e*x^2 + d)*e*x - (2*e*x^2 + d)*sqrt(e)))/(c*sqrt(e)), 1/4*(sq
rt(1/2)*c*sqrt(-e)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e
 - ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*
c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*
c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*
(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)
*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a
^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d
*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e + ((a*b^2*c^3
 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^
2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c +
a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4
- 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*
c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 -
 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b
^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 + 2*sqrt(1/2)*
sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*
c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2
*c^3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt
((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*
b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*
e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 -
4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) + ((b^5 - 5*
a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*
d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a
*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c
^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 -
 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 -
4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2
*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (
a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c
^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) - sqrt(
1/2)*c*sqrt(-e)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e -
((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2
 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2
)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^
3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^
2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*
c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e
+ (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e + ((a*b^2*c^3 -
4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 -
```

```
 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2
*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2
*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6
)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*
a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^3
- a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 - 2*sqrt(1/2)*sqr
t(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2
 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^
3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a
^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c
)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e +
 (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a
^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) + ((b^5 - 5*a*b
^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e
 + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^
2 - 2*a^2*c)*e - ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)
*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*
a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a
*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^
5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2
*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2
- 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) - sqrt(1/2
)*c*sqrt(-e)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + ((b
^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 -
4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d
^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c
^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e
^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5
)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (
a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e - ((a*b^2*c^3 - 4*a
^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*
a^3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^
2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b
^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d
^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3
*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^3 - a
^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e
*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2 -
5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 +
 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a^2*
b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d
*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b
^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*
b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - ((b^5 - 5*a*b^3*
c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e +
(a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 -
 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*
e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b
^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^
7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 -
 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^
2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4
*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) + sqrt(1/2)*c
*sqrt(-e)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + ((b^2*
```

```
c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a
^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2
- 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5
- 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2
- 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e
^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b
^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e - ((a*b^2*c^3 - 4*a^2*
c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3
*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*
d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*
c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*
e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^
5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^3 - a^2*
b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^
2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2 - 5*a
*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*
a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a^2*b^2
*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)
/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*
c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c
^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - ((b^5 - 5*a*b^3*c +
 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e + (a^
2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*
a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e +
 (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*
c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*
d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*
a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c
^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*
b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) + 4*arctan(sqrt(
-e)*x/sqrt(e*x^2 + d)))/(c*sqrt(-e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)

[Out] Integral(x**4/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.381 $\int \frac{x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx$

Optimal. Leaf size=240

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/
(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) + (S
qrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqr
t[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 0.907831, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/
(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) + (S
qrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqr
t[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi in Sympy [A] time = 78.5665, size = 219, normalized size = 0.91

$$-\frac{\sqrt{b-\sqrt{-4ac+b^2}}\operatorname{atanh}\left(\frac{x\sqrt{be-2cd-e\sqrt{-4ac+b^2}}}{\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{d+ex^2}}\right)}{\sqrt{-4ac+b^2}\sqrt{be-2cd-e\sqrt{-4ac+b^2}}}+\frac{\sqrt{b+\sqrt{-4ac+b^2}}\operatorname{atanh}\left(\frac{x\sqrt{be-2cd+e\sqrt{-4ac+b^2}}}{\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{d+ex^2}}\right)}{\sqrt{-4ac+b^2}\sqrt{be-2cd+e\sqrt{-4ac+b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] -sqrt(b - sqrt(-4*a*c + b**2))*atanh(x*sqrt(b*e - 2*c*d - e*sqrt(
-4*a*c + b**2))/(sqrt(b - sqrt(-4*a*c + b**2))*sqrt(d + e*x**2)))
/(sqrt(-4*a*c + b**2)*sqrt(b*e - 2*c*d - e*sqrt(-4*a*c + b**2)))
+ sqrt(b + sqrt(-4*a*c + b**2))*atanh(x*sqrt(b*e - 2*c*d + e*sqrt
(-4*a*c + b**2))/(sqrt(b + sqrt(-4*a*c + b**2))*sqrt(d + e*x**2))
)/(sqrt(-4*a*c + b**2)*sqrt(b*e - 2*c*d + e*sqrt(-4*a*c + b**2)))
```

Mathematica [A] time = 0.184479, size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[x^2/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[x^2/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.024, size = 161, normalized size = 0.7

$$-\frac{1}{2}\sqrt{e}\sum_{\_R=RootOf\left(c\_Z^4+(4be-4cd)\_Z^3+(16ae^2-8bde+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z+cd^4\right)}\frac{\_R^2-2\_Rd+d^2}{\_R^3c+3\_R^2be-3\_R^2cd+8\_Rae^2-4\_Rbde+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] -1/2*e^(1/2)*sum((_R^2-2*_R*d+d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+
8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/
2)-x*e^(1/2))^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2
-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(cx^4+bx^2+a)\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

Fricas [A] time = 4.08212, size = 4583, normalized size = 19.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3
- 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*
c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2
*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)
*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4
*a^2*c)*e^2))*log((((b^2*c - 4*a*c^2)*d^3 - (b^3 - 4*a*b*c)*d^2*e
 + (a*b^2 - 4*a^2*c)*d*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2
*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^
2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x^2 +
 2*a*d^2 - (b*d^2 - 4*a*d*e)*x^2 + 2*sqrt(1/2)*((b^2 - 4*a*c)*d^2
*x - ((b^3*c - 4*a*b*c^2)*d^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2
*e + 3*(a*b^3 - 4*a^2*b*c)*d*e^2 - 2*(a^2*b^2 - 4*a^3*c)*e^3)*sqr
t(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b
^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3
 + (a^2*b^2 - 4*a^3*c)*e^4))*x)*sqrt(e*x^2 + d)*sqrt(-(b*d - 2*a*
e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2
*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2
)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^
2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2
- (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)))/x^2) - 1/4*sqrt(
1/2)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c
)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4
- 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2
*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/(
(b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e
^2))*log((((b^2*c - 4*a*c^2)*d^3 - (b^3 - 4*a*b*c)*d^2*e + (a*b^2
 - 4*a^2*c)*d*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c -
 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*
b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x^2 + 2*a*d^2
- (b*d^2 - 4*a*d*e)*x^2 - 2*sqrt(1/2)*((b^2 - 4*a*c)*d^2*x - ((b^
3*c - 4*a*b*c^2)*d^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e + 3*(a
*b^3 - 4*a^2*b*c)*d*e^2 - 2*(a^2*b^2 - 4*a^3*c)*e^3)*sqrt(d^2/((b
^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*
b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b
^2 - 4*a^3*c)*e^4))*x)*sqrt(e*x^2 + d)*sqrt(-(b*d - 2*a*e + ((b^2
```

```
*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*
sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e +
 (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*
e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 -
4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt
(-(b*d - 2*a*e - ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (
a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*
c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*
(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c -
4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log(
-(((b^2*c - 4*a*c^2)*d^3 - (b^3 - 4*a*b*c)*d^2*e + (a*b^2 - 4*a^2
*c)*d*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c
^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*
a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x^2 - 2*a*d^2 + (b*d^2
 - 4*a*d*e)*x^2 + 2*sqrt(1/2)*((b^2 - 4*a*c)*d^2*x + ((b^3*c - 4*
a*b*c^2)*d^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e + 3*(a*b^3 - 4
*a^2*b*c)*d*e^2 - 2*(a^2*b^2 - 4*a^3*c)*e^3)*sqrt(d^2/((b^2*c^2 -
 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c -
8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a
^3*c)*e^4))*x)*sqrt(e*x^2 + d)*sqrt(-(b*d - 2*a*e - ((b^2*c - 4*a
*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2
/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 -
2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a
^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)
*d*e + (a*b^2 - 4*a^2*c)*e^2)))/x^2) + 1/4*sqrt(1/2)*sqrt(-(b*d -
 2*a*e - ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 -
4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*
b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 -
 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)
*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log(-(((b^2*
c - 4*a*c^2)*d^3 - (b^3 - 4*a*b*c)*d^2*e + (a*b^2 - 4*a^2*c)*d*e^
2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*
e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)
*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x^2 - 2*a*d^2 + (b*d^2 - 4*a*d
*e)*x^2 - 2*sqrt(1/2)*((b^2 - 4*a*c)*d^2*x + ((b^3*c - 4*a*b*c^2)
*d^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e + 3*(a*b^3 - 4*a^2*b*c
)*d*e^2 - 2*(a^2*b^2 - 4*a^3*c)*e^3)*sqrt(d^2/((b^2*c^2 - 4*a*c^3
)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^
2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^
4))*x)*sqrt(e*x^2 + d)*sqrt(-(b*d - 2*a*e - ((b^2*c - 4*a*c^2)*d^
2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c
^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*
c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 -
 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (
a*b^2 - 4*a^2*c)*e^2)))/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.382 $\int \frac{1}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=243

$$\frac{2c\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{2c\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

```
[Out] (2*c*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b -
 Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b
- Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (
2*c*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b +
Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 0.459747, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{2c\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{2c\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*c*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b -
 Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b
- Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (
2*c*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b +
Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi in Sympy [A] time = 70.8734, size = 226, normalized size = 0.93

$$-\frac{2c\,\mathrm{atanh}\left(\frac{x\sqrt{be-2cd+e\sqrt{-4ac+b^2}}}{\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\sqrt{be-2cd+e\sqrt{-4ac+b^2}}}+\frac{2c\,\mathrm{atanh}\left(\frac{x\sqrt{be-2cd-e\sqrt{-4ac+b^2}}}{\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\sqrt{be-2cd-e\sqrt{-4ac+b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] -2*c*atanh(x*sqrt(b*e - 2*c*d + e*sqrt(-4*a*c + b**2))/(sqrt(b +
sqrt(-4*a*c + b**2))*sqrt(d + e*x**2)))/(sqrt(b + sqrt(-4*a*c + b
**2))*sqrt(-4*a*c + b**2)*sqrt(b*e - 2*c*d + e*sqrt(-4*a*c + b**2
))) + 2*c*atanh(x*sqrt(b*e - 2*c*d - e*sqrt(-4*a*c + b**2))/(sqrt
(b - sqrt(-4*a*c + b**2))*sqrt(d + e*x**2)))/(sqrt(b - sqrt(-4*a*
c + b**2))*sqrt(-4*a*c + b**2)*sqrt(b*e - 2*c*d - e*sqrt(-4*a*c +
 b**2)))
```

Mathematica [A] time = 0.0995064, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[1/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.021, size = 151, normalized size = 0.6

$$-2\,e^{3/2}\sum_{\_R=RootOf\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+(16\,ae^2-8\,bde+6\,cd^2)\_Z^2+(4\,bd^2e-4\,cd^3)\_Z+cd^4\right)}\frac{\_R\ln\left(\left(\sqrt{ex^2+d}-x\sqrt{e}\right)^2-\right.}{\_R^3c+3\,\_R^2be-3\,\_R^2cd+8\,\_R\,ae^2-4\,\_R\,bde+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] -2*e^(3/2)*sum(_R/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b
*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-x*e^(1/2))^2-_
R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)
*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)\sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

Fricas [A] time = 10.3954, size = 6152, normalized size = 25.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*
c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqr
t((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4
 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*
a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a
^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e
 + (a^2*b^2 - 4*a^3*c)*e^2))*log(-(2*a*c^2*d^2 - 2*a*b*c*d*e + ((
a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2
*b^2*c - 4*a^3*c^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^
2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d
^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 -
 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - (b*c^2*d^2 + 4*a*
b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d
)*((2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*a^3*b*c^2)
*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4
*a^4*b*c)*e^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c
^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^
4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*
e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - ((a*b^2*c - 4*a^2*c^2)*d*e - (a
*b^3 - 4*a^2*b*c)*e^2)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^
2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3
```

```
*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a
^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^
3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a
^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*
a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)))/x^2) - 1/4*sqrt(1/2)*sq
rt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^
3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b
*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c -
 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2
 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a
*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*
a^3*c)*e^2))*log(-(2*a*c^2*d^2 - 2*a*b*c*d*e + ((a*b^2*c^2 - 4*a^
2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*c - 4*a^3*c
^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2
 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4
- 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^
3 + (a^4*b^2 - 4*a^5*c)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c
 + 4*a*c^2)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c
^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e + (a^2*b^
4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4*a^4*b*c)*e^3)*x
*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*
d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c
- 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 -
 4*a^5*c)*e^4)) - ((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c
)*e^2)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)
*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c
^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*
(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*
c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c
)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (
a^2*b^2 - 4*a^3*c)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*c*d - (b^
2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d
*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2
)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^
3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 -
4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c
^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log
(-(2*a*c^2*d^2 - 2*a*b*c*d*e - ((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*
b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*e^2)*x^2*s
qrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^
4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c -
8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4
*a^5*c)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)
*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^2 - 4*a^3*c^3)*
d^3 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c
- 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4*a^4*b*c)*e^3)*x*sqrt((c^2*d^2 -
 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3
*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2
*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4))
+ ((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*sqrt(-
(b*c*d - (b^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 -
4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d
*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a
^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2
```

```
*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2
*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*
c)*e^2)))/x^2) + 1/4*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (
(a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 -
4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2
- 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 -
 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3
 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3
 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log(-(2*a*c^2*d^2 -
 2*a*b*c*d*e - ((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*
c^2)*d^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2
*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c
 - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e
^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) -
(b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 - 2*sqrt(1/
2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3
*c - 4*a^3*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e
^2 - (a^3*b^3 - 4*a^4*b*c)*e^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2
*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2
)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^
3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) + ((a*b^2*c - 4*
a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*sqrt(-(b*c*d - (b^2 -
2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e +
 (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((
a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e
+ (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^
4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*
d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)))/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.383 $\int \frac{1}{x^2\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=280

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{adx}$$

```
[Out] -(Sqrt[d + e*x^2]/(a*d*x)) - (c*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 -
4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*
c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (c*(1 - b/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt
[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*
Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 1.36195, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{adx}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[d + e*x^2]/(a*d*x)) - (c*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 -
4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*
c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (c*(1 - b/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt
[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*
Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi in Sympy [A] time = 152.541, size = 269, normalized size = 0.96

$$\frac{c\left(b-\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{x\sqrt{be-2cd+e\sqrt{-4ac+b^2}}}{\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{d+ex^2}}\right)}{a\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\sqrt{be-2cd+e\sqrt{-4ac+b^2}}} - \frac{c\left(b+\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{x\sqrt{be-2cd-e\sqrt{-4ac+b^2}}}{\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}\sqrt{be-2cd-e\sqrt{-4ac+b^2}}} - \frac{\sqrt{d+ex^2}}{adx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] c*(b - sqrt(-4*a*c + b**2))*atanh(x*sqrt(b*e - 2*c*d + e*sqrt(-4*
a*c + b**2))/(sqrt(b + sqrt(-4*a*c + b**2))*sqrt(d + e*x**2)))/(a
*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)*sqrt(b*e - 2*c
*d + e*sqrt(-4*a*c + b**2))) - c*(b + sqrt(-4*a*c + b**2))*atanh(
x*sqrt(b*e - 2*c*d - e*sqrt(-4*a*c + b**2))/(sqrt(b - sqrt(-4*a*c
 + b**2))*sqrt(d + e*x**2)))/(a*sqrt(b - sqrt(-4*a*c + b**2))*sqr
t(-4*a*c + b**2)*sqrt(b*e - 2*c*d - e*sqrt(-4*a*c + b**2))) - sqr
t(d + e*x**2)/(a*d*x)
```

Mathematica [A] time = 0.606775, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/(x^2*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.031, size = 197, normalized size = 0.7

$$-\frac{1}{adx}\sqrt{ex^2+d} + \frac{1}{2a}\sqrt{e}\sum_{\_R=RootOf\left(c\_Z^4+(4be-4cd)\_Z^3+(16ae^2-8bde+6cd^2)\_Z^2+(4bd^2e-4cd^3)\_Z+cd^4\right)}\frac{\_R^2c+2\,(2be-cd)\_R+c}{\_R^3c+3\_R^2be-3\_R^2cd+8\_Rae^2-4\_Rbde}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] -(e*x^2+d)^(1/2)/a/d/x+1/2/a*e^(1/2)*sum((_R^2*c+2*(2*b*e-c*d)*_R
+c*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*
c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-x*e^(1/2))^2-_R),_R=Root
Of(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b
*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)\sqrt{ex^2 + d}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^2), x)
```

Fricas [A] time = 6.18433, size = 8682, normalized size = 31.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*a*d*x*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b
^2*c + 2*a^2*c^2)*e - ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4
*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2
*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e +
 (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3
)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*
c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2
 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^
4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*log((((a^3*b^2*c^3 - 4*a^4
*c^4)*d^3 - (a^3*b^3*c^2 - 4*a^4*b*c^3)*d^2*e + (a^4*b^2*c^2 - 4*
a^5*c^3)*d*e^2)*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 -
 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4
*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c
- 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^
2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)) + 2
*(a*b^2*c^3 - a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 2*a^2*b*c^3)*d*e - ((
b^3*c^3 - a*b*c^4)*d^2 - (b^4*c^2 + 2*a*b^2*c^3 - 4*a^2*c^4)*d*e
```

```
+ 4*(a*b^3*c^2 - 2*a^2*b*c^3)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 +
 d)*(((a^4*b^3*c^2 - 4*a^5*b*c^3)*d^3 - 2*(a^4*b^4*c - 5*a^5*b^2*
c^2 + 4*a^6*c^3)*d^2*e + (a^4*b^5 - 5*a^5*b^3*c + 4*a^6*b*c^2)*d*
e^2 - (a^5*b^4 - 6*a^6*b^2*c + 8*a^7*c^2)*e^3)*x*sqrt(((b^4*c^2 -
 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^
3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4
*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*
a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 +
(a^8*b^2 - 4*a^9*c)*e^4)) + ((a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c
^4)*d^2 - (2*a*b^5*c - 11*a^2*b^3*c^2 + 12*a^3*b*c^3)*d*e + (a*b^
6 - 6*a^2*b^4*c + 8*a^3*b^2*c^2)*e^2)*x)*sqrt(-((b^3*c - 3*a*b*c^
2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - ((a^3*b^2*c - 4*a^4*c^2)
*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(
((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 +
 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*
b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a
^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*
c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^
2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)))/x^2) -
 sqrt(1/2)*a*d*x*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c
+ 2*a^2*c^2)*e - ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*
b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3
+ a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6
 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4
 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8
*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*
a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c
)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*log((((a^3*b^2*c^3 - 4*a^4*c^4)
*d^3 - (a^3*b^3*c^2 - 4*a^4*b*c^3)*d^2*e + (a^4*b^2*c^2 - 4*a^5*c
^3)*d*e^2)*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b
^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*
b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a
^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2
*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)) + 2*(a*b
^2*c^3 - a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 2*a^2*b*c^3)*d*e - ((b^3*c
^3 - a*b*c^4)*d^2 - (b^4*c^2 + 2*a*b^2*c^3 - 4*a^2*c^4)*d*e + 4*(
a*b^3*c^2 - 2*a^2*b*c^3)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(
((a^4*b^3*c^2 - 4*a^5*b*c^3)*d^3 - 2*(a^4*b^4*c - 5*a^5*b^2*c^2 +
 4*a^6*c^3)*d^2*e + (a^4*b^5 - 5*a^5*b^3*c + 4*a^6*b*c^2)*d*e^2 -
 (a^5*b^4 - 6*a^6*b^2*c + 8*a^7*c^2)*e^3)*x*sqrt(((b^4*c^2 - 2*a*
b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*
e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*
c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b
^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*
b^2 - 4*a^9*c)*e^4)) + ((a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d
^2 - (2*a*b^5*c - 11*a^2*b^3*c^2 + 12*a^3*b*c^3)*d*e + (a*b^6 - 6
*a^2*b^4*c + 8*a^3*b^2*c^2)*e^2)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d
- (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - ((a^3*b^2*c - 4*a^4*c^2)*d^2
- (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4
*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^
2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c
^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^
4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*
e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (
```

a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)))/x^2) + sqrt
(1/2)*a*d*x*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a
^2*c^2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*
d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2
*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*
a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*
(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*
c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c
)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e
 + (a^4*b^2 - 4*a^5*c)*e^2))*log(-(((a^3*b^2*c^3 - 4*a^4*c^4)*d^3
 - (a^3*b^3*c^2 - 4*a^4*b*c^3)*d^2*e + (a^4*b^2*c^2 - 4*a^5*c^3)*
d*e^2)*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c
 - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*
c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b
*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^
7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)) - 2*(a*b^2*c
^3 - a^2*c^4)*d^2 + 2*(a*b^3*c^2 - 2*a^2*b*c^3)*d*e + ((b^3*c^3 -
 a*b*c^4)*d^2 - (b^4*c^2 + 2*a*b^2*c^3 - 4*a^2*c^4)*d*e + 4*(a*b^
3*c^2 - 2*a^2*b*c^3)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^
4*b^3*c^2 - 4*a^5*b*c^3)*d^3 - 2*(a^4*b^4*c - 5*a^5*b^2*c^2 + 4*a
^6*c^3)*d^2*e + (a^4*b^5 - 5*a^5*b^3*c + 4*a^6*b*c^2)*d*e^2 - (a^
5*b^4 - 6*a^6*b^2*c + 8*a^7*c^2)*e^3)*x*sqrt(((b^4*c^2 - 2*a*b^2*
c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e +
(b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)
d^4 - 2(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c
 - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2
- 4*a^9*c)*e^4)) - ((a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d^2 -
 (2*a*b^5*c - 11*a^2*b^3*c^2 + 12*a^3*b*c^3)*d*e + (a*b^6 - 6*a^2
*b^4*c + 8*a^3*b^2*c^2)*e^2)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b
^4 - 4*a*b^2*c + 2*a^2*c^2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a
^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2
 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*
c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 -
 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 -
2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3
+ (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*
b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)))/x^2) - sqrt(1/2
)*a*d*x*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c
^2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e
+ (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4
)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^
4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6
*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)
*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^
4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (
a^4*b^2 - 4*a^5*c)*e^2))*log(-(((a^3*b^2*c^3 - 4*a^4*c^4)*d^3 - (
a^3*b^3*c^2 - 4*a^4*b*c^3)*d^2*e + (a^4*b^2*c^2 - 4*a^5*c^3)*d*e^
2)*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3
*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)
*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2
)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^
3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)) - 2*(a*b^2*c^3 -
 a^2*c^4)*d^2 + 2*(a*b^3*c^2 - 2*a^2*b*c^3)*d*e + ((b^3*c^3 - a*b
*c^4)*d^2 - (b^4*c^2 + 2*a*b^2*c^3 - 4*a^2*c^4)*d*e + 4*(a*b^3*c^

```
2 - 2*a^2*b*c^3)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^
3*c^2 - 4*a^5*b*c^3)*d^3 - 2*(a^4*b^4*c - 5*a^5*b^2*c^2 + 4*a^6*c
^3)*d^2*e + (a^4*b^5 - 5*a^5*b^3*c + 4*a^6*b*c^2)*d*e^2 - (a^5*b^
4 - 6*a^6*b^2*c + 8*a^7*c^2)*e^3)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3
+ a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6
 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4
 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8
*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*
a^9*c)*e^4)) - ((a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d^2 - (2*
a*b^5*c - 11*a^2*b^3*c^2 + 12*a^3*b*c^3)*d*e + (a*b^6 - 6*a^2*b^4
*c + 8*a^3*b^2*c^2)*e^2)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 -
 4*a*b^2*c + 2*a^2*c^2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b
^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2
*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)
*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a
^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^
7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a
^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3
- 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)))/x^2) + 4*sqrt(e*x^2
 + d))/(a*d*x)
```

Sympy **[F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{d + ex^2}\,(a + bx^2 + cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS **[F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.384 $\int \frac{1}{x^4\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=341

$$\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{b\sqrt{d+ex^2}}{a^2dx}+\frac{2e\sqrt{d+ex^2}}{3ad^2x}-\frac{\sqrt{d+ex^2}}{3adx^3}$$

```
[Out] -Sqrt[d + e*x^2]/(3*a*d*x^3) + (b*Sqrt[d + e*x^2])/(a^2*d*x) + (2
*e*Sqrt[d + e*x^2])/(3*a*d^2*x) + (c*(b + (b^2 - 2*a*c)/Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqr
t[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b - Sqrt[b^
2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (c*(b - (b
^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/
(a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e])
```

Rubi [A] time = 1.77365, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{b\sqrt{d+ex^2}}{a^2dx}+\frac{2e\sqrt{d+ex^2}}{3ad^2x}-\frac{\sqrt{d+ex^2}}{3adx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(3*a*d*x^3) + (b*Sqrt[d + e*x^2])/(a^2*d*x) + (2
*e*Sqrt[d + e*x^2])/(3*a*d^2*x) + (c*(b + (b^2 - 2*a*c)/Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqr
t[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b - Sqrt[b^
2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (c*(b - (b
^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/
(a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.76783, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/(x^4*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[1/(x^4*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.05, size = 248, normalized size = 0.7

$$-\frac{1}{3\,adx^3}\sqrt{ex^2+d}+\frac{2\,e}{3\,ad^2x}\sqrt{ex^2+d}$$
$$-\frac{1}{2\,a^2}\sqrt{e}\sum_{\_R=RootOf\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+(16\,ae^2-8\,bde+6\,cd^2)\_Z^2+(4\,bd^2e-4\,cd^3)\_Z+cd^4\right)}\frac{bc\_R^2+2\,\left(-2\,ace+2\,b^2e-bcd\right.}{\_R^3c+3\,\_R^2be-3\,\_R^2cd+8\,\_R\,ae^2-4\,\_R\,bd}$$
$$+\frac{b}{a^2dx}\sqrt{ex^2+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] -1/3*(e*x^2+d)^(1/2)/a/d/x^3+2/3*e*(e*x^2+d)^(1/2)/a/d^2/x-1/2/a^
2*e^(1/2)*sum((b*c*_R^2+2*(-2*a*c*e+2*b^2*e-b*c*d)*_R+b*c*d^2)/(_
R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^
2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-x*e^(1/2))^2-_R),_R=RootOf(c*_Z^4+
(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c
*d^3)*_Z+c*d^4))+b*(e*x^2+d)^(1/2)/a^2/d/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4+bx^2+a)\sqrt{ex^2+d}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^4), x)
```

Fricas [A] time = 58.1071, size = 11052, normalized size = 32.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^4),x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(1/2)*a^2*d^2*x^3*sqrt(-((b^5*c - 5*a*b^3*c^2 + 5*a^2
*b*c^3)*d - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*e - ((a
^5*b^2*c - 4*a^6*c^2)*d^2 - (a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2
- 4*a^7*c)*e^2)*sqrt(((b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6
*a^3*b^2*c^5 + a^4*c^6)*d^2 - 2*(b^9*c - 7*a*b^7*c^2 + 16*a^2*b^5
*c^3 - 13*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + (b^10 - 8*a*b^8*c + 22
*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*e^2)/((a^10*b^2*c^
2 - 4*a^11*c^3)*d^4 - 2*(a^10*b^3*c - 4*a^11*b*c^2)*d^3*e + (a^10
*b^4 - 2*a^11*b^2*c - 8*a^12*c^2)*d^2*e^2 - 2*(a^11*b^3 - 4*a^12*
b*c)*d*e^3 + (a^12*b^2 - 4*a^13*c)*e^4)))/((a^5*b^2*c - 4*a^6*c^2
)*d^2 - (a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 4*a^7*c)*e^2))*log
(-(((a^5*b^2*c^4 - 4*a^6*c^5)*d^3 - (a^5*b^3*c^3 - 4*a^6*b*c^4)*d
^2*e + (a^6*b^2*c^3 - 4*a^7*c^4)*d*e^2)*x^2*sqrt(((b^8*c^2 - 6*a*
b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6)*d^2 - 2*(b^9*
c - 7*a*b^7*c^2 + 16*a^2*b^5*c^3 - 13*a^3*b^3*c^4 + 3*a^4*b*c^5)*
d*e + (b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4
```

```
*b^2*c^4)*e^2)/((a^10*b^2*c^2 - 4*a^11*c^3)*d^4 - 2*(a^10*b^3*c -
 4*a^11*b*c^2)*d^3*e + (a^10*b^4 - 2*a^11*b^2*c - 8*a^12*c^2)*d^2
*e^2 - 2*(a^11*b^3 - 4*a^12*b*c)*d*e^3 + (a^12*b^2 - 4*a^13*c)*e^
4)) + 2*(a*b^4*c^4 - 3*a^2*b^2*c^5 + a^3*c^6)*d^2 - 2*(a*b^5*c^3
- 4*a^2*b^3*c^4 + 3*a^3*b*c^5)*d*e - ((b^5*c^4 - 3*a*b^3*c^5 + a^
2*b*c^6)*d^2 - (b^6*c^3 - 9*a^2*b^2*c^5 + 4*a^3*c^6)*d*e + 4*(a*b
^5*c^3 - 4*a^2*b^3*c^4 + 3*a^3*b*c^5)*e^2)*x^2 + 2*sqrt(1/2)*sqrt
(e*x^2 + d)*(((a^6*b^4*c^2 - 6*a^7*b^2*c^3 + 8*a^8*c^4)*d^3 - (2*
a^6*b^5*c - 13*a^7*b^3*c^2 + 20*a^8*b*c^3)*d^2*e + (a^6*b^6 - 6*a
^7*b^4*c + 6*a^8*b^2*c^2 + 8*a^9*c^3)*d*e^2 - (a^7*b^5 - 7*a^8*b^
3*c + 12*a^9*b*c^2)*e^3)*x*sqrt(((b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*
b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6)*d^2 - 2*(b^9*c - 7*a*b^7*c^2 +
 16*a^2*b^5*c^3 - 13*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + (b^10 - 8*a
*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*e^2)/((
a^10*b^2*c^2 - 4*a^11*c^3)*d^4 - 2*(a^10*b^3*c - 4*a^11*b*c^2)*d^
3*e + (a^10*b^4 - 2*a^11*b^2*c - 8*a^12*c^2)*d^2*e^2 - 2*(a^11*b^
3 - 4*a^12*b*c)*d*e^3 + (a^12*b^2 - 4*a^13*c)*e^4)) + ((a*b^7*c^2
 - 7*a^2*b^5*c^3 + 13*a^3*b^3*c^4 - 4*a^4*b*c^5)*d^2 - (2*a*b^8*c
 - 16*a^2*b^6*c^2 + 39*a^3*b^4*c^3 - 29*a^4*b^2*c^4 + 4*a^5*c^5)*
d*e + (a*b^9 - 9*a^2*b^7*c + 27*a^3*b^5*c^2 - 31*a^4*b^3*c^3 + 12
*a^5*b*c^4)*e^2)*x)*sqrt(-((b^5*c - 5*a*b^3*c^2 + 5*a^2*b*c^3)*d
- (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*e - ((a^5*b^2*c -
 4*a^6*c^2)*d^2 - (a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 4*a^7*c)
*e^2)*sqrt(((b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c
^5 + a^4*c^6)*d^2 - 2*(b^9*c - 7*a*b^7*c^2 + 16*a^2*b^5*c^3 - 13*
a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + (b^10 - 8*a*b^8*c + 22*a^2*b^6*c
^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*e^2)/((a^10*b^2*c^2 - 4*a^11
*c^3)*d^4 - 2*(a^10*b^3*c - 4*a^11*b*c^2)*d^3*e + (a^10*b^4 - 2*a
^11*b^2*c - 8*a^12*c^2)*d^2*e^2 - 2*(a^11*b^3 - 4*a^12*b*c)*d*e^3
 + (a^12*b^2 - 4*a^13*c)*e^4)))/((a^5*b^2*c - 4*a^6*c^2)*d^2 - (a
^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 4*a^7*c)*e^2)))/x^2) - 3*sqr
t(1/2)*a^2*d^2*x^3*sqrt(-((b^5*c - 5*a*b^3*c^2 + 5*a^2*b*c^3)*d -
 (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*e - ((a^5*b^2*c -
4*a^6*c^2)*d^2 - (a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 4*a^7*c)*
e^2)*sqrt(((b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^
5 + a^4*c^6)*d^2 - 2*(b^9*c - 7*a*b^7*c^2 + 16*a^2*b^5*c^3 - 13*a
^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + (b^10 - 8*a*b^8*c + 22*a^2*b^6*c^
2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*e^2)/((a^10*b^2*c^2 - 4*a^11*
c^3)*d^4 - 2*(a^10*b^3*c - 4*a^11*b*c^2)*d^3*e + (a^10*b^4 - 2*a^
11*b^2*c - 8*a^12*c^2)*d^2*e^2 - 2*(a^11*b^3 - 4*a^12*b*c)*d*e^3
+ (a^12*b^2 - 4*a^13*c)*e^4)))/((a^5*b^2*c - 4*a^6*c^2)*d^2 - (a^
5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 4*a^7*c)*e^2))*log(-(((a^5*b^
2*c^4 - 4*a^6*c^5)*d^3 - (a^5*b^3*c^3 - 4*a^6*b*c^4)*d^2*e + (a^6
*b^2*c^3 - 4*a^7*c^4)*d*e^2)*x^2*sqrt(((b^8*c^2 - 6*a*b^6*c^3 + 1
1*a^2*b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6)*d^2 - 2*(b^9*c - 7*a*b^7
*c^2 + 16*a^2*b^5*c^3 - 13*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + (b^10
 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*e
^2)/((a^10*b^2*c^2 - 4*a^11*c^3)*d^4 - 2*(a^10*b^3*c - 4*a^11*b*c
^2)*d^3*e + (a^10*b^4 - 2*a^11*b^2*c - 8*a^12*c^2)*d^2*e^2 - 2*(a
^11*b^3 - 4*a^12*b*c)*d*e^3 + (a^12*b^2 - 4*a^13*c)*e^4)) + 2*(a*
b^4*c^4 - 3*a^2*b^2*c^5 + a^3*c^6)*d^2 - 2*(a*b^5*c^3 - 4*a^2*b^3
*c^4 + 3*a^3*b*c^5)*d*e - ((b^5*c^4 - 3*a*b^3*c^5 + a^2*b*c^6)*d^
2 - (b^6*c^3 - 9*a^2*b^2*c^5 + 4*a^3*c^6)*d*e + 4*(a*b^5*c^3 - 4*
a^2*b^3*c^4 + 3*a^3*b*c^5)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)
```

```
*(((a^6*b^4*c^2 - 6*a^7*b^2*c^3 + 8*a^8*c^4)*d^3 - (2*a^6*b^5*c -
 13*a^7*b^3*c^2 + 20*a^8*b*c^3)*d^2*e + (a^6*b^6 - 6*a^7*b^4*c +
6*a^8*b^2*c^2 + 8*a^9*c^3)*d*e^2 - (a^7*b^5 - 7*a^8*b^3*c + 12*a^
9*b*c^2)*e^3)*x*sqrt(((b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6
*a^3*b^2*c^5 + a^4*c^6)*d^2 - 2*(b^9*c - 7*a*b^7*c^2 + 16*a^2*b^5
*c^3 - 13*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + (b^10 - 8*a*b^8*c + 22
*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*e^2)/((a^10*b^2*c^
2 - 4*a^11*c^3)*d^4 - 2*(a^10*b^3*c - 4*a^11*b*c^2)*d^3*e + (a^10
*b^4 - 2*a^11*b^2*c - 8*a^12*c^2)*d^2*e^2 - 2*(a^11*b^3 - 4*a^12*
b*c)*d*e^3 + (a^12*b^2 - 4*a^13*c)*e^4)) + ((a*b^7*c^2 - 7*a^2*b^
5*c^3 + 13*a^3*b^3*c^4 - 4*a^4*b*c^5)*d^2 - (2*a*b^8*c - 16*a^2*b
^6*c^2 + 39*a^3*b^4*c^3 - 29*a^4*b^2*c^4 + 4*a^5*c^5)*d*e + (a*b^
9 - 9*a^2*b^7*c + 27*a^3*b^5*c^2 - 31*a^4*b^3*c^3 + 12*a^5*b*c^4)
*e^2)*x)*sqrt(-((b^5*c - 5*a*b^3*c^2 + 5*a^2*b*c^3)*d - (b^6 - 6*
a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*e - ((a^5*b^2*c - 4*a^6*c^2)
*d^2 - (a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 4*a^7*c)*e^2)*sqrt(
((b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^
6)*d^2 - 2*(b^9*c - 7*a*b^7*c^2 + 16*a^2*b^5*c^3 - 13*a^3*b^3*c^4
 + 3*a^4*b*c^5)*d*e + (b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3
*b^4*c^3 + 9*a^4*b^2*c^4)*e^2)/((a^10*b^2*c^2 - 4*a^11*c^3)*d^4 -
 2*(a^10*b^3*c - 4*a^11*b*c^2)*d^3*e + (a^10*b^4 - 2*a^11*b^2*c -
 8*a^12*c^2)*d^2*e^2 - 2*(a^11*b^3 - 4*a^12*b*c)*d*e^3 + (a^12*b^
2 - 4*a^13*c)*e^4)))/((a^5*b^2*c - 4*a^6*c^2)*d^2 - (a^5*b^3 - 4*
a^6*b*c)*d*e + (a^6*b^2 - 4*a^7*c)*e^2)))/x^2) + 3*sqrt(1/2)*a^2*
d^2*x^3*sqrt(-((b^5*c - 5*a*b^3*c^2 + 5*a^2*b*c^3)*d - (b^6 - 6*a
*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*e + ((a^5*b^2*c - 4*a^6*c^2)*
d^2 - (a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 4*a^7*c)*e^2)*sqrt((
(b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6
)*d^2 - 2*(b^9*c - 7*a*b^7*c^2 + 16*a^2*b^5*c^3 - 13*a^3*b^3*c^4
+ 3*a^4*b*c^5)*d*e + (b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*
b^4*c^3 + 9*a^4*b^2*c^4)*e^2)/((a^10*b^2*c^2 - 4*a^11*c^3)*d^4 -
2*(a^10*b^3*c - 4*a^11*b*c^2)*d^3*e + (a^10*b^4 - 2*a^11*b^2*c -
8*a^12*c^2)*d^2*e^2 - 2*(a^11*b^3 - 4*a^12*b*c)*d*e^3 + (a^12*b^2
 - 4*a^13*c)*e^4)))/((a^5*b^2*c - 4*a^6*c^2)*d^2 - (a^5*b^3 - 4*a
^6*b*c)*d*e + (a^6*b^2 - 4*a^7*c)*e^2))*log((((a^5*b^2*c^4 - 4*a^
6*c^5)*d^3 - (a^5*b^3*c^3 - 4*a^6*b*c^4)*d^2*e + (a^6*b^2*c^3 - 4
*a^7*c^4)*d*e^2)*x^2*sqrt(((b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^
4 - 6*a^3*b^2*c^5 + a^4*c^6)*d^2 - 2*(b^9*c - 7*a*b^7*c^2 + 16*a^
2*b^5*c^3 - 13*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + (b^10 - 8*a*b^8*c
 + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*e^2)/((a^10*b
^2*c^2 - 4*a^11*c^3)*d^4 - 2*(a^10*b^3*c - 4*a^11*b*c^2)*d^3*e +
(a^10*b^4 - 2*a^11*b^2*c - 8*a^12*c^2)*d^2*e^2 - 2*(a^11*b^3 - 4*
a^12*b*c)*d*e^3 + (a^12*b^2 - 4*a^13*c)*e^4)) - 2*(a*b^4*c^4 - 3*
a^2*b^2*c^5 + a^3*c^6)*d^2 + 2*(a*b^5*c^3 - 4*a^2*b^3*c^4 + 3*a^3
*b*c^5)*d*e + ((b^5*c^4 - 3*a*b^3*c^5 + a^2*b*c^6)*d^2 - (b^6*c^3
 - 9*a^2*b^2*c^5 + 4*a^3*c^6)*d*e + 4*(a*b^5*c^3 - 4*a^2*b^3*c^4
+ 3*a^3*b*c^5)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^6*b^4*
c^2 - 6*a^7*b^2*c^3 + 8*a^8*c^4)*d^3 - (2*a^6*b^5*c - 13*a^7*b^3*
c^2 + 20*a^8*b*c^3)*d^2*e + (a^6*b^6 - 6*a^7*b^4*c + 6*a^8*b^2*c^
2 + 8*a^9*c^3)*d*e^2 - (a^7*b^5 - 7*a^8*b^3*c + 12*a^9*b*c^2)*e^3
)*x*sqrt(((b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5
 + a^4*c^6)*d^2 - 2*(b^9*c - 7*a*b^7*c^2 + 16*a^2*b^5*c^3 - 13*a^
3*b^3*c^4 + 3*a^4*b*c^5)*d*e + (b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2
 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*e^2)/((a^10*b^2*c^2 - 4*a^11*c
```

^3)*d^4 - 2*(a^10*b^3*c - 4*a^11*b*c^2)*d^3*e + (a^10*b^4 - 2*a^11*b^2*c - 8*a^12*c^2)*d^2*e^2 - 2*(a^11*b^3 - 4*a^12*b*c)*d*e^3 + (a^12*b^2 - 4*a^13*c)*e^4)) - ((a*b^7*c^2 - 7*a^2*b^5*c^3 + 13*a^3*b^3*c^4 - 4*a^4*b*c^5)*d^2 - (2*a*b^8*c - 16*a^2*b^6*c^2 + 39*a^3*b^4*c^3 - 29*a^4*b^2*c^4 + 4*a^5*c^5)*d*e + (a*b^9 - 9*a^2*b^7*c + 27*a^3*b^5*c^2 - 31*a^4*b^3*c^3 + 12*a^5*b*c^4)*e^2)*x)*sqrt(-((b^5*c - 5*a*b^3*c^2 + 5*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*e + ((a^5*b^2*c - 4*a^6*c^2)*d^2 - (a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 4*a^7*c)*e^2)*sqrt(((b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6)*d^2 - 2*(b^9*c - 7*a*b^7*c^2 + 16*a^2*b^5*c^3 - 13*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + (b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*e^2)/((a^10*b^2*c^2 - 4*a^11*c^3)*d^4 - 2*(a^10*b^3*c - 4*a^11*b*c^2)*d^3*e + (a^10*b^4 - 2*a^11*b^2*c - 8*a^12*c^2)*d^2*e^2 - 2*(a^11*b^3 - 4*a^12*b*c)*d*e^3 + (a^12*b^2 - 4*a^13*c)*e^4)))/((a^5*b^2*c - 4*a^6*c^2)*d^2 - (a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 4*a^7*c)*e^2)))/x^2) - 3*sqrt(1/2)*a^2*d^2*x^3*sqrt(-((b^5*c - 5*a*b^3*c^2 + 5*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*e + ((a^5*b^2*c - 4*a^6*c^2)*d^2 - (a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 4*a^7*c)*e^2)*sqrt(((b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6)*d^2 - 2*(b^9*c - 7*a*b^7*c^2 + 16*a^2*b^5*c^3 - 13*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + (b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*e^2)/((a^10*b^2*c^2 - 4*a^11*c^3)*d^4 - 2*(a^10*b^3*c - 4*a^11*b*c^2)*d^3*e + (a^10*b^4 - 2*a^11*b^2*c - 8*a^12*c^2)*d^2*e^2 - 2*(a^11*b^3 - 4*a^12*b*c)*d*e^3 + (a^12*b^2 - 4*a^13*c)*e^4)))/((a^5*b^2*c - 4*a^6*c^2)*d^2 - (a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 4*a^7*c)*e^2))*log((((a^5*b^2*c^4 - 4*a^6*c^5)*d^3 - (a^5*b^3*c^3 - 4*a^6*b*c^4)*d^2*e + (a^6*b^2*c^3 - 4*a^7*c^4)*d*e^2)*x^2*sqrt(((b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6)*d^2 - 2*(b^9*c - 7*a*b^7*c^2 + 16*a^2*b^5*c^3 - 13*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + (b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*e^2)/((a^10*b^2*c^2 - 4*a^11*c^3)*d^4 - 2*(a^10*b^3*c - 4*a^11*b*c^2)*d^3*e + (a^10*b^4 - 2*a^11*b^2*c - 8*a^12*c^2)*d^2*e^2 - 2*(a^11*b^3 - 4*a^12*b*c)*d*e^3 + (a^12*b^2 - 4*a^13*c)*e^4)) - 2*(a*b^4*c^4 - 3*a^2*b^2*c^5 + a^3*c^6)*d^2 + 2*(a*b^5*c^3 - 4*a^2*b^3*c^4 + 3*a^3*b*c^5)*d*e + ((b^5*c^4 - 3*a*b^3*c^5 + a^2*b*c^6)*d^2 - (b^6*c^3 - 9*a^2*b^2*c^5 + 4*a^3*c^6)*d*e + 4*(a*b^5*c^3 - 4*a^2*b^3*c^4 + 3*a^3*b*c^5)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^6*b^4*c^2 - 6*a^7*b^2*c^3 + 8*a^8*c^4)*d^3 - (2*a^6*b^5*c - 13*a^7*b^3*c^2 + 20*a^8*b*c^3)*d^2*e + (a^6*b^6 - 6*a^7*b^4*c + 6*a^8*b^2*c^2 + 8*a^9*c^3)*d*e^2 - (a^7*b^5 - 7*a^8*b^3*c + 12*a^9*b*c^2)*e^3)*x*sqrt(((b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6)*d^2 - 2*(b^9*c - 7*a*b^7*c^2 + 16*a^2*b^5*c^3 - 13*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + (b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*e^2)/((a^10*b^2*c^2 - 4*a^11*c^3)*d^4 - 2*(a^10*b^3*c - 4*a^11*b*c^2)*d^3*e + (a^10*b^4 - 2*a^11*b^2*c - 8*a^12*c^2)*d^2*e^2 - 2*(a^11*b^3 - 4*a^12*b*c)*d*e^3 + (a^12*b^2 - 4*a^13*c)*e^4)) - ((a*b^7*c^2 - 7*a^2*b^5*c^3 + 13*a^3*b^3*c^4 - 4*a^4*b*c^5)*d^2 - (2*a*b^8*c - 16*a^2*b^6*c^2 + 39*a^3*b^4*c^3 - 29*a^4*b^2*c^4 + 4*a^5*c^5)*d*e + (a*b^9 - 9*a^2*b^7*c + 27*a^3*b^5*c^2 - 31*a^4*b^3*c^3 + 12*a^5*b*c^4)*e^2)*x)*sqrt(-((b^5*c - 5*a*b^3*c^2 + 5*a^2*b*c^3)*d - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2

```
- 2*a^3*c^3)*e + ((a^5*b^2*c - 4*a^6*c^2)*d^2 - (a^5*b^3 - 4*a^6*
b*c)*d*e + (a^6*b^2 - 4*a^7*c)*e^2)*sqrt(((b^8*c^2 - 6*a*b^6*c^3
+ 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6)*d^2 - 2*(b^9*c - 7*a*
b^7*c^2 + 16*a^2*b^5*c^3 - 13*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + (b
^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4
)*e^2)/((a^10*b^2*c^2 - 4*a^11*c^3)*d^4 - 2*(a^10*b^3*c - 4*a^11*
b*c^2)*d^3*e + (a^10*b^4 - 2*a^11*b^2*c - 8*a^12*c^2)*d^2*e^2 - 2
*(a^11*b^3 - 4*a^12*b*c)*d*e^3 + (a^12*b^2 - 4*a^13*c)*e^4)))/((a
^5*b^2*c - 4*a^6*c^2)*d^2 - (a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2
- 4*a^7*c)*e^2)))/x^2) + 4*((3*b*d + 2*a*e)*x^2 - a*d)*sqrt(e*x^2
 + d))/(a^2*d^2*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.385 $\int \frac{1}{x^6\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=443

$$-\frac{(b^2-ac)\sqrt{d+ex^2}}{a^3dx}-\frac{c\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2be\sqrt{d+ex^2}}{3a^2d^2x}+\frac{b\sqrt{d+ex^2}}{3a^2dx^3}-\frac{8e^2\sqrt{d+ex^2}}{15ad^3x}+\frac{4e\sqrt{d+ex^2}}{15ad^2x^3}-\frac{\sqrt{d+ex^2}}{5adx^5}$$

```
[Out] -Sqrt[d + e*x^2]/(5*a*d*x^5) + (b*Sqrt[d + e*x^2])/(3*a^2*d*x^3)
+ (4*e*Sqrt[d + e*x^2])/(15*a*d^2*x^3) - ((b^2 - a*c)*Sqrt[d + e*
x^2])/(a^3*d*x) - (2*b*e*Sqrt[d + e*x^2])/(3*a^2*d^2*x) - (8*e^2*
Sqrt[d + e*x^2])/(15*a*d^3*x) - (c*(b^2 - a*c + (b*(b^2 - 3*a*c))
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^3*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) -
(c*(b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c
]]*Sqrt[d + e*x^2])])/(a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 3.27137, antiderivative size = 443, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207

$$-\frac{(b^2-ac)\sqrt{d+ex^2}}{a^3dx}-\frac{c\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2be\sqrt{d+ex^2}}{3a^2d^2x}+\frac{b\sqrt{d+ex^2}}{3a^2dx^3}-\frac{8e^2\sqrt{d+ex^2}}{15ad^3x}+\frac{4e\sqrt{d+ex^2}}{15ad^2x^3}-\frac{\sqrt{d+ex^2}}{5adx^5}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^6*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(5*a*d*x^5) + (b*Sqrt[d + e*x^2])/(3*a^2*d*x^3)
+ (4*e*Sqrt[d + e*x^2])/(15*a*d^2*x^3) - ((b^2 - a*c)*Sqrt[d + e*
x^2])/(a^3*d*x) - (2*b*e*Sqrt[d + e*x^2])/(3*a^2*d^2*x) - (8*e^2*
Sqrt[d + e*x^2])/(15*a*d^3*x) - (c*(b^2 - a*c + (b*(b^2 - 3*a*c))
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^3*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) -
(c*(b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c
]]*Sqrt[d + e*x^2])])/(a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**6/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.942758, size = 0, normalized size = 0.

$$\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/(x^6*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[1/(x^6*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.042, size = 350, normalized size = 0.8

$$-\frac{1}{5\,ad x^5}\sqrt{e x^2+d}+\frac{4\,e}{15\,a d^2 x^3}\sqrt{e x^2+d}-\frac{8\,e^2}{15\,a d^3 x}\sqrt{e x^2+d}-\frac{-ac+b^2}{a^3 dx}\sqrt{e x^2+d} -\frac{1}{2\,a^3}\sqrt{e}\sum_{\_R=RootOf\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+(16\,ae^2-8\,bde+6\,cd^2)\_Z^2+(4\,bd^2e-4\,cd^3)\_Z+cd^4\right)}\frac{c\left(ac-b^2\right)\_R^2+2\left(4\,abce-ac^2d-2\,b^3e+b\right.}{\_R^3c+3\,\_R^2be-3\,\_R^2cd+8\,\_R\,ae^2-4\,\_R\,bd} +\frac{b}{3\,a^2dx^3}\sqrt{e x^2+d}-\frac{2\,be}{3\,a^2d^2x}\sqrt{e x^2+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] -1/5*(e*x^2+d)^(1/2)/a/d/x^5+4/15*e*(e*x^2+d)^(1/2)/a/d^2/x^3-8/1
5*e^2*(e*x^2+d)^(1/2)/a/d^3/x-(-a*c+b^2)*(e*x^2+d)^(1/2)/a^3/d/x-
1/2/a^3*e^(1/2)*sum((c*(a*c-b^2)*_R^2+2*(4*a*b*c*e-a*c^2*d-2*b^3*
e+b^2*c*d)*_R+a*c^2*d^2-b^2*c*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+
8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/
2)-x*e^(1/2))^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2
-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))+1/3*b*(e*x^
2+d)^(1/2)/a^2/d/x^3-2/3*b*e*(e*x^2+d)^(1/2)/a^2/d^2/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4+bx^2+a)\sqrt{ex^2+d}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^6),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^6), x)
```

Fricas [A] time = 94.5071, size = 13497, normalized size = 30.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^6),x, algorithm="fricas")
```

```
[Out] -1/60*(15*sqrt(1/2)*a^3*d^3*x^5*sqrt(-((b^7*c - 7*a*b^5*c^2 + 14*
a^2*b^3*c^3 - 7*a^3*b*c^4)*d - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2
- 16*a^3*b^2*c^3 + 2*a^4*c^4)*e - ((a^7*b^2*c - 4*a^8*c^2)*d^2 -
(a^7*b^3 - 4*a^8*b*c)*d*e + (a^8*b^2 - 4*a^9*c)*e^2)*sqrt(((b^12*
c^2 - 10*a*b^10*c^3 + 37*a^2*b^8*c^4 - 62*a^3*b^6*c^5 + 46*a^4*b^
4*c^6 - 12*a^5*b^2*c^7 + a^6*c^8)*d^2 - 2*(b^13*c - 11*a*b^11*c^2
 + 46*a^2*b^9*c^3 - 91*a^3*b^7*c^4 + 86*a^4*b^5*c^5 - 34*a^5*b^3*
c^6 + 4*a^6*b*c^7)*d*e + (b^14 - 12*a*b^12*c + 56*a^2*b^10*c^2 -
128*a^3*b^8*c^3 + 148*a^4*b^6*c^4 - 80*a^5*b^4*c^5 + 16*a^6*b^2*c
^6)*e^2)/((a^14*b^2*c^2 - 4*a^15*c^3)*d^4 - 2*(a^14*b^3*c - 4*a^1
5*b*c^2)*d^3*e + (a^14*b^4 - 2*a^15*b^2*c - 8*a^16*c^2)*d^2*e^2 -
 2*(a^15*b^3 - 4*a^16*b*c)*d*e^3 + (a^16*b^2 - 4*a^17*c)*e^4)))/(
(a^7*b^2*c - 4*a^8*c^2)*d^2 - (a^7*b^3 - 4*a^8*b*c)*d*e + (a^8*b^
2 - 4*a^9*c)*e^2))*log((((a^7*b^2*c^5 - 4*a^8*c^6)*d^3 - (a^7*b^3
*c^4 - 4*a^8*b*c^5)*d^2*e + (a^8*b^2*c^4 - 4*a^9*c^5)*d*e^2)*x^2*
sqrt(((b^12*c^2 - 10*a*b^10*c^3 + 37*a^2*b^8*c^4 - 62*a^3*b^6*c^5
 + 46*a^4*b^4*c^6 - 12*a^5*b^2*c^7 + a^6*c^8)*d^2 - 2*(b^13*c - 1
1*a*b^11*c^2 + 46*a^2*b^9*c^3 - 91*a^3*b^7*c^4 + 86*a^4*b^5*c^5 -
 34*a^5*b^3*c^6 + 4*a^6*b*c^7)*d*e + (b^14 - 12*a*b^12*c + 56*a^2
*b^10*c^2 - 128*a^3*b^8*c^3 + 148*a^4*b^6*c^4 - 80*a^5*b^4*c^5 +
16*a^6*b^2*c^6)*e^2)/((a^14*b^2*c^2 - 4*a^15*c^3)*d^4 - 2*(a^14*b
^3*c - 4*a^15*b*c^2)*d^3*e + (a^14*b^4 - 2*a^15*b^2*c - 8*a^16*c^
2)*d^2*e^2 - 2*(a^15*b^3 - 4*a^16*b*c)*d*e^3 + (a^16*b^2 - 4*a^17
*c)*e^4)) + 2*(a*b^6*c^5 - 5*a^2*b^4*c^6 + 6*a^3*b^2*c^7 - a^4*c^
8)*d^2 - 2*(a*b^7*c^4 - 6*a^2*b^5*c^5 + 10*a^3*b^3*c^6 - 4*a^4*b*
c^7)*d*e - ((b^7*c^5 - 5*a*b^5*c^6 + 6*a^2*b^3*c^7 - a^3*b*c^8)*d
^2 - (b^8*c^4 - 2*a*b^6*c^5 - 10*a^2*b^4*c^6 + 20*a^3*b^2*c^7 - 4
*a^4*c^8)*d*e + 4*(a*b^7*c^4 - 6*a^2*b^5*c^5 + 10*a^3*b^3*c^6 - 4
*a^4*b*c^7)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^8*b^5*c^2
 - 7*a^9*b^3*c^3 + 12*a^10*b*c^4)*d^3 - (2*a^8*b^6*c - 15*a^9*b^4
*c^2 + 30*a^10*b^2*c^3 - 8*a^11*c^4)*d^2*e + (a^8*b^7 - 7*a^9*b^5
*c + 11*a^10*b^3*c^2 + 4*a^11*b*c^3)*d*e^2 - (a^9*b^6 - 8*a^10*b^
4*c + 18*a^11*b^2*c^2 - 8*a^12*c^3)*e^3)*x*sqrt(((b^12*c^2 - 10*a
*b^10*c^3 + 37*a^2*b^8*c^4 - 62*a^3*b^6*c^5 + 46*a^4*b^4*c^6 - 12
*a^5*b^2*c^7 + a^6*c^8)*d^2 - 2*(b^13*c - 11*a*b^11*c^2 + 46*a^2*
b^9*c^3 - 91*a^3*b^7*c^4 + 86*a^4*b^5*c^5 - 34*a^5*b^3*c^6 + 4*a^
6*b*c^7)*d*e + (b^14 - 12*a*b^12*c + 56*a^2*b^10*c^2 - 128*a^3*b^
8*c^3 + 148*a^4*b^6*c^4 - 80*a^5*b^4*c^5 + 16*a^6*b^2*c^6)*e^2)/(
(a^14*b^2*c^2 - 4*a^15*c^3)*d^4 - 2*(a^14*b^3*c - 4*a^15*b*c^2)*d
^3*e + (a^14*b^4 - 2*a^15*b^2*c - 8*a^16*c^2)*d^2*e^2 - 2*(a^15*b
^3 - 4*a^16*b*c)*d*e^3 + (a^16*b^2 - 4*a^17*c)*e^4)) + ((a*b^10*c
^2 - 10*a^2*b^8*c^3 + 35*a^3*b^6*c^4 - 51*a^4*b^4*c^5 + 29*a^5*b^
2*c^6 - 4*a^6*c^7)*d^2 - (2*a*b^11*c - 22*a^2*b^9*c^2 + 88*a^3*b^
7*c^3 - 155*a^4*b^5*c^4 + 114*a^5*b^3*c^5 - 24*a^6*b*c^6)*d*e + (
a*b^12 - 12*a^2*b^10*c + 54*a^3*b^8*c^2 - 112*a^4*b^6*c^3 + 104*a
^5*b^4*c^4 - 32*a^6*b^2*c^5)*e^2)*x)*sqrt(-((b^7*c - 7*a*b^5*c^2
+ 14*a^2*b^3*c^3 - 7*a^3*b*c^4)*d - (b^8 - 8*a*b^6*c + 20*a^2*b^4
*c^2 - 16*a^3*b^2*c^3 + 2*a^4*c^4)*e - ((a^7*b^2*c - 4*a^8*c^2)*d
^2 - (a^7*b^3 - 4*a^8*b*c)*d*e + (a^8*b^2 - 4*a^9*c)*e^2)*sqrt(((
b^12*c^2 - 10*a*b^10*c^3 + 37*a^2*b^8*c^4 - 62*a^3*b^6*c^5 + 46*a
^4*b^4*c^6 - 12*a^5*b^2*c^7 + a^6*c^8)*d^2 - 2*(b^13*c - 11*a*b^1
1*c^2 + 46*a^2*b^9*c^3 - 91*a^3*b^7*c^4 + 86*a^4*b^5*c^5 - 34*a^5
*b^3*c^6 + 4*a^6*b*c^7)*d*e + (b^14 - 12*a*b^12*c + 56*a^2*b^10*c
^2 - 128*a^3*b^8*c^3 + 148*a^4*b^6*c^4 - 80*a^5*b^4*c^5 + 16*a^6*
```

```
b^2*c^6)*e^2)/((a^14*b^2*c^2 - 4*a^15*c^3)*d^4 - 2*(a^14*b^3*c -
4*a^15*b*c^2)*d^3*e + (a^14*b^4 - 2*a^15*b^2*c - 8*a^16*c^2)*d^2*
e^2 - 2*(a^15*b^3 - 4*a^16*b*c)*d*e^3 + (a^16*b^2 - 4*a^17*c)*e^4
)))/((a^7*b^2*c - 4*a^8*c^2)*d^2 - (a^7*b^3 - 4*a^8*b*c)*d*e + (a
^8*b^2 - 4*a^9*c)*e^2)))/x^2) - 15*sqrt(1/2)*a^3*d^3*x^5*sqrt(-((
b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 - 7*a^3*b*c^4)*d - (b^8 - 8*
a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 2*a^4*c^4)*e - ((a^7*
b^2*c - 4*a^8*c^2)*d^2 - (a^7*b^3 - 4*a^8*b*c)*d*e + (a^8*b^2 - 4
*a^9*c)*e^2)*sqrt(((b^12*c^2 - 10*a*b^10*c^3 + 37*a^2*b^8*c^4 - 6
2*a^3*b^6*c^5 + 46*a^4*b^4*c^6 - 12*a^5*b^2*c^7 + a^6*c^8)*d^2 -
2*(b^13*c - 11*a*b^11*c^2 + 46*a^2*b^9*c^3 - 91*a^3*b^7*c^4 + 86*
a^4*b^5*c^5 - 34*a^5*b^3*c^6 + 4*a^6*b*c^7)*d*e + (b^14 - 12*a*b^
12*c + 56*a^2*b^10*c^2 - 128*a^3*b^8*c^3 + 148*a^4*b^6*c^4 - 80*a
^5*b^4*c^5 + 16*a^6*b^2*c^6)*e^2)/((a^14*b^2*c^2 - 4*a^15*c^3)*d^
4 - 2*(a^14*b^3*c - 4*a^15*b*c^2)*d^3*e + (a^14*b^4 - 2*a^15*b^2*
c - 8*a^16*c^2)*d^2*e^2 - 2*(a^15*b^3 - 4*a^16*b*c)*d*e^3 + (a^16
*b^2 - 4*a^17*c)*e^4)))/((a^7*b^2*c - 4*a^8*c^2)*d^2 - (a^7*b^3 -
 4*a^8*b*c)*d*e + (a^8*b^2 - 4*a^9*c)*e^2))*log((((a^7*b^2*c^5 -
4*a^8*c^6)*d^3 - (a^7*b^3*c^4 - 4*a^8*b*c^5)*d^2*e + (a^8*b^2*c^4
 - 4*a^9*c^5)*d*e^2)*x^2*sqrt(((b^12*c^2 - 10*a*b^10*c^3 + 37*a^2
*b^8*c^4 - 62*a^3*b^6*c^5 + 46*a^4*b^4*c^6 - 12*a^5*b^2*c^7 + a^6
*c^8)*d^2 - 2*(b^13*c - 11*a*b^11*c^2 + 46*a^2*b^9*c^3 - 91*a^3*b
^7*c^4 + 86*a^4*b^5*c^5 - 34*a^5*b^3*c^6 + 4*a^6*b*c^7)*d*e + (b^
14 - 12*a*b^12*c + 56*a^2*b^10*c^2 - 128*a^3*b^8*c^3 + 148*a^4*b^
6*c^4 - 80*a^5*b^4*c^5 + 16*a^6*b^2*c^6)*e^2)/((a^14*b^2*c^2 - 4*
a^15*c^3)*d^4 - 2*(a^14*b^3*c - 4*a^15*b*c^2)*d^3*e + (a^14*b^4 -
 2*a^15*b^2*c - 8*a^16*c^2)*d^2*e^2 - 2*(a^15*b^3 - 4*a^16*b*c)*d
*e^3 + (a^16*b^2 - 4*a^17*c)*e^4)) + 2*(a*b^6*c^5 - 5*a^2*b^4*c^6
 + 6*a^3*b^2*c^7 - a^4*c^8)*d^2 - 2*(a*b^7*c^4 - 6*a^2*b^5*c^5 +
10*a^3*b^3*c^6 - 4*a^4*b*c^7)*d*e - ((b^7*c^5 - 5*a*b^5*c^6 + 6*a
^2*b^3*c^7 - a^3*b*c^8)*d^2 - (b^8*c^4 - 2*a*b^6*c^5 - 10*a^2*b^4
*c^6 + 20*a^3*b^2*c^7 - 4*a^4*c^8)*d*e + 4*(a*b^7*c^4 - 6*a^2*b^5
*c^5 + 10*a^3*b^3*c^6 - 4*a^4*b*c^7)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(
e*x^2 + d)*(((a^8*b^5*c^2 - 7*a^9*b^3*c^3 + 12*a^10*b*c^4)*d^3 -
(2*a^8*b^6*c - 15*a^9*b^4*c^2 + 30*a^10*b^2*c^3 - 8*a^11*c^4)*d^2
*e + (a^8*b^7 - 7*a^9*b^5*c + 11*a^10*b^3*c^2 + 4*a^11*b*c^3)*d*e
^2 - (a^9*b^6 - 8*a^10*b^4*c + 18*a^11*b^2*c^2 - 8*a^12*c^3)*e^3)
*x*sqrt(((b^12*c^2 - 10*a*b^10*c^3 + 37*a^2*b^8*c^4 - 62*a^3*b^6*
c^5 + 46*a^4*b^4*c^6 - 12*a^5*b^2*c^7 + a^6*c^8)*d^2 - 2*(b^13*c
- 11*a*b^11*c^2 + 46*a^2*b^9*c^3 - 91*a^3*b^7*c^4 + 86*a^4*b^5*c^
5 - 34*a^5*b^3*c^6 + 4*a^6*b*c^7)*d*e + (b^14 - 12*a*b^12*c + 56*
a^2*b^10*c^2 - 128*a^3*b^8*c^3 + 148*a^4*b^6*c^4 - 80*a^5*b^4*c^5
 + 16*a^6*b^2*c^6)*e^2)/((a^14*b^2*c^2 - 4*a^15*c^3)*d^4 - 2*(a^1
4*b^3*c - 4*a^15*b*c^2)*d^3*e + (a^14*b^4 - 2*a^15*b^2*c - 8*a^16
*c^2)*d^2*e^2 - 2*(a^15*b^3 - 4*a^16*b*c)*d*e^3 + (a^16*b^2 - 4*a
^17*c)*e^4)) + ((a*b^10*c^2 - 10*a^2*b^8*c^3 + 35*a^3*b^6*c^4 - 5
1*a^4*b^4*c^5 + 29*a^5*b^2*c^6 - 4*a^6*c^7)*d^2 - (2*a*b^11*c - 2
2*a^2*b^9*c^2 + 88*a^3*b^7*c^3 - 155*a^4*b^5*c^4 + 114*a^5*b^3*c^
5 - 24*a^6*b*c^6)*d*e + (a*b^12 - 12*a^2*b^10*c + 54*a^3*b^8*c^2
- 112*a^4*b^6*c^3 + 104*a^5*b^4*c^4 - 32*a^6*b^2*c^5)*e^2)*x)*sqr
t(-((b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 - 7*a^3*b*c^4)*d - (b^8
 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 2*a^4*c^4)*e - (
(a^7*b^2*c - 4*a^8*c^2)*d^2 - (a^7*b^3 - 4*a^8*b*c)*d*e + (a^8*b^
2 - 4*a^9*c)*e^2)*sqrt(((b^12*c^2 - 10*a*b^10*c^3 + 37*a^2*b^8*c^
```

```
4 - 62*a^3*b^6*c^5 + 46*a^4*b^4*c^6 - 12*a^5*b^2*c^7 + a^6*c^8)*d
^2 - 2*(b^13*c - 11*a*b^11*c^2 + 46*a^2*b^9*c^3 - 91*a^3*b^7*c^4
+ 86*a^4*b^5*c^5 - 34*a^5*b^3*c^6 + 4*a^6*b*c^7)*d*e + (b^14 - 12
*a*b^12*c + 56*a^2*b^10*c^2 - 128*a^3*b^8*c^3 + 148*a^4*b^6*c^4 -
 80*a^5*b^4*c^5 + 16*a^6*b^2*c^6)*e^2)/((a^14*b^2*c^2 - 4*a^15*c^
3)*d^4 - 2*(a^14*b^3*c - 4*a^15*b*c^2)*d^3*e + (a^14*b^4 - 2*a^15
*b^2*c - 8*a^16*c^2)*d^2*e^2 - 2*(a^15*b^3 - 4*a^16*b*c)*d*e^3 +
(a^16*b^2 - 4*a^17*c)*e^4)))/((a^7*b^2*c - 4*a^8*c^2)*d^2 - (a^7*
b^3 - 4*a^8*b*c)*d*e + (a^8*b^2 - 4*a^9*c)*e^2)))/x^2) + 15*sqrt(
1/2)*a^3*d^3*x^5*sqrt(-((b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 - 7
*a^3*b*c^4)*d - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^
3 + 2*a^4*c^4)*e + ((a^7*b^2*c - 4*a^8*c^2)*d^2 - (a^7*b^3 - 4*a^
8*b*c)*d*e + (a^8*b^2 - 4*a^9*c)*e^2)*sqrt(((b^12*c^2 - 10*a*b^10
*c^3 + 37*a^2*b^8*c^4 - 62*a^3*b^6*c^5 + 46*a^4*b^4*c^6 - 12*a^5*
b^2*c^7 + a^6*c^8)*d^2 - 2*(b^13*c - 11*a*b^11*c^2 + 46*a^2*b^9*c
^3 - 91*a^3*b^7*c^4 + 86*a^4*b^5*c^5 - 34*a^5*b^3*c^6 + 4*a^6*b*c
^7)*d*e + (b^14 - 12*a*b^12*c + 56*a^2*b^10*c^2 - 128*a^3*b^8*c^3
 + 148*a^4*b^6*c^4 - 80*a^5*b^4*c^5 + 16*a^6*b^2*c^6)*e^2)/((a^14
*b^2*c^2 - 4*a^15*c^3)*d^4 - 2*(a^14*b^3*c - 4*a^15*b*c^2)*d^3*e
+ (a^14*b^4 - 2*a^15*b^2*c - 8*a^16*c^2)*d^2*e^2 - 2*(a^15*b^3 -
4*a^16*b*c)*d*e^3 + (a^16*b^2 - 4*a^17*c)*e^4)))/((a^7*b^2*c - 4*
a^8*c^2)*d^2 - (a^7*b^3 - 4*a^8*b*c)*d*e + (a^8*b^2 - 4*a^9*c)*e^
2))*log(-(((a^7*b^2*c^5 - 4*a^8*c^6)*d^3 - (a^7*b^3*c^4 - 4*a^8*b
*c^5)*d^2*e + (a^8*b^2*c^4 - 4*a^9*c^5)*d*e^2)*x^2*sqrt(((b^12*c^
2 - 10*a*b^10*c^3 + 37*a^2*b^8*c^4 - 62*a^3*b^6*c^5 + 46*a^4*b^4*
c^6 - 12*a^5*b^2*c^7 + a^6*c^8)*d^2 - 2*(b^13*c - 11*a*b^11*c^2 +
 46*a^2*b^9*c^3 - 91*a^3*b^7*c^4 + 86*a^4*b^5*c^5 - 34*a^5*b^3*c^
6 + 4*a^6*b*c^7)*d*e + (b^14 - 12*a*b^12*c + 56*a^2*b^10*c^2 - 12
8*a^3*b^8*c^3 + 148*a^4*b^6*c^4 - 80*a^5*b^4*c^5 + 16*a^6*b^2*c^6
)*e^2)/((a^14*b^2*c^2 - 4*a^15*c^3)*d^4 - 2*(a^14*b^3*c - 4*a^15*
b*c^2)*d^3*e + (a^14*b^4 - 2*a^15*b^2*c - 8*a^16*c^2)*d^2*e^2 - 2
*(a^15*b^3 - 4*a^16*b*c)*d*e^3 + (a^16*b^2 - 4*a^17*c)*e^4)) - 2*
(a*b^6*c^5 - 5*a^2*b^4*c^6 + 6*a^3*b^2*c^7 - a^4*c^8)*d^2 + 2*(a*
b^7*c^4 - 6*a^2*b^5*c^5 + 10*a^3*b^3*c^6 - 4*a^4*b*c^7)*d*e + ((b
^7*c^5 - 5*a*b^5*c^6 + 6*a^2*b^3*c^7 - a^3*b*c^8)*d^2 - (b^8*c^4
- 2*a*b^6*c^5 - 10*a^2*b^4*c^6 + 20*a^3*b^2*c^7 - 4*a^4*c^8)*d*e
+ 4*(a*b^7*c^4 - 6*a^2*b^5*c^5 + 10*a^3*b^3*c^6 - 4*a^4*b*c^7)*e^
2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^8*b^5*c^2 - 7*a^9*b^3*c
^3 + 12*a^10*b*c^4)*d^3 - (2*a^8*b^6*c - 15*a^9*b^4*c^2 + 30*a^10
*b^2*c^3 - 8*a^11*c^4)*d^2*e + (a^8*b^7 - 7*a^9*b^5*c + 11*a^10*b
^3*c^2 + 4*a^11*b*c^3)*d*e^2 - (a^9*b^6 - 8*a^10*b^4*c + 18*a^11*
b^2*c^2 - 8*a^12*c^3)*e^3)*x*sqrt(((b^12*c^2 - 10*a*b^10*c^3 + 37
*a^2*b^8*c^4 - 62*a^3*b^6*c^5 + 46*a^4*b^4*c^6 - 12*a^5*b^2*c^7 +
 a^6*c^8)*d^2 - 2*(b^13*c - 11*a*b^11*c^2 + 46*a^2*b^9*c^3 - 91*a
^3*b^7*c^4 + 86*a^4*b^5*c^5 - 34*a^5*b^3*c^6 + 4*a^6*b*c^7)*d*e +
 (b^14 - 12*a*b^12*c + 56*a^2*b^10*c^2 - 128*a^3*b^8*c^3 + 148*a^
4*b^6*c^4 - 80*a^5*b^4*c^5 + 16*a^6*b^2*c^6)*e^2)/((a^14*b^2*c^2
- 4*a^15*c^3)*d^4 - 2*(a^14*b^3*c - 4*a^15*b*c^2)*d^3*e + (a^14*b
^4 - 2*a^15*b^2*c - 8*a^16*c^2)*d^2*e^2 - 2*(a^15*b^3 - 4*a^16*b*
c)*d*e^3 + (a^16*b^2 - 4*a^17*c)*e^4)) - ((a*b^10*c^2 - 10*a^2*b^
8*c^3 + 35*a^3*b^6*c^4 - 51*a^4*b^4*c^5 + 29*a^5*b^2*c^6 - 4*a^6*
c^7)*d^2 - (2*a*b^11*c - 22*a^2*b^9*c^2 + 88*a^3*b^7*c^3 - 155*a^
4*b^5*c^4 + 114*a^5*b^3*c^5 - 24*a^6*b*c^6)*d*e + (a*b^12 - 12*a^
2*b^10*c + 54*a^3*b^8*c^2 - 112*a^4*b^6*c^3 + 104*a^5*b^4*c^4 - 3
```

2*a^6*b^2*c^5)*e^2)*x)*sqrt(-((b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c
^3 - 7*a^3*b*c^4)*d - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*
b^2*c^3 + 2*a^4*c^4)*e + ((a^7*b^2*c - 4*a^8*c^2)*d^2 - (a^7*b^3
- 4*a^8*b*c)*d*e + (a^8*b^2 - 4*a^9*c)*e^2)*sqrt(((b^12*c^2 - 10*
a*b^10*c^3 + 37*a^2*b^8*c^4 - 62*a^3*b^6*c^5 + 46*a^4*b^4*c^6 - 1
2*a^5*b^2*c^7 + a^6*c^8)*d^2 - 2*(b^13*c - 11*a*b^11*c^2 + 46*a^2
*b^9*c^3 - 91*a^3*b^7*c^4 + 86*a^4*b^5*c^5 - 34*a^5*b^3*c^6 + 4*a
^6*b*c^7)*d*e + (b^14 - 12*a*b^12*c + 56*a^2*b^10*c^2 - 128*a^3*b
^8*c^3 + 148*a^4*b^6*c^4 - 80*a^5*b^4*c^5 + 16*a^6*b^2*c^6)*e^2)/
((a^14*b^2*c^2 - 4*a^15*c^3)*d^4 - 2*(a^14*b^3*c - 4*a^15*b*c^2)*
d^3*e + (a^14*b^4 - 2*a^15*b^2*c - 8*a^16*c^2)*d^2*e^2 - 2*(a^15*
b^3 - 4*a^16*b*c)*d*e^3 + (a^16*b^2 - 4*a^17*c)*e^4)))/((a^7*b^2*
c - 4*a^8*c^2)*d^2 - (a^7*b^3 - 4*a^8*b*c)*d*e + (a^8*b^2 - 4*a^9
*c)*e^2)))/x^2) - 15*sqrt(1/2)*a^3*d^3*x^5*sqrt(-((b^7*c - 7*a*b^
5*c^2 + 14*a^2*b^3*c^3 - 7*a^3*b*c^4)*d - (b^8 - 8*a*b^6*c + 20*a
^2*b^4*c^2 - 16*a^3*b^2*c^3 + 2*a^4*c^4)*e + ((a^7*b^2*c - 4*a^8*
c^2)*d^2 - (a^7*b^3 - 4*a^8*b*c)*d*e + (a^8*b^2 - 4*a^9*c)*e^2)*s
qrt(((b^12*c^2 - 10*a*b^10*c^3 + 37*a^2*b^8*c^4 - 62*a^3*b^6*c^5
+ 46*a^4*b^4*c^6 - 12*a^5*b^2*c^7 + a^6*c^8)*d^2 - 2*(b^13*c - 11
*a*b^11*c^2 + 46*a^2*b^9*c^3 - 91*a^3*b^7*c^4 + 86*a^4*b^5*c^5 -
34*a^5*b^3*c^6 + 4*a^6*b*c^7)*d*e + (b^14 - 12*a*b^12*c + 56*a^2*
b^10*c^2 - 128*a^3*b^8*c^3 + 148*a^4*b^6*c^4 - 80*a^5*b^4*c^5 + 1
6*a^6*b^2*c^6)*e^2)/((a^14*b^2*c^2 - 4*a^15*c^3)*d^4 - 2*(a^14*b^
3*c - 4*a^15*b*c^2)*d^3*e + (a^14*b^4 - 2*a^15*b^2*c - 8*a^16*c^2
)*d^2*e^2 - 2*(a^15*b^3 - 4*a^16*b*c)*d*e^3 + (a^16*b^2 - 4*a^17*
c)*e^4)))/((a^7*b^2*c - 4*a^8*c^2)*d^2 - (a^7*b^3 - 4*a^8*b*c)*d*
e + (a^8*b^2 - 4*a^9*c)*e^2))*log(-(((a^7*b^2*c^5 - 4*a^8*c^6)*d^
3 - (a^7*b^3*c^4 - 4*a^8*b*c^5)*d^2*e + (a^8*b^2*c^4 - 4*a^9*c^5)
*d*e^2)*x^2*sqrt(((b^12*c^2 - 10*a*b^10*c^3 + 37*a^2*b^8*c^4 - 62
*a^3*b^6*c^5 + 46*a^4*b^4*c^6 - 12*a^5*b^2*c^7 + a^6*c^8)*d^2 - 2
*(b^13*c - 11*a*b^11*c^2 + 46*a^2*b^9*c^3 - 91*a^3*b^7*c^4 + 86*a
^4*b^5*c^5 - 34*a^5*b^3*c^6 + 4*a^6*b*c^7)*d*e + (b^14 - 12*a*b^1
2*c + 56*a^2*b^10*c^2 - 128*a^3*b^8*c^3 + 148*a^4*b^6*c^4 - 80*a^
5*b^4*c^5 + 16*a^6*b^2*c^6)*e^2)/((a^14*b^2*c^2 - 4*a^15*c^3)*d^4
- 2*(a^14*b^3*c - 4*a^15*b*c^2)*d^3*e + (a^14*b^4 - 2*a^15*b^2*c
- 8*a^16*c^2)*d^2*e^2 - 2*(a^15*b^3 - 4*a^16*b*c)*d*e^3 + (a^16*
b^2 - 4*a^17*c)*e^4)) - 2*(a*b^6*c^5 - 5*a^2*b^4*c^6 + 6*a^3*b^2*
c^7 - a^4*c^8)*d^2 + 2*(a*b^7*c^4 - 6*a^2*b^5*c^5 + 10*a^3*b^3*c^
6 - 4*a^4*b*c^7)*d*e + ((b^7*c^5 - 5*a*b^5*c^6 + 6*a^2*b^3*c^7 -
a^3*b*c^8)*d^2 - (b^8*c^4 - 2*a*b^6*c^5 - 10*a^2*b^4*c^6 + 20*a^3
*b^2*c^7 - 4*a^4*c^8)*d*e + 4*(a*b^7*c^4 - 6*a^2*b^5*c^5 + 10*a^3
*b^3*c^6 - 4*a^4*b*c^7)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((
(a^8*b^5*c^2 - 7*a^9*b^3*c^3 + 12*a^10*b*c^4)*d^3 - (2*a^8*b^6*c
- 15*a^9*b^4*c^2 + 30*a^10*b^2*c^3 - 8*a^11*c^4)*d^2*e + (a^8*b^7
- 7*a^9*b^5*c + 11*a^10*b^3*c^2 + 4*a^11*b*c^3)*d*e^2 - (a^9*b^6
- 8*a^10*b^4*c + 18*a^11*b^2*c^2 - 8*a^12*c^3)*e^3)*x*sqrt(((b^1
2*c^2 - 10*a*b^10*c^3 + 37*a^2*b^8*c^4 - 62*a^3*b^6*c^5 + 46*a^4*
b^4*c^6 - 12*a^5*b^2*c^7 + a^6*c^8)*d^2 - 2*(b^13*c - 11*a*b^11*c
^2 + 46*a^2*b^9*c^3 - 91*a^3*b^7*c^4 + 86*a^4*b^5*c^5 - 34*a^5*b^
3*c^6 + 4*a^6*b*c^7)*d*e + (b^14 - 12*a*b^12*c + 56*a^2*b^10*c^2
- 128*a^3*b^8*c^3 + 148*a^4*b^6*c^4 - 80*a^5*b^4*c^5 + 16*a^6*b^2
*c^6)*e^2)/((a^14*b^2*c^2 - 4*a^15*c^3)*d^4 - 2*(a^14*b^3*c - 4*a
^15*b*c^2)*d^3*e + (a^14*b^4 - 2*a^15*b^2*c - 8*a^16*c^2)*d^2*e^2
- 2*(a^15*b^3 - 4*a^16*b*c)*d*e^3 + (a^16*b^2 - 4*a^17*c)*e^4))

```
- ((a*b^10*c^2 - 10*a^2*b^8*c^3 + 35*a^3*b^6*c^4 - 51*a^4*b^4*c^5
 + 29*a^5*b^2*c^6 - 4*a^6*c^7)*d^2 - (2*a*b^11*c - 22*a^2*b^9*c^2
 + 88*a^3*b^7*c^3 - 155*a^4*b^5*c^4 + 114*a^5*b^3*c^5 - 24*a^6*b*
c^6)*d*e + (a*b^12 - 12*a^2*b^10*c + 54*a^3*b^8*c^2 - 112*a^4*b^6
*c^3 + 104*a^5*b^4*c^4 - 32*a^6*b^2*c^5)*e^2)*x)*sqrt(-((b^7*c -
7*a*b^5*c^2 + 14*a^2*b^3*c^3 - 7*a^3*b*c^4)*d - (b^8 - 8*a*b^6*c
+ 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 2*a^4*c^4)*e + ((a^7*b^2*c -
4*a^8*c^2)*d^2 - (a^7*b^3 - 4*a^8*b*c)*d*e + (a^8*b^2 - 4*a^9*c)*
e^2)*sqrt(((b^12*c^2 - 10*a*b^10*c^3 + 37*a^2*b^8*c^4 - 62*a^3*b^
6*c^5 + 46*a^4*b^4*c^6 - 12*a^5*b^2*c^7 + a^6*c^8)*d^2 - 2*(b^13*
c - 11*a*b^11*c^2 + 46*a^2*b^9*c^3 - 91*a^3*b^7*c^4 + 86*a^4*b^5*
c^5 - 34*a^5*b^3*c^6 + 4*a^6*b*c^7)*d*e + (b^14 - 12*a*b^12*c + 5
6*a^2*b^10*c^2 - 128*a^3*b^8*c^3 + 148*a^4*b^6*c^4 - 80*a^5*b^4*c
^5 + 16*a^6*b^2*c^6)*e^2)/((a^14*b^2*c^2 - 4*a^15*c^3)*d^4 - 2*(a
^14*b^3*c - 4*a^15*b*c^2)*d^3*e + (a^14*b^4 - 2*a^15*b^2*c - 8*a^
16*c^2)*d^2*e^2 - 2*(a^15*b^3 - 4*a^16*b*c)*d*e^3 + (a^16*b^2 - 4
*a^17*c)*e^4)))/((a^7*b^2*c - 4*a^8*c^2)*d^2 - (a^7*b^3 - 4*a^8*b
*c)*d*e + (a^8*b^2 - 4*a^9*c)*e^2)))/x^2) + 4*((10*a*b*d*e + 8*a^
2*e^2 + 15*(b^2 - a*c)*d^2)*x^4 + 3*a^2*d^2 - (5*a*b*d^2 + 4*a^2*
d*e)*x^2)*sqrt(e*x^2 + d))/(a^3*d^3*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**6/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^6),x, algorithm="giac")
```

```
[Out] Timed out
```

3.386 $\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=350

$$\frac{2\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}}+\frac{2\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}}-\frac{d^2x}{e\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{ce^{3/2}}$$

```
[Out] -((d^2*x)/(e*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + (2*(b^2
- a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(2*c*d - (b - Sqrt[b
^2 - 4*a*c])*e)^(3/2)) + (2*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(
Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b + Sqrt[b
^2 - 4*a*c]]*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)^(3/2)) + ArcTanh
[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(c*e^(3/2))
```

Rubi [A] time = 10.5529, antiderivative size = 507, normalized size of antiderivative = 1.45, number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241

$$\frac{\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,(ae^2-bde+cd^2)}+\frac{\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,(ae^2-bde+cd^2)}-\frac{d^2x}{e\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}-\frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}\,(ae^2-bde+cd^2)}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d^2*x)/(e*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + ((b^2*d
- a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/
(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(c*Sqrt[b - Sqrt[
b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*
d*e + a*e^2)) + ((b^2*d - a*c*d - a*b*e + (b^3*d - 3*a*b*c*d - a*
b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + S
qrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])]])/(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2
- 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + (d^2*ArcTanh[(Sqrt[e]*x)/
Sqrt[d + e*x^2]])/(e^(3/2)*(c*d^2 - b*d*e + a*e^2)) - ((b*d - a*e
)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(c*Sqrt[e]*(c*d^2 - b*d*e
 + a*e^2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.08464, size = 0, normalized size = 0.

$$\int \frac{x^6}{(d+ex^2)^{3/2}\,(a+bx^2+cx^4)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.051, size = 480, normalized size = 1.4

$$-\frac{x}{ce}\frac{1}{\sqrt{ex^2+d}}+\frac{1}{c}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{3}{2}}-\frac{bx}{c^2d}\frac{1}{\sqrt{ex^2+d}}$$
$$+8\frac{e^{3/2}ab}{c^2\left(4\,ae^2-4\,bde+4\,cd^2\right)\left(2\,ex^2-2\,\sqrt{e}\sqrt{ex^2+d}x+2\,d\right)}$$
$$+8\frac{\sqrt{e}ad}{c\left(4\,ae^2-4\,bde+4\,cd^2\right)\left(2\,ex^2-2\,\sqrt{e}\sqrt{ex^2+d}x+2\,d\right)}$$
$$-8\frac{\sqrt{e}b^2d}{c^2\left(4\,ae^2-4\,bde+4\,cd^2\right)\left(2\,ex^2-2\,\sqrt{e}\sqrt{ex^2+d}x+2\,d\right)}$$
$$+2\frac{\sqrt{e}}{c\left(4\,ae^2-4\,bde+4\,cd^2\right)}\sum_{\_R=RootOf\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+(16\,ae^2-8\,bde+6\,cd^2)\_Z^2+(4\,bd^2e-4\,cd^3)\_Z+cd^4\right)}\frac{\left(\left(abe+acd-b^2d\right)\_R^2+2\,(\right.}{\_R^3c\,\cdot}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/c*x/e/(e*x^2+d)^(1/2)+1/c/e^(3/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2)
)-1/c^2*b*x/d/(e*x^2+d)^(1/2)+8/c^2*e^(3/2)/(4*a*e^2-4*b*d*e+4*c*
d^2)/(2*e*x^2-2*e^(1/2)*(e*x^2+d)^(1/2)*x+2*d)*a*b+8/c*e^(1/2)/(4
*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/2)*(e*x^2+d)^(1/2)*x+2*d)
*a*d-8/c^2*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/2)*(
e*x^2+d)^(1/2)*x+2*d)*b^2*d+2/c*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)
*sum(((a*b*e+a*c*d-b^2*d)*_R^2+2*(2*a^2*e^2-3*a*b*d*e-a*c*d^2+b^2
*d^2)*_R+a*b*d^2*e+a*c*d^3-b^2*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d
+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1
/2)-x*e^(1/2))^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^
2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^6}{(cx^4+bx^2+a)(ex^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(d+ex^2)^{\frac{3}{2}}(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**6/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.387 $\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=360

$$-\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,(ae^2-bde+cd^2)}-\frac{\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,(ae^2-bde+cd^2)}+\frac{dx}{\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}$$

```
[Out] (d*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - ((b*d - a*e - (
b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d
 + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) - ((b*d - a*e + (b^2*d
- 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + S
qrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 3.2369, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$-\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,(ae^2-bde+cd^2)}-\frac{\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,(ae^2-bde+cd^2)}+\frac{dx}{\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (d*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - ((b*d - a*e - (
b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d
 + e*x^2])]])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) - ((b*d - a*e + (b^2*d
- 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + S
qrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])]])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.808117, size = 0, normalized size = 0.

$$\int \frac{x^4}{(d+ex^2)^{3/2}\,(a+bx^2+cx^4)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.042, size = 338, normalized size = 0.9

$$\frac{x}{cd}\frac{1}{\sqrt{ex^2+d}} - 8\,\frac{e^{3/2}a}{c\,(4\,ae^2-4\,bde+4\,cd^2)\left(2\,ex^2-2\,\sqrt{e}\sqrt{ex^2+d}x+2\,d\right)}$$
$$+\,8\,\frac{\sqrt{e}bd}{c\,(4\,ae^2-4\,bde+4\,cd^2)\left(2\,ex^2-2\,\sqrt{e}\sqrt{ex^2+d}x+2\,d\right)}$$

$$-2\,\frac{\sqrt{e}}{4\,ae^2-4\,bde+4\,cd^2}\sum_{\_R=RootOf\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+(16\,ae^2-8\,bde+6\,cd^2)\_Z^2+(4\,bd^2e-4\,cd^3)\_Z+cd^4\right)}\frac{\left((ae-bd)\_R^2+2\,d\,(-3\,ae+bd\right.}{\_R^3c+3\,\_R^2be-3\,\_R^2cd\,-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/c*x/d/(e*x^2+d)^(1/2)-8/c*e^(3/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*
e*x^2-2*e^(1/2)*(e*x^2+d)^(1/2)*x+2*d)*a+8/c*e^(1/2)/(4*a*e^2-4*b
*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/2)*(e*x^2+d)^(1/2)*x+2*d)*b*d-2*e^(
1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)*sum(((a*e-b*d)*_R^2+2*d*(-3*a*e+b*
d)*_R+a*d^2*e-b*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_
R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-x*e^(1/2))^
2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d
^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(cx^4 + bx^2 + a)(ex^2 + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

Fricas [A] time = 69.2811, size = 19524, normalized size = 54.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(1/2)*(c*d^3 - b*d^2*e + a*d*e^2 + (c*d^2*e - b*d*e^2 +
a*e^3)*x^2)*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^
3 - 3*(a*b^2 - 2*a^2*c)*d^2*e - ((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3
*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4
*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^
2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a
^3*b^2 - 4*a^4*c)*e^6)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 -
(b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(
5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*
c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6
)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3
```

```
+ 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c +
6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*
a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6
- 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7
 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*
b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5
*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (
a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2
- 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2
- (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2
*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^
2 - 4*a^4*c)*e^6))*log(-(6*a^3*b*d^3*e - 6*a^4*d^2*e^2 - 2*(a^2*b
^2 - a^3*c)*d^4 + ((a*b^2*c^3 - 4*a^2*c^4)*d^7 - 3*(a*b^3*c^2 - 4
*a^2*b*c^3)*d^6*e + 3*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^5*e
^2 - (a*b^5 + 2*a^2*b^3*c - 24*a^3*b*c^2)*d^4*e^3 + 3*(a^2*b^4 -
3*a^3*b^2*c - 4*a^4*c^2)*d^3*e^4 - 3*(a^3*b^3 - 4*a^4*b*c)*d^2*e^
5 + (a^4*b^2 - 4*a^5*c)*d*e^6)*x^2*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^
4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)
*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/((b^2*c^6 - 4*a*c^7)*d^
12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5
 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c
^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 -
 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5
+ (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c
^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*
c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8
- 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*
b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c
)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)) + (15*a^3*b*d^2*e^2 - 12*a^
4*d*e^3 + (a*b^3 - a^2*b*c)*d^4 - (7*a^2*b^2 - 4*a^3*c)*d^3*e)*x^
2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*
c^5)*d^8 - (3*b^5*c^2 - 16*a*b^3*c^3 + 16*a^2*b*c^4)*d^7*e + (3*b
^6*c - 9*a*b^4*c^2 - 16*a^2*b^2*c^3 + 16*a^3*c^4)*d^6*e^2 - (b^7
+ 6*a*b^5*c - 40*a^2*b^3*c^2)*d^5*e^3 + 5*(a*b^6 - a^2*b^4*c - 12
*a^3*b^2*c^2)*d^4*e^4 - (11*a^2*b^5 - 32*a^3*b^3*c - 48*a^4*b*c^2
)*d^3*e^5 + (13*a^3*b^4 - 48*a^4*b^2*c - 16*a^5*c^2)*d^2*e^6 - 8*
(a^4*b^3 - 4*a^5*b*c)*d*e^7 + 2*(a^5*b^2 - 4*a^6*c)*e^8)*x*sqrt(-
(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d
^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)
/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(
5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 -
5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^
4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3
- 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 34
0*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30
*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c
^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*
c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10
- 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)) + (
(b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^5 - (5*a*b^4 - 22*a^2*b^2*c + 8
*a^3*c^2)*d^4*e + 9*(a^2*b^3 - 4*a^3*b*c)*d^3*e^2 - 6*(a^3*b^2 -
4*a^4*c)*d^2*e^3)*x)*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*
a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e - ((b^2*c^3 - 4*a*c^4)*d^6
 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2
```

```
*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b
^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d
*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d
^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^
5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/((b^2*c^6 - 4*a*c^7)*d^12
- 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 -
8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)
*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*
(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (
b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)
*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3
)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 1
0*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4
 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d
*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3 - 4*a*c^4)*d^6 - 3*
(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)
*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 -
3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5
+ (a^3*b^2 - 4*a^4*c)*e^6)))/x^2) - sqrt(1/2)*(c*d^3 - b*d^2*e +
a*d*e^2 + (c*d^2*e - b*d*e^2 + a*e^3)*x^2)*sqrt(-(3*a^2*b*d*e^2 -
 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e - ((
b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c
 - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b
*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(
a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)*sqrt(-(18*a
^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 +
6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/((b^
2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4
*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b
^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4
*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*
a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3
*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*
b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 -
4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*
d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(
a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c
^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*
a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)
*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b
^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6))*log(-(6*a^3*b*d
^3*e - 6*a^4*d^2*e^2 - 2*(a^2*b^2 - a^3*c)*d^4 + ((a*b^2*c^3 - 4*
a^2*c^4)*d^7 - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*d^6*e + 3*(a*b^4*c - 3
*a^2*b^2*c^2 - 4*a^3*c^3)*d^5*e^2 - (a*b^5 + 2*a^2*b^3*c - 24*a^3
*b*c^2)*d^4*e^3 + 3*(a^2*b^4 - 3*a^3*b^2*c - 4*a^4*c^2)*d^3*e^4 -
 3*(a^3*b^3 - 4*a^4*b*c)*d^2*e^5 + (a^4*b^2 - 4*a^5*c)*d*e^6)*x^2
*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2
*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d
^4*e^2)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*
e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5
*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2
*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b
^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c
^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5
```

*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)) + (15*a^3*b*d^2*e^2 - 12*a^4*d*e^3 + (a*b^3 - a^2*b*c)*d^4 - (7*a^2*b^2 - 4*a^3*c)*d^3*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^8 - (3*b^5*c^2 - 16*a*b^3*c^3 + 16*a^2*b*c^4)*d^7*e + (3*b^6*c - 9*a*b^4*c^2 - 16*a^2*b^2*c^3 + 16*a^3*c^4)*d^6*e^2 - (b^7 + 6*a*b^5*c - 40*a^2*b^3*c^2)*d^5*e^3 + 5*(a*b^6 - a^2*b^4*c - 12*a^3*b^2*c^2)*d^4*e^4 - (11*a^2*b^5 - 32*a^3*b^3*c - 48*a^4*b*c^2)*d^3*e^5 + (13*a^3*b^4 - 48*a^4*b^2*c - 16*a^5*c^2)*d^2*e^6 - 8*(a^4*b^3 - 4*a^5*b*c)*d*e^7 + 2*(a^5*b^2 - 4*a^6*c)*e^8)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)) + ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^5 - (5*a*b^4 - 22*a^2*b^2*c + 8*a^3*c^2)*d^4*e + 9*(a^2*b^3 - 4*a^3*b*c)*d^3*e^2 - 6*(a^3*b^2 - 4*a^4*c)*d^2*e^3)*x)*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e - ((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)))/x^2) - sqrt(1/2)*(c*d^3 - b*d^2*e + a*d*e^2 + (c*d^2*e - b*d*e^2 + a*e^3)*x^2)*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e + ((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*

```
b^2 - 4*a^4*c)*e^6)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^
4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a
^2*b^2 - 2*a^3*c)*d^4*e^2)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5
 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d
^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 1
5*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a
*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b
^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6
*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 +
15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5
 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^
2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6
*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4
*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (
b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c
- 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 -
 4*a^4*c)*e^6))*log(-(6*a^3*b*d^3*e - 6*a^4*d^2*e^2 - 2*(a^2*b^2
- a^3*c)*d^4 - ((a*b^2*c^3 - 4*a^2*c^4)*d^7 - 3*(a*b^3*c^2 - 4*a^
2*b*c^3)*d^6*e + 3*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^5*e^2
- (a*b^5 + 2*a^2*b^3*c - 24*a^3*b*c^2)*d^4*e^3 + 3*(a^2*b^4 - 3*a
^3*b^2*c - 4*a^4*c^2)*d^3*e^4 - 3*(a^3*b^3 - 4*a^4*b*c)*d^2*e^5 +
 (a^4*b^2 - 4*a^5*c)*d*e^6)*x^2*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d
^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^
5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/((b^2*c^6 - 4*a*c^7)*d^12
- 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 -
8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)
*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*
(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (
b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)
*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3
)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 1
0*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4
 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d
*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)) + (15*a^3*b*d^2*e^2 - 12*a^4*d
*e^3 + (a*b^3 - a^2*b*c)*d^4 - (7*a^2*b^2 - 4*a^3*c)*d^3*e)*x^2 +
 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5
)*d^8 - (3*b^5*c^2 - 16*a*b^3*c^3 + 16*a^2*b*c^4)*d^7*e + (3*b^6*
c - 9*a*b^4*c^2 - 16*a^2*b^2*c^3 + 16*a^3*c^4)*d^6*e^2 - (b^7 + 6
*a*b^5*c - 40*a^2*b^3*c^2)*d^5*e^3 + 5*(a*b^6 - a^2*b^4*c - 12*a^
3*b^2*c^2)*d^4*e^4 - (11*a^2*b^5 - 32*a^3*b^3*c - 48*a^4*b*c^2)*d
^3*e^5 + (13*a^3*b^4 - 48*a^4*b^2*c - 16*a^5*c^2)*d^2*e^6 - 8*(a^
4*b^3 - 4*a^5*b*c)*d*e^7 + 2*(a^5*b^2 - 4*a^6*c)*e^8)*x*sqrt(-(18
*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6
+ 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/((
b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b
^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a
*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 -
 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 4
0*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a
^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^
3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2
- 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2
)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6
*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)) - ((b^
```

```
5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^5 - (5*a*b^4 - 22*a^2*b^2*c + 8*a^
3*c^2)*d^4*e + 9*(a^2*b^3 - 4*a^3*b*c)*d^3*e^2 - 6*(a^3*b^2 - 4*a
^4*c)*d^2*e^3)*x)*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b
*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e + ((b^2*c^3 - 4*a*c^4)*d^6 -
3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^
3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4
- 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^
5 + (a^3*b^2 - 4*a^4*c)*e^6)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*
e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e
 - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/((b^2*c^6 - 4*a*c^7)*d^12 - 6
*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a
^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^
9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^
7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8
 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^
6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d
^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(
2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 -
18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^
11 + (a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^
3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^
4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a
^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (
a^3*b^2 - 4*a^4*c)*e^6)))/x^2) + sqrt(1/2)*(c*d^3 - b*d^2*e + a*d
*e^2 + (c*d^2*e - b*d*e^2 + a*e^3)*x^2)*sqrt(-(3*a^2*b*d*e^2 - 2*
a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e + ((b^2
*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c -
3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^
2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2
*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)*sqrt(-(18*a^3*
b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(
a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/((b^2*c
^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^
4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*
c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^
3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3
*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^
2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3
*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a
^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3
*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5
*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3
- 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b
^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^
3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3
- 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6))*log(-(6*a^3*b*d^3*
e - 6*a^4*d^2*e^2 - 2*(a^2*b^2 - a^3*c)*d^4 - ((a*b^2*c^3 - 4*a^2
*c^4)*d^7 - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*d^6*e + 3*(a*b^4*c - 3*a^
2*b^2*c^2 - 4*a^3*c^3)*d^5*e^2 - (a*b^5 + 2*a^2*b^3*c - 24*a^3*b*
c^2)*d^4*e^3 + 3*(a^2*b^4 - 3*a^3*b^2*c - 4*a^4*c^2)*d^3*e^4 - 3*
(a^3*b^3 - 4*a^4*b*c)*d^2*e^5 + (a^4*b^2 - 4*a^5*c)*d*e^6)*x^2*sq
rt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^
2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*
e^2)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e +
```

```
3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^
3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^
2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*
c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2
- 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c
- 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b
^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^
5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e
^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12))
 + (15*a^3*b*d^2*e^2 - 12*a^4*d*e^3 + (a*b^3 - a^2*b*c)*d^4 - (7*
a^2*b^2 - 4*a^3*c)*d^3*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^
4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^8 - (3*b^5*c^2 - 16*a*b^3*c^3
+ 16*a^2*b*c^4)*d^7*e + (3*b^6*c - 9*a*b^4*c^2 - 16*a^2*b^2*c^3 +
 16*a^3*c^4)*d^6*e^2 - (b^7 + 6*a*b^5*c - 40*a^2*b^3*c^2)*d^5*e^3
 + 5*(a*b^6 - a^2*b^4*c - 12*a^3*b^2*c^2)*d^4*e^4 - (11*a^2*b^5 -
 32*a^3*b^3*c - 48*a^4*b*c^2)*d^3*e^5 + (13*a^3*b^4 - 48*a^4*b^2*
c - 16*a^5*c^2)*d^2*e^6 - 8*(a^4*b^3 - 4*a^5*b*c)*d*e^7 + 2*(a^5*
b^2 - 4*a^6*c)*e^8)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (
b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5
*a^2*b^2 - 2*a^3*c)*d^4*e^2)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c
^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)
*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 +
 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6
*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a
*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 -
 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7
+ 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b
^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*
b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a
^6*b^2 - 4*a^7*c)*e^12)) - ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^5 -
 (5*a*b^4 - 22*a^2*b^2*c + 8*a^3*c^2)*d^4*e + 9*(a^2*b^3 - 4*a^3*
b*c)*d^3*e^2 - 6*(a^3*b^2 - 4*a^4*c)*d^2*e^3)*x)*sqrt(-(3*a^2*b*d
*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d^2*
e + ((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*
(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24
*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4
 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)*sqrt(
-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*
d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2
)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*
(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 -
 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c
^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3
 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 3
40*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 3
0*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*
c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b
*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10
 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))/(
(b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*
c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*
b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*
(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)))/x^2) + 4
*sqrt(e*x^2 + d)*d*x)/(c*d^3 - b*d^2*e + a*d*e^2 + (c*d^2*e - b*d
```

```
*e^2 + a*e^3)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(d+ex^2)^{\frac{3}{2}}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**4/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.388 $\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=333

$$\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,(ae^2-bde+cd^2)}+\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,(ae^2-bde+cd^2)}-\frac{ex}{\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}$$

```
[Out] -((e*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + (c*(d - (b*d
 - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 -
 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(S
qrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
]*(c*d^2 - b*d*e + a*e^2)) + (c*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b +
 Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c
]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2
))
```

Rubi [A] time = 1.70365, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,(ae^2-bde+cd^2)}+\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,(ae^2-bde+cd^2)}-\frac{ex}{\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((e*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + (c*(d - (b*d
 - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 -
 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(S
qrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
]*(c*d^2 - b*d*e + a*e^2)) + (c*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b +
 Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c
]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2
))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.686821, size = 0, normalized size = 0.

$$\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.037, size = 252, normalized size = 0.8

$$-8\,\frac{d\sqrt{e}}{(4\,ae^2-4\,bde+4\,cd^2)\left(2\,ex^2-2\,\sqrt{e}\sqrt{ex^2+d}x+2\,d\right)}$$

$$-2\,\frac{\sqrt{e}}{4\,ae^2-4\,bde+4\,cd^2}\sum_{\_R=RootOf\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+(16\,ae^2-8\,bde+6\,cd^2)\_Z^2+(4\,bd^2e-4\,cd^3)\_Z+cd^4\right)}\frac{\left(\_R^2cd+2\,\left(2\,ae^2-cd^2\right)\_R}{\_R^3c+3\,\_R^2be-3\,\_R^2cd+8\,\_}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -8*e^(1/2)*d/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/2)*(e*x^2+
d)^(1/2)*x+2*d)-2*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)*sum((_R^2*c*d
+2*(2*a*e^2-c*d^2)*_R+c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a
*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-x*e
^(1/2))^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d
*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(cx^4 + bx^2 + a)(ex^2 + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

Fricas [A] time = 89.8779, size = 19097, normalized size = 57.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*(c*d^3 - b*d^2*e + a*d*e^2 + (c*d^2*e - b*d*e^2 +
 a*e^3)*x^2)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (
a*b^2 - 2*a^2*c)*e^3 + ((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*
a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b
^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c -
 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 -
4*a^4*c)*e^6)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3
 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/((b^2*c^6 -
 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 -
18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4
- 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^
5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c
^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^
```

```
3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2
 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c
^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9
 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3
 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3 - 4*
a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c
^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^
3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*
a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6))*log((2*a*c^3*d^4 - 6*a
^2*c^2*d^2*e^2 + 2*a^2*b*c*d*e^3 + ((b^2*c^4 - 4*a*c^5)*d^7 - 3*(
b^3*c^3 - 4*a*b*c^4)*d^6*e + 3*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4
)*d^5*e^2 - (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^4*e^3 + 3*(a*b
^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^4 - 3*(a^2*b^3*c - 4*a^3*
b*c^2)*d^2*e^5 + (a^3*b^2*c - 4*a^4*c^2)*d*e^6)*x^2*sqrt((c^4*d^6
 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^
2*b*c*d*e^5 + a^2*b^2*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5
 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d
^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 1
5*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a
*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b
^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6
*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 +
15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5
 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^
2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6
*b^2 - 4*a^7*c)*e^12)) - (b*c^3*d^4 - 4*a*c^3*d^3*e - 3*a*b*c^2*d
^2*e^2 - 4*a^2*b*c*e^4 + (a*b^2*c + 12*a^2*c^2)*d*e^3)*x^2 + 2*sq
rt(1/2)*sqrt(e*x^2 + d)*(((b^3*c^4 - 4*a*b*c^5)*d^8 - (3*b^4*c^3
 - 8*a*b^2*c^4 - 16*a^2*c^5)*d^7*e + (3*b^5*c^2 + 4*a*b^3*c^3 - 64
*a^2*b*c^4)*d^6*e^2 - (b^6*c + 17*a*b^4*c^2 - 72*a^2*b^2*c^3 - 48
*a^3*c^4)*d^5*e^3 + 10*(a*b^5*c - a^2*b^3*c^2 - 12*a^3*b*c^3)*d^4
*e^4 - (a*b^6 + 17*a^2*b^4*c - 72*a^3*b^2*c^2 - 48*a^4*c^3)*d^3*e
^5 + (3*a^2*b^5 + 4*a^3*b^3*c - 64*a^4*b*c^2)*d^2*e^6 - (3*a^3*b^
4 - 8*a^4*b^2*c - 16*a^5*c^2)*d*e^7 + (a^4*b^3 - 4*a^5*b*c)*e^8)*
x*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2
*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/((b^2*c^6 - 4*a*c^7)*d^
12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5
 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c
^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 -
 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5
+ (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c
^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*
c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8
- 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*
b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c
)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)) - ((b^2*c^3 - 4*a*c^4)*d^5
- 4*(a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 + (a*b^3*c - 4*a^2*b*c^2)*d^2
*e^3 + 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^
5)*x)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 -
 2*a^2*c)*e^3 + ((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3
)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*
a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*
c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c
)*e^6)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^
```

2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)))/x^2) - sqrt(1/2)*(c*d^3 - b*d^2*e + a*d*e^2 + (c*d^2*e - b*d*e^2 + a*e^3)*x^2)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + ((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6))*log((2*a*c^3*d^4 - 6*a^2*c^2*d^2*e^2 + 2*a^2*b*c*d*e^3 + ((b^2*c^4 - 4*a*c^5)*d^7 - 3*(b^3*c^3 - 4*a*b*c^4)*d^6*e + 3*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^5*e^2 - (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^4*e^3 + 3*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^4 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e^5 + (a^3*b^2*c - 4*a^4*c^2)*d*e^6)*x^2*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)) - (b*c^3*d^4 - 4*a*c^3*d^3*e - 3*a*b*c^2*d^2*e^2 - 4*a^2*b*c*e^4 + (a

*b^2*c + 12*a^2*c^2)*d*e^3)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((
b^3*c^4 - 4*a*b*c^5)*d^8 - (3*b^4*c^3 - 8*a*b^2*c^4 - 16*a^2*c^5)
*d^7*e + (3*b^5*c^2 + 4*a*b^3*c^3 - 64*a^2*b*c^4)*d^6*e^2 - (b^6*
c + 17*a*b^4*c^2 - 72*a^2*b^2*c^3 - 48*a^3*c^4)*d^5*e^3 + 10*(a*b
^5*c - a^2*b^3*c^2 - 12*a^3*b*c^3)*d^4*e^4 - (a*b^6 + 17*a^2*b^4*
c - 72*a^3*b^2*c^2 - 48*a^4*c^3)*d^3*e^5 + (3*a^2*b^5 + 4*a^3*b^3
*c - 64*a^4*b*c^2)*d^2*e^6 - (3*a^3*b^4 - 8*a^4*b^2*c - 16*a^5*c^
2)*d*e^7 + (a^4*b^3 - 4*a^5*b*c)*e^8)*x*sqrt((c^4*d^6 - 6*a*c^3*d
^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5
+ a^2*b^2*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6
)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10
*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 -
15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 3
0*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^
2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*
a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6
- 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3
*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*
c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7
*c)*e^12)) - ((b^2*c^3 - 4*a*c^4)*d^5 - 4*(a*b^2*c^2 - 4*a^2*c^3)
*d^3*e^2 + (a*b^3*c - 4*a^2*b*c^2)*d^2*e^3 + 3*(a^2*b^2*c - 4*a^3
*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*x)*sqrt(-(b*c^2*d^3 - 6*
a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + ((b^2*c^3 -
4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^
2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3
e^3 + 3(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 -
4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)*sqrt((c^4*d^6 - 6*a*
c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d
*e^5 + a^2*b^2*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*
b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2
- 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*
c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^
2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c -
30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7
+ 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2
*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^
4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8
*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 -
4*a^7*c)*e^12)))/((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^
3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2
*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3
*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*
c)*e^6)))/x^2) - sqrt(1/2)*(c*d^3 - b*d^2*e + a*d*e^2 + (c*d^2*e
- b*d*e^2 + a*e^3)*x^2)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*
c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 - ((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b
^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d
^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*
a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 +
(a^3*b^2 - 4*a^4*c)*e^6)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*
c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/
((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5
*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5
*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4
- 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 -

```
 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340
*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*
a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^
2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c
^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 -
 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))/((b
^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c
- 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*
c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a
^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6))*log((2*a*c^
3*d^4 - 6*a^2*c^2*d^2*e^2 + 2*a^2*b*c*d*e^3 - ((b^2*c^4 - 4*a*c^5
)*d^7 - 3*(b^3*c^3 - 4*a*b*c^4)*d^6*e + 3*(b^4*c^2 - 3*a*b^2*c^3
- 4*a^2*c^4)*d^5*e^2 - (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^4*e
^3 + 3*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^4 - 3*(a^2*b^3
*c - 4*a^3*b*c^2)*d^2*e^5 + (a^3*b^2*c - 4*a^4*c^2)*d*e^6)*x^2*sq
rt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2
*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 -
 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8
*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*
d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(
b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b
^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*
d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)
*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10
*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4
- 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*
e^11 + (a^6*b^2 - 4*a^7*c)*e^12)) - (b*c^3*d^4 - 4*a*c^3*d^3*e -
3*a*b*c^2*d^2*e^2 - 4*a^2*b*c*e^4 + (a*b^2*c + 12*a^2*c^2)*d*e^3)
*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^3*c^4 - 4*a*b*c^5)*d^8 -
(3*b^4*c^3 - 8*a*b^2*c^4 - 16*a^2*c^5)*d^7*e + (3*b^5*c^2 + 4*a*b
^3*c^3 - 64*a^2*b*c^4)*d^6*e^2 - (b^6*c + 17*a*b^4*c^2 - 72*a^2*b
^2*c^3 - 48*a^3*c^4)*d^5*e^3 + 10*(a*b^5*c - a^2*b^3*c^2 - 12*a^3
*b*c^3)*d^4*e^4 - (a*b^6 + 17*a^2*b^4*c - 72*a^3*b^2*c^2 - 48*a^4
*c^3)*d^3*e^5 + (3*a^2*b^5 + 4*a^3*b^3*c - 64*a^4*b*c^2)*d^2*e^6
- (3*a^3*b^4 - 8*a^4*b^2*c - 16*a^5*c^2)*d*e^7 + (a^4*b^3 - 4*a^5
*b*c)*e^8)*x*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3
+ 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/((b^2*c^6 -
4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 1
8*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 -
 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5
)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^
4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3
 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2
- 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^
3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9
+ 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3
- 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)) + ((b^2*c^3 - 4*
a*c^4)*d^5 - 4*(a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 + (a*b^3*c - 4*a^2
*b*c^2)*d^2*e^3 + 3*(a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*
a^3*b*c)*e^5)*x)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2
 - (a*b^2 - 2*a^2*c)*e^3 - ((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2
- 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2
- (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2
*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^
```

```
2 - 4*a^4*c)*e^6)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3
*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/((b^2*c
^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^
4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*
c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^
3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3
*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^
2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3
*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a
^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3
*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5
*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3
- 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b
^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^
3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3
- 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)))/x^2) + sqrt(1/2)*
(c*d^3 - b*d^2*e + a*d*e^2 + (c*d^2*e - b*d*e^2 + a*e^3)*x^2)*sqr
t(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)
*e^3 - ((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e +
 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c -
 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*
e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)*sq
rt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2
*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 -
 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8
*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*
d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(
b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b
^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*
d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)
*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10
*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4
- 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*
e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3 - 4*a*c^4)*d^6 - 3*(
b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*
d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3
*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 +
 (a^3*b^2 - 4*a^4*c)*e^6))*log((2*a*c^3*d^4 - 6*a^2*c^2*d^2*e^2 +
 2*a^2*b*c*d*e^3 - ((b^2*c^4 - 4*a*c^5)*d^7 - 3*(b^3*c^3 - 4*a*b*
c^4)*d^6*e + 3*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^5*e^2 - (b^5
*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^4*e^3 + 3*(a*b^4*c - 3*a^2*b^2
*c^2 - 4*a^3*c^3)*d^3*e^4 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e^5 +
 (a^3*b^2*c - 4*a^4*c^2)*d*e^6)*x^2*sqrt((c^4*d^6 - 6*a*c^3*d^4*e
^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^
2*b^2*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^
11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*
b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*
a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^
2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^
4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*
b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15
*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c -
 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)
*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*
```

```
e^12)) - (b*c^3*d^4 - 4*a*c^3*d^3*e - 3*a*b*c^2*d^2*e^2 - 4*a^2*b
*c*e^4 + (a*b^2*c + 12*a^2*c^2)*d*e^3)*x^2 - 2*sqrt(1/2)*sqrt(e*x
^2 + d)*(((b^3*c^4 - 4*a*b*c^5)*d^8 - (3*b^4*c^3 - 8*a*b^2*c^4 -
16*a^2*c^5)*d^7*e + (3*b^5*c^2 + 4*a*b^3*c^3 - 64*a^2*b*c^4)*d^6*
e^2 - (b^6*c + 17*a*b^4*c^2 - 72*a^2*b^2*c^3 - 48*a^3*c^4)*d^5*e^
3 + 10*(a*b^5*c - a^2*b^3*c^2 - 12*a^3*b*c^3)*d^4*e^4 - (a*b^6 +
17*a^2*b^4*c - 72*a^3*b^2*c^2 - 48*a^4*c^3)*d^3*e^5 + (3*a^2*b^5
+ 4*a^3*b^3*c - 64*a^4*b*c^2)*d^2*e^6 - (3*a^3*b^4 - 8*a^4*b^2*c
- 16*a^5*c^2)*d*e^7 + (a^4*b^3 - 4*a^5*b*c)*e^8)*x*sqrt((c^4*d^6
- 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2
*b*c*d*e^5 + a^2*b^2*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5
- 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^
10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15
*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*
b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^
6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*
(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 1
5*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5
- 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2
*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*
b^2 - 4*a^7*c)*e^12)) + ((b^2*c^3 - 4*a*c^4)*d^5 - 4*(a*b^2*c^2 -
 4*a^2*c^3)*d^3*e^2 + (a*b^3*c - 4*a^2*b*c^2)*d^2*e^3 + 3*(a^2*b^
2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*x)*sqrt(-(b*c
^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 -
((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4
*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2
*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3
*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)*sqrt((c^4
*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 -
6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3
*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^
6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3
 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c +
 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26
*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6
 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^
7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3
*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^
5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 +
(a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2
 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2
 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^
2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b
^2 - 4*a^4*c)*e^6)))/x^2) + 4*sqrt(e*x^2 + d)*e*x)/(c*d^3 - b*d^2
*e + a*d*e^2 + (c*d^2*e - b*d*e^2 + a*e^3)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(d+ex^2)^{\frac{3}{2}}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**2/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.389 $\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=341

$$-\frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,(ae^2-bde+cd^2)}-\frac{c\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,(ae^2-bde+cd^2)}+\frac{e^2x}{d\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}$$

```
[Out] (e^2*x)/(d*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - (c*(e - (2*
c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(
Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e]*(c*d^2 - b*d*e + a*e^2)) - (c*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b
+ Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(Sqrt[b + Sqrt[b^2 - 4*a*
c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^
2))
```

Rubi [A] time = 2.02382, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,(ae^2-bde+cd^2)}-\frac{c\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,(ae^2-bde+cd^2)}+\frac{e^2x}{d\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (e^2*x)/(d*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - (c*(e - (2*
c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(
Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e]*(c*d^2 - b*d*e + a*e^2)) - (c*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b
+ Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(Sqrt[b + Sqrt[b^2 - 4*a*
c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^
2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.734579, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.03, size = 246, normalized size = 0.7

$$32\,\frac{e^{3/2}}{(16\,ae^2-16\,bde+16\,cd^2)\left(2\,ex^2-2\,\sqrt{e}\sqrt{ex^2+d}x+2\,d\right)}$$

$$+8\,\frac{e^{3/2}}{16\,ae^2-16\,bde+16\,cd^2}\sum_{\_R=RootOf\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+(16\,ae^2-8\,bde+6\,cd^2)\_Z^2+(4\,bd^2e-4\,cd^3)\_Z+cd^4\right)}\frac{\left(\_R^2c+2\,(2\,be-3\,cd)\_\right.}{\_R^3c+3\,\_R^2be-3\,\_R^2cd+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 32*e^(3/2)/(16*a*e^2-16*b*d*e+16*c*d^2)/(2*e*x^2-2*e^(1/2)*(e*x^2
+d)^(1/2)*x+2*d)+8*e^(3/2)/(16*a*e^2-16*b*d*e+16*c*d^2)*sum((_R^2
*c+2*(2*b*e-3*c*d)*_R+c*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a
*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-x*e
^(1/2))^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d
*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4+bx^2+a)(ex^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^2)^{\frac{3}{2}}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(1/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.390 $\int \frac{1}{x^2(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=339

$$-\frac{2c^2\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}}-\frac{2c^2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}}+\frac{ex(cd-be)}{ad\sqrt{d+ex^2}\left(e(ae-bd)+cd^2\right)}+\frac{-d-2ex^2}{ad^2x\sqrt{d+ex^2}}$$

```
[Out] (e*(c*d - b*e)*x)/(a*d*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[d + e*x^2]
) + (-d - 2*e*x^2)/(a*d^2*x*Sqrt[d + e*x^2]) - (2*c^2*(1 + b/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)
/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b - Sqrt
[b^2 - 4*a*c]]*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)^(3/2)) - (2*c^
2*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*
Sqrt[b + Sqrt[b^2 - 4*a*c]]*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)^(
3/2))
```

Rubi [A] time = 6.18784, antiderivative size = 462, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276

$$-\frac{c\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}-\frac{c\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(ae^2-bde+cd^2\right)}-\frac{e^2}{dx\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{d+ex^2}(cd-be)}{adx\left(ae^2-bde+cd^2\right)}-\frac{2e^3x}{d^2\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(e^2/(d*(c*d^2 - b*d*e + a*e^2)*x*Sqrt[d + e*x^2])) - (2*e^3*x)/
(d^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - ((c*d - b*e)*Sqrt
[d + e*x^2])/(a*d*(c*d^2 - b*d*e + a*e^2)*x) - (c*(c*d - b*e + (b
*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (
b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d
+ e*x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt
[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) - (c*(c*d - b*e - (b*c
*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d +
e*x^2])])/(a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.12516, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(d+e x^2\right)^{3/2} \left(a+b x^2+c x^4\right)} \, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.046, size = 387, normalized size = 1.1

$$-\frac{1}{adx}\frac{1}{\sqrt{ex^2+d}}-2\frac{ex}{ad^2\sqrt{ex^2+d}}-8\frac{e^{3/2}b}{a\left(4\,ae^2-4\,bde+4\,cd^2\right)\left(2\,ex^2-2\,\sqrt{e}\sqrt{ex^2+d}x+2\,d\right)}$$
$$+8\frac{\sqrt{e}cd}{a\left(4\,ae^2-4\,bde+4\,cd^2\right)\left(2\,ex^2-2\,\sqrt{e}\sqrt{ex^2+d}x+2\,d\right)}$$
$$-2\frac{\sqrt{e}}{a\left(4\,ae^2-4\,bde+4\,cd^2\right)}\sum_{\_R=RootOf\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+(16\,ae^2-8\,bde+6\,cd^2)\_Z^2+(4\,bd^2e-4\,cd^3)\_Z+cd^4\right)}\frac{\left(c\left(be-cd\right)\_R^2+2\,\left(-2\,ace\right.\right.}{\_R^3c+3\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a/d/x/(e*x^2+d)^(1/2)-2/a*e/d^2*x/(e*x^2+d)^(1/2)-8/a*e^(3/2)/
(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/2)*(e*x^2+d)^(1/2)*x+2*
d)*b+8/a*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/2)*(e*
x^2+d)^(1/2)*x+2*d)*c*d-2/a*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)*sum
((c*(b*e-c*d)*_R^2+2*(-2*a*c*e^2+2*b^2*e^2-3*b*c*d*e+c^2*d^2)*_R+
b*c*d^2*e-c^2*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*
b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-x*e^(1/2))^2-
_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2
)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4+bx^2+a)(ex^2+d)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (d + ex^2)^{\frac{3}{2}} (a + bx^2 + cx^4)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(1/(x**2*(d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)*x^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.391 $\int \frac{1}{x^4(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=419

$$\frac{2c^2\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}}+\frac{2c^2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}}-\frac{ex\left(ace+b^2(-e)+bcd\right)}{a^2d\sqrt{d+ex^2}\left(e(ae-bd)+cd^2\right)}+\frac{2ex(4ae+3bd)}{3a^2d^3\sqrt{d+ex^2}}+\frac{4ae+3bd}{3a^2d^2x\sqrt{d+ex^2}}-\frac{1}{3adx^3\sqrt{d+ex^2}}$$

```
[Out] -1/(3*a*d*x^3*Sqrt[d + e*x^2]) + (3*b*d + 4*a*e)/(3*a^2*d^2*x*Sqr
t[d + e*x^2]) + (2*e*(3*b*d + 4*a*e)*x)/(3*a^2*d^3*Sqrt[d + e*x^2
]) - (e*(b*c*d - b^2*e + a*c*e)*x)/(a^2*d*(c*d^2 + e*(-(b*d) + a*
e))*Sqrt[d + e*x^2]) + (2*c^2*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - S
qrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b - Sqrt[b^2 - 4*a
*c]]*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)^(3/2)) + (2*c^2*(b - (b^
2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2
- 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(
a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e)^(3/2))
```

Rubi [A] time = 12.8139, antiderivative size = 647, normalized size of antiderivative = 1.54, number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$

$$\frac{\sqrt{d+ex^2}\left(ace+b^2(-e)+bcd\right)}{a^2dx\left(ae^2-bde+cd^2\right)}+\frac{c\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}+\frac{c\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(ae^2-bde+cd^2\right)}+\frac{2e\sqrt{d+ex^2}(cd-be)}{3ad^2x\left(ae^2-bde+cd^2\right)}-\frac{e^2}{3dx^3\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{d+ex^2}(cd-be)}{3adx^3\left(ae^2-bde+cd^2\right)}+\frac{4e^3}{3d^2x\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}+\frac{8e^4x}{3d^3\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -e^2/(3*d*(c*d^2 - b*d*e + a*e^2)*x^3*Sqrt[d + e*x^2]) + (4*e^3)/
(3*d^2*(c*d^2 - b*d*e + a*e^2)*x*Sqrt[d + e*x^2]) + (8*e^4*x)/(3*
d^3*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - ((c*d - b*e)*Sqrt[
d + e*x^2])/(3*a*d*(c*d^2 - b*d*e + a*e^2)*x^3) + (2*e*(c*d - b*e
)*Sqrt[d + e*x^2])/(3*a*d^2*(c*d^2 - b*d*e + a*e^2)*x) + ((b*c*d
- b^2*e + a*c*e)*Sqrt[d + e*x^2])/(a^2*d*(c*d^2 - b*d*e + a*e^2)*
x) + (c*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3
*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 -
 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a
^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e]*(c*d^2 - b*d*e + a*e^2)) + (c*(b*c*d - b^2*e + a*c*e - (b^2
*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4
*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.36056, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(d+ex^2\right)^{3/2} \left(a+bx^2+cx^4\right)} \, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x]
```

Maple [C] time = 0.052, size = 541, normalized size = 1.3

$$-\frac{1}{3\,adx^3}\frac{1}{\sqrt{ex^2+d}}+\frac{4\,e}{3\,ad^2x}\frac{1}{\sqrt{ex^2+d}}+\frac{8\,e^2x}{3\,ad^3}\frac{1}{\sqrt{ex^2+d}}$$
$$-8\,\frac{e^{3/2}c}{a\,(4\,ae^2-4\,bde+4\,cd^2)\left(2\,ex^2-2\,\sqrt{e}\sqrt{ex^2+d}x+2\,d\right)}$$
$$+8\,\frac{e^{3/2}b^2}{a^2\,(4\,ae^2-4\,bde+4\,cd^2)\left(2\,ex^2-2\,\sqrt{e}\sqrt{ex^2+d}x+2\,d\right)}$$
$$-8\,\frac{\sqrt{e}bcd}{a^2\,(4\,ae^2-4\,bde+4\,cd^2)\left(2\,ex^2-2\,\sqrt{e}\sqrt{ex^2+d}x+2\,d\right)}$$
$$-2\,\frac{\sqrt{e}}{a^2\,(4\,ae^2-4\,bde+4\,cd^2)}\sum_{\_R=RootOf\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+(16\,ae^2-8\,bde+6\,cd^2)\_Z^2+(4\,bd^2e-4\,cd^3)\_Z+cd^4\right)}\frac{\left(c\left(ace-b^2e+bcd\right)\_R^2+\right.}{}$$
$$+\frac{b}{a^2dx}\frac{1}{\sqrt{ex^2+d}}+2\,\frac{bex}{a^2d^2\sqrt{ex^2+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/3/a/d/x^3/(e*x^2+d)^(1/2)+4/3/a/d^2*e/x/(e*x^2+d)^(1/2)+8/3/a/
d^3*e^2*x/(e*x^2+d)^(1/2)-8/a*e^(3/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(
2*e*x^2-2*e^(1/2)*(e*x^2+d)^(1/2)*x+2*d)*c+8/a^2*e^(3/2)/(4*a*e^2
-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/2)*(e*x^2+d)^(1/2)*x+2*d)*b^2-8
/a^2*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/2)*(e*x^2+
d)^(1/2)*x+2*d)*b*c*d-2/a^2*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)*sum
((c*(a*c*e-b^2*e+b*c*d)*_R^2+2*(4*a*b*c*e^2-3*a*c^2*d*e-2*b^3*e^2
+3*b^2*c*d*e-b*c^2*d^2)*_R+a*c^2*d^2*e-b^2*c*d^2*e+b*c^2*d^3)/(_R
^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2
*e-c*d^3)*ln(((e*x^2+d)^(1/2)-x*e^(1/2))^2-_R),_R=RootOf(c*_Z^4+(
4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*
d^3)*_Z+c*d^4))+b/a^2/d/x/(e*x^2+d)^(1/2)+2*b/a^2*e/d^2*x/(e*x^2+
d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(cx^4+bx^2+a)(ex^2+d)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)*x^4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)*x^4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.392 $\int \frac{(fx)^m(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=243

$$\frac{2c(fx)^{m+1}\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{m+1}{2};1,-q;\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)} - \frac{2c(fx)^{m+1}\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{m+1}{2};1,-q;\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] (2*c*(f*x)^(1 + m)*(d + e*x^2)^q*AppellF1[(1 + m)/2, 1, -q, (3 +
m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(Sqrt[b^
2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^2)/d)^q) -
 (2*c*(f*x)^(1 + m)*(d + e*x^2)^q*AppellF1[(1 + m)/2, 1, -q, (3 +
 m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(Sqrt[b
^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^2)/d)^q)
```

Rubi [A] time = 1.4892, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{2c(fx)^{m+1}\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{m+1}{2};1,-q;\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)} - \frac{2c(fx)^{m+1}\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{m+1}{2};1,-q;\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(f*x)^(1 + m)*(d + e*x^2)^q*AppellF1[(1 + m)/2, 1, -q, (3 +
m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(Sqrt[b^
2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^2)/d)^q) -
 (2*c*(f*x)^(1 + m)*(d + e*x^2)^q*AppellF1[(1 + m)/2, 1, -q, (3 +
 m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(Sqrt[b
^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^2)/d)^q)
```

Rubi in Sympy [A] time = 114.954, size = 202, normalized size = 0.83

$$-\frac{2c(fx)^{m+1}\left(1+\frac{ex^2}{d}\right)^{-q}\left(d+ex^2\right)^q\operatorname{appellf}_1\left(\frac{m}{2}+\frac{1}{2},1,-q,\frac{m}{2}+\frac{3}{2},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}},-\frac{ex^2}{d}\right)}{f\left(b+\sqrt{-4ac+b^2}\right)(m+1)\sqrt{-4ac+b^2}}+\frac{2c(fx)^{m+1}\left(1+\frac{ex^2}{d}\right)^{-q}\left(d+ex^2\right)^q\operatorname{appellf}_1\left(\frac{m}{2}+\frac{1}{2},1,-q,\frac{m}{2}+\frac{3}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{ex^2}{d}\right)}{f\left(b-\sqrt{-4ac+b^2}\right)(m+1)\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] -2*c*(f*x)**(m + 1)*(1 + e*x**2/d)**(-q)*(d + e*x**2)**q*appellf1
(m/2 + 1/2, 1, -q, m/2 + 3/2, -2*c*x**2/(b + sqrt(-4*a*c + b**2))
, -e*x**2/d)/(f*(b + sqrt(-4*a*c + b**2))*(m + 1)*sqrt(-4*a*c + b
**2)) + 2*c*(f*x)**(m + 1)*(1 + e*x**2/d)**(-q)*(d + e*x**2)**q*a
ppellf1(m/2 + 1/2, 1, -q, m/2 + 3/2, -2*c*x**2/(b - sqrt(-4*a*c +
 b**2)), -e*x**2/d)/(f*(b - sqrt(-4*a*c + b**2))*(m + 1)*sqrt(-4*
a*c + b**2))
```

Mathematica [A] time = 0.106386, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(ex^2+d\right)^q}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^q (fx)^m}{cx^4+bx^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^2+d)^q (fx)^m}{cx^4+bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^q (fx)^m}{cx^4+bx^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

3.393 $\int \frac{x^7 (d+ex^2)^q}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=312

$$\frac{\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-be+\sqrt{b^2-4ac}\,e}\right)}{2c(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{(be+cd)(d+ex^2)^{q+1}}{2c^2e^2(q+1)}+\frac{(d+ex^2)^{q+2}}{2ce^2(q+2)}$$

```
[Out] -((c*d + b*e)*(d + e*x^2)^(1 + q))/(2*c^2*e^2*(1 + q)) + (d + e*x
^2)^(2 + q)/(2*c*e^2*(2 + q)) + ((a - b^2/c + (b*(b^2 - 3*a*c))/(
c*Sqrt[b^2 - 4*a*c]))*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1
+ q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)
])/(2*c*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + ((a - b^2/
c - (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(d + e*x^2)^(1 + q)*
Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e)])/(2*c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)*(1 + q))
```

Rubi [A] time = 1.80059, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-be+\sqrt{b^2-4ac}\,e}\right)}{2c(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{(be+cd)(d+ex^2)^{q+1}}{2c^2e^2(q+1)}+\frac{(d+ex^2)^{q+2}}{2ce^2(q+2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((c*d + b*e)*(d + e*x^2)^(1 + q))/(2*c^2*e^2*(1 + q)) + (d + e*x
^2)^(2 + q)/(2*c*e^2*(2 + q)) + ((a - b^2/c + (b*(b^2 - 3*a*c))/(
```

```
c*Sqrt[b^2 - 4*a*c]))*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1
+ q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)
])/(2*c*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + ((a - b^2/
c - (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(d + e*x^2)^(1 + q)*
Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e)])/(2*c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)*(1 + q))
```

Rubi in Sympy [A] time = 96.978, size = 289, normalized size = 0.93

$$\frac{\left(d+ex^2\right)^{q+2}}{2ce^2\left(q+2\right)}+\frac{\left(d+ex^2\right)^{q+1}\left(b\left(-3ac+b^2\right)-\sqrt{-4ac+b^2}\left(-ac+b^2\right)\right){}_2F_1\left(\left.\begin{matrix}1,q+1\\q+2\end{matrix}\right|\frac{c\left(-2d-2ex^2\right)}{be-2cd-e\sqrt{-4ac+b^2}}\right)}{2c^2\left(q+1\right)\sqrt{-4ac+b^2}\left(2cd-e\left(b-\sqrt{-4ac+b^2}\right)\right)}-\frac{\left(d+ex^2\right)^{q+1}\left(b\left(-3ac+b^2\right)+\sqrt{-4ac+b^2}\left(-ac+b^2\right)\right){}_2F_1\left(\left.\begin{matrix}1,q+1\\q+2\end{matrix}\right|\frac{c\left(-2d-2ex^2\right)}{be-2cd+e\sqrt{-4ac+b^2}}\right)}{2c^2\left(q+1\right)\sqrt{-4ac+b^2}\left(2cd-e\left(b+\sqrt{-4ac+b^2}\right)\right)}-\frac{\left(d+ex^2\right)^{q+1}\left(be+cd\right)}{2c^2e^2\left(q+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] (d + e*x**2)**(q + 2)/(2*c*e**2*(q + 2)) + (d + e*x**2)**(q + 1)*
(b*(-3*a*c + b**2) - sqrt(-4*a*c + b**2)*(-a*c + b**2))*hyper((1,
 q + 1), (q + 2,), c*(-2*d - 2*e*x**2)/(b*e - 2*c*d - e*sqrt(-4*a
*c + b**2)))/(2*c**2*(q + 1)*sqrt(-4*a*c + b**2)*(2*c*d - e*(b -
sqrt(-4*a*c + b**2)))) - (d + e*x**2)**(q + 1)*(b*(-3*a*c + b**2)
 + sqrt(-4*a*c + b**2)*(-a*c + b**2))*hyper((1, q + 1), (q + 2,),
 c*(-2*d - 2*e*x**2)/(b*e - 2*c*d + e*sqrt(-4*a*c + b**2)))/(2*c*
*2*(q + 1)*sqrt(-4*a*c + b**2)*(2*c*d - e*(b + sqrt(-4*a*c + b**2
)))) - (d + e*x**2)**(q + 1)*(b*e + c*d)/(2*c**2*e**2*(q + 1))
```

Mathematica [A] time = 1.35108, size = 367, normalized size = 1.18

$$2^{-q-2}\left(d+ex^2\right)^q\left(\left(\left(b^2\sqrt{e^2\left(b^2-4ac\right)}-ac\sqrt{e^2\left(b^2-4ac\right)}+3abce+b^3(-e)\right)\left(\frac{c\left(d+ex^2\right)}{-\sqrt{e^2(b^2-4ac)}+be+2cex^2}\right)^{-q}{}_2F_1\left(-q,-q;1-q;\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2^(-2 - q)*(d + e*x^2)^q*(((-(b^3*e) + 3*a*b*c*e + b^2*Sqrt[(b^2
 - 4*a*c)*e^2] - a*c*Sqrt[(b^2 - 4*a*c)*e^2])*Hypergeometric2F1[-
q, -q, 1 - q, (2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(-(b*e) + S
qrt[(b^2 - 4*a*c)*e^2] - 2*c*e*x^2)])/((c*(d + e*x^2))/(b*e - Sqr
t[(b^2 - 4*a*c)*e^2] + 2*c*e*x^2))^q + ((b^3*e - 3*a*b*c*e + b^2*
Sqrt[(b^2 - 4*a*c)*e^2] - a*c*Sqrt[(b^2 - 4*a*c)*e^2])*Hypergeome
tric2F1[-q, -q, 1 - q, (-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(
b*e + Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*e*x^2)])/((c*(d + e*x^2))/(b*
e + Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*e*x^2))^q))/(c^3*Sqrt[(b^2 - 4*
a*c)*e^2]*q)
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int \frac{x^{7} \left(e x^{2}+d\right)^{q}}{c x^{4}+b x^{2}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e x^{2}+d\right)^{q} x^{7}}{c x^{4}+b x^{2}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^q*x^7/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q*x^7/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e x^{2}+d\right)^{q} x^{7}}{c x^{4}+b x^{2}+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q*x^7/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q*x^7/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q x^7}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q*x^7/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q*x^7/(c*x^4 + b*x^2 + a), x)
```

3.394 $\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=255

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-be+\sqrt{b^2-4ac}e}\right)}{2c(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(d+ex^2\right)^{q+1}{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}+\frac{\left(d+ex^2\right)^{q+1}}{2ce(q+1)}$$

```
[Out] (d + e*x^2)^(1 + q)/(2*c*e*(1 + q)) + ((b - (b^2 - 2*a*c)/Sqrt[b^
2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q
, (2*c*(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)])/(2*c*(2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + ((b + (b^2 - 2*a*c)/
Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q
, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/
(2*c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q))
```

Rubi [A] time = 1.16036, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-be+\sqrt{b^2-4ac}e}\right)}{2c(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(d+ex^2\right)^{q+1}{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}+\frac{\left(d+ex^2\right)^{q+1}}{2ce(q+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```


```
[Out] (d + e*x^2)^(1 + q)/(2*c*e*(1 + q)) + ((b - (b^2 - 2*a*c)/Sqrt[b^
2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q
, (2*c*(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)])/(2*c*(2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + ((b + (b^2 - 2*a*c)/
Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q
, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/
(2*c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q))
```

Rubi in Sympy [A] time = 93.4113, size = 241, normalized size = 0.95

$$\frac{\left(d+ex^2\right)^{q+1}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right){}_2F_1\left(\begin{matrix}1,q+1\\q+2\end{matrix}\middle|\frac{c\left(-2d-2ex^2\right)}{be-2cd+e\sqrt{-4ac+b^2}}\right)}{2c(q+1)\sqrt{-4ac+b^2}\left(2cd-e\left(b+\sqrt{-4ac+b^2}\right)\right)} - \frac{\left(d+ex^2\right)^{q+1}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right){}_2F_1\left(\begin{matrix}1,q+1\\q+2\end{matrix}\middle|\frac{c\left(-2d-2ex^2\right)}{be-2cd-e\sqrt{-4ac+b^2}}\right)}{2c(q+1)\sqrt{-4ac+b^2}\left(2cd-e\left(b-\sqrt{-4ac+b^2}\right)\right)} + \frac{\left(d+ex^2\right)^{q+1}}{2ce(q+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] (d + e*x**2)**(q + 1)*(-2*a*c + b**2 + b*sqrt(-4*a*c + b**2))*hyp
er((1, q + 1), (q + 2,), c*(-2*d - 2*e*x**2)/(b*e - 2*c*d + e*sqr
t(-4*a*c + b**2)))/(2*c*(q + 1)*sqrt(-4*a*c + b**2)*(2*c*d - e*(b
 + sqrt(-4*a*c + b**2)))) - (d + e*x**2)**(q + 1)*(-2*a*c + b**2
- b*sqrt(-4*a*c + b**2))*hyper((1, q + 1), (q + 2,), c*(-2*d - 2*
e*x**2)/(b*e - 2*c*d - e*sqrt(-4*a*c + b**2)))/(2*c*(q + 1)*sqrt(
-4*a*c + b**2)*(2*c*d - e*(b - sqrt(-4*a*c + b**2)))) + (d + e*x*
*2)**(q + 1)/(2*c*e*(q + 1))
```

Mathematica [A] time = 1.8908, size = 362, normalized size = 1.42

$$\frac{2^{-q-2}\left(d+ex^2\right)^q\left(-\left(b\sqrt{e^2\left(b^2-4ac\right)}+2ace+b^2(-e)\right)\left(\frac{c\left(d+ex^2\right)}{-\sqrt{e^2\left(b^2-4ac\right)}+be+2cex^2}\right)^{-q}{}_2F_1\left(-q,-q;1-q;\frac{2cd-be+\sqrt{\left(b^2-4ac\right)e^2}}{-2cex^2-be+\sqrt{\left(b^2-4ac\right)e^2}}\right)\right.}{c^2q\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2^(-2 - q)*(d + e*x^2)^q*((2^(1 + q)*c*Sqrt[(b^2 - 4*a*c)*e^2]*q
*(d + e*x^2))/(e*(1 + q)) - ((-(b^2*e) + 2*a*c*e + b*Sqrt[(b^2 -
4*a*c)*e^2])*Hypergeometric2F1[-q, -q, 1 - q, (2*c*d - b*e + Sqrt
[(b^2 - 4*a*c)*e^2])/(-(b*e) + Sqrt[(b^2 - 4*a*c)*e^2] - 2*c*e*x^
2)])/((c*(d + e*x^2))/(b*e - Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*e*x^2)
)^q - ((b^2*e - 2*a*c*e + b*Sqrt[(b^2 - 4*a*c)*e^2])*Hypergeometr
ic2F1[-q, -q, 1 - q, (-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(b*
e + Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*e*x^2)])/((c*(d + e*x^2))/(b*e
+ Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*e*x^2))^q))/(c^2*Sqrt[(b^2 - 4*a*
```

```
c)*e^2]*q)
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int \frac{x^5 (e x^2 + d)^q}{c x^4 + b x^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)

[Out] int(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e x^2 + d)^q x^5}{c x^4 + b x^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q*x^5/(c*x^4 + b*x^2 + a),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^q*x^5/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e x^2 + d)^q x^5}{c x^4 + b x^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q*x^5/(c*x^4 + b*x^2 + a),x, algorithm="fricas")

[Out] integral((e*x^2 + d)^q*x^5/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q x^5}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q*x^5/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q*x^5/(c*x^4 + b*x^2 + a), x)
```

3.395 $\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=209

$$-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-be+\sqrt{b^2-4ac}\,e}\right)}{2(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$

```
[Out] -((1 - b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1
[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2 - 4*a
*c]*e)])/(2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) - ((1 +
b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 +
 q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]
)/(2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q))
```

Rubi [A] time = 0.72123, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-be+\sqrt{b^2-4ac}\,e}\right)}{2(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((1 - b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1
[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2 - 4*a
*c]*e)])/(2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) - ((1 +
b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 +
 q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]
)/(2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q))
```

Rubi in Sympy [A] time = 66.313, size = 206, normalized size = 0.99

$$-\frac{\left(b-\sqrt{-4ac+b^2}\right)\left(d+ex^2\right)^{q+1}{}_2F_1\left(\begin{matrix}1,q+1\\ q+2\end{matrix}\middle|\frac{c\left(-2d-2ex^2\right)}{be-2cd-e\sqrt{-4ac+b^2}}\right)}{2\left(q+1\right)\sqrt{-4ac+b^2}\left(be-2cd-e\sqrt{-4ac+b^2}\right)}-\frac{\left(b+\sqrt{-4ac+b^2}\right)\left(d+ex^2\right)^{q+1}{}_2F_1\left(\begin{matrix}1,q+1\\ q+2\end{matrix}\middle|\frac{c\left(-2d-2ex^2\right)}{be-2cd+e\sqrt{-4ac+b^2}}\right)}{2\left(q+1\right)\sqrt{-4ac+b^2}\left(2cd-e\left(b+\sqrt{-4ac+b^2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] -(b - sqrt(-4*a*c + b**2))*(d + e*x**2)**(q + 1)*hyper((1, q + 1)
, (q + 2,), c*(-2*d - 2*e*x**2)/(b*e - 2*c*d - e*sqrt(-4*a*c + b*
*2)))/(2*(q + 1)*sqrt(-4*a*c + b**2)*(b*e - 2*c*d - e*sqrt(-4*a*c
 + b**2))) - (b + sqrt(-4*a*c + b**2))*(d + e*x**2)**(q + 1)*hype
r((1, q + 1), (q + 2,), c*(-2*d - 2*e*x**2)/(b*e - 2*c*d + e*sqrt
(-4*a*c + b**2)))/(2*(q + 1)*sqrt(-4*a*c + b**2)*(2*c*d - e*(b +
sqrt(-4*a*c + b**2))))
```

Mathematica [A] time = 0.715017, size = 303, normalized size = 1.45

$$\frac{2^{-q-2}\left(d+ex^2\right)^q\left(\left(\sqrt{e^2\left(b^2-4ac\right)}-be\right)\left(\frac{c\left(d+ex^2\right)}{-\sqrt{e^2\left(b^2-4ac\right)}+be+2cex^2}\right)^{-q}{}_2F_1\left(-q,-q;1-q;\frac{2cd-be+\sqrt{\left(b^2-4ac\right)e^2}}{-2cex^2-be+\sqrt{\left(b^2-4ac\right)e^2}}\right)+\left(\sqrt{e^2\left(b^2-\right.}\right.\right.}{cq\sqrt{e^2\left(b^2-4ac\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2^(-2 - q)*(d + e*x^2)^q*(((-(b*e) + Sqrt[(b^2 - 4*a*c)*e^2])*Hy
pergeometric2F1[-q, -q, 1 - q, (2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*
e^2])/(-(b*e) + Sqrt[(b^2 - 4*a*c)*e^2] - 2*c*e*x^2)])/((c*(d + e
*x^2))/(b*e - Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*e*x^2))^q + ((b*e + S
qrt[(b^2 - 4*a*c)*e^2])*Hypergeometric2F1[-q, -q, 1 - q, (-2*c*d
+ b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(b*e + Sqrt[(b^2 - 4*a*c)*e^2] +
 2*c*e*x^2)])/((c*(d + e*x^2))/(b*e + Sqrt[(b^2 - 4*a*c)*e^2] + 2
*c*e*x^2))^q))/(c*Sqrt[(b^2 - 4*a*c)*e^2]*q)
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{x^3 \left(e x^2 + d\right)^q}{c x^4 + b x^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e x^2 + d\right)^q x^3}{c x^4 + b x^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q*x^3/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q*x^3/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e x^2 + d\right)^q x^3}{c x^4 + b x^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q*x^3/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q*x^3/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^q x^3}{cx^4 + bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q*x^3/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q*x^3/(c*x^4 + b*x^2 + a), x)
```

3.396 $\int \frac{x\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=197

$$\frac{c\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{(q+1)\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{c\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-be+\sqrt{b^2-4ac}e}\right)}{(q+1)\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}$$

```
[Out] -((c*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*
(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)])/(Sqrt[b^2 - 4*
a*c]*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q))) + (c*(d + e*x^
2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4*a*c]*(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)*(1 + q))
```

Rubi [A] time = 0.820165, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12

$$\frac{c\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{(q+1)\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{c\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-be+\sqrt{b^2-4ac}e}\right)}{(q+1)\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((c*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*
(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)])/(Sqrt[b^2 - 4*
a*c]*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q))) + (c*(d + e*x^
2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4*a*c]*(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)*(1 + q))
```

Rubi in Sympy [A] time = 71.8548, size = 175, normalized size = 0.89

$$\frac{c\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(\begin{matrix}1,q+1\\q+2\end{matrix}\middle|\frac{c\left(-2d-2ex^2\right)}{be-2cd+e\sqrt{-4ac+b^2}}\right)}{(q+1)\sqrt{-4ac+b^2}\left(2cd-e\left(b+\sqrt{-4ac+b^2}\right)\right)}-\frac{c\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(\begin{matrix}1,q+1\\q+2\end{matrix}\middle|\frac{c\left(-2d-2ex^2\right)}{be-2cd-e\sqrt{-4ac+b^2}}\right)}{(q+1)\sqrt{-4ac+b^2}\left(2cd-e\left(b-\sqrt{-4ac+b^2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] c*(d + e*x**2)**(q + 1)*hyper((1, q + 1), (q + 2,), c*(-2*d - 2*e
*x**2)/(b*e - 2*c*d + e*sqrt(-4*a*c + b**2)))/((q + 1)*sqrt(-4*a*
c + b**2)*(2*c*d - e*(b + sqrt(-4*a*c + b**2)))) - c*(d + e*x**2)
**(q + 1)*hyper((1, q + 1), (q + 2,), c*(-2*d - 2*e*x**2)/(b*e -
2*c*d - e*sqrt(-4*a*c + b**2)))/((q + 1)*sqrt(-4*a*c + b**2)*(2*c
*d - e*(b - sqrt(-4*a*c + b**2))))
```

Mathematica [A] time = 0.485026, size = 261, normalized size = 1.32

$$\frac{e2^{-q-1}\left(d+ex^2\right)^q\left(\left(\frac{c\left(d+ex^2\right)}{-\sqrt{e^2(b^2-4ac)}+be+2cex^2}\right)^{-q}{}_2F_1\left(-q,-q;1-q;\frac{2cd-be+\sqrt{(b^2-4ac)e^2}}{-2cex^2-be+\sqrt{(b^2-4ac)e^2}}\right)-\left(\frac{c\left(d+ex^2\right)}{\sqrt{e^2(b^2-4ac)}+be+2cex^2}\right)^{-q}{}_2F_1\left(-\right.\right.}{q\sqrt{e^2\left(b^2-4ac\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2^(-1 - q)*e*(d + e*x^2)^q*(Hypergeometric2F1[-q, -q, 1 - q, (2*
c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(-(b*e) + Sqrt[(b^2 - 4*a*c)
*e^2] - 2*c*e*x^2)]/((c*(d + e*x^2))/(b*e - Sqrt[(b^2 - 4*a*c)*e^
2] + 2*c*e*x^2))^q - Hypergeometric2F1[-q, -q, 1 - q, (-2*c*d + b
*e + Sqrt[(b^2 - 4*a*c)*e^2])/(b*e + Sqrt[(b^2 - 4*a*c)*e^2] + 2*
c*e*x^2)]/((c*(d + e*x^2))/(b*e + Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*e
*x^2))^q))/(Sqrt[(b^2 - 4*a*c)*e^2]*q)
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \frac{x\left(ex^2+d\right)^q}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^q x}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^q*x/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q*x/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^2+d)^q x}{cx^4+bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^q*x/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q*x/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^q x}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q*x/(c*x^4 + b*x^2 + a),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^q*x/(c*x^4 + b*x^2 + a), x)
```

3.397 $\int \frac{(d+ex^2)^q}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=261

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(d+ex^2\right)^{q+1} {}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-be+\sqrt{b^2-4ac}e}\right)}{2a(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1} {}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2a(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{\left(d+ex^2\right)^{q+1} {}_2F_1\left(1,q+1;q+2;\frac{ex^2}{d}+1\right)}{2ad(q+1)}$$

```
[Out] (c*(1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F
1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2 - 4*
a*c]*e)])/(2*a*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (c*
(1 - b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1
, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)])/(2*a*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q)) - ((d +
e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^2)/d])
/(2*a*d*(1 + q))
```

Rubi [A] time = 1.21648, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(d+ex^2\right)^{q+1} {}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-be+\sqrt{b^2-4ac}e}\right)}{2a(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1} {}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2a(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{\left(d+ex^2\right)^{q+1} {}_2F_1\left(1,q+1;q+2;\frac{ex^2}{d}+1\right)}{2ad(q+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (c*(1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F
1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2 - 4*
```

```
a*c]*e)])/(2*a*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (c*
(1 - b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1
, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)])/(2*a*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q)) - ((d +
e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^2)/d])
/(2*a*d*(1 + q))
```

Rubi in Sympy [A] time = 105.931, size = 243, normalized size = 0.93

$$\frac{c\left(b-\sqrt{-4ac+b^2}\right)\left(d+ex^2\right)^{q+1}{}_2F_1\left(\begin{matrix}1,q+1\\q+2\end{matrix}\middle|\frac{c(-2d-2ex^2)}{be-2cd+e\sqrt{-4ac+b^2}}\right)}{2a(q+1)\sqrt{-4ac+b^2}\left(be-2cd+e\sqrt{-4ac+b^2}\right)}+\frac{c\left(b+\sqrt{-4ac+b^2}\right)\left(d+ex^2\right)^{q+1}{}_2F_1\left(\begin{matrix}1,q+1\\q+2\end{matrix}\middle|\frac{c(-2d-2ex^2)}{be-2cd-e\sqrt{-4ac+b^2}}\right)}{2a(q+1)\sqrt{-4ac+b^2}\left(2cd-e\left(b-\sqrt{-4ac+b^2}\right)\right)}-\frac{\left(d+ex^2\right)^{q+1}{}_2F_1\left(\begin{matrix}1,q+1\\q+2\end{matrix}\middle|1+\frac{ex^2}{d}\right)}{2ad(q+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**q/x/(c*x**4+b*x**2+a),x)
```

```
[Out] c*(b - sqrt(-4*a*c + b**2))*(d + e*x**2)**(q + 1)*hyper((1, q + 1
), (q + 2,), c*(-2*d - 2*e*x**2)/(b*e - 2*c*d + e*sqrt(-4*a*c + b
**2)))/(2*a*(q + 1)*sqrt(-4*a*c + b**2)*(b*e - 2*c*d + e*sqrt(-4*
a*c + b**2))) + c*(b + sqrt(-4*a*c + b**2))*(d + e*x**2)**(q + 1)
*hyper((1, q + 1), (q + 2,), c*(-2*d - 2*e*x**2)/(b*e - 2*c*d - e
*sqrt(-4*a*c + b**2)))/(2*a*(q + 1)*sqrt(-4*a*c + b**2)*(2*c*d -
e*(b - sqrt(-4*a*c + b**2)))) - (d + e*x**2)**(q + 1)*hyper((1, q
 + 1), (q + 2,), 1 + e*x**2/d)/(2*a*d*(q + 1))
```

Mathematica [A] time = 0.110124, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex^2\right)^q}{x\left(a+bx^2+cx^4\right)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[(d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)), x]
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^q}{x\,(cx^4+bx^2+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x)
```

```
[Out] int((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^2+d)^q}{cx^5+bx^3+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q/(c*x^5 + b*x^3 + a*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**q/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q}{(cx^4+bx^2+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x), x)
```

3.398 $\int \frac{(d+ex^2)^q}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=321

$$-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-be+\sqrt{b^2-4ac}\,e}\right)}{2a^2(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2a^2(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}+\frac{b\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{ex^2}{d}+1\right)}{2a^2d(q+1)}+\frac{e\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(2,q+1;q+2;\frac{ex^2}{d}+1\right)}{2ad^2(q+1)}$$

```
[Out] -(c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hyp
ergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - b*e +
Sqrt[b^2 - 4*a*c]*e)])/(2*a^2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
*(1 + q)) - (c*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^
(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*a^2*(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)*(1 + q)) + (b*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1
, 1 + q, 2 + q, 1 + (e*x^2)/d])/(2*a^2*d*(1 + q)) + (e*(d + e*x^2
)^(1 + q)*Hypergeometric2F1[2, 1 + q, 2 + q, 1 + (e*x^2)/d])/(2*a
*d^2*(1 + q))
```

Rubi [A] time = 1.57045, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-be+\sqrt{b^2-4ac}\,e}\right)}{2a^2(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2a^2(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}+\frac{b\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{ex^2}{d}+1\right)}{2a^2d(q+1)}+\frac{e\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(2,q+1;q+2;\frac{ex^2}{d}+1\right)}{2ad^2(q+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hyp
ergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - b*e +
Sqrt[b^2 - 4*a*c]*e)])/(2*a^2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
*(1 + q)) - (c*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^
(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*a^2*(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)*(1 + q)) + (b*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1
, 1 + q, 2 + q, 1 + (e*x^2)/d])/(2*a^2*d*(1 + q)) + (e*(d + e*x^2
)^(1 + q)*Hypergeometric2F1[2, 1 + q, 2 + q, 1 + (e*x^2)/d])/(2*a
*d^2*(1 + q))
```

Rubi in Sympy [A] time = 141.294, size = 303, normalized size = 0.94

$$\frac{e\left(d+ex^2\right)^{q+1}{}_2F_1\left(\begin{matrix}2,q+1\\q+2\end{matrix}\middle|1+\frac{ex^2}{d}\right)}{2ad^2\left(q+1\right)}+\frac{b\left(d+ex^2\right)^{q+1}{}_2F_1\left(\begin{matrix}1,q+1\\q+2\end{matrix}\middle|1+\frac{ex^2}{d}\right)}{2a^2d\left(q+1\right)}$$
$$-\frac{c\left(d+ex^2\right)^{q+1}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right){}_2F_1\left(\begin{matrix}1,q+1\\q+2\end{matrix}\middle|\frac{c\left(-2d-2ex^2\right)}{be-2cd+e\sqrt{-4ac+b^2}}\right)}{2a^2\left(q+1\right)\sqrt{-4ac+b^2}\left(be-2cd+e\sqrt{-4ac+b^2}\right)}$$
$$-\frac{c\left(d+ex^2\right)^{q+1}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right){}_2F_1\left(\begin{matrix}1,q+1\\q+2\end{matrix}\middle|\frac{c\left(-2d-2ex^2\right)}{be-2cd-e\sqrt{-4ac+b^2}}\right)}{2a^2\left(q+1\right)\sqrt{-4ac+b^2}\left(2cd-e\left(b-\sqrt{-4ac+b^2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**q/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] e*(d + e*x**2)**(q + 1)*hyper((2, q + 1), (q + 2,), 1 + e*x**2/d)
/(2*a*d**2*(q + 1)) + b*(d + e*x**2)**(q + 1)*hyper((1, q + 1), (
q + 2,), 1 + e*x**2/d)/(2*a**2*d*(q + 1)) - c*(d + e*x**2)**(q +
1)*(-2*a*c + b**2 - b*sqrt(-4*a*c + b**2))*hyper((1, q + 1), (q +
 2,), c*(-2*d - 2*e*x**2)/(b*e - 2*c*d + e*sqrt(-4*a*c + b**2)))/
(2*a**2*(q + 1)*sqrt(-4*a*c + b**2)*(b*e - 2*c*d + e*sqrt(-4*a*c
+ b**2))) - c*(d + e*x**2)**(q + 1)*(-2*a*c + b**2 + b*sqrt(-4*a*
c + b**2))*hyper((1, q + 1), (q + 2,), c*(-2*d - 2*e*x**2)/(b*e -
 2*c*d - e*sqrt(-4*a*c + b**2)))/(2*a**2*(q + 1)*sqrt(-4*a*c + b*
*2)*(2*c*d - e*(b - sqrt(-4*a*c + b**2))))
```

Mathematica [A] time = 0.988913, size = 420, normalized size = 1.31

$$\frac{2^{-q-2}\left(d+ex^2\right)^q\left(\left(b\sqrt{e^2\left(b^2-4ac\right)}-2ace+b^2e\right)\left(\frac{c\left(d+ex^2\right)}{-\sqrt{e^2(b^2-4ac)}+be+2cex^2}\right)^{-q}{}_2F_1\left(-q,-q;1-q;\frac{2cd-be+\sqrt{(b^2-4ac)e^2}}{-2cex^2-be+\sqrt{(b^2-4ac)e^2}}\right)+\left(b\right.}{a^2q\sqrt{e^2\left(b^2-4ac\right)}}$$

$$+\frac{\left(d+ex^2\right)^q\left(\frac{d}{ex^2}+1\right)^{-q}\left(aq\,{}_2F_1\left(1-q,-q;2-q;-\frac{d}{ex^2}\right)-b(q-1)x^2\,{}_2F_1\left(-q,-q;1-q;-\frac{d}{ex^2}\right)\right)}{2a^2(q-1)qx^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((d + e*x^2)^q*(a*q*Hypergeometric2F1[1 - q, -q, 2 - q, -(d/(e*x^
2))] - b*(-1 + q)*x^2*Hypergeometric2F1[-q, -q, 1 - q, -(d/(e*x^2
))]))/(2*a^2*(-1 + q)*q*(1 + d/(e*x^2))^q*x^2) + (2^(-2 - q)*(d +
 e*x^2)^q*(((b^2*e - 2*a*c*e + b*Sqrt[(b^2 - 4*a*c)*e^2])*Hyperge
ometric2F1[-q, -q, 1 - q, (2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])
/(-(b*e) + Sqrt[(b^2 - 4*a*c)*e^2] - 2*c*e*x^2)])/((c*(d + e*x^2)
)/(b*e - Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*e*x^2))^q + ((-(b^2*e) + 2
*a*c*e + b*Sqrt[(b^2 - 4*a*c)*e^2])*Hypergeometric2F1[-q, -q, 1 -
 q, (-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(b*e + Sqrt[(b^2 - 4
*a*c)*e^2] + 2*c*e*x^2)])/((c*(d + e*x^2))/(b*e + Sqrt[(b^2 - 4*a
*c)*e^2] + 2*c*e*x^2))^q))/(a^2*Sqrt[(b^2 - 4*a*c)*e^2]*q)
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q}{x^3\left(cx^4+bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] int((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q}{(cx^4+bx^2+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex^2+d\right)^q}{cx^7+bx^5+ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="fricas")

[Out] integral((e*x^2 + d)^q/(c*x^7 + b*x^5 + a*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**q/x**3/(c*x**4+b*x**2+a),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q}{(cx^4+bx^2+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^3), x)
```

3.399 $\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=339

$$\frac{x\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{x\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(\sqrt{b^2-4ac}+b\right)}-\frac{bx(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}{}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c^2}+\frac{x^3(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}{}_2F_1\left(\frac{3}{2},-q;\frac{5}{2};-\frac{ex^2}{d}\right)}{3c}$$

```
[Out] ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^
q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -
((e*x^2)/d)])/(c^2*(b - Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + (
(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q
*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -(
(e*x^2)/d)])/(c^2*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) - (b
*x*(d + e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(
c^2*(1 + (e*x^2)/d)^q) + (x^3*(d + e*x^2)^q*Hypergeometric2F1[3/2
, -q, 5/2, -((e*x^2)/d)])/(3*c*(1 + (e*x^2)/d)^q)
```

Rubi [A] time = 1.41638, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296

$$\frac{x\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{x\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(\sqrt{b^2-4ac}+b\right)}-\frac{bx(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}{}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c^2}+\frac{x^3(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}{}_2F_1\left(\frac{3}{2},-q;\frac{5}{2};-\frac{ex^2}{d}\right)}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^
q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -
((e*x^2)/d)])/(c^2*(b - Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + (
(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q
*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -(
(e*x^2)/d)])/(c^2*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) - (b
*x*(d + e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(
c^2*(1 + (e*x^2)/d)^q) + (x^3*(d + e*x^2)^q*Hypergeometric2F1[3/2
, -q, 5/2, -((e*x^2)/d)])/(3*c*(1 + (e*x^2)/d)^q)
```

Rubi in Sympy [A] time = 163.142, size = 308, normalized size = 0.91

$$-\frac{bx\left(1+\frac{ex^2}{d}\right)^{-q}\left(d+ex^2\right)^q {}_2F_1\left(\begin{matrix}-q,\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|-\frac{ex^2}{d}\right)}{c^2}+\frac{x^3\left(1+\frac{ex^2}{d}\right)^{-q}\left(d+ex^2\right)^q {}_2F_1\left(\begin{matrix}-q,\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|-\frac{ex^2}{d}\right)}{3c}+\frac{x\left(1+\frac{ex^2}{d}\right)^{-q}\left(d+ex^2\right)^q\left(b\left(-3ac+b^2\right)+\sqrt{-4ac+b^2}\left(-ac+b^2\right)\right)\operatorname{appellf}_1\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}},-\frac{ex^2}{d}\right)}{c^2\left(b+\sqrt{-4ac+b^2}\right)\sqrt{-4ac+b^2}}-\frac{x\left(1+\frac{ex^2}{d}\right)^{-q}\left(d+ex^2\right)^q\left(b\left(-3ac+b^2\right)-\sqrt{-4ac+b^2}\left(-ac+b^2\right)\right)\operatorname{appellf}_1\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{ex^2}{d}\right)}{c^2\left(b-\sqrt{-4ac+b^2}\right)\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] -b*x*(1 + e*x**2/d)**(-q)*(d + e*x**2)**q*hyper((-q, 1/2), (3/2,)
, -e*x**2/d)/c**2 + x**3*(1 + e*x**2/d)**(-q)*(d + e*x**2)**q*hyp
er((-q, 3/2), (5/2,), -e*x**2/d)/(3*c) + x*(1 + e*x**2/d)**(-q)*(
d + e*x**2)**q*(b*(-3*a*c + b**2) + sqrt(-4*a*c + b**2)*(-a*c + b
**2))*appellf1(1/2, 1, -q, 3/2, -2*c*x**2/(b + sqrt(-4*a*c + b**2
)), -e*x**2/d)/(c**2*(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2
)) - x*(1 + e*x**2/d)**(-q)*(d + e*x**2)**q*(b*(-3*a*c + b**2) -
sqrt(-4*a*c + b**2)*(-a*c + b**2))*appellf1(1/2, 1, -q, 3/2, -2*c
*x**2/(b - sqrt(-4*a*c + b**2)), -e*x**2/d)/(c**2*(b - sqrt(-4*a*
c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.31375, size = 0, normalized size = 0.

$$\int \frac{x^6\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]

[Out] Integrate[(x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int \frac{x^{6} \left(e\,x^{2}+d\right)^{q}}{c\,x^{4}+b\,x^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)

[Out] int(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e x^{2}+d\right)^{q} x^{6}}{c x^{4}+b x^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q*x^6/(c*x^4 + b*x^2 + a),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^q*x^6/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(e x^{2}+d\right)^{q} x^{6}}{c x^{4}+b x^{2}+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q*x^6/(c*x^4 + b*x^2 + a),x, algorithm="fricas")

[Out] integral((e*x^2 + d)^q*x^6/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^q x^6}{cx^4 + bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q*x^6/(c*x^4 + b*x^2 + a),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^q*x^6/(c*x^4 + b*x^2 + a), x)
```

3.400 $\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=273

$$-\frac{x\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}-\frac{x\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(\sqrt{b^2-4ac}+b\right)}+\frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}{}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c}$$

```
[Out] -(((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1
[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d
)])/(c*(b - Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q)) - ((b + (b^2 -
 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3
/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c*(b + Sq
rt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + (x*(d + e*x^2)^q*Hypergeome
tric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(c*(1 + (e*x^2)/d)^q)
```

Rubi [A] time = 1.19441, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{x\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}-\frac{x\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(\sqrt{b^2-4ac}+b\right)}+\frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}{}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1
[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d
)])/(c*(b - Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q)) - ((b + (b^2 -
 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3
/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c*(b + Sq
```

```
rt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + (x*(d + e*x^2)^q*Hypergeome
tric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(c*(1 + (e*x^2)/d)^q)
```

Rubi in Sympy [A] time = 150.094, size = 240, normalized size = 0.88

$$\frac{x\left(1+\frac{ex^2}{d}\right)^{-q}\left(d+ex^2\right)^q {}_2F_1\left(\begin{matrix}-q,\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle| -\frac{ex^2}{d}\right)}{c} - \frac{x\left(1+\frac{ex^2}{d}\right)^{-q}\left(d+ex^2\right)^q\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{appellf}_1\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}},-\frac{ex^2}{d}\right)}{c\left(-4ac+b^2+b\sqrt{-4ac+b^2}\right)} - \frac{x\left(1+\frac{ex^2}{d}\right)^{-q}\left(d+ex^2\right)^q\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{appellf}_1\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{ex^2}{d}\right)}{c\left(-4ac+b^2-b\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] x*(1 + e*x**2/d)**(-q)*(d + e*x**2)**q*hyper((-q, 1/2), (3/2,), -
e*x**2/d)/c - x*(1 + e*x**2/d)**(-q)*(d + e*x**2)**q*(-2*a*c + b*
*2 + b*sqrt(-4*a*c + b**2))*appellf1(1/2, 1, -q, 3/2, -2*c*x**2/(
b + sqrt(-4*a*c + b**2)), -e*x**2/d)/(c*(-4*a*c + b**2 + b*sqrt(-
4*a*c + b**2))) - x*(1 + e*x**2/d)**(-q)*(d + e*x**2)**q*(-2*a*c
+ b**2 - b*sqrt(-4*a*c + b**2))*appellf1(1/2, 1, -q, 3/2, -2*c*x*
*2/(b - sqrt(-4*a*c + b**2)), -e*x**2/d)/(c*(-4*a*c + b**2 - b*sq
rt(-4*a*c + b**2)))
```

Mathematica [A] time = 0.136161, size = 0, normalized size = 0.

$$\int \frac{x^4\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[(x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{x^4 (ex^2 + d)^q}{cx^4 + bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)

[Out] int(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^q x^4}{cx^4 + bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q*x^4/(c*x^4 + b*x^2 + a),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^q*x^4/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^2 + d)^q x^4}{cx^4 + bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q*x^4/(c*x^4 + b*x^2 + a),x, algorithm="fricas")

[Out] integral((e*x^2 + d)^q*x^4/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q x^4}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q*x^4/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q*x^4/(c*x^4 + b*x^2 + a), x)
```

3.401 $\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=162

$$\frac{x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}}-\frac{x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -((x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c]), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(1 + (e*x^2)/d)
^q)) + (x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(1 + (e*x^
2)/d)^q)
```

Rubi [A] time = 0.729731, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}}-\frac{x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c]), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(1 + (e*x^2)/d)
^q)) + (x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(1 + (e*x^
2)/d)^q)
```

Rubi in Sympy [A] time = 69.1076, size = 134, normalized size = 0.83

$$-\frac{x\left(1+\frac{ex^2}{d}\right)^{-q}\left(d+ex^2\right)^q\operatorname{appellf}_1\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{ex^2}{d}\right)}{\sqrt{-4ac+b^2}}+\frac{x\left(1+\frac{ex^2}{d}\right)^{-q}\left(d+ex^2\right)^q\operatorname{appellf}_1\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}},-\frac{ex^2}{d}\right)}{\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] -x*(1 + e*x**2/d)**(-q)*(d + e*x**2)**q*appellf1(1/2, 1, -q, 3/2,
 -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -e*x**2/d)/sqrt(-4*a*c + b*
*2) + x*(1 + e*x**2/d)**(-q)*(d + e*x**2)**q*appellf1(1/2, 1, -q,
 3/2, -2*c*x**2/(b + sqrt(-4*a*c + b**2)), -e*x**2/d)/sqrt(-4*a*c
 + b**2)
```

Mathematica [A] time = 0.104587, size = 0, normalized size = 0.

$$\int \frac{x^2 \left(d+e x^2\right)^q}{a+b x^2+c x^4} \, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[(x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int \frac{x^2 \left(e x^2+d\right)^q}{c x^4+b x^2+a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e x^2+d\right)^q x^2}{c x^4+b x^2+a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q*x^2/(c*x^4 + b*x^2 + a),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^q*x^2/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex^2+d\right)^q x^2}{cx^4+bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q*x^2/(c*x^4 + b*x^2 + a),x, algorithm="fricas")

[Out] integral((e*x^2 + d)^q*x^2/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q x^2}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q*x^2/(c*x^4 + b*x^2 + a),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^q*x^2/(c*x^4 + b*x^2 + a), x)
```

3.402 $\int \frac{(d+ex^2)^q}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=190

$$-\frac{2cx\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[Out] (-2*c*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - S
qrt[b^2 - 4*a*c]), -((e*x^2)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*
a*c])*(1 + (e*x^2)/d)^q) - (2*c*x*(d + e*x^2)^q*AppellF1[1/2, 1,
-q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/((b^2
 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q)
```

Rubi [A] time = 0.715909, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{2cx\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^q/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-2*c*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - S
qrt[b^2 - 4*a*c]), -((e*x^2)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*
a*c])*(1 + (e*x^2)/d)^q) - (2*c*x*(d + e*x^2)^q*AppellF1[1/2, 1,
-q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/((b^2
 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q)
```

Rubi in Sympy [A] time = 61.7594, size = 163, normalized size = 0.86

$$-\frac{2cx\left(1+\frac{ex^2}{d}\right)^{-q}\left(d+ex^2\right)^q\operatorname{appellf}_1\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}},-\frac{ex^2}{d}\right)}{-4ac+b^2+b\sqrt{-4ac+b^2}}-\frac{2cx\left(1+\frac{ex^2}{d}\right)^{-q}\left(d+ex^2\right)^q\operatorname{appellf}_1\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{ex^2}{d}\right)}{-4ac+b^2-b\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] -2*c*x*(1 + e*x**2/d)**(-q)*(d + e*x**2)**q*appellf1(1/2, 1, -q,
3/2, -2*c*x**2/(b + sqrt(-4*a*c + b**2)), -e*x**2/d)/(-4*a*c + b*
*2 + b*sqrt(-4*a*c + b**2)) - 2*c*x*(1 + e*x**2/d)**(-q)*(d + e*x
**2)**q*appellf1(1/2, 1, -q, 3/2, -2*c*x**2/(b - sqrt(-4*a*c + b*
*2)), -e*x**2/d)/(-4*a*c + b**2 - b*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.0482326, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x^2)^q/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x]
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int((e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^2 + d)^q}{cx^4 + bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^q}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

3.403 $\int \frac{(d+ex^2)^q}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=264

$$-\frac{cx\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(b-\sqrt{b^2-4ac}\right)}-\frac{cx\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(\sqrt{b^2-4ac}+b\right)}-\frac{\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}{}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\frac{ex^2}{d}\right)}{ax}$$

```
[Out] -((c*(1 + b/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -
q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a*(b
- Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q)) - (c*(1 - b/Sqrt[b^2 - 4
*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a*(b + Sqrt[b^2 - 4*a*c])*(1
+ (e*x^2)/d)^q) - ((d + e*x^2)^q*Hypergeometric2F1[-1/2, -q, 1/2,
 -((e*x^2)/d)])/(a*x*(1 + (e*x^2)/d)^q)
```

Rubi [A] time = 1.16722, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\frac{cx\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(b-\sqrt{b^2-4ac}\right)}-\frac{cx\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(\sqrt{b^2-4ac}+b\right)}-\frac{\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}{}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\frac{ex^2}{d}\right)}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((c*(1 + b/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -
q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a*(b
- Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q)) - (c*(1 - b/Sqrt[b^2 - 4
*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b +
```

```
Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a*(b + Sqrt[b^2 - 4*a*c])*(1
+ (e*x^2)/d)^q) - ((d + e*x^2)^q*Hypergeometric2F1[-1/2, -q, 1/2,
 -((e*x^2)/d)])/(a*x*(1 + (e*x^2)/d)^q)
```

Rubi in Sympy [A] time = 138.045, size = 228, normalized size = 0.86

$$\frac{cx\left(1+\frac{ex^2}{d}\right)^{-q}\left(b-\sqrt{-4ac+b^2}\right)\left(d+ex^2\right)^q\operatorname{appellf}_1\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}},-\frac{ex^2}{d}\right)}{a\left(-4ac+b^2+b\sqrt{-4ac+b^2}\right)}+\frac{cx\left(1+\frac{ex^2}{d}\right)^{-q}\left(b+\sqrt{-4ac+b^2}\right)\left(d+ex^2\right)^q\operatorname{appellf}_1\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{ex^2}{d}\right)}{a\left(-4ac+b^2-b\sqrt{-4ac+b^2}\right)}-\frac{\left(1+\frac{ex^2}{d}\right)^{-q}\left(d+ex^2\right)^q{}_2F_1\left(\begin{matrix}-q,-\frac{1}{2}\\ \frac{1}{2}\end{matrix}\middle|-\frac{ex^2}{d}\right)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**q/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] c*x*(1 + e*x**2/d)**(-q)*(b - sqrt(-4*a*c + b**2))*(d + e*x**2)**
q*appellf1(1/2, 1, -q, 3/2, -2*c*x**2/(b + sqrt(-4*a*c + b**2)),
-e*x**2/d)/(a*(-4*a*c + b**2 + b*sqrt(-4*a*c + b**2))) + c*x*(1 +
 e*x**2/d)**(-q)*(b + sqrt(-4*a*c + b**2))*(d + e*x**2)**q*appell
f1(1/2, 1, -q, 3/2, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -e*x**2/
d)/(a*(-4*a*c + b**2 - b*sqrt(-4*a*c + b**2))) - (1 + e*x**2/d)**
(-q)*(d + e*x**2)**q*hyper((-q, -1/2), (1/2,), -e*x**2/d)/(a*x)
```

Mathematica [A] time = 0.102576, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex^2\right)^q}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[(d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)), x]
```

Maple [F] time = 0.091, size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q}{x^2\left(cx^4+bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x)

[Out] int((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q}{\left(cx^4+bx^2+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex^2+d\right)^q}{cx^6+bx^4+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="fricas")

[Out] integral((e*x^2 + d)^q/(c*x^6 + b*x^4 + a*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**q/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^q}{(cx^4 + bx^2 + a)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^2), x)
```

3.404 $\int \frac{(d+ex^2)^q}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=328

$$\frac{cx\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{cx\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(\sqrt{b^2-4ac}+b\right)}+\frac{b(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}{}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\frac{ex^2}{d}\right)}{a^2x}-\frac{(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}{}_2F_1\left(-\frac{3}{2},-q;-\frac{1}{2};-\frac{ex^2}{d}\right)}{3ax^3}$$

```
[Out] (c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1
[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d
)])/(a^2*(b - Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + (c*(b - (b^
2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q
, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a^2*(b
 + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) - ((d + e*x^2)^q*Hyperge
ometric2F1[-3/2, -q, -1/2, -((e*x^2)/d)])/(3*a*x^3*(1 + (e*x^2)/d
)^q) + (b*(d + e*x^2)^q*Hypergeometric2F1[-1/2, -q, 1/2, -((e*x^2
)/d)])/(a^2*x*(1 + (e*x^2)/d)^q)
```

Rubi [A] time = 1.31291, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{cx\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{cx\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(\sqrt{b^2-4ac}+b\right)}+\frac{b(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}{}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\frac{ex^2}{d}\right)}{a^2x}-\frac{(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}{}_2F_1\left(-\frac{3}{2},-q;-\frac{1}{2};-\frac{ex^2}{d}\right)}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1
[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d
)])/(a^2*(b - Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + (c*(b - (b^
2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q
, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a^2*(b
 + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) - ((d + e*x^2)^q*Hyperge
ometric2F1[-3/2, -q, -1/2, -((e*x^2)/d)])/(3*a*x^3*(1 + (e*x^2)/d
)^q) + (b*(d + e*x^2)^q*Hypergeometric2F1[-1/2, -q, 1/2, -((e*x^2
)/d)])/(a^2*x*(1 + (e*x^2)/d)^q)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**2+d)**q/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.295477, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex^2\right)^q}{x^4\left(a+bx^2+cx^4\right)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[(d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)), x]
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q}{x^4\left(cx^4+bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] int((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^4),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ex^2+d)^q}{cx^8+bx^6+ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^4),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q/(c*x^8 + b*x^6 + a*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**2+d)**q/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q}{(cx^4+bx^2+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^4),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^4), x)
```

3.405 $\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=40

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{1-c^4x^4}}{cx\sqrt{\frac{1}{c^2x^2}+1}}\right)}{c}$$

```
[Out] -(ArcTanh[Sqrt[1 - c^4*x^4]/(c*Sqrt[1 + 1/(c^2*x^2)]*x)]/c)
```

Rubi [A] time = 0.275145, antiderivative size = 44, normalized size of antiderivative = 1.1, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$-\frac{x\sqrt{\frac{1}{c^2x^2}+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + 1/(c^2*x^2)]/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -((Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[Sqrt[1 - c^2*x^2]])/Sqrt[1 + c
^2*x^2])
```

Rubi in Sympy [A] time = 20.0079, size = 56, normalized size = 1.4

$$-\frac{x\sqrt{1+\frac{1}{c^2x^2}}\operatorname{atanh}\left(\frac{\sqrt{-c^4x^4+1}}{c\sqrt{x^2+\frac{1}{c^2}}}\right)}{c\sqrt{x^2+\frac{1}{c^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+1/c**2/x**2)**(1/2)/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] -x*sqrt(1 + 1/(c**2*x**2))*atanh(sqrt(-c**4*x**4 + 1)/(c*sqrt(x**
2 + c**(-2))))/(c*sqrt(x**2 + c**(-2)))
```

Mathematica [A] time = 0.0767345, size = 79, normalized size = 1.98

$$\frac{x\sqrt{\frac{1}{c^2x^2}+1}\left(\log\left(c^2x^3+x\right)-\log\left(c^2x^2+\sqrt{c^2x^2+1}\sqrt{1-c^4x^4}+1\right)\right)}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + 1/(c^2*x^2)]/Sqrt[1 - c^4*x^4],x]
```

```
[Out] (Sqrt[1 + 1/(c^2*x^2)]*x*(Log[x + c^2*x^3] - Log[1 + c^2*x^2 + Sq
rt[1 + c^2*x^2]*Sqrt[1 - c^4*x^4]]))/Sqrt[1 + c^2*x^2]
```

Maple [C] time = 0.083, size = 101, normalized size = 2.5

$$-\frac{x\,csgn\left(c^{-1}\right)}{\left(c^2x^2+1\right)c}\sqrt{\frac{c^2x^2+1}{c^2x^2}}\sqrt{-c^4x^4+1}\ln\left(2\,\frac{1}{xc^2}\left(csgn\left(c^{-1}\right)c\sqrt{-\frac{c^2x^2-1}{c^2}}+1\right)\right)\frac{1}{\sqrt{-\frac{c^2x^2-1}{c^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] -((c^2*x^2+1)/c^2/x^2)^(1/2)*x*(-c^4*x^4+1)^(1/2)*csgn(1/c)*ln(2*
(csgn(1/c)*c*(-1/c^2*(c^2*x^2-1))^(1/2)+1)/x/c^2)/(c^2*x^2+1)/(-1
/c^2*(c^2*x^2-1))^(1/2)/c
```

Maxima [A] time = 0.762961, size = 42, normalized size = 1.05

$$-\frac{\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|}+\frac{2}{|x|}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(1/(c^2*x^2) + 1)/sqrt(-c^4*x^4 + 1),x, algorithm="maxima")
```

```
[Out] -log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x))/c
```

Fricas [A] time = 0.292123, size = 162, normalized size = 4.05

$$-\frac{\log\left(\frac{c^2x^2+\sqrt{-c^4x^4+1}\,cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{c^2x^2+1}\right)-\log\left(-\frac{c^2x^2-\sqrt{-c^4x^4+1}\,cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{c^2x^2+1}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(1/(c^2*x^2) + 1)/sqrt(-c^4*x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/2*(log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) + 1)/(c^2*x^2 + 1)) - log(-(c^2*x^2 - sqrt(-c^4*x^4 + 1)
*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{-(cx-1)(cx+1)(c^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/c**2/x**2)**(1/2)/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(1 + 1/(c**2*x**2))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*
x**2 + 1)), x)
```

GIAC/XCAS [A] time = 0.299391, size = 78, normalized size = 1.95

$$\frac{\left(\ln\left(\sqrt{2}+1\right)-\ln\left(\sqrt{2}-1\right)-\ln\left(\sqrt{-c^2x^2+1}+1\right)+\ln\left(-\sqrt{-c^2x^2+1}+1\right)\right)|c|\operatorname{sign}(x)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(1/(c^2*x^2) + 1)/sqrt(-c^4*x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/2*(ln(sqrt(2) + 1) - ln(sqrt(2) - 1) - ln(sqrt(-c^2*x^2 + 1) +
1) + ln(-sqrt(-c^2*x^2 + 1) + 1))*abs(c)*sign(x)/c^2
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
          # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```